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Helping Others or Helping Oneself? International Subsidies and the Provision of Global Public Goods

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This paper explores the welfare effects of international subsidies designed to expedite the production of global public goods. It distinguishes between the impact subsidies exert on behaviour and the impact subsidies exert on welfare. Subsidies that encourage recipients to contribute to the provision of global public goods can be designed to maximise the welfare of donor countries. While these optimal subsidies achieve a Pareto efficient allocation of resources, all the efficiency gains are appropriated by donor countries. If equity is irrelevant, optimal subsidies are higher for high-income recipients than for low-income recipients.

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1. Introduction

Nation states must act collectively if they are to make a meaningful response to global challenges. Global challenges take different forms. Diseases are disseminated internationally, financial instabilities are transmitted across integrated markets and climate changes threaten the world environment. Nation states produce global public goods when their response provides benefits that are experienced worldwide (Sandler 2003:131).

Nation states' willingness to act collectively depends on the nature of the public good to be produced (Sandler 2004) and on incentives provided by international treaties. International collaboration is more successful when treaties offer side-payments to mobilise collective action (e.g. Barrett 1999, 2003). The

Montreal Protocol offers assistance to countries that agree to reduce emissions from ozone-depleting substances. The Global Environmental Facility assists countries that use expensive substitutes for fossil fuels and practice energy conservation. The Group of Seven and the European Union offers subsidies to countries to reduce the risk of nuclear accidents. The World Heritage Fund provides subsidies to countries to protect their national heritage when this also generates global benefits. More recently, the U.N. Climate Change Conference at Cancun has agreed to establish a Green Climate Fund and, by 2020, this Fund will disburse $100 billion to help countries reduce carbon emissions and to help countries adapt to climate change.

The extent to which these subsidies can be described as aid is moot. When King (2006: 371) reflected on subsidies offered to cover incremental costs (i.e. the "...additional costs of pursuing one course of action rather than another.."), he insisted that these payments "...are not conventional aid..."; they are provided to make "...international co-operation more incentive compatible". Sandler (1997) drew attention to the proportion of bilateral and multilateral aid transfers that are offered to encourage developing countries to contribute to the provision of global public goods (e.g. to encourage developing countries to sustain rain forests, or to administer the vaccination programmes required to eradicate disease). These transfers are increasing as a share of official overseas development assistance, but it is far from obvious that they should be described as aid (Raffer 1999; World Bank 2001, te Velde et al 2002; Anand 2004; UNIDO 2008).

In this paper the objective is to explore the welfare effects of international subsidies offered to mobilise collective action. Will they increase the provision of a global public good? Will they achieve a Pareto-efficient allocation of resources? Will they ensure that every nation state that responds to a global challenge receives a meaningful share of the efficiency gains delivered by collective action?

An established literature already explores the impact of income redistribution on the supply of a public good. Some question whether lump sum transfers are likely to have any impact at all (e.g. Warr 1983; Bergstrom et al 1986). Others focus on the question of whether governments are able to rely on distortionary taxation to increase the supply of a public good (e.g. Bernheim 1986). In this context, Andreoni and Bergstrom (1996) demonstrate that governments can offer tax-financed subsidies to increase the provision of a public good if voluntary contributors take the government's budget constraint into account.

This paper has a narrower remit. It focuses on voluntary co-operation between nation states that respect each others' national sovereignty (there is no international government able to levy coercive taxation). In this paper the question is whether treaties and aid packages can be designed to mobilise collective action. The intention is to analyse the impact of subsidies in treaties when treaties are analysed as stylised tax-subsidy arrangements.

In the absence of an international government, tax is the cost that the nation state incurs when it voluntarily signs a treaty. It is possible to analyse treaties that impose a cost as a tax payment to a common budget (to finance the provision of a global public good). It is also possible to analyse treaties that impose a tax (more generally) as a direct cost that must be incurred to fulfil treaty obligations. As an example, consider the cost that nation states incur when they agree to reduce carbon emissions (in order to fulfil their obligations to an international environment treaty). The same stylised tax-subsidy model can be applied if nation states make tax payments to a common fund, or if nation states incur tax costs as direct contributions to provide a public good. This paper analyses tax costs as contributions to produce a global public good with a summation technology (Sandler 2004).

The next section of the paper illustrates the main issues by focussing on the cost-sharing arrangement implicit in an international treaty. A developed country (country 1) is able to offer a cost-sharing arrangement to induce a developing country (country 2) to make a contribution to the provision of a global public good, even when the developing country is able to free-ride on the provision of the global public good by the developed country. Studies have already considered the impact of cost-sharing arrangements on the supply of a public good (Guttman 1978; 1987 considers a cost-sharing arrangement in a two-stage game) but, in this paper, the intention is to focus on the distribution of the efficiency gains that collective action can deliver.

Section three of the paper considers the properties of the tax-subsidy arrangement that mobilises collective action to deliver all of the efficiency gains to the country that pays the subsidy. The two-country model is generalised to an n country model. Counter-intuitively, only the donor country (the country that pays the subsidy) gains (even though collective action successfully produces a Pareto-efficient quantity of the public good).

Section four of the paper focuses on the pattern of subsidies when donor countries mobilise collective action and appropriate all of the efficiency gains. The pattern of subsidies differs systematically from the pattern that would emerge if donor countries' only concern was to alleviate poverty in developing countries.

More generally, section five of the paper considers the properties of international treaties that mobilise collective action to deliver a meaningful share of the welfare gains to every country that acts collectively.

