Optimisation methodology to uncover robust low energy designs that accounts for occupant behaviour or other unknowns

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Abstract

The use of software to aid in the design of buildings or to show compliance is now commonplace. This has led several authors to investigate the potential for using such software to automatically optimise a design, or to generate a variety of near-optimal designs. One area where this approach has been found useful is in minimising annual energy demand. It is known that any estimate of demand will depend not only on the architecture and constructions used, but on the preferences and behaviours of the occupants. This suggests which design is truly optimal will also depend on occupant behaviour. In this paper optimisation is carried out for an array of different occupant behaviours based on real records. It is found that the resultant designs are more robust in terms of predicted heating energy use and overheating than when only a single behaviour is considered. It is recommended that in future all such optimisations are made using a realistic spectrum of behaviours, and that the approach is expanded to include other elements of design that might show variance during construction, for example, U-values and air tightness. This, it is hoped, will reduce some of the risks of designing and asking people to occupy very low energy buildings. Importantly, it is found that the near-optimal building designs found under variable occupancy present different characteristics than when only a single statement of occupancy is used. Being cognisant of this reduces the potential for inappropriate designs to be created that rely on a serendipitous arrangement of design and occupancy parameters that might not be met on site or by the occupants.

Keywords: Optimisation, CEES, low energy buildings, energy, occupants, behaviour

1 Introduction

Improvements in building standards and codes have contributed to the creation of new design philosophies that require, at least in theory, much reduced levels of heating and cooling to maintain a comfortable internal environment. However, as can be expected, the final, monitored, thermal environment and energy consumption will to a large extent depend upon the behaviour of the occupants. There has been little use of the limited data concerning actual energy habits and routines observed in the home; therefore, it is rare that modelling or optimisation is completed under a range of occupant behavioural scenarios.

Several of the most popular approaches to low-energy design, including Passivhaus, gain much of their space heating requirement from incidental gains, require very low levels of uncontrolled infiltration, and have low capacity heating/cooling systems. This suggests that such designs might be more sensitive than older buildings to the behaviour of the occupants, and leads to concern over their applicability to a wider audience, including their use in social housing.

It is known from the CEPHEUS study [1], among others, that the in-use energy demand of low-energy housing is indeed sensitive to occupant behaviour, with demand differing by more than a factor of three (see Figure 1), yet even in these buildings it is unusual for designers to model with more than one set of occupant behaviours. At least within academia this mono-behavioural paradigm would appear to be changing, with some suggesting a move to a more human-centric view of modelling with a large number

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of simulations being run so as to explore the sensitivities of a design to the demands, desires and vagaries of occupants [2]. As Figure 2 shows, such simulations have proved surprisingly accurate at matching the distribution of energy use found in collections of real low-energy buildings.

The potential for using optimisation within the modelling environment has a long history in building science; as early as 1956 Speyer investigated the optimal use and storage of solar energy [3]. Jurovics presented a method of optimising energy-efficient buildings in [4]. More specific works are found in the literature, concentrating on energy systems or constructions, such as the work of Bloomfield and Fisk on optimising the heating plant of a building [5], Michelson’s multivariate optimisation of solar water heating using direct methods [6], and Marks’ multi-objective optimisation of the building envelope [7]. A more holistic approach is found in more recent publications, such as the work of Coley and Schukart [8], Peippo et. al. [9] or Magnier and Haghighat [10]; these take into account many parameters involved in building design including architectural characteristics and those related to energy systems.

This work attempts to meld human-centric modelling with optimisation to create designs that are not only low-energy and comfortable given a single representation of behaviour, but low-energy and comfortable when presented with a range of typical behaviours found within a society.

The predicted energy use and levels of comfort found from this multi-run, human-centric, modelling is used as the objective function within an optimisation algorithm to locate within the space of possible designs buildings that are simultaneously low-energy and comfortable for a wide range of occupant behaviours. This has required the creation of a new evolutionary algorithm capable of producing results in a reasonable time on a desk-top computer of a form that might be used by a practising engineer or architect.

