Multi-criteria selection of building materials
Dr Daniel Maskell¹, Dr Andrew Thomson² and Prof Pete Walker¹

¹BRE Centre for innovative Construction Materials,
Department of Architecture and Civil Engineering,
University of Bath,
Bath,
United Kingdom

²MODCELL,
Bristol,
United Kingdom

Corresponding Author: Dr Daniel Maskell, D.Maskell@bath.ac.uk

Reference
https://doi.org/10.1680/jcoma.16.00064

Abstract

Rational selection of building materials for their optimal performance and minimal environmental impact is complex, as materials are multi-functional. Beyond the primary function for which a material or product has been identified, such as insulation, the contribution to additional aspects of building performance may be conservatively overlooked. For example, important properties, such as latent and hygrothermal performance, are rarely considered when selecting insulation materials. Consequently, if materials are to be used optimally, there is a need for reliable and robust ranking methods based on a multi-criteria analysis. This will help to facilitate the most optimal selection of materials for building performance, environmental impact and occupant well-being. This paper describes four statistical methods of comparing and ranking different building materials. A critical appraisal of these methods is presented following implementation with a reference material data set. Finally, recommendations for future development and adoption of material selection tools are outlined.
1 Introduction

The selection of building materials is common to all construction projects, but the decisions that guide these choices will vary in each situation. While building materials are required to conform to certain legislative requirements, for example, in the UK these include Building Regulations, Energy Related Products Regulations and Hazardous Waste, there is still a wide choice of available products. Material selection is informed by multiple criteria, with a trade-off commonly occurring between cost and performance. Historically, material selection was predominantly based on minimising cost alone (Kishk et al., 2003). Increasingly, the environmental impact of materials and the energy use of a building after completion are also important to designers and clients, becoming ‘quasiregulatory’ requirements. Exploiting the full potential of materials is therefore becoming a key factor in material selection, particularly where they can provide multiple solutions. Currently material selection remains limited to specific performance criteria, and a narrow range of properties dominate the selection process (Wastiels et al., 2007). These performance criteria are those specified by legislative drivers, ensuring compliance, with the presence of additional non-mandatory performance criteria varying. This range of both mandatory and additional performance criteria, inherently has varying units, drivers or performance references that make comparison of materials challenging. However, Keysar and Pearce (2007) comment that material selection tools can allow for greater innovation, overcoming the common practice of selecting materials based on past experience alone (Yang and Ogunkah, 2013).

A well-established method of comparing the environmental impact of materials is the Building Research Establishment Environmental Assessment Method (Breeam) Green guide, which considers over 1500 specifications. The method predominantly considers environmental impact and is based on the BRE’s environmental profiles methodology, which in turn is an adapted life cycle assessment (LCA). Breeam considers 13 categories of environmental impact and applies characterisation factors that convert the various impacts for comparison. Weightings are applied to each of the 13 categories that are subsequently
combined into a single score, measured as ‘ecopoints’. As typical with LCA methods, a functional unit is specified that allows for a method of comparison considering technical performance. However, functional units are largely one-dimensional, suitable when considering only one performance aspect at a time. Other methods have also been developed, including leadership in energy and environmental design, reviewed by Keysar and Pearce (2007). Although commonly used, these methods consider only the environmental profile and do not directly consider cost or multiple technical performances. Giudice et al. (2005) demonstrated how mechanical properties could be integrated with environmental performance. Multi-objective analysis was used to compare materials across different categories. While Giudice et al. (2005) presented a simple mathematical method of multi-objective analysis, Azapagic (1999) used Pareto-optimum solutions. However, Azapagic (1999) commented that Pareto-optimum solutions do not represent the best solutions and instead considered multi-criteria analysis (MCA).

Both Cooper (1999) and Kohler (1999) commented that material evaluation is typically only concerned with single criteria, such as mechanical or thermal performance, indoor environmental impact or embodied environmental impact. This approach can clearly lead to over-specification and redundancy, which will impact on cost and embodied environmental impact. As such, there has been increasing research into multi-performance materials where a single material can be utilised for a variety of functions. An example would be the use of thermal insulation not only to regulate thermal comfort, but also as acoustic insulation, and for vapour condensation prevention and potential fire protection and structural integrity (Al-Homoud, 2005). This requires the specification and selection of any material to be based on multiple criteria. The range of factors and measurements of various properties further compounds the complexity of this multi-criteria decision. The thermal properties of materials
are influenced by a variety of factors including thermal resistivity and thermal storage by latent heat.

