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Modelling the U.S. sovereign credit rating

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Abstract

This paper proposes a new methodology for generating sovereign credit ratings. These are determined by mapping the probability that the debt-GDP ratio might exceed a maximum debt limit at some point in the future into a credit rating. The debt limit can be either ad hoc or based on the financial ability of a government to change fiscal policy in the future to meet its outstanding obligations. When applied to quarterly U.S. data from 1970 to 2011, two clear instances are found in which the U.S. sovereign credit rating would have been downgraded on this basis: during the 1970s oil crisis and in the aftermath of the Lehman collapse in 2008. This result is robust to several alternative views on the maximum borrowing capacity of the U.S. economy.

JEL Classification: E62, H30, H60

Keywords: Credit risk, Sovereign risk, fiscal limits, default probability

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1 Introduction

Credit ratings for both private sector and sovereign debt have come under intense scrutiny since the onset of the 2008 financial crisis. Credit rating agencies (CRAs) have been criticized for failing to identify the amount of risk accumulated by mortgage-backed securities in the United States during the lending boom of the 2000s. Following the recent downgrades of a number of eurozone sovereigns, CRAs have been accused of exacerbating the eurozone debt crisis and contributing to the rise of the cost of borrowing above sustainable levels in several European countries.

In 2011, concerns about the adequacy of the operating procedures of the leading CRAs have been expressed by the Security and Exchange Commission in a report on the CRAs' ability to make timely and accurate disclosures, and to allay fears about potential conflicts of interest.¹ At the same time, the European Commission issued a proposal for stricter rules for CRAs to make them more transparent, accountable and to increase competition in the sector. The Commission's proposal stressed the role of conflict of interest, political interference and inefficiencies in existing CRAs methodologies. It also suggested the creation of an European ratings foundation in order to counter the influence of U.S.-based CRAs and that individual investors should determine their own independent evaluation of credit ratings (European Commission, 2011).² In his review of the regulatory structure of CRAs, White (2010) also argues in favor of investors seeking their own independent assessment of the credit rating as a way of reducing reliance on CRAs. It is widely thought, however, that it would be too costly for individual investors to make their own credit evaluations. Cost is also the major reason why the European Commission abandoned its plan to create a new CRA.

The aim of this paper is to show how it would be possible to provide measures of sovereign credit ratings that are transparent, independent and timely. Transparency refers to the ease of the general public to access and to reproduce credit ratings, and to the ability of the public to make its own judgments about their validity. Independence reflects the derivation of sovereign credit ratings that are model-based rather than driven by the subjective evaluation of analysts. The evaluation can be updated systematically using the latest available data, and is timely for this reason. The measure is inexpensive to produce, and can even be automated. Given these properties, we argue that such a procedure can provide benchmark statistics for sovereign credit ratings that could be especially useful not only for individual investors and independent (national and supra-national) agencies, but also for policy makers and CRAs. Duan and Van Laere (2012) recently argued that the provision of sovereign credit ratings for private sector businesses is a public good that can be made available to financial market participants through the creation of a publicly-funded infrastructure. The

¹See Security and Exchange Commission (2011)

²Similar reactions from policy makers and commentators were prompted following the Latin American defaults in the early 1980s, the 1997 Asian banking crisis and the default of Enron in 2001.

model-based measure of the sovereign credit ratings serves a similar purpose but without requiring the creation of expensive infrastructures.

The methodology is an adaptation to government debt of Merton's (1974)'s measure of distance-to-default and default probability. This has three key elements: a forecast of the level of indebtedness over a given time horizon; an estimate of the uncertainty surrounding this forecast; and a measure of the maximum borrowing capacity of the sovereign country. We calculate the time-varying forecasts of the debt-GDP ratio and their volatility using a VAR model based on rolling-window estimation, hereafter a ROVAR model. Rolling-window estimation, favoured by Stock and Watson (2007, 2008) and Orphanides and Wei (2012) among the others, has the advantage of taking account of time-variation in the VAR coefficients and in their volatility over the sample period without taking a specific stance on the source of time-variation. In this way forecasts of the debt-GDP ratio, the uncertainty surrounding them and the implied default probability may be up-dated each period. This procedure is computationally inexpensive as the model can be estimated using standard classical methods. Having determined the default probability profile, this can be mapped into a credit rating using historic information on sovereign default probabilities and credit ratings.

The analysis requires the definition of a debt threshold (debt-GDP limit) beyond which a default event is assumed to occur. We implement two alternative views about the default threshold. The first, based on ad-hoc values, is entirely agnostic about the economic rationale for the debt-GDP limit. Its usefulness is in providing preliminary evidence on the likely values of the model-based sovereign credit rating and on its sensitivity to forecasts of the debt-GDP ratio, macroeconomic volatility and the debt threshold itself. The second employs a real business cycle model with an elastic labor supply and distortionary taxation to derive the debt-GDP limit. This is based on the notion that governments default (either formally or de facto) only when they are not able to meet their financial obligations through using their fiscal instruments. We employ four alternative definitions of the debt limit, depending on whether future fiscal policy changes are anticipated or unanticipated and, in the second case, if they stem from changes in expenditure policy, tax policy or both. We apply this methodology to the U.S. sovereign credit rating for the period 1969:4 to 2011:2 for both ad-hoc and theory-based debt-GDP limits, constructing quarterly time-series of credit ratings for short-term and long-term U.S. debt.

It might be argued that the use of a debt limit based exclusively on a government's financial ability to generate fiscal savings provides too narrow a view of the borrowing capacity of a country, and that taking into account additional factors might result in either higher or lower debt limits. A government could, for example, either inflate away its debt obligations or alter the maturity structure of debt. Credit ratings may also be influenced by a country's default history and a government's willingness to meet its financial obligations. There are two main reasons why we choose not to incorporate these possible extensions into our proposed methodology. First, it would significantly complicate the analysis. Second, adopting a narrow and specific view of the maximum borrowing

capacity has the advantage that the resulting credit rating has an unambiguous economic interpretation that can be easily communicated to investors and the general public. Any differences between the model-based and the official credit ratings might indicate the impact of factors, in addition to those associated with fiscal policy, that have contributed to the subjective judgment of CRAs. These discrepancies are likely to be smaller for advanced countries with stable political, independent central banks committed to maintain price stability, and fiscal authorities interested in retain reputation in sovereign bond markets, than for other countries.³

U.S. Treasury securities have long been considered risk-free assets. Historically, they have received the highest credit-quality rating by all CRAs. Since the latest global financial crisis, however, both prominent economists (see, for example, Buiter (2010)) and market participants have increasingly taken the view that U.S. bonds are no longer risk-free assets. The change in the market sentiment is reflected in the fast-growing trend in the price of credit default swaps (CDS) for U.S. sovereign bonds, an indicator of the market's perception of the U.S. government creditworthiness.

This is illustrated very clearly in Figure 1 which shows the price of U.S. Treasury securities CDSs over the three and half years (from January 2008 to June 2011) before Standard & Poor's downgraded the U.S. sovereign credit rating by one notch from its highest ranking on August 5, 2011.⁴ Early in 2008, the 5-year U.S. sovereign CDSs traded below 10 basis points (bps). The price rose substantially in July 2008 when IndyMac Bank collapsed, and rose further in September 2008 when Lehman Brothers declared bankruptcy and AIG attempted to negotiate a bridging loan from the Federal Reserve. CDS prices also increased in early 2009 to just below 100 bps and, after a sharp decline to about 30 bps in the first half of the 2009, again increased steadily. By the end of June 2011 U.S. bonds CDS traded at about 51 bps, twice as much as German sovereign CDS (26 bps) and close to that of Japan CDS (52 bps).⁵ Notwithstanding this sharp deterioration of the market's perception of its creditworthiness over this period, U.S. government debt has received the highest quality ranking by all CRAs during this period.⁶

In contrast, the model-based sovereign credit rating derived in this paper appears to be more in line with the market's perception about the creditworthiness of the U.S. government. The results suggest that the U.S. sovereign credit

³In the U.S., as in most advanced countries, honouring sovereign debt is a constitutional obligation. This was recently restated by the U.S. Treasury's General Counsel George W. Madison, see The New York Times, July 8 2011.

⁴The data are from Datastream (Thomson Reuters CDS), accessed on July 11, 2012. The sample includes daily observations from January 7, 2008 to June 30, 2011. No data on U.S. sovereign CDS is available prior January 2008. The U.S. macroeconomic data is only available up to 2011:2, therefore defining the end of the CDS sample period.

⁵On July 11, 2012, the 5-year CDSs were traded at about 47 bps, while the price on German CDS was about 32 bps and that of Japan CDS is 59 bps.

⁶In 2011, the main CRAs (Fitch Ratings, Moody's and Standard & Poor's) expressed concern about the medium-term perspectives of the U.S. fiscal outlook and lowered the outlook of the U.S. sovereign debt to negative. All three CRAs also began issuing warnings about a possible (though limited to one or two notches) downgrade of the U.S. sovereign credit rating.

rating would have been of the highest quality for most of the last 40 years with the exception of two periods where the credit rating is lower: the oil crisis in the early 1970s and in the aftermath of IndyMac Bank and Lehman Brothers collapse. The extent and duration of these downgrades depend on the level of the debt-GDP limit, and in no case does the credit rating fall to speculative grade. The differences between the model-based and the official credit ratings illustrate the point made earlier, that factors other than the ability of to generate fiscal savings may have influenced the judgment of the CRAs in these two instances. A possible explanation for the differences between the credit ratings of the early 1970s is that the Fed printed money to purchase U.S. Treasury securities. A major factor affecting the official credit ratings in the post-Lehman period discrepancy is likely to be confidence that the U.S. government would honour its financial liabilities. Nonetheless, the U.S. credit rating was downgraded in 2011:3, an event anticipated by about two years by the model-based credit rating.

Fig. 1. U.S. sovereign CDS, mid rate spread. Source: Datastream (July 2012).

