Strategy to Achieve Phase Matching Condition for Third Harmonic Generation in All-Solid Photonic Crystal Fibers

Jinxu Bai, Wenquan Xing, Chang Yang, Yanfeng Li, and David M. Bird

Abstract—All-fiber based devices for third harmonic generation in the ultraviolet range are highly desirable. Using a fundamental wavelength of 1.06 μm as an example, we show numerically how the phase matching condition for third harmonic generation can be achieved between an index-guided fundamental HE_{11} mode for the infrared source and a bandgap-guided higher-order HE_{12} mode for the ultraviolet radiation in all-solid photonic crystal fibers. The fiber parameters are first determined by an analytical effective index model and then improved by numerical calculations.

Index Terms—photonic crystal fibers, photonic bandgaps, third harmonic generation, phase matching, fiber design

I. INTRODUCTION

There has been interest in the generation of third harmonics in the important ultraviolet (UV) range in a fiber format with pulsed fiber or microchip lasers as the source and tapered fibers as the nonlinear medium [1-3]. The waveguiding properties of the fibers imply good mode confinement and an extended interaction length, and at the same time a high pump intensity can be achieved due to the small fiber diameter. These factors can compensate for the weak nonlinearity of the fiber material—typically silica, with the lowest-order nonlinearity being the third-order. Thus, UV radiation could be generated from fibers of centimeter length [2, 3]. A fiber frequency converter requires fewer optical components and makes alignment easier than in a two-stage scheme using nonlinear bulk crystals with second harmonic generation first and then sum frequency conversion.

Fig. 1 Schematic of the ASPCFs with Ge-doped high-index rods of diameter d and pitch Λ in silica. The doped core region with diameter D allows more defect mode control.

The phase matching condition (PMC) for third harmonic generation (THG) from fibers requires the two modes involved to have the same effective index. Phase-matched THG from the fundamental infrared (IR) source to the third-harmonic UV is not possible in a conventional fiber when both the IR and UV are guided in the same fundamental spatial mode, because material dispersion and waveguiding effects combined make the third harmonic have a higher effective index than the fundamental [1]. In tapered fibers [2, 3], PMC was achieved between a fundamental HE_{11} mode for the IR and a higher-order HE_{12} mode for the UV which has the largest mode overlap with the HE_{12} mode among all possible higher-order modes [1]. Alternatively, PMC could be achieved between a pair of HE_{11} modes in all-solid photonic crystal fibers (ASPCFs) with interstitial air holes (IAHs) [4], where the IR mode is index-guided and the UV mode is bandgap-guided. The problems with tapered fibers are that the uniformity of the half-micron sized tapers has to be strictly controlled and the limited taper length puts a constraint on the conversion efficiency [2, 3]. The difficulty with the latter design is that the IAHs are in the 100 nm range and also need precise control. Besides, phase matching is only marginally possible at wavelengths around 1 μm [4].

In this letter, we propose a design scheme for THG in ASPCFs, where PMC is achieved between an index-guided fundamental HE_{11} mode for the IR source and a bandgap-guided higher-order HE_{12} mode for the UV light. We show that an analytical effective index model (EIM) gives fast and accurate fiber parameters approximation, which is then confirmed by numerical simulations. The use of an all-solid structure with material doping could make this design easier to be realized and might also be able to eliminate the observed
second harmonic generation in tapered fibers [2,3], which could be attributed to the surface nonlinearity present at the glass/air interface.

Fig 2. DOS map calculated for an ASPCF with $d=0.7\Lambda$ and nominal refractive indices of $n_{\text{rod}}=1.48$ and $n_{\text{clad}}=1.45$. Grey scale represents DOS of propagating states (white = high DOS) and the bandgaps are shown in red. The horizontal line is the core line (here, $n_{\text{core}}=1.47$ for illustration). The three dots illustrate the modes considered for THG.