Multi-criteria decision making or multi-criteria decision analysis explicitly considers multiple criteria and the analytics behind the decision making (Tzeng and Huang, 2011). Ding (2008) critiques environmental assessment tools, suggesting the adoption of a sustainability index model that combines multiple criteria. The method, however, only considers four main criteria related to sustainability with no consideration of technical performance and utilises a simplistic summation of weighted factors. A rational, transparent and numerically robust method of comparing material across multiple criteria is required.

The selection of building materials to provide multiple design solutions is important for improving the efficiency and performance of construction and buildings, respectively. An

2 Characterising materials for selection

Building materials are used in a broad range of applications to meet the performance and comfort requirements of an internal space. Different disciplines within the construction industry focus on how to best identify materials that will fulfil these needs. Each discipline focusses on particular characteristics, to inform the selection of a material for a particular application. The methods described in this paper seek to broaden the focus of material selection by enabling the consideration of additional characteristics. Therefore, for the purposes of rationalising the selection of a material using statistical methods, the characteristics can be grouped into categories that are likely to include, but not exclusively, the following

- Acoustic,
- Chemical,
- Fire,
- Hygric,
- Indoor Air Quality,
- Life Cycle Impact,
- Mechanical,
- Microbiological,
These categories seek to capture the primary areas of focus when identifying materials as a part of a building design. Within each of these categories there are a range of criteria that can characterise the material. The categories used are dependent on the application, and it is expected that additional categories and criteria may develop further as new materials and building design approaches advance.

The characterisation of materials involves mixed and varied data types. Typically, data would be classified as primary, having been determined through testing. However, the characterisation of materials could also rely on secondary data, such as those collected indirectly from the literature. There is, of course, potential for bias within primary and secondary data sets. This would include the selection of test parameters and methods. These factors could lead to a 'hierarchy of data' based on provenance. However, the extent of this created hierarchy can result in a subjective bias, which would need to be considered if implemented. In addition, there are quantitative and qualitative data types. Stevens (1946) proposes four typologies for the scale of measurement: 'nominal', 'ordinal', 'interval' and 'ratio', that unify qualitative and quantitative data. The majority of measurements associated with building materials are of a ratio type because discrete measurements from a continuous scale are obtained. An inherent property of the ratio type is a discrete and non-arbitrary zero value that allows for a range of descriptive and inferential statistics to be used.

Material testing standards provide a consistent approach to obtaining performance data and typically provide a method of additional analysis for interpretation of results. Typically, the analysis would convert a continuous data output into discrete data, preventing the potential subjectivity of material characterisation. For example, the compressive strength of insulation materials is determined at the 10% strain level. This allows the conversion of a continuous stress–strain behaviour of the materials to be quantified into discrete values such as the
compressive strength and elastic modulus. However, some methods of characterisation cannot simply be interpreted and individually quantified; remaining either continuous or unquantified. Typical methods of chemical characterisation including Fourier transform infrared spectroscopy (FTIR), dynamic vapour sorption and other spectroscopies produce spectra that cannot be rationalised for methods of comparison without potential subjectivity. However, there are a range of spectra that can be analysed, leading to multiple ratio data types including X-ray diffraction and X-ray fluorescence. These singular characterisation methods therefore result in quantification of a range of individual properties. However, this would result in numerous data points for a single selection criteria and could skew the effect of a single criterion if considering a form of an aggregate score through effective double counting.

Collation of the numerical characterisation of different materials results in a performance matrix. This matrix consists of raw data that have variable units and different variance, which makes it difficult to compare based solely on judgement. Goodwin and Wright (2004) discuss the problems associated with decision making based on human judgement and conclude that it should be avoided. Therefore, a consistent, transparent and rigorous numerical approach would be preferred.