This paper starts with a background section in which the methods used are explained, including: robust optimisation, evolutionary algorithms, the changing environments evolutionary algorithm, and the creation of realistic behavioural patterns using Markov chains. Following this, the chosen optimisation problem is described. The paper finishes with the presentation and discussion of results and conclusions. An appendix describing the evolutionary algorithm in detail is also included.

2 Background

2.1 Robust optimisation

Numeric optimisation is used to find the best combination of parameters that solve a given problem. Optimisation is often applied to find the minimum or maximum of a function termed the objective, or cost function. This function represents a model of the real system and will therefore be subject to uncertainties and inaccuracies. Robust optimisation implies finding the optimum of a given function subjected to such uncertainties.

The uncertainties present in an optimisation problem were classified by Beyer and Sendhoff [12] as:
A) Changing environmental and operating conditions. The objective value is not dependent on the decision variables included in the model solely; it also depends on a set of parameters that are not decision variables and cannot be determined, but have a substantial influence on the results. For example, if the objective function is the deflection of a telephone steel post to be minimised, and the decision variables represent the geometry of the section of the pole, variations in the mechanical properties of the steel may affect the deflection of the pole for a given wind load, even if the geometry is the same.

B) Production tolerances and actuator imprecisions. The decision variables chosen in the optimisation cannot always be achieved with enough accuracy in reality due to workmanship or other issues. In the example of the pole, one of the dimensions of the section of the pole might not be accurately achieved due to irregularities in the manufacturing process.

C) Uncertainties in the system output. The objective function can have mistaken outputs due to inaccuracy in the mathematical models or the measuring devices involved. For the previous example, misleading results from the software used to calculate the wind loads affecting the pole will generate mistaken deflection estimates for a given wind speed.

D) Feasibility uncertainties. These uncertainties are applied to the constraints and not the objective function, resulting in an altered decision space. These uncertainties represent the variation of the boundaries that define the range of feasible solutions. In our example, there are only standard diameters available for the pole.

All four of these uncertainties are present in most real-world optimisations to some extent and many authors have considered them when applying optimisation (see Beyer’s and Sendhoff’s review).

Figure 2 - Uncertainties in an optimisation problem (adapted from Beyer and Sendhoff (2007) [12]).

Many optimisation strategies are found in the literature; Evolutionary Algorithms (EA) have gained a substantial following and can be found in a large number of publications including those concerning the built environment [10, 13-20]. EAs do not require knowledge of the global form of the objective function
as they belong to the group of direct methods: meaning that they only require the value of the objective function at the points it is evaluated during the search.

EAs are natural candidates for robust optimisation, because their internal operators rely substantially on redundancy, which naturally averages the effect of uncertainties. For a review on the use of EAs in robust optimisation, see [21].

The problem that will be studied in this paper will take into account uncertainties of type A -under Beyer and Sendhoff’s classification- specifically, occupants’ behaviour, but could be adapted to cover the other categories, when information about the probability distribution of those uncertainties is available.

2.2 Evolutionary Algorithms

Evolutionary Algorithms (EAs) are optimisation techniques that mimic the principles of natural evolution to find the optima of a given function. EAs are direct stochastic methods that do not require knowledge about the function to be optimised, i.e. they can be used for black-box type problems.

EAs are population based algorithms, which means that several solutions are evaluated in each iteration of the optimisation. Each population produces a new population by means of crossover and/or mutation, these being mathematical operators that have much in common with their organic equivalents: crossover creates new individuals (solutions) from two or more parents, whereas mutation creates random modifications of an individual.

To allow such algorithms to move the population towards optimal solutions a selection mechanism is required. This operator allows strong individuals to be retained (survive) over weak ones.

The EA used in this paper has been created after consideration of the question at hand, namely the impact of occupant behaviour on optimised building design, but it is a general method and could be used in other problems where a solution has to be able to “survive” in different environments. Its main differences from most EA’s are that it uses only mutation and not crossover; and that the number of individuals that survive each generation can vary between populations. For general information about EAs, see [22]. The new EA created for this work is outlined in the following section, with a more detailed description in the appendix.

