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Quality Design and Environmental Implications of Green Consumerism in Remanufacturing

ABSTRACT

We study quality design and the environmental consequences of green consumerism in a remanufacturing context. Specifically, a firm has the option to design a non-remanufacturable or a remanufacturable product and to specify a corresponding quality, and the design choices affect both the production costs and consumer valuations associated with the product. On the cost side, remanufacturable products cost more to produce originally, but less to remanufacture, than non-remanufacturable products cost to produce. Analogously, on the consumer side, remanufacturable products are valued more, but remanufactured products are valued less, than non-remanufacturable products are valued. Given this, we investigate the environmental consequences of designing for remanufacturability by first defining a measure of environmental impact that ultimately is a function of what is produced and how much is produced, and then applying that measure to assess the environmental impact associated with the firm's optimal strategy relative to the environmental impact associated with the firm's otherwise optimal strategy if a non-remanufacturable product were designed and produced.

Keywords: Product Design; Quality; Remanufacturing; Environmental Friendliness; Green Consumerism

1 INTRODUCTION

"Green" has become a buzzword that penetrates daily life. Among consumers, in particular, there is a growing trend to incorporate socially responsible considerations into purchasing decisions and to buy eco-friendly products accordingly (Layton, 2008; Grekova et al., 2014; Eurobarometer, 2008, 2009). This trend is referred to as "green consumerism" and is conceptualized as 'a personal ethical orientation or as a set of pro-environmental personal values and attitudes that inform a particular form of socially conscious or socially responsible decision making' (Moisander and Pesonen, 2002, pp.329). Basically, green consumers are those who are willing to trade-off, to varying degrees, conventional economic attributes of a product, such as price and quality, for example, for environmentally friendly features. Indeed, a significant portion of consumers even are willing to pay a premium for products with these features. Based on their recent meta-analysis of approximately 80 empirical studies published or presented between 1996 and 2012, for example, Tully and Winer (2014) conclude that upwards of 60% of respondents are willing to pay such a premium and that, on average, the premium that consumers are willing to pay is 16.8%. Thus, the green segment is becoming increasingly important to firms.

Recognizing these shifts in the marketplace, firms understandably are redesigning products to include features that would appeal to green consumers. Remanufacturing is an example of a process that creates such appeal. Remanufacturable products are generally considered to be not only environmentally friendly because they lead to reduced waste by encouraging practices such as reverse logistics

(Guide, 2000; Lai et al., 2013), but also profitable because they translate into lower production costs (Caterpillar Press Release, 2005; Lebreton and Tuma, 2006; Robotis et al., 2012; Sabharwal and Garg, 2013). Nevertheless, whereas the economic benefits of remanufacturing have been studied extensively, the environmental implications remain unclear. One possible reason for this ambiguity stems from reports that firms proliferated across many industries are guilty of greenwashing in the sense that they ride the wave of green consumerism without necessarily considering whether or not their actions actually benefit the environment (Orange, 2010). Thus, the interplay of how remanufacturability, on the one hand, and green consumerism, on the other hand, affect the environment requires more rigorous examination.

In this paper, we study a firm's quality design problem in a remanufacturing context given that the firm's market is defined by green consumers. More importantly, we examine the environmental consequences associated with the resulting optimal design. In particular, we investigate the conditions under which a firm designs its products to be remanufacturable. Accordingly, we develop a model to characterize the firm's optimal product portfolio of new and remanufactured products, and establish the corresponding optimal product design decisions. Then, we assess the environmental impact of these results. Consequently, we ascertain the extent to which profitability and environmental friendliness are complementary, and in doing so, we identify key drivers that would make remanufacturing practices more environmentally friendly.

We approach these issues by modeling a firm's quality design and remanufacturing decisions when consumers are heterogeneous in their willingness to pay for product quality. On the product design front, the firm must choose to design either a non-remanufacturable product or a remanufacturable product; and the firm must also specify the corresponding product quality, which we define as a single dimensional vertical characteristic as in Moorthy (1984). These design choices impact both the manufacturer's cost structure and consumers' valuations of the product. On the cost side, a remanufacturable product costs more to produce originally, but less to remanufacture, as compared to the cost of producing a non-remanufacturable product (Debo et al., 2005, 2006). On the consumer side, a remanufacturable product is valued more by consumers (Tully and Winer, 2014; Sengupta, 2011; Harris Interactive, 2013), as compared to the valuation of a non-remanufacturable product, but a remanufactured one is valued less (Guide and Li, 2010; Michaud and Llerena, 2011). The firm thus must consider these trade-offs and optimally choose the design and corresponding quality and price. Note that our model therefore applies to cases in which remanufacturability is an upfront decision made as part of the design process, which is true for many firms; although firms sometimes choose to remanufacture some of their product lines well after product introduction, such a case is beyond the scope of our study.

To operationalize our problem of quality design for a green market with a parsimonious model that captures remanufacturing fundamentals, we follow the lead of Ferrer and Swaminathan (2006) and Atasu et al. (2008) by formulating a two-stage analytic framework. At the beginning of stage 1, the firm first determines whether to design a remanufacturable product or a non-remanufacturable product and, correspondingly, establishes the quality of the chosen product. Then the firm sets the selling price for the product and sells an amount accordingly, as dictated by the specified consumer

market's heterogeneity. Finally, to conclude stage 1, consumers who purchase the product extract its consumption value and then either discard the remains (which is the case if the product was designed to be non-remanufacturable) or return the remains (which is the case if the product was designed to be remanufacturable). The amount of re-collected remains, if applicable, thus establishes a supply constraint on the number of units that can be remanufactured for resale. Given that, at the beginning of stage 2, the firm's decision is to set its optimal product portfolio, that is, to determine how many units of new versus remanufactured products to produce and what associated prices to set accordingly for sale of each product type in stage 2.

By jointly studying quality design for a green market and product design for remanufacturability to assess and evaluate the associated environmental consequences, our model offers two benefits that constitute its primary contribution. The first benefit of our model is the incorporation of product quality as a vertical attribute in a remanufacturing context. Specifically, we formulate our model by explicitly building not only the firm's cost structure but also consumers' valuation preferences on quality. As a result, we find that, everything else being equal, the firm would couple increased remanufacturing with higher product quality. This mirrors empirical evidence that not only suggests a strong link between product quality and environmental performance (e.g., Wiengarten and Pagell, 2012; Oakley, 1993), but also suggests that consumers are interested in environmental performance and quality as interrelated dimensions of their willingness to pay (Jaffry et al., 2004). In addition, we find that product quality provides a demand lever for manipulating the product mix of new versus remanufactured products offered in stage 2. For example, if demand is not affected by quality, then the firm could reap the cost benefits of remanufacturing in stage 2 only by increasing sales of new products in stage 1 through lowering price (Yalabik et al., 2013). However, with quality dependent demand, the firm would decrease new products sales and increase product quality instead, and in doing so, the firm could charge a higher profit margin from each product without lowering price.

The second benefit of our model is the explicit inclusion of the notion of environmental impact, a quality-dependent analytical measure, to quantitatively capture the environmental consequences of designing for remanufacturability. This environmental measure provides a mechanism to assess the ecological footprint of product design and quality in a remanufacturing context. Specifically, our measure represents the total resources acquired from, and wastes discarded to, the environment during the planning horizon. Moreover, this measure is robust in the sense that various weights can be assigned to the different stages of the product life cycle without altering the insights. Perhaps most notable among these insights is that environmental impact could increase significantly if consumers are green to the extent that they value the idea that a product *can be* remanufactured but not to the extent that they also value the fact that the product *has been* remanufactured. This subtle distinction can be particularly detrimental to the environment if consumers value remanufactur<u>able</u> products on the one hand, but significantly *devalue* remanufactur<u>ed</u> products on the other hand. Given strong empirical evidence that environmentally friendly products are favored by consumers (e.g., Yoo and Kwak, 2009; Davis et al., 1995; Laroche et al., 2001) while remanufactured products are perceived as lower quality (e.g., Michaud and Llerena, 2011; Hazen et al., 2012), this somewhat counterintuitive result suggests that environmental friendliness is not necessarily a synonym for remanufacturing, and it

reinforces the idea that consumption rather than production, per se, is the enemy of the environment. Thus, to paraphrase Orange (2010), the first step to a better environment is to reduce, not to recycle or to reuse.

In a similar vein, we also find that a lower production cost or a higher remanufacturing cost saving may not necessarily benefit the environment, despite increasing profit for the firm. Intuitively, a lower production cost (for new products) attracts the firm to increase the volume of new products, which thereby consumes more virgin resources and results in more discarded waste. Similarly, a higher cost saving from remanufacturing attracts the firm to design higher product quality, which again ultimately results in a more negative impact to the environment. This result is an example of Jevons paradox (Alcott, 2005). As such, it echoes discussions, many in industry journals, that warn against wholesale adoption of practices such as remanufacturing and recycling without considering industry dynamics and product properties (e.g., Volokh and Scarlett, 1997; Reich, 2004; Griff, 2003) by suggesting that it is in the interest of the environment for production technologies not to be too cost efficient.

The remainder of this paper is organized as follows. In §2, we review the literature and position our paper accordingly. In §3, we specify and discuss our model primitives, and we formulate and solve the firm's resulting profit maximization problem by mapping out and cataloging different quality design and remanufacturing strategies, we compare the different strategies to determine the firm's optimal decisions, and we explore implications accordingly. Section 4 defines the measure that we use to evaluate the impact on the environment resulting from the firm's optimal strategy. We discuss the scope and applicability of our model in §5, and we conclude the paper in §6. Proofs of propositions appear in Appendix.

2   RELATION TO LITERATURE

Our research relates to two streams of literature. The first stream is the literature on product design and segmentation, which we trace to Mussa and Rosen (1978) and to Moorthy (1984). See also Yayla-Kullu et al. (2011) and the references therein. This stream studies the optimal quality and pricing decisions of a product line that is differentiated by quality when serving consumers heterogeneous in their willingness to pay. Results indicate that a low-quality product cannibalizes the sales of a high-quality product and, as a result, the firm responds by increasing the differentiation between the high- and low-quality products. Our problem is similar to this product line problem in that the new and remanufactured products in our model can be considered as the high- and low-quality products in the product line problem. However, our problem is also different from the product line problem in two respects. The first difference is the presence of remanufacturing supply constraint that restricts the number of remanufactured products to be no greater than the number of re-collected used products available. The second difference is that the consumer perception of quality of the two products is not the same. In our remanufacturing setting, both products have the same quality level. Thus, differentiation between products is due to the fact that consumers discount their valuation of remanufactured products because these products were used previous to being remanufactured.

The second stream is the literature on remanufacturing. Within this realm, there are two different

foci. One focus is defined primarily by operational concerns. This focus typically assumes price, demand, and remanufacturability to be exogenous, and generally applies to issues such as logistics, production planning, and inventory control (Fleischmann et al., 1997; Toktay et al., 2000; Ferguson et al., 2011; Bulmuş et al., 2013; Hsueh, 2011; Vercraene et al., 2014). The other focus is defined primarily by pricing or remanufacturability concerns and generally applies to such issues as market segmentation or growth (Debo et al., 2005, 2006; Ferrer and Swaminathan, 2006; Atasu et al., 2008; Atasu and Souza, 2011; Xiong et al., 2014) and competition (Majumder and Groenevelt, 2001; Ferrer and Swaminathan, 2006, 2010; Ferguson and Toktay, 2006; Mitra and Webster, 2008; Kleber et al., 2011; Choi et al., 2013). This focus typically models rich interactions between price and quantity trade-offs. In particular, on the one hand, the firm has incentive to lower the first-stage new product price to boost sales in order to achieve cost savings realized by selling remanufactured products in the second stage. However, on the other hand, such remanufacturing would cannibalize new product sales in the second stage, thus creating a disincentive for remanufacturing. Considering these trade-offs, Majumder and Groenevelt (2001) analyze competition between an OEM and a local firm. They find that a social planner should provide incentives to the OEM to increase remanufacturing. Debo et al. (2005) study segmentation and remanufacturability in an infinite-horizon setting and find that both fixed and variable costs affect remanufacturing in a negative way. Debo et al. (2006) similarly study life-cycle dynamics of new and remanufactured products; they find that the interplay between the diffusion rate and the likelihood of repeat purchases is a fundamental driver for investing in remanufacturing capacity. Ferrer and Swaminathan (2006, 2010) study both finite-horizon and infinite-horizon problems in a competition setting and conclude that remanufacturing decisions become stable after initial states. Ferguson and Toktay (2006) investigate remanufacturing as a strategy to deter market entry. Common to these models is the assumption that both new and remanufactured products are indistinguishable or that remanufactured products are discounted relative to new products. None of these models, however, define product quality as a vertical attribute that affects demand. In contrast, our model includes product quality as a decision in addition to price that affects demand.

