Electric Vehicle Charging in Smart Grid: A Spatial-temporal Simulation Method

Yue Xiang\textsuperscript{a}, Zhuozhen Jiang\textsuperscript{b, *}, Chenghong Gu\textsuperscript{c}, Fei Teng\textsuperscript{d}, Xiangyu Wei\textsuperscript{a}, Yang Wang\textsuperscript{a}

\textsuperscript{a} College of Electrical Engineering, Sichuan University, Chengdu 610065, China
\textsuperscript{b} State Grid Chongqing Electric Power Company, Chongqing 400014, China
\textsuperscript{c} Department of Electronic and Electrical Engineering, University of Bath, Bath BA2 7AY, UK
\textsuperscript{d} Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2AZ, UK

Abstract Electric vehicles (EVs) play an important role in the future energy system. The large-scale adoption of moving EV load significantly accelerates the integration of transportation and distribution systems. The method to simulate the mobility and charging of a single or aggregated EVs is the key to analyze EVs’ flexibility on the operation of distribution network. Considering the integrated impacts from both the transportation and power systems, and the uncertainty of user’s driving behavior and charging intention, this paper proposes a spatial-temporal simulation method based on the vehicle-transportation-grid trajectory. The trajectory can not only describe the destination location and time like the trip chain, but also give the key information including the driving path in a whole travel process. The driving, parking, and charging are analyzed by the proposed spatial-temporal simulation method. It models the driving behavior based on statistical results and transportation systems, EV energy consumption pattern based on battery energy, and the charging demand based on the user’s subjective intention at the coupled systems. Finally, a 30-node transportation system is developed and integrated with a 33-bus distribution network to illustrate the proposed method. Two typical days, “workday” and “holiday”, are simulated and compared under different EV penetration levels (0\%, 20\%, 50\% and 100\%), different trip chain ratio (the ratio of 3-trip chains is 50\%, 70\%, 90\%) to demonstrate the effectiveness of the spatial-temporal simulation method.

Keywords electric vehicle; distribution network; trajectory; transportation system; spatial-temporal modeling; charging load

1 Introduction

To reduce greenhouse gas emissions by replacing traditional combustion-engine driven vehicles, the increasing charging demand of moving electric vehicles (EVs) significantly accelerates the integration of transportation systems.
and power systems [1]. In order to realize the target of reducing CO2 emissions by 14% in 2020, the UK Government has supported EV’s participation in the future transport sector [2]. Governments and renewable energy companies have taken great efforts to promote the development of EVs. Due to the introduced uncertainty and flexibility of the EV charging load, distribution network operation will be greatly impacted by the large-scale EV charging power. The normal operation of the power system will be fundamentally affected by a new load peak, consisting of uncontrolled EV charging load and the original load peak. The research on modeling EV charging load underlies analyzing the impact on the operation of power systems, charging facility planning and charging ordered control [3].

As the participants of the urban transportation system and distribution network, EVs are driven in the road and charged in the grid. The chief aim of charging load modeling for EV is to infer the driving behavior and predict the spatial-temporal distribution of charging load based on the limited data. Numerous studies have been devoted to modeling EV load based on statistical methods, considering probability distributions of partial random factors [4], queuing theory [5], trip chain and Monte Carlo method [6]. Queuing theory was discussed in [5] to predict EV charging demand. A charging probability model was presented based on queuing theory in [7], with the hypothesis that the time of EV arriving at the charging station obeys Poisson distribution. Accordingly, a two-stage Poisson distributed charging station model with clustering parameters was proposed, which could effectively describe the clustering characteristics of charging load [8]. Monte Carlo method is usually combined with probability statistics results to determine the charging location and time according to the probability distribution of the driving law of traditional fuel vehicles or a few EVs, so as to simulate the driving and charging process of EVs [9]. A charging model considering the random features of EV travel and charging patterns are presented in [10] to determine the EV charging load profile and the impact. With a predefined charging location and time, the travel survey data are used directly for EV load modeling in previous studies [11], [12]. However, the impact of the transportation system is not fully considered, and the randomness in daily travel could not be accurately presented by them.

EVs are closely dependent on the transportation system and distribution network, allowing the collaborative charging prediction of the transportation system and distribution network. Therefore, the research on EV charging load modeling needs to consider the coupling of vehicle-transportation-grid system. With the introduction of the origin-destination (OD) method, travel demand and a spatial-temporal model was acquired and established, which
combines the transportation and distribution networks [13]. However, it’s not easy to acquire the real OD traffic-flow data at any locations. To solve this problem, the trip chain [14] is utilized to describe travel demand in this paper, with the consideration of the randomness of spatial-temporal charging load assisted by the household travel survey data [15]. The trip chain refers to the connection form of individuals for different purposes in a certain time complete one or more activities sequentially [16]. Commonly, the whole trip chain is obtained by Monte Carlo methods and the probability distributions from historical data, including travel time, departure time, mileage and destination.