To the best of our knowledge, this paper is the first to attempt to use macroeconomic models to determine sovereign credit ratings. As outlined in sections 2 and 3, there is a large theoretical literature on sovereign default risk, but little attention has been paid to how this translates into a sovereign credit rating. The empirical literature on sovereign credit rating has focused either on deter-

mining good predictors of official sovereign credit ratings, or how (and to what extent) financial markets and macroeconomic variables react to changes in sovereign credit ratings. In contrast to this literature, this paper seeks to provide an alternative measure of credit ratings to those of the CRAs.

The paper proceeds as follows. Section 2 provides some background information on sovereign credit ratings and the methodologies followed by CRAs. Section 3 discusses how we map the probability of default into credit ratings. Section 4 describes how we adapt Merton’s model in order to compute sovereign default probability. Section 5 specifies the ROVAR model used for the derivation of the debt-GDP forecasts and their uncertainty. Section 6 carries out a preliminary analysis of the U.S. sovereign credit rating by combining estimates from the ROVAR with ad-hoc values of the debt-GDP limit. Section 7 assesses various different measures of the debt-GDP limit obtained from a standard dynamic stochastic general equilibrium (DSGE) model and recalculates the U.S. sovereign credit rating. In Section 8 we summarize the methodology, the main results and draw some conclusions about its use. In the paper we refer to four appendices, which are available in the working paper version of this article, see Polito and Wickens (2012, b). Appendix A describes the ROVAR model. Appendix B provides details of the data. Appendix C includes details on the solution of the DSGE model used for the determination of the theory-based default limits. Appendix D describes the algorithm used for computing the distribution of the steady-state values of the theory-based debt-GDP limits.

2 Sovereign credit ratings and the CRAs’ procedures

A sovereign credit rating is broadly defined as an opinion about the likelihood of default of a government, based on the perceived ability and willingness of the government to meet its financial obligations.⁷ A government is able to repay its debt when it can change its policy to generate enough savings to service its obligations. Whether a government wants to repay its liabilities depends on a number of factors that go beyond the ability to generate revenue. These include the existence of binding constraints to enforce debt repayment, sovereign reputation, whether debt holders are either domestic or foreigners, the maturity structure of government debt and the share of government debt held by domestic banks.⁸

⁷This section draws on Gaillard (2012).

⁸The distinction between willingness and ability to pay debt, first highlighted by Eaton and Gersovitz (1981), is also central in the economic literature. The literature on willingness initially looked at the role of sanctions (Sachs, 1984), sovereign reputation (Eaton and Fernandez, 1995) and asymmetric information (Gertler and Rogoff, 1990) for the assessment of sovereign risk. More recent contributions have pointed out the role of debt maturity (Cole and Kehoe, 2000) and secondary markets (Broner, Martin and Ventura, 2010) in determining sovereign default events. There is a vast economic literature on the theoretical determinants of the ability to pay and its empirical measurement. Polito and Wickens (2012, a) provide an up-to-date survey.

The definition of a sovereign credit rating has evolved over time. CRAs currently view a sovereign credit rating as being closely related to a government's ability to meet its financial obligations. The main CRAs also issue separate sovereign recovery ratings for government bonds that reflect the willingness to repay debt. Credit ratings are assigned both to countries and bonds. The former reflects a government's overall financial capacity to pay its financial obligations; the latter measures the quality of securities issued by a country. In addition to evaluating countries and bonds over the long period, CRAs also assign short-term ratings to sovereign issuers and short-term bonds, where short-term is deemed to be a period of time of 12 months or less.

Although they vary across CRAs and have changed considerably over time, sovereign rating methodologies are ultimately based on the individual judgments of rating analysts. For example, no CRA uses a mathematical formula or an economic model to measure the credit rating. Instead, CRAs have sovereign risk units in charge of assigning new credit ratings and of monitoring and reviewing the existing ratings. Ratings are issued following a request for a rating made by a sovereign country.⁹ The CRA's team of analysts meets with the leading officials of a country, collects data and then formulates a preliminary rating recommendation, which is subsequently ratified (via a voting system) by the members of a rating committee within the CRA. The resulting rating is first notified to the issuer and then disseminated to the public through the media. Fitch Ratings and Standard & Poor's allow the rating issued by the rating committee to be appealed before dissemination, while Moody's does not. Economic and political factors and qualitative and quantitative criteria are employed by the CRAs to determine the credit rating. Their use vary across CRAs and this has changed over time. No information is provided on how each criterion and variable is weighted in the final determination of the overall credit rating.¹⁰

3 Credit ratings and the probability of default

Credit ratings reflect the probability of default. In order to translate our measure of the probability of default we require a mapping of the probability of default into a credit rating. This may be derived from data made available by the CRAs on the historic experience of many countries. The first three columns in Table 3 summarize the letter-rating scale that we employ as a benchmark for the numerical analysis. This includes 19 categories ranging from Aaa (highest quality and minimum credit risk) to C (lowest quality assigned to bonds or countries already in default). Ratings from Aaa to Baa3 are termed investment grade, whereas ratings below Baa3 are categorized as speculative grade.

⁹Originally CRAs were paid by market investors for issuing credit rating (the so-called "subscriber-pays" model). Since the early 1990s, however, CRAs are paid for their services directly by sovereign countries ("issuer-pays" model).

¹⁰There is an extensive empirical literature on the determinants, accuracy and effects of sovereign credit ratings, in particular with reference to developing countries. Examples include Eichengreen and Porters (1986); Cantor and Packer (1996); Ferreira and Gama (2007); Hill, Brooks and Faff (2010); Afonso, Gomes and Rother (2011) and Chen et al. (2013).

Table 1
Sovereign credit rating scale and cumulative default probabilities.

Category	Rating		Cumulative default probability			
	Long-term	Short-term	1-year	5-year	10-year	average
Investment grade	Aaa	Prime - 1	4.99E-4	4.99E-4	4.99E-4	4.99E-4
	Aa1	Prime - 1	0.026	0.245	0.317	0.208
	Aa2	Prime - 1	0.053	0.490	0.634	0.415
	Aa3	Prime - 1	0.079	0.736	0.952	0.623
	A1	Prime - 1	0.106	0.981	1.269	0.830
	A2	Prime - 1/2	0.132	1.226	1.586	1.038
	A3	Prime - 1/2	0.159	1.471	1.903	1.245
	Baa1	Prime - 2	0.185	1.717	2.221	1.453
	Baa2	Prime- 2 or 3	0.212	1.962	2.538	1.660
	Baa3	Prime-3	0.238	2.207	2.855	1.868
Speculative grade	Ba1	Not Prime	0.415	3.950	8.197	3.942
	Ba2	Not Prime	0.592	5.692	13.540	6.017
	Ba3	Not Prime	0.769	7.435	18.882	8.092
	B1	Not Prime	1.643	9.989	20.785	10.196
	B2	Not Prime	2.517	12.542	22.687	12.299
	B3	Not Prime	3.391	15.096	24.590	14.403
	Caa	Not Prime	10.139	21.005	27.334	19.607
	Ca	Not Prime	16.888	26.914	30.079	24.812
	C	Not Prime	23.636	32.823	32.823	30.016

This table reports the letter-rating scale employed for the numerical analysis and the corresponding cumulative default probabilities after 1, 5 and 10 years. The last column includes the average cumulative default probability over the 10-year horizon. The default probabilities are calculated by interpolating original data available from Moody's (www.moody.com).

Short-term ratings are related to long-term ratings and range from Prime, which implies minimum credit risk for obligations of short-term maturity (further distinguished in Prime-1, Prime-2 and Prime-3) to Not Prime.¹¹ CRAs provide separate ratings for debt issued in local currency (LC) or in foreign currency (FC). For the purpose of this paper we abstract from this distinction; while this is particularly relevant for emerging economies, it is less important for advanced economies like the U.S. Recent evidence suggests that LC and FC sovereign credit ratings have converged over time with the main CRAs currently regarding governments equally likely to default on both domestic and foreign liabilities (see Gaillard 2011, p24).

In mapping default probabilities into a credit rating we proceed as though a credit rating reflects exclusively the ability of a government to repay debt as

¹¹ Although this rating scale is closely related to that currently employed by Moody's, it also approximates the grading scales employed by other CRAs. The actual rating scale used by Moody's includes 21 categories as the Caa rating is split into Caa1, Caa2 and Caa3. Fitch's scale includes 24 categories from AAA to D, while S&P scale is based on 23 categories from AAA to D.

measured through the probability of default. We have noted that there is not a clear-cut definition of sovereign credit rating, but by associating credit ratings with this definition of the probability of default we obtain a rating consistent with the current orientation of the main CRAs.

The final four columns of Table 3 report the cumulative probability of default after one, five, ten years as well as the unweighted average for a 10-year period associated to each of the 19 letter grades. The 1-year scale is used later to derive the measure of the short-term rating, while the 5-year, 10-year and average scales are used to measure long-term ratings. The default probabilities are derived by interpolating through annual data on 10-year cumulative default probabilities and the corresponding sovereign credit ratings over the period 1983-2011 available in Moody's (2011).¹² Similar tables are provided by Standard & Poor's and Fitch Ratings. The table shows a clear negative relation between the sovereign credit rating and the probability of default. For example, a Baa rating implies a probability of default of 2.207 per cent by the end of year 5, while a Ba rating is linked to a higher cumulative default probability (7.435 per cent) over the same period of time.

The accuracy and interpretation of the default probabilities in Table 3 require further comment. The default probabilities taken from Moody's (2011) are calculated by forming cohorts of sovereign issuers on the basis of their original ratings and then dividing the number of defaults in the cohort over a specific period of time by the initial size of the cohort. They therefore provide an ex-post measure of the default probability that is not entirely consistent with that required by an ex-ante analysis of the sovereign credit rating. Concerning their reliability, the data in Table 3 may understate the default probability profile attached to each credit rating as they exclude the default episodes that occurred between World War I and World War II, and from the second half of the 1970s to the early 1980s. It is also possible that these data may overstate the probability of sovereign default probability as they are based on actual default episodes which include instances of the inability and unwillingness to repay debt.