2.3 The Changing Environments Evolutionary Strategy

The problem explored requires solutions that are robust over different environments thereby generating robust solutions for uncertainties of type A. To make this possible the EA evaluates the population under a different environmental condition at each generation, with all members of the population being subjected to the same environment. The variety of different environments used being drawn from the set of known possible environments. This approach has been called the Changing Environment Evolutionary Strategy (CEES). A detailed explanation about the CEES can be found in the appendix; however, the main characteristics are explained in the following.

Within the CEES, a specific single environment (for example a set of occupant behaviours including occupancy density; times of occupancy; use of electrical items within the building) is selected for all members of the population to be evaluated against; this environment is changed between generations. Solutions that show a poor performance for any environment, do not survive to future generations and solutions that do survive are therefore tested against a large variety of scenarios.

Some EAs use a tolerance measurement or the observation of a slowing rate in the improvement between generations as the termination criterion [23]; the CEES however, continues until a fixed number of generations (and therefore environments) have been assessed, to ensure the population is evaluated under a wide range of environmental conditions during the run.

The CEES has no crossover operator in this work, but it could be implemented to accelerate convergence (see any text on evolutionary computation, for example [24]). However, this could lead to premature convergence, and therefore the creation of a solution that has not been tested for a variety of scenarios.

The strength of the mutation operator in the CEES automatically declines as the run progresses. This constantly reduces the size of the space being searched and promotes the discovery of local optima. This mutation mechanism was used by Michalewicz and Janilow [25], and performs well when carrying out optimisations with a fixed number of generations.

The selection operator is deterministic: any solution that is found to be outside the range of allowable fitness is excluded. The acceptable range is found for each generation by taking a fixed percentage of the range of fitness of all solutions at that specific time; only the solutions that lie within this acceptable interval will survive to the next population.
The number of generations needed to achieve convergency was tested on a trial run. Once the the maximum number of iterations was known, the number of scenarios was chosen, applying the rule:

\[ \text{scenarios} = \frac{\text{Max. generations}}{4}. \]  

(1)

The factor to calculate the number of scenarios (4 in this case) is the way of determining the robustness of the solutions; the higher that value, the more likely it is that the final individuals had gone through a given scenario in the last runs.

This means that the uncertainty – in this case occupants’ profile- that can take an infinite number of configurations, was sampled to a finite set of profile ensuring that the distribution was properly represented (having more average than extreme profiles as indicated by the probability distribution, see Fig. 1).

The CEES has allowed the authors to compare the differences between building designs that are intended to perform well under a range of different occupants’ behaviours, and designs optimised solely for a single, specific, occupant behaviour.

2.4 Occupant behaviour as environmental parameters for building simulation

As previously discussed, the results obtained from single runs of typical building models potentially contain unexamined uncertainties due to only considering one representation of occupant behaviour. Here we optimise allowing for multiple representations of occupants including changes to occupancy and incidental gains from electrical items. These incidental gains are substantial when compared to the annual space conditioning gains required by low energy building and therefore need to be accurately accounted for.

Occupancy and appliance-use profiles were generated for this work by a third-party tool [26], which uses a first-order Markov-Chain technique, and details of occupancy and energy use at a ten-minute resolution [27]. The Markov-Chain technique is an established stochastic method for generating data [28]. A first-order Markov-chain means that the state of the system during the next period is dependent only on the current state, not on any preceding states. For example, the probability of whether an occupant is in the building is correlated only with whether they were present during the last ten minutes, not whether they were present during the ten minutes before that.

The representation of occupancy in the model provides the primary method for creating synthetic occupancy and electricity demand data with appropriate aggregate daily profiles. As an example, from Table 1 we see that if a two-person household is unoccupied (the number of active occupants = 0) at 21:00 then there is an 89.2% chance the house will still be unoccupied at 21:10.

Table 1
Example transition probability matrix for a two-person household on weekdays, including activity probability. Data from Richardson et al. (2009) [11].