Running in the transportation system, the driving paths of EVs are affected by the network structure and traffic flow. With the increase of driving mileage and the decrease of energy consumption, EVs will charge at plugin infrastructure connected to the distribution network. EVs integrate transportation system and distribution network closely. In fact, the charging demand is positively correlated with driving mileage. The driving mileage is almost obtained by probability distribution function (PDF) [17,18] in the relative scientific literature. In most literature, it is shown that the log-normal distribution function is a proper choice for modeling of the distance traveled by EV. However, this mileage generation method is not enough accurate for a special and diverse traffic network. This paper puts forward an approach to compute the actual mileage of EVs driving in the street, instead of PDF.

EVs closely integrate transportation system and distribution network. In order to spatially and temporally describe the features of EVs in travel and charging demand more actually and the simulation results are more accurate, the relationship between the traffic flow and power flow should be fully discovered. In general, the best way to obtain more practical prediction results is to simulate the whole process of traveling and charging of EVs. In this context, this paper proposes a novel spatial-temporal simulation method based on the vehicle-transportation-grid trajectory to simulate EV charging load with the spatial-temporal characteristics by the means of transportation system model and the trip chain. One the one hand, it takes the coupling relation of the integrated system into consideration, which having a simple mechanism. On the other hand, the driving, parking, charging of a whole travel process can be analyzed with the information of driving path, destination, charging location and time. The results demonstrate the driving trajectory and spatial-temporal charging demand trajectory can be well reflected by the proposed method. In addition to providing more accurate results and the description for a special traffic network, i.e., the proposed model can easily be extended to arbitrary traffic network, the case analyses are intuitive and comprehensive.

The rest of this paper is as follows. Section II introduces the vehicle-transportation-grid trajectory framework of
the spatial-temporal simulation method. In Section III, the transportation system model is introduced, including the network topology and speed-flow simplified model. In Section IV, the spatial-temporal simulation method based on the vehicle-transportation-grid trajectory is discussed, including how to simulate the travel behavior, driving path, charging decision-making process. A system integrated of both transportation network and distribution network is provided in Section V to prove the effectiveness of the proposed method. Section VI concludes the paper.

2 Research framework

In this section, the framework of the proposed spatial-temporal simulation method is firstly introduced to illustrate the vehicle-transportation-grid trajectory concept and the vehicle-transportation-grid trajectory.

![Vehicle-transportation-grid trajectory diagram](https://example.com/vehicle-trajectory.png)

Fig. 1. Vehicle-transportation-grid trajectory

The proposed vehicle-transportation-grid trajectory can be seen in Fig. 1. The purple projection of the spatial plane represents the EV driving trajectory, while the blue line represents the EV parking duration at the corresponding site. Moreover, the green line indicates the charging duration at the corresponding site, and the red solid line indicates the
EV travel duration from the previous node to the node q. The auxiliary red line means time difference and the black line indicates that EV is driving. The trajectory can not only describe the destination like the trip chain, but also give the key information including driving path, travel time, parking duration time in a whole travel process.

Suppose the EV travels along the trip chain “home (H) - workplace (W) - leisure-related location (O) -home (H)” on the main urban roads in Fig.1. The spatial and temporal attributes of any node Q in a path can be described by the points $Q_1=(x_1,y_1,z_1)$, $Q_2=(x_2,y_2,z_2)$, $Q_3=(x_3,y_3,z_3)$ in the trajectory. x and y represent the location of transportation nodes. Moreover, $z_1$, $z_2$ denote the time to reach or leave node q separately, and $z_3$ denotes the end time of charging at node q. The state of charge of EV at time $z$ is represented by $c$. Equation $Q_1=Q_2=Q_3$ represents that EV is not parking at the node. The state variable of EVs can be expressed by the above variables, including location and energy information, which is shown in Table I.

When EV needs to charge, it looks for charging facilities in the transportation network, which are connected to the distribution network buses. The coupling relationship between the transportation system and distribution network are represented by the black dotted line.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location of transportation system node n</td>
<td>$q^n(x^n,y^n)$</td>
</tr>
<tr>
<td>EV parking at node n</td>
<td>$Q^n(x^n,y^n,z^n_1,c^n_1)$</td>
</tr>
<tr>
<td>EV leaves from node n</td>
<td>$Q^n(x^n,y^n,z^n_2,c^n_2)$</td>
</tr>
<tr>
<td>EV parking duration time at node n</td>
<td>$z^n_2-z^n_1$</td>
</tr>
<tr>
<td>EV charging duration time at node n</td>
<td>$z^n_3-z^n_1$</td>
</tr>
</tbody>
</table>

According to the vehicle-transportation-grid trajectory, three indexes are proposed to quantify the EV spatial-temporal characteristics and describe the driving, parking and charging states. The three indexes include the expected number of EV driving (END), the expected number of EV parking (ENP), and the expected number of EV charging (ENC):

$$END^{(n,n+1)}(t) = \frac{1}{N} \sum_{n=1}^{N} Num_{h_n}^{D^{(n,n+1)}}(t)$$  \hspace{1cm} (1)$$

$$ENP^{(n)}(t) = \frac{1}{N} \sum_{n=1}^{N} Num_{h_n}^{P^{(n)}}(t)$$  \hspace{1cm} (2)$$

$$ENC^{(n)}(t) = \frac{1}{N} \sum_{n=1}^{N} Num_{h_n}^{C^{(n)}}(t)$$  \hspace{1cm} (3)$$

where, END represents the dynamic transportation flow of EVs, $Num_{h_n}^{D^{(n,n+1)}}(t)$ is the number of EV from a transportation node n to the next node n+1 at time t, EDP is designed to represent the space-time varying number of
parking EVs, $Num_{n,t}^{P}$ is the number of parking EV at node $n$ and time $t$, EDC is developed to represent the space-time varying aggregated EV load, $Num_{n,t}^{C}$ is the number of EV charging at node $n$ and time $t$, $N$ is the maximum number of Monte Carlo simulation.