Commentators are critical that international subsidies offered to developing countries to induce them to act collectively are not really offering aid. They are concerned that these subsidies are displacing conventional aid. This paper identifies an even more worrying concern. International subsidies can be offered to mobilise developing countries and to maximise the welfare of the countries that pay the subsidies. They can be designed to deliver all of the efficiency gains to the countries that pay the subsidies. This observation is important. It is not sufficient to argue that citizens in developing countries value the global public good. It is not sufficient to argue that countries that receive subsidies are always able to reject the offer of subsidies. The message to policymakers is that it is necessary to examine the properties of tax-subsidy arrangements implicit in international treaties. When are they likely to mobilise collective action? When are they likely to deliver all of the welfare gains to the nation states that pay international subsidies?

2. International subsidies to mobilise developing countries.

In this section of the paper the intention is to illustrate the welfare effects of a subsidy offered by a developed country (country 1) to encourage a developing country (country 2) to contribute to the provision of a global public good (x). In countries 1 and 2, citizens derive utility from a global public good x and from a private good y. We follow the usual convention by assuming that each country maximises a utility function that is "…representative of the welfare of the nation's population" (Sandler 1993:449).

To begin, consider the way that a developed country might induce a developing country to contribute to the provision of x even though the developed country already provides $\bar{x}$ unilaterally. Let m_1 and m_2 denote the incomes of countries 1 and 2. The utility levels of the two countries are:

$$u_1(\bar{x}, m_1 - \bar{x}) = \bar{u}_1 \qquad (1)$$

$$u_2(\bar{x}, m_2) = \bar{u}_2 \qquad (2)$$

where we assume that the marginal cost of production is equal to one. This starting point resembles the problem discussed by developed countries at the UN Climate Conferences in Copenhagen and Cancun. Many developed countries are already committed to measures to reduce carbon emissions (as signatories to the Kyoto Protocol). At the Copenhagen Conference (December 2009) their objective was to encourage developing countries to make a similar commitment. They decided to offer developing countries subsidies if developing countries would also agree to incur costs to mitigate the problem of global warming.

In Figure 1 the marginal cost of providing the global public good (x) is MC. Country 1, or country 2, can pay the full marginal costs of providing the good, or both can agree to share the marginal cost. If they agree to share the marginal costs, country 1 would pay a cost-share between τ_1^0 and τ_1^{100} (on the vertical axis) and country 2 would pay the remaining cost-share between $(1-\tau_1^0)$ and $(1-\tau_1^{100})$.

The indifference curves illustrate the preferences of each country. At any output, country 1 is better off on indifference curve I_1^2 than on I_1^1 because country 1 pays a lower tax share for each unit of x (conversely, at any output, country 2 is worse off on indifference curves that lie below I_2^1). The initial situation is point a. Country 1 provides $x^1 = \bar{x}$ units (at point a country 1's marginal benefit of the public good equals marginal cost). At point a country 2 benefits as a free rider. Can a cost-sharing arrangement be designed to induce country 2 to act collectively?

Both countries might enjoy a higher welfare level by acting collectively. There are potential efficiency gains if they agree to share costs to produce a higher level of output. In Figure 1 each country can achieve a higher level of welfare in the ellipse described between points a and c. These efficiency gains are maximised if countries chose a cost-sharing arrangement at a point on the contract curve (not drawn) in the ellipse. When the slopes of the indifference curves are identical, Samuelson's (1954) conditions are satisfied (Mueller 2003).

The question is whether there is a process that will mobilise collective action. In theory, it is possible to rely on the process described by Lindahl (1919). An impartial auctioneer suggests a cost-sharing arrangement and asks each country if it prefers more, or less, of the public good. By a tâtonnement process, the auctioneer increases the cost-share for the country that prefers a higher output of the public good, and decreases the cost-share for the country that prefers a lower output. If there is a cost-sharing arrangement at which neither country has a preference for more, or less, of the good, the outcome is Pareto optimal (Johansen 1963). The cost-shares are Lindahl prices; at these prices each nation state equates its marginal benefit to its marginal cost. To achieve a Lindahl equilibrium, each country must answer honestly, even though there are incentives to under-reveal demand (Cornes and Sandler 1996).

By comparison, the process considered in this paper is quite different. In the first instance, it is the developed country that takes the initiative. As it is the convention that a donor (prepared to pay the subsidy) offers a subsidy, the developed country enjoys a first move advantage. The developed country acts as a Stackelberg leader.[1]

The second difference is that, when the developed country sets the subsidy, it is also able to determine the cost-sharing arrangement. As each country's cost-share depends on the mix of tax and subsidy, the developed country is able to set the subsidy to determine the cost-sharing arrangement (receipt of the subsidy is contingent on willingness to pay an agreed tax contribution to the public good). If the developed country sets the subsidy to ensure that every nation shares the efficiency gains delivered by collective action, the cost-sharing arrangement will leave each country on the contract curve within the ellipse (described between points a and c). However, the developed country can also set the subsidy to maximise its own welfare on the contract curve at point b in Figure 1.

3. International subsidies that maximize donors' welfare.

How can a cost-sharing arrangement be produced to mobilise collective action at point b in Figure 1? In this section of the paper attention focuses on the properties of a tax-subsidy arrangement to maximize the welfare of the country that pays the subsidy.

Country 1 and country 2 consume y_1 and y_2 of the private good. The analysis is premised on the following assumptions:

[Assumption 1] $u_1(x,y_1)$ and $u_2(x,y_2)$ are increasing quasi-concave functions.

[Assumption 2] the cost function of the public good is linearly increasing and is given by $c(x)=x$, where the constant marginal cost of production equals one.

[1] Participation in the Lindahl process is different. When the auctioneer suggests different cost-sharing arrangements there is no suggestion that the donor country is more likely to reap all of the efficiency gains.