<table>
<thead>
<tr>
<th>Current state (at 21:00)</th>
<th>Number of active occupants</th>
<th>Next state (at 21:10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0.892</td>
<td>0.082</td>
</tr>
<tr>
<td>1</td>
<td>0.038</td>
<td>0.878</td>
</tr>
<tr>
<td>2</td>
<td>0.003</td>
<td>0.043</td>
</tr>
</tbody>
</table>

To create the high-resolution appliance-use profiles Richardson et al. use the household journal data to define ‘activity profiles’, where a particular listed activity has associated appliances with a certain chance of a switch-on event occurring. Combined with details of the mean power use and cycle lengths for each appliance (from various sources, see [29]), load profiles are thereby stochastically generated. An example electric load and occupancy profile can be seen in Figure 3; these profiles are useful since they have been shown [26] to display similar statistical characteristics to the measured UK appliance-use and occupancy patterns used to create them, and hence they can be used to represent a large range of likely UK behaviours.
3 Methodology

Optimisation of a building model can be used to calculate the best architectural/constructional parameters that optimise a given general objective function (e.g. heating, emissions, comfort levels, etc.). Optimisation can also be used for more specific problems when the design is in a more developed stage and the variables are more specific (for example studying only window shape and size to optimise lighting quality as in [18]).

As previously mentioned, occupants’ behaviour can have substantial impact on building’s performance. If a building design is optimised with the aim of targeting low-energy consumption under single specific conditions of use, the solution could be misleading, as different occupants may render the solutions sub-optimal, or non-acceptable. One example being a dwelling with a specific, targeted, thermal mass that relies on the thermal capacity of partitions to store precisely the heat that is generated from electric appliances during the day.

The occupants’ behaviours are considered as the uncertainties of this work, being them considered as environmental parameters i.e. uncertainties of type A. This work allowed the evaluation of the differences between two optimal buildings obtained in two different manners:

1) Fixed environmental variables: the optimisation is done for a single given statement of occupant behaviour.
2) Variable environmental variables to generate robust solutions: the algorithm will only consider solutions that are near-optimal under a range of occupant behaviours.

The CEES was applied to a building design problem to obtain robust solutions (case 2) and a generic EA to obtain optimal solutions for one behavioural pattern (case 1). As the aim of this work is to compare the impact of considering only one occupant behaviour on the optimisation, a general building optimisation problem was used. The methodology could be used for more specific problems in which the building is at a later stage of design.

The building that was optimised is constrained to maintain floor area and height (but not aspect ratio); apart from those restrictions, all variables may change via the decision space described in Table 2.

The optimisation generates solutions that provide general architectural rules to designing low-energy buildings that are robust against changes to occupant behaviour. The approach can therefore be seen as an attempt to reduce risk. As cast, it is aimed at optimising an early-stage design where few decisions have been made, and flexibility still exists.

The conductivity and capacity of the internal partitions of the building have been included in the problem as these parameters are understood to have an important role in low-energy buildings [30]. The real variables can take any value in the interval defined by the lower bound and the upper bound. The wall construction can only be A, B, C or D from Table 3. The insulation used has a thermal conductivity of 0.05 W/(mK).
EnergyPlus was used to evaluate the different solutions. The base model is a two storey dwelling located in London that has been modelled as a single zone. An ideal heating system with a set-point of 23°C was used. Ventilation occurred when the house reached a temperature in excess of 27°C and the occupants were present, thus simulating the opening of windows by the tenants (this, like the set point, is itself a behaviour and could have been included within the changing environment, however there was limited data on this behaviour so it was fixed). To simulate ventilation, the equations from section 16 of [31] were used; these equations take into account the air flow driven by wind and by thermal forces due to the stack effect. The effective opening area was related to the size of the windows selected in each solution.

Ventilation motivated to improve air quality (such opening windows in the morning) has not been separately modelled. This ventilation is hard to model as it is highly dependent on the habits of tenants, and little data exist. More complete modelling of this kind of ventilation is suggested for further work as one of the environmental parameters (see [32] or [33] for an elaborated model of windows opening).