Nonetheless, in our view, the historical default probabilities represent the best option available for mapping the probability of sovereign default into a

¹²The original data cannot be used directly because they are available on an annual basis and only for Aa, A, Baa, Ba, B and Caa-C ratings. In addition a zero-default probability is attached to Aaa-A ratings for the whole 10-year horizon and Aaa-Baa ratings for the 1-year horizon. We use a two-stages linear interpolation to retrieve this missing information. First, we interpolate vertically (for each year) to derive the default probability associated with each of the 19 letter grades. We assume that the available ratings - Aa, A, Baa, Ba, B and Caa-C - correspond respectively to Aa3, A3, Baa3, Ba3, B3 and C in table 3. We also replace the value of 0 for Baa in year 1 with 0.476/2, i.e. half of the value in the following year. This initial interpolation has the effect of assigning, for each year, nonzero default probabilities for ratings Aaa-Baa3 in year 1, and ratings Aaa-A3 in subsequent years. Having derived a mapping of the probability of default into credit ratings in each of the 10 years, we then derive a quarterly mapping for the whole 10-year period by a further interpolation. This second round of interpolation is carried out assuming that in the first year the default probability at the beginning of the first quarter is 0. We have also replaced the default probabilities at the end of the first year for Aaa ratings from 0.000E-20 to 4.99E-4, as the model typically yields nonzero default probability for Aaa ratings.

sovereign credit rating. CRAs also publish default probabilities associated with corporate credit ratings. These would also suffer of the same limitations as sovereign ratings. Moreover, as sovereign defaults occur less frequently than corporate defaults, corporate default rates cannot be used to map sovereign credit ratings. Rather than use historic data, one could specify a rating scale based on pre-defined cumulative default probabilities. This would, however, be an ad hoc choice and there is no good reason to prefer it to that based on historic data.

In the following sections we describe and implement a model for computing ex-ante default probabilities over a 10-year forecast period. The model-based measure of the sovereign credit rating is then obtained by comparing the ex-ante default probabilities with the ex-post values in Table 3.

4 Measuring the probability of sovereign default

The conceptual framework for the determination of the probability of sovereign default is adapted for use on government debt from Merton's (1974) model of credit risk. The starting point is the one-period government budget constraint (GBC) which describes the intertemporal dynamics of government debt. Expressed as a proportion of nominal GDP, the GBC can be written as:

$$\frac{d_t}{y_t} + (1 + \rho_t) \frac{b_{t-1}}{y_{t-1}} = \frac{b_t}{y_t}, \quad (1)$$

where y_t is real GDP, $\frac{d_t}{y_t}$ is the primary deficit-GDP ratio, ρ_t is the output-adjusted real interest rate on government debt and $\frac{b_t}{y_t}$ is the debt-GDP ratio. The primary deficit $\frac{d_t}{y_t}$ is defined as the difference between government expenditure in goods and services ($\frac{g_t}{y_t}$) plus transfers ($\frac{z_t}{y_t}$), both expressed as a proportion of GDP, and the government revenue-GDP ratio ($\frac{v_t}{y_t}$). The discount rate ρ_t is the nominal interest rate on government bonds (i_t^b) less the inflation rate (π_t) and the growth rate of GDP (γ_t). By expanding the variable $\frac{d_t}{y_t}$ to include seigniorage revenue, equation (1) can also represent the consolidated government budget constraint. Defining $\delta_t = \frac{1}{1+\rho_t}$ as the one period discount factor, the h -period ahead forward solution to the GBC is:

$$\frac{b_t}{y_t} = -E_t \sum_{s=1}^h \frac{d_{t+s}/y_{t+s}}{\prod_{j=1}^s \delta_{t+j}} + E_t \frac{b_{t+h}/y_{t+h}}{\prod_{j=1}^h \delta_{t+j}}, \quad (2)$$

where E_t denotes the mathematical expectation conditional on information available at time t . Given a target level of the debt-GDP ratio $\frac{b_{t+h}}{y_{t+h}}$, this equation can be written as

$$\frac{b_t}{y_t} + E_t \sum_{s=1}^h \frac{d_{t+s}/y_{t+s}}{\prod_{j=1}^s \delta_{t+j}} - E_t \frac{b_{t+h}/y_{t+h}}{\prod_{j=1}^h \delta_{t+j}} = E_t \frac{b_{t+h}/y_{t+h}}{\prod_{j=1}^h \delta_{t+j}} - E_t \frac{b_{t+h}/y_{t+h}}{\prod_{j=1}^h \delta_{t+j}}.$$

This measures the deviation of the discounted value of the future debt-GDP ratio (equivalent to the sum of the current debt-GDP ratio and the present value of future expected primary surpluses) from the discounted value of the debt-GDP target.¹³

To measure the probability of sovereign default we reinterpret the target debt-GDP ratio as a default threshold reflecting either the willingness or the ability of a government to meet its financial obligations. Default is presumed if the expected value of the debt-GDP ratio exceeds this threshold level. The default threshold is interpreted as the market (or an analyst's) expectation of the value of the debt-GDP ratio beyond which the debt cannot be rolled over, thereby bringing about a default event. A default event occurs when the government fails to make scheduled payments (either partially or in full) on the principal or the interest. Typically, this leads to a modification of the terms of the debt contract which either reduces the value of the bonds issued, or extends their maturity, or reduces the interest rate.

To implement this we use the h -period ahead solution of the GBC in equation (2) given by

$$\frac{b_{t+h}}{y_{t+h}} = - \sum_{j=1}^h \left[\Pi_{s=1}^j (1 + \rho_{t+s}) \frac{d_{t+j}}{y_{t+j}} \right] + \Pi_{s=1}^h (1 + \rho_{t+s}) \frac{b_t}{y_t},$$

where the right-hand side is the cumulative saving generated by current and future primary surpluses from t to $t+h$ plus the interest cost of rolling-over the current debt-GDP ratio until period $t+h$.

The probability of sovereign default by period $t+h$ (hazard rate) is the probability of not defaulting prior to year $t+h$ but defaulting in year $t+h$, and hence is given by

$$p_{t,t+h} = p_{t+h} (1 - p_{t+h-1}) (1 - p_{t+h-2}) \dots (1 - p_{t+1}), \quad (3)$$

where p_{t+h} denotes the probability of defaulting in period $t+h$ given information up to period t . This is measured by

$$p_{t+h} = \Pr \left(- \sum_{j=1}^h \left[\Pi_{s=1}^j (1 + \rho_{t+s}) \frac{d_{t+j}}{y_{t+j}} \right] + \Pi_{s=1}^h (1 + \rho_{t+s}) \frac{b_t}{y_t} \geq \frac{\overline{b_{t+h}}}{y_{t+h}} \middle| \frac{b_t}{y_t} = \frac{b}{y} \right),$$

where $\Pr(\cdot)$ could be either an empirical density function or assumed to be a known theoretical distribution.¹⁴ Hence,

$$p_{t+h} = \Pr \left(\frac{b_{t+h}}{y_{t+h}} \geq \frac{\overline{b_{t+h}}}{y_{t+h}} \middle| \Phi_t \right), \quad (4)$$

¹³This equation is at the basis of numerous short- and medium-term indicators of the fiscal stance proposed in the economic literature and used by policy makers; for example, the tax-gap indicators pioneered by Blanchard (1990) and Blanchard et al. (1990); the S1 and S2 indicators in European Commission (2007), the projections of the fiscal stance in Congressional Budget Office (2010), and model-based measures of the fiscal stance proposed by Polito and Wickens (2011) and Polito and Wickens (2012, a).

¹⁴To implement the credit rating we assume a normal distribution.

where $\Pr(\cdot)$ is assumed to be the normal probability density function and Φ_t denotes information available at time t . The debt-GDP ratio at time $t + 1$ may be decomposed into

$$\frac{b_{t+1}}{y_{t+1}} = E_t \frac{b_{t+1}}{y_{t+1}} + \xi_{t+1}$$

where $E_t \frac{b_{t+1}}{y_{t+1}}$ is the expectation of the debt-GDP ratio by the end of period $t + 1$ conditional on information available in t , and ξ_{t+1} is the corresponding innovation in period $t + 1$. The latter may be written as

$$\xi_t = \sigma_t \varepsilon_t,$$

where $\varepsilon_t \sim i.i.d. (0, 1)$. It then follows that the debt-GDP ratio for period $t + h$ may be written as

$$\begin{aligned} \frac{b_{t+h}}{y_{t+h}} &= E_t \frac{b_{t+h}}{y_{t+h}} + \eta_{t+h} \\ \eta_{t+h} &= \sum_{s=1}^h \xi_{t+s} \end{aligned}$$

where $V_t(\eta_{t+h}) = \sigma_{\eta, t+h}^2 = \sum_{s=1}^h \sigma_{t+s}^2$ is the conditional variance of the debt-GDP ratio.

The probability of sovereign default in period $t + h$ given information in period t is therefore

$$p_{t+h} = \Pr(-DD_{t+h} \leq \zeta_{t+h} | \Phi_t), \quad (5)$$

with

$$\zeta_{t+h} = \frac{\eta_{t+h}}{\sigma_{\eta, t+h}}$$

and

$$DD_{t+h} = \frac{E_t \frac{b_{t+h}}{y_{t+h}} - \overline{\frac{b_{t+h}}{y_{t+h}}}}{\sigma_{\eta, t+h}} \quad (6)$$

denoting the distance-to-default of sovereign debt, a measure, in terms of standard deviations, of the size of the increase in the expected debt-GDP ratio between t and $t + h$ that would cause default in period $t + h$. The probability of default increases due to a narrowing of $E_t \frac{b_{t+h}}{y_{t+h}} - \overline{\frac{b_{t+h}}{y_{t+h}}}$, to the gap between the expected debt-GDP ratio and the default threshold level, and to increased uncertainty surrounding the forecasts of the debt-GDP ratio (σ_{t+h}). As the base year changes, causing changes in the estimates of the conditional expectation and variance of the debt-GDP ratio, this probability will change over time. From equation (3), the cumulative default probability (i.e. the probability of default in any period between t and $t + h$) is

$$p_{t, t+h}^c = \sum_{j=1}^h p_{t, t+j}, \quad (7)$$

which is calculated assuming a standard cumulative normal distribution. This provides a measure of the cumulative default probability compatible with that used in Table 3.