Both spatial attributes and time attributes can be represented by the trajectory. For spatial attributes, it can reflect the spatial behaviours of user travel, including destination, driving path, etc. For time attributes, it can reflect the changes in user travel time, including real-time power, arrival time, departure time, etc. Conclusively, it can: i) well describe the space-time characteristics of EVs in the coupled transportation and distribution networks; ii) visualize the degree of charging load and transportation congestion. The framework of EV charging load modeling integrated with the trajectory is as follows:

- According to the survey data, the vehicle travel location is classified, and the lognormal probability function is used to fit the start travel time and time for going home.
- The daily destinations start from the home are generated by Monte Carlo sampling method.
- A simple or complex trip chain is established.
- For an EV $i$-th trip destination $q_i^{t,n}$, combined with specific transportation system model, Dijkstra's shortest path method and speed-flow model are utilized to determine its acquisition of travel route set $R_{i,n}^{t}$ and velocity.
- A charging demand model based on the fuzzy theory is established so that the end charging time $z_3$ and the charging position $q(x,y)$ of the EV are determined.
- According to the $z$, $c$, and the $(i+1)$th travel distance, decide whether mid-way charging is required.
- According to the daily driving/parking/ charging states of an EV, the points $Q_1$, $Q_2$, and $Q_3$ of the every transportation node $q$ are determined to construct a trajectory.
- Finally, by the repeated sampling, the spatial-temporal data of the charging load of each EV is obtained.
- The spatial-temporal distribution of the charging load of the EV in different regions is obtained.

3 Transportation system modeling

This section expounds how to model transportation network. The urban roads are described by the mathematical method in part A. Simplified speed-flow model is presented to describe the vehicle speed with the time-varying road congestion level. Part C provides a path selection method for several routes between source and destination.
3.1 Transportation network Topology

In this paper, graph theory [19] is used to model the two-way transportation network. $G=(V, E)$ is used to represent the map of the transportation network. It consists of the vertex set $V$ and connected edge set $E$. The matrix $D(G)$ is a $Na \times Na$ matrix of the road weight ($Na$ is the number of vertices) used to describe the distance between every two vertices. Taking Fig. 2 as an example, the intersections of multiple road segments constitute a vertex set $V={1,2,3,4,5}$. The transportation network contains five road segments, represented as $E=\{d_{12}, d_{14}, d_{23}, d_{24}, d_{34}, d_{25}, d_{35}\}$. The corresponding matrix $D$ is represented by (4). Once matrix $D$ is generated, the shortest travel path $R_n$ between the vertices is obtained by the shortest path algorithm [20].

$$D(G) = \begin{bmatrix}
0 & d_{12} & \text{inf} & d_{14} & \text{inf} \\
 d_{12} & 0 & d_{23} & d_{24} & d_{25} \\
 \text{inf} & d_{23} & 0 & d_{34} & d_{35} \\
 d_{14} & d_{24} & d_{34} & 0 & \text{inf} \\
 \text{inf} & d_{25} & d_{35} & \text{inf} & 0
\end{bmatrix} \quad (4)$$

where, $d_{nm}$ is the length of the link between vertex $n$ and vertex $m$, inf indicates that the two vertices are not adjacent or have no direct link.

![Fig. 2. Transportation network topology](image)

3.2 Simplified speed-flow model

In the urban transportation network, vehicle speed is closely related to the degree of road congestion. Considering the scale of existing urban vehicles and the development of transportation networks, a simplified speed-flow model is introduced to simulate the vehicle driving process. In this paper, the saturation of different roads in a time period is set to a uniform value, assuming that the saturation of different roads has a generally consistent temporal trend. At time $t$, the vehicle travel speed $v_{n1n2}(t)$ on the link between the adjacent nodes $n_1$ and $n_2$ is described by the formula $(5)-(6)$ [21].

$$r(t) = w_{n_1n_2}(t)/C_{n_1n_2} \quad (5)$$

$$v_{n_1n_2}(t) = v_{n_1n_2}^0 \left(1 + \left(r(t)\right)^{1/v'(r)}\right) \quad (6)$$

where, $r(t)$ is the road saturation at time $t$, $w_{n_1n_2}(t)$ is the road $(n_1, n_2)$ segment flow at time $t$. $C_{n_1n_2}$ is the road
capacity, $j$, $k$, $l$ are the adaptive coefficients, and $v_{n_{n}}^{0}$ is the free speed of the corresponding section.