3 Multiple Criteria Analysis

There is a range of performance criteria to consider when selecting a material. Even when only considering legislature compliant building materials, optimal selection will depend on maximising multiple desirable criteria, which can often be conflicting. Currently, selection by a designer usually relies on a judgemental choice rather than any traceable process. In contrast, MCA uses explicit objectives and measurable criteria to assess the success of
meeting objectives (Dodgson et al., 2009). Dodgson et al. (2009) summarise the benefits that MCA has over informal judgement decisions

- Both numerical methods and implementation are transparent and explicit;
- Weightings may be applied, but are equally transparent and rational;
- Measurements of performance can come from a variety of expert sources;
- The objectives of the analysis can be easily changed to suit appropriateness;
- An audit trail can be easily developed.

The use of a MCA approach for the selection of materials is therefore preferable. The MCA techniques utilise the performance matrix developed through the characterisation of materials and apply numerical analysis, typically involving methods of scoring and weighting. Some of the different types of MCA that can be adopted (Cinelli et al., 2014) are highlighted below

- Multi attribute utility theory (Keeney and Raiffa, 1993),
- Analytical Hierarchy process (Satay 1980 and Satay 2005),
- Elimination and choice expressing the reality (Roy, 1996 and Figueira et al., 2013)
  Preference ranking organization method for enrichment of evaluations (Brans and Mareschal, 1986).

Of the many MCA methods available, this paper considers four approaches, each with increasing complexity. The result of each is an aggregate score for each material across the ten categories set out above. This allows for the ranking of materials within each category. All of the methods use consistent numerical scales to reflect whether better performance is associated with a decreasing or increasing material property value.

Any method of selection, whether qualitative or quantitative, is reliant on the quality of the data and the completeness of the data sets. Discussion regarding the quality of the data is
outside the scope of this paper, but any statistical methods developed must consider incomplete data sets and sample size. It is unlikely that all materials considered in a given comparison will have the same quality of parametric data sets. Therefore, any statistical method adopted must consider and mitigate dominance from varied data sets. Small sample sizes are often difficult when considering precision. Small sample sizes can occur within the individual measurements and the range of materials considered. This paper assumes that individual measurements are statistically robust, with other parameters discussed within each comparison method below.

3.1 Ranking Method

The ranking method scores each material based on its rank in each category according to the performance of that material relative to the others (Equation 1). For each property/category the ranks were summed and divided by the number of properties measured giving the average score based on rank (Equation 3). This removed dominance based on inconsistent and incomplete data sets

\[ r_{ijk} = \text{rank}(X_{ijk}) \]  

\[ W_j^* (r, W) = \begin{cases} W_j & r_j \neq 0 \\ 0 & \text{otherwise} \end{cases} \]  

\[ \bar{r}_{ik} = \frac{1}{\sum_{j=1}^{n} W_j} \sum_{j=1}^{n} W_j \times r_{ijk} \]  

where;

- \( X_{ijk} \) is the measured value for material \( i \) of test \( j \) of property \( k \),
- \( s_{ijk} \) is the scored value for material \( i \) of test \( j \) of property \( k \),
- \( W_j \) is the weighting applied to test \( j \), discussed in Sect. 3.6.
- \( W_j^* \) is the function of weighting applied to test \( j \), discussed in Sect. 3.6.

3.2 Scoring Method

The scoring method calculates the difference in performance between the material and a control for that group. The chosen ‘control’ is dependent on the decision maker and could be
real or contrived. It could represent an existing material to which other materials are compared, or it could be the average result for each parameter. For each material this difference was calculated from Equation 4, and was summed and divided by the number of properties measured, giving the average score based on the differences from Equation 4. This average difference was calculated for each of the ten property categories and subsequently ranked.

\[
s_{ijk} = \frac{x_{ijk} - c_{jk}}{c_{jk}}
\]  

(4)

\[
W_j^*(s, W) = \begin{cases} 
W_j & \text{if } s_j \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]  

(5)

\[
\bar{s}_{ik} = \frac{1}{\sum_{j=1}^{m} w_j} \sum_{j=1}^{m} w_j s_{ijk}
\]