More recently, Atasu and Souza (2011) study a firm's quality choice with regards to different types of product recovery. They find that while product recovery generally increases the firm's profitability, different forms of product recovery impact the environment differently. In particular, they find that although remanufacturing is a product recovery strategy that provides environmental benefits, recycling is a product recovery strategy that can contribute to environmental damage. As such, Atasu and Souza (2011) is particularly instrumental to our work because it provides the inspiration and basis for both the product-design cost function as well as the analytical measure of environmental damage that we adopt, but our work differs from theirs in three important respects. First, we use a two-stage model to capture the trade-off between current and future profits. Second, we incorporate the notion that green consumers are willing to pay more for products that can be remanufactured upon return than they are for products that cannot be remanufactured, everything else being equal. This assumption is consistent with the evidence suggesting that consumers are willing to pay a premium for products with green features (Tully and Winer, 2014; Harris Interactive, 2013), given the stipulation that re-manufacturability constitutes one such green feature. We make this stipulation because of indirect

support in the form of industry initiatives that identify organizations with established recycling activities and that encourage consumers to patron those organizations (such as, for example, the Electronics Takeback Coalition (`http://www.electronicstakeback.com`, 2014)). We note, however, that the extent to which consumers can observe remanufacturability and the extent to which they are willing to pay premiums for this specific feature may vary from one product to another. Third, we consider remanufacturable products more costly to produce than non-remanufacturable ones. This assumption is consistent with Debo et al. (2005, 2006) and reflects the intuition that advanced components (i.e., components more durable than the original item) might be required in such circumstances.

In addition to these modeling-focused research streams, remanufacturing has been investigated empirically as well. The empirical remanufacturing literature can be roughly split into three categories relevant to this paper. First, consumer perceptions of remanufactured products (e.g., Hazen et al., 2012) and green consumerism (e.g., Prothero, 1990) have been investigated widely, the former in the operations management literature, and the latter in the marketing and psychology literatures. These literatures, in particular, describe the impacts and drivers of green consumerism, that is the notion that consumers, all else being equal, will prefer environmentally friendly products. Although the exact motivations behind the phenomenon are beyond our scope, the resulting behavior is explicitly built into our model. The second stream of empirical research relevant to this work is that which investigates the barriers to the adoption of remanufacturing (e.g., Besch, 2005) and closed-loop systems (e.g., Miemczyk, 2008). This body of literature looks at both the institutional and market limitations to operating systems such as those supporting remanufacturing. Thus, our work both feeds off of and contributes to this literature. Finally, there is a body of literature that investigates the impact of environmentally friendly practices on profits (e.g., Ageron et al., 2012; Gimenez et al, 2012; Wong et al., 2012), which we contribute to by identifying the conditions under which remanufacturing is profitable, regardless of whether or not it is environmentally friendly.

3   THE MODEL

We specify the modeling assumptions that define the firm, the consumers, and the decision making framework in §3.1. Then we develop and conditionally solve the firm's design for remanufacturing problem based first on the stipulation that the firm does not remanufacture any products in stage 2 (§3.2) and subsequently on the stipulation that the firm does remanufacture products in stage 2 (§3.3). We compare the results in §3.2 and §3.3 to develop the optimal quality design in §3.4.

3.1   Modeling Assumptions

The Firm. We define the firm as a profit maximizer that can both *manufacture new* products and *remanufacture used* ones. The firm makes design choices concerning product quality and remanufacturability. We model product quality, denoted by q, as a one-dimensional vertical measure representing all the components/attributes that consumers prefer. We model remanufacturability, denoted by k, such that $k = 0$ represents a *non-remanufacturable* product and $k = 1$ represents a *remanufacturable* product. Once determined, the product design cannot be changed. Moreover, a remanufacturable

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product can be remanufactured at most once.

Given product quality q and remanufacturability k, the firm incurs a variable cost for each unit of its product produced[1]. Consistent with the product design literature, the variable cost of production is quadratic in quality, but the magnitude of this variable cost depends on whether the unit produced is a new non-remanufacturable product, a new remanufacturable product, or a remanufactured product. Specifically, we use q^2 to model the variable cost per unit to produce a new *non-remanufacturable* product of quality q, we use $(1 + c_1)q^2$ to model the per unit variable cost to produce a new *remanufacturable* product (i.e., we use $c_1 q^2$ to denote the cost premium associated with producing a new remanufacturable unit over producing a new non-remanufacturable unit), and we use $(1 - c_2)q^2$ to model the per unit variable cost to produce a *remanufactured* product (i.e., we use $c_2 q^2$ to denote the cost savings associated with remanufacturing a used unit over producing a new non-remanufacturable unit). Note that $c_1 \geq 0$, which means that producing a new product that *can* be remanufactured is (weakly) more expensive than producing a new product that *cannot* be remanufactured, and that $0 \leq c_2 \leq 1$, which means that *remanufacturing* a used product is (weakly) less expensive than *manufacturing* a new product. Our assumptions are consistent with Debo et al. (2005, 2006).

Given this construct, to facilitate our presentation, we denote $g_n(k) = 1 + c_1 k$ for $k = 0, 1$ as the production cost coefficient associated with a new product, which depends on whether or not the new product is remanufacturable, and we denote $g_r = 1 - c_2$ as the remanufacturing cost coefficient, which applies only when $k = 1$. Please see Table 1 below for a summary of this nomenclature.

Consumers. We model heterogenous green consumerism where consumers differ vertically in their willingness to pay, and they differentiate whether a product is new and *non-remanufacturable*, new and *remanufacturable*, or used but *remanufactured*. Specifically, we model consumers' valuation for a new *non-remanufacturable* product of quality q as vq, where v follows a uniform distribution along $[0, 1]^2$. Comparatively, we model consumers' valuation for a new *remanufacturable* product of quality q as $(1 + \theta)vq$ (i.e., we use θvq to denote consumers' valuation premium associated with purchasing a new remanufacturable product over a new non-remanufacturable one), and we model consumers' valuation for a *remanufactured* product as $(1 - \alpha)vq$ (i.e., we use αvq to denote the valuation discount associated with purchasing a remanufactured product over a new non-remanufacturable one). Consistent with Tully and Winer (2014) and Harris Interactive (2013), we define $\theta \geq 0$ to mean that consumers in a green market value a new product that *can* be remanufactured (weakly) more than one that *cannot* be remanufactured; and, consistent with Atasu et al. (2008) and Guide and Li (2010), we define $\alpha \geq 0$ to mean that consumers in a green market nevertheless value a used product that *has been* remanufactured (weakly) less than a new one. Analogous to $g_n(k)$ and g_r, we denote $f_n(k) = 1 + \theta k$ for $k = 0, 1$ and $f_r = 1 - \alpha$ to facilitate our presentation. Please see Table 1 for a summary of this nomenclature.

Consistent with similar derivations in Ferguson and Toktay (2006) and Ferrer and Swaminathan

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[1]Although in some remanufacturing models the firm incurs a fixed cost associated with developing and choosing production technology, we adopt the convention of Atasu et al. (2008) and normalize this fixed cost to zero. However, incorporating a fixed cost such as this in our model is straightforward.

[2]In this paper, we assume a uniform distribution for consumer valuation of quality for several reasons: 1) it allows us to focus on the cannibalization between new and remanufactured products; 2) it enables us to derive closed form results; 3) it is a standard assumption in the quality design literature.

Table 1: Model parameters definition

Notation	Definition	Range	References
$f_n(k)$	$1 + \theta k$	≥ 1	Tully and Winer (2014); Harris Interactive (2013)
f_r	$1 - \alpha$	$(0, 1]$	Atasu et al. (2008); Guide and Li (2010)
$g_n(k)$	$1 + c_1 k$	≥ 1	Debo et al. (2005, 2006)
g_r	$1 - c_2$	$(0, 1)$	Debo et al. (2005, 2006)

(2006), we assume that consumers purchase to maximize their non-negative surplus, which is defined by the difference between valuation and price paid. Let p_n and p_r be the price of a new and a remanufactured product, respectively. Then, the surplus derived by a consumer of type v from buying a new and a remanufactured product characterized by q and k is $f_n(k)vq - p_n$ and $f_r vq - p_r$, respectively. If only new products are available, then consumers choose to purchase the new product if $f_n(k)vq - p_n \geq 0$. If both new and remanufactured products are available, then consumers choose the new product over the remanufactured one if $f_n(k)vq - p_n \geq f_r vq - p_r$ and vice versa, but if consumers cannot derive nonnegative surplus from either product then they will remain inactive (i.e., they will purchase neither product), which occurs for consumers with low valuation as shown in Figure 1.

In the first stage, only new products can be sold regardless of product design. Thus the demand for new products in the first stage, d_{n1}, is

$$d_{n1} = 1 - \frac{p_{n1}}{f_n(k)q} \tag{1}$$

where p_{n1} is the new product price in the first stage, q is the quality, and $k = 0, 1$ is the remanufacturability. If the product is designed to be non-remanufacturable (i.e., if $k = 0$), then in the second stage, again only new products can be sold. In this case, the corresponding second stage demand is similar to (1): $d_{n2} = 1 - \frac{p_{n2}}{f_n(0)q}$, where p_{n2} is the second stage new-product price. If the product is designed to be remanufacturable (i.e., if $k = 1$), then in the second stage the firm must choose whether or not to remanufacture. If the firm chooses not to remanufacture any units in the second stage, then only new products can be sold and the corresponding demand is again similar to (1). In contrast, if the firm chooses to remanufacture, then a fraction of consumers purchase new products, a fraction of consumers purchase remanufactured products, and the remainder of consumers purchase no products[3].

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[3]Given that $\alpha \geq 0$ by assumption, consumers who buy new products have higher v than those who buy remanufactured
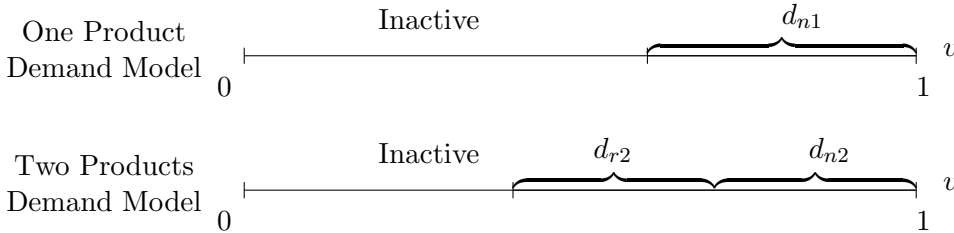

Figure 1: Heterogeneous consumers

In this case, the corresponding second stage demands for new and remanufactured products, d_{n2} and d_{r2}, respectively, are

$$d_{n2} = 1 - \frac{p_{n2} - p_{r2}}{(f_n(1) - f_r)q}, \text{ and}$$

$$d_{r2} = \frac{p_{n2} - p_{r2}}{(f_n(1) - f_r)q} - \frac{p_{r2}}{f_r q} \tag{2}$$

where p_{r2} is the remanufactured product price in the second stage.

The Decision Framework. Consistent with the related remanufacturing literature (Majumder and Groenevelt, 2001; Ferrer and Swaminathan, 2006; Ray et al., 2005; Atasu et al., 2008), we use a two-stage model to formulate the firm's design for remanufacturing problem. At the beginning of stage 1, the firm first chooses its product quality and remanufacturability. Then the firm sets a corresponding selling price and produces only new products. At the end of stage 1, these units either are discarded (which is the case if $k = 0$) or are re-collected after use (which is the case if $k = 1$)[4]. Finally, in stage 2, the firm sets the prices for new and (or) remanufactured products and produces the corresponding quantities subject to the remanufacturing supply constraint imposed by the new product sales in the first stage.

Given this, we next develop and solve the firm's profit maximization problem for given quality q to establish a strategy space for product quality that depends on whether or not remanufacturing takes place. We first investigate the case in which there is no remanufacturing (i.e., the case in which $d_{r2} = 0$). We refer to this case as the *Non-Remanufacturing* strategy. Then we study the case in which there is remanufacturing (i.e., the case in which $d_{r2} > 0$). We refer to this case as the *Remanufacturing* strategy. Later, in §3.4, we compare the profit of the optimal Remanufacturing strategy to that of the optimal Non-Remanufacturing strategy to establish conditions indicating when it is optimal to design a remanufacturable product for the green market.

3.2 Non-Remanufacturing Strategy

We define the Non-Remanufacturing strategy as one in which only new products are sold in each period. In other words, by definition of this strategy, $d_{r2} = 0$. Nevertheless, at the design stage, two substrategies exist: either design the product to be non-remanufacturable ($k = 0$), which we refer to as the *Baseline* substrategy (B); or design the product to be remanufacturable ($k = 1$), which we refer to as the *Premium* substrategy (M). In either substrategy, the demand function is the same for both stage 1 and stage 2 and is given by (1). Accordingly, if the firm implements the Non-Remanufacturing strategy, then $p_{n1} = p_{n2} = p$ and the total profit for the two stages is

$$\Pi^{NR}(p, q, k) = 2 \left(1 - \frac{p}{f_n(k)q} \right) (p - g_n(k)q^2), \tag{3}$$

ones. This is depicted in Figure 1, which illustrates that d_{n2} is to the right of d_{r2}.