The number of non-comfortable hours that a building design will have during its operation is a crucial factor for the success of low-energy buildings. Buildings that overheat, or operate at a range of temperatures that the occupants find unpleasant will be considered a failure in design; and will worsen the image of the designer and low-energy design architecture. Designers have to make sure that a building design will perform well in any scenario.

To evaluate comfort, the ASHRAE Standard 55 [31] was used. With the building simulator reporting the number of non-comfortable hours that have been experienced in each solution when the building was occupied. Standard 55 could be considered too strict for domestic dwellings, so a solution is allowed to have up to ten per cent of hours (i.e. 876) reaching non-comfortable conditions before it is considered non-viable (to see the definition of comfortable hours under Standard 55 see [31]).

The objective function to be minimised is the heating demand over a year in kWh/m² (no cooling has been considered). When a solution presents more than 876 non-comfortable hours the solution is considered non-comfortable and therefore not acceptable. To discard solutions that are considered non-comfortable, the heating demand is multiplied by a penalty factor of a hundred. That way, the objective function of a non-comfortable low-heating solution will always be worse than an average comfortable solution. A penalty factor of a hundred is sufficient to make the objective value of a non-valid solution larger than any other valid solution in the search space. If the heating demand turns to be zero for one solution, and this solution is non-comfortable, the solution will be assign an objective value of 100. With such a penalty factor, the solutions that violate the condition of non-comfortable hours are unlikely to be preserved in the next generation.
4 Results and Discussion

The optimisation took 24 hours to run when using a population of 100 individuals and 200 generations (the machine was a desktop Intel core 2 duo at 2.93 GHz, single threaded code). This should be compared to 12 hours if only single occupant behaviour was considered. The differences on time is because with the CEES, the solutions that survive from one generation to the next still need to be evaluated in opposition with the “static” algorithm.

The algorithms were run for 200 generations and each converged to a single solution: The two final solutions obtained can be seen in Table 4.

Table 4.
Final solutions obtained with single occupant behaviour and with CEES. IP: Internal partitions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Type</th>
<th>Single occupancy</th>
<th>CEES</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infiltration</td>
<td>Real</td>
<td>0.030</td>
<td>0.030</td>
<td>ach</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>Real</td>
<td>0.67</td>
<td>0.47</td>
<td>m/m</td>
</tr>
<tr>
<td>Fenestration, North</td>
<td>Real</td>
<td>0.0500</td>
<td>0.0500</td>
<td>m²/m²</td>
</tr>
<tr>
<td>Fenestration, South</td>
<td>Real</td>
<td>0.0582</td>
<td>0.0575</td>
<td>m²/m²</td>
</tr>
<tr>
<td>Fenestration, East</td>
<td>Real</td>
<td>0.0500</td>
<td>0.0500</td>
<td>m²/m²</td>
</tr>
<tr>
<td>Fenestration, West</td>
<td>Real</td>
<td>0.0500</td>
<td>0.0500</td>
<td>m²/m²</td>
</tr>
<tr>
<td>Wall Type</td>
<td>Symbolic</td>
<td>Construction A</td>
<td>Construction C</td>
<td></td>
</tr>
<tr>
<td>Insulation</td>
<td>Real</td>
<td>499.6</td>
<td>482.6</td>
<td>mm</td>
</tr>
<tr>
<td>Conductivity of IP</td>
<td>Real</td>
<td>0.20</td>
<td>1.04</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>Capacity of IP</td>
<td>Real</td>
<td>2649</td>
<td>2047</td>
<td>J/(kgK)</td>
</tr>
<tr>
<td>Max. Heating Pow.</td>
<td>Real</td>
<td>200.0</td>
<td>1497</td>
<td>W</td>
</tr>
</tbody>
</table>

The two solutions were tested (simulated) using forty four different profiles that constituted the environmental parameter spectrum (see formula 1 in section 2.3). Each solution was simulated to obtain the heating demand and the number of non-comfortable hours per year. To understand the risk of carrying out a building design optimisation with a single occupancy, we have shown the heating demand and non-comfortable hours, for the occupancy profiles that can be found in a 2σ interval of the normal distribution (this interval represents 68% of the possible occupancy profiles i.e. 30 profiles). The motivation for this is that extreme profiles outside this range are less likely to happen, and robust designs that will perform well are difficult to find with any method. These results are shown in Figure 4. It can be seen that even only considering the profiles within the 2σ range, the design of the building obtained with “traditional” optimisation can become non-comfortable for one third of the behavioural patterns. This represents a great risk for the design team.