[Assumption 3] $m_1 \geq m_2$ and $\dfrac{\partial u_1}{\partial x}(x,y_1) > \dfrac{\partial u_2}{\partial x}(x,y_2)$ for all $x > 0$.

[Assumption 4] $\dfrac{\partial u_1}{\partial x}(0,m_1) = \infty$ for all $y_1 \geq 0$.

[Assumption 5] Country 1 makes a once for all (take it or leave it) offer that country 2 accepts or rejects.

[Assumption 6] lump-sum transfers of wealth between the two countries are not available.

A1-A2 are standard assumptions that ensure that the first order conditions for Pareto efficiency are sufficient. Assumption A3 ensures that in a decentralized country decision process, (the competitive equilibrium where each country takes as given the amount of x purchased by the other country), only country 1 spends on the public good while country 2 free-rides. Assumption A4 ensures that there is always a finite amount of x consumed. A5 amounts to assuming that the two countries play a Stackelberg game. This assumption is relaxed in section 5 where we consider treaties that allow country 2 to be strictly better off than under (2). In a theoretical world lump-sum contributions T_1 and T_2 are possible but here the analysis focuses on tax contributions that are proportional to incomes (m_1 and m_2) in the two countries. A6 is thus introduced to capture some realism in international negotiations.

The feature that is common in the treaties analysed in this paper is the per-unit subsidy $s \in [0,1]$ paid by the developed country to the developing country. As noted, the tax element in the treaty is the cost that a signatory agrees to incur to produce a global public good. To begin, it is helpful to focus on a tax rate, t, that depends on income:

(i) If the costs of providing the global public good are financed by a common budget, the tax is the payment that each country contributes to the budget. When the tax rate is t, each country contributes tm_1 and tm_2 respectively. Total tax revenue ($tm_1 + tm_2$) together with the subsidy sx (paid by the developed country) is available to finance the provision of the global public good.

(ii) If the costs of providing the public good are incurred as direct costs (to fulfil treaty obligations), the direct contribution costs are tm_1 and tm_2. These are the tax costs that country 1 and country 2 incur directly (e.g. by reducing carbon emissions). With a summation technology these contributions, tm_1 and tm_2, together with the subsidy sx, are available to produce the global public good.

The analysis begins with the assumption that the developed country will incur a higher cost (tm_1) than the developing country (tm_2). This may often be the case (e.g. in international environment treaties, the costs of reducing carbon emissions are likely to be higher in countries that have higher industrial output). It is also the case that the argument in this paper is stronger if it appears to be the case that the donor country is willing to accept higher tax costs. In section 5 of the paper this assumption (that nation state contributions are linear functions of income) can be relaxed.

The first objective is to compare welfare effects when a developed country 'generously' offers to incur a higher cost ($tm_1 > tm_2$) and to finance a subsidy sx. If the developed country (country 1) is intent on maximising its own utility, it will offer a treaty that will (at minimum) maintain country 2's initial welfare. If $\bar{u}_2$ is the utility country 2 achieves while free-riding, the resulting participation constraint from the perspective of country 1 is:

$$u_2[x, m_2(1-t)] = \bar{u}_2. \tag{3}$$

such that,

$$\frac{dt}{dx}|_{\bar{u}_2} = \frac{1}{m_2}\frac{\partial u_2/\partial x}{\partial u_2/\partial y_2} \tag{4}$$

As this quantity is positive, country 1 will only raise the tax rate t in exchange for an agreement that there will be a higher level of global activity. If the total cost of production is simply the sum of individual contributions, the resource constraint takes the form:

$$t(m_1+m_2)+sx = x \tag{5}$$

The developed country's objective (country 1) is to maximize, by choice of x,t and s, its utility function $u_1[x, m_1(1-t)-sx]$ subject to the above participation and resource constraints.

When focussing on the properties of the optimal tax-subsidy mechanism that maximises the donor country's welfare, let μ denote the Lagrange multiplier on the participation constraint, and let λ denote the corresponding multiplier on the resource constraint. The associated Lagrangian for the optimum choice of x,t and s, from the perspective of country 1, can thus be written as follows:

$$\begin{aligned} L(x,t,s;\mu,\lambda) = {} & u_1[x, m_1(1-t)-sx]+ \\ & +\mu[u_2[x, m_2(1-t)]-\bar{u}_2]+ \\ & +\lambda[t(m_1+m_2)-x(1-s)] \end{aligned} \tag{6}$$

First order conditions for the optimum choice of x, t and s entail:

$$\frac{\partial L}{\partial x} = \frac{\partial u_1}{\partial x} + \mu \frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y_1} s - \lambda(1-s) = 0 \tag{7}$$

$$\frac{\partial L}{\partial t} = -m_1 \frac{\partial u_1}{\partial y_1} - \mu m_2 \frac{\partial u_2}{\partial y_2} + \lambda(m_1 + m_2) = 0 \tag{8}$$

$$\frac{\partial L}{\partial s} = -\frac{\partial u_1}{\partial y_1} x + \lambda x = 0 \tag{9}$$

By defining $\Pi(x,y) = \dfrac{\partial u/\partial x}{\partial u/\partial y}$ as the marginal rate of substitution between the two goods, we obtain the following result (see the appendix for a derivation):

Proposition 1 *There exists a Pareto efficient treaty* $(\hat{x},\hat{t},\hat{s})$ *such that the donor country reaps all the welfare gains. At the point* $(\hat{x},\hat{t},\hat{s})$ *satisfying the first order conditions [7-9]above, we have the equalities*

$$\Pi_1\left(\hat{x}, m_1(1-\hat{t}) - \hat{s}\hat{x}\right) + \Pi_2\left(\hat{x}, m_2(1-\hat{t})\right) = 1 \tag{10}$$

$$\lambda = \frac{\partial u_1}{\partial y_1} \tag{11}$$

$$\mu = \frac{\partial u_1/\partial y_1}{\partial u_2/\partial y_2} \tag{12}$$

where Π_i *is the marginal rate of substitution of country* i *between the global public good and disposable income and* 1 *is the marginal cost of production of the global public good* x.