II. RESULTS AND DISCUSSION

We consider the possibility of phase-matched THG in ASPCFs for a pump wavelength of $1.06\ \mu m$, where compact and efficient pulsed fiber and microchip lasers are readily available. ASPCFs [4, 5] are a class of PCFs [6] that are formed by a periodically arranged microstructure in the transverse direction, most often doped rods in silica. Shown in Fig. 1 is the schematic of the ASPCFs considered in this work, which consist of Ge-doped high-index rods (index $n_{\text{rod}}$) in a silica background (index $n_{\text{silica}}$) and a region with a different level of doping as the core (index $n_{\text{core}}$). The other parameters that characterize the fibers are rod diameter $d$, pitch $\Lambda$, and core diameter $D$.

According to the antiresonant reflecting optical waveguide model [7, 8], the photonic bandgaps in ASPCFs arise due to the anti-resonances of the modes of the individual high-index cladding rods. Such a description is exemplified by the density of states (DOS) map shown in Fig. 2 calculated by a plane wave expansion method [9] for a fiber structure of $d=0.7\Lambda$, $n_{\text{rod}}=1.48$, and $n_{\text{silica}}=1.45$. The bandgaps are shown in red and the rod modes in the weak-guidance approximation are labeled along the side.

When the local refractive index of the core is higher, the donor-like defect will produce guided modes from the neighboring lower photonic bands [10]. The effective index of the guided modes can be tuned with a variation of the index of the core. If the defect index is higher than the effective index of the cladding, the mode is typically described as an index-guided mode [6]. It is even possible to have index- and bandgap-guided modes simultaneously at wavelengths within the bandgaps [10, 11]. Such a possible scenario is illustrated in Fig. 2, where there is an index-guided fundamental HE_{11} mode at the IR source wavelength and there are two bandgap-guided modes at the third harmonic: a fundamental HE_{11} mode and a higher-order HE_{12} mode. Note that material dispersion is neglected in Fig. 2 and the index difference between the HE_{11} mode at the source wavelength and the HE_{12} mode at the harmonic wavelength, induced by the waveguiding effect, can be sufficient to compensate for the material dispersion of silica for a source wavelength at $1.06\ \mu m$ ($\sim 0.026$ for silica [4]). The index difference between the pair of HE_{11} modes is not large enough because the high-index core also causes the HE_{11} mode index at the third harmonic to be elevated. Therefore, our aim will be to find the fiber parameters that can fulfill the PMC for THG between the IR HE_{11} mode and the UV HE_{12} mode.

There are a whole family of parameters to be determined: rod diameter $d$ and index $n_{\text{rod}}$, pitch $\Lambda$, and defect diameter $D$ and index $n_{\text{core}}$. Tuning each parameter by a numerical procedure would be a tedious process. The advantage with ASPCFs, however, is that the bandgap structure can be calculated based on analytical means [12]. Accordingly, our strategy is to use an analytical method—EIM [13] to first obtain the approximate parameters and then use a more accurate numerical method to fine tune the parameters. The EIM models the complicated ASPCF in Fig. 1 as a step-index fiber. The equivalent step-index fiber has the same core radius and refractive index as the original ASPCF but with an effective cladding index $n_{\text{clad}}$, which defines the lower range of operation for each mode (HE_{11} or HE_{12} mode), and is taken as the effective index of the top of the LP_{01} and LP_{02} bands in Fig. 2, respectively.

With the EIM, we can quickly calculate the bandgap structure and the bandgap edges (and hence $n_{\text{clad}}$) for a particular ASPCF, where all refractive indices are assumed to be constant for a certain rod diameter $d$ and defect diameter $D$/A. Then, the effective mode indices of the HE_{11} and HE_{12} modes are compared to obtain the required difference to compensate for that due to material dispersion. Once appropriate normalized values are found, the fiber parameters are converted to realistic ones for an IR wavelength of $1.06\ \mu m$.