[4]Throughout our analysis, we implicitly assume 100% collection rate for the remanufacturable products at the end of stage 1 and, similar to Atasu et al. (2008), we assume that the collection cost is linear in the quantity collected and is included in the production cost $g_n q^2$. For the case in which the collection rate is less than 100%, our results are not significantly altered as long as the collection rate is not too small.

which is concave in p for given quality q and remanufacturability k. Optimizing (3) over p for given q and k, we have $p(q, k) = \frac{(f_n(k) + g_n(k)q)q}{2}$. Substituting this for p in (3), the resulting profit written as a function of q and k, is

$$\Pi^{NR}(q, k) = \frac{q(f_n(k) - g_n(k)q)^2}{2f_n(k)}. \tag{4}$$

Let $q^{NR}(k)$ denote the optimal product quality as a function of k given that the Non-Remanufacturing strategy is implemented. Then, optimizing (4) over q yields $q^{NR}(k) = \frac{f_n(k)}{3g_n(k)}$ for $k = 0, 1$. This, from (4), implies that

$$\Pi^{NR}(k) \equiv \Pi^{NR}(q^{NR}(k), k) = \frac{2f_n(k)^2}{27g_n(k)} = \begin{cases} \frac{2}{27}, & \text{if } k = 0; \\ \frac{2(1+\theta)^2}{27(1+c_1)}, & \text{if } k = 1. \end{cases} \tag{5}$$

Given (5), let $\Pi^B \equiv \Pi^{NR}(k = 0)$ denote the firm's profit if the Baseline substrategy is implemented, and let $\Pi^M \equiv \Pi^{NR}(k = 1)$ denote the firm's profit if the Premium substrategy is implemented. Then, comparing Π^B to Π^M leads directly to the following proposition.

Proposition 1 (Optimal Non-Remanufacturing Strategy). *Let $(k^{NR*}, q^{NR*}, p^{NR*})$ denote the optimal design and pricing decisions, given that $d_{r2} = 0$; and let d^{NR*} and Π^{NR*} denote the associated two-stage total demand and profit, respectively. Then, the Premium substrategy is optimal (i.e., $k^{NR*} = 1$) if and only if $1 + c_1 < (1 + \theta)^2$. Accordingly,*

	k^{NR*}	q^{NR*}	p^{NR*}	d^{NR*}	Π^{NR*}
$1 + c_1 < (1 + \theta)^2$	1	$q^M = \frac{(1+\theta)}{3(1+c_1)}$	$\frac{2(1+\theta)^2}{9(1+c_1)}$	$\frac{2}{3}$	$\Pi^M = \frac{2(1+\theta)^2}{27(1+c_1)}$
$1 + c_1 \geq (1 + \theta)^2$	0	$q^B = \frac{1}{3}$	$\frac{2}{9}$	$\frac{2}{3}$	$\Pi^B = \frac{2}{27}$

Recall that $1 + \theta$ represents a consumer's valuation coefficient for a remanufacturable product and that $1 + c_1$ represents the corresponding cost coefficient for the remanufacturable product. Thus, in essence, Proposition 1 indicates that if the marginal valuation for a remanufacturable product justifies the marginal cost of producing the product, then designing for remanufacturing is justified even if no units are actually remanufactured subsequently. In other words, even without remanufacturing, it could be in the firm's interest to design a remanufacturable product if that is what consumers value. This suggests that the valuation premium θ is more representative of the halo effect of green *consumerism*, or the willingness to pay for the idealistic promise of a green world, than it is representative of actual greenness, per se (Layton, 2008). In this sense, θ is akin to consumers "talking a green talk", but it is not indicative of whether or not consumers are willing to follow through by actually "walking a green walk".

3.3 Remanufacturing Strategy

We define the Remanufacturing strategy as one in which remanufactured products are sold in the second stage. In other words, by definition of this strategy, $d_{r2} > 0$. Given this strategy, the firm designs

the new product to be remanufacturable (i.e., $k = 1$). In stage 2, then, the firm optimally determines the product portfolio of new and remanufactured products knowing that the remanufactured products will cannibalize the demand for new products. We therefore categorize the Remanufacturing strategy into two substrategies depending on the firm's level of remanufacturing in stage 2 given that all used units are returned at the end of stage 1. If all returned used products are remanufactured (i.e., if $d_{n1} = d_{r2} > 0$), then we say the firm implements the *Complete Remanufacturing* (CR) substrategy. If fewer than all returned used products are remanufactured (i.e., if $d_{n1} > d_{r2} > 0$), then we say the firm implements the *Partial Remanufacturing* (PR) substrategy. In either of these substrategies, the demand functions for stage 1 and stage 2 are given by (1) and (2), respectively. Accordingly, if the firm implements the Remanufacturing strategy, then the total profit for the two stages is

$$\Pi^R(p_{n1}, p_{n2}, p_{r2}, q) = d_{n1}(p_{n1} - g_n q^2) + d_{n2}(p_{n2} - g_n q^2) + d_{r2}(p_{r2} - g_r q^2) \tag{6}$$

where $k = 1$ by definition, and $f_n \equiv f_n(1) = 1 + \theta$ and $g_n \equiv g_n(1) = 1 + c_1$. Given (6), note that the Remanufacturing strategy would be implemented only if $g_r/f_r < g_n/f_n$ because, otherwise, the Remanufacturing strategy would be dominated by the Non-Remanufacturing strategy of Section 3.2. (Please see Lemma 1 in Appendix for details.) Thus, in mapping out the Remanufacturing strategy, we implicitly assume that $g_r/f_r < g_n/f_n$ to ensure that $d_{r2} > 0$. Optimizing (6) for given q, subject to the supply constraint $d_{r2} \leq d_{n1}$, thus yields Lemma 2, which also is provided in Appendix. Lemma 2 characterizes $(p_{n1}^R(q), p_{n2}^R(q), p_{r2}^R(q))$, the optimal two-stage pricing decisions as functions of q, given that the Remanufacturing strategy is implemented, as well as $(d_{n1}^R(q), d_{n2}^R(q), d_{r2}^R(q))$, the corresponding optimal demands. This leads to the following proposition.

Proposition 2. *If the Remanufacturing strategy is implemented, then the optimal two-stage prices and demands, as functions of q, are such that:*

i) $\frac{\partial p_{n2}^R(q)}{\partial q} \geq \left(\frac{\partial p_{n1}^R(q)}{\partial q}, \frac{\partial p_{r2}^R(q)}{\partial q} \right) > 0;$

ii) $\left(\frac{\partial d_{n2}^R(q)}{\partial q}, \frac{\partial d_{n1}^R(q)}{\partial q} \right) \leq 0$, *whereas* $\frac{\partial d_{r2}^R(q)}{\partial q} > 0$ *for* $q < q_1$ *but* $\frac{\partial d_{r2}^R(q)}{\partial q} < 0$ *for* $q > q_1$,

where $q_1 = \frac{f_n f_r (f_n - f_r)}{f_n^2 (g_n - g_r) - (f_n - f_r)^2 g_n}$ *denotes the quality threshold above which* $d_{r2}^R(q) = d_{n1}^R(q)$, *but below which* $d_{r2}^R(q) < d_{n1}^R(q)$.

Proposition 2 (i) shows that the prices for both new and remanufactured products increase in quality, given that the Remanufacturing strategy is implemented. Moreover, the stage 2 new product price increases faster than both the stage 1 new product price and the stage 2 remanufactured product price. These are the effects of both cost saving and cannibalization. First, cost saving negatively affects the stage 1 new product price because high cost saving warrants more used units to be re-collected for remanufacturing purpose, whereas it does not directly affect the stage 2 new product price because these products will not be remanufactured at the end of stage 2. Second, cannibalization positively affects the stage 2 new product price because fewer new products mean consumers with higher valuation are targeted, whereas it negatively affects the stage 2 remanufactured product price because more remanufactured products mean consumers with higher valuation are targeted.

As Proposition 2 also illustrates, if the Remanufacturing strategy is implemented, then the sales volume of new products in both stage 1 and stage 2 decreases as quality increases. In contrast, the corresponding sales volume of remanufactured products first increases and then decreases in q. Intuitively, when quality is sufficiently low (specifically, when $q < q_1$), the firm does not remanufacture all of the returned used products from stage 1. In that case, an increase in quality means larger cost savings from remanufacturing, hence the firm remanufactures more. However, when quality is sufficiently high (specifically, when $q > q_1$), the firm remanufactures all of the returned used products from stage 1. In that case, an increase in quality not only affects the profit of remanufactured products in stage 2, but also that of new products in stage 1. And because the marginal value of remanufacturing a unit is smaller than the marginal loss of producing a unit of new product, the firm remanufactures less. Given (6), let $\Pi^R(q) = \Pi^R(p_{n1}^R(q), p_{n2}^R(q), p_{r2}^R(q), q)$ denote the profit associated with the Remanufacturing strategy, reduced to a function of q only. Lemma 3, also provided in Appendix, establishes that $\Pi^R(q)$ is continuous and unimodal in q. Maximizing $\Pi^R(q)$ accordingly leads to the following proposition.

Proposition 3 (Optimal Remanufacturing Strategy). *Let $(q^{R*}, p_{n1}^{R*}, p_{n2}^{R*}, p_{r2}^{R*})$ denote the optimal design and pricing decisions given that $d_{r2} > 0$; and let d^{R*} and Π^{R*} denote the associated two-stage total demand and profit, respectively. Then by definition $k^{R*} = 1$, and*

$$
q^{R*} = \begin{cases} q_S = \frac{f_n+f_r}{3(g_n+g_r)}, & 0 \le g_r \le g_{rS}; \\[2mm] q_C = \frac{f_n^2(g_n+g_r)+g_n(f_n^2-f_r^2)}{3(f_ng_n^2+f_rg_r^2+(f_n-f_r)(g_n+g_r)^2)}\left(2 - \sqrt{1 - \frac{3(f_n^2+f_nf_r-f_r^2)(f_rg_n-f_ng_r)^2}{(f_n^2(g_n+g_r)+g_n(f_n^2-f_r^2))^2}}\right), & g_{rS} < g_r \le g_{rC} \\[2mm] q_P = \frac{2f_nf_r(f_n-f_r)g_n}{3((f_rg_n-f_ng_r)^2+2f_r(f_n-f_r)g_n^2)}\left(2 - \sqrt{1 - \frac{3(f_rg_n-f_ng_r)^2}{2f_r(f_n-f_r)g_n^2}}\right), & g_{rC} < g_r < g_{rP}; \end{cases}
$$

where $g_{rS} = \frac{f_rg_n}{f_n} - \frac{2(f_n^2-f_r^2)g_n}{f_n(3f_n-2f_r)}$, $g_{rC} = \frac{f_rg_n}{f_n} - \frac{4f_rg_n(f_n-f_r)}{2f_n^2+3f_nf_r-3f_r^2}$, and $g_{rP} = \frac{f_rg_n}{f_n}$. Accordingly, for $j = n1, n2, r2$:

	p_j^{R*}	d^{R*}	Π^{R*}
$0 \le g_r \le g_{rS}$	$p_j^R(q_S)$	$2d_{n1}^R(q_S)$	$\Pi^R(q_S)$
$g_{rS} < g_r \le g_{rC}$	$p_j^R(q_C)$	$2d_{n1}^R(q_C) + d_{n2}^R(q_C)$	$\Pi^R(q_C)$
$g_{rC} < g_r < g_{rP}$	$p_j^R(q_P)$	$d_{n1}^R(q_P) + d_{n2}^R(q_P) + d_{r2}^R(q_P)$	$\Pi^R(q_P)$

Note that $g_{rS} < g_{rC} < g_{rP}$. Thus, Proposition 3 characterizes the optimal remanufacturing quality based on the remanufacturing cost g_r, indicating that this quality increases as g_r decreases, which is intuitive. Accordingly, from Proposition 2, the prices increase, and, correspondingly, the sales volumes of new products decrease while the sales volume of remanufactured products first increases and then decreases. Notice also from Proposition 3 that if the Remanufacturing strategy is implemented, then g_{rC} establishes the critical remanufacturing cost threshold below which the Complete Remanufacturing substrategy is optimal ($d_{r2}^* = d_{n1}^*$), and above which the Partial Remanufacturing substrategy is optimal ($0 < d_{r2}^* < d_{n1}^*$). Similarly, g_{rS} establishes the critical remanufacturing cost threshold below which it is optimal to *exclusively* remanufacture all of the returned used products from stage 1 ($d_{r2}^* = d_{n1}^*$).

3.4 Optimal Design for A Green Market

In this section, we compare the profit associated with the optimal Remanufacturing strategy (Proposition 3) to the profit associated with the optimal Non-Remanufacturing strategy (Proposition 1) to determine conditions for optimality. Specifically, we answer three questions: Given the market of green consumers, when is it optimal to *remanufacture*? When is it optimal to remanufacture *everything*? When is it optimal to design for remanufacturability but *not* to remanufacture?

