AIDS and Dualism

Ethiopia’s Burden under Rational Expectations

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Abstract

An AIDS epidemic threatens Ethiopia with a long wave of premature adult mortality, and thus with an enduring setback to capital formation and economic growth. The authors develop a two-sector model with three overlapping generations and intersectorally mobile labor, in which young adults allocate resources under rational expectations. They calibrate the model to the demographic and economic data, and perform simulations for the period ending in 2100 under alternative assumptions about mortality with and without the epidemic. Although the epidemic does not bring about a catastrophic economic collapse, which is hardly possible in view of Ethiopia’s poverty and high background adult mortality, it does cause a permanent, downward displacement of the path of output per head, amounting to 10 percent in 2100. An externally funded program to combat the disease is socially very profitable.

This paper—a product of the Global HIV/AIDS Program, Human Development Network—is part of a larger effort in the World Bank to assess the economic effects of HIV/AIDS, especially in Africa. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The authors may be contacted at clive.bell@urz.uni-heidelberg.de or anastasios.koukoumelis@urz.uni-heidelberg.de.
AIDS and Dualism: Ethiopia’s Burden under Rational Expectations

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1 Introduction

Ethiopia is a so-called ‘next-wave’ country where HIV/AIDS is concerned: populous, in the early- to mid-stages of an epidemic, and with a government that some thought to be rather tardy in recognizing the threat the disease poses (NIC, 2002). Some 1.2 million Ethiopians died of AIDS-related causes from the onset of the epidemic in the early 1980s until the end of 2006, 111,000 alone in the latter year (MoH, 2006). In 2007 980,000 adults and children were living with HIV (UNAIDS, 2008). It is estimated that in 2006 life expectancy at birth was lowered, purely as a result of AIDS, by 4.2 years and 880,000 children had been orphaned by the disease.

A burgeoning AIDS epidemic is not Ethiopia’s only burden. It is one of the world’s poorest countries, with low levels of physical and human capital: adults of working age have, on average, about one year of schooling. The economy also exhibits strong dualism, with clear differences between rural and urban families’ living standards and opportunities. High levels of fertility compound matters by increasing population pressure on land, and so threaten to degrade the natural resource base. Both developments are closely intertwined with stagnant agricultural productivity and internal migration flows.¹ These features of the Ethiopian economy must be incorporated into any analysis of its long-term development prospects, overshadowed as they are by the AIDS epidemic.

Ethiopia merits attention as the object of study, not only for the above reasons in themselves, but also for certain contrasts that it offers in relation to the much more developed economies of southern Africa, in which the epidemic is both mature and severe. It can be argued that the wave of premature adult mortality that is now sweeping over the latter is so great as to threaten a dramatic fall in productivity over the longer run. Ethiopia’s poverty leaves little scope for such a fall. Yet two questions remain. First,

¹See, inter alia, Hurni (1993), Grepperud (1996), and Shiferaw and Holden (1999).
does the AIDS epidemic pose a formidable obstacle to Ethiopia’s attaining long-run growth? Second, is it socially profitable to commit substantial resources to combating the epidemic, even if its effects on the long-run rate of growth are modest? We seek to answer both these questions, and in so doing to shed some light on the damage inflicted by the pandemic on other parts of sub-Saharan Africa with similar characteristics.

To tackle this task, we employ an overlapping-generations (OLG) model of a dual economy, in which the two sectors produce the same aggregate good and young adults who grew up in villages can seek their fortunes in the towns. The model conforms to the Ethiopian setting and experience in two ways: first, the growth of income depends heavily on human capital formation, and second, the availability of land influences not only intra-family allocations regarding education and child labor, but also population movements. It draws on Bell, Devarajan and Gersbach (2006), whereby the extension to two sectors and three generations greatly enlarges the set of variables over whose future course agents must form their (rational) expectations. The resulting structure, when calibrated to Ethiopia, is then used to simulate the path of the economy under alternative demographic developments, namely, with and without the AIDS epidemic.

There is, of course, a substantial literature on the macro-economics of AIDS, but, to our knowledge, not a single formal application to Ethiopia. Among a number of notable studies, Cuddington (1993a) and Cuddington and Hancock (1994) use standard neoclassical (Solovian) models to simulate the impact of AIDS on macroeconomic aggregates. The epidemic affects both the size and quality (health and accumulated experience) of the labor force and the accumulation of physical capital (to the extent that higher medical expenditures are financed out of domestic savings). In closely related frameworks, Over (1992) allows for infection to be concentrated among educated

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2 "The fact that the majority of Ethiopians remains uneducated explains in part the poor development performance of the country over a prolonged period of time." (Degefe et al., 2002: 218).

3 Abate (1995: 312) reports a statistically significant correlation between average land-holding and urban population. Golini et al. (2001) and Mberu (2006) provide thorough analyses of internal migration dynamics.
workers, and Cuddington (1993b) and Cuddington and Hancock (1995) incorporate unemployment (workers who die from AIDS are replaced by the unemployed) and the operation of dual labor markets (workers unable to secure employment in the formal sector find lower-productivity employment in the informal sector). The macroeconomic impact of AIDS depends, naturally enough, on the values of the parameters describing the distribution of its prevalence among classes of workers, the speed of adjustment in the labor market, and the proportion of health care costs that is financed out of savings.

Cuddington, Hancock and Rogers (1994) add epidemiological elements to the one-sector neoclassical growth model, and investigate the effects of health-sector policies that either reduce the rate of transmission of HIV or increase the life expectancy of AIDS patients. In this extended model, which allows for interactions between demographic and economic variables and the existence of multiple equilibria, cost-effective policy interventions may propel the economy into a steady state in which the prevalence rate is lower – or even zero. Finally, Young (2005) embeds a sub-model that endogenizes labor force participation, fertility, and education decisions in a Solovian framework with a constant savings rate. Higher mortality induces lower fertility, both directly and in response to the resulting higher wages, and so keeps the population below its current size for 70 years. This contraction of labor supply outweighs the reduced accumulation of human capital among orphaned children, and individuals in the surviving population (and succeeding generations) are, on average, correspondingly better endowed with resources and enjoy higher levels of income.

A few country-specific studies are based on computable general equilibrium models (Kambou, Devarajan and Over, 1992; Arndt and Lewis, 2001; and Arndt, 2003). Their dynamic structure is similar to that of the standard neoclassical growth model, but they go a step in the direction of realism by allowing some sectoral disaggregation of GDP, and hence of disproportionate effects across sectors (those that use factors which
are more heavily affected by the epidemic tend to contract. As is well known, the estimation and calibration of such models is very demanding of data, and numerous assumptions about the constituent structural relationships are necessary.

Studies that employ OLG models often focus on how the epidemic influences human capital accumulation. In Bell, Bruhns and Gersbach (2006) and Bell, Devarajan and Gersbach (2006), the accumulation of human capital is the force that generates economic growth over the long run. Premature adult mortality destroys existing skills and experience, and weakens the mechanism through which knowledge is transmitted from parents to children. With reduced life-time resources, the level of schooling may fall among affected families. Even when both parents survive, their expectations concerning mortality rates among their own and their children’s generation will still influence decisions concerning current investments in schooling. The system normally exhibits two or more equilibria, and if nothing is done to combat a sufficiently large and sustained shock to mortality, the economy may fall into a poverty trap.

Corrigan, Glomm and Mendez (2004) concentrate on the fall in life expectancy, which decreases the incentive to invest in all forms of capital, and the associated creation of a large number of orphans. HIV-infected individuals face medical expenditures and do not save during adulthood; uninfected individuals save a portion of their income for old-age consumption. The level of human capital accumulation depends on whether parents die prematurely due to HIV/AIDS, and on the amount of time that children spend pursuing formal education rather than working. The macroeconomic consequences of the epidemic are large, especially if the mortality shock persists.

The plan of the paper is as follows. Section 2 begins by setting out the basic approach, followed by brief accounts of the sub-models of the rural and urban sectors, and of the momentary equilibrium of the system, in which rural-urban migration under rational expectations plays a central role. The section concludes with an examination of the system’s long-run behavior. The calibration of the system to the data for Ethiopia is
treated at some length in Section 3, thereby laying the groundwork for the results in Section 4. The latter begins by setting out the sequences of the exogenous variables, and then analyzes and compares the reference case, in which the epidemic runs its projected course, with the counter-factual, in which it never breaks out. Turning to policy, the question of what objectives should be pursued is addressed in Section 5, the relation between public spending and mortality is estimated in Section 6, and the social profitability of a particular policy program funded by aid is evaluated in Section 7. Some concluding comments are drawn together in Section 8.

2 The model in outline

The basic economic unit is an extended family household, whose members pool their resources. The following assumptions apply to urban and rural households alike. Individuals live for up to three periods: childhood, adulthood and old age. Children may divide their time between working and learning and all children within the family are given the same schooling. On reaching adulthood, these individuals are homogeneous in skills and preferences, and supply their labor completely inelastically. Those who were born in villages also decide, at the very outset, whether they will stay or seek their fortune in the towns, where some members of the extended family are already established. Wherever young adults settle down, they raise children for altruistic as well as self-interested reasons. A social rule determines the level of consumption in the third period of life, with young adults making transfers out of the family’s current income to their parents. The old make no bequests, except of land.

4The details of the algorithm for computing a rational expectations sequence are consigned to Appendix B.
5Schaffner (2004: 13), for example, reports that 89% of orphans live with their relatives.
6According to the CSA (2001), in 2001, 49.0% of children aged 5 to 14 and 67.4% of those aged 15 to 17 were working. ‘All studies made on child labor in urban areas identified household poverty as the major and primary cause of child labor’. (Tegenu, 2003: 28).
In what follows, we begin by describing demography and household behavior in the rural and urban sectors of the economy, which produce the same aggregate good, but are connected by the migration of young adults. This requires a full set of demographic accounts for the entire economy, in which the profile of age-specific mortality rates plays a leading role. Momentary equilibrium is brought about by a level of migration such that young adults who have grown up in villages are indifferent between migrating to the towns and staying on the family farm. The sequence of momentary equilibria is such that agents’ expectations are realized along the whole path.

2.1 The rural sector

The rural sector (labelled 1) is made up of a very large number of identical families. Shortly after the beginning of each period, when the surviving children from the previous period have just attained adulthood, these young adults decide whether to migrate. Only after choosing where to live do they form unions and have children, in rural areas \( N_{c,t} \) per union in period \( t \). Death can come early to these adults, first, just after they have had their children, and then once more later, just before reaching old age. Denote these probabilities by \( q_{ae}^{c,1,t} \) and \( q_{al}^{c,1,t} \), respectively, so that the survival rate among all young rural adults, conditional on their surviving childhood, is \( (1 - q_{ae}^{a,1,t}) \equiv (1 - q_{ae}^{c,1,t})(1 - q_{al}^{c,1,t}) \).

Under the social rules, those young adults who survive the early phase are charged with caring for all children, natural and adopted alike. Normalizing each household around a surviving young ‘couple’, \( N_{a,1,t} = 2 \ \forall \ t \), let \( N_{c,1,t}^{c} / (1 - q_{ae}^{c,1,t}) \) denote the number of children raised by such a couple in period \( t \). The representative rural household is completed by \( N_{o,1,t} \) elders, all of whom die at the end of period \( t \).