**Table 1:**

<table>
<thead>
<tr>
<th>$k_{\text{A}}$ for IR</th>
<th>$k_{\text{A}}$ for UV</th>
<th>HE_{11} mode</th>
<th>HE_{12} mode</th>
<th>Index difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>22.5</td>
<td>1.46393</td>
<td>1.43737</td>
<td>0.03020</td>
</tr>
<tr>
<td>8.0</td>
<td>24.0</td>
<td>1.46400</td>
<td>1.43717</td>
<td>0.02683</td>
</tr>
<tr>
<td>8.5</td>
<td>25.5</td>
<td>1.46408</td>
<td>1.44019</td>
<td>0.02389</td>
</tr>
<tr>
<td>9.0</td>
<td>27.0</td>
<td>1.46417</td>
<td>1.44285</td>
<td>0.02132</td>
</tr>
</tbody>
</table>

Table 1 lists the results calculated for an ASPCF with $d=0.7\Lambda$, $D=1.1\Lambda$, $n_{\text{silica}}=1.45$, $n_{\text{rod}}=1.48$, and $n_{\text{core}}=1.47$. It is seen that the required index difference is obtained at around $k_{\text{A}}\Lambda=8.0$. This will yield $\Lambda=1.36\ \mu m$ for an IR wavelength of $1.06\ \mu m$ and $d$ and $D$ are scaled accordingly. The level of GeO$_2$ doping required to achieve the indices of the rods and the defect are calculated based on [14] to be 20 mol% and 16 mol%, respectively.

When the material dispersion of all constituents [14] is taken into consideration, the phase matching curves obtained by the EIM and by a finite difference method (FDM) are compared in Fig. 3(a). It is seen that the EIM gives a somewhat longer phase-matched wavelength than the FDM by ~3 nm because the EIM is less accurate in calculating the HE_{12} mode index. The phase-matched wavelength can be finely tuned by the FDM. The problem with the above set of parameters is that both modes are weakly confined. This is because the core-cladding difference for the HE_{11} mode is low and the HE_{12} mode is close...
to the bandgap edge. Better mode confinement can be achieved by making the index of the core higher while varying its size to get the desired index difference. Shown in Fig. 3(b) is another set of fiber parameters that fulfill the PMC: \(d=0.7\mu m\), \(D=1.0\mu m\) and \(\Lambda=1.25 \mu m\) with a nominal core index \(n_{core}=1.485\) (doping level 22 mol\% [14]). This time, \(\Lambda\) has been chosen slightly smaller than that predicted by the EIM due to the numerical difference.

With the above parameters, the high-index rods are around 1 \(\mu m\), and the sizeable structure without small air holes, which tend to collapse during drawing, will make fabrication easier. A precise control of the various parameters like fiber size and index contrast is still needed. However, as can be seen in Fig. 4 for this ASPCF, although the HE\(_{11}\) mode can be confined in the first several rings of rods, its spreading would imply a small mode overlap, which is estimated to be more than 100 times smaller than in [1]. This could be compensated for by a higher pump power, which can be a problem in tapered fibers, or other fiber designs can be envisaged, like high-index contrast ASPCFs [5]. These will be a subject of further study.

### III. CONCLUSION

We have demonstrated that it is possible to achieve phase matching for THG in ASPCFs between an index-guided HE\(_{11}\) mode and a bandgap-guided HE\(_{12}\) mode. The analytical solution by EIM gives fast and accurate fiber parameter estimations, as verified by numerical calculations. The design strategy could be useful in other design situations.

![Fig. 3 Comparison of the phase matching curves as a function of the source wavelength calculated by EIM and finite difference method. (a) ASPCF with \(d=0.7\mu m\), \(D=1.1\mu m\), and \(\Lambda=1.36 \mu m\). GeO\(_2\) doping is 20 mol\% for the rods and 16 mol\% for the core. (b) ASPCF with \(d=0.7\mu m\), \(D=1.0\mu m\), and \(\Lambda=1.25 \mu m\). GeO\(_2\) doping is 20 mol\% for the rods and 22 mol\% for the core.](image)

![Fig. 4 Field distribution of Ex-component of the x-polarized (a) HE\(_{11}\) mode and (b) HE\(_{12}\) mode for the ASPCF of Fig. 3 (b).](image)

### REFERENCES