3.3 Path selection method

The starting point $i$ and destination $j$ of each trip can be generated by the trip chain. The known EV starting and destination points can be represented by the driving path $R={i,..., e, f,..., j}$ composed of vertices and edges. However, it is common to find several routes between $i$ and $j$. Suppose that the driver will not go around the original road and choose the shortest path. The shortest driving path set $R$ and total mileage $D_{ij}$ can be obtained by Dijkstra shortest path method. Dijkstra algorithm utilizes labeling to find the shortest path from $i$ to $j$, which is the least weight (element in road weight matrix $D$). Each iteration produces a permanent label. $p(j)$ is the label of $i$; $q(j)$ is the parent vertex of $j$. The specific steps are as followed:

1) Initialization, $l(j)=0$;

2) Iteration. Update $l(j)$ and $q(j)$. Existing set $S$ and set $T$, $S \cap T= \phi$, $S \cup T$ includes all vertices. If $i \in T$ and $l(i)$ is shortest, add $i$ to $S$. For all $j$ vertices of $T$, if $l(j) > l(i)+D(i,j)$, update $l(j)$ and $q(j)$. $l(j)=l(i)+D(i,j)$, $q(j)=j$.

3) Repetition step 2) until all vertices are in set $S$.

4 Charging load model

A complete journey of EV consists of the travel plan, driving route, charging time and location. This section describes how to simulate driving, parking, and charging based on vehicle-transportation-grid trajectory. In part A, the travel demand model is represented, including destination, departure time. The electricity consumption is determined by part B. The last two give methods of calculating EV charging load.

4.1 Travel Demand model

The trip chain can well describe the daily travel behaviours of users. This section combines the travel chain, conducts the user travel demand simulation under the transportation network constraints, and obtains the vehicle route. Travel chain refers to the form of connection where individuals travel for different purposes in a certain time order to complete one or more activities [22]. In general, private EV users travel between certain nodes of the urban transportation network, and their daily travel destinations are relatively fixed. Three major destinations attributes are categorized: family, work, business (leisure). It can be concluded that EV travels between these three destinations
and the charging behaviour may occur at the passing-through nodes.

In the trip chain, it is impossible to charge a few extremely short stops (such as staying at the school when taking a child to school on the way to work). Table II shows the combinations of various travel chains with the home as the starting point [23], where, H, W, and R represent for the residential area, working area, and business area respectively, while HW represents traveling from the residential area to the work area. For various travel patterns, user travel requirements can be expressed by state parameters such as the starting and stopping place and distance, starting and stopping time and length of each trip in Table III, as shown in Eq. (7).

\[
TC = \{L_0, L_p, X_d, T_0, T_p, X_s, T_s\}
\]  

(7)

Table II. Combinations of daily trip chains

<table>
<thead>
<tr>
<th>Serial Number N</th>
<th>Simple Chain (Nr=2)</th>
<th>Serial Number N</th>
<th>Complex Chain (Nr=3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work day</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>HW-WH</td>
<td>4</td>
<td>HW-WR-RH</td>
</tr>
<tr>
<td>2</td>
<td>HH-HH</td>
<td>5</td>
<td>HR-RW-WH</td>
</tr>
<tr>
<td>3</td>
<td>HR-RH</td>
<td>6</td>
<td>HH-HW-WH</td>
</tr>
<tr>
<td>4</td>
<td>HW-WH</td>
<td>7</td>
<td>HW-WH-HH</td>
</tr>
<tr>
<td>5</td>
<td>HH-WW-WH</td>
<td>8</td>
<td>HW-WW-WH</td>
</tr>
<tr>
<td>Holiday</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>HH-HH</td>
<td>9</td>
<td>HH-HH-HH</td>
</tr>
<tr>
<td>3</td>
<td>HR-RH</td>
<td>10</td>
<td>HR-RR-HH</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>12</td>
<td>HH-RR-HH</td>
</tr>
</tbody>
</table>

Table III. State parameters of EVs

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Instruction</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_i)</td>
<td>Departure place for route (i)</td>
<td>(Q^{-1}(x^{-},y^{-}))</td>
</tr>
<tr>
<td>(L'_{p,i})</td>
<td>Parking place for route (i)</td>
<td>(Q^{-1}(x',y'))</td>
</tr>
<tr>
<td>(X'_{d,i})</td>
<td>Mileage for route (i)</td>
<td>(\sum_{a\in[1]} \left( x'<em>{i,a}, y'</em>{i,a}, x^{'-}_{i,a} \right) )</td>
</tr>
<tr>
<td>(T_0)</td>
<td>Departure time for route (i)</td>
<td>(Z_0^{-1})</td>
</tr>
<tr>
<td>(T'_{p,i})</td>
<td>stop time for route (i)</td>
<td>(Z'_0^{-1})</td>
</tr>
<tr>
<td>(X'_{s,i})</td>
<td>stop duration for route (i)</td>
<td>( Z'<em>{i} \cdot Z</em>{0}^{-1})</td>
</tr>
<tr>
<td>(Cap_t)</td>
<td>electricity of time (t)</td>
<td>(c^{-1}(z=t))</td>
</tr>
</tbody>
</table>