The population evolves as follows. Let all children born in villages stay there until adulthood, and denote the probability that a child born in period \( t \) will survive into adulthood in period \( t + 1 \) by \( 1 - q_{l,1,t}^{c} \). Of the \( (1 - q_{l,1,t-1})N_{c,1,t-1} \) children born to a
couple in period \( t - 1 \) who attain adulthood in period \( t \), let \( M_a^t \) choose to migrate. Denoting by \( N_{f,t}^t \) the number of unions formed in the rural sector, just after the start of period \( t \), the above normalization to surviving couples implies that the number of representative families (households) in the rural sector, \( F_{1,t} \), changes according to

\[
F_{1,t} = (1 - q_{1,t}^{ae}) \cdot N_{f,t}^t = (1 - q_{1,t}^{ae}) \cdot \frac{(1 - q_{1,t-1}^c)N_{1,t-1}^c - M_a^t}{2} \cdot N_{f,t-1}^t. 
\]

The number of elders per household in period \( t \) is thus given by

\[
N_{o,t}^t = (1 - q_{1,t-1}^{el})N_{1,t-1}^a \cdot (F_{1,t-1}/F_{1,t}). 
\]

Each young adult possesses \( \lambda_{1,t} \) efficiency units of labor, which is a natural measure of human capital. This is formed in a process whereby the benefits of formal education are complemented by the quality of child-rearing, which depends on the human capital of all the adults in the family.\(^7\) Let \( e_{1,t} \in [0,1] \) denote the fraction of childhood devoted to formal education, the residual being allocated to work. Choosing the iso-elastic form \( b_1 \cdot (e_{1,t})^{\theta_1} \) for the ‘educational technology’, let a child’s endowment of labor (measured in efficiency units) on reaching adulthood in period \( t + 1 \) be given by

\[
\lambda_{1,t+1} = (\lambda_{1,t}N_{1,t}^a + \phi \lambda_{1,t-1}N_{1,t}^o) \cdot b_1 \cdot (e_{1,t})^{\theta_1} + 1, 
\]

where the product on the RHS implies that formal education and the quality of child-rearing are indeed complements. It is plausible that \( \phi < 1 \), that is, grandparents contribute less, \textit{cet. par.}, than parents to the children’s potential capacity to build human capital. Some education is required if \( \lambda_{1,t+1} \) is to exceed unity, the human capital of an uneducated young adult.\(^8\)

\(^7\)For empirical evidence on the latter point, see Christiaensen and Alderman (2002) and Woldehanna \textit{et al.} (2005).

\(^8\)This is just a convenient normalization.
The aggregate good is produced by means of human capital and land. The higher the level of rural-urban migration, the larger is the number of elders associated with each rural young couple. The migrants’ departure also relieves pressure on the land, for those young adults who remain will have sole claim to the family’s holding, $\ell_t$. Given a fixed area available for cultivation, $\ell_t$ evolves according to

$$\ell_{t+1} = \left( \frac{F_{1,t}}{F_{1,t+1}} \right) \cdot \ell_t,$$

where the sequence $\{\ell_t\}_{t=1}^T$ is anchored to the observed value of $\ell_1$.

The resulting output is distributed among the members of the family as follows. Let $c_{1,t}$ denote the consumption of a young adult, and let children and old adults consume the constant fractions $\beta$ and $\rho$, respectively, of $c_{1,t}$, whereby the migrants have no further claim on the household’s resources. The parameters $\beta$ and $\rho$ define a social sharing rule.

Subject to this rule, all allocative decisions within the family lie in the hands of the young adults. Their choices determine not only all family members’ current levels of consumption, but also their children’s level of human capital on reaching adulthood, and hence the family’s consumption possibilities in the next period. It is thus implied that, although the Ethiopian government is committed to the provision of free and compulsory primary education for all children, school attendance in practice depends on the household’s income, its needs for child labor (which conflict with schooling), and parental awareness and appreciation of the benefits of education (Schaffner, 2004: 1). Those young adults who remain in the village choose an optimal allocation $(c_{1,t}^0, e_{1,t}^0 | M_t^0)$, which depends on the level of full income, the relative price of education, the strength of their altruism towards their children, and their (subjective) estimates of the levels of mortality that will afflict them later in young adult life and, in turn, their children in adulthood. In choosing this optimum, they take the number of departing
migrants as parametrically given. The formal statement of their decision problem is given in Appendix A.

By way of closing remarks, first, it is assumed that parents cannot borrow to finance schooling (Schaffner, 2004: 17). Second, the only form of investment is education, the costs of which are the children’s potential contributions to current output. Indeed, rural households in Ethiopia save little (Tengenu, 2003: 15) and cannot purchase formal insurance against risk (Dercon, 2004). Third, since all households are assumed to be identical, there is no trade in the services of labor and land.\footnote{Land is state-owned and distributed equally among farmers on a strict usufruct basis by local councils, though leasing is allowed (OPM, 2004: 3). Wage labor, whether in agriculture or otherwise, remains relatively rare, and many off-farm activities (such as selling home-made products) are closely linked to agricultural activities (Dercon, 2004: Sec. 5). Indeed, according to a survey conducted by the Ministry of Labor in 1996, 44% of rural households reported non-agricultural sources of income, but these sources accounted for only 10% of total income (Degefe and Nega, 2000: 179).}

### 2.2 The urban sector

The number of children (per representative family) born in the urban sector (labelled 2) in period \( t - 1 \) that survive into young adulthood in period \( t \) is \((1 - q^c_{2,t-1}) N^c_{2,t-1} \).

The total number of migrants arriving at the start of period \( t \) is \( M^a_t N^f_{1,t-1} \). Hence, the total number of young adults who form unions in urban areas at this juncture is \((1 - q^c_{2,t-1}) N^c_{2,t-1} N^f_{2,t-1} + M^a_t N^f_{1,t-1} \). Given the definition of \( F_{i,t} \) as the number of surviving couples, we have

\[
F_{2,t} = (1 - q^ae_{2,t}) \cdot \frac{(1 - q^c_{2,t-1}) N^c_{2,t-1} N^f_{2,t-1} + M^a_t N^f_{1,t-1}}{2} \\
= \frac{(1 - q^ae_{2,t})}{2} \cdot \left[ (1 - q^c_{2,t-1}) N^c_{2,t-1} F_{2,t-1} + \frac{M^a_t F_{1,t-1}}{1 - q^ae_{1,t-1}} \right]. \tag{5}
\]

The number of elders per urban household is, accordingly,

\[
N^o_{2,t} = (1 - q^ol_{2,t-1}) N^o_{2,t-1} \cdot \left( F_{2,t-1} / F_{2,t} \right). \tag{6}
\]
In effect, migrants join the established members of their extended kin groups, contributing their labor and making the conventional claims on the common pot. They are also assumed to adopt their urban counterparts’ preferences, including those concerning fertility, and make the same contributions to savings for the next period and to the formation of human capital among the upcoming generation.

The urban sector differs from its rural counterpart in two other respects. First, as there is some participation in secondary and higher education, one needs to distinguish between children of primary school age (5-14 years) and youths aged 15 to 24. Extending the notation of Section 2.1, let there be \( N_{cp}^2, t \) of the former, with net attendance ratio \( e_{p, t}^2 \), and \( N_{cs}^2, t \) of the latter, with net attendance ratio \( e_{s, t}^2 \), where \( N_c^2, t = N_{cp}^2, t + N_{cs}^2, t \) in order to make up the entire 20-year generation. Second, although families working in the industrial and services sectors are landless, they may be endowed instead with physical capital, which is accumulated through the savings of young adults. As in the rural sector, the technology for producing the aggregate good exhibits constant returns to scale, but now in efficiency units of labor and physical capital.

Analogously to eq. (3), the formation of human capital is assumed to be governed by

\[
\lambda_{2,t+1} = \left[ \frac{(\lambda_{2,t} + \xi_t \lambda_{1,t}) N_a^2}{1 + \xi_t} + \frac{\phi(\lambda_{2,t-1} + \xi_{t-1} \lambda_{1,t-1}) N_a^2}{1 + \xi_{t-1}} \right] \cdot b_2 \left( e_{p, t}^2 + e_{s, t}^2 \right)^{\theta_2} + 1 \quad (7)
\]

where

\[
\xi_t = \frac{M_t^a \cdot (N_{1,t}^f / N_{2,t}^f)}{(1 - q_{2,t-1}^a)(1 - q_{2,t-1}^{ae}) N_{c,t-1}^2} \quad (8)
\]
defines the ratio of newly arrived young migrants from the villages to the ‘native young townies’. In effect, primary and post-primary education are treated as perfect substitutes. This is certainly a strong assumption, but so long as \( e_{p, t}^2 \) is close to unity, it is not particularly objectionable.

The level of physical capital in the next period, \( k_{t+1} \), depends on savings in the
current period, \(s_t\), it being assumed that the whole of the current capital stock lasts for just the twenty years spanned by each new generation. Recalling that all young adults are assumed to have their children before some of them are carried off in the early phase of adulthood (to be precise, the proportion \(q_{2,t}^{ac}\)), and that the surviving children themselves form couples at the start of the next period, the capital owned by each representative urban household, normalized to a surviving couple, evolves according to

\[
k_{t+1} = \frac{1 - q_{2,t}^{ac}}{1 - q_{2,t+1}^{ac}} \cdot \frac{2N^a_{2,t}s_t}{(1 - q_{2,t}) (N^c_{2,t} + N^s_{2,t})}.
\]

(9)

The young adults’ preferences are, in principle at least, defined over three goods: the levels of consumption in young adulthood and old age, \(c_{2,t}\) and \(\rho c_{2,t+1}\), respectively, and the human capital attained by their school-age children on attaining full adulthood, \(\lambda_{2,t+1}\), which they may appreciate in both phases of their own lives. The latter quantity depends heavily on the children’s education, \((e_{2,t}^p, e_{2,t}^s)\), and it produces two kinds of pay-offs, namely, in the form of altruism and selfishly, inasmuch as an increase in \(\lambda_{2,t+1}\) will also lead to an increase in \(\rho c_{2,t+1}\) under the said social rules.

Although the pooling arrangement implicit in the extended family structure eliminates the risk that orphaned children will be left to fend for themselves, others remain. A young adult still faces uncertainty about whether he or she will actually survive into the last phase of life, and whether his or her children will do likewise, conditional on their reaching adulthood in their turn. There is also arguably uncertainty about future demographic developments, which influence the levels of \(c_{2,t+1}\) and \(\lambda_{2,t+1}\).

Young adults in the current period must form expectations about the level of consumption in old age should they survive into that phase of life. Forecasting consumption in the next period is a formidable task, involving forecasting \(s_{t+1}\), \(e_{2,t+1}^p\), \(e_{2,t+1}^s\) and \(\xi_{t+1}\), all under rational expectations about the behavior of subsequent generations. Like their rural counterparts, their task is to choose an optimal plan, albeit a complicated
one to compute: \([c_{2,t}, s_t, e^p_{2,t}, e^s_{2,t}|e^p_{1,t}, \xi_t, E_t(s_{t+1}, e^p_{2,t+1}, e^s_{2,t+1}, \xi_{t+1})]\).

2.3 Momentary equilibrium

At the beginning of each period, the economy’s endowments of productive resources and their distribution among households are given. The two sectors are connected by the flow of young rural migrants into the towns, the level of which all households take as parametrically given when making their decisions concerning the allocation of full income between investment and consumption. The central question in the determination of momentary equilibrium, therefore, is this: what governs the size of this flow? In keeping with the general approach adopted here, we assume that on reaching adulthood, members of the rising generation in villages decide whether to migrate to the towns on the basis of the expected lifetime utilities, \(E_t u_i (i = 1, 2)\), offered by the two locations. In each and every period, the endogenous variables of the system must satisfy the condition

\[
E_t u_1(c^0_{1,t}, e^p_{1,t}) + d^u = E_t u_2(c^0_{2,t}, e^p_{2,t}, e^s_{2,t}), \tag{10}
\]

where \(d^u \geq 0\) represents the value of those net compensating advantages of rural life not covered in \(E_t u_1\). Observe that agents’ (rational) expectations concerning future developments that have a bearing on current decisions appear in \(E_t u_2\). The sequence of equilibria satisfying eq. (10) is anchored to the observed levels of investment in education in the year 2000 \((t = 1)\).