To summarize, equations (5) and (6) show that three fundamental pieces of information are required to measure the probability of sovereign default: a forecast of the debt-GDP ratio over a specific time-horizon, an estimate of the uncertainty surrounding this forecast, and a value for the debt-GDP default threshold. The first two may be determined from a macroeconomic model of the economy. This is described in the next section.

5 Forecasting sovereign debt and volatility

5.1 The model

An economic analysis of the probability of sovereign default and credit ratings requires the specification of a model of the economy in order to forecast the future evolution of the debt-GDP ratio. This could be a structural model of the economy such as a DSGE model or, as in this section, a time series model such as VAR.

In an analysis of the forecasting properties of DSGE models Wickens (2014) has shown that there is little difference in the forecasting properties of DSGE and VAR models. Both forecast well when the economy is growing smoothly but forecast badly when the economy experiences turning points. One reason for poor forecasts from DSGE models is the difficulty of forecasting exogenous variables for which, by definition, there is no theory and so they must be forecast by time series models. The reason why an unrestricted VAR model forecasts as well as a DSGE model is that when the exogenous variables are represented by a VAR, the solution to a DSGE model is a restricted VAR. If these restrictions are correct then the forecasts from a DSGE model will have smaller forecast error variances than those from an unrestricted VAR, but if the restrictions are incorrect then an unrestricted VAR will give less biased forecasts.

There are further advantages to using a VAR. It is theory-free and is easy to implement empirically, both of which are attractive features for market participants seeking to determine their own credit-rating measure. In order to improve the forecasting performance we take account of the uncertainty arising from changes in the processes generating the exogenous variables and any variations in the parameters by estimating the VAR using a rolling-window data period, or ROVAR. Rolling analyses of time-series models are often used in finance to assess a model's stability over time. Stock and Watson (2007, 2008) have used rolling-window estimation to forecast U.S. inflation, while Orphanides and Wei (2012) have demonstrated the effectiveness of rolling-window estimation in capturing time-variation in the parameters and (the volatility of) the residuals in macro-finance models. Recently, Canova and Ferroni (2012) have employed rolling samples to evaluate within a medium-scale DSGE model the impact of policy shocks on the dynamic of inflation. Using a ROVAR it is easy

and simple to continually up-date the probability distribution of the debt-GDP ratio on the latest data.

An alternative type of VAR to a ROVAR could also be used. In the monetary policy literature on the determinants of the great acceleration and the great moderation Primiceri (2005) has used a reduced form VAR with time-varying parameters and stochastic volatility (TVP-SV-VAR), and Sims and Zha (2006) have used a VAR with Markow-switching parameters and volatility (MS-VAR). There are, however, two main drawbacks to these VAR models. First, they can only be estimated using Bayesian methods as the analytical form of the likelihood function is not tractable under the assumption that the covariance structure of the shocks follows a stochastic volatility process. Even then, the estimation is generally not straightforward as it depends crucially on the specific assumptions made about the priors; it is also time-intensive, especially for medium- and large-scale models. Second, forecasts and measures of uncertainty from both the TVP-SV-VAR and the MS-VAR are influenced by the particular view taken of the process driving the variance structure of the shocks, whether this is gradual, as in the TVP-SV-VAR, or driven by a Markow-switching process, as in the MS-VAR. An agnostic view on volatility may be preferable. Kapetanios et al. (2012) finds that during the latest financial crisis macroeconomic forecasts obtained from a ROVAR significantly outperform those the TVP-SV-VAR, while being in line with those from the MS-VAR. Forecasts from the three models are instead similar during periods of macroeconomic stability.

Following the VAR literature on time-varying macroeconomic volatility, we specify a ROVAR model with two lags and a constant term. This is estimated using a moving-data window of 40 quarters. At each step of the estimation we derive pseudo out-of-sample forecasts. In essence, we estimate the VAR using data from $t - 40$ until date t and then compute h -period ahead point forecasts and forecasts errors. These two steps are repeated until t reaches the end of the sample.¹⁵ The vector of variables included in the ROVAR is

$$\mathbf{x}_t = \left[\gamma_t \quad \pi_t \quad \frac{d_t}{y_t} \quad \frac{b_t}{y_t} \quad r_t^s \quad r_t^l \quad cp_t \right]. \quad (8)$$

The first four variables - the growth rate of real GDP, the inflation rate (GDP deflator), the primary deficit-GDP ratio and the debt-GDP ratio - allow the model to satisfy the GBC, equation (1).¹⁶ The variables r_t^s and r_t^l measure the Federal funds rate and long-term interest rate on government bonds respectively. These capture the links between the debt-GDP dynamics, monetary policy and the term structure. The last variable, cp_t , is the growth rate of crude oil price, used to proxy changes in commodity prices. This exogenous variable captures the impact of global economic factors on the domestic macroeconomic and the fiscal outlook. Reinhart and Rogoff (2008), pp.6 document that "peaks and

¹⁵ Appendix A describes the specification of the ROVAR model and the determination of the recursive forecasts with more details.

¹⁶ This is because there is always an unique nominal rate i_t^b that satisfies (1) which it is not therefore necessary to include explicitly in the model.

troughs in commodity price cycles appear to be leading indicators of peaks and troughs in the capital flow cycle, with troughs typically resulting in multiple defaults".¹⁷ The specification of the variables in $\mathbf{x}_t$ is typical of VAR analyses of fiscal shocks and business cycle fluctuations; see for example Fatas and Mihov (2001), Canzonieri et al. (2002) and Chung and Leeper (2007).

5.2 Data and stylized facts

Fig. 2. U.S. aggregate macroeconomic (left axis) and volatility (right axis) 1969:4-2011:2. St. Dev. is standard deviation of the 1-quarter ahead forecast error from ROVAR model.

The data are quarterly for the U.S. from 1969:4 to 2011:2 and are shown in Figure 2 (left axis). Four features of these data are worth bearing in mind when interpreting the empirical analysis of the U.S. sovereign credit rating in the next two sections. The first is the so-called great macroeconomic acceleration and moderation. This is reflected in the higher growth rate of GDP, in the inflation rate and in nominal interest rates in the 1970s and early 1980s relative to the post-1985 period. The second is the great deterioration of the fiscal stance reflected in the upward trend in the debt-GDP ratio since the early 1980s and in the greater amplitude of swings in the primary deficit-GDP ratio starting from the second half of the 1980s.¹⁸ The third is the large fall of the growth rate of GDP and further deterioration of the fiscal stance since the 2007-2009 global financial crisis. The fourth feature is the large spike in oil inflation that

¹⁷Full details on the data are in Appendix B.

¹⁸Like the great moderation, the great deterioration of the fiscal stance is also typical of most of European countries, see Polito and Wickens (2011).

occurred in the first quarter of 1974, though the series appear to be stationary throughout the 1970-2011 period.

Figure 2 (right axis) reports estimates of the time-varying volatility of the variables included in the ROVAR, measured by the standard deviation of the 1-quarter ahead forecast error. The most striking feature is that over the past 40 years there have been large fluctuations not only in the volatility of U.S. macroeconomic variables (output, inflation and interest rates) but also in the volatility of fiscal and global variables. In addition, these fluctuations appear to be highly correlated. For example, the great moderation coincides with a large fall in the volatility of the rate of change in oil prices. The volatility of both output and inflation has steadily (though not sharply) increased since the late 1990s and this coincides with a gradual surge in the volatility in the inflation rate in oil prices over the same period. There is also a strong positive correlation between the volatilities of global and fiscal variables. The volatility of the fiscal variables (particularly the deficit-GDP ratio) increased during the 1970s', declined during the 1980s, then increased during the 1990s and 2000s and has picked again during the latest financial crisis. This evidence highlights the significance of fiscal and global factors in determining domestic macroeconomic volatility. It also points to the need to capture time-variation in macroeconomic volatility in the forecasting model.

6 Credit ratings with ad-hoc debt limits

We now estimate the probability of default and map this into a credit rating using ad-hoc time-invariant debt-GDP values as proxies for the default threshold. We use these results as a benchmark. They abstract from considerations about whether the debt limit reflects the ability or the willingness of the U.S. government to meet its financial obligations, or about the impact on the sovereign credit rating of time-varying debt-GDP limits. The latter is investigated in the next section.

The sovereign credit rating is determined as follows. First we obtain time-varying debt-GDP forecasts ($E_t \frac{b_{t+h}}{y_{t+h}}$) and forecast-errors standard deviations ($\sigma_{\eta,t+h}$) from the ROVAR for t ranging quarterly from 1969:4 to 2011:2 and over a 10-year horizon, i.e. $h = 1, \dots, 40$. We then use these to compute the probability of default in each quarter of the forecasting horizon. As the ad-hoc debt-GDP threshold does not change over time, effectively, we compute the probability of default in period $t + h$ from

$$p_{t+h} = \Pr \left(\frac{E_t \frac{b_{t+h}}{y_{t+h}} - \bar{y}}{\sigma_{\eta,t+h}} \leq \eta_{t+h} \right). \quad (9)$$

We have simulated the model over a wide range of debt-GDP limits, from 100 to 500 per cent, but for illustrative purposes report only the results obtained for debt-GDP limits equal to 150, 200 and 300 per cent which are sufficient

to highlight the main results.¹⁹ The corresponding hazard rates can then be calculated using equation (3) and converted into cumulative default probabilities using equation (7). Finally, the model-based measure of the credit rating is derived by mapping this probability into an initial sovereign credit rating using Table 3. As this may be too sensitive to swings in debt-GDP forecasts and uncertainty, we later revise this initial rating to provide a smoother rating which we call the final rating.