The role of the subsidy is crucial in ensuring that the Samuelson condition (10) characterizing Pareto efficiency of the solution $(\hat{x},\hat{t},\hat{s})$ is satisfied. In the absence of a subsidy, $s=0$, the first order condition (9) does not arise and the terms $s\partial u_1/\partial y_1$ and λs vanish from (7). In the resulting optimum a weighted sum of the marginal rates of substitution of the two countries is equated to the marginal cost of production, thus breaking the required Samuelson optimality condition[2]. It should then be clear that in the framework of Proposition 1, aid would have little to do with altruism, or caring for country 2. Instead, aid, i.e. the subsidy, is used as a corrective mechanism to mitigate the distortion that is caused by free-riding behaviour.

[2] More specifically, in the absence of subsidies we obtain

$$\lambda = \theta_1 \frac{\partial u_1}{\partial y_1} + \mu \theta_2 \frac{\partial u_2}{\partial y_2}$$

where $\theta_i = m_i/(m_1+m_2)$. The optimum level of x then equates the sum of marginal utilities $\dfrac{\partial u_1}{\partial x} + \mu \dfrac{\partial u_2}{\partial x}$ to this resulting value of λ.

Define $v_1(m_1,m_2,\bar{u}_2)=u_1(\hat{x},\hat{t},\hat{s})$ as country 1's maximized utility function in relation to the problem (6). We state below properties of $v_1(m_1,m_2,\bar{u}_2)$ which will be further explored in the sections below:

Corollary 1 Let $v_1(m_1,m_2,\bar{u}_2)$ *denote country 1's indirect utility function. Then*

(i) $\dfrac{\partial v_1}{\partial m_2}=\lambda>0$

(ii) $\dfrac{\partial v_1}{\partial \bar{u}_2}=-\mu<0$

so that country 1's welfare is increasing in country 2's income, and decreasing in country 2's reservation utility.

Thus, other things equal, country 1 would benefit more by engaging in a treaty with a richer free-riding country. Furthermore, since $\dfrac{\partial v_1}{\partial \bar{u}_2}<0$, there would appear to be a range of Pareto-efficient treaties, some of which may increase country 2's welfare. One such case is a treaty where both countries share the welfare gains (section 5).

3.1 A multi-country world.

The above results are robust when generalising the model beyond a two-country world. We consider a world consisting of two homogeneous groups of countries, say North and South, where within each group preferences and endowments are identical [3]. Specifically, we assume there are n_1 rich countries with incomes m_1 and identical preferences $u_1(x,m_1)$ and n_2 poor countries with incomes m_2 and identical preferences $u_2(x,m_2)$ [4].

There are now n_2 identical participation constraints, for each type 2 country, identical to (3), while the new resource constraint is of the form:

$$t(n_1m_1+n_2m_2)=x(1-n_1s) \tag{13}$$

The associated Lagrangian with the global community consisting of n_1 rich countries and n_2 poor countries is:

$$\begin{aligned} L = {} & u_1(x,m_1(1-t)-sx) \\ & +\mu[u_2(x,m_2(1-t))-\bar{u}_2] \\ & +\lambda[t(n_1m_1+n_2m_2)-x(1-n_1s)] \end{aligned} \tag{14}$$

[3] The authors are grateful for the advice that an anonymous referee offered on the approach that might be employed to analyse a multi-country model.

[4] The assumption of two homogeneous groups is further relaxed in section 5 where we consider various generalisations of our analysis.

First order conditions for the optimum choice of x,t, and s entail:

$$\frac{\partial L}{\partial x} = \frac{\partial u_1}{\partial x} - s\frac{\partial u_1}{\partial y_1} + \mu\frac{\partial u_2}{\partial x} - \lambda(1-n_1 s) = 0 \tag{15}$$

$$\frac{\partial L}{\partial t} = -m\frac{\partial u_1}{\partial y_1} - m_2\mu\frac{\partial u_2}{\partial y_2} + \lambda(n_1 m_1 + n_2 m_2) = 0 \tag{16}$$

$$\frac{\partial L}{\partial s} = -\frac{\partial u_1}{\partial y_1}x + n_1\lambda x = 0 \tag{17}$$

Letting Π_i denote a representative country $i's$ marginal rate of substitution, we generalize the result of Proposition 1 in the context of a multi-country world as follows:

Proposition 2 *There exists a Pareto efficient treaty such that the donor community reaps all the welfare gains. At the point* $(\hat{x},\hat{t},\hat{s})$ *satisfying the first order conditions* $[15-17]$ *above, we have the equalities*

$$n_1\Pi_1\left(\hat{x}, m_1\left(1-\hat{t}\right) - \hat{s}\hat{x}\right) + n_2\Pi_2\left(\hat{x}, m_2\left(1-\hat{t}\right)\right) = 1 \tag{18}$$

$$\lambda = \frac{1}{n_1}\frac{\partial u_1}{\partial y_1} \tag{19}$$

$$\mu = \frac{1}{n_1}\frac{\partial u_1}{\partial y_1} / \frac{1}{n_2}\frac{\partial u_2}{\partial y_2} \tag{20}$$

Once again, the optimum is the Samuelson condition (18) that characterizes Pareto efficiency. The weighted sum of marginal rates of substitution equals the marginal rate of transformation. The bilateral model (discussed earlier) is helpful when analysing bilateral aid arrangements. The multi-country model can be employed when exploring the properties of tax-subsidy arrangements in multilateral international treaties. The multi-country model illustrates the advantage that developed countries are able to exploit as countries that offer subsidies (as altruists, able to determine implicit cost-sharing arrangements). Even in the multi-country model, developed countries can offer cost-sharing arrangements that are designed to maximise their own welfare.