Considering the transportation network constraints, the MC method is used to simulate the travel of users, then some key elements of EV paths and the whole trajectory are obtained. Assuming that the driver does not detour the original path in the trajectory, and the shortest path, which can be obtained by Dijkstra shortest path method is chosen [16]. The simulation process of travel demand is shown in Figure 3.
The duration of $i$ journey $X^i$, the parking time $T_p^i$ and the next departure time $T_{0}^{i+1}$ are as follows:

$$X^i = \sum_{h=1}^{g} \frac{d_h}{V_h(t)}$$  \hspace{1cm} (8)

$$T_p^i = T_0^i + X^i + T_{mid}^i$$  \hspace{1cm} (9)

$$T_{0}^{i+1} = T_p^i + T_s^i$$  \hspace{1cm} (10)

where, $g$ is the number of sections; $d_h$ is the length of road $h$, $V_h(t)$ is speed in road $h$, $T_{mid}$ is the duration of charging in midway, $T_p^i$ is parking time, $T_s^i$ is parking duration.

4.2 Electricity consumption model based on travel demand

The realization of EV's transportation function is satisfied by electricity consumption. When EV is driving, its power consumption model is established. This paper assumes that the power consumption of EV is constant. With the increase of driving distance $X_d^i$, the real-time power $Cap_t$ of EV decreases linearly. Before reaching the next parking place, the charging state of EV can be determined by the following formula:

$$Cap_{T_p}^i = (Cap_{T_0}^i - \frac{X_d^i \cdot w}{C}) \cdot 100$$  \hspace{1cm} (11)

where, $Cap_{T_p}^i$ is the SOC of parking EV at trip $i$ finished, $Cap_{T_0}^i$ is the initial SOC of EV at trip $i$, $X_d^i$ is the driving distance at trip $i$, $w$ is the power consumption per distance, $C$ is the battery capacity of EVs.
4.3 Charging demand model based on user's intention

The user's charging demand is closely related to whether the EV state of charge can meet the following travel journey. The charging behaviors of EV users are uncertain. According to their subjective intentions, there are two states: charging immediately after the end of driving and charging when the driving demand cannot be met [24]. It is assuming that the user would not cancel the scheduled journey due to the lack of EV Cap. Whether to charge or not depends on how rich the SOC it is for the next trip. In general, when Cap, does not meet the next travel demand, they will choose to charge. When Cap, meets the next trip demand, the users’ choices are uncertain. And the more sufficient the Cap, is compared with the next trip demand, the weaker the user's charging demand.

Fuzzy theory is used to describe the charging intention of EV users [25]. The index "Electricity Satisfaction $U_f$" is defined to measure the current electricity demand for the next trip. The EV user will choose to charge or not according to the SOC($Cap_i$), which is determined by formulas (12)-(14).

$$U_f = \frac{Cap_{Tp_i} \times C}{w \times X_{d_{i+1}}}$$  \hspace{1cm} (12)

where, $Cap_{Tp_i}$ is the SOC of parking EV at the destination $i$, $X_{d_{i+1}}$ is the estimated distance of the next trip.

If $M$ represents a fuzzy set with ‘charging demand’, then the membership function of $M$ can be determined by:

$$M(U_f) = \begin{cases} 
1, & U_f < l \\
m_1, & l < U_f < u \\
0, & U_f > u 
\end{cases} \hspace{1cm} (13)$$

$$m_1 = \sin\left[\frac{\pi}{4} + \frac{\pi}{4} \times \left(\frac{1}{u-l} \times \left(\frac{u+l}{2} - U_f\right)\right)\right] \hspace{1cm} (14)$$

where, $M(U_f)$ is the degree of membership, designed to represent the probability of users charging demand as (13), whose range is [0, 1]. $l$ is the lower coefficient, $U_f < l$ represents that the SOC cannot meet the demand of next trip, i.e. EV users must charge, $u$ is the upper coefficient, $U_f \geq u$ represents the SOC is sufficient for next trip, EV users will not choose to charge, $m_1$ indicates vague charging demand.

Once the EV user determines to charge, the charging mode needs to be selected. Considering some factors such as charging price and battery life, the default is slow-charging mode. The charging conditions for a single EV are assumed as follows: During the parking time, if the slow-charging power cannot meet the charging demand, the fast-charging power is used. Otherwise, the slow-charging power is used. If formula (15) is satisfied, the EV chooses to charge at the $i_{th}$ parking place by fast-charging power.
where, $P_s$ is the slow-charging power, $T_i$ is the parking time at the destination $i$, $Cap_{r_i}$ is the remaining energy when reaching the location $i$.

Assuming that travel is not abandoned because of the insufficient electricity. If EV cannot arrive at the destination with full capacity for a long mileage trip, the user will choose to charge during the journey. However, it takes unnecessary time for charging in midway. Therefore, considering battery safety and user mileage anxiety, users will only choose midway charging under the condition of (16).

$$Cap_i^l < Cap_m$$

where, $Cap_m$ is the real time electricity when SOC reaches the threshold, and the threshold is set to uniform distribution between 0.15 and 0.3 [23].