6.1 Initial credit rating

Fig. 3. Initial model-based U.S. sovereign credit rating (short and long term) under ad-hoc debt-GDP limit, 1969:4-2011:2. Actual corresponds to Moody's (2011) historical credit rating.

Figure 3 plots the initial model-based sovereign credit rating together with the historic U.S. rating as reported in Moody's (2011). Short- and long-term sovereign credit ratings are obtained by comparing the cumulative default probability profile derived from the model over horizons of 1 (top panel), 5 (middle panel) and 10 years (bottom panel) with those corresponding to columns 4, 5 and 6 in Table 3. The short-term credit rating derived from the model is Prime 1 for the whole sample period, regardless of the default threshold. This coincides

¹⁹Sovereign credit ratings obtained from other values of the debt-GDP limit within the 100-500 per cent range can be made available upon request.

with the actual rating, indicating that the model attaches a zero probability to a U.S. default within 1-year over the whole sample period. For a 5-year horizon the initial rating is lowered from Aaa to C in the first two quarters of 1974 regardless of the debt-GDP limit. A downgrade from Aaa to C also occurs following the bankruptcy of Lehman Brothers in September 2008, though its duration reduces as the debt-GDP limit increases.²⁰ Using a 10-year horizon the model gives the same downgrade in early 1974 as for the 5-year horizon, but the downgrade to C is longer-lasting (for five consecutive quarters, starting from 2008:4) during the latest financial crisis for any of the debt-GDP limits considered. The model also lowers the credit rating in 2008:2 to C, Ba3 and Aa1 if the debt-GDP limits are, respectively, 150, 200 and 300.²¹

6.2 Final credit rating

²⁰This downgrade lasts four quarters (from 2008:4 to 2009:3) when $\frac{\bar{b}}{y} = 150$ per cent, three quarters (2009:1-2009:3) when $\frac{\bar{b}}{y} = 200$ per cent and two quarters (2009:1-2009:2) with $\frac{\bar{b}}{y} = 300$ per cent.

²¹With $\frac{\bar{b}}{y} = 200$ per cent, further downgrades from Aaa to Aa1 occur in 1987:1, 1992:2, 1993:1, 1997:2 and 1997:3. With the lowest debt-GDP threshold, $\frac{\bar{b}}{y} = 150$ per cent, the model yields further downgrades from Aaa in 1970:1 (to Aa1), 1987:1 (to Aa1), in the first two quarters of 1992 (to Aa1 and Baa1 respectively), during 1992:4-1993:2 (Aa1, C and Aa1 respectively), in 1997:2 (to Aa1). In addition, the credit rating remains at Aa1 from 2010:1 onward.

Fig. 4. Final model-based U.S. sovereign credit rating (short and long term) under ad-hoc debt-GDP limit, 1969:4-2011:2. Actual corresponds to Moody’s (2011) historical credit rating.

This initial sovereign credit rating is very sensitive to fluctuations in the debt-GDP forecast and may be very volatile especially during periods of high macroeconomic instability. This is in contrast with the operating procedures of CRAs, which typically revise sovereign credit ratings twice a year rather than on a quarterly basis and place a sovereign credit rating under review in the short term, perhaps before changing it later.²² Sovereign credit ratings, at least for industrialized countries, tend to be altered gradually rather than suddenly.²³ We therefore provide a final sovereign credit rating, which is less volatile than the initial credit rating, by imposing the constraint that quarterly changes in the sovereign credit rating may occur only gradually. This is calculated each quarter from the initial rating as follows: in the first period of the sample the final credit rating is set equal to the initial credit rating; if the new initial credit rating (from the second period onwards) is the same as the previous quarter’s initial rating, the new final rating is set equal to the previous quarter’s final rating; if the new initial credit rating is higher (lower) than the previous period’s initial rating then the new final credit rating is upgraded (downgraded) by one notch.

²²This is referred to “creditwatch” by S&P, “watchlist” by Moody’s and “ratingwatch” by Fitch. Possible rating changes can be either upgrade, or downgrade, or with uncertain direction uncertain. Sovereign credit ratings also can be changed without being first under review.

²³See Sovereign Bond Rating Histories, in Appendix III of Moody’s (2011).

The final credit ratings are reported in Figure 4. We observe no change in the final short-term credit rating compared to the initial measure as this is Prime-1 throughout. With a 5-year horizon, the model would have downgraded U.S. sovereign debt from Aaa to Aa1 in 1974:1 and to Aa2 in 1974:2, regardless of the debt-GDP limit, and it would have restored the Aaa credit rating gradually over the next two quarters. The final rating further downgrades U.S. debt during the latest financial crisis. The starting date and the magnitude of the downgrade vary with the debt-GDP limit but, unlike the initial rating, in no case is the U.S. credit rating lowered below speculative grade.²⁴

Using a 10-year horizon has the effect of deepening the reduction and duration of the downgrade during the latest financial crisis. The lowest value reached is A2 in 2009:4 regardless of the debt-GDP limit. With the lowest debt-GDP limit the credit rating is lowered to A1 in 2008:1 but is restored to Aaa in the following quarter. It then reduces to A2 in 2009:4 and reaches A1 in 2010:4 where it remains until 2011:2. The same pattern is found with the intermediate and high debt-GDP limits, except that a Aaa rating is restored in 2011:2 and 2011:1. With a 150 per cent debt-GDP limit, minor downgrades occur in the mid 1980s and during the 1990s; with the lowest debt-GDP limit, the credit rating is lowered to Aa1 in 1987:1, 1992:1, 1992:3 1997:2 and 1997:3, and is Aa2 in 1992:2; and with the intermediate debt-GDP limit the credit rating is Aa1 in 1987:1, 1992:2, 1993:1, 1997:1 and 1997:2.

To summarize, the model-based final sovereign credit rating identifies two episodes over the past 40 years in which U.S. debt would have been downgraded if based on our measure: the first coincides with the oil crisis starting in October 1973; the second coincides with the period after the collapse of IndyMac Bank and Lehman Brothers. These findings are generally robust to the specification of the debt-GDP limit within the range 150-300 per cent. The extent and duration of the downgrade varies with the debt-GDP limit and the time horizon. A mild downgrade is indicated following the earlier 1990s recession and in the latest 1980s and 1990s. Clearly, the model-based measure of the U.S. sovereign credit rating is related to key events of the U.S. macroeconomic history, with downgrades typically being associated with episodes of economic recessions, swings in commodity prices and large increase in government spending. This differ from the credit ratings issued by the main CRAs. As noted in the introduction, the CRAs have given U.S. debt the highest credit rating throughout the whole 1969:4-2011:2 period. Standard & Poor only lowered this by one notch in its rating on August 5, 2011. This was about 12 quarters after the collapse of IndyMac Bank and Lehman Brothers.

²⁴With the lowest and intermediate debt-GDP limits, the credit rating is gradually lowered from Aaa in 2008:3 to A1 in 2009:3 and recovers the Aaa status by 2010:3. With the 300 per cent debt-GDP limit, the credit rating is gradually lowered from Aaa in 2008:4 to Aa2 in 2009:2 and the Aaa rating is restored by 2009:4.

7 Credit ratings with theory-based debt limits

7.1 The debt-GDP limit

The previous analysis of sovereign credit ratings is based on an ad-hoc choice of the debt-GDP limit. We now consider a limit derived from the intertemporal government budget constraint (IGBC) embedded in a microfounded DSGE model of the economy. The debt-GDP limit may be obtained either directly from the steady-state value of the IGBC, or by maximizing this with respect to tax rates, or by minimizing the steady-state IGBC with respect to expenditure.²⁵ All these notions of the maximum borrowing capacity compare the outstanding liabilities of the government with the market expectation of future saving that the government can generate by either anticipated or unanticipated fiscal policy changes. Default is deemed to occur whenever the present value of this expected saving is lower than the existing debt-GDP ratio.

The starting point for the derivation of a debt-GDP limit is the one-period GBC in equation (1). The solution to an infinite horizon DSGE model constrains the government from rolling over its liabilities forever and implies that the current period debt-GDP ratio must satisfy the intertemporal government budget constraint limit (IGBCL) every period. Denoting this level of the debt-GDP ratio as $\frac{b_t}{y_t}^{IGBCL}$, the resulting equilibrium condition is

$$\frac{b_t}{y_t}^{IGBCL} = -E_t \sum_{j=1}^{\infty} \frac{\frac{g_{t+j}}{y_{t+j}} + \frac{z_{t+j}}{y_{t+j}} - \frac{v_{t+j}}{y_{t+j}}}{\prod_{s=1}^j (1 + \rho_{t+s})}, \quad (10)$$

where g_t is government expenditures on goods and services, z_t is transfers, v_t is government revenues and ρ_t is the discount rate defined earlier. The right-hand side of equation (10) is the market's expectation of the present value of current and future primary surpluses given information up to time t , and is a measure of a government's borrowing capacity based on the market's anticipation of the future evolution of fiscal policy.

Other measures of the debt-GDP limit exploit the fact that unanticipated changes in fiscal policy may increase a government's ability to raise further debt.

²⁵There are several alternatives to using the IGBC to measure the debt-GDP limit of a government. The IGBC approach is related to Sachs (1989)'s concept of debt Laffer curve based on the notion of debt overhang, a situation where high debt levels (relative to the repayment capacity of a country) result in efficiency losses due, for example, to rescheduling negotiations of debt obligations, changes in trade patterns, or crowding out of investment. The quantification of these costs is however not straightforward. Cole and Kehoe (2000)'s model of liquidity crisis can be used to derive a debt-GDP threshold below which governments are always solvent and default never occurs. However, default above this threshold is undetermined as it is subject to whether a country can still avoid a liquidity crisis. There is also an extensive empirical literature that quantifies "safe" debt-GDP thresholds. These however vary from country to country, change over time, and crucially depend on the default history of a country (see, for example, Reinhart, Rogoff and Savastano (2003) for emerging markets and Reinhart and Rogoff (2011) for advanced economies).