4. International subsidies when developing countries' incomes increase.

In section 3 of the paper a developed country (country 1) encouraged a developing country (country 2) to commit resources to provide a public good and to appropriate all of the efficiency gains. How will s and t change as the developing country becomes richer? How will they change if the marginal utility a developing country derives from the public good increases at all levels of x?

When we parametrize the utility functions of the two countries using quasi-linear preferences, income effects are absent from the demand for the global public good. Once again, the intention is to illustrate the possibility that aid is motivated by the incentive to mobilise collective action, rather than by any consideration of international equity. The utility functions of the developed country (country 1) and the developing country (country 2) are:

$$u_1(x,y_1) = y_1 + \gamma \log x \tag{21}$$

$$u_2(x,y_2) = y_2 + \beta \log x \tag{22}$$

where $\gamma > \beta$. This means that country 1 derives higher marginal utility from the global public good at all levels of x.

Under private (national) provision, country 1 provides an amount $\bar{q}_1 = \gamma$, whereas country 2 free rides ($\bar{q}_2 = 0$). At this competitive equilibrium $\bar{x} = \bar{q}_1 + \bar{q}_2 = \gamma$ and $\bar{u}_1(x,y_1) = (m_1 - \gamma) + \gamma \log\gamma$ while $\bar{u}_2(x,y_2) = m_2 + \beta\log\gamma$. The constrained optimization problem of the previous section using quasi-linear preferences can be parametrized to describe the optimum subsidy country 1 would offer. The associated Lagrangian is thus:

$$\begin{aligned} L(x,t,s;\mu,\lambda) = {} & m_1(1-t) - sx + \gamma\log x + \\ & + \mu[m_2(1-t) + \beta\log x - \bar{u}_2] + \\ & + \lambda[t(m_1+m_2) - x(1-s)] \end{aligned} \tag{23}$$

Assuming the exogenous parameters γ, β, m_1, m_2 are chosen in a way that the optimum values of s and t are in the range $[0,1]$, we can ignore boundary solutions in the resolution of the above problem. Let $\theta_2 = m_2/(m_1+m_2)$ denote the income share of country 2 in the world distribution of income. At a point $(\hat{x}, \hat{t}, \hat{s})$ satisfying the first order conditions in relation to the Lagrangian (23) we thus obtain:

$$\hat{x} = (\gamma + \beta) \tag{24}$$

$$\hat{t} = \frac{\beta}{m_2} \log\left(\frac{\hat{x}}{\gamma}\right) \tag{25}$$

$$\hat{s} = 1 - \left(\frac{\beta}{\theta_2 \hat{x}}\right) \log\left(\frac{\hat{x}}{\gamma}\right) \tag{26}$$

$$\frac{\partial u_1}{\partial y_1} = \frac{\partial u_2}{\partial y_2} = \lambda \tag{27}$$

$$\lambda = \mu = 1 \tag{28}$$

Note that in the solution to this problem $\lambda = \mu = 1$. This result is a consequence of the assumption of quasi-linear preferences where the marginal utility of income is constant and equal to one at all income levels.

With quasi-linear preferences the optimum price $\hat{p}=(1-\hat{s})$ is a function $\hat{p}(1,\gamma,\beta,\theta_2)$, where 1 is the marginal cost of producing the public good, γ and β measure the relative valuations of the two countries for the public good [5], and $\theta_2=1-\theta_1$ is a measure of income inequality in the two-country world. Thus, $\hat{p}(1,\gamma,\beta,\theta_2)$ decreases with the income share θ_2 of the free-riding country. Other things equal, the optimum subsidy to a middle-income country would be greater than the optimum subsidy to a low-income country. This seemingly paradoxical result is best understood with reference to (4). In the absence of income effects $\partial u_2/\partial y_2=1$ and the optimum value $\hat{x}$ is independent of m_2. Thus, the MRS of country 2 (at constant utility $\bar{u}_2$) becomes flatter as m_2 increases. Consequently, the optimum $\hat{t}$ that secures country 2's participation falls, and as a result $\hat{s}$ must rise to ensure that the resource constraint is satisfied. On the other hand, $\partial\hat{s}/\partial\beta<0$: the more the free-riding country values the public good, the higher the price $(1-\hat{s})$ it would be asked to pay. Thus, in this two-country world characterized by quasi-linear preferences, the optimum subsidy to country 2 is shaped by incentive considerations, and this is likely to be sharply different from subsidies premised on the usual altruism arguments that are put forward in the aid literature.

Proposition 3 *At a point $(\hat{x},\hat{t},\hat{s})$ satisfying the first order conditions in relation to the Lagrangian (23) where countries 1 and 2 have quasi-linear preferences, the optimum price of the global public good is a function $\hat{p}(1,\gamma,\beta,\theta_2)$, such that:*

(i) $\hat{p}$ decreases with the income share θ_2 of the free riding country.

Also,

(ii) $\hat{p}$ increases with β, the intensity of the free-riding country's preference for the public good.