(6)

where;

\( X_{ijk} \), is the measured value for material \( i \) of test \( j \) of property \( k \),
\( C_{jk} \), is the control value of test \( j \) of material group \( k \),
\( s_{ijk} \), is the scored value for material \( i \) of test \( j \) of property \( k \),
\( W_j \), is the weighting applied to test \( j \), discussed in Sect. 3.6
\( W_j^* \), is the function of weighting applied to test \( j \), discussed in Sect. 3.6

3.3 Difference Range Method

The scoring method of calculating the differences can be normalised to allow for meaningful summation of the units. This difference was compared with the range of the difference and interpolated linearly based on Equations 7 and 8

\[
d_{ijk} = y_0 + (y_1 - y_0) \left( \frac{x - x_0}{x_1 - x_0} \right)
\]

(7)

\[
y_0(s) = \begin{cases} 
0 & \text{if } s_{ijk} > 0 \\
-1 & \text{otherwise}
\end{cases}
\]

\[
y_1(s) = \begin{cases} 
1 & \text{if } s_{ijk} > 0 \\
0 & \text{otherwise}
\end{cases}
\]
\[ x_0(s) = \begin{cases} 
0 & \text{if } s_{ijk} > 0 \\
\min(s_{jk}) & \text{if otherwise} 
\end{cases} \]

\[ x_1(s) = \begin{cases} 
\max(s_{jk}) & \text{if } s_{ijk} > 0 \\
0 & \text{if otherwise} 
\end{cases} \]  \hspace{1cm} (8)

\[ \overline{d}_{ik} = \frac{1}{\sum_{j=1}^{n} w_j} \sum_{j=1}^{n} w_j d_{ijk} \]  \hspace{1cm} (9)

where:

- \( d_{ijk} \) is the scored value for material \( i \) of test \( j \) of property \( k \) as defined in Eqtn. 4,
- \( d_{ijk}^* \) is the interpolated value for material \( i \) of test \( j \) of property \( k \),
- \( w_j \) is the weighting applied to test \( j \), discussed in Sect. 3.6
- \( w_j^* \) is the function of weighting applied to test \( j \), discussed in Sect. 3.6

In the general case, this results in a bi-linear graph in an equivalent range of negative unity to positive unity, as shown in Figure 1.

Thus a score of

- 0 would represent the same value as that achieved by the control.
- -1 would represent the maximum negative difference in performance compared to the control,
- +1 would represent the maximum positive difference in performance compared to the control.
Where all materials performed better than the control, the interpreted range, as shown in Figure 2 was between

- 0 representing a hypothetical material equivalent to the control and
- +1 representing the maximum positive difference in performance compared to the control.

Where all materials did not better the performance of the control, the interpreted range, as shown in Figure 3, was between:

- 0 representing a hypothetical material equivalent to the control and
- -1 representing the maximum negative difference in performance compared to the control.
Figure 3 Scalar assignment when all values are less than zero

3.4 z-Value Method

The z-value is also commonly referred to as the standard score and considers the variance of the data. Typically, calculation of standard deviation (used for the z-value method) of the values is based on the difference of the arithmetic mean, whereas this calculation considers the difference from the control of each material category.

\[
z_{ijk} = \frac{x_{ijk} - c_{jk}}{\sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_{ijk} - c_{jk})^2}}
\]

(10)

\[
W_j^*(z, W) = \begin{cases} W_j & \text{if } z_j \neq 0 \\ 0 & \text{otherwise} \end{cases}
\]

(11)

\[
\bar{z}_{ik} = \frac{1}{\sum_{j=1}^{n} W_j} \sum_{j=1}^{n} W_j x z_{ijk}
\]

(12)
\[ W_k^*(z, W) = \begin{cases} W_k & \text{if } z_k \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (13) \]

\[ z_i = \frac{1}{\sum_{j=1}^{l_i} W_j} \sum_{k=1}^{n_k} W_k \times z_{i,k} \quad (14) \]

where:

- \( X_{ijk} \) is the measured value for material \( i \) of test \( j \) of property \( k \),
- \( C_{jk} \) is the control value of test \( j \) of material group \( k \),
- \( z_{ijk} \) is the z-value for material \( i \) of test \( j \) of property \( k \),
- \( W_j \) is the weighting applied to test \( j \), discussed in Sect. 3.6
- \( W_k \) is the weighting applied to property \( k \), discussed in Sect. 3.6
- \( W_j^* \) is the function of weighting applied to test \( j \), discussed in Sect. 3.6
- \( W_k^* \) is the function of weighting applied to property \( k \), discussed in Sect. 3.6

3.5 Discussion

The ranking method is numerically the simplest method of comparison among all the materials. The best-performing material, such as the one with the highest compressive strength, or the lowest thermal conductivity (depending on the objectives) will always be given a score of 1, with the least given a score of \( n \), where \( n \) is the number of materials being analysed. The scores are based on an interval level of measurement and do not account for any relative differences between the properties. Therefore, there is an inconsistent definition of difference between the levels of the rank and between the scores, which results in an inconsistent aggregate score.

The scoring method helps to address shortcomings of simple ranking, as it results in a continuous score that represents the relative improvement in performance. However, the scoring method is strongly influenced by dominance factors resulting in the likelihood of the average summed results being skewed by a single property, such as quoting strengths in kilo Pascals or mega Pascals. While the difference has been rationalised by dividing with
respect to the control value, the variation of this difference between each category will
dominate any summation score. Therefore, a method of comparison between all the
materials that accounts for the differences and normalises them needed to be developed,
resulting in the difference range methods and z-score method.

Both the difference range methods and z-score method attempt to remove the effects of
dominance. This allows the relative performances to be compared without significant skew
from an individual property. It further allows for summation across all property categories to
give an aggregate score. The difference range method interpolates the difference on a scale
of −1 to 1, where 0 represents the control. The z-value represents the number of modified
standard deviations that a value is above the control specimens and assumes a normal
distribution. The z-value method is regarded as numerically more robust. The difference
range method completely removes the dominance through its interpolation. However, this
assumption does not account for the inherent spread of values as with the z-value method.
Therefore, the maximum difference is always given a value of 1, regardless of its relation to
the standard deviation. This has the effect of removing any material that performs
significantly differently from the others and magnifies the relative difference of materials
which perform similarly within a particular category. The z-value allows for the variation in
relative performance but also standardises the value, therefore allowing significant
performance to be captured without skew effects from dominance.

3.6 Weighting

Weighting is the numerical process that allows for the relative importance of a particular
property or category to be shifted. The introduction of weighting will inherently introduce
judgemental decision about the relative importance, which is difficult to avoid. However,
clear and consistent applications of weighting allow for auditing of the decision maker
choices.
The test parameters are weighted for importance separately and the properties group can also be further weighted. The weighting per test simply multiplies the value score from the selection method by a constant weight for the criterion as in Equations 3, 6 and 11. A weighted average is then taken for each property category. This method of applying weights and taking an average weighting can then be applied for each property category to give an overall weighted score for the material.

4 Implementation

The various methods of MCA discussed above in Section 3 were implemented with an existing data set of eco-innovative building materials. The data set consisted of eight insulation materials, with one of the eight designated as representative of an industry standard and therefore control. The materials were tested across the ten categories discussed in Section 3 with 48 material characteristics identified and investigated. Some of these characteristics resulted in non-discrete data, and therefore were not considered for the MCA. Other characteristics gave multiple data points for each material and were further subdivided. Therefore, the data sets contained 47 discrete data points for each material.

The issue of identifying discrete data was notably more difficult for the chemical composition of materials. The chemical category consisted of tests that included FTIR and Raman spectroscopy that could not be reduced to a single ‘ratio’ data type. In other areas such as indoor air quality assessment, testing included organic content, total volatile organic compounds (TVOC) emission rate and formaldehyde emission rate and each test provided multiple data points for assessment. TVOC emission rate was tested at 3 and 28 d which doubled the data gathered for this parameter. The measurements considered the emission in relation to the exposed surface area, volume and mass of the specimen. This further tripled the data associated with each measurement. Therefore, the measurement of TVOC (as well as formaldehyde) produces six times the number of results compared with other parameters such as organic content. This could potentially lead to TVOC and formaldehyde
measurements dominating the aggregate score for the category due to the effective multiple (six times) counting. The introduction of weightings on each measurement can account for this dominance by scaling down each of the measurements so the sum of the weights is unity.