2.4 The system’s long-run behavior

The dual economy formulated above constitutes a system of simultaneous, non-linear, second-order difference equations, whose (exogenous) ‘forcing functions’ are the se-
quences of fertility, $\{N_{i,t}\}_{t=1}^T$, age-specific mortality, $\{q_{i,t}\}_{t=1}^T$, and total factor productivity, $\{A_{i,t}\}_{t=1}^T$, which are common knowledge to all economic agents. There is no closed-form solution in the general case, and all the results presented in subsequent sections necessarily relate to transitional paths that are anchored to the initial conditions in the period 1980-2000. Even in 2100, the last period of the sequences that are computed under rational expectations, the economy is still far removed from a state of steady growth in which the rural sector has shrunk to a mere appendage and rural-urban migration has come to an end. In order to understand and interpret these results, however, it is instructive to examine the steady-state path that will be asymptotically attained.

The steady state in question is characterized by the said exogenous variables taking stationary values such that there is also a stationary population, and by parents choosing a full education for their children. On reaching this state, the system becomes tractable and linear, as we now demonstrate. Starting with the rural sector, and noting that there is now no rural-urban migration, eq. (3) specializes to

$$\lambda_{1,t+1} = 2(\lambda_{1,t} + \phi(1 - q_{1l}^{al})\lambda_{1,t-1}) \cdot b_1 + 1. \quad (11)$$

With a stationary landholding and a Cobb-Douglas production technology, output per family will grow at a steady rate equal to the rate of growth of $A_{1,t}$ plus the product of the growth rate of $\lambda_{1,t}$ and the elasticity of output with respect to inputs of labor measured in efficiency units. Turning to the urban sector, eq. (7) specializes to

$$\lambda_{2,t+1} = 2(\lambda_{2,t} + \phi(1 - q_{2l}^{al})\lambda_{2,t-1}) \cdot b_2 + 1. \quad (12)$$

In a steady state, savings decisions will be such that $k_t$ grows at the same rate as $\lambda_{2,t}$, and with constant returns to scale in production, output per family will grow at the

\footnote{With a stationary population, $q_{i,t}^{al} = q_{i}^{al} \forall i, t.$}
sum of that rate and that of $A_{2,t}$. The last step is to check that the urban sector is indeed growing faster than the rural sector, thereby verifying that dualism has come to an end. All this will be done in Section 4.

### 3 Calibration

Ethiopia’s turbulent history in the second half of the 20th Century and the paucity of relevant data pose formidable challenges to all forms of calibration and rule out any that are heavily based on historical series.\(^{11}\) The method adopted here involves anchoring the whole system to the observed values of key variables in the year 2000, which is the starting point of all the simulations in the sections that follow.\(^{12}\) This is a minimum requirement; but with a plethora of parameters to be determined, it is far from sufficient. It is therefore supplemented by ‘guesstimates’ based on what appear to be reasonable values or regularities in cross-country data.

#### 3.1 The rural sector

The values of the following variables in 2000 are observed: enrollments, average family landholding, and all demographic magnitudes. In the absence of any estimates of the years of completed schooling for this cohort, we have used USAID’s (2005) estimate of the net attendance ratio among children of primary school age, namely, $e_{P,1} = 0.243$. The average family landholding, $\ell_1$, is 1.02 hectares (OPM, 2004: 3). We define as

\(^{11}\)In particular, it is difficult to estimate the key relationship between changes in human capital and economic performance. For example, enrollment rates increased by roughly 15% under both the imperial regime, which was brought down by a coup in 1974, and the junta that followed (Negash, 1990), but GDP per capita increased under the first and decreased under the second. Similar difficulties arise in evaluating the contribution of investment in physical capital. An examination of the data set assembled by Nehru and Dhareshwar (1993) reveals that the growth rate of GDP per capita was negatively correlated with the level of average gross domestic fixed investment.

\(^{12}\)The choice of base year naturally involves some compromises. Our choice was heavily influenced by the availability of demographic, health and educational data. For example, 2000 is the only year for which data on rural and urban attendance ratios are available.
‘children’ those persons falling between the ages of 5 and 24 years, whose number must be normalized to that of surviving young adults: recall that $N_{i,t}^c \equiv (1 - q_{i,t}^{ae})N_{i,t}^c$. As the main costs of schooling are the opportunity costs of this group’s time and very young children make limited claims on consumption, we take no account of the latter in our calculations. Hence, the numbers of children, adults and elders in each family are assigned according to the relative sizes of the cohorts 5-24, 25-44 and 45+, so that the span of each generation is effectively 20 years. Recalling the normalization $N_{1,t}^a = 2 \forall t$, we obtain $N_{1,1}^c = 4.64$ and $N_{1,1}^o = 1.40$ from CSA (2000). The premature mortality rates are $q_{1,1}^c = 0.062$ and $q_{1,1}^o = 0.206$, where the latter implies $N_{1,1}^c = (1 - 0.1089) \times 4.64 = 4.135$.

We now turn to those parameters that must be set by guess and judgment. Beginning with the production and consumption of the aggregate good, the values of $\lambda_{1,t}$ are not, of course, directly observable. In view of the very limited levels of education and literacy of the rural population even up to the present, but with some progress over the past generation, eqs. (3) and (7) suggest values of $\lambda_1$ in 1980 and 2000 that are close to one: let $\lambda_{1,0} = 1.15$ and $\lambda_{1,1} = 1.40$. With these values as reference, we set the human capital of a child at $\gamma = 0.4$.\footnote{Cockburn (2002) claims that in rural Ethiopia “the marginal productivity of children is roughly one-third to one-half that of male adults”} An adult’s labor capacity is assumed to depreciate at the rate $\delta = 0.2$ per generation. The technology is taken to be Cobb-Douglas. The initial value of the efficiency parameter can be chosen quite freely, since it does not influence the steady-state behavior of $\lambda_{1,t}$; hence, $A_{1,1} = 1$. Given the fact that, in poor countries, the share of (imputed) rent in agricultural value added lies between 0.3 and 0.4, we choose $1 - \alpha_1 = 0.333$, where $\alpha_1$ denotes the elasticity of output with respect to inputs of labor measured in efficiency units. In the absence of data on the consumption of children and elders relative to young adults, we postulate $\beta = \rho = 0.6$.

If parents choose to give their children a full primary education, consumption and
income can grow so long as the transmission of knowledge across generations, as represented by the parameter $b_1$, is sufficiently strong, even if $A_{1,t}$ grows slowly and the size of the holding falls. In this connection, grandparents are assumed to be less capable than parents in imparting knowledge: let $\phi = 0.75$. When schooling is incomplete, the parameter $\theta_1$ comes into play. We choose $b_1 = 0.35$ and $\theta_1 = 0.4$.

We are not aware of any micro-economic studies of rural household’s preferences over current consumption and children’s educational attainment. We assume that the preferences are Stone-Geary, with ‘taste’ parameters $\omega$ and $1 - \omega$, respectively. In view of the fact that these households are very poor and that investment in education is not wholly altruistic, but also produces a pay-off for the parents in old age, we set $\omega = 0.8$. The ‘subsistence’ parameter $c_{1,\text{min}}$ is a flexible component of the algorithm that yields the rational expectations path of the economy.\(^{14}\) Finally, $e_{1,\text{min}}^p$ is set to an arbitrarily small value, namely 0.001.

### 3.2 The urban sector

The starting values of education are $e_{2,1}^p = 0.740$ and $e_{2,1}^s = 0.502$ (USAID, 2005). Where the magnitude of $k_1$ is concerned, recall that the household’s savings in period $t$ form the entire investment fund for the next period. Since capital, so defined, is a stock, the capital-output ratio has the dimension of time, the unit of which in this framework is 20 years.\(^{15}\) Hence, the usual magnitude of 3 (years) translates here into 0.15. The value of $k_1$ actually chosen, namely, 1.0, satisfies the requirement that the path of $k_t$ thereafter be tolerably smooth. It yields, along with the initial values of other variables and various parameters, the rather modest capital-output ratio of

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\(^{14}\)The resulting value is reported in the notes to Table 2.

\(^{15}\)Marquetti (2004) provides estimates of the net fixed standardized capital stock for the period 1963-2000, based on the sum of depreciated past aggregate investment (the perpetual inventory method), including both gross residential capital formation and changes in stocks. He assumes that equipment and structures represent 20 and 80 per cent, respectively, of gross capital formation, and that both categories of asset have the same life of 14 years.
2.57 (= 20 × 1.00/6.964) years (see Table 2). With \( N^a_{2,t} = 2 \forall t \), CSA (2000) yields \( N^c_{2,1} = 4.15 \) and \( N^o_{2,1} = 0.990 \). The adult premature mortality rate is set with that in the rural sector, where HIV-prevalence rates are much lower, so as to yield the estimated national average. We obtain \( q^a_{2,1} = 0.299 \) (we assume \( N^{cp}_{2,t} = N^{cs}_{2,t} = N^c_{2,t}/2 \forall t \)).

We retain the values of \( \gamma, \beta \) and \( \rho \) in the rural sector. Given the much higher educational attainments of town-dwellers, the initial value of \( \lambda_{2,t} \) should be pitched well above its rural counterpart.\(^{16}\) In the light of the prevailing differences in the quality of schooling and the attainment in vocational and higher education, it seems safe to conclude that in the year 2000, the level of human capital among adults in the cities was at least twice that among those in the country: we set \( \lambda_{2,0} = 2.3 \) and \( \lambda_{2,1} = 2.8 \). We are no longer at liberty to choose the efficiency parameter \( A_{2,1} \) as we please; for there are (crude) estimates of the productivity differential between the two sectors, and these must be largely respected. This consideration yields \( A_{2,1} = 2.0 \).

Since inputs of labor fully reflect human capital, we choose \( \alpha_2 = 0.667 \).

Turning to the preferences of young urban adults, let these be Stone-Geary once more, with parameters \( \omega_j \ (j = 1, 2, 3, 4) \), which relate to current and future consumption and the two levels of education, respectively. The parameters \( \omega_3 \) and \( \omega_4 \) are endogenously determined so that the optimal values of primary and secondary education in period 1 are equal to 0.740 and 0.502, respectively. The remaining \( \omega_i \) parameters are obtained from the normalization \( \omega_1 + \omega_2 + \omega_3 + \omega_4 = 1 \) and the rate of pure impatience in consumption, \( 1 - \omega_2/\omega_1 \). We are unaware of any attempts to estimate this rate for Ethiopia, but for the U.S., Fullerton and Rogers (1993) find that it lies between 0.25% and 0.5% per annum. With this as a guide, we set \( \omega_2/\omega_1 = 0.85 \).

Finally, \( e^{p}_{2,min} = e^{s}_{2,min} = e^{p}_{1,min} \), and the subsistence parameter \( c_{2,min} \) is assumed to about 17% larger than its rural counterpart.\(^{17}\)

\(^{16}\)Adult illiteracy rates in urban and rural Ethiopia in 1999 were 28.0% and 77.9%, respectively, (ILO, 2005).