To illustrate, Table 1 reports the findings of a calibration of the preferences and endowments of the two countries. Throughout the calibration we set the preference parameters $\gamma=1$, $\beta=0.5$ and $m_1=3$ in relation to (21). We report the optimum values $\hat{x}$, $\hat{t}$ and $\hat{s}$ as we vary the income share θ_2 of the free-rider.

[Insert Table 1 here]

For a given value of β, as country 1 becomes richer the price subsidy $\hat{s}$ is increased, while the tax $\hat{t}$ is reduced. Taking for instance the value $\beta=0.50$, we have $\hat{s}=0.46$ when $\theta_2=0.25$, while

[5] At any level of global public activity x^o, $\dfrac{\partial u_1/\partial x}{\partial u_2/\partial x}|_{x=x^o}=\gamma/\beta$.

the price subsidy rises to $\hat{s}=0.59$ when $\theta_2=0.33$. From Corollary 3.2, we know that country 1's welfare rises with m_2 so that the increased subsidy is all the more profitable from the perspective of country 1.

When the welfare of the donor country depends on the efficiency gains derived from collective action, the welfare of the donor country increases with the income of the developing country. This prediction (that a higher subsidy will be offered as the recipient's income increases) resonates with discussions of the importance of mobilising such countries as China and India to make a meaningful response to the challenge of global warming.

As β rises, country 1 reduces the subsidy element and offers a contract with a higher $\hat{t}$. For instance, when $\beta=0.75$ and $\theta_2=0.25$ (results not shown in the table), the price subsidy is virtually nil ($\hat{s}=0.04$) while $\hat{t}=0.42$.

5. International treaties designed to share efficiency gains.

It is now possible to compare the characteristics of tax-subsidy arrangements that maximise the welfare of one country (as in sections 2, 3 and 4) with characteristics of tax-subsidy arrangements that share the gains of collective action. This section of the paper focuses on outcomes that exist within the ellipse illustrated in Figure 1. In the empirical calibrations in section 4 the tax is relatively large and the subsidy is relatively small (when the cost-sharing function delivers all of the efficiency gains to the developed country).

Of course, more generally, the policy instruments available to negotiating parties may differ from those that have been considered so far (there may be other policy instruments than just a price subsidy and a tax). Also, more generally, it may not always be possible to achieve Samuelson first best efficiency (as there may also be second best considerations).

This section of the paper considers a general approach. Firstly, we assume there are three homogenous groups of countries [6] (say high, middle and low income countries) of respective sizes n_1,n_2 and n_3. Within a group $i=1,2,3$, a given country has a utility function $u_i(x,y_i)$. In this approach any cost-sharing arrangement is dependent on three functions $E_i(x,\eta)$, $i=1,2,3$, where η is a $k-$dimensional vector of parameters (to be chosen optimally) in a

[6] It will be clear from the first order conditions below that the analysis is equally applicable in the context of $h=3,4,...$ groups of homogeneous countries. The value of $h=3$ is chosen here for expositional purposes.

way that $\sum_{i=1}^{3} n_i E_i(x,\eta) = x$. $E_i(x,\eta)$ is the total amount spent by a representative country i on the global public good, so that $y_i = m_i - E_i(x,\eta)$. In the two-country discussion considered so far in the paper $\eta = (t,s)$, $E_1(x,\eta) = sx + tm_1$ and $E_2(x,\eta) = tm_2$. Another example, discussed in section 2, is one where τ is the cost share of country 1, $E_1(x,\eta) = \tau x$ and $E_2(x,\eta) = (1-\tau)x$.

Assume again that only rich countries provide x in the absence of an international treaty, and let $\bar{u}_3 = u_3[x, m_3]$ denote the reservation utility of a free-riding low-income country. An efficient treaty, given the policy instruments $E_i(x,\eta)$ will maximize the welfare of the high-income community, $u_1[x, m_1 - E_1(x,\eta)]$, given that middle and low income countries are willing to participate in the treaty, i.e. provided $u_2[x, m_2 - E_2(x,\eta)] = u_2^o \geq \bar{u}_2$, and $u_3[x, m_3 - E_3(x,\eta)] = u_3^o \geq \bar{u}_3$. By varying the levels of u_2^o and u_3^o, we can envisage treaties that will share efficiency gains from those that benefit only one party. Set $\mu_1 = 1$ and define the Lagrangian of the problem as follows:

$$\begin{aligned} L(x,\eta;\mu_2,\mu_3,\lambda) = {} & \mu_1 u_1[x, m_1 - E_1(x,\eta)] + \\ & + \mu_2 [u_2[x, m_2 - E_2(x,\eta)] - u_2^o] + \\ & + \mu_3 [u_3[x, m_3 - E_3(x,\eta)] - u_3^o] + \\ & + \lambda \left[\sum_{i=1}^{3} n_i E_i(x,\eta) - x \right] \end{aligned} \tag{29}$$

There are $k+4$ first order conditions: one for the optimum choice of x, the level of global public activity, one equation in relation to each of the parameters in η, one pertaining to the resource constraint and, finally, two equations defining the welfare types 2 and 3 countries derive from the treaty:

$$\frac{\partial L}{\partial x} = \sum_{i=1}^{3} \mu_i \left(\frac{\partial u_i}{\partial x} - \frac{\partial u_i}{\partial y_i} \frac{\partial E_i}{\partial x} \right) + \lambda \left(\sum_{i=1}^{3} n_i \frac{\partial E_i}{\partial x} - 1 \right) = 0 \tag{30}$$

$$\frac{\partial L}{\partial \eta_j} = -\sum_{i=1}^{3} \mu_i \left(\frac{\partial u_i}{\partial y_i} \frac{\partial E_i}{\partial \eta_j} \right) + \lambda \sum_{i=1}^{3} n_i \frac{\partial E_i}{\partial \eta_j} = 0 \quad j = 1,\ldots,k \tag{31}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{3} n_i E_i(x,\eta) - x = 0 \tag{32}$$

$$\frac{\partial L}{\partial \mu_i} = u_i[x, m_i - E_i(x,\eta)] - u_i^o = 0 \quad i = 2,3. \tag{33}$$