Solutions (buildings) with very low heating demand were obtained with both algorithms (see Figure 4 top), the best approaching the energy performance of Passivhaus. This means, accounting for occupancy changes does not automatically imply the implementation of poor-efficiency houses.

There are several points that can be extracted from Table 4: In both cases too large windows are undesirable; although they provide “free heating” through solar gains, much of this does not occur when needed; instead, it is likely that the solar gains arise when other gains (e.g. electrical gains) are also contributing to a potentially uncomfortable temperature. To release the excess of heat from the coincidental gains, solar gains will eventually trigger ventilation, and the free heat is lost through openings.

The CEES was able to adjust the conductivity and the thermal capacity of the partitions to make the best use of free gains. Although the CEES’s final solution has less insulation than the solution obtained with a single statement of occupancy the greater thermal mass in conjunction with the higher thermal conductivity of partitions makes up for this and provides with a design with low heating demand. Although the solution obtained with the CEES makes a better use of the gains, designers that want to produce designs robust to different occupants will need to consider that they come at the expense of increasing the heating demand by around a 25% over that possible design for a single occupants’ profile; however, in gross terms, this increment is small due to the low demands achieved by the designs.
Figure 4 - Heating demand (top) and non-comfortable hours (bottom) of the solutions obtained with the two optimisation methods, when simulated with 30 profiles that belong to the 2σ interval of the distribution of profiles. (Occupant profiles have been ordered by accumulated electricity use).

The improvement given by the CEES is clear when the number of non-comfortable hours is examined (Figure 4 bottom). The building obtained with the CEES is found to always outperform the one produced with only a single pattern of occupancy, and only violates the condition of comfort, very slightly, for three profiles (profiles 26, 27 and 28; 873, 873 and 908 hours respectively).

5 Conclusions

We consider that the methodology presented here is a first step to more elaborate optimisation mechanisms that will provide building designers with solutions robust against the many parameters which are in truth unknown or ill-defined during the design stage, may vary during value engineering or construction, or dependent on the behaviour of occupants over the many decades a building may last, without the need to simulate each possible combination of parameters.

As this optimisation technique requires only an acceptable increase in computational time, and produces more robust solutions than those obtained using ordinary optimisation methods, its use may be of interest not just to building scientists, but to practising engineers. It is recommended that in future all such optimisation runs are made using a realistic spectrum of behaviours (including set-points), and that the approach is expanded to include other elements of design that might show variance during construction, for example, U-values and air tightness. This it is hoped will reduce some of the risks associated with designing and asking people to occupy very low energy buildings.

6 Acknowledgement

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Appendix: The Changing Environment Evolutionary Strategy

1 The Changing Environment Evolutionary Strategy: CEES

The optimisation procedure that was investigated in this paper is aimed at dealing with environmental uncertainties. As it is slightly different than the canonical form of ES, the algorithm will be described in this appendix.

The impact of environmental parameter in uncertain optimisation is usually unknown; however, it is possible to know the range of values that it may take, and its probability distribution. That information allows the implementation of a check of robustness for all generations in the CEES.

The CEES shares with the canonical form of ESs the real-encoded variables, the random mutation at a phenotypic level and the elitism. Elitism is the characteristic of the algorithm that makes solutions that perform well stay over generations (for more details about the canonical form of ESs see [22]).

To produce robust solutions, the populations are evaluated at each generation with a different environmental parameter. This continuous evaluation of the solutions in different environmental conditions makes those individuals that survive more resistant to environmental changes.