![Figure 4 Properties considered and data measured in each category](image)

Figure 4 Properties considered and data measured in each category

Weightings also allow the decision maker to assign relative importance to different test parameters. An example could be through the measurement of different mechanical properties such as compressive, flexural and shear strength and their associated moduli. The decision maker could assign a higher weighting to the compressive strength. The effect of this weighting will vary depending on the individual performance matrix generated. The effect of weighting across each method is observed in Figure 4, where the weighting applied for the compression test was doubled. Materials D through to G had a higher compressive strength compared with the control but this did not directly translate into an increase in score. This is due to the effect of weighted average scores being used, which reduce the
influence of an individual weighting. While the change of scores may appear significant, the associated ranking of the materials by each method did not change.

Weighted averages also allow for incomplete data sets to be used without dominance effects. All the methods assume a score of zero if there was no measurement of data, and the weighting is not considered for the count of the weighted average. For the scoring, difference range and z-value methods, this has the implication that a material that was not tested has the equivalent performance as the control material. This can have significant implications across all methods that are not possible to quantify with the exception of the z-value method. As the z-score is an expression within the z-distribution that shows the number of standard deviations the material is from the control, it can be quoted to a probability and therefore, the level of risk of assuming a z-score of 0. By definition there is 50% probability that the true score is either larger or smaller than the z-score of 0. There is a 68% likelihood that the actual score is within a single z-score of the control specimen and a 10% risk that the real z-score would be ±1.645.

**Figure 5 Effect of weighting (Note: compressive strength was not investigated for material A)**
The assumption of equal distribution around the control material for a given property is suitable only for the z-value method. The bi-linear interpolation resulting from the difference range method presented in Equation 7 can disproportionately skew results when values are above and below, as in Figure 1. An example is observed with the compressive strengths of insulation materials. The compressive strength results and the subsequent analysis are presented in Table 1. Materials D, F and G are all forms of rigid insulation material compared with the others that are fibrous or loose fill (product A).

### Table 1 Compressive strength Analysis

<table>
<thead>
<tr>
<th>Product Name</th>
<th>Compressive strength (kPa)</th>
<th>$r_{ijk}$</th>
<th>$s_{ijk}$</th>
<th>$d_{ijk}$</th>
<th>$z_{ijk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>0.125</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>N.I.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.038</td>
<td>6</td>
<td>-0.70</td>
<td>-0.7909</td>
<td>-0.0004</td>
</tr>
<tr>
<td>C</td>
<td>0.015</td>
<td>7</td>
<td>-0.88</td>
<td>-1.0000</td>
<td>-0.0005</td>
</tr>
<tr>
<td>D</td>
<td>81.5</td>
<td>3</td>
<td>651</td>
<td>0.2312</td>
<td>0.3868</td>
</tr>
<tr>
<td>E</td>
<td>0.754</td>
<td>4</td>
<td>5.03</td>
<td>0.0018</td>
<td>0.0030</td>
</tr>
<tr>
<td>F</td>
<td>301</td>
<td>2</td>
<td>2410</td>
<td>0.8559</td>
<td>1.4321</td>
</tr>
<tr>
<td>G</td>
<td>352</td>
<td>1</td>
<td>2820</td>
<td>1.0000</td>
<td>1.6732</td>
</tr>
</tbody>
</table>

Insulation materials B and C have a marginally lower compressive strength than the control as demonstrated by the scoring and z-value method of analysis in Table 1. This is in contrast to the rigid materials that are significantly stronger than the control provided by the same method of analyses. Observation of the results from the scoring method also indicates the difficulties of dominance through the range of results achieved when the material is standardised with respect to the control specimen.
Material C has the lowest compressive strength and therefore has been assigned the value of −1 according to the difference range method. However, considering the spread of the results, this resulted in a z-value of −0.0005, which is compared with the equivalent maximum value of compressive strength, material G having a $d_{ijk}$ value of 1, but a z-value of 1.67. This has the effect of reducing the significance of higher compressive strength values and inflating the marginally poorer strengths, as demonstrated in Figure 5.