\(^{17}\)According to the 2000 Welfare Monitoring Survey (World Bank, 2003), the poorest 20% of house-
3.3 Population

Knowing the size of the rural population in the year 2000, \( P_{1,1} \), as well as \( N_{1,1}^c / (1 - q_{1,1}^{ae}) \), \( N_{1,1}^a \) and \( N_{1,1}^o \), we can obtain \( F_{1,1} \) from

\[
P_{1,t} = \left( N_{1,t}^c + N_{1,t}^a + N_{1,t}^o \right) F_{1,t}.
\]

(13)

Using eq. (2), we then obtain \( F_{1,0} \).

Similarly, for the urban population, we have

\[
P_{2,t} = \left( N_{2,t}^c + N_{2,t}^a + N_{2,t}^o \right) F_{2,t}
\]

(14)

which yields \( F_{2,1} \); \( F_{2,0} \) then follows from eq. (6). For any sequence \( \{M_t^a\}_{t=1}^T \), it is then possible to calculate all the demographic variables \( F_{i,t} \) and \( P_{i,t} \) \( (i = 1, 2) \) in all subsequent periods. Note that the mortality rates in the early and late phases of adulthood, \( q_{ae}^{i,t} \) and \( q_{al}^{i,t} \), are assumed to be equal for all \( i \) and \( t \).

\[18\] holds in both sectors spent ‘almost the same amount’ (Birr 49 compared to Birr 42).

Values of \( q_{c}^{i,0} \) and \( q_{a}^{i,0} \) are also required. These are calculated using the following mortality rates from World Bank (2003):

<table>
<thead>
<tr>
<th></th>
<th>Adult female</th>
<th>Adult male</th>
<th>Infant</th>
<th>Under-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>400.98</td>
<td>490.97</td>
<td>143.00</td>
<td>213.00</td>
</tr>
<tr>
<td>2000</td>
<td>535.00</td>
<td>594.00</td>
<td>117.00</td>
<td>174.00</td>
</tr>
</tbody>
</table>

where \( q_{a}^{i,t} \) is the average over the two sexes. Child mortality in 1980 is proportional to child mortality in 2000, that is, \( q_{1,0}^a = q_{2,0}^a = (213/174) \cdot 0.062 = 0.0759 \). Similarly, for premature adult mortality in 1980, assuming that the sectoral mortality profiles do not differ: \( q_{1,0}^{ae} = q_{2,0}^{ae} = [(400.98 + 490.97)/(535.00 + 594.00)] \cdot 0.200 = 0.158 \). Our data indicate fewer children in period 0 than in period 1, but the World Bank (2003) reports total fertility rates of 6.60 in 1980 and 5.65 in 2000. Note that \( P_{1,t} \) corresponds to the population aged 5 or more years.
4 The results

Given the initial conditions, the system evolves under the influence of demographic developments and (disembodied) technical progress, which are treated as exogenous. Recalling Section 2.4, these are the sequences of fertility, \( \{N_{c,i,t}\}_{t=0}^{T} \), age-specific mortality, \( \{q_{i,t}\}_{t=0}^{T} \), and total factor productivity, \( \{A_{i,t}\}_{t=1}^{T} \). In order to establish the effects of the epidemic, we need these sequences for two disease environments: first, there is the counter-factual (denoted by \( D = 0 \)), in which the epidemic never appears; second, there is its actual and projected course (denoted by \( D = 1 \)).

We begin with demography. Although there are projections for the overall Ethiopian population by age group till 2050 (UN, 2004), no such projections exist for the two sectors separately. To produce them, we assume that there will be a complete demographic transition to stationary fertility levels in five periods. To be precise, \( N_{c,i,t} \) is assumed to decline linearly from its current value \( (t = 1) \) to 2.041 at the start of period 5 (see Table 1). The series for premature mortality among adults, \( q_{i,t}(D = 1) \), was estimated on the following basis. (i) Drawing on the experience from the epidemic elsewhere in Africa, Loewenson and Whiteside (1997) conclude that: the spread of HIV becomes exponential when about 2% of the sexually active population are infected; the ensuing period until the peak is reached is comparatively short, probably 10 years or less; and the epidemic may then become endemic, with perhaps about 10% of the sexually active population being HIV-positive at any one time. (ii) The (estimated) upswing in prevalence rates in Ethiopia’s rural sector (see Figure 1) will probably gain momentum; for urbanization is commonly associated with return flows, including those associated with the agricultural peak periods, all of which intensify the spread of HIV/AIDS (Degefe and Nega, 2000: 74). A notable feature of \( D = 1 \) is that mortality reaches its peak before 2020. The counterfactual series for \( q_{i,t}(D = 0) \) has been

19 This is consistent with Hladik et al. (2006). Their analysis suggests an ‘expanding HIV epidemic in rural and all Ethiopia, but a possible decline in some urban areas’.
interpolated from the W.H.O. life tables for Ethiopia for the year 2000.

Where the growth of total factor productivity (TFP) is concerned, we proceed as follows. The growth rate of \( A_{1,t} \), denoted by \( g_{A_{1,t}} \), is set at a level such that the resulting rates of income and consumption growth in agriculture, together with that of rural-urban migration, remain plausible. If, at first glance, the annual rate of 1% (i.e., 22% over 20 years) looks rather low, it should be recalled that the accumulation of human capital will also contribute to the growth of individual productivity. In the urban sector, we choose, parsimoniously, \( g_{A_{2,t}} = 0 \). This apparently startling assumption does not rule out the growth of individual productivity, even if physical capital per head stays constant. For, as shown below, the parameters of the educational technology allow unbounded growth of individual human capital when all children are fully educated, and the production technology is Cobb-Douglas.

There remains the (exogenous) quantity \( d^u \) in eq. (10). Since (expected) ‘utils’ are not observable and the purist position that \( d^u = 0 \) is a useful benchmark, even when many factors have been omitted from the model, we sought results with \( d^u \) as close to zero as the algorithm would permit when given a day or two in order to converge. With \( d^u = 0.5 \), convergence became very slow indeed, and there we left it – provisionally. As it turns out, the resulting value of \( c_{1,1} \) would have to be about 17% larger in order that a young rural adult be indifferent between the two locations, which seems quite modest and so justifies the decision to proceed with \( d^u = 0.5 \).

As a final preliminary, we calculate the rate of growth in the steady-state setting of Section 2.4. With the parameter values adopted above, eq. (11) becomes, for \( D = 1 \),

\[
\lambda_{1,t+1} = 2(\lambda_{1,t} + 0.75(1 − 0.05)\lambda_{1,t-1}) \cdot 0.35 + 1,
\]

whose relevant characteristic solution is \( \lambda_{1,t} = \lambda_{1,0}(1.139)^t \). Hence, the sector grows asymptotically at the rate 0.315 \((= 0.222 + 0.667 \times 0.139)\) per generation, or 1.38% p.a.
For the urban sector, eq. (12) yields $\lambda_{2,t} = \lambda_{2,0}(1.368)^{t}$, or 1.58% p.a., which is also the whole economy’s asymptotic rate, and which is finally approached from above.

4.1 The projected course with AIDS

Table 2 presents the results for the reference case $D = 1$. In the rural sector, the human capital of a young adult grows, on average, at about 32% per period over the whole time horizon, which is over twice the asymptotic rate. This outcome is the immediate result of two developments. First, there is the rise in educational attainments early on, with full primary education attained by period 3. Second, after heavy out-migration in period 2, the number of elders per family, all of whom contribute to the capacity of their grandchildren to build human capital, reaches a minimum in period 3, and then increases substantially.

Land per family decreases until period 5, the consequence of a fixed overall cultivable area, high fertility in the early periods and insufficient relief from outmigration. The upturn is then quite sharp. With $N_{t}^{a} = 2 \forall t$ and $(1 - q_{1,1}^{c})N_{1,1}^{c} = 3.879$, i.e., with the size of the cohort of rural young adults expected to almost double over the period 2000-2020, then under any reasonable assumptions about the growth of TFP, agriculture will fail to generate much of a livelihood for them, and spurred on by a narrowing of the gap between urban and rural mortality rates, outmigration is indeed heavy in period 2. With the relief offered by this alternative, consumption per young adult grows throughout, increasing by about 22% in the first period, and then accelerating to 55% in the fifth, yielding an average rate over the whole horizon of about 39% per period, thus exceeding the asymptotic rate of 31.5%.

Turning to developments in the towns, despite migration throughout, only 60% of the population is urban in period 6.\textsuperscript{20} Even so, this internal ‘brain drain’ of young

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\textsuperscript{20}In reality, the rate of urbanization is affected not only by the number of young adults who move from their villages to the cities, but also by population growth in some rural settlements, which
adults has its drawbacks, not only for rural families, but also for urban ones. For there are substantial differences in the levels of young adults’ human capital across the two sectors, so that the arrival of migrants reduces the average level of human capital in urban areas and slows down the rate of growth of $\lambda_{2,t}$. In any event, $e_{2,t}^{s}$ and $e_{2,t}^{a}$ both reach unity in period 3, so that thereafter, it is only the contribution of the family’s adults to the children’s capacity to build human capital that affects the evolution of $\lambda_{2,t}$. As $\xi_{t}$ peaks in period 2, the influence of the term $(\lambda_{1,t}^{s}\xi_{t} + \phi\lambda_{1,t-1}^{s}\xi_{t-1}) / (1 + \xi_{t})$ weakens in later periods, and the growth rate of $\lambda_{2,t}$ pick ups strongly: the average growth rate of $\lambda_{2,t}$ over the whole horizon is about 23% per period, well below its asymptotic rate. Thus, while the level of urban young adults’ human capital in the cities is always higher than their rural cousins’, the ratio thereof falls from 2.0 in period 1 to 1.44 in period 6.

A salient feature of the urban sector is the use of physical capital in production. Premature adult mortality has an adverse impact on the savings of young adults, and yet $s_{t}$ drops by 20% between periods 1 and 2, before starting to accelerate, attaining almost two and a half times its initial level in period 6. Other factors are also at work, of course; so that a comparison with the counter-factual $D = 0$ is needed to settle matters (see Section 4.2 below). The consumption of a young adult increases six-fold.

A brief comment on the main national aggregates is also required. From Table 2 it is seen that the total population of those aged 5 and older increases by a factor of 4.3 over the time horizon from 2000 to 2100. If this seems rather staggering, the reader is reminded that the Ethiopian population doubled over the past 25 years alone. Given the country’s very youthful population, only a much faster demographic transition or a sharp reduction in completed fertility due to the HIV/AIDS epidemic will impose a much less numerous population in the steady-state.

The other index of central importance is GDP per head. Let $Y_{1,t} \equiv y_{1,t}F_{1,t}$ and gradually acquire the organizational characteristics and judicial status of towns.
$Y_{2,t} \equiv y_{2,t}F_{2,t}$ denote the levels of aggregate income in the rural and urban sectors, respectively. Their sum, $Y_t \equiv Y_{1,t} + Y_{2,t}$, is aggregate income (GDP) when the two sectors are combined, and $y'_t \equiv Y_t / P_t$ is income per capita in the overall economy. The latter increases by 50% in the first period, decelerates to about 30% in the next period, in part under the delayed effects of high mortality in period 1, and then picks up again to 55-60% in the final three – again, in part, as a result of the decrease in premature adult mortality. In this connection, it may be helpful to express $y'_t$, which relates to an indivisible 20-year period, as a more familiar annual quantity. The level of GDP per head in the year 2000 was $725$ (Heston et al., 2006). Scaling in the same way, $y'_6$ then corresponds to $5428$ in 2100, which implies an average annual growth rate of 2.01% over the entire horizon. This would be a decided improvement over Ethiopia’s performance in the last century, but it is rather modest by the standards set by many Asian countries.