The CEES can be written in the pseudo-code shown in Eq. (A.1).

\[ t := 0; \]
\[ e := \text{sample } \alpha \]
\[ \text{initialize } P(0) := \{ x_1(0), ..., x_\mu(0) \}; \]
\[ \textbf{while } (t \leq T) \textbf{ do} \]
\[ \quad e := \text{sample } \alpha \]
\[ \quad \text{evaluate } P(t) := \{ f(x_1(t), e), ..., f(x_\mu(t), e) \} \]
\[ \quad \text{rank } P(t) \]
\[ \quad \text{select } P'(t) \text{ among } P(t) \]
\[ \quad i := 1 \]
\[ \quad k := 1 \]
\[ \quad \lambda := \text{size of } P'(t) \]
\[ \quad \textbf{while } (k < \mu - \lambda) \textbf{ do} \]
\[ \quad \quad \text{x}_{\lambda+k}(t) = \text{mutation of } x_i(t) \]
\[ \quad \quad i := i + 1 \]
\[ \quad \quad k := k + 1 \]
\[ \quad \quad \text{if}(i = \lambda) \]
\[ \quad \quad \quad i = 1 \]
\[ \quad \textbf{fi} \]
\[ \quad \textbf{od} \]
\[ \quad t := t + 1 \]
\[ \quad \textbf{od} \]

where \( x_i(t) \) represents solution \( i \) at generation \( t \), \( e \) is the environmental parameter, \( T \) is the maximum number of generations, \( \mu \) is the population size, \( P(t) \) is the set of solutions (population) at generation \( t \) and \( f \) is the fitness function.

It can be seen in Eq. (A.1) that, in the CEES, the worst individuals are eliminated leaving empty slots in the population that are filled with mutations of the best individuals of the population. This mechanism maintains good individuals for a large number of generations; that way, they can be eliminated for only two reasons: because better individuals have appeared; or, because a specific environmental parameter makes them poor individuals.

1.1 Selection operator in the CEES

The selection operator in the CEES is deterministic as in any Evolutionary Strategies (ES) (see [22]). However, instead of selecting the \( \lambda \) best individuals, a range of fitness is calculated between the best and the worst individual of the population, and the individuals belonging to a given percentage of that range are selected. This threshold was fixed at 3% as the penalty function makes the range rather large and a low number needs to be chosen to make sure there is enough selection pressure.
This way of defining the selection operator ensures that individuals located away from the area of high fitness are eliminated rapidly, i.e., individuals that have a poor fitness when the environment changes are removed. An illustration of the operator compared to a rank-based selection operator can be found in Figure A.1. For more information about selection operators see [34].

Figure A.1. Selection operator based on Range (Right) and Rank (Centre), from an original population shown in the left. Range selection: individuals that belong within 75% of the fitness range are selected. Ranking selection: the 75% best individuals are retained. It can be seen that the individual excluded in the centre by the ranking selection is almost as good as the selected ones; this does not happen with the range selection.

1.2 Mutation operator in the CEES

The CEES uses the mutation operator as the only exploratory operator. The mutation operator used in this work is the one introduced by Michalewicz in his work [25]. It consists in modifying the selected individual by adding or subtracting to the decision variable a scaled random number. The scope of the modification of the decision variables decrease with the generation number, making the search coarser at the beginning than at the end of the optimisation.

The mathematical formulation of this operator is as follows: for a given parent \( x \), the decision variable \( x_k \) is mutated with one of the equations (depending on the value of the random number \( r \), see below):

\[
x'_k = x_k + \Delta(t, x_k^U - x_k) \\
x'_k = x_k + \Delta(t, x_k + x_L^L)
\]

where \( \Delta(t, x) \) is a function that returns a value in the range from \( [0, y] \) such that \( y \) approaches zero as \( t \) increases. \( t \) is the generation number, and the function \( \Delta(t, x) \) is defined as follows:

\[
\Delta(t, x) = yr \left( 1 - \frac{t}{T} \right)^b
\]

The function \( \Delta(t, x) \) is used as the step size of the mutation, \( r \) is a random number from the interval \([0, 1]\), \( T \) the maximum generation, and \( b \) a parameter determining the degree of non-uniformity [25].
REFERENCES


