Fabric formed concrete: physical modelling for assessment of digital form finding methods

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Abstract
Fabric formwork is a novel concrete construction method which replaces conventional prismatic moulds with lightweight, high strength sheets of fabric. The geometry of fabric formed structures is therefore dictated by the behaviour of fabric under hydrostatic loading. While there are numerous examples of digital and physical modelling of this problem, there have only been limited efforts to link the two through measurement. In this investigation, a number of small scale fabric formed beams were manufactured using both ‘free hanging’ and ‘keel mould’ methods, and the resulting forms were accurately measured with a digital 3D scanner. Computational form finding tools were also developed, enabling a comparison to be made between the predicted and build geometries. This allowed assessment of both the accuracy of the construction methods and the limitations of the form finding techniques used. The data collected provides a useful assessment of existing form finding techniques and will be used as a reference data set as these are developed further.

1 Introduction
The success of recent research and legislation in improving the operational efficiency of buildings has led to a shift from operational to embodied energy as the main contributor to whole-life energy use (Vukotic et al. 2010). Since the structure is a major contributor to total embodied energy in typical reinforced concrete buildings (Cole and Kernan 1996), there is a real opportunity for reduced environmental impact through efficient structural use of materials.

Fabric formwork offers many practical advantages; it is easily transported, creates a durable concrete finish (Orr et al. 2013) and allows elegant structural optimisation through the use of non-uniform geometry (Orr et al. 2011). Challenges include repeatability, reinforcement strategy and modelling serviceability. The technique has previously been applied to beams, trusses, columns, wall panels, foundations and shells, and the development of accurate tools to analyse and optimise these structures is an area of ongoing research (Veenendaal and Block 2012). A key challenge is the reliable and accurate prediction of the shape of the fabric mould when subject to hydrostatic pressure from wet concrete and constrained by boundary conditions imposed in the manufacturing set-up. This is a prerequisite for structural analysis and interface with building elements. The accuracy of form finding methods must be confirmed through comparison with physical measurements, which have in previous research been limited to single dimensions or 2D sections only (Orr 2012, 424-426). This paper aims to:

- Provide 3D data from physical models for the assessment of digital form finding techniques.
- Highlight practical issues associated with the construction of fabric formed concrete beams.
- Assess the suitability of digital 3D scanning as a means of measuring physical models.
- Investigate construction tolerance for fabric formwork construction and the impact on final geometry.
- Identify the limitations of an analytical form finding method based on a 2D approach.

2 Physical Modelling

2.1 Key principles and approach
Simplicity, repeatability and accuracy were the key requirements for the fabrication process in order to achieve useful results for analysis. Since the behaviour of the fabric was to be modelled computationally, minimising ambiguity of the boundary and support conditions was also essential. For this investigation, avoiding wrinkling of the fabric was important due to the significant complexity this adds to computational modelling. The fabric was therefore prestressed in the longitudinal direction.
Producing small-scale models saves manufacturing time, complexity and cost, and the deformation of the fabric under hydrostatic loading is physically identical to that of a full scale beam albeit with reduced fluid pressures and fabric forces. Although any fluid could have been used to load the fabric, concrete was chosen due to its ready availability, low cost, high density and strength when set. Optimised structural applications require reinforcement which follows the beam profile. Conventional rebar (bent to shape) has been successfully used, and there is potential for simplified construction using flexible reinforcing materials such as prestressing tendons or fibre-reinforced polymers (Orr et al. 2011).

The basic design featured a rectangular sheet of fabric fixed along its two long edges to a flat, stiff plywood board with a rectangular opening cut from its centre. The fabric drops through this opening creating a down stand beam, and the fixing points are varied to create differing beam geometries and to control the fabric prestress in both longitudinal and transverse directions. In total, four concrete casts were made, which fall into two distinct construction categories; free hanging and keel mould. Typical formwork arrangements for these are shown in Fig. 1 and Fig. 2 respectively.