In the context of our model, remanufacturing means that $d_{r2} > 0$ in an optimal solution. Thus, to answer this question, we directly compare the profit associated with the optimal Remanufacturing strategy (Proposition 3) to the profit associated with the optimal Non-Remanufacturing strategy (Proposition 1). This yields the following proposition.

Proposition 4. *If $g_r/f_r < g_n/f_n < f_n$, then it is optimal to remanufacture; that is, $k^* = 1$ and $d_{r2}^* > 0$.*

Proposition 4 provides a sufficient condition indicating when the Remanufacturing strategy in Proposition 3 is optimal. Notice that this condition is two-fold: First, for remanufacturing to be optimal, it must be worthwhile to produce a remanufacturable product in the first place (re: $g_n/f_n < f_n$). But that is not enough. Second, for remanufacturing to be optimal, it must also be worthwhile to actually remanufacture the remanufacturable products (re: $g_r/f_r < g_n/f_n$). The first half of this sufficient condition ($g_n/f_n < f_n$) is equivalent to that in Proposition 1, thus indicating when it is worthwhile to design for remanufacturability regardless of whether or not remanufacturing will actually occur. For insight into the second half of the sufficient condition, we note that the ratio g_i/f_i can be interpreted as a market cost efficiency ratio in the following sense: Consider a consumer of type v. If that consumer were to purchase a new product with quality q, which would cost $g_n q^2$ to produce, then the corresponding consumer valuation would be $f_n vq$. Thus, the cost per unit valuation of this new product is $g_n q^2/(f_n vq) = g_n q/(f_n v)$. Likewise, the cost per unit valuation of the corresponding remanufactured product is $g_r q/(f_r v)$. Therefore, Proposition 4 essentially indicates that it is optimal to remanufacture if it is more cost efficient, as measured against market valuation, to *remanufacture* a remanufacturable unit of quality q than it is to *manufacture* a remanufacturable unit of quality q (i.e., if $g_r q/(f_r v) < g_n q/(f_n v)$ or $g_r/f_r < g_n/f_n$). Note that this condition is consistent with those in Atasu et al. (2008) and in Ferrer and Swaminathan (2010), both of which consider models of exogenous quality for the special case in which $f_n = g_n = 1$. Thus, Proposition 4 validates our model and extends their results to our case of endogenous quality.

In the context of our model, remanufacturing everything means that $0 < d_{r2} = d_{n1}$ in an optimal solution. Thus, to answer this question, we again compare the profit associated with the optimal Remanufacturing strategy (Proposition 3) to the profit associated with the optimal Non-Remanufacturing strategy (Proposition 1). This yields the following proposition.

Proposition 5. *If $g_r/f_r \leq g_{rC}/f_r < g_n/f_n < f_n$, then it is optimal to remanufacture everything; that is, $d_{r2}^* = d_{n1}^* > 0$.*

Proposition 5 provides a sufficient condition indicating when remanufacturing all of the available supply of re-collected used products is optimal. Notice that, again, this condition is two-fold.

Consistent with Proposition 4, for complete remanufacturing to be optimal, not only must it be worthwhile to first produce remanufacturable products and then to sell remanufactured products (re: $g_r/f_r < g_n/f_n < f_n$), but also it must be worthwhile to remanufacture all of the used products (re: $g_r \leq g_{rC}$). Intuitively, for complete remanufacturing to be optimal, the market cost efficiency ratio of a remanufactured product must be sufficiently lower than that of a new product, namely $g_r/f_r \leq g_{rC}/f_r = g_n/f_n - \frac{4g_n(f_n-f_r)}{2f_n^2+3f_nf_r-3f_r^2} < g_n/f_n$. Otherwise, the potential cannibalization of new product sales would dominate the potential benefits of selling a marginal unit of the products, in which case the firm would be better served by not remanufacturing everything.

Propositions 4 and 5 describe scenarios under which remanufacturing is a profitable activity. Perhaps not surprisingly, remanufacturing is profitable if the cost of doing so is advantageous with respect to the market for remanufactured goods. More surprising is the lack of empirical analysis of the profitability of remanufacturing operations. For instance, Seitz (2007) notes that organizations may not analyze the economic costs and benefits of remanufacturing when it is not their core business, in which case customer orientation might be the bigger justification. Our results suggest that the two dimensions, cost advantage and customer orientation, work together to make remanufacturing profitable.

In the context of our model, designing for remanufacturability but not remanufacturing means that $k = 1$ but $d_{r2} = 0$ in an optimal solution. Thus, to answer this question, first recall from Lemma 1 (in Appendix) that $d_{r2} = 0$ if and only if $g_n/f_n \leq g_r/f_r$. Comparing this with Proposition 1 leads to the following.

Proposition 6. *If and only if $g_n/f_n < \min[f_n, g_r/f_r]$, then it is optimal to design for remanufacturability but not to remanufacture; that is, $k^* = 1$ and $d_{r2}^* = 0$.*

Thus, consistent with Proposition 4, Proposition 6 indicates that, if the market cost efficiency ratio of a new remanufacturable product is lower than that of a remanufactured product (i.e., if $g_n/f_n \leq g_r/f_r$), then it is not worthwhile to remanufacture any units even though it is worthwhile to design remanufacturable products to begin with (i.e., even though $g_n/f_n < f_n$). Basically, under these conditions, optimality dictates leveraging the valuation premium associated with the market's green consumerism, but then avoiding cannibalization by not remanufacturing any units to sell at a discount. Thus, in markets where consumers value environmental attributes in a product (such as remanufacturability) but do not value reused components (such as in remanufactured goods), the ability to remanufacture itself may have adverse implications for the environment. This is in particular an issue for remanufacturing practice because remanufactured products are often seen as lower-quality relative to their new counterparts (Michaud and Llerena, 2011; Hazen et al., 2012).

3.5 Discussion on the Optimal Quality and Demand

Note that, by combining Proposition 1 with Propositions 4–6, we can write the optimal quality (denoted by q^*), given the condition that $g_n/f_n < f_n$, as follows:

$$q^* = \begin{cases} q^{R*}, & \text{if } g_r/f_r < g_n/f_n; \\ q^M = \frac{f_n}{3g_n}, & \text{if } g_r/f_r \geq g_n/f_n. \end{cases} \tag{7}$$
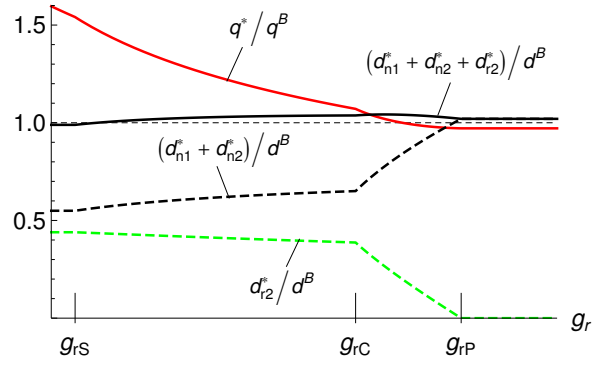
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Figure 2: Illustration of optimal remanufacturing quality and demand

As a stepping stone to better understand the impact of endogenous quality in the context of our model, particularly with regard to its effect on the optimal remanufacturability (k^*), we graphically compare the optimal quality from (7) to $q^B = 1/3$, which from Proposition 1, denotes the quality that corresponds to the Baseline substrategy and, as such, represents the conditionally optimal quality given that $k = 0$. Specifically, in Figure 2, we graph the ratio q^*/q^B as a function of g_r. In a similar vein, we also graph in Figure 2 the ratio of the corresponding total demands $(d_{n1}^* + d_{n2}^* + d_{r2}^*)/d^B$ as well as the ratios of the corresponding new product demands $(d_{n1}^* + d_{n2}^*)/d^B$ and remanufactured product demands d_{r2}^*/d^B. In producing these graphs, we set $f_n = 1.2$, $f_r = 0.8$, and $g_n = 1.1$. These parameters serve as our illustrative and representative case for Figure 2, as well as for the figures that follow unless otherwise stated. Examining first the graph of q^*/q^B, we find that, as the remanufacturing cost g_r increases, the optimal quality decreases until it reaches its minimal level at $g_r = g_{rP}$. At this point, $q^*/q^B = f_n/g_n$; thus, $q^* > q^B$ is assured if $f_n/g_n > 1$. Examining second the graphs of $(d_{n1}^* + d_{n2}^*)/d^B$, d_{r2}^*/d^B and $(d_{n1}^* + d_{n2}^* + d_{r2}^*)/d^B$, we find that a smaller volume of new products are sold when an optimal strategy is implemented than otherwise would be sold if $k = 0$, which is intuitive. However, a (weakly) larger total volume of products are sold than otherwise would be sold if $k = 0$. This thus reflects the conventional wisdom that a firm reaches a larger consumer base when implementing a remanufacturing strategy (Souza, 2008).

4 ENVIRONMENTAL FRIENDLINESS

In this section, we address the extent to which remanufacturable products designed for a green market are indeed environmentally friendly, given that they are optimal to produce. In other words, we assess the environmental friendliness of $k^* = 1$. To this end, we first define an environmental measure that we refer to as *environmental impact*. We then apply this measure to assess the environmental impact associated with the optimal strategy (q^*) discussed in Proposition 3, and we compare that to the environmental impact associated with the Baseline substrategy (q^B) discussed in Proposition 6, which represents the conditionally optimal strategy given that $k = 0$. Specifically, we make comparisons to assess when it is environmentally friendly to remanufacture. In this context, the lower is the

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environmental impact, the more environmentally friendly is the strategy.

To define environmental impact, we follow the leads of Atasu and Souza (2011) and Agrawal et al. (2011) by adopting a single aggregated measure to represent the total resources acquired from, and wastes discarded to, the environment during a product's life cycle. In that vein, we assume that the environmental impact of extracting the resources for manufacturing a product of quality q is $e_p q$, where e_p represents an environmental impact coefficient; and we similarly assume that the environmental impact associated with consuming, remanufacturing (if applicable), and disposing the remains of a product of quality q is $e_u q$, $e_r q$, and $e_d q$, respectively. In this context, we define environmental impact to be linear in q, everything else being equal, to reflect the notion that quality is a one-dimensional vertical measure of a product and, thus, higher quality typically represents the need for larger quantities of virgin materials. Accordingly, for given quality q and remanufacturability k, the environmental impact associated with stage 1 of our model is

$$\Delta_1 = d_{n1} e_p q + d_{n1} e_u q + d_{n1}(1-k) e_d q,$$

where $d_{n1} e_p q$ denotes the environmental impact caused by the extraction of resources required to produce d_{n1} units of new product with quality q at the start of stage 1, $d_{n1} e_u q$ denotes the collateral environmental impact associated with the consumption of those units, and $d_{n1}(1-k) e_d q$ denotes the environmental impact caused by waste disposal if the used d_{n1} units are not re-collected at the end of stage 1 (i.e., if $k = 0$). Analogously, the environmental damage associated with stage 2 of our model is

$$\Delta_2 = d_{n2} e_p q + d_{r2} e_r q + (d_{n1} - d_{r2}) k e_d q + (d_{n2} + d_{r2}) e_u q + (d_{n2} + d_{r2}) e_d q,$$

where $d_{n2} e_p q$ denotes the environmental impact caused by the extraction of resources required to produce d_{n2} units of new product with quality q at the start of stage 2, $d_{r2} e_r q$ and $(d_{n1} - d_{r2}) k e_d q$ denote the associated impact at the start of stage 2 resulting from remanufacturing d_{r2} units and disposing the remaining $(d_{n1} - d_{r2})$ units, respectively, if the used d_{n1} units are re-collected at the end of stage 1 (i.e., if $k = 1$), $(d_{n2} + d_{r2}) e_u q$ denotes the total impact associated with stage 2 consumption, and $(d_{n2} + d_{r2}) e_d q$ denotes the impact caused by waste disposal at the end of stage 2.

Given Δ_1 and Δ_2, the total environmental impact for the two stages, which we denote $\Delta(q)$, is

$$\Delta(q) = \Delta_1 + \Delta_2 = ((d_{n1} + d_{n2}) e_1 + d_{r2} e_2) q \tag{8}$$

where $e_1 = e_p + e_u + e_d$ and $e_2 = e_r + e_u$ represent the per-unit environmental impact associated with a new and a remanufactured product, respectively. Thus, $e_1 > e_2$ is always true by definition. An example of such aggregated measures of environmental impact is the carbon footprint of a product, which is calculated by considering the amount of greenhouse gases emitted during the life-cycle of the product and converting to the equivalent CO_2 emissions (for example $kgCO_2 e$). In that spirit, Table 2 provides a summary of e_1 and e_2 values derived for a sampling of products, taking $kgCO_2 e$ as the unit of measurement. It is interesting to note from Table 2 that e_2/e_1 has a wide spread of values across the range $[0, 1]$ based on the impacts in different stages (represented by e_p, e_u, e_r, and e_d).