The charging location is the nearest transportation node to the destination where the energy doesn’t achieve $Cap_m$, i.e., the charging location $s$ is determined by equations (17)–(19). The fast charging mode is selected for the middle charging, and the duration time of the middle charging is given by formula (20).

$$X_s^{Cap} = (Cap_i^l - Cap_m) \cdot C/h$$

$$\sum_{h=1}^{s} d_h < X_d^{Cap}, s_n \in \{1, 2, ..., g\}$$

$$s = R'(s_n)$$

$$T_{mid} = (1 - Cap_i^l + \frac{h \cdot \sum_{h=1}^{s} d_h}{C}) \cdot C / P_q$$

where, $X_s^{Cap}$ is the travel distance corresponding to the real time energy $Cap_i$ sunk to $Cap_m$, $\sum d_h$ is the distance of the $s_n$ section in a trip, $\{1,2,..,N\}$ is the set satisfying inequality (18), $R'(s_n)$ represents the node number of the $s_n$-th node in the route, $P_q$ is the fast-charging power, all other variables are as the previous definitions.

The specific steps of the charging demand simulation are shown in Figure 4.
4.4 Charging load calculation by the spatial-temporal simulation method

Monte Carlo method is used to calculate the charging power of EVs in the test area within the typical day for analysis of spatial-temporal characteristic. It is worth noting that the simulation of charging for a single EV in one day can be used to generate sequential data for evaluation or planning. The simulation time interval is 15 minutes. The specific steps are as follows:

1) For a single EV, simulate the travel and charging process, and establish a travel demand model based on transportation system constraints (part A), a power consumption model based on travel demand (part B), and a charging demand model based on subjective intention of the user (part C).

2) The variables of the whole trajectory are obtained for each trip, such as the start charging time, the charging location, the charging power, and the charging duration. The specific simulation processes are shown in preamble Fig.3 and Fig.4.

3) In each Monte Carlo simulation, for $M$ EVs, step 1 and 2 are repeated $M$ times. The charging demand of each vehicle is recorded and the total load in the distribution network are calculated according to the coupling relationship between the transportation system nodes and the distribution network buses.
4) For the distribution network bus \( a \), the total charging load \( P_a(t) \) at time \( t \) can be expressed by (21), \( P_m^a(t) \) is the charging power of the \( m \)-th EV at bus \( a \).

\[
P_a(t) = \sum_{m=1}^{M} P_m^a(t)
\]  

(21)

5) According to the daily charging power superposition of the \( N_b \) buses, the total charging load \( P_{\text{total}}(t) \) of the planning area can be obtained by (22).

\[
P_{\text{total}}(t) = \sum_{a=1}^{N} P_a(t)
\]  

(22)

6) After completing a Monte Carlo simulation, store the total charging load \( P_{\text{total}}(t) \) in the distribution network charging power matrix \( H(N_b \times 96) \). When the maximum number of simulations \( N \) is reached or the convergence condition is met, the simulation terminates. The convergence condition is:

\[
\max[|U_r^H - U_{r-1}^H|] < \varepsilon_1
\]  

(23)

where, \( H_r \) represents the corresponding column vector in the charging power matrix \( H \) at time \( t \), \( U_{r-1}^H \) represents the mean value of each time after the \( r \)th Monte Carlo simulation.

5 Test cases

5.1 Case overview and parameter settings

The transportation system and distribution network topology are shown in Figure 5. The nodal couple relationship of the two could be seen in Appendix Table A. The road network consists of 30 buses and 52 roads. This area is divided into residential area 1 (transportation nodes 1-11), residential area 2 (transportation nodes 12-17), work area (transportation node 18-22), commercial area 1 for leisure (transportation node 23-26), commercial area 2 for leisure (including transportation node 27-30). The distribution network including 33 nodes. The road length is represented in Appendix Table B. The road saturation in different period is listed in Appendix Table C. The number of EVs refers to the private car data of a city in 2015, which is 2.32 million. Thus, the EV ownership is assumed to be 2.4 million in the region. The parameter of EV could be seen in Appendix Table D. The initial SOC is set to the level of electricity after full of energy. The lower bound coefficient \( e = 1.2 \), the upper bound coefficient \( u = 2 \), and the time interval of the
sequential simulation are 15 minutes. In order to reduce the calculation time and simplify the optimization process, it is assumed that the travel characteristics of private electric vehicles are similar to those of fuel vehicles. A part of trip chains in Table I are utilized to illustrate the model and solution method in this paper. The time of first travel and return home obey normal distribution, and the leisure time obeys uniform distribution $U(1, 2)$.[23].

5.2 Results analysis

The simulated charging demand a week in the region is shown in Figure 6. The results of different days are similar due to the randomness of the Monte Carlo method. For workdays, although the maximum value is different, the trend of the curve is the same on each day, which contains double peaks. Figure 7 presents the charging power of several nodes in different regions. Node 1 is in a residential area, Node 18 is in a work area, and Node 30 is in a commercial area. Two important points are noted about this figure: load demand has different characteristics and the results obtained by simulation method has uncertainties (i.e., daily load profile are not the same shape, especially demand of the third day at node 28 compared with the others).
The charging demand of each bus in the region on typical workday and holiday are shown in Figure 8. The red and black line on the 3D surface represents the travel route of an EV, which corresponds to information of trajectory and SOC. \( \varepsilon \) is the convergence accuracy of the simulation, set to \( \varepsilon=0.1 \), and the maximum Monte Carlo simulation times \( N \) is 1000.
It can be seen from Figure 8 that the spatial-temporal charging demand of every node in the region. On a workday, charging demand concentrates in 18-22 nodes at 07:00-11:00, who are in the working area; at 17:00-21:00, charging demand in residential and commercial is more concentrated. On holiday, at 08:00-14:00, charge demand in the residential is clear, caused by that the EV users choose to travel in the morning (about 06:00-12:00). The charging demand in the commercial area on holiday is obviously higher than that on workdays, and the charging demand in the work area is contrary. Compared with the holiday, the peak load of the workday (6298.382kW) is 3.8 times that of the holiday(1651.405kW), the total average load demand of workday (174.479MW) is 72.01% higher.
than that of holiday (101.435MW).