<table>
<thead>
<tr>
<th>Beam label</th>
<th>Construction</th>
<th>Opening width (mm)</th>
<th>Top of concrete (mm)</th>
<th>Longitudinal prestrain</th>
<th>Transverse prestrain</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Free hanging</td>
<td>100</td>
<td>50</td>
<td>2%</td>
<td>-</td>
</tr>
<tr>
<td>K1</td>
<td>Keel mould</td>
<td>50</td>
<td>50</td>
<td>2%</td>
<td>0%</td>
</tr>
<tr>
<td>K2</td>
<td>Keel mould</td>
<td>50</td>
<td>50</td>
<td>2%</td>
<td>1%</td>
</tr>
<tr>
<td>K3</td>
<td>Keel mould with ribs</td>
<td>50</td>
<td>50</td>
<td>3%</td>
<td>1%</td>
</tr>
</tbody>
</table>

Fig. 1 Plan and sections showing formwork layout used for free hanging beam F1

Fig. 2 Plan and sections showing formwork layout used for keel mould beams
2.3 Free hanging beam F1

The simplest construction method for a fabric formed beam uses fabric freely hanging between two supporting edges. The amount of fabric drawn into the opening dictates the section depth, which was controlled by creating a waist in the fabric fixing pattern as shown in Fig. 1. A sinusoidal pattern was chosen with a waist of 25 mm per side in order to avoid sudden changes of orientation which could lead to local wrinkling of the fabric.

2.4 Keel mould beams K1, K2 and K3

A development in the construction of variable section fabric formed beams was the keel mould. An additional piece of rigid formwork (the keel) allows the fabric to be pulled downwards creating a taller section with straighter sides, as shown in Fig. 2. The beam depth is controlled precisely by the shape of the keel which in this case was parabolic with a maximum depth of 60 mm. A single sheet of fabric was used which is drawn beneath the keel. Spacers between the plywood allowed the fabric to slide freely, and all edges were sanded smooth to further reduce friction.

Using a keel mould allows biaxial prestressing of the fabric along the entire length of the beam. Beam K1 was constructed with zero transverse prestrain, meaning that the fixing pattern was dictated by the unstressed width of the fabric, assuming a straight line in the hydrostatic region. The transverse prestrain could be controlled by shifting this fixing pattern relative to the opening. This was increased to 1% for beam K2 in order to investigate the effect on final geometry.

In order to create a more complex modelling scenario, triangular ribs were incorporated in Beam K3. Five ribs per side were cut out of the main plywood formwork, with lengths proportional to the depth of the beam in each location. It was found that a significant longitudinal prestrain of 3% was required to fully eliminate wrinkling. Due to the increased width of the fabric, additional fixing points were required along the base of the keel.

![Fig. 3 Formwork for (left) and finished (right) beam K2](image)

2.2 Fabric

2.2.1 Selection

Previous full-scale applications of fabric formwork have used polyester or polypropylene fabrics, often geotextiles. These are affordable, widely available and have the correct porosity to support the concrete while allowing excess water to bleed through, with advantages for surface finish and durability (Orr et al. 2013). Small scale models feature smaller hydrostatic pressures and fabric spans, meaning that the tension is significantly lower than would be found in a full scale beam. It was therefore decided that a more compliant fabric would be required in order to achieve measurable strains. The fabric chosen is woven from nylon fibres and is used in the construction of hot air balloons. It is highly resistant to tearing and features a silicone-elastomer coating which reduces friction. This is particularly important since it informs the assumption of zero friction between the fabric and plywood.

2.2.2 Fixing method

Several methods for fixing the fabric to the plywood were considered, including the use of staples, pins, small nails and adhesives. Nails were preferred due to the high level of precision possible as well as strength. One issue to be resolved was that of slip, caused by elongation of the perforation in the fabric upon application of force. This was resolved by wrapping the fabric around a length of 4 mm diameter wire rope and fixing the nail through two points, thus reducing the stress on each perforation.
The fixing process began with the insertion of the nails at evenly spaced points along the fabric edge. Corresponding fixing points were then measured and marked out on the plywood using a braddle to create a small hole, allowing a quick and precise insertion of each nail. Experimentation with fabric orientation revealed that aligning the warp and fill directions in the transverse and longitudinal directions respectively was preferential for eliminating slack regions.