The above equations define, in abstract form, necessary conditions for Pareto efficiency given general cost functions $E_i(x,\eta)$. To make these optimality conditions more transparent, consider, for illustrative purposes, the case where the parties agree on a linear cost sharing rule such that $E_i(x,\eta)=\tau_i x$. The above first order conditions specialize to a seven equation system, characterizing the optimum values of x, τ_1,τ_2 and τ_3:

$$\frac{\partial L}{\partial x} = \sum_{i=1}^{3}\mu_i\left(\frac{\partial u_i}{\partial x}-\frac{\partial u_i}{\partial y_i}\tau_i\right)+\lambda\left(\sum_{i=1}^{3}n_i\tau_i-1\right)=0 \tag{34}$$

$$\frac{\partial L}{\partial \tau_j} = -\mu_i\frac{\partial u_i}{\partial y_i}x+\lambda\sum_{i=1}^{3}n_i x=0 \quad j=1,2,3 \tag{35}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{3}n_i\tau_i x-x=0 \tag{36}$$

$$\frac{\partial L}{\partial \mu_i} = u_i[x,m_i-\tau_i x]-u_i^o=0 \quad i=2,3. \tag{37}$$

The solution to this problem is one where a first-best allocative efficiency is obtained, i.e. a tangency on the contract curve in Figure 1.

$$\sum_{i=1}^{3}n_i\Pi_i[x^*,m_i-E_i(x^*,\tau_i^*)] = 1 \tag{38}$$

$$\lambda = \frac{1}{n_1}\frac{\partial u_1}{\partial y_1} \tag{39}$$

$$\mu_i = \frac{n_i}{n_1}\frac{\partial u_1/\partial y_1}{\partial u_i/\partial y_i} \quad i=2,3 \tag{40}$$

Furthermore, for $i=2,3$ the welfare of the rich community increases in community i's income, but decreases in the welfare level community i secures for itself.

If, again for illustrative purposes, we return to the two-country case ($n_1=n_2=1$, $n_3=0$, $\tau_1=\tau$ and $\tau_2=1-\tau$) where both countries have quasi-linear preferences, we find from (34)

$$\frac{\gamma}{x}-\tau+\mu\left(\frac{\beta}{x}-(1-\tau)\right)=0 \tag{41}$$

while (35) entails $\mu=1$. The optimum thus has the same efficient level of global public activity as encountered before, namely $x^*=\gamma+\beta$. From the participation constraint (37), we obtain τ as a function of u_2^o:

$$\tau^*=1-\left(\frac{\beta\log x^*+m_2-u_2^o}{x^*}\right) \tag{42}$$

The above equation captures in a simple fashion several of the themes developed in the paper, namely other things equal (i)

country 1's welfare increases with m_2, (*ii*) country 1's cost share rises in the utility u_2^o country 2 will manage to secure itself, and finally (iii) country 1's cost share is decreasing in β, the free-rider's valuation of the public good. We next proceed to illustrate these findings with the help of a further calibration exercise.

In the calibrations of Table 2 we set $m_1 = 3$, $\gamma = 1$ and $\beta = 0.5$ as in the calibrations of Table 1, and we fix the income share of country 2 at $\theta_2 = 0.25$. Our purpose here is to calculate the optimum cost-share τ^* and the welfare u_1 of country 1 as we vary u_2^o. We normalize the welfare level of both countries by their reservation utilities (1) and (2), so that $u_2/\bar{u}_2 = 1$ corresponds to a treaty where all the welfare gains accrue to country 1, and $u_1/\bar{u}_1 = 1$ corresponds to a treaty where country 2 reaps all the welfare gains.

[Insert Table 2 here]

The results of the first row of Table 2 correspond to those of the first row of Table 1 where $\hat{x} = 1.50$, $\hat{s} = 0.46$, $\hat{t} = 0.20$ and the developed country reaps all the welfare gains. Likewise, the results of the fourth row of Table 2 where $\tilde{x} = 1.50$, $\tilde{s} = 0.75$, $\tilde{t} = 0.09$ are those where country 2 appropriates all the welfare gains from the treaty. In the first row of Table 2, $\tau^* = 0.86$, whereas in the fourth row, the cost share of country 1 rises to $\tau^* = 0.93$. Between these two limiting cases, there exist treaties that benefit both parties. In the third row for instance, $u_1/\bar{u}_1 = 1.03$, $u_2/\bar{u}_2 = 1.05$ and country 1's cost-share stands at $\tau^* = 0.90$. We can now quite simply envisage treaties that benefit both parties by continuously varying $u_2/\bar{u}_2$, while ensuring that the resulting value of τ^* remains between these critical bounds of 0.86 and 0.93.

6. Conclusions.

This paper describes international treaties that mobilise developing countries to contribute to the provision of a global public good when developing countries can free-ride on developed countries' contributions to produce a global public good. The good news is that, even in the absence of an international authority (that is able to tax nation states), international treaties are able to rely on tax-subsidy arrangements to produce cost-sharing functions that will mobilise voluntary collective action. When subsidies increase as output increases, subsidies can be introduced to mobilise collective action.