Unanticipated changes in policy are unpredictable: however market participants can determine the maximum impact of any surprise change in government expenditure and tax policy if these were to occur. This logic is at the basis of three other definitions of the debt-GDP limit.

The first is obtained by adapting Aiyagari (1994)'s natural debt limit (NDL) for government policy. In a DSGE model, the NDL corresponds to the representative household's wealth and comes from the requirement that it must be feasible for the household to repay its state contingent debt in every possible state by eliminating its expenditures. When applied to fiscal policy, the NDL is obtained by setting to zero all sources of government expenditure in (10). The implied debt-GDP limit, denoted by $\frac{b_t}{y_t}^{NDL}$, then corresponds to the expected present value of current and future tax revenue and is given by

$$\frac{b_t}{y_t}^{NDL} = E_t \sum_{j=1}^{\infty} \frac{\frac{v_{t+j}}{y_{t+j}}}{\prod_{s=1}^j (1 + \rho_{t+s})}. \quad (11)$$

This debt-GDP limit precludes a government from being able to finance higher debt levels from unanticipated reductions in expenditure; unanticipated increases in tax revenues or changes in monetary policy must occur in order to increase the debt-GDP limit.

The second debt limit has been suggested recently by Davig, Leeper and Walker (2010, 2011) and Cochrane (2011). The starting point is the notion that distortionary taxation implies that there exists a set of tax rates that maximizes tax revenues. For this to happen taxes must satisfy a Laffer effect. The debt-GDP limit then emerges as the expected present value of future primary deficits under the assumption that tax revenue is maximized in each period and there will be no unanticipated changes in the conduct of government expenditure policy. Davig, Leeper and Walker (2010) refer to this as the fiscal limit (FL). This is given by

$$\frac{b_t}{y_t}^{FL} = E_t \sum_{j=1}^{\infty} \frac{\frac{d_{t+j}^{FL}}{y_{t+j}}}{\prod_{s=1}^j (1 + \rho_{t+s})}, \quad (12)$$

where $\left\{ \frac{d_{t+j}^{FL}}{y_{t+j}} = \frac{g_{t+j}}{y_{t+j}} + \frac{z_{t+j}}{y_{t+j}} - \frac{v_{t+j}^{\max}}{y_{t+j}} \right\}_{j=0}^{\infty}$ denotes the sequence of primary deficit achievable when government tax revenues are maximized. It disregards the potential impact of unanticipated changes in government expenditure. Thus Davig, Leeper and Walker's (2010) fiscal limit occurs when the government no longer has the ability to increase its borrowing capacity by increasing tax revenues. Nonetheless, it could still either change its expenditure policy or use monetary policy, or both.

A further possibility is to maximize tax revenues whilst setting government expenditure and transfers equal to zero in (12). This gives the maximum debt-

GDP limit (MDL):

$$\frac{b_t}{y_t}^{MDL} = E_t \sum_{j=1}^{\infty} \frac{j \frac{v_{t+j}^{\max}}{y_{t+j}}}{\prod_{s=1}^j (1 + \rho_{t+s})}. \quad (13)$$

When the limit in (13) is reached, a government can no longer use unanticipated fiscal policy to finance more debt and would need to resort to monetary policy.

Implementing these four measures of the debt-GDP limit presents a number of problems. For example, computation of the debt-GDP limit in equation (10) may be problematic when using historic data from countries that have run persistent deficits in the past as the debt-GDP limit implied by the projection of future deficits may be negative. This point is illustrated in the next section. The debt-GDP limits based on (11) and (13) are always positive and the maximization of tax revenue in (12) is also likely to produce a positive debt-GDP limit. The fiscal changes required to achieve the debt-GDP limits in (11), (12) and (13) may not, however, be politically feasible.²⁶

We implement the debt-GDP limits in equations (10), (11), (12) and (13) by including distortionary taxation in an otherwise standard real business cycle model with an elastic labor supply. Davig, Leeper and Walker (2010) have used a similar model to compute the debt-GDP limit for the U.S. economy, but extended to include nominal rigidities. They assume that labor and capital income are taxed at the same rate and do not consider revenue generated by the taxation of consumption. This model is closely related to the neoclassical growth model employed by Trabandt and Uhlig (2011) to estimate Laffer curves for the U.S. and for individual EU countries. We depart from Davig, Leeper and Walker (2010) in that we consider separately taxation on consumption, labor and capital income. We also depart from Trabandt and Uhlig (2011) because we derive debt-GDP limits rather than Laffer hills alone. Another difference is that we use the model to derive a quarterly time series of debt-GDP limits over the past 40 years by allowing taxes, expenditures and technological progress in the U.S. to change and to have time-varying volatility.

7.2 The model

The model is defined in real terms. We assume that households derive utility from consumption c_t and leisure $1 - n_t$, and want to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t),$$

where E_0 denotes a mathematical expectation conditioned on time 0 information; $\beta \in (0, 1)$ is the household discount factor; $u(\cdot)$ is a twice continuously

²⁶Davig, Leeper and Walker (2010) discuss the use of logistic functions to compute feasible debt-GDP limits. But this is beyond the scope of our paper.

differentiable, increasing, strictly concave utility function; and n_t denotes labor. Households face a sequence of budget constraints

$$(1 + \tau_t^c)c_t + k_t + b_t = (1 - \tau_t^n)w_t n_t + [1 + (r_t^k - \delta)(1 - \tau_t^k)]k_{t-1} + (1 + r_t^b)b_{t-1} + z_t,$$

where k_t , b_t , w_t , r_t^k , r_t^b , z_t , δ , τ_t^c , τ_t^n and τ_t^k denote physical capital, government bonds, wages, the gross real rate of return on capital, the real rate of return on bonds, government transfers, the rate of physical depreciation of capital, a tax rate on consumption, a tax rate on labor income and a tax rate on net income from capital (as in Prescott (2002)). Output is generated by a Hicks-neutral Cobb-Douglas production function

$$y_t = A_t k_t^\alpha n_t^{1-\alpha},$$

where A_t denotes technological progress and α is the income share of capital. The stock of physical capital evolves according to

$$k_t - k_{t-1} = x_t - \delta k_{t-1},$$

where x_t denotes investment. The government budget constraint (GBC) is

$$g_t + (1 + r_t^b)b_{t-1} + z_t = v_t + b_t$$

where g_t is government expenditure on goods and services and $v_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k (r_t^k - \delta) k_{t-1}$ is total tax revenue. As we treat the U.S. as a closed economy, the national income identity is

$$y_t = c_t + g_t + x_t.$$

The model excludes the possibility that a government could inflate away its debt obligations. Although this may not be an appropriate assumption for all countries, it is more suitable for the U.S. for the following reasons for this. First, the fiscal consolidation strategies to reduce budget deficits in advanced economies recently proposed by the IMF explicitly exclude inflation (seigniorage revenue) as a policy instrument, see Cottarelli (2010). Second, in the U.S., monetary policy is been delegated to the Fed which has adopted an implicit low inflation target. This leaves little scope for the Federal government to raise unanticipated seigniorage revenues through the inflation tax.²⁷ Third, historically, the contribution of inflation to the U.S. budget deficit has been negligible, see King (1995) and Woodford (1996). The real rate r_t^b captures the impact of inflation on payments to bondholders. This too has made little contribution to the U.S. budget deficit since the 1970s, see Hall and Sargent (2011).

Like Davig, Leeper and Walker (2010, 2011) and the baseline analysis of Bi (2011), the model does not incorporate a default risk premium in r_t^b . In principle,

²⁷The effects of anticipated inflation are implicitly accounted for in the ROVAR forecast of the debt-GDP ratio.

as the probability of default approaches close to unity - but not before - it may be expected that purchasers of government debt would demand a risk premium.²⁸ As this would increase the cost of borrowing close to default, it would also lower the debt-GDP limit. In practice, because according to the model the probability of sovereign default by the U.S. is close to zero, this adjustment is unnecessary. Although for the U.S. we may ignore such considerations of risk in the theoretical model of the debt-GDP limit, the debt-GDP ratio and the interest rate forecasted from the VAR are not constrained in this way. The use of ad-hoc debt-GDP limits in section 6 is, of course, free from this problem.

In stationary equilibrium, the debt-GDP ratio implied by (10) is:

$$\frac{b}{y}^{IGBCL} = \frac{1}{\rho} \left(\frac{v}{y} - \frac{g}{y} - \frac{z}{y} \right), \quad (14)$$

where $\rho = r^b - \gamma$, with $r^b = i^b - \pi$, is the output-adjusted real rate of return on government bonds. The maximum debt-GDP ratio sustainable under distortionary taxation is obtained at the peak of the Laffer hill associated with tax rates on labor and capital income alone. The consumption tax rate is held at its current level since in a standard real business cycle model the tax revenue generated by the consumption tax increases monotonically with the consumption tax rate, implying that there is no interior maximum.²⁹ For this reason Davig, Leeper and Walker (2010) exclude consumption taxation from their model on the debt limit; and Trabandt and Uhlig (2011) look only at Laffer curves generated by taxation of income from labor and capital.

Appendix C shows that assuming

$$u(c_t, 1 - n_t) = \log c_t + \psi \log(1 - n_t),$$

the first-order conditions for households and firms imply that the steady-state value of tax revenues as a proportion of GDP (the average tax rate) can be computed as:

$$\frac{v}{y} = \tau^c \chi \left(\frac{1}{\varphi^k} - 1 \right) + \tau^n (1 - \alpha) + \tau^k \alpha \left[1 - \delta \left(\frac{\beta^{-1} - 1}{1 - \tau^k} + \delta \right)^{-1} \right], \quad (15)$$

²⁸To illustrate, in order for the expected pay-off when the probability of default next period is p to be the same as when the probability is zero, the risk premium ξ must satisfy $1 + r^b = (1 + r^b + \xi)(1 - p)$ giving a risk premium of $\xi = \frac{p(1+r^b)}{1-p}$, where r^b is the risk-free return.