2.2.3 Fabric testing
The mechanical properties of woven fabrics are non-linear, time dependant and anisotropic. Typical fabrics, including that used in this investigation, feature two perpendicular weave directions known as warp and fill (or weft). The stiffness is higher in the warp direction since these fibres are held straight during the weaving process. The form finding method described in Section 3 of this paper models the fabric in one dimension only (the warp direction in this case), and allows for extension of the fabric caused by tensile forces. Tensile tests were therefore carried out on six fabric samples in order to determine the stress-strain relationship in each direction. Each 200 mm long and 50 mm wide sample was held in place using rubber-lined clamps to eliminate slippage. Fig. 4 shows the results obtained along with the polynomial curve fit which was later used in form finding. Although strains as high as 27% were achieved during testing, only behaviour at the low strains relevant for this application are shown.

Fig. 4  Fabric tensile test set-up (left) and results with polynomial curve fit (right)

2.5 Measurement and digitisation of geometry

2.5.1 Scanning
After construction, each cast was scanned using an Artec 3D 'Eva' digital scanner (Artec Group 2015). This device projects a structured image onto a surface, and calculates 3D topography by analysing the distortions of the reflected image. Data from multiple scans are combined to create a single digital 3D model, providing complete geometrical data for further analysis. An example is shown in Fig. 5.

Fig. 5  Digital 3D model of beam K3 after scanning

2.5.2 Scan Accuracy
The accuracy of the scans was investigated through comparison with physical measurements. These were taken from the plywood formed region of the beams, where the geometry is uniform and surfaces
are smooth. The scanner specifications quote an accuracy of 0.1 mm per point and 0.03% over a 1000 mm distance.

Width measurements were made using callipers of 0.01 mm precision at seven points along each beam. These were then compared with samples taken in the same location from the 3D scans. The results can be seen in Fig. 6. The average error was 0.06 mm (showing a slight tendency for the scanned widths to be larger than those measured) with a standard deviation of 0.15 mm. This is within the expected accuracy based on the specifications given. Both sets of measurements showed similar levels of variation, suggesting a similar accuracy. It is interesting to note the variation in width along each beam as an indicator of construction tolerances, in this case showing an average variation of 0.73 mm using formwork constructed as parallel.

Fig. 6 Comparison of physically measured and scanned cast width

All the casts were constructed using the same shuttering boards, and hence have a similar length. Since the length could not be physically measured with callipers, a steel rule was used to confirm a length of 630 mm in every case to an estimated accuracy of ± 0.5 mm. This was compared to the length measured by the scan, taken as the average from nine sample points. The maximum discrepancy was 0.55 mm (beam K1), but this is within the expected range considering both the scan and steel rule accuracy. The scans were therefore confirmed as being sufficiently accurate for the investigations being carried out. In a full-scale application the measured inaccuracies would be proportionally less significant.

2.5.3 Registration

After scanning, the digital meshes were imported into the 3D modelling software Rhino3D. These were then aligned with the global coordinate axes for comparison with predicted geometry in a process known as registration. This task is non-trivial since there are no well-defined straight edges or flat surfaces in the scanned object, and the precision of the construction is unknown.

In order to ensure accuracy and repeatability, an automated process for scan registration was developed utilising the Galapagos tool within Grasshopper, a parametric modelling plug-in for Rhino3D. This uses a genetic algorithm to optimise a fitness function (single value) based on any number of numeric input variables. In this case, the fitness value used was the sum of distances between the mesh and each of a set of defined points representing a perfect fit. The optimal solution is therefore one which minimises this value, through manipulation of the six input variables representing translation and rotation about each axis.

3 Digital form finding method

3.1 Theory & approach

Computational predictions of the shape of the fabric mould under hydrostatic loading were made by adapting a 2D sectional method previously developed at the University of Bath (Foster 2010). For each beam, 60 sections were analysed from which a full 3D surface was interpolated within Rhino3D. This is a first step towards the development of fully 3D form finding methods in future work.