4.2 The counterfactual: no outbreak

Table 3 presents the results for the benchmark $D = 0$, which differs from $D = 1$ only in the series $\{q_{a,t}\}$, with the economy’s initial total endowments being unaffected. The lower levels of premature adult mortality encourage investment in both physical and human capital in both sectors, and so lead to somewhat faster growth. With the urban sector as the main motor of the economy, migration is higher throughout, the rate being notably greater in period 1, when mortality is highest under $D = 1$. The end result is that in period 6, about 65% of the population is urban.

Going into the essential details, full education is reached in the same periods as under $D = 1$, but $e_{1,t}$, $e_{2,t}$, and $e_{2,t}$ are higher in the interim. The general course of $k_t$ is also similar to, but lies above that under $D = 1$. The courses of the fixed factor land, $\ell_t$, are virtually identical up to period 4, but begin to diverge quite markedly thereafter. The
upshot is that the consumption enjoyed by a young rural adult in period 6 is a mere 8.5% higher than under $D = 1$. The consumption enjoyed by a young urban adult, whose level grows six-fold over the whole horizon, is but 5.0% higher than under $D = 1$. GDP per head in 2100 is about 10% higher in the counter-factual. This difference seems rather small, but it is permanent, as can be seen by examining the difference equations (3) and (7), in which divergences in the transitional dynamics produce cumulative level effects.\footnote{The characteristic roots are the same if the mortality rates are likewise.} The trajectories of all the main variables under $D = 0$ and $D = 1$ are depicted for comparison in Figure 2. What is striking is that the effects of the epidemic on the levels of per capita endowments and GDP are still growing in 2100, even though the epidemic has weakened to an endemic state by period 5. Also striking is the finding that the epidemic has virtually no effect on the size of the total population, even at the close of the horizon in 2100. The reasons for this are, first, the assumption that while the epidemic kills off young adults,\footnote{Recall that $P_t$ is defined not to include those young adults who die at the start of period $t$.} it has no effect on net reproduction in each sector, and second, that urban fertility is lower, so that faster urbanization under the counterfactual leads to lower fertility in aggregate.

5 Policy objectives

What objectives commend themselves in the present setting, where the population is growing rapidly, but is also beginning to experience a wave of premature adult mortality? In each period, the level of human capital attained by the rising generation of young adults has a powerful influence on both output and the formation of human capital, not only in the present, but also in the future – provided they and their offspring do not die prematurely in adulthood. Mortality on the scale that now rules in Ethiopia is also strongly undesirable in itself on purely ethical grounds, a consideration that

\[21\]
high levels of premature adult mortality undermine the process through which human capital is formed and also hinder the accumulation of physical capital. In particular, it can certainly be argued that the level of social welfare ought to be higher, *cet. par.*, when any cohort is larger and none is smaller.

In Section 4, we concentrated heavily on the stream of losses in GDP per head, which, by definition, accords no weight whatsoever to changes in the sizes of cohorts – or, for that matter, to those in the size of the whole population. It turns out, however, that the assumptions concerning fertility and the timing of premature mortality leave but little scope for the population to vary much in the face of the epidemic. Net reproduction in the villages and towns, respectively, is independent of premature adult mortality, so that, in each period, the level of fertility in the aggregate depends only on the distribution of the population of young adults between town and country, and hence on rural-urban migration, whose level is endogenous. In the long run, after the epidemic has subsided and fertility has reached stationary levels, it is quite possible that the population will be smaller in the counterfactual without the epidemic; for urban fertility is lower than rural fertility in the transition and there is more migration. Granted these assumptions, therefore, the use of GDP per head as the basic argument of the social welfare function is relatively innocuous – provided, of course, the use of measures based on current aggregate output is admissible in this context. As it turns out, the size of the population does enter into the calculations, for reasons that will become clear in the sections that follow.

6 Public spending and mortality

The next step is to introduce public policy explicitly aimed at altering the course of mortality in the presence of the epidemic. In order to produce the corresponding variants of the benchmark $D = 1$, it is necessary to establish the relationship between
the level of spending on measures to contain the epidemic and treat those infected, and premature mortality. We draw on the procedure and estimates in Bell, Devarajan and Gersbach (2006), a brief sketch of which will suffice here.

It is argued that the efficacy of spending depends directly on the number of families, \(F_t\), that make up the whole society. Aggregate spending is then written as \(F_t \cdot G_t\), where \(G_t\) is the level of spending per family in period \(t\). Let such aggregate expenditures produce a pure public good, so that ridding the relationship of the size of the population, we write

\[
q_t(D = 1) = q_t(G_t; D = 1),
\]  \hspace{1cm} (15)

where the superscript ‘\(a\)’ to \(q_t\) can now be suppressed without ambiguity and the function \(q_t(G_t; D = 1)\) is to be interpreted as the efficiency frontier of the set of all measures that can be undertaken to reduce \(q_t\). We also assume, and this seems sensible, that the effects of these expenditures last for only the period in which they are made.

Our definition of the benchmark \(D = 1\) implies that \(q_t(0; D = 1)\) should yield the estimates in Table 1. A second, plausible, condition is that arbitrarily large spending on combating the epidemic should lead to the restoration of the status quo ante, that is, \(q_t(\infty; D = 1) = q_t(D = 0)\) in all periods, again as set out in Table 1. The four-parameter logistic,

\[
q_t(G_t; D = 1) = d_t - \frac{1}{a_t + c_t e^{-b_t G_t}},
\]  \hspace{1cm} (16)

possesses an asymptote and sufficient curvature. With four parameters to be determined, the above conditions must be augmented by two others.

One way of proceeding is to pose the question: what is the marginal effect of efficient spending on \(q_t\) in high- and low-prevalence environments, respectively? That is to say, we need estimates of the derivatives of \(q_t(G_t; D = 1)\) at \(G_t = 0\) and some value of \(G_t\) that corresponds to heavy spending, when the scope for using cheap interventions has been exhausted. Here, we draw on the costs of preventing a case of
AIDS or saving a disability-adjusted-life-year (hereinafter, a DALY), as estimated by Marseille, Hofmann and Kahn (2002, Table 1) using various methods. These authors put the average annual cost per DALY saved of a diverse bundle of HIV-prevention interventions (e.g., targeting sex workers and condom promotion, voluntary counseling and testing, ensuring a safe blood supply) at $12.50, whereby it may be remarked that, for these particular interventions, the assumption that $G_t$ produces a pure public good does not seem to be wide of the mark.

Following the purposive and determined implementation of the full battery of preventive measures, the remaining intervention is to treat the infected. Opportunistic infections can be treated in the later stages of the disease, and the onset of full-blown AIDS can be delayed for some years through the controlled use of anti-retroviral therapies. Such measures will do little to reduce $q_t$ as strictly defined here, but by keeping infected individuals healthier and extending life somewhat, they will raise lifetime income and improve the parental care enjoyed by children in the affected families. In the context of the model, therefore, it seems perfectly defensible to interpret these gains in longevity as a reduction in $q_t$. The estimated average annual cost of the recommended first-line HAART regimens is about $360 per person (WHO, 2005). It is assumed here that the latter is the cost of the efficient, marginal form of intervention when determined and extensive efforts at prevention are already being undertaken.

With an additional assumption about how quickly diminishing returns to preventive measures set in as the prevalence rate falls (see the note to Table 4), the above conditions yield the values of the parameters $a_t$, $b_t$, $c_t$ and $d_t$. The years in which such a program might be undertaken fall into the decades beginning in 2000 and closing in 2040 (see below). The resulting values of the parameters are set out in Table 4, and the associated functions $q_t(G_t; D = 1)$ are plotted in Figure 3.
7 The policy program

The instruments potentially at the government’s disposal are: (i) the set of all measures designed to contain the disease and to treat those who become infected; (ii) measures aimed at supporting the victims’ families, especially in the form of consumption and school-attendance subsidies: and (iii) the taxes or – better still – outside aid needed to finance the spending under items (i) and (ii). The introduction of any of these measures brings us at once to private decisions, for these respond to the program of taxes and spending that the government chooses. Specifying all this in detail is a complicated matter (for a full account, see Bell, Devarajan and Gersbach [2006]), and no attempt will be made to implement the full procedure here.

We adopt, instead, a simpler approach, which not only does away with the complications that arise when the magnitudes of fiscal variables are endogenous, but is also quite plausible in the light of the international community’s willingness to shoulder a large part of the costs of combating the epidemic in poor countries. The proposal is that donors would commit themselves to making outright grants to the government of Ethiopia for this purpose. It would be unrealistic to expect indefinite budgetary support for such programs. Given the course of the epidemic as expressed by \( q_t(D = 1) \), the worst should be over by 2040 (the start of period 3), so that one can entertain the possibility of a stream of outright grants that ends in 2039. Let this stream be denoted by \( B = (B_1, B_2) \). In each period, the said amount is allocated to measures designed to combat the epidemic, as summarized in the function \( q_t(G_t; D = 1) \), where \( G_t = B_t/F_t, \ (t = 1, 2) \). The model is then run with the corresponding exogenous mortality profile \( \{q_t(G_t; D = 1)\}_{t=1}^{t=7} \).

The next step is to name sums of money. We suppose, perhaps rather optimistically, that a spending program of $100 million a year in real terms will commence in 2008 and continue until 2040. Since the model’s unit time period is 20 years, this translates
into two sub-programs. The first translates, somewhat roughly, into an outlay of $60 million a year, starting in 2000 and ending in 2019: equivalently, about $4.20 is spent annually per family, whereby some allowance has been made for population growth over that interval.\textsuperscript{23} The second runs from 2020 until 2040. Expenditure per family falls to $4.03 a year for that period, again making allowance for population growth. These two elements constitute the spending program $B$. The evolution of the main variables under this program is set out in Table 5. As intuition would suggest, the reductions in mortality lead to a slightly more rapid accumulation of human and physical capital than under $D = 1$, but leave the population size virtually unchanged, as slightly higher levels of migration induce lower aggregate fertility.

In order to assess the social profitability of this particular program $B$, we need an index of welfare. A very conservative choice is confined to the resulting change in GDP per head in each period relative to the reference case $D = 1$ without any intervention (see Table 2), a choice that pays no heed to the premature deaths warded off by $B$. The absolute values of GDP per head and the series of corresponding differences are reported in Table 6, in both absolute and proportional terms. The next step is to make these differences in 20-year per capita incomes commensurable with the aggregate program $B$. This involves, first, multiplying each by the size of the corresponding population, and second, converting the resulting sums into annual flows of constant dollars. Recalling that GDP per head in 2000 was $725 annually (Heston et al., 2006), $y_1'(D = 1) = 0.488$ implies an ‘exchange rate’ of 29,710 in our 20-year unit period.

A social discount rate, $r$, is also needed. The real, long-term annual yield offered by good quality paper in international capital markets is about 4%, so this is the minimum real rate that public investment should yield. As a further check, results are also reported for the more stringent standard of 5% annually; but our preferred

\textsuperscript{23}In the year 2000, there were about 28.5 million Ethiopians between the ages of 15 and 44 (UN, 2004), the groups which make up the overwhelming majority of the sexually active population. These numbers have been normalized to yield the numbers of families as defined in Section 2.1.
rate is the lower one. These rates yield the two columns of aggregate present values reported in Table 6. Adding up each column, we obtain the aggregate gross benefits of the program, as implied by our choice of welfare measure. The present value of the aggregate costs is obtained by discounting the stream of annual (aggregate) outlays under $B = (B_1, B_2)$. Table 7 presents the corresponding findings using full income per head.\textsuperscript{24} The resulting ratio of benefits to costs ranges from 3.5 to 1 to 5.3 to 1, depending on the discount rate and, to a much lesser extent, the choice of income measure employed. These levels of social profitability seem high indeed, especially when one considers that the positive net benefits accruing after period 7 have been left out. As argued above, the said level effects are permanent, and if, conservatively, the differences in period 6 continued indefinitely, the associated discounted stream of benefits would contribute about another 0.7 units when $r = 0.04$.