Regarding the END index, the spatial distribution of EV driving and parking is obtained by 1000 Monte Carlo simulations, as shown in Figure 9 and Figure 10. The real line in Figure 9 represents the road, and the dot represents the equivalent number of EV driving on the road in a day. The circle in Figure 10 represents the equivalent number of EV parking on the node in a day. Gray represents residential areas, black represents business areas, and white represents work areas, K represents 1000 unit.

The EV travel trajectory is mainly concentrated in residential to commercial roads (6-24, 7-25, 8-26, 15-28, 16-29) and commercial to work area roads (24-19, 25-20, 26-21, 19-28, 20-29) as Fig.9. Furthermore, the EV for travel on workday is obviously more than that on holiday. Fig. 11 demonstrates that the fast charging load is mainly
concentrated in the commercial area.

![Fig. 11. Numbers of EVs with fast-charging power in a day](image)

The definition of EV permeability $P_{EV}$ is the ratio of charging load peak to base load peak. According to the results of the spatial-temporal distribution of charging load, the total charging demand with different EV permeability is given in Fig. 12. It can be seen that the overlapping of 18:00-20:00 charging load and traditional load forms a peak at night, and the peak load of the system (19.864MW) increases by 20.05% compared with the peak value of basic load (16.547MW) on the workday, for 20% permeability of EV. The peak load (15.402MW) increases by 38.06% compared with the peak value of basic load (14.562MW) on holiday for 5.77% permeability of EV. With the increasing EV permeability, the peak-valley difference of the overall load gradually increases from 9.67MW to 13.43MW on holiday and from 9.98MW to 27.63MW. Furthermore, the midday load on the holiday (4.21MW) increased significantly compared to that on the workday (0.11MW).

![Fig.12 The load curve of different penetrations](image)

In order to verify that the charging load may change with the ratio of the daily trip chain, the following three cases are studied.

1) Case 1: The proportion of EV traveling by 2-trip chains on workday is 50%, others in 3-trip chains.
2) Case 2: The proportion of EV traveling by 2-trip chains on workday is 30%, others in 3-trip chains.

3) Case 3: The proportion of EV traveling by 2-trip chains on workday is 10%, others in 3-trip chains.

From the three cases, the nodal and total charging demand could be obtained in Figure 13. The peak charging power of three cases is 24.727MW (at 19:00), 26.792MW (at 20:00), 29.358MW (at 20:00) respectively. The total power of three cases is 174.5MW, 186.7MW, 198.7MW respectively. The total charging power increases with the 3-trip chains, which increases by 600kW per one percent approximately. Furthermore, as a result of leisure activities, the charging demand at 5:00-12:00 of case1 is more than those of case 2 and case 3. Thus, the EV users with leisure plan have more charging demand because of the longer distance.

![Fig. 13. The charging power of different trip chain ratio](image)

6 Conclusion

This paper presents a spatial-temporal simulation method based on the vehicle-transportation-grid trajectory, and the validity is verified by a transportation and distribution coupled test case. The results show that the proposed vehicle-transportation-grid trajectory and its indices of the proposed method can reasonably describe the driving, parking and charging state of EV, and the proposed method fully considers the transportation system constraints, the travel demand, and uncertain charging choices of users. Two typical days, “workday” and “holiday”, were simulated and compared under different EV penetration levels (0%, 20%, 50% and 100%), different trip chain ratio (the ratio of 3-trip chains is 50%, 70%, 90%) to verify the effectiveness of the spatial-temporal simulation method. The driving, parking, and charging of EVs have different distributions in residential working and commercial areas.
in the workday or holiday. The charging demand could change as the result of the ratio of the trip chain and the EV penetration. Those results are obtained by America national household travel survey (NHTS) data. During the regional diversity, more traffic survey combined with the local situation should be carried out. In addition, more key factors need to be considered in future research, including battery technology, diversified roads, and long-distance routes.

Acknowledgment

This work was supported by the Scientific Foundation for the National Natural Science Foundation of China (51807127), the Sichuan Science and Technology Program (2019YFH0171) and International Visiting Program for Excellent Young Scholars of Sichuan University.