The shape of the fabric is found by stepping from the opening edge in small segments of equal length $\delta L$, using a predetermined start angle $\theta_0$. Since all hydrostatic loads are normal to the fabric and
friction between the fabric and plywood is ignored, the tension is uniform along the entire section. The change in angle at each node can be calculated by equilibrium in the direction normal to the fabric as in Equation (1). In this way, the shape of the fabric can be fully traced using Equation (2) and the parameters in Fig. 7 (left).

\[ \delta \theta_n = \frac{\rho g z_n \delta L}{T} \] (1)

\[(x_n, y_n) = \left( \delta L \sum_{i=1}^{n} \cos \left( \theta_0 - \sum_{j=1}^{i-1} \delta \theta_j \right), \delta L \sum_{i=1}^{n} \sin \left( \theta_0 - \sum_{j=1}^{i-1} \delta \theta_j \right) \right) \] (2)

For a given formwork geometry, fluid height and fluid density (2400 kg/m³), the shape of the fabric is defined by the tension T, the start angle \( \theta_0 \) and the fabric length L as described in Fig. 7 (centre and right). The basic iterative procedure is shown schematically in Fig. 8. This extends the approach used in existing research, which does not allow for the deformation of the fabric and takes the depth of the beam as a known value. The algorithms for free hanging and keel mould beams followed a similar approach, albeit with different criteria for the iterative steps reflecting the specific geometry of each construction method. These were developed in the C# programming language.

\[ \text{Concrete level} \]

\[ \begin{array}{c}
(x, y) \\
(\delta L) \\
(\delta \theta) \\
\theta_0 \\
\end{array} \]

\[ \begin{array}{c}
(x_n, y_n) \\
T \\
\rho g z \delta L \\
\delta \theta_n \end{array} \]

\[ \text{Edge of opening} \]

\[ \begin{array}{c}
\theta_0 \\
\text{decreasing} \\
\text{start angle} \theta_0 \end{array} \]

\[ \text{decreasing} \]

\[ \text{fabric tension T} \]

Fig. 7 Discretised model for form finding (left), effect of modifying the start angle (centre) and effect of modifying the fabric tension (right)

\[ \begin{array}{c}
\text{START} \\
\text{Estimate initial values of } \theta_0 \text{ and } T \text{ based on } \delta L \text{ a simplified geometrical model} \\
\text{Calculate the expected length of the fabric in the hydrostatic region } L \text{ based on the} \\
\text{formwork geometry and allowing for tensile strains} \\
\text{Step from edge of opening along total hydrostatic length } L \text{ (Equation (2))} \\
\text{Subtract error from } \theta_0 \text{ to find new value} \nonumber
\text{no} \Rightarrow \text{Check rotation of section is within tolerance} \nonumber
\text{yes} \Rightarrow \text{Recalculate } T \text{ using linear interpolation} \\
\text{no} \Rightarrow \text{Check end coordinates are within tolerance} \nonumber
\text{yes} \Rightarrow \text{END} \\
\end{array} \]

Fig. 8 Iterative procedure for form finding of each 2D section

4 Results & discussion

4.1 Comparison of predicted and built geometry

Fig. 9 shows the minimum distance between the predicted and built geometry, with positive values indicating that the build geometry was larger than that predicted. The form finding input parameters for geometry, material properties and hydrostatic loading were based on design values. These were not adjusted to achieve a best fit solution, although this could potentially be automated using an optimisation technique in future work.
4.2 Sources of error and effect on geometry

There are sources of error inherent in the each of the variables affecting the geometry of the casts, as well as in the scanning itself. Whilst simplistic in its assumptions, the computational model developed can give an insight into the sensitivity of the final geometry to each error source and hence identify the most important parameters affecting the final construction tolerance. Table 2 shows the results from an investigation where a number of input parameters were varied from their assumed values and the resulting maximum displacements of the predicted shape were recorded. It can be seen that for both types of construction, the error in the fixing location of the fabric has the greatest potential to affect the geometry. This is also a parameter with a significant amount of uncertainty due to the accumulation of error from multiple processes, including the measurement and marking of the fabric, insertion of the nails into the fabric, measurement and marking of fixing locations and slip at the fixing points. Apart from this, the only other significant error source is the location of the plywood itself, including the opening edges and the width and depth of the keel where applicable. The magnitude of the hydrostatic loading and elongation of the fabric were shown to have a much smaller impact on overall geometry. It is possible therefore that modelling the fabric strain precisely from test data may have been unnecessary, since a simplified linear elastic model would have been sufficiently accurate.