The bad news is that, international treaties that offer subsidies to mobilise collective action can be designed to deliver all of the efficiency gains to the countries that pay the subsidies. In the absence of an international authority to monitor and to regulate the inducements implicit in these tax-subsidy arrangements, international treaties may deliver all of the welfare gains to the donor countries. If it is a convention that the altruist offers the subsidy, developed countries are able to make the first move. They are at an institutional advantage. As donors they set the subsidy. As 'donors' they offer the tax-subsidy arrangement implicit in an international treaty. They are able to set the subsidy to mobilise collective action and to appropriate the efficiency gains. In the absence of regulation, international treaties that 'generously' offer subsidies to mobilise collective action can be designed to maximise the welfare of the nation states that pay the subsidies.

When considering the pattern of international subsidies it appears counter-intuitive that developed countries might offer higher subsidies to developing countries that have higher levels of income. This is at odds with expectations based on the assumption that developed countries offer subsidies to alleviate poverty. However, it is consistent with expectations premised on the assumption that developed countries offer subsidies to maximise their own welfare.

More generally it is possible to compare the properties of Pareto-efficient treaties that advantage some countries with the properties of Pareto-efficient treaties that advantage all of the nation states that act collectively. The calibrations in this paper (premised, albeit, on quasi-linear preferences) illustrate the sensitivity of the distribution of welfare gains to changes in tax and subsidy arrangements. The properties of these tax-subsidy arrangements are important if policymakers are to ensure that every nation state receives a meaningful share of the efficiency gains that they generate when they act collectively.

The conclusions in this paper are also relevant when analysing bilateral and multilateral aid arrangements. Critics have argued that, when these subsidies displace conventional aid, the arrangements are unethical and inefficient (e.g. Anand 2004). One advantage of the analysis presented in this paper is that it is possible to address each concern separately. There are ethical considerations even when international treaties (and aid packages) achieve a Pareto-efficient outcome.

While the analysis suggests that the developed countries that have already complied with the Kyoto treaty will be able to offer a subsidy that will encourage developing countries to reduce carbon emissions (e.g. as discussed at the U.N. Climate Conferences), it calls in question the impact that a subsidy will exert on the distribution of the efficiency gains. It offers a different perspective on the emphasis placed on encouraging commitments from middle-income developing countries.

The analysis presented in this paper can be extended in different ways, e.g.

(i) The analysis is premised on a summation technology (Sandler 2004) and it is possible to compare the outcomes that would emerge if there were different technologies to produce a global public good (e.g. see Ihori 1996 and Jayaraman and Kanbur 1999).

(ii) The paper explores the normative effects of subsidies and these are important if a political economy model is to be developed to analyse the pressures that are likely to be exerted by different groups in donor countries and in recipient countries. The gains reported in this paper are likely to be appropriated by groups that are able to exert political influence (e.g. Jones 2006).

While many future developments are possible, the important message is that the subsidies developed countries offer to mobilise developing countries can also be designed to appropriate the efficiency gains that are delivered by collective action. It is far from obvious that international subsidies provide any meaningful assistance to recipient countries. If policymakers ignore the possibilities illustrated in this paper, donors are able to employ international subsidies to help themselves, rather than to help others.

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Appendix

In this appendix we gather proofs of our main results.

Proof of Proposition 1 From (9), we have $\frac{\partial u_1}{\partial y_1}=\lambda$. Substituting for $\frac{\partial u_1}{\partial y_1}$ in (8), we find that $\mu m_2 \frac{\partial u_2}{\partial y_2}=\lambda m_2$; viz. (12) . Finally, substituting for the marginal utilities of both countries' incomes using (11-12) in the first order condition (7), and dividing throughout by λ, we obtain the Samuelson condition (10). $\square$

Proof of Corollary 1 $v_1(m_1,m_2,\bar{u}_2)$ is a maximum value function for the problem (6), where both Lagrange multipliers are strictly positive. From the envelope theorem we readily obtain:

$$\frac{\partial v_1}{\partial m_2}=\frac{\partial L}{\partial m_2}=\mu\frac{\partial u_2}{\partial y_2}(1-\hat{t})+\lambda\hat{t} \tag{43}$$

From (12) $\mu\frac{\partial u_2}{\partial y_2}=\lambda$. Hence $\frac{\partial v_1}{\partial m_2}=\lambda(1-\hat{t})+\lambda\hat{t}=\lambda$ as required. This establishes (i).

Applying the envelope theorem a second time we have

$$\frac{\partial v_1}{\partial \bar{u}_2}=\frac{\partial L}{\partial \bar{u}_2}=-\mu \tag{44}$$

Thus (ii) is also proven. $\square$

Proof of Proposition 2 Consider the Lagrangian associated with the multi-country model. This would take the form:

$$
\begin{aligned}
L = {} & n_1 u_1(x, m_1(1-t) - sx) + \\
& + \sum_{i=1}^{n_2} \mu_i^* [u_2(x, m_2(1-t)) - \bar{u}_2] \\
& + \lambda [t(n_1 m_1 + n_2 m_2) - x(1 - n_1 s)]
\end{aligned} \tag{45}
$$

We note that whatever allocation $\left(\hat{x}, \hat{t}, \hat{s}\right)$ maximizes $ku_1(x, y_1)$ (where $k > 0$), also maximizes $u_1(x, y_1)$. Furthermore, we define $\mu = \sum_{i=1}^{n_2} \mu_i^*$. Thus, the point $\left(\hat{x}, \hat{t}, \hat{s}\right)$ solution to the above Lagrangian is also solution to (14). $\square$

$\blacksquare$