Table 2: Environmental parameters examples

Product Type (Unit)	e_p	e_r	e_d	e_u	e_1	e_2	e_2/e_1
					CO$_2$e(kg/unit)		
Paper (ton)[a]	955	680	553	118	1626	789	0.49
Tire (ton)[b]	3177	1977	34	14,100	17,311	16,077	0.93
Cartridge (per)[c]	3.97	1.8	0.0425	45	49.0125	46.8	0.95
Wood (ton)[d]	666	122	851	0	1517	122	0.08
iPad (per)[f]	115.2	14.4	0.0099	41.6	156.8099	56	0.36

a. 1 ton = 200,000 sheets. Assumes office printing. e_u from `hp.com` (Hewlett Packard 2014); e_p, e_r, e_d from DEFRA (DEFRA, 2014).

b. Assumes life of 30,000 miles. e_u from `http://shrinkthatfootprint.com/` (2014); e_p, e_r, e_d from DEFRA (DEFRA, 2014).

c. Assumes one remanufacture cycle. 10,000 pages printed. e_u from `hp.com` (Hewlett Packard, 2014); e_p, e_r, e_d from DEFRA (DEFRA, 2014) assuming 1.25kg plastics per cartridge (ecycler.com, 2011).

d. Assumes no usage impacts. e_p, e_r, e_d from DEFRA (DEFRA, 2014).

f. e_p, e_r, and e_u from Apple (2012), e_d from DEFRA (DEFRA, 2014).

Note that (8) reflects the fact that $(1-k)d_{r2} = 0$ because $k = 0$ requires that $d_{r2} = 0$ by definition. Notice therefore that the waste resulting from the disposal of remanufactured products in stage 2 is offset by the waste avoided by not having to dispose the remanufacturable products re-collected for remanufacturing in stage 1. Thus, as (8) illustrates, what ultimately impacts the environment are three factors: (i) *what* is produced (which is represented by product quality q), (ii) *how much new* is produced (which is represented by $d_{n1} + d_{n2}$), and (iii) *how much used* is remanufactured (which is represented by d_{r2}). Note that, as a result, the cannibalization of new products by remanufactured products in stage 2 is a good thing from the standpoint of environmental friendliness, to the extent that $e_2/f_r < e_1/f_n$. Note also that we write environmental impact $\Delta(q)$ as a function of q because both $(d_{n1} + d_{n2})$ and d_{r2} ultimately are functions of q as per Propositions 1 and 3.

Given (8), we begin our exploration by asking if it necessarily is environmentally friendly to remanufacture everything given that it is optimal to do so. To address this question, recall that, in the context of our model, remanufacturing everything means that $0 < d_{r2} = d_{n1}$ in an optimal solution. Thus, we compare $\Delta(q^*|(k^* = 1, d_{r2}^* = d_{n1}^*))$ to $\Delta(q^B)$.

Proposition 7. *If $g_r \leq g_{rC}$ and $g_n/f_n < \min[f_n, \frac{f_n(2f_n^2+3f_nf_r-3f_r^2)}{(2f_n-f_r)^2(f_n+f_r)^2}((2f_n-f_r)+e_2/e_1f_n)]$, then $\Delta(q^*) > \Delta(q^B)$.*

Proposition 7 demonstrates that, indeed, it is not necessarily environmentally friendly to remanufacture even if it means remanufacturing everything. Surprisingly, but not entirely unexpectedly, this would be the case if the production technology is especially cost efficient (in the sense that g_n/f_n is relatively low). Intuitively, if the market cost efficiency ratio of a new remanufacturable product is low, then it is optimal not only to remanufacture everything when $g_r \leq g_{rC}$, but also to set quality relatively high. As a result, the associated environmental impact ends up greater than it otherwise would be if $k = 0$.

This suggests that cost efficiency potentially affects the environment differently than it affects the
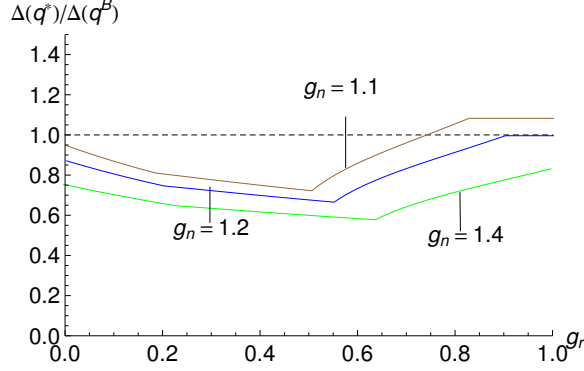
Figure 3: Illustration of environmental impact for varying production cost parameters

firm. To the firm, cost efficiency translates into increased profit. To the environment, however, cost efficiency translates into higher quality which in turn leads to greater environmental impact. Therefore, although it is in the best interest of the firm to pursue such efficiencies, it is not particularly in the best interest of the environmentalist for the firm to achieve such pursuits. This is yet another example of Jevons paradox.

Proposition 7 suggests that cost efficiency of new remanufacturable products (i.e., low g_n/f_n) potentially could result in negative consequences to the environment. However, these consequences are driven by higher product quality rather than larger remanufacturing volume. In other words, although remanufacturing everything may result in more negative consequences to the environment than otherwise would result if the product were designed not to be remanufacturable in the first place (Proposition 7), it turns out that remanufacturing available units generally is more environmentally friendly than not remanufacturing available units if the product is designed to be remanufacturable. Given that previous literature discussing the practical implications of closed-loop systems (e.g., Volokh and Scarlett, 1997; Reich, 2004; Griff, 2003) warns that the environmental benefits of such practices should not be taken for granted, highlighting the importance of product attributes in making this determination, our results underscore this warning, and the sensitivity analysis that follows helps illustrate it.

Through sensitivity analysis, we investigate the impact of production costs (g_n and g_r) and consumer valuations (f_n and f_r) on the environmental friendliness of an optimal policy. Specifically, we fix two of the four parameters while varying the other two, and we compare $\Delta(q^*)$, the environmental impact associated with the optimal strategy, to $\Delta(q^B)$, the environmental impact associated with the conditionally optimally strategy given that $k = 0$. To present the results of these comparisons, given Table 2, we set $e_2/e_1 = 0.30$ for illustrative purposes; however, we note that the graphs and discussions that follow are representative of $0 \leq e_2/e_1 < 1$.

Figure 3 shows graphs of $\Delta(q^*)/\Delta(q^B)$ as a function of remanufacturing cost g_r for three different new product production costs $g_n = 1.1, 1.2$, and 1.4. In the figure, if $\Delta(q^*)/\Delta(q^B) > 1$, then the optimal strategy causes higher environmental impact than the baseline strategy. Given that, Figure 3 demonstrates that a reduction in the remanufacturing cost (g_r) yields a decrease in environmental impact if $g_r > g_{rC}$, but it yields an increase in environmental impact if $g_r \leq g_{rC}$. Intuitively,
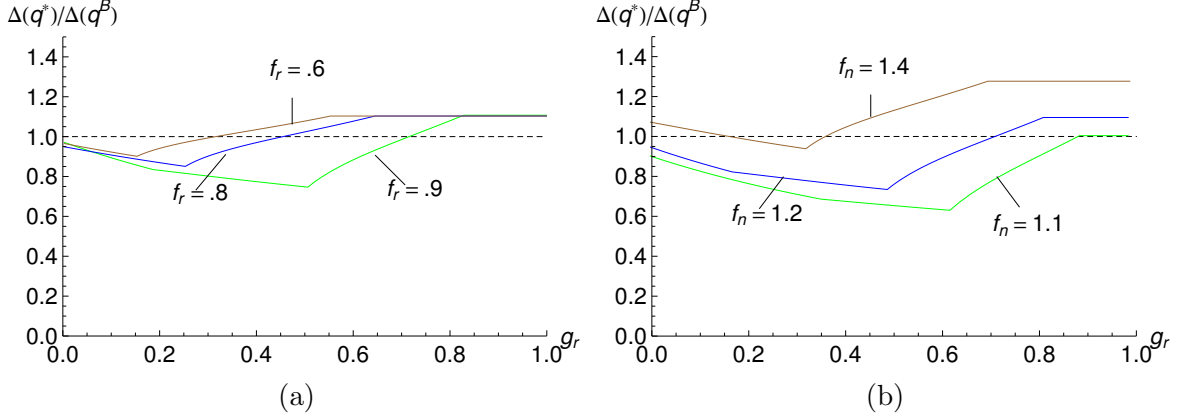
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Figure 4: Illustration of environmental impact for representative valuation parameters

$g_r > g_{rC}$ means that partial (or) no remanufacturing is optimal. In this case, reducing the remanufacturing cost increases the volume of remanufacturing, which increases cannibalization of new products (smaller d_{n2}), thus reducing the impact on the environment. However, $g_r \leq g_{rC}$ means complete remanufacturing is optimal. In this case, further reductions in g_r do not affect cannibalization because all available units already are being remanufactured. Rather, such reductions in g_r result in higher quality, thus increasing the impact on the environment, potentially to the extent that complete remanufacturing becomes less environmentally friendly than the Baseline strategy would otherwise dictate (re: Proposition 7).

Figure 3 also illustrates that environmental impact increases as g_n decreases. This observation reflects two effects: First, a lower g_n means a higher profit margin for new remanufacturable products. Second, a lower g_n also means a smaller relative cost difference between new and remanufactured products, $g_n - g_r$. These two effects thus combine to make remanufacturing less attractive. Hence, the firm manufactures more new products and remanufactures less used ones. As a result, cannibalization decreases, thus increasing the impact on the environment.

In contrast, as illustrated by Figure 4, which provides graphs of $\Delta(q^*)/\Delta(q^B)$ as a function of g_r for three values of f_r ($f_r = 0.6, 0.8,$ and 0.9) and for three values of f_n ($f_n = 1.1, 1.2,$ and 1.4), environmental impact decreases as consumers' willingness to pay for remanufactured products (f_r) increases, but it increases as consumers' willingness to pay for new products (f_n) increases. Intuitively, as Figure 4 (a) reflects, the firm is more reluctant to remanufacture for lower f_r. Thus, reductions in f_r decrease cannibalization, thus increasing the impact on the environment. Similarly, as Figure 4 (b) reflects, a higher f_n translates not only into a greater reluctance to remanufacture but also into a desire for higher quality. Hence, increases in f_n decrease cannibalization and increase quality, thus creating a reinforcing effect increasing the impact on the environment.

These results together suggest that inconsistency in consumers' valuations between what *can be* remanufactured relative to what *has been* remanufactured contributes to environmental impact. Basically, the bigger is the difference, $(f_n - f_r)$, whether it is due to increases in f_n or to decreases in f_r, the larger is the environmental impact that results from an optimal solution. In other words, although valuing remanufactur*ed* products leads to smaller environmental impact, valuing remanufactur*able*

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products leads to greater environmental impact. These results also suggest that whether or not the two bottom lines — profit and environment — align with each other depends on the specific type of products. On the one hand, factors pertaining to remanufactured products yield the same effects on both bottom lines. Increases in profit are achieved through remanufacturing more used products, and the resulting strong cannibalization leads to less damage to the environment. On the other hand, factors pertaining to new remanufacturable products may affect the firm and the environment differently. This is because increases in profit are achieved through offering more new products and remanufacturing less, and the weakened cannibalization leads to more damage to the environment.

5   EXTENSIONS

In §4, we demonstrated that higher product quality is one reason why the environmental impact associated with a remanufacturable product, when such a design is optimal, could be larger than that associated with a non-remanufacturable product, depending on the circumstances. Thus, as our first extension, to test the extent to which this result depends on the endogeneity of quality in our model, we revisit the comparison of environmental impact behind §4 but this time we assume exogenous quality instead. Accordingly, for the purpose of this section, let $\Delta^*(q)$ denote the environmental impact associated with the optimal two-stage pricing and' remanufacturability strategy for given q, and let $\Delta^B(q)$ denote the environmental impact associated with the conditionally optimal pricing policy, given that $k = 0$, for given q.

Proposition 8. *If $g_n/f_n < 1$ and $g_r/f_r > \frac{g_n}{f_n} - \frac{2e_1(f_n - f_r)(f_n - g_n)}{f_n(e_1 f_r - e_2 f_n)}$, then $\Delta^*(q) > \Delta^B(q)$ as long as $q < \frac{f_n f_r(f_n - f_r)}{f_n^2(g_n - g_r) - (f_n - f_r)^2 g_n}$.*

As Proposition 8 demonstrates, even for an exogenous quality, it is not necessarily environmentally friendly to remanufacture. In particular, remanufacturing leads to larger environmental impact if the remanufacturing cost is sufficiently high. In this case, Proposition 8 dictates that the firm will design for remanufacturability due to the low cost efficiency ratio for new remanufacturable products, but it will not remanufacture enough due to the high cost efficiency ratio for the remanufactured products. As a result, the level of cannibalization is not enough for the environment to benefit. If, however, the firm does remanufacture enough of the re-collected units, which would be the case, for example, if the remanufacturing cost saving is relatively high (i.e., if g_r is relatively low), then it would be especially environmentally friendly to remanufacture.