Are there reasons to doubt the robustness of this finding? In view of the discussion in Section 6, the most compelling point to be made against them is that the effectiveness of spending on prevention when prevalence rates are relatively high and levels of spending are rather low has been over-estimated. Suppose, therefore, that the (absolute) slopes of the $q_t(·)$ functions at zero levels of spending were only one half those derived in Section 6. Since the changes in $q_t$ under the program are small anyway, it follows from the approximately linear responses of the whole system in any sufficiently small neighborhood that the benefits in any period would be only half as large. That is to say, the social benefit-cost ratio would then range from at least 1.75 to 1 up to 2.65 to 1 or more. On this basis, we can still conclude with some confidence that the program analyzed in this chapter would yield a large social return for each dollar spent.

A second important consideration is whether we have adopted an unduly pessimistic view of Ethiopia’s growth prospects over the long run. We have therefore experimented by increasing the “transmission” parameter $b_2$ from 0.45 to 0.505, which raises the

\textsuperscript{24}Full income, $y''_t$, is equal to $(y_1,t|e_{1,t}=0 F_{1,t} + y_2,t|e_{2,t}=e_{2,t}=0 F_{2,t}))/P_t \forall t \geq 1.$
asymptotic rate of growth of GDP per capita from 1.58% p.a. to 2.0% p.a. (see eq. (12)) and yields an average rate of 2.18% over the 100-year horizon.\textsuperscript{25} The program to combat AIDS becomes even more profitable: the benefit-cost ratio with respect to GDP per capita (full income), assuming a 5% discount rate, is now 5.11 (4.90). The reason is as follows. Faster urban growth induces more migration, with unchanged mortality. The program to combat AIDS yields sharper reductions in mortality per $ spent in the towns, where the prevalence rates are much higher, than in the villages. Hence, the pay-off to this form of intervention rises as the underlying rate of growth of the urban sector increases. If one wants to go further and make the case that an asymptotic rate of, say, 2.5% p.a. is attainable, this would simply reinforce our finding.

8 Conclusions

Taking a long-term view, the effects of the AIDS epidemic in Ethiopia estimated here might, at first glance, seem rather small. There is no threat of an economic collapse, and the size of the population, which increases somewhat more than fourfold over the century 2000 to 2100, is scarcely affected – though many meet an early death along the way. At 2.01% over the entire horizon, the average annual rate of growth of output per head would be a decided improvement over Ethiopia’s performance in the last century, however modest that may appear in comparison with East Asia or even some fully developed countries. Under the counter-factual, in which there is no outbreak, output per head at the end of the horizon is, moreover, only 10% higher.

Yet such a view savors of complacency. First, the threat of an economic collapse, using output per head as yardstick, cannot be that imminent when the economy is already mired in poverty and backwardness. Second, the said difference in output per head is permanent, a consequence of the weaker inter-generational transmission

\footnote{Results for a range of values of $b_2$ are depicted in Figure 4.}
of human capital during the transitional phase. Third, premature adult mortality would be high in Ethiopia even if there had been no outbreak, so that AIDS effectively ‘competes’ with other mortal afflictions. Fourth, the projected course of the epidemic is much less severe than that now ravaging southern Africa. All this stands in stark contrast to South Africa in particular, a middle-income country which enjoyed rather low adult mortality in 1990, when the adult prevalence rate was about 1%, but is now experiencing a great wave of mortality, with the prevalence rate in ante-natal clinics already exceeding 25%. Fifth, in following the common practice of measuring national economic well-being in terms of GDP per capita – admittedly with misgivings – we have neglected the fact that premature death is also something to be avoided in itself, not only for the infected individuals themselves, but also on account of its consequences for their dependent children, towards whom all parents presumably have feelings of love and altruism.
Table 1: Exogenous demographic variables.

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*Note:* $D = D^*$ is an hypothetical scenario with an epidemic and equal mortality rates across sectors. $D = 1$ denotes the actual case with epidemic.
Table 2: The evolution of the economy and demographic variables, $D = 1$.

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1 The rural sector: $A_{1,1} = 1.000, g_{A_{1,t}} = 0.222, \lambda_{1,0} = 1.150, \omega = 0.800, c_{1,\min} = 0.500, c^p_{1,\min} = 0.001, b_1 = 0.350, \theta_1 = 0.400$.

2 The urban sector: $A_{2,1} = 2.000, g_{A_{2,t}} = 0.000, \lambda_{2,0} = 2.300, \omega_1 = 0.539, \omega_2 = 0.458, \omega_3 = 0.001, \omega_4 = 0.001, c_{2,\min} = 0.584, c^p_{2,\min} = c^e_{2,\min} = 0.001, b_2 = 0.450, \theta_2 = 0.400$.

$\dagger$ $du = 0.500, \alpha_i = 0.667, \gamma = 0.40, \delta = 0.20, \beta = 0.60, \rho = 0.60, \phi = 0.75$. $y_t'$ grows at 1.98% per annum. $F_{1,t}, P_{1,t}$ and $P_t$ in millions.
Table 3: The evolution of the economy and demographic variables, $D = 0$.

| $t$ | $\lambda_{1,t}$ | $\ell_t$ | $c_{1,t}$ | $e_{1,t}^p$ | $y_{1,t}$ | $\lambda_{2,t}$ | $k_t$ | $c_{2,t}$ | $s_t$ | $e_{2,t}^p$ | $e_{2,t}^e$ | $y_{2,t}$ | $y_t' | y_t''$
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<td>2.083</td>
<td>2.083</td>
<td>1.701</td>
<td>0.508</td>
<td>0.220</td>
<td>16.49</td>
<td>23.55</td>
<td>99.37</td>
<td>128.04</td>
<td>227.41</td>
</tr>
<tr>
<td>6</td>
<td>0.020</td>
<td>0.040</td>
<td>2.083</td>
<td>2.083</td>
<td>2.529</td>
<td>1.693</td>
<td>0.451</td>
<td>0.158</td>
<td>12.78</td>
<td>27.27</td>
<td>84.50</td>
<td>157.49</td>
</tr>
</tbody>
</table>

1 The rural sector: $A_{1,1} = 1.000$, $g_{A_{1,t}} = 0.222$, $\lambda_{1,0} = 1.150$, $\omega = 0.800$, $c_{1,min} = 0.500$, $e_{1,min}^p = 0.001$, $b_1 = 0.350$, $\theta_1 = 0.400$.
2 The urban sector: $A_{2,1} = 2.000$, $g_{A_{2,t}} = 0.000$, $\lambda_{2,0} = 2.300$, $\omega_1 = 0.539$, $\omega_2 = 0.458$, $\omega_3 = 0.001$, $\omega_4 = 0.001$, $c_{2,min} = 0.584$, $e_{2,min}^p = e_{2,min}^e = 0.001$, $b_2 = 0.450$, $\theta_2 = 0.400$.
† $du = 0.500$, $\alpha_i = 0.667$, $\gamma = 0.40$, $\delta = 0.20$, $\beta = 0.60$, $\rho = 0.60$, $\phi = 0.75$. $y_t'$ grows at 2.06% per annum. $F_{1,t}$, $P_{1,t}$ and $P_t$ in millions.
Table 4: Parameters of the functions $q_t(G_t)$.

<table>
<thead>
<tr>
<th></th>
<th>rural sector</th>
<th>urban sector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t = 1$</td>
<td>$t = 2$</td>
</tr>
<tr>
<td>$a_t$</td>
<td>3.939</td>
<td>5.213</td>
</tr>
<tr>
<td>$b_t$</td>
<td>0.078</td>
<td>0.107</td>
</tr>
<tr>
<td>$c_t$</td>
<td>1.109</td>
<td>1.467</td>
</tr>
<tr>
<td>$d_t$</td>
<td>0.404</td>
<td>0.312</td>
</tr>
<tr>
<td>$\nu_t$</td>
<td>16.398</td>
<td>11.970</td>
</tr>
</tbody>
</table>

Note $\nu_t$ denotes the level of spending that would restore $D = 0$ if there were no diminishing returns to preventive measures; $m$ is the multiple of $\nu_t$ at which HAART becomes cost-effective.
Table 5: The evolution of the economy and demographic variables under program B.

<table>
<thead>
<tr>
<th>t</th>
<th>$\lambda_{1,t}$</th>
<th>$\ell_{t}$</th>
<th>$c_{1,t}$</th>
<th>$e_{1,t}$</th>
<th>$y_{1,t}$</th>
<th>$\lambda_{2,t}$</th>
<th>$k_{t}$</th>
<th>$c_{2,t}$</th>
<th>$s_{t}$</th>
<th>$e_{2,t}$</th>
<th>$e_{2,t}^{p}$</th>
<th>$y_{2,t}$</th>
<th>$y_{t}$</th>
<th>$y_{t}^{''}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.400</td>
<td>1.020</td>
<td>0.573</td>
<td>0.251</td>
<td>3.211</td>
<td>2.800</td>
<td>1.000</td>
<td>0.786</td>
<td>1.477</td>
<td>0.737</td>
<td>0.500</td>
<td>6.931</td>
<td>0.489</td>
<td>0.513</td>
</tr>
<tr>
<td>2</td>
<td>1.806</td>
<td>0.693</td>
<td>0.701</td>
<td>0.717</td>
<td>3.564</td>
<td>3.127</td>
<td>1.450</td>
<td>1.111</td>
<td>1.198</td>
<td>0.934</td>
<td>0.634</td>
<td>7.208</td>
<td>0.734</td>
<td>0.804</td>
</tr>
<tr>
<td>3</td>
<td>2.500</td>
<td>0.423</td>
<td>0.904</td>
<td>1.000</td>
<td>4.211</td>
<td>3.180</td>
<td>1.416</td>
<td>1.469</td>
<td>1.309</td>
<td>1.000</td>
<td>1.000</td>
<td>17.395</td>
<td>2.362</td>
<td>2.436</td>
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<tr>
<td>4</td>
<td>3.284</td>
<td>0.338</td>
<td>1.312</td>
<td>1.000</td>
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<td>4.646</td>
<td>1.788</td>
<td>2.111</td>
<td>1.685</td>
<td>1.000</td>
<td>1.000</td>
<td>12.194</td>
<td>1.476</td>
<td>1.551</td>
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<tr>
<td>5</td>
<td>4.279</td>
<td>0.282</td>
<td>2.012</td>
<td>1.000</td>
<td>8.838</td>
<td>5.993</td>
<td>2.630</td>
<td>3.106</td>
<td>2.392</td>
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<td>1.000</td>
<td>26.538</td>
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<tr>
<td>6</td>
<td>5.578</td>
<td>0.408</td>
<td>3.121</td>
<td>1.000</td>
<td>14.683</td>
<td>7.992</td>
<td>4.539</td>
<td>4.551</td>
<td>3.563</td>
<td>1.000</td>
<td>1.000</td>
<td>142.31</td>
<td>51.15</td>
<td>142.31</td>
</tr>
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</table>

1. The rural sector: $A_{1,1} = 1.000$, $g_{A_{1,t}} = 0.222$, $\lambda_{1,0} = 1.150$, $\omega = 0.800$, $c_{1,min} = 0.500$, $e_{1,min} = 0.001$, $b_{1} = 0.350$, $\theta_{1} = 0.400$.
2. The urban sector: $A_{2,1} = 2.000$, $g_{A_{2,t}} = 0.000$, $\lambda_{2,0} = 2.300$, $\omega_{1} = 0.539$, $\omega_{2} = 0.458$, $\omega_{3} = 0.001$, $\omega_{4} = 0.001$, $c_{2,min} = 0.584$, $e_{2,min}^{p} = e_{2,min} = 0.001$, $b_{2} = 0.450$, $\theta_{2} = 0.400$.
3. $du = 0.500$, $\alpha_{1} = 0.667$, $\gamma = 0.40$, $\delta = 0.20$, $\beta = 0.60$, $\rho = 0.60$, $\phi = 0.75$. $y_{t}^{'}$ grows at 1.98% per annum. $F_{i,t}$, $P_{i,t}$ and $P_{i}$ in millions.