References


## Appendix

### TABLE A. Corresponding node-bus connecting distribution network and transportation system

<table>
<thead>
<tr>
<th>Node#</th>
<th>Bus#</th>
<th>Node#</th>
<th>Bus#</th>
<th>Node#</th>
<th>Bus#</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>12</td>
<td>12</td>
<td>23</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>13</td>
<td>13</td>
<td>24</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>14</td>
<td>14</td>
<td>25</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>15</td>
<td>15</td>
<td>26</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>16</td>
<td>16</td>
<td>27</td>
<td>26</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
<td>17</td>
<td>29</td>
<td>28</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>18</td>
<td>28</td>
<td>29</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>21</td>
<td>19</td>
<td>5</td>
<td>30</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>22</td>
<td>20</td>
<td>23</td>
<td>31</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>21</td>
<td>18</td>
<td>32</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>17</td>
<td>22</td>
<td>27</td>
<td>33</td>
<td>-</td>
</tr>
</tbody>
</table>

### TABLE B. Road length

<table>
<thead>
<tr>
<th>Road</th>
<th>length(km)</th>
<th>Road</th>
<th>length(km)</th>
<th>Road</th>
<th>length(km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,2)</td>
<td>10.2</td>
<td>(9,11)</td>
<td>10.1</td>
<td>(18,27)</td>
<td>9.7</td>
</tr>
<tr>
<td>(1,4)</td>
<td>10.4</td>
<td>(10,11)</td>
<td>9.0</td>
<td>(19,20)</td>
<td>10.0</td>
</tr>
<tr>
<td>(2,3)</td>
<td>10.3</td>
<td>(10,21)</td>
<td>10.1</td>
<td>(19,24)</td>
<td>10.4</td>
</tr>
<tr>
<td>(2,5)</td>
<td>10.5</td>
<td>(11,22)</td>
<td>10.2</td>
<td>(19,28)</td>
<td>8.8</td>
</tr>
<tr>
<td>(2,6)</td>
<td>10.6</td>
<td>(12,13)</td>
<td>10.3</td>
<td>(20,21)</td>
<td>9.1</td>
</tr>
<tr>
<td>(3,4)</td>
<td>10.4</td>
<td>(12,16)</td>
<td>10.6</td>
<td>(20,25)</td>
<td>10.5</td>
</tr>
<tr>
<td>(3,7)</td>
<td>10.7</td>
<td>(12,17)</td>
<td>10.7</td>
<td>(20,29)</td>
<td>8.9</td>
</tr>
<tr>
<td>(4,8)</td>
<td>10.8</td>
<td>(13,14)</td>
<td>10.4</td>
<td>(21,22)</td>
<td>10.2</td>
</tr>
<tr>
<td>(4,9)</td>
<td>9.9</td>
<td>(13,15)</td>
<td>10.5</td>
<td>(21,25)</td>
<td>10.5</td>
</tr>
<tr>
<td>(5,6)</td>
<td>10.6</td>
<td>(14,15)</td>
<td>9.5</td>
<td>(22,30)</td>
<td>11.0</td>
</tr>
<tr>
<td>(5,23)</td>
<td>10.3</td>
<td>(14,28)</td>
<td>9.7</td>
<td>(23,24)</td>
<td>10.4</td>
</tr>
<tr>
<td>(6,7)</td>
<td>9.7</td>
<td>(15,16)</td>
<td>8.6</td>
<td>(24,25)</td>
<td>9.5</td>
</tr>
<tr>
<td>(6,24)</td>
<td>9.4</td>
<td>(15,28)</td>
<td>8.8</td>
<td>(25,26)</td>
<td>9.6</td>
</tr>
<tr>
<td>(7,8)</td>
<td>10.8</td>
<td>(16,17)</td>
<td>10.7</td>
<td>(27,28)</td>
<td>9.8</td>
</tr>
<tr>
<td>(7,25)</td>
<td>9.5</td>
<td>(16,29)</td>
<td>8.9</td>
<td>(28,29)</td>
<td>8.9</td>
</tr>
<tr>
<td>(8,10)</td>
<td>10.0</td>
<td>(17,30)</td>
<td>11.0</td>
<td>(29,30)</td>
<td>11.0</td>
</tr>
<tr>
<td>(8,26)</td>
<td>9.6</td>
<td>(18,19)</td>
<td>8.9</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(9,10)</td>
<td>9.9</td>
<td>(18,23)</td>
<td>10.3</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### TABLE C. Road saturation in different time periods

<table>
<thead>
<tr>
<th>period</th>
<th>saturation</th>
<th>period</th>
<th>saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>00:00-07:00</td>
<td>0.2</td>
<td>14:00-17:00</td>
<td>0.3</td>
</tr>
<tr>
<td>07:00-09:00</td>
<td>0.5</td>
<td>17:00-19:00</td>
<td>0.5</td>
</tr>
<tr>
<td>09:00-12:00</td>
<td>0.3</td>
<td>19:00-23:00</td>
<td>0.3</td>
</tr>
<tr>
<td>12:00-14:00</td>
<td>0.4</td>
<td>23:00-24:00</td>
<td>0.2</td>
</tr>
</tbody>
</table>

### TABLE D. Parameter of EV

<table>
<thead>
<tr>
<th>Power consumption per km (kWh/km)</th>
<th>Capacity (kwh)</th>
<th>Common charging power (kw)</th>
<th>Fast charging power (kw)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>20</td>
<td>7</td>
<td>30</td>
</tr>
</tbody>
</table>