**Spatio-Temporal Nonlinear Optics in Arrays of Subwavelength Waveguides**

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**Abstract:**
Spectral broadening in an array of subwavelength silicon waveguides pumped with fs pulses is studied. Adjusting input pulse position, different spectral patterns are observed and explained using the resonant emission from temporal supermode solitons.

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The interplay of spatial and temporal effects in nonlinear systems is an active and challenging research topic spanning several areas of physics [1]. Our aim is to deal with nonlinear propagation of electromagnetic spatiotemporal wave packets, where the ratios between the diffraction (spatial scale) and dispersion (temporal scale) lengths and the characteristic nonlinear length are critical for the system dynamics. Recent progress in fabrication has made it possible to achieve very large values of anomalous GVD induced by the strong geometric confinement in subwavelength silicon waveguides (photonic wires) [2], thereby creating an opportunity to investigate whether an array of such waveguides could be used as an experimental platform to study spatiotemporal nonlinear dynamics [3].

Below, we study spatiotemporal effects in coupled silicon waveguides leading to spectral broadening with a pronounced dependence on the spatial symmetry of the eigenmodes of the array (supermodes). We explain features of the observed spectra through the formation of temporal solitons in the individual supermodes and emission of the associated dispersive radiation.

Our system is an array of three coupled silicon wires. Each wire is 220 nm thick, 380 nm wide and 3.2 mm long with the zero-GVD wavelength at $\lambda=1.62 \text{ \mu m}$. The wall-to-wall separation between the wires is 600 nm. At this separation the wires are coupled through the evanescent tails, and the mode profiles and dispersion of the supermodes can be approximated using a set of coupled nonlinear Schrödinger equations:

$$\partial_t E_n = iD(i\partial_z)E_n + i\hat{C}(i\partial_z)(E_{n-1} + E_{n+1}) + i\gamma(1 + i\varepsilon_{pa})|E_n|^2 E_n - (\varepsilon + \sigma_{fcc}Q_n)E_n$$

where $n=1,2,3$ numbers the wires and $E_n$ is the amplitude of the electric field, $E_0$, $E_i \equiv 0$; $\gamma$ is the nonlinear coefficient, $\varepsilon_{pa}$ is the two photon absorption (TPA) coefficient, $\varepsilon$ is the linear loss, $Q_n$ is the free carrier density, and $\sigma_{fc}$ is the free carrier scattering coefficient. $\hat{D}(i\partial_z)$ and $\hat{C}(i\partial_z)$ are the polynomial operators.

**Fig. 1.** Calculated temporal (a,b,c) and spectral (d,e,f) evolution of the $S_1(a,d)$, $S_2(b,e)$ and $S_3 (c,f)$ supermodes along the array under the edge excitation with $P_{av}=70 \mu W$. In order to simultaneously show formation of quasisolitons and dispersive radiation in time domain, the panels (a-c) are plotted on the linear scale in the soliton area ($t>-1 \text{ ps}$) and on the log scale for $t<-1 \text{ ps}$, where the radiation can be seen. In the spectral plots $R_1$, $R_2 \approx 1.72 \text{ \mu m}$ and $R_3$ correspond to the intramode resonances of the respectively numbered supermodes, while $R_4$ and $R_5$ to the intermode ones.

The array supports three supermodes with the electric field parallel to the substrate: The $S_1$ mode is symmetric with respect to the central wire and has all three wires excited in phase; the $S_2$ mode is antisymmetric with zero intensity in the middle wire; the $S_3$ mode is symmetric as $S_1$, but the edge wires are out of phase with the central one.

Mathematically the supermodes are expressed as $E_{nj} = x_{nj}e^{i\delta_j - i\delta}$, $x_{nj} = \sqrt{1/2} \sin(n\pi/4)$ and $\delta$ is the detuning from the pump frequency. The corresponding propagation constants are $D_j(\delta) = D(\delta) + 2C(\delta)\cos(j\pi/4)$. $D(\delta)$
and $C(\delta)$ are obtained by replacing $i\partial_t$ with $\delta$ in the operators $\hat{D}$ and $\hat{C}$. $D(\delta)$ is dispersion in a single wire. $C(\delta)$ is the coupling coefficient characterizing discrete diffraction in our system. The supermodal GVDs can be very different due to strong dependence of the discrete diffraction (coupling) coefficient on the frequency. Our geometry features strong coupling between temporal and spatial degrees of freedom.

There are two types of soliton solutions in our system: temporal solitonic supermodes and spatio-temporal solitons. The power threshold required for the latter is out of our excitation conditions, so that, we expect solitonic supermodes to play a dominant role. Note that the term quasi-solitons is more appropriate for our system, since TPA is appreciable and therefore solitons decay adiabatically. One criterion of this is emission of the resonant Cherenkov radiation into the normal GVD range, which happens only if an emitting pulse forms a soliton [4]. Thus presence or absence of the Cherenkov radiation in the output spectrum can be used as a soliton signature.

We numerically model Eq. (1) with two types of initial conditions, when the input pulse is coupled into the edge and the central wire. For the former, a superposition of all three supermodes is excited; for the latter, only symmetric modes $S_{1,3}$ are excited. Fig. 1 shows temporal and spectral evolutions of the individual modes along the array. One can see that all three supermodes emit resonant radiation at a certain stage of their evolution. In the spectral domain, this radiation forms peaks. If only the central wire is pumped the $S_2$ mode is not excited.

In our experiments, we had used 120 fs pulses at 1.54 $\mu$m. Pulses were generated by an optical parametric amplifier (OPA) and were coupled in and out of the array using singlet objective lenses (numerical aperture 0.65). The pump spot diameter illuminating the array was estimated in our case at $\sim$1.45 $\mu$m. The transmitted light was then sent into an optical spectrum analyzer. Fig. 2 shows significant spectral broadening observed with increasing pump power. In our system, unlike in fibers, we can use an additional degree of control over pump and adjust its position across the array face, i.e., shift it from the edge to the center of the array, thereby changing the relative strength of the excited supermodes. Doing exactly this, we observed transition between the spectra shown in Figs. 2a and 2b. A notable and reproducible feature of these measurements is that for the edge excitation the radiation band around $\lambda \approx 1.72$ $\mu$m disappears from the spectrum, see spectral intervals marked with the white arrows in Figs. 2a and 2b. This part of the spectrum is associated with the radiation emitted by the temporal solitons excited in the antisymmetric $S_2$ mode. $S_2$ mode is prohibited, if the pump beam is aligned with the geometrical center of symmetry of the array, so that the pump has nonzero projections only on the symmetric $S_1$ and $S_3$ modes. In order to compare our measurements and modeling results, we monitored numerical output in one of the edge wires. The resulting spectrum vs the input power is plotted in Fig. 2 for the edge (Fig. 2c) and center excitations (Fig. 2d). One can see very good qualitative agreement with the experimental measurements in Figs.2a and 2b.

In summary, pronounced spectral broadening, soliton generation and emission of Cherenkov radiation have been reported in the regime, when several supermodes are excited by a femtosecond input pulse in an array of coupled subwavelength silicon waveguides. We continue to investigate the array structures and are likely to present results going beyond the one described above.

References