<table>
<thead>
<tr>
<th>t</th>
<th>$q_{1,t}$</th>
<th>$q_{2,t}$</th>
<th>$q_{2,t}^{a}$</th>
<th>$N_{1,t}^{c}$</th>
<th>$N_{1,t}^{o}$</th>
<th>$N_{2,t}^{o}$</th>
<th>$N_{2,t}^{o}$</th>
<th>$M_{t}^{a}$</th>
<th>$\xi_{t}$</th>
<th>$F_{1,t}$</th>
<th>$F_{2,t}$</th>
<th>$P_{1,t}$</th>
<th>$P_{2,t}$</th>
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<td>0.174</td>
<td>0.174</td>
<td>3.840</td>
<td>0.423</td>
<td>4.51</td>
<td>0.65</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>0.062</td>
<td>0.193</td>
<td>0.285</td>
<td>4.603</td>
<td>4.110</td>
<td>1.400</td>
<td>0.990</td>
<td>0.415</td>
<td>0.899</td>
<td>5.88</td>
<td>1.19</td>
<td>47.09</td>
<td>8.47</td>
<td>55.56</td>
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<td>0.201</td>
<td>0.201</td>
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<td>3.486</td>
<td>1.221</td>
<td>1.102</td>
<td>1.442</td>
<td>8.66</td>
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<td>61.79</td>
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<td>0.126</td>
<td>0.162</td>
<td>3.304</td>
<td>3.012</td>
<td>1.126</td>
<td>1.066</td>
<td>0.192</td>
<td>0.109</td>
<td>14.18</td>
<td>8.42</td>
<td>91.16</td>
<td>51.15</td>
<td>142.31</td>
</tr>
<tr>
<td>4</td>
<td>0.031</td>
<td>0.104</td>
<td>0.139</td>
<td>2.709</td>
<td>2.586</td>
<td>1.494</td>
<td>1.046</td>
<td>0.490</td>
<td>0.306</td>
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<td>14.73</td>
<td>110.04</td>
<td>82.95</td>
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<tr>
<td>5</td>
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<td>0.100</td>
<td>0.100</td>
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<td>2.151</td>
<td>1.836</td>
<td>1.283</td>
<td>0.427</td>
<td>0.217</td>
<td>18.29</td>
<td>21.30</td>
<td>109.53</td>
<td>115.74</td>
<td>225.28</td>
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<tr>
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<td>0.100</td>
<td>0.100</td>
<td>2.151</td>
<td>2.151</td>
<td>2.357</td>
<td>1.625</td>
<td>0.390</td>
<td>0.167</td>
<td>14.73</td>
<td>24.87</td>
<td>95.85</td>
<td>143.64</td>
<td>239.49</td>
</tr>
</tbody>
</table>
Table 6: GDP per head with and without the programme B.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$y'(B)$</th>
<th>$y'(D = 1)$</th>
<th>difference</th>
<th>% change</th>
<th>present value$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.48893</td>
<td>0.48801</td>
<td>0.00092</td>
<td>0.19</td>
<td>1.523</td>
</tr>
<tr>
<td>2</td>
<td>0.73379</td>
<td>0.73313</td>
<td>0.00066</td>
<td>0.09</td>
<td>0.817</td>
</tr>
<tr>
<td>3</td>
<td>0.96072</td>
<td>0.95633</td>
<td>0.00439</td>
<td>0.46</td>
<td>3.870</td>
</tr>
<tr>
<td>4</td>
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<td>1.47142</td>
<td>0.00445</td>
<td>0.30</td>
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<tr>
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<td>2.35664</td>
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<td>0.25</td>
<td>1.685</td>
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<td>6</td>
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<td>3.65225</td>
<td>0.00620</td>
<td>0.17</td>
<td>0.876</td>
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</table>

Benefit-cost ratio: 5.30 3.75

$^1$ Present values are reported in billions of dollars.

Note $y'_t$ series from Tables 5 and 2.

Table 7: Full income with and without the programme B.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$y''(B)$</th>
<th>$y''(D = 1)$</th>
<th>difference</th>
<th>% change</th>
<th>present value$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.51331</td>
<td>0.51220</td>
<td>0.00111</td>
<td>0.22</td>
<td>1.741</td>
</tr>
<tr>
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<td>0.80352</td>
<td>0.00035</td>
<td>0.04</td>
<td>0.416</td>
</tr>
<tr>
<td>3</td>
<td>1.03734</td>
<td>1.03307</td>
<td>0.00428</td>
<td>0.41</td>
<td>3.591</td>
</tr>
<tr>
<td>4</td>
<td>1.55136</td>
<td>1.54712</td>
<td>0.00424</td>
<td>0.27</td>
<td>2.207</td>
</tr>
<tr>
<td>5</td>
<td>2.43638</td>
<td>2.43074</td>
<td>0.00564</td>
<td>0.23</td>
<td>1.565</td>
</tr>
<tr>
<td>6</td>
<td>3.74082</td>
<td>3.73477</td>
<td>0.00605</td>
<td>0.16</td>
<td>0.814</td>
</tr>
</tbody>
</table>

Benefit-cost ratio: 4.89 3.49

$^1$ Present values are reported in billions of dollars.

Note $y''_t$ series from Tables 5 and 2.
Figure 1: Estimated and projected adult HIV prevalence rates in Ethiopia, 1990-2010 (MoH, 2006).
Figure 2: $D = 1$ (solid line) and $D = 0$ (dashed line).
Figure 3: Values of the functions $q_t(G_t)$. 
Figure 4: Benefit-cost ratios as a function of $b_2$. 
A Microfoundations

We lay out the microfoundations of the model in greater detail, beginning with rural households.

Children are less productive than uneducated adults; they provide at most \( \gamma \in (0, 1) \) efficiency units of labor. In a setting with three overlapping generations, each period is taken to span 20 years. Thus, with at most 10 years of primary education as the only option in rural areas, the labor actually supplied by the household in period \( t \) is

\[
\frac{\gamma(2 - e_{p,1,t}^{c})N_{1,t}^{c}}{2} + \lambda_{1,t}N_{1,t}^{a} + (1 - \delta)\lambda_{1,t-1}N_{1,t}^{o} = \Lambda_{1,t} - \frac{\gamma e_{p,1,t}^{c}N_{1,t}^{c}}{2},
\]

where \( \Lambda_{1,t} \) is the family’s total endowment of labor in efficiency units and \( \delta (> 0) \) denotes the depreciation rate of labor ‘capacity’ across the last two periods of life.

The technology for producing the aggregate good is assumed to be Cobb-Douglas in form, with constant returns to scale in labor and land. The level of output produced by a family in period \( t \) is thus given by

\[
y_{1,t} = A_{1,t} \left( \Lambda_{1,t} - \frac{\gamma e_{p,1,t}^{c}N_{1,t}^{c}}{2} \right)^{\alpha_{1}} \ell_{t}^{1-\alpha_{1}},
\]

where \( A_{1,t} \) and \( \ell_{t} \) denote the level of technical efficiency and the size of the family’s agricultural land holding, respectively. Under the social sharing rule, the upper boundary of the rural household’s budget set in the space of \( (c_{1,t}, e_{p,1,t}) \) becomes

\[
A_{1,t} \left( \Lambda_{1,t} - \frac{\gamma e_{p,1,t}^{c}N_{1,t}^{c}}{2} \right)^{\alpha_{1}} \ell_{t}^{1-\alpha_{1}} - \left( \beta N_{1,t}^{c} + N_{1,t}^{a} + \rho N_{1,t}^{o} \right) c_{1,t} = 0. \tag{17}
\]

The parents’ preferences over current consumption and the level of education attained by their children are influenced by the expectations that they form in period \( t \) about the premature mortality that will afflict their children as adults in period \( t + 1 \). Let
these preferences take the Stone-Geary form:

$$E_t u_1(c_{1,t}, e^p_{1,t}) = \omega \ln (c_{1,t} - c_{1,\text{min}}) + (1 - \omega)(1 - q^e_{1,t+1}) \ln (e^p_{1,t} + e^p_{1,\text{min}}),$$

where $(1 - q^e_{1,t+1})$ is the (point) expected survival rate in the next period, as formed in the minds of the young adults when they make their decisions in period $t$, $c_{1,\text{min}} > 0$ is an endogenously determined subsistence level of consumption (see Appendix B), $e_{1,\text{min}} > 0$ is likewise defined with respect to education, and $\omega$ is a ‘taste’ parameter.

The presence of inter-generational altruism, in the sense that parents place some value on their children’s education, ensures that the process of human capital accumulation may get underway and so lead to sustained growth. Observe, however, that $c_{1,\text{min}} > 0$ and $e_{1,\text{min}} > 0$ imply that whereas consumption is a necessary good, education is a luxury. Hence, there is the possibility that a household is so poor that the children will not be sent to school, that is, the parents’ altruism will not actually be operative. A poverty trap may, therefore, arise.

The family’s decision problem is

$$\max_{(c_{1,t}, e^p_{1,t}, M^p_t)} E_t u_1 \text{ s.t. } (17), (c_{1,t}, e^p_{1,t}) \geq 0, e^p_{1,t} \leq 1,$$  

where it should be noted once more that they take this decision after the migrants have departed for the towns.

Turning to the urban sector, the production technology is also Cobb-Douglas. Analogously to eq. (17), the upper boundary of the urban household’s budget constraint is

$$A_{2,t} \left[ \frac{\gamma(2 - e^p_{2,t} - e^s_{2,t})(N_{2,t}^{cp} + N_{2,t}^{cs})}{2} + (\lambda_{2,t} + \xi_{t}\lambda_{1,t})N_{2,t}^{a} + (1 - \delta)(\lambda_{2,t-1} + \xi_{t-1}\lambda_{1,t-1})N_{2,t}^{o} \right]^{\alpha_2} (k_{t})^{1 - \alpha_2}$$

$$- \left\{ \beta (N_{2,t}^{cp} + N_{2,t}^{cs}) + N_{2,t}^{a} + \rho N_{2,t}^{o} \right\} c_{2,t} + N_{2,t}^{a} s_t \right) = 0,$$  

where

$$A_{2,t} \left[ \frac{\gamma(2 - e^p_{2,t} - e^s_{2,t})(N_{2,t}^{cp} + N_{2,t}^{cs})}{2} + (\lambda_{2,t} + \xi_{t}\lambda_{1,t})N_{2,t}^{a} + (1 - \delta)(\lambda_{2,t-1} + \xi_{t-1}\lambda_{1,t-1})N_{2,t}^{o} \right]^{\alpha_2} (k_{t})^{1 - \alpha_2}$$

$$- \left\{ \beta (N_{2,t}^{cp} + N_{2,t}^{cs}) + N_{2,t}^{a} + \rho N_{2,t}^{o} \right\} c_{2,t} + N_{2,t}^{a} s_t \right) = 0.$$  

45
where $s_t$ denotes the level of savings per young adult. These savings are available to the family as physical capital in the next period.