To graphically illustrate these effects, we plot, in Figure 5, graphs of environmental impact for $q = 1/3$ (which corresponds to the conditionally optimal quality when $k = 0$) and $q = f_n/(3g_n) = .36$ (which corresponds to the conditionally optimal quality when $k = 1$ but $d_{r2} = 0$), given the representative case in which $f_n = 1.2$, $f_r = 0.8$, $g_n = 1.1$, and $e_2/e_1 = 0.30$. We find that higher quality generally results in higher environmental impact if g_r is either large or small. However, we also find that higher quality leads to lower environmental impact if g_r is moderate. Intuitively, if g_r is moderate, then higher quality means that the firm remanufactures a higher percentage of re-collected units due to the correspondingly smaller volume of new product sales. This increases cannibalization,
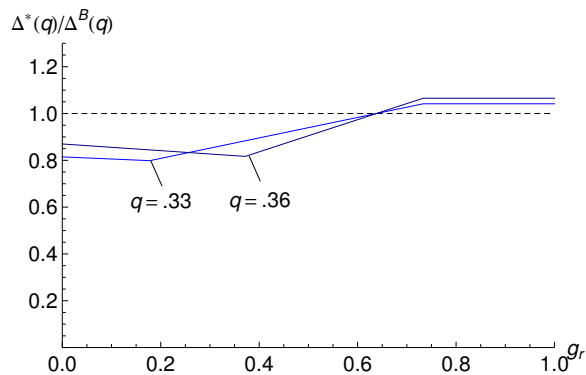
Figure 5: Illustration of environmental impact for exogenous quality

Table 3: Marginal effect of consumer valuation on environmental impact

		θ				
		0.1	0.2	0.3	0.4	0.5
	0.1	0.08, 0.01	0.08, 0.01	0.17, 0.03	0.13, 0.08	0.12, 0.05
	0.2	0.08, 0.01	0.15, 0.10	0.12, 0.06	0.11, 0.04	0.11, 0.03
α	0.3	0.13, 0.07	0.12, 0.05	0.11, 0.04	0.10, 0.03	0.10, 0.02
	0.4	0.11, 0.04	0.10, 0.03	0.10, 0.02	0.10, 0.02	0.09, 0.00
	0.5	0.10, 0.02	0.10, 0.02	0.09, 0.00	0.09, 0.00	0.09, 0.00

thus decreasing impact on the environment. These results suggest that quality and environmental performance are dependent on each other and the two need to be managed together rather than in isolation. This explains findings in empirical literature that suggests a strong link between quality and environmental performance (e.g., Wiengarten and Pagell, 2012; Oakley, 1993). There is also evidence that consumers' willingness to pay for a product is dependent on environmental performance and quality as interrelated factors (Jaffry et al., 2004), which is further supported by our analysis.

In §4, we also demonstrated that consumers who are green only to the extent that they value the idea that a product *can be* remanufactured, but not to the extent that they particularly value a product that *has been* remanufactured could lead to unintended consequences in the form of increased environmental impact. Thus, as a natural extension, we explore the potential value of educating consumers to be more green. Specifically, we focus on the question of what would be more environmentally beneficial, a decrease in f_n (i.e., a decrease in θ) or an increase in f_r (i.e., a decrease in α). To this end, we numerically evaluate changes to environmental impact relative to changes in θ and in α, given that the optimal policy is implemented. Specifically, we set $c_1 = 0.2$, $c_2 = 0.5$, $e_1 = .5$, and $e_2 = 0.15$, and we compute $\frac{\partial \Delta(q^*)}{\partial \theta}$ and $\frac{\partial \Delta(q^*)}{\partial \alpha}$ for values of θ and α such that $(\theta, \alpha) \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$, where $\Delta(q)$ is defined by 8. Table 3 provides a matrix of results where each cell presents $(\frac{\partial \Delta(q^*)}{\partial \theta}$, $\frac{\partial \Delta(q^*)}{\partial \alpha})$ for specified values of θ and α. According to Table 3, increases in θ result in increases to environmental impact regardless of the value of α. Similarly, increases in α result in increases in environmental impact for most values of θ, although increases in α yield no effect to environmental impact if both θ and α are relatively large. Thus, as a general rule, there is value (to the environmentalist) in educating consumers to be less environmentally conscious (smaller θ) and more environmentally

active (smaller α). In other words, it is in the environment's better interest to have consumers not necessarily value so highly that which *can be* remanufactured, but rather to value highly that which *has been* remanufactured. Moreover, notice that $\frac{\partial \Delta(q^*)}{\partial \theta} > \frac{\partial \Delta(q^*)}{\partial \alpha}$. Thus, if the choice must be made between educating consumers against the ills of overvaluing that which *can* be remanufactured and the ills of undervaluing that which *has been* remanufactured, the environment would be better served by investing in the former.

6 CONCLUSION

We have studied the quality design and environmental consequences of green consumerism, which is characterized by higher consumer valuations for environmentally friendly features. In this context, a firm has the option to design a non-remanufacturable or a remanufacturable product and to specify a corresponding product quality, and these design choices affect both the consumer valuations and the production costs associated with the product. On the consumer side, remanufacturable products are valued more, but remanufactured products are valued less, than non-remanufacturable products are. Analogously, on the cost side, remanufacturable products cost more to produce originally, but less to remanufacture, than non-remanufacturable products do. Given this, we investigate the environmental consequences of remanufacturing by first defining a measure of environmental impact that, ultimately, is a function of what is produced and how much is produced, and then applying that measure to assess the environmental impact associated with the firm's optimal strategy relative to the environmental impact associated with the firm's otherwise optimal strategy if a non-remanufacturable product were designed and produced.

Our results indicate that production efficiencies generally lead to profitability, which is consistent with the remanufacturing literature, but it does not necessarily translate to environmentally friendliness. Specifically, we find that if the production cost for each remanufacturable product is lower, then, everything else being equal, the firm offers more new remanufacturable product and less remanufactured products. Consequently, the larger total sales volume translates into increased damage on the environment, which is consistent of Jevons paradox.

In a related vein, we find that the impacts of consumer preferences and production technology on profitability align with those on environmental friendliness only when such impacts are manifested through remanufactured products rather than through new remanufacturable products. On the valuation side, we find that if green consumers value a product that *has been* remanufactured, then the resulting cannibalization will benefit the environment in the form of waste disposal reductions. However, if green consumers overvalue the idea that a product *can be* remanufactured, then the firm's optimal mix of new versus remanufactured products will not necessarily achieve the level of cannibalization required to benefit the environment in the form of waste disposal reductions. Consequently, under such circumstances, green consumerism produces a halo effect that translates into increased harmful impact on the environment.

Extrapolating, these implications indicate that the interactions between product quality and environmental performance need to be investigated further. Much of the current thinking is centered

around the idea that good quality will bring along environmental performance, however our findings propose a richer relationship between the two. In turn, the potential adverse impact of green consumerism on environmental performance has now been suggested both theoretically and empirically, and awaits further confirmation. Indeed, echoing Atasu et al. (2008) and Seitz (2007), we note here that more comprehensive empirical research regarding remanufacturable product cost structures also is required. Nevertheless, we acknowledge that the challenges inherent in collecting such accurate data seem to stem from organizational uncertainty regarding economic valuation of remanufacturing activities (Seitz, 2007) and mismanagement of remanufacturing operations (Guide et al., 2006).

Acknowledgments

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APPENDIX: PROOFS OF PROPOSITIONS

As building blocks for proving Propositions 2–7, we first establish three lemmas. The proofs of Lemmas 1 and 2 are combined and follow the statement of Lemma 2. The proof of Lemma 3 directly follows its statement. For Lemmas 1–3, let $p_{ij}^R(q), d_{ij}^R(q)$ denote the optimal Remanufacturing strategy price and corresponding demand as a function of q for product $i = n, r$ in stage $j = 1, 2$, respectively. Moreover, let $q_1 = \frac{f_n f_r (f_n - f_r)}{f_n^2 (g_n - g_r) - (f_n - f_r)^2 g_n}$ denote the quality threshold above which $d_{r2}^R(q) = d_{n1}^R(q)$, $q_2 = \frac{(f_n^2 - f_r^2)}{f_n g_n - f_r g_r}$ denote the quality threshold above which $d_{r2}^R(q) = d_{n1}^R(q)$ and $d_{n2}^R(q) = 0$, and $q_3 = \frac{(f_n + f_r)}{(g_n + g_r)}$ denote the quality threshold above which $d_{r2}^R(q) = d_{n1}^R(q) = d_{n2}^R(q) = 0$.

Lemma 1. *If $k = 1$, then for given $q < q_3$, $d_{r2}(q) > 0$ if and only if $g_r/f_r < g_n/f_n$.*

Lemma 2. *If $k = 1$ and $g_r/f_r < g_n/f_n$, then $q^{R*} < q_3$ and, for $i = n, r$ and $j = 1, 2$, $p_{ij}^R(q), d_{ij}^R(q)$ are as follows:*

(i) for $0 < q < q_1$,

$$d_{n1}^R(q) = \frac{f_n - g_n q}{2 f_n}; \quad d_{n2}^R(q) = \frac{f_n - f_r - (g_n - g_r) q}{2(f_n - f_r)}; \quad d_{r2}^R(q) = \frac{(f_r g_n - f_n g_r) q}{2 f_r (f_n - f_r)};$$
$$p_{n1}^R(q) = p_{n2}^R(q) = \frac{(f_n + g_n q) q}{2}; \quad p_{r2}^R(q) = \frac{q(f_r + g_r q)}{2}; \tag{9}$$

(ii) for $q_1 \leq q < q_2$,

$$d_{n1}^R(q) = d_{r2}^R(q) = \frac{f_n(f_n - g_n q) + (f_r g_n - f_n g_r) q}{2(f_n^2 + f_r(f_n - f_r))}; \quad d_{n2}^R(q) = \frac{f_n(f_n - g_n q) - f_r(f_r - g_r q)}{2(f_n^2 + f_r(f_n - f_r))};$$
$$p_{n1}^R(q) = \frac{(f_n(f_n + g_n q) + 2 f_r(f_n - f_r) - (f_r g_n - f_n g_r) q) f_n q}{2(f_n^2 + f_r(f_n - f_r))}; \quad p_{n2}^R(q) = \frac{(f_n + g_n q) q}{2};$$
$$p_{r2}^R(q) = \frac{(f_n(f_n + g_n q) - (f_n - f_r)(f_n - f_r - (g_n + g_r) q)) f_r q}{2(f_n^2 + f_r(f_n - f_r))} \tag{10}$$

(iii) for $q_2 \leq q < q_3$,

$$d_{n1}^R(q) = d_{r2}^R(q) = \frac{f_n + f_r - (g_n + g_r) q}{2(f_n + f_r)}; \quad d_{n2}^R(q) = 0;$$
$$p_{n1}^R(q) = \frac{f_n}{f_r} p_{r2}^R(q) = \frac{f_n q(f_n + f_r + (g_n + g_r) q)}{2(f_n + f_r)}; p_{n2}^R(q) = p_{r2}^R(q) + (f_n - f_r) q \tag{11}$$

Proof of Lemmas 1–2: Here we prove our results using the lagrangian method. The corresponding lagrangian of profit (6) for given q is

$$\mathcal{L}(p_{n1}, p_{n2}, p_{r2}) = (p_{n1} - g_n q^2 + \lambda) \left(1 - \frac{p_{n1}}{f_n q}\right) + \left(1 - \frac{p_{n2} - p_{r2}}{(f_n - f_r) q}\right)(p_{n2} - g_n q^2 + \mu)$$
$$+ \left(\frac{p_{n2} - p_{r2}}{(f_n - f_r) q} - \frac{p_{r2}}{f_r q}\right)(p_{r2} - g_r q^2 - \lambda + \eta)$$

where λ, μ and η are lagrangian coefficients for conditions $d_{r2} \leq d_{n1}$, $d_{n2} \geq 0$, and $d_{r2} \geq 0$ respectively. The corresponding KKT conditions are

$$\frac{\partial \mathcal{L}}{\partial p_{n1}} = 1 - \frac{p_{n1}}{f_n q} - \frac{(p_{n1} - g_n q^2 - \lambda)}{f_n q} = 0$$

$$\frac{\partial \mathcal{L}}{\partial p_{n2}} = 1 - \frac{p_{n2} - p_{r2}}{(f_n - f_r)q} - \frac{(p_{n2} - g_n q^2 + \mu)}{(f_n - f_r)q} + \frac{(p_{r2} - g_r q^2 - \lambda + \eta)}{(f_n - f_r)q} = 0$$

$$\frac{\partial \mathcal{L}}{\partial p_{r2}} = \frac{(p_{n2} - g_n q^2 + \mu)}{(f_n - f_r)q} + \frac{p_{n2} - p_{r2}}{(f_n - f_r)q} - \frac{p_{r2}}{f_r q} - \frac{f_n(p_{r2} - g_r q^2 - \lambda + \eta)}{f_r(f_n - f_r)q} = 0$$

as well as the corresponding orthogonal conditions. The corresponding solutions are

$$p_{n1}(q) = \frac{(f_n + g_n q)q - \lambda}{2}; \quad d_{n1}(q) = \frac{(f_n - g_n q)q + \lambda}{2f_n q};$$

$$p_{n2}(q) = \frac{(f_n + g_n q)q - \mu}{2}; \quad d_{n2}(q) = \frac{(f_n - f_r)q - (g_n - g_r)q^2 + \mu + \lambda - \eta}{2(f_n - f_r)q};$$

$$p_{r2}(q) = \frac{(f_r + g_r q)q + \lambda - \eta}{2}; \quad d_{r2}(q) = \frac{(f_r g_n - f_n g_r)q^2 - f_n \lambda - f_r \mu + f_n \eta}{2f_r(f_n - f_r)q}$$

To prove Lemma 1, we first prove sufficient and then necessary conditions. To that end, we first assume $d_{r2}(q) > 0$ which implies $\eta = 0$. Thus

$$0 < d_{r2}(q) = \frac{(f_r g_n - f_n g_r)q^2 - f_n \lambda - f_r \mu}{2f_r(f_n - f_r)q} \Rightarrow f_r g_n - f_n g_r > \frac{f_n \lambda + f_r \mu}{q^2} \geq 0$$

which completes the sufficient condition.