²⁹Similarly, inflation has no interior maximum when introduced in a general equilibrium model using a cash-in-advance constraint on consumption expenditure. For further discussion on this see Chiari, Christiano and Kehoe (1996).

where

$$\chi = \frac{(1 - \tau^N)}{\psi(1 + \tau^C)} (1 - \alpha) \quad (16)$$

$$k = \frac{\mu + (1 + \tau^c)g}{[(1 + \tau^c)\Omega + \mu\varphi]} \quad (17)$$

$$\mu = \frac{1}{\psi} (1 - \tau^n) (1 - \alpha) A \varphi^{-\alpha} \quad (18)$$

$$\Omega = A \varphi^{1-\alpha} - \delta \quad (19)$$

$$\varphi = \left[\frac{\beta^{-1} - 1 + \delta (1 - \tau^k)}{\alpha A (1 - \tau^k)} \right]^{\frac{1}{1-\alpha}}. \quad (20)$$

Substituting equation (15) into (14) gives the steady-state debt-GDP limit implied by the IGBC in (10). To implement a time-varying version of this debt-GDP limit, we employ time-varying values of the steady-state levels of the three tax rates τ^c , τ^n and τ^k , and the expenditure processes for $\frac{g}{y}$ and $\frac{z}{y}$. These steady-state values are estimated by their rolling-window means and hence form a time-series. Further detail is given in appendix D (see steps 2 - 4 in particular).

The steady-state debt-GDP limit corresponding to (11) is derived as

$$\frac{b}{y}^{NDL} = \frac{1}{\rho} \frac{v}{y} \quad (21)$$

where $\frac{v}{y}$ is still calculated from (15). The time-varying version of this debt-GDP limit is calculated using rolling-window means of the three tax rates τ^c , τ^n and τ^k . The steady-state debt-GDP limit corresponding to (12) is

$$\frac{b}{y}^{FL} = \frac{1}{\rho} \left(\frac{v^{\max}}{y} - \frac{g}{y} - \frac{z}{y} \right) \quad (22)$$

where

$$\frac{v^{\max}}{y} = \tau^c \chi \left(\frac{1}{\varphi k} - 1 \right) + \tau^{n,\max} (1 - \alpha) + \tau^{k,\max} \alpha \left[1 - \delta \left(\frac{\beta^{-1} - 1}{1 - \tau^k} + \delta \right)^{-1} \right]$$

is the maximum steady-state revenue-GDP ratio obtained by the government through distortionary taxation. This is implemented by using the time-varying steady state value of τ^c and the time-varying maximum levels of $\tau^{n,\max}$ and $\tau^{k,\max}$. The steady-state equivalent of equation (13) is obtained by setting $\frac{g}{y} = \frac{z}{y} = 0$ in (22) to give

$$\frac{b}{y}^{MDL} = \frac{1}{\rho} \frac{v^{\max}}{y}. \quad (23)$$

We note that since the debt-GDP limits are based on the stationary equilibrium solution, they would be unaffected if the model above is extended to

include nominal (price and wages) rigidities, imperfect competition in the production sector, and the maturity structure of government debt. These changes would impact on the short-term dynamics of the endogenous variables, but not on the determinants of the steady-state debt-GDP limits in equation (14), (21), (22) and (23). We also note that the analysis of debt limits is not unconnected with the literature on fiscal sustainability. The latter focuses on the intertemporal budget constraint (IGBC), the measure of the debt limit, which is based on extrapolating the anticipated fiscal policy stance. The other debt limits take account potential changes in the fiscal stance designed to increase the borrowing capacity above the levels predicted by the IGBC.

7.3 Evolution of debt-GDP limits

Fig. 5. Left panel: Components of the U.S. government budget; rolling window (10-year) means of government expenditure-GDP ratio $\frac{g_t}{y_t}$, transfers-GDP ratio $\frac{z_t}{y_t}$, actual revenue-GDP ratio $\frac{v_t}{y_t}$ and maximum revenue-GDP ratio $\frac{v_t}{y_t}^{\max}$. Right panel: evolution of debt limits. 1969:4 to 2011:2.

Quarterly time series of debt-GDP limits for the period 1969:4-2011:2 are obtained through numerical simulation of the model in section 6.2. These data are generated from steady-state values of the ratios of government expenditure to GDP and transfers to GDP, technological progress and actual tax rates using 40-period rolling window of sample means. Time-varying volatility is introduced by simulating the stochastic processes of the ratios of government expenditure to GDP and transfers to GDP and technological progress from AR(1) processes

driven by shocks with time-varying volatility using a 40-period rolling window of sample standard deviations derived from the original data. All other parameters are calibrated on an annual basis using standard values for the U.S. The Markov Chain Monte Carlo algorithm used for the simulation is described in Appendix D. As a result, in each period, we obtain a distribution of the debt-GDP limit. The differences between the four measures of the steady state debt-GDP limits depend on the evolution of their main components.

Rolling window means of the ratios of government expenditure to GDP, transfers to GDP, tax revenues to GDP and the maximum revenue to GDP are shown in the left-panel of Figure 7 for the sample period 1969:4-2011:2. While government transfers show a slight upward trend, government expenditures and both measures of the tax revenues have remained relatively stable. Higher government transfers lower the debt-GDP limits obtained from (10) and (12), but do not affect those from (11) and (13). As government expenditures, actual and maximum tax revenues are relatively stable over this sample period, we would expect the debt-GDP limits from (10) and (12) to decline over time, and those from (11) and (13) to be relatively stable. The steady-state values of the four debt-GDP limits are shown in the right-panel of Figure 7 for the period 1969:4-2011:2. The four measures give a very different picture of the likely evolution of the borrowing capacity of the U.S. economy. The highest debt-GDP limit is given by $\frac{b}{y}^{MDL}$ in which the debt limit is on average 7.5 times U.S. GDP. The next highest is $\frac{b}{y}^{NDL}$ on average 5.7 times the U.S. GDP. Both $\frac{b}{y}^{MDL}$ and $\frac{b}{y}^{NDL}$ are relatively stable over the sample period. This is consistent with the relative stability of the actual and maximum ratios of tax revenue to GDP. In contrast, the other two debt-GDP limits, $\frac{b}{y}^{FL}$ and $\frac{b}{y}^{IGBCL}$ are lower and decline over time due to the upward trend in the ratio of government transfers to GDP. These results highlight several practical issues in measuring the debt-GDP limit from the macroeconomic theory. The average debt-GDP limit obtained from the IGBC is only 0.56 and becomes negative in the last 10 quarters of the sample. The empirical results in section 5, based on the ad-hoc default thresholds, suggest that this definition of the debt-GDP limit would imply a U.S. sovereign credit rating of speculative grade (or possibly C) throughout most of the sample period and therefore is unsuitable. Davig, Leeper and Walker (2010) argue that the fiscal limit derived from (12) may overstate the true debt-GDP limit since governments would face increasing political pressures as they increase taxes in order to reach the peak of the Laffer hill.³⁰ Similar considerations apply to the debt-GDP limits implied by (11) and (13), not least because their empirical estimates appear to be even more implausible. Nevertheless, it is still of interest to compute the sovereign credit rating associated with these measures of the maximum borrowing capacity.

³⁰It is also true, however, that the same debt-GDP limit can be achieved through an appropriate fiscal policy mix including government expenditure cuts as well as increasing taxes.

Fig. 6. Standard deviations of the distribution of the debt limits for the U.S. economy, 1969:4-2011:4.

Figure 8 plots the time-varying volatilities (standard deviation) of the four debt limits. These show how uncertainty surrounding the measurement of the maximum borrowing capacity of the U.S. economy has changed over time. This volatility fluctuates markedly over time. The increase in uncertainty surrounding the borrowing capacity of the U.S. economy during the latest financial crisis is even greater. The volatilities of the four debt limits move similarly. IGBCL is more volatile than NDL because it includes the effect on the borrowing capacity of government expenditure, which is excluded from NDL. This is also why FL is more volatile than MDL. FL and MDL have the largest volatilities due to the greater uncertainty of unanticipated than anticipated tax revenues.

Fig. 7. State-dependent probability density functions (PDFs) and cumulative density functions (CDFs) of the fiscal limit $\frac{b}{y}^{FL}$ at different points of the 1969:4-2011:2 sample period.

These results indicate significant shifts and changes in the shape over time in the distributions of the four debt limits. Figure 9 shows the probability density functions (PDFs) and cumulative density functions (CDFs) of the debt limits at six different points in time.³¹ The distributions shift to the left over time, thereby lowering the average debt-GDP limit. The largest shifts occurred during the 1970s and in the latest financial crisis. Changes in dispersion of the PDFs, reflecting time-varying volatility in the shocks, are also evident, with the greatest dispersion occurring in the 1970s and since the year 2007. The CDFs measure the probability of sovereign default (i.e. the probability that the debt-GDP ratio is above the fiscal limit in equation (22)) over different values of the debt-GDP ratio. The state of the economy at the time has a clear impact on this probability. For example, throughout the whole period 1969:4-2007:1 and with a debt-GDP of 150%, the probability of default at every point has been zero, but by 2011 it reached about 5%.³²

7.4 Credit rating

³¹Similar results can be observed for the other three debt limits; these are omitted for reason of space but they are available upon request from the authors.

³²Using a microfounded model with fixed parameters, time-invariant volatilities, and calibrated using average data for 8 advanced countries, Bi (2011) finds that the average fiscal limit is close to the 150 per cent debt-GDP threshold we compute for the U.S. by the end of the sample period.

Fig. 8. Model-based U.S. sovereign credit rating for different time horizons and under the fiscal limit (FL), natural debt limit (NDL) and maximum debt limit (MDL).

We now recalculate the U.S. sovereign credit rating for the period 1969:4-2011:2 over 1, 5 and 10-year horizons based on the mean values of the FL, NDL and MDL theoretical debt-GDP limits and on the average default probability for the whole 10-year period. The results are shown in Figure 10. The corresponding letter ratings are reported in Tables 2 and 3.