Let the preferences of a young adult at time $t$ be represented as follows:

$$E_t u_2(c_{2,t}, e^{p}_{2,t}, e^{s}_{2,t}, e^{p}_{2,t+1}, e^{s}_{2,t+1}) = \omega_1 \ln(c_{2,t} - c_{2,min}) + \omega_2 (1 - q_{2,t}^a) E_t [\ln(\rho c_{2,t+1} - c_{2,min})] + \omega_3 (1 - q_{2,t+1}^a) \ln(e^{p}_{2,t} + e^{p}_{2,min}) + \omega_4 (1 - q_{2,t+1}^a) \ln(e^{s}_{2,t} + e^{s}_{2,min}).$$

where the third and fourth terms on the RHS represent altruism towards her children in the form of the levels of their primary and secondary education. It should be noted, first, that the 'pay-offs' in the event that the parent should die prematurely (with probability $q_{2,t}^a$), or that the children, in their turn, should die prematurely in adulthood (with probability $q_{2,t+1}^a$), have been normalized to zero, and second, that $\rho c_{2,t+1}$ is also, in general, a random variable for those who survive into old age, by virtue of the fact that its level depends on a whole variety of future economic and demographic developments.

With this scheme of expectations, the household’s decision problem at time $t$ is

$$\max_{[c_{2,t}, s_{t}, e^{p}_{2,t}, e^{s}_{2,t}, \xi_{t}, E_t(s_{t+1}, e^{p}_{2,t+1}, e^{s}_{2,t+1}, \xi_{t+1})]} E_t u_2 \text{ s.t. (19), } (c_{2,t}, s_{t}, e^{p}_{2,t}, e^{s}_{2,t}) \geq 0, \ (e^{p}_{2,t}, e^{s}_{2,t}) \leq 1.$$

(20)

B Algorithm

Where the rational expectations sequence is concerned, inspection of problems (18) and (20) reveals that we need the forecasts $E_t s_{t+1}$, $E_t e^{p}_{2,t+1}$, $E_t e^{s}_{2,t+1}$ (from which, for any given migration series, optimal or otherwise, we may obtain $E_t c_{2,t+1}$ from eq. (19)) and $E_t \xi_{t+1}$ (which, from eq. (8), depends on $E_t M_{t+1}^a$). In addition, when $t = 1$ the solution must yield the observed levels of educational attainment. To summarize, the
following must hold:

- $E_t(M_{t+1}^a) = M_{t+1}^{a0}$ (where $M_{t+1}^{a0}$ is the value that satisfies eq. (10) in period $t + 1$),
- $E_t s_{t+1} = s_{t+1}, E_t(e_{2,t+1}^p) = e_{2,t+1}^{p0}, E_t(e_{2,t+1}^s) = e_{2,t+1}^{s0},$
- $e_{1,1}^0 = 0.243, e_{2,1}^p = 0.740$ and $e_{2,1}^s = 0.502.$

The algorithm set out below describes our estimation approach for $D = 1$ (i.e., the scenario wherein all conditions specified above are satisfied). It proceeds in five stages. In the first, households are assumed to form stationary expectations where future levels of savings and education are concerned. Given the resulting values, a rational expectations path with respect to migration flows is computed. In the following three stages, these restrictions to stationary expectations are gradually lifted, and a full rational expectations path in migration and urban consumption is sought.

The utility function parameters $\omega_3$ and $\omega_4$ are gradually adjusted in stages 1 to 4 so as to satisfy the boundary conditions at $t = 1$ at the end of each stage. If, during this adjustment process, either $\omega_3$ or $\omega_4$ becomes very small, the system of first-order conditions in the two sectors and eq. (10) often fails to converge to a solution. For this reason, floor values are specified for these parameters, namely, $\bar{\omega}_3$ and $\bar{\omega}_4$, and whenever either $\omega_3$ or $\omega_4$ becomes smaller than the corresponding floor value, all $\omega_i$ are restored to their starting/provisional values and the process starts again with a lower value of $d^n$. The following table gives the parameters' starting and floor values used to produce the results reported in Table 2. Stage 5 replicates stage 4 (i.e., the solution of the full rational expectations system) for increasingly smaller values of $d^n$.

<table>
<thead>
<tr>
<th>$d^n$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
<th>$\bar{\omega}_3$</th>
<th>$\bar{\omega}_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td>0.5360</td>
<td>0.4556</td>
<td>0.0047</td>
<td>0.0037</td>
<td>0.00425</td>
<td>0.00325</td>
</tr>
</tbody>
</table>

\(^{26}\)No floor values are specified at this stage (which explains why the adopted $\omega_3$ and $\omega_4$ values are smaller than $\bar{\omega}_3$ and $\bar{\omega}_4$).
The utility differential decreases at this final stage up to some user-specified $\tilde{d}_u$ value (which equals 0.50 in the paper, but could also be zero).

Another complication arises from the fact that we need to know the population structures under $D = D^*$ and $D = 0$ in order to calculate the mortality profile under $D = 1$. But the population structure depends on (among other factors) migration, and the solutions of problems (18) and (20) under $D = D^*$ and $D = 0$ require values of certain parameters (e.g., $\omega_j$ ($j = 1, 2, 3, 4$) and $c_{i,min}$ ($i = 1, 2$)), which we obtain from the analysis of the factual scenario.

To overcome this difficulty, we employed an iterative process, where, using a provisional $q_{a,t}^u(D = 1)$ series, we estimate all the parameters under $D = 1$. We then use these parameters to find all demographic variables under $D = D^*$ and $D = 0$, and employ the latter to recalculate $q_{a,t}^u(D = 1)$ using eqs. (21) to (24)). This process is repeated till the difference between the provisional and recalculated $q_{a,t}^u(D = 1)$ series becomes very small (in the present case, smaller than $1/10^5$).

To obtain a separate premature adult mortality series for each sector, we solve the following system of equations:

\[
q_{a,t}^u(D = D^*) = \frac{P_{1,t}(D = D^*) q_{a,1,t}^u(D = D^*) + P_{2,t}(D = D^*) q_{a,2,t}^u(D = D^*)}{P_{1,t}(D = D^*) + P_{2,t}(D = D^*)} \tag{21}
\]

\[
R_t = \frac{q_{a,2,t}^u(D = D^*) - q_{a,2,t}^u(D = 0)}{q_{a,1,t}^u(D = D^*) - q_{a,1,t}^u(D = 0)} \tag{22}
\]

\[
q_{a,t}^u(D = 0) = \frac{P_{1,t}(D = 0) q_{a,1,t}^u(D = 0) + P_{2,t}(D = 0) q_{a,2,t}^u(D = 0)}{P_{1,t}(D = 0) + P_{2,t}(D = 0)} \tag{23}
\]

\[
q_{a,1,t}^u(D = 0) = m \cdot q_{a,2,t}^u(D = 0) \tag{24}
\]

Eq. (21) states that the economy-wide premature adult mortality rate is a weighted average of the premature adult mortality rates in the two sectors, with weights equal to the respective population sizes (the estimation results when $D = D^*$ are available from the authors upon request). Eq. (22) relates the excess mortality due to the epidemic in the urban sector to the excess mortality due to the epidemic in the rural sector. We assume that $R_t$ changes linearly from 3 in period 1 to 2 in period 4 (thereafter the two sectors have the same premature adult mortality rates). Eq. (23) replicates eq. (21) for the No-AIDS scenario. Finally, eq. (24) specifies the relationship between premature adult mortality in the two sectors in the absence of an epidemic. We assume that $m = 1.1$, that is, premature adult mortality would be 10% higher in the villages than in the cities, which, in view of the existing differences in the provision of medical care, seems plausible.
1: Start with a set of provisional values for $d^u$, $\omega_j$ ($j = 1, 2, 3, 4$), $\bar{\omega}_j$ ($j = 3, 4$) and
\$
\{M^a_t\}_{t=0}^{T+2}.
\$
2: **for** stage = 1 to 4 **do**
3:   **repeat**
4:     **repeat**
5:       Find $c_{1, min}$ such that $e_{1,0}^p = 0.243^{29}$ and set $c_{2, min} = 1.167 \times c_{2, min}$.
6:     **for** $t = 1$ to $T + 1$ **do**
7:       Solve the first order conditions in the two sectors and eq. (10) imposing the restrictions:
8:         $E_t(s_{t+1}) = s^0_t$, $E_t(e_{s,t+1}^p) = e_{s,t}^p$ and $E_t(e_{s,t+1}^s) = e_{s,t}^s$ if stage = 1.
9:         $E_t(e_{s,t+1}^p) = e_{s,t}^p$ and $E_t(e_{s,t+1}^s) = e_{s,t}^s$ if stage = 2.
10:        $E_t(e_{s,t+1}^p) = e_{s,t}^p$ if stage = 3.
11: **until** \$
\{M^a_t\}_{t=0}^{T+2} = \{M^a_t\}_{t=0}^{T+2} \\text{or a linear combination of } \{M^a_t\}_{t=0}^{T+2} \text{and } \{M^a_t\}_{t=0}^{T+2}.
\$
12: **if** $e_{2,1}^p \neq 0.740$ or $e_{2,1}^s \neq 0.502$ **then**
13: **if** $\omega_3 > \bar{\omega}_3$ and $\omega_4 > \bar{\omega}_4$ **then**
14:     adjust the urban utility function $\omega_i$ parameters.$^{30}$
15: **else**
16:     reduce the value of $d^u$.
17: **end if**
18: **end if**
19: **until** $e_{2,1}^p = 0.740$ and $e_{2,1}^s = 0.502$.
20: **end for**
21: **while** $d^u \geq \bar{d^u}$ **do** **// stage = 5**
22: **repeat**
23: **repeat**
24:     Replicate line 5.
25: **for** $t = 1$ to $T + 1$ **do**
26:       Solve the first-order conditions in the two sectors and eq. (10).
27: **end** line 5.
28: **end** for
29: Replicate line 10.
30: **until** \$
\{M^a_t\}_{t=0}^{T+2} = \{M^a_t\}_{t=0}^{T+2}.
\$
31: **if** $e_{2,1}^p \neq 0.740$ or $e_{2,1}^s \neq 0.502$ **then**
32:     adjust the urban utility function $\omega_i$ parameters.
33: **end if**
34: **until** $e_{2,1}^p = 0.740$ and $e_{2,1}^s = 0.502$.
35: reduce the value of $d^u$.
36: **end while
We report \( T = 6 \) periods in the tables. In order to have rational expectations in period 6 we need to obtain optimal values in period 7, and optimization in period 7 requires the calculation of demographic variables in period 8. Through iteration we obtain \( E_7(\xi_8) = \xi_8 \). We also assume that \( E_7(c_{2,8}^e) = E_7(c_{2,8}^s) = 1 \). Finally, \( E_7(s_{8}) \) is obtained from a trend of \( s_{t}^e \) for \( t = 1, \ldots, 6 \).

That is, for \( e_{1,1}^1 = 0.243 \), solve the first-order conditions in the rural sector in terms of \( c_{1,\text{min}} \) and \( c_{1,1}^1 \).

Two parameters change freely: \( \omega_3 \) is increasing (decreasing) if \( e_{2,1}^0 < 0.740 \) (\( > 0.740 \)), and/or \( \omega_4 \) is increasing (decreasing) if \( e_{2,1}^0 < 0.502 \) (\( > 0.502 \)). The remaining \( \omega_i \) parameters are estimated from the equations \( \omega_2/\omega_1 = 0.85 \) and \( \omega_1 + \omega_2 + \omega_3 + \omega_4 = 1 \).