Next, assume $f_r g_n > f_n g_r$, which implies $q_1 < \frac{(f_n - f_r)}{(g_n - g_r)} < q_2 < \frac{f_n}{g_n} < q_3$. We consider four cases based on the multipliers λ and μ: (1) if $\mu > 0$, then $d_{n2}(q) = 0$, which implies $d_{r2}(q) = \frac{(f_r - g_r q)q + \mu}{2f_r q} > 0$; (2) $\lambda = \mu = 0$, then $d_{r2}(q) > \frac{f_n \eta}{2f_r(f_n - f_r)q} \geq 0$; (3) if $\lambda > 0$ and $\mu = 0$, then $d_{n1}(q) = d_{r2}(q) > 0$, thereby completing the proof of Lemma 1.

To prove Lemma 2, note that $d_{r2}(q) > 0$ (by definition); thus, $\eta = 0$ and $f_r g_n > f_n g_r$ (by Lemma 1). Accordingly,

$$0 \leq d_{n1}(q) - d_{r2}(q) = \frac{q_1 - q}{2q_1} + \frac{(f_n^2 + f_n f_r - f_r^2)\lambda}{2f_n f_r(f_n - f_r)q} + \frac{\mu}{2(f_n - f_r)q} \tag{12}$$

Notice from (12) that $d_{n1}(q) - d_{r2}(q)$ is decreasing in q. There are four possibilities.

Case I: If $0 < q < q_1$, then $q < \frac{f_n - f_r}{g_n - g_r} < \frac{f_n}{g_n}$. Accordingly, $d_{n1}(q) - d_{r2}(q) > \frac{(f_n^2 + f_n f_r - f_r^2)\lambda}{2f_n f_r(f_n - f_r)q} + \frac{\mu}{2(f_n - f_r)q} \geq 0$, and $d_{n2}(q) > \frac{\mu + \lambda}{2(f_n - f_r)q} \geq 0$. These inequalities imply that $\lambda = 0$ and $\mu = 0$, respectively. Thus, the KKT conditions above directly imply (9).

Case II: If $q_1 \leq q < q_2$, then

$$\lambda = \frac{2f_n f_r (f_n - f_r)q}{(f_n^2 + f_r(f_n - f_r))}(d_{n1}(q) - d_{r2}(q) + \frac{q - q_1}{2q_1}) - \frac{f_n f_r \mu}{f_n^2 + f_r(f_n - f_r)},$$

$$\Rightarrow d_{n2}(q) = \frac{f_n f_r (d_{n1}(q) - d_{r2}(q))}{(f_n^2 + f_r(f_n - f_r))} + \frac{(f_n g_n - f_r g_r)(q_2 - q)}{2(f_n^2 + f_r(f_n - f_r))} + \frac{(f_n + f_r)\mu}{2(f_n^2 + f_r(f_n - f_r))q}$$

$$> \frac{(f_n + f_r)\mu}{2(f_n^2 + f_r(f_n - f_r))q} \geq 0$$

Thus, $\mu = 0$ and correspondingly, $\lambda = \frac{2f_n f_r (f_n - f_r)q}{(f_n^2 + f_r(f_n - f_r))}(d_{n1}(q) - d_{r2}(q) + \frac{q - q_1}{2q_1})$. Notice that if $d_{n1}(q) > d_{r2}(q)$, then $\lambda > 0$ which is a contradiction. Therefore, $d_{n1}(q) = d_{r2}(q)$ and $\lambda = \frac{2f_n f_r (f_n - f_r)q}{(f_n^2 + f_r(f_n - f_r))}\frac{q - q_1}{2q_1}$. Correspondingly, the KKT conditions above directly imply (10).

Case III: If $q_2 \leq q < q_3$, then $q > \frac{f_n - f_r}{g_n - g_r}$. Assume $\lambda = 0$. If $\lambda = 0$, then on the one hand, (12) becomes $0 \leq d_{n1}(q) - d_{r2}(q) = \frac{q_1 - q}{2q_1} + \frac{\mu}{2(f_n - f_r)q}$, which implies $\mu > 0$. But if $\lambda = 0$, then on the other hand, $d_{n2}(q) = \frac{(g_n - g_r)}{2(f_n - f_r)}\left(\frac{f_n - f_r}{g_n - g_r} - q\right) + \frac{\mu}{2(f_n - f_r)q} > 0$ which, in turn, implies $\mu = 0$, a contradiction. Thus $\lambda > 0$, which implies $d_{n1}(q) = d_{r2}(q)$. Correspondingly,

$$0 < \lambda = \frac{f_n f_r (f_n - f_r)q}{(f_n^2 + f_r(f_n - f_r))}\frac{(q - q_1)}{q_1} - \frac{f_n f_r \mu}{f_n^2 + f_r(f_n - f_r)}$$

$$\mu = \frac{2(f_n^2 + f_r(f_n - f_r))q d_{n2}(q) + (f_n g_n - f_r g_r)(q - q_2)q}{(f_n + f_r)}$$

Notice that if $d_{n2}(q) > 0$, then $\mu > 0$, a contradiction. Therefore, $d_{n2}(q) = 0$, and thus, $\mu = \frac{(f_n g_n - f_r g_r)(q - q_2)q}{f_n + f_r} > 0$. Correspondingly, the KKT conditions above directly imply (11).

Case IV: If $q \geq q_3$, then $q > \frac{f_n}{g_n}$. Accordingly, $0 \leq d_{n1}(q) < \frac{\lambda}{2f_n q}$. This implies that $\lambda > 0$ and $d_{n1}(q) = d_{r2}(q)$. Similar to Case III, we also have $\mu > 0$. Thus the KKT conditions again reduce to (11). But notice that, given (11), $q \geq q_3$ implies that $d_{n1}(q) = d_{n2}(q) = d_{r2}(q) = 0$. Thus, $q^{R*} < q_3$. $\qquad\square$

Lemma 3. *If $k = 1$ and $g_r/f_r < g_n/f_n$, then $\Pi^R(q)$ is unimodal in q over $q \in [0, q_3)$.*

Proof of Lemma 3: From Lemma 2 and equation (6), $\Pi^R(q)$ reduces to

$$\Pi^R(q) = \begin{cases} \Pi_P(q) = \dfrac{2q(f_n - g_n q)^2}{4f_n} + \dfrac{(f_r g_n - f_n g_r)^2 q^3}{4f_n f_r(f_n - f_r)} & q < q_1 \\[2ex] \Pi_C(q) = \dfrac{(f_n - g_n q)^2 q}{4f_n} + \dfrac{q(f_n(f_n - g_n q) + (f_r g_n - f_n g_r)q)^2}{4f_n(f_n^2 + f_r(f_n - f_r))} & q_1 \leq q < q_2 \\[2ex] \Pi_S(q) = \dfrac{q(f_n + f_r - (g_n + g_r)q)^2}{4(f_n + f_r)} & q_2 \leq q < q_3 \end{cases} \tag{13}$$

Given (13), it is easy to establish that $\Pi^R(q)$ is continuous and differentiable over $[0, q_3)$. To prove Lemma 3, we show that $\Pi^R(q)$ is unimodal over $q \in [0, q_3)$ if $\Pi_i(q)$ for $i = P, C, S$ is such that $\Pi_P(q)$ is unimodal over $(0, q_1)$, $\Pi_C(q)$ is unimodal over $[q_1, q_2)$, and $\Pi_S(q)$ is unimodal over $[q_2, q_3)$.

Next, we establish that $\Pi_P(q)$ is unimodal over $(0, q_1)$. To that end, we define the following

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notations:

$$\gamma = \frac{f_r}{f_n}; \beta = \frac{g_r}{g_n}; \quad a_i = i + \frac{(\gamma - \beta)^i}{\gamma(1-\gamma)} > i; \Rightarrow a_i' = -\frac{i(\gamma-\beta)^{i-1}}{\gamma(1-\gamma)}, \quad i = 1, 2$$

Accordingly, we have $q_1 = \frac{f_n}{g_n}\frac{\gamma(1-\gamma)}{\gamma(1-\gamma)+\gamma-\beta}$, which implies $\frac{f_n}{g_n} = a_1 q_1$.

$$\Pi_P(q) \propto \frac{2f_n^2}{g_n^2}q - 4\frac{f_n}{g_n}q^2 + q^3 a_2$$

$$\Rightarrow \frac{\partial \Pi_P(q)}{\partial q} \propto \frac{2f_n^2}{g_n^2} - 8\frac{f_n}{g_n}q + 3a_2 q^2 = 3a_2(q - \hat{q}^-)(q - \hat{q}^+)$$

where $\hat{q}^\pm = \frac{a_1(4\pm\sqrt{16-6a_2})}{3a_2}q_1$. Notice, if $3a_2 \geq 8$, then $\frac{\partial \Pi_P(q)}{\partial q} > 0$ over $[0, q_1)$, in which case $\Pi_P(q)$ is unimodal. Thus, consider if $3a_2 < 8$. Let $H = \sqrt{16 - 6a_2}$, and $\hat{q}^- = \frac{a_1(4-H)}{3a_2}q_1 < \hat{q}^+ = \frac{a_1(4+H)}{3a_2}q_1$. Then to show that $\Pi_P(q)$ is unimodal, it suffices to show that $\hat{q}^+ \geq q_1$. To that end, if $\hat{q}^- \geq q_1$, then $\hat{q}^+ > \hat{q}^- \geq q_1$. Otherwise, if $\hat{q}^- < q_1$, then $H > \frac{4a_1-3a_2}{a_1}$, or $\beta > \gamma\frac{(3\gamma-1)(2-\gamma)}{2+3\gamma(1-\gamma)}$. In this case, we have $\hat{q}^+ \geq q_1 \iff K(\beta) = \frac{a_1}{a_2}\frac{4+H}{3} \geq 1$. Thus, to complete the proof that $\Pi_P(q)$ is unimodal, it suffices to show that $\min K(\beta) \geq 1$. We consider two cases. First, if $4a_1 \geq 3a_2$, then $K(\beta) \geq \frac{4a_1}{3a_2} \geq 1$. Next, if $4a_1 < 3a_2$, then it suffices to show that (1) $K(\beta)$ is unimodal in β, and (2) $\min[K(\beta_{\max}), K(\beta_{\min})] \geq 1$. To that end, we establish (2) first: $K(\beta_{\max}) = K(\gamma) = 1$; if $\gamma \geq 1/3$, then $K(\beta_{\min}) = K(\gamma\frac{(3\gamma-1)(2-\gamma)}{2+3\gamma(1-\gamma)}) = 1$; and if $\gamma < 1/3$, then

$$K(\beta_{\min}) = K(0) \geq 1 \iff \lim_{\beta \to 0} H^2 \geq \lim_{\beta \to 0}(\frac{3a_2}{a_1} - 4)^2$$

$$\iff \lim_{\beta \to 0} 8a_1 - 2a_1^2 - 3a_2 \geq 0 \iff \lim_{\beta \to 0} a_1(5 - 2a_1) = a_1\frac{1-3\gamma}{1-\gamma} \geq 0$$

Finally, we establish that $K(\beta)$ is, indeed, unimodal in β, thereby completing the proof that $\Pi_P(q)$ is unimodal over $[0, q_1)$. Let $Z = K\frac{a_2}{a_1}$. Hence, $Z' = Z(\frac{K'}{K} + X)$ and $Z'' = \frac{(Z')^2}{Z} + Z(\frac{K''}{K} - (\frac{K'}{K})^2 + X')$, where $X = \frac{a_2'}{a_2} - \frac{a_1'}{a_1}$. Correspondingly, we have

$$3Z - 4 = H = \sqrt{16 - 6a_2} > 0 \Rightarrow Z'(3Z - 4) = -a_2' > 0$$

$$\Rightarrow Z''(3Z - 4) + 3(Z')^2 = -a_2''$$

$$\Rightarrow \frac{K''}{K}\Big|_{K'=0} = -\frac{a_2'' + Z(3Z - 4)X' + 2(3Z - 2)\frac{(Z')^2}{Z}}{Z(3Z - 4)} < 0$$

This implies that $K(\beta)$ is unimodal, thereby completing the proof that $\Pi_P(q)$ is unimodal in q over $(0, q_1)$.