![Graph](image)

**Figure 6 Comparison between results of the difference range and z-Value methods**

The difference range method could be modified to remove this bi-linear effect through consideration of the maximum modulus. Equation 7 can be simplified and reduced to the following form

$$d_{ijk} = \frac{s_{ijk}}{\max{|k_{ij}|}}$$  \hspace{1cm} (15)

This method normalises the range of difference and represents the relative performance of the materials on a linear scale. This allows for an aggregation within each category and across categories. However, the linear interpolation can again have the effect of inflating or diluting the effect of the spread of the results compared with the z-value method. A
representation of this can be seen in Figure 5 where the results of the compression test are compared with the TVOC measurement. Both results were analysed and the resulting $d_{ijk}$ and $z_{ijk}$ values plotted. The gradient of the resulting lines represents the relationship between $d_{ijk}$ and $z_{ijk}$ across the two testing categories. As expected, the values do not equate and therefore the interpolated scores from either Equations 7 or 15 have no relation to other values from the other test parameters. The aggregate score is the average of the scores for measurement of compressive strength and TVOC, where the $d_{ijk}$ was calculated from Equation 15. While the $R^2$ value of $0.96$ indicates a very strong correlation between the two scoring methods, it indicates that the aggregation of the same scoring method across different properties results in non-related values. This anomaly will be exacerbated with increasing aggregation of differing properties. This is due to the aggregation of linear interpolation scores having no mathematical meanings whereas the z-value method can always be related to the standard deviations of individual properties and therefore aggregated categories. Therefore, while the interpolated values, especially as given from Equation 15, are numerically simpler, the z-value method is recommended.

There is a whole field of mathematics on MCA that has the potential to be utilised within civil engineering and particularly material selection decisions. While the mathematics presented in this paper is numerically simple, allowing for widespread and transparent adoption, more complex computational methods could be adopted. The mathematical approaches can be easily adopted by different members of a design team including architects, structural engineers, mechanical engineers and contractors. Each will have different objectives that can be adopted within the framework presented here. Adoption of a unified mathematical framework can easily be shared among the design team, with each in addition to external experts contributing to the values and weightings. No other framework of analysis of building materials across a limitless variety of categories and properties with such transparency exists.
5 Summary and Conclusions

This paper has presented methods of MCA for a range of characterisation properties of construction materials. Due to the growing appreciation of the environmental impact of the materials used within the construction industry, material selection increasingly considers multiple properties. While some methods exist that consider environmental impact these are generally decoupled from the technical performance of the material. Nonetheless, there is an increasing breadth of technical measures that can be considered by decision makers.

This paper presents ten categories that could be considered during material selection and therefore material testing characterisation. Within these categories there can be a wide range of test properties that produce a variety of different data types. Due to the numerical methods used in the analyses, only single ratio type data points can be used. These data can be compiled together into a performance matrix for a given material. The performance matrix will give a holistic overview of the material that can be used for simplistic material selection.

Several methods were considered for MCA, based on the principles of the multi-attribute utility theory and analytical hierarchy process. Of the four methods considered, only two could be aggregated to allow for the desired multiple comparisons, of which the z-value method is more mathematically robust. The method compares materials by calculating the standard score compared with a control material rather than the average properties. This allows for variation to be accounted for without dominance effects. Transparent weighting was implemented to allow decision makers to modify the relative importance of different test parameters. Weighted averages were taken, which allows for comparison between incomplete data sets.
The outcome of this paper is a mathematical model based on established statistical methods, which can be used to compare building materials objectively. While this model could be developed further to include more complex approaches within MCA, its current ease and transparency allows potential widespread adoption through many different disciplines within civil engineering.

6 Acknowledgements

The research leading to these results has received funding from the European Union’s Seventh Framework Programme (FP7/2007-2013) under grant agreement no 609234.
7 References