Table 2  Comparative sensitivity of final geometry to error sources

<table>
<thead>
<tr>
<th>Error source</th>
<th>Estimated uncertainty</th>
<th>Maximum effect on final geometry (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>F1</td>
</tr>
<tr>
<td>Fixing position of fabric (per side)</td>
<td>+3 mm/6.56</td>
<td>-2.56</td>
</tr>
<tr>
<td></td>
<td>-3 mm/6.36</td>
<td>+2.00</td>
</tr>
<tr>
<td>Location of plywood edges</td>
<td>±2 mm/2.00</td>
<td>±2.00</td>
</tr>
<tr>
<td>Depth of concrete</td>
<td>±5 mm/0.23</td>
<td>±0.10</td>
</tr>
<tr>
<td>Concrete density</td>
<td>±100 kg/m$^3$/0.10</td>
<td>±0.10</td>
</tr>
<tr>
<td>Fabric strain</td>
<td>±5%/0.22</td>
<td>±0.22</td>
</tr>
</tbody>
</table>

For all beams the built section was predominantly larger than predicted. It is likely that this was a partly a result of slippage at the fixing points, causing more fabric to be drawn into the opening than expected. This hypothesis is supported by the fact that the highest error of this type was observed in beam K2 where the large transverse prestrain exerts a higher force on the nails and perforations. The geometry has also been shown to be particularly sensitive to inaccuracies of this nature. It is also possible that the opening was wider than the design value. Using callipers this was found to be the case for beam F1 by between 0.6 mm and 1.6 mm. The opening width for all other beams was within 0.4 mm of that assumed and is therefore unlikely to have affected the geometry significantly.
Since each beam is doubly symmetric, any variation between quadrants must be attributed to construction. This is particularly visible for keel mould beams K1 and K2. It could be that friction acting on the fabric beneath the keel resulted in differing tensions on either side of the beam, or that the keel was not aligned precisely centrally.

### 4.3 Limitations of form finding approach

The form finding method used models the fabric in 2D sections, where the hydrostatic pressure on the fabric is resisted by the product of tension and curvature in one orientation only. In reality there is an additional resistance from the longitudinal curvature and tension, and it would be expected that the accuracy of the method reduces in regions with double curvature.

The clearest example of this are the ribs of beam K3. From inspection of the model we can see that the longitudinal exceeds the transverse curvature in this region. By ignoring the large contribution this makes to support the fluid, the model gives a wildly inaccurate prediction of geometry, assuming a much deeper section than was observed. The central region of beam F1 also has a potentially significant curvature in the longitudinal direction which is greater towards the middle of the section. This may explain why the measured geometry was smaller than predicted in the centre of this beam.

Beam F1 showed an error at each end where the constructed beam was larger than predicted. Since the fabric was only fixed along the long edges, there was a tendency for it to dip inwards at each end causing more fabric to be drawn into the opening. This behaviour cannot be captured by a 2D modelling approach, but could potentially be mitigated by fixing the fabric along all four edges.

### 5 Conclusions & further work

In this investigation four small scale fabric formed casts were constructed using methods which potentially have practical applications in the creation of optimised concrete beams. The casts were digitally scanned to provide 3D measurement data, the accuracy of which was confirmed using physical measurements. Computational form finding methods were developed based on a 2D approach, enabling a comparison between predicted and built geometry to be made. This highlighted both the limitations of the form finding method and the sources of inaccuracy in the manufacturing technique. The form finding method was shown to be reliable only where the fabric is predominantly singly-curved, as would be expected from the assumptions involved. The geometric precision of the fabric boundary conditions was found to be critical for control of the beam geometry.

The next stage of this research will extend the form finding techniques into three dimensions, aiming to capture the observed fabric behaviour more fully. The data collected will provide an essential resource for ongoing assessment and development of these techniques, with the aim of developing a tool which enables confident design of efficient concrete structures using fabric formwork.

### References

Foster, R. 2010. "Form Finding And Analysis Of Fabric Formed Concrete Beams." MEng, Department of Architecture & Civil Engineering, The University of Bath.