Next, we follow a similar analysis to establish that $\Pi_C(q)$ is unimodal in q over $[q_1, q_2)$. Let

$$b_i = 1 + \frac{(1-\gamma+\beta)^i}{1+\gamma(1-\gamma)}; \quad b_i' = \frac{i(1-\gamma+\beta)^{(i-1)}}{1+\gamma(1-\gamma)} \quad i = 0, 1, 2$$

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Note that $b_0 > b_1 > b_2$. Accordingly, we have $q_2 = \frac{f_n}{g_n}\frac{1-\gamma^2}{1-\beta\gamma}$, which implies $\frac{f_n}{g_n} = \frac{1-\beta\gamma}{1-\gamma^2}q_2$.

$$\Pi_C(q) \propto \frac{b_0 f_n^2}{g_n^2}q - 2b_1\frac{f_n}{g_n}q^2 + b_2 q^3$$

$$\Rightarrow \frac{\partial \Pi_C(q)}{\partial q} \propto \frac{b_0 f_n^2}{g_n^2} - 4b_1\frac{f_n}{g_n}q + 3b_2 q^2 = 3b_2(q - \hat{q}^-)(q - \hat{q}^+)$$

where $\hat{q}^\pm \equiv \frac{1-\beta\gamma}{1-\gamma^2}\frac{2b_1 - \sqrt{4b_1^2 - 3b_0 b_2}}{3b_2}q_2$. Notice that if $4b_1^2 \leq 3b_0 b_2$, then $\frac{\partial \Pi_C(q)}{\partial q} > 0$ over $[q_1, q_2)$, in which case $\Pi_C(q)$ is unimodal. Thus, consider if $4b_1^2 > 3b_0 b_2$. Let $H = \sqrt{4b_1^2 - 3b_0 b_2} > 0$. Then, to show that $\Pi_C(q)$ is unimodal, it suffices to show that $\hat{q}^+ \geq q_2$. To that end, if $\hat{q}^- \geq q_2$, then $\hat{q}^+ > \hat{q}^- \geq q_2$. Otherwise, if $\hat{q}^- < q_2$, then $H > 2b_1 - 3b_2\frac{(1-\gamma^2)}{(1-\beta\gamma)}$, or $\beta > \frac{3\gamma-2}{3-2\gamma}$. In this case, we have $\hat{q}^+ \geq q_2 \iff K(\beta) = \frac{1-\beta\gamma}{1-\gamma^2}\frac{b_0}{(2b_1-H)} \geq 1$. Thus, to complete the proof that $\Pi_C(q)$ is unimodal, it suffices to show that $\min K(\beta) \geq 1$. We consider two cases. First, if $2b_1 \geq 3b_2\frac{(1-\gamma^2)}{(1-\beta\gamma)}$, then $K(\beta) \geq \frac{1-\beta\gamma}{1-\gamma^2}\frac{2b_1}{3b_2} \geq 1$. Next, if $2b_1 < 3b_2\frac{(1-\gamma^2)}{(1-\beta\gamma)}$, then it suffices to show that (1) $K(\beta)$ is unimodal in β, and (2) $\min[K(\beta_{\max}), K(\beta_{\min})] \geq 1$. To that end, we establish (2) first: $K(\beta_{\max}) = K(\gamma) = 1$; if $\gamma \geq 2/3$, then $K(\beta_{\min}) = K\left(\frac{3\gamma-2}{3-2\gamma}\right) = 1 + \frac{2(4\gamma(1-\gamma)+(3\gamma-2))}{10-17\gamma+8\gamma^2} \geq 1$; and if $\gamma < 2/3$, then

$$K(\beta_{\min}) = K(0) \geq 1 \iff H^2 \geq (3b_2(1-\gamma^2) - 2b_1)^2$$

$$\iff b_0 \leq (1-\gamma^2)(4b_1 - 3(1-\gamma^2)b_2) \iff \gamma(1+\gamma)(3\gamma-2) \leq 0 \iff \gamma \leq 2/3$$

Finally, we establish that $K(\beta)$ is unimodal in β, thereby completing the proof that $\Pi_C(q)$ is unimodal over $[q_1, q_2)$. Let $Z \equiv \frac{(1-\gamma^2)K}{(1-\beta\gamma)}$. Hence $\frac{Z'}{Z} = \frac{K'}{K} + \frac{\gamma}{(1-\beta\gamma)}$. Correspondingly, we have

$$b_2 Z - 2b_1 = H = \sqrt{4b_1^2 - 3b_0 b_2} > 0$$

$$\Rightarrow \frac{Z'}{Z} = \frac{2 - (1-\gamma+\beta)Z}{H}b_1' \Rightarrow \frac{K'}{K} = \frac{2(1+\gamma) - (1+\beta)Z}{(1-\beta\gamma)(b_2 Z - 2b_1)}$$

$$\Rightarrow \frac{K''}{K}\bigg|_{K'=0} = -\frac{(1+\gamma)Z}{(1-\beta\gamma)^2(b_2 Z - 2b_1)} < 0$$

This implies that $K(\beta)$ is unimodal, thereby completing the proof that $\Pi_C(q)$ is unimodal over $[q_1, q_2)$.

Next, to complete the proof of Lemma 2, we establish that $\Pi_S(q)$ is unimodal in q over $[q_2, q_3)$. Accordingly, we have $q_3 = \frac{f_n}{g_n}\frac{(1+\gamma)}{(1+\beta)}$, which implies $\frac{f_n}{g_n} = \frac{(1+\beta)}{(1+\gamma)}q_3$.

$$\Pi_S(q) \propto (1+\gamma)\frac{f_n^2}{g_n^2}q - 2(1+\beta)\frac{f_n}{g_n}q^2 + \frac{(1+\beta)^2}{(1+\gamma)}q^3$$

$$\Rightarrow \frac{\partial \Pi_S(q)}{\partial q} \propto (1+\gamma)\frac{f_n^2}{g_n^2} - 4(1+\beta)\frac{f_n}{g_n}q + 3\frac{(1+\beta)^2}{(1+\gamma)}q^2 = 3\frac{(1+\beta)^2}{(1+\gamma)}\left(\frac{q_3}{3} - q\right)(q_3 - q)$$

Thus $\frac{\partial \Pi_S(q)}{\partial q} > 0$ for $q < \frac{q_3}{3}$, and $\frac{\partial \Pi_S(q)}{\partial q} < 0$ for $\frac{q_3}{3} < q < q_3$, which implies that $\Pi_S(q)$ is unimodal in q over $[q_2, q_3)$, thereby completing the proof for Lemma 3. $\qquad\square$

Proof of Proposition 2: From Lemma 1, we know that $g_r/f_r < g_n/f_n$, and from Lemma 2, we

know that prices and demands are defined over three segments: $(0, q_1)$, $[q_1, q_2)$, and $[q_2, q_3)$. We first establish the continuity of both price and demand, and then we analyze the impact of q for each segment. At $q = q_1$, we have $\mu = 0$ and $\lim_{q \to q_1^-} \lambda = 0 = \lim_{q \to q_1^+} \lambda$. Similarly, at $q = q_1$, we have $\lim_{q \to q_2^-} \mu = 0 = \lim_{q \to q_2^+} \mu$ and $\lim_{q \to q_1^-} \lambda = \frac{(f_r g_n - f_n g_r)(1 - f_r)(f_n^2 - f_r^2)}{[g_n(f_n + f_r) - f_r(g_n + g_r)]^2} = \lim_{q \to q_1^+} \lambda$. Therefore, for $i = n1, n2, r2$, p_i^R and d_i^R are continuous given the lagrange solution in the proof of Lemma 1–3.

Given the continuity of $p_i^R(q)$ and $d_i^R(q)$, we discuss the impact of q for each segment. First, for $q < q_1$, we have

$$
\frac{\partial p_{n2}^R(q)}{\partial q} = \frac{\partial p_{n1}^R(q)}{\partial q} = \frac{(f_n + 2g_n q)}{2} > \frac{\partial p_{r2}^R(q)}{\partial q} = \frac{(f_r + 2g_r q)}{2} > 0;
$$
$$
\frac{\partial d_{n2}^R(q)}{\partial q} = -\frac{(g_n - g_r)}{2(f_n - f_r)} < -\frac{g_n}{2f_n} = \frac{\partial d_{n1}^R(q)}{\partial q} < 0; \quad \frac{\partial d_{r2}^R(q)}{\partial q} = \frac{(f_r g_n - f_n g_r)}{2f_n(f_n - f_r)} > 0.
$$

Secondly, for $q_1 \le q < q_2$, we have

$$
\frac{\partial p_{n2}^R(q)}{\partial q} = \frac{(f_n + 2g_n q)}{2} > \frac{\partial p_{n1}^R(q)}{\partial q} = \frac{f_n[f_n(f_n + 2g_n q) + 2(f_n - f_r)f_r - 2(f_r g_n - f_n g_r)q]}{2(f_n^2 + f_r(f_n - f_r))} > 0;
$$
$$
\frac{\partial p_{n2}^R(q)}{\partial q} = \frac{(f_n + 2g_n q)}{2} > \frac{\partial p_{r2}^R(q)}{\partial q} = \frac{f_r[(f_n - f_r)(f_r + 2(g_n + g_r)q) + f_n(f_r + 2g_n q)]}{2(f_n^2 + f_r(f_n - f_r))} > 0;
$$
$$
\frac{\partial d_{n1}^R(q)}{\partial q} = \frac{\partial d_{r2}^R(q)}{\partial q} = -\frac{((f_n - f_r)g_n + f_n g_r)}{2(f_n^2 + f_r(f_n - f_r))} < 0; \quad \frac{\partial d_{n2}^R(q)}{\partial q} = -\frac{(f_n g_n - f_r g_r)}{2(f_n^2 + f_r(f_n - f_r))} < 0.
$$

Finally, for $q_2 \le q < q_3$, we have

$$
\frac{\partial p_{n2}^R(q)}{\partial q} = \frac{f_r(f_n - f_r) + 2(f_n^2 + f_r(g_n + g_r)q)}{2(f_n + f_r)} >
$$
$$
\frac{\partial p_{n1}^R(q)}{\partial q} = \frac{f_n}{f_r} \frac{\partial p_{r2}^R(q)}{\partial q} = \frac{f_n((f_n + f_r) + 2(g_n + g_r)q)}{2(f_n + f_r)} > 0
$$
$$
\frac{\partial d_{n1}^R(q)}{\partial q} = \frac{\partial d_{r2}^R(q)}{\partial q} = -\frac{(g_n + g_r)}{2(f_n + f_r)} < 0 = \frac{\partial d_{n2}^R(q)}{\partial q}
$$

$\square$

Proof of Proposition 3: Given Lemma 3 and its proof, $q^{R*} = q_S \equiv \frac{q_3}{3}$ if and only if $q_S \ge q_2$, which is equivalent to $g_r \le g_{rS}$. Similarly, $q^{R*} = q_C \equiv \frac{2b_1 - \sqrt{4b_1^2 - 3b_0 b_2}}{3b_2} \frac{f_n}{g_n}$ if and only if $q_1 \le q_C^R < q_2$, which is equivalent to $g_{rS} < g_r \le g_{rC}$. Finally, $q^{R*} = q_P^R \equiv \frac{4 - \sqrt{16 - 6a_2}}{3a_2} \frac{f_n}{g_n}$ if and only if $q_P^R < q_1$, which is equivalent to $g_{rC} < g_r < \frac{f_r g_n}{f_n}$. $\square$

Proof of Proposition 4: Assume that $g_r/f_r < g_n/f_n < f_n$. Then, two possibilities exist in an optimal solution: either $d_{r2}^* = 0$ or $d_{r2}^* > 0$. If $d_{r2}^* = 0$, then $k^* = 1$ by Proposition 1. If $d_{r2}^* > 0$, then $k^* = 1$ by definition. Thus, $k^* = 1$ is always true. Given this, Lemma 1 implies $d_{r2}^* > 0$. $\square$

Proof of Proposition 5: As per the proof of Proposition 4, $g_n/f_n < f_n$ implies $k^* = 1$, and $g_r/f_r < g_n/f_n$ implies $d_{r2}^* > 0$ by Lemma 1. Given this, Proposition 3 applies, which indicates that $g_r \le g_{rC}$ implies $d_{r2}^* = d_{n1}^* > 0$. $\square$

Proof of Proposition 6: As