Like the model-based credit ratings based on ad hoc limits, those based on theoretical limits exhibit deeper and more prolonged downgrades as the time-horizon increases and the debt limit becomes more stringent than the historic ratings. We find, first, that throughout the whole sample period over a short time horizon the credit rating is Prime-1 credit rating regardless of the choice of the debt limit (top-left panel). Second, in mid-1974, regardless of the choice of the debt limit, under any horizon longer than 5 year, the model-based credit rating is downgraded by two notches from triple-A. The duration of the downgrade is relatively short as the triple-A rating is restored by 1974:4. Third, over the 10-year horizon (bottom-left panel) the credit rating is downgraded from triple-A by 1 notch and for 1 quarter in 1992:2, 1993:1 and 1997:2. Fourth, under any horizon longer than 5 year and regardless of the choice of debt limit, during the latest financial crisis the triple-A rating is downgraded. For the 5-year horizon, when using the FL, the U.S. credit rating is downgraded from triple-A in the last quarter of 2008, it reaches its lowest level (A1) in 2009:3 and the triple-A rating is restored by 2012:3. When using the NDL and MDL the triple-A rating is still reduced by one notch in the early part of 2009. Under the 10-year horizon and the FL, the model-based credit rating is downgraded from triple-A first by one notch in 2008:2 and subsequently by 5 notches from 2008:4 to 2009:4; by

Table 2
Model-based U.S. sovereign credit rating.

Date	FL	Date	NDL	Date	MDL	Date	FL	Date	NDL	Date	MDL
Short-term (1-year ahead)						Long-term (10-year ahead)					
69:4	Aaa	69:4	Aaa	69:4	Aaa	69:4	Aaa	69:4	Aaa	69:4	Aaa
Medium-term (5-year ahead)						74:1	Aa1	74:1	Aa1	74:1	Aa1
69:4	Aaa	69:4	Aaa	69:4	Aaa	74:2	Aa2	74:2	Aa2	74:2	Aa2
74:1	Aa1	74:1	Aa1	74:1	Aa1	74:3	Aa1	74:3	Aa1	74:3	Aa1
74:2	Aa2	74:2	Aa2	74:2	Aa2	74:4	Aaa	74:4	Aaa	74:4	Aaa
74:3	Aa1	74:3	Aa1	74:3	Aa1	92:2	Aa1	08:4	Aa1	08:4	Aa1
74:4	Aaa	74:4	Aaa	74:4	Aaa	92:3	Aaa	09:1	Aa2	09:1	Aa2
08:4	Aa1	09:1	Aa1	09:1	Aa1	93:1	Aa1	09:2	Aa3	09:2	Aa3
09:1	Aa2	09:3	Aaa	09:2	Aaa	93:2	Aaa	09:3	A1	09:3	A1
09:2	Aa3					97:2	Aa1	09:4	Aa3	09:4	Aa3
09:3	A1					97:4	Aaa	10:1	Aa2	10:1	Aa2
09:4	Aa3					08:2	Aa1	10:2	Aa1	10:2	Aa1
10:1	Aa2					08:3	Aaa	10:3	Aaa	10:3	Aaa
10:2	Aa1					08:4	Aa1				
10:3	Aaa					09:1	Aa2				
						09:2	Aa3				
						09:3	A1				
						09:4	A2				
						10:1	A1				
						10:2	Aa3				
						10:3	Aa2				
						10:4	Aa1				

This table reports the model-based measure of the U.S. sovereign credit rating. The rating is calculated (i) considering the cumulative default probability over the 1, 5 and 10-year horizons and (ii) for three alternative measures of the debt limit: the fiscal limit (FL), natural debt limit (NDL) and maximum debt limit (MDL).

the end of the sample the triple-A rating is not restored. When using the NDL and MDL, the downgrades start in 2008:4 and continue to reach A1 by 2009:3 before recovering the triple-A mark in 2010:3. Fifth, when using the average default probability over the 10-year horizon (bottom-right panel and Table 3), the credit rating is lowered from triple-A only in 1974 and during the latest financial crisis.

8 Summary and conclusions

8.1 Methodology

This paper has proposed a model-based measure of a country's sovereign credit rating based solely on its ability to meet its financial obligations using fiscal policy. Such a measure is suited to a country that is strongly committed to

Table 3
Model-based U.S. sovereign credit rating.

Date	FL	Date	NDL	Date	MDL
Average over 10-year horizon					
69:4	Aaa	69:4	Aaa	69:4	Aaa
74:1	Aa1	74:1	Aa1	74:1	Aa1
74:2	Aa2	74:2	Aa2	74:2	Aa2
74:3	Aa1	74:3	Aa1	74:3	Aa1
74:4	Aaa	74:4	Aaa	74:4	Aaa
08:2	Aa1	08:4	Aa1	08:4	Aa1
08:3	Aaa	09:1	Aa2	09:1	Aa2
08:4	Aa1	09:2	Aa3	09:2	Aa3
09:1	Aa2	09:3	A1	09:3	A1
09:2	Aa3	09:4	Aa3	09:4	Aa3
09:3	A1	10:1	Aa2	10:1	Aa2
09:4	A2	10:2	Aa1	10:2	Aa1
10:1	A1	10:3	Aaa	10:3	Aaa
10:2	Aa3				
10:3	Aa2				
10:4	Aa1				
11:1	Aaa				

This table reports the model-based measure of the U.S. sovereign credit rating. The rating is calculated considering the average cumulative default probability over the 10-year horizon and (ii) for three alternative measures of the debt limit: the fiscal limit (FL), natural debt limit (NDL) and maximum debt limit (MDL).

achieving an inflation target such as most developed economies and, in particular, the United States and countries in the E.U. It is less suited to countries that might choose to inflate away their debt, a course of action that some emerging market economies might prefer. The analysis does not explain the difference between the model-based and the official credit ratings provided by the CRAs. The latter takes account of additional factors to those involved in determining fiscal savings, in particular, political considerations which may affect a country's willingness to pay, and its unwillingness to default.

The proposed measure estimates the probability that a country's debt-GDP ratio will exceed a given limit during a given time horizon and maps this into a credit rating using Moody's historic records on sovereign default probability and credit ratings. The measure requires a forecast of the debt-GDP ratio and its forecast-error variance, together with an assessment of the debt-GDP limit beyond which default is deemed to occur. The forecasts can be derived from either a structural or a reduced-form economic model. We have chosen to use a VAR as it has been found to forecast at least as well as a typical

DSGE model, and is much easier to implement. The VAR we have used has two desirable features: it includes variables that make it consistent with the government budget constraint, the key determinant of debt, and it allows for structural change in the economy through time-variation both in the parameters and in volatility. We have suggested that a simple way to estimate such a VAR, that reproduces features usually captured by more complex estimation methods, is by rolling-window estimation (i.e. a ROVAR).

How to determine the debt-GDP limit is more problematic. We have considered two possibilities: the use of ad-hoc limits and limits derived from macroeconomic theory that are based either on anticipated or on unanticipated changes in future fiscal policy. The former implies that the debt-GDP limits must satisfy a government's intertemporal budget constraint. The latter yields three weaker definitions of the debt-GDP limits, depending whether government expenditure is reduced to the minimum (NDL), or tax revenue is increased to the maximum (FL) or both (MDL). Each of these is implemented empirically by simulating a DSGE model that is calibrated to the U.S. economy.

8.2 Main findings

A number of factors affect the probability of default and hence the credit ratings. In particular, the lower the debt limit, the longer the time horizon and the greater is macroeconomic uncertainty as measured by the volatility of macroeconomic shocks, the higher is the probability of default and hence the likelihood that the model will determine a reduction of the credit rating.

During the period from 1969:4 - 2011:2 the U.S. experienced two major oil crises, six macroeconomic recessions, an unprecedented banking crisis and significant domestic fiscal reforms, yet throughout U.S. Treasury securities received the highest credit-quality rating by all of the CRAs. Our measure, which focuses solely on fiscal considerations and hence is more narrowly based than the official ratings, is more sensitive to these events as it identifies at least two clear episodes over the past 40 years in which U.S. debt would have been downgraded: the first coincides with the oil crisis starting in October 1973; the second with the period after the collapse of IndyMac Bank and Lehman Brothers. These results are generally robust to the specification of the debt-GDP limit, whether ad-hoc or theoretical. The downgrade in the early 1970s is by two notches and lasts for two to three quarters; during the latest financial crisis the extent and duration of the downgrade predicted by the model varies with the debt-GDP limit and the time horizon, but never falls as far as speculative grade.

A by-product of our analysis is the insight provided of the borrowing capacity of an economy through the estimation of theoretical debt-GDP limits. Temporary reductions in the limit are often associated with economic crises and large increases in government expenditures as in the U.S. in the early 1970s and late 2000s. They also contributed to the credit downgrades. More permanent reductions in the limit - as in the U.S. over the last 40 years due in large part to a steady increase in government transfers as a proportion of GDP - may significantly erode a government's borrowing capacity and hence jeopardize its

ability to maintain a Aaa rating indefinitely.

8.3 Assessment

One of the aims of this paper was to show how it is possible to derive sovereign ratings in a simple, transparent and relative inexpensive way. The results are very encouraging for those who wish to calculate their own credit ratings on a frequent basis.

Nonetheless, the procedure is mechanical and does not take into account several potentially important considerations which cause the markets to be either more or less fearful of sovereign default. Principal among these is the underlying political context. It may, for example, be politically infeasible to attain a debt-GDP limit due to the degree of fiscal austerity entailed. Further, defaulting is a policy instrument and it may be optimal to default before reaching the debt-GDP limit.

The analysis of debt limits has focused on the use of fiscal policy and has excluded the use of higher inflation to erode the real burden of government debt. This seems appropriate for the U.S. and for most OECD countries who adhere to inflation targeting, but may be less appropriate for countries that are willing to inflate away their nominal domestic-currency denominated liabilities which many emerging market countries may prefer. We note, however, that although higher U.S. inflation may raise its debt limit, our model-based measure downgrades for 1974 and following the collapse of Lehman occur both under ad-hoc limits and the highly implausible MDL debt limit.

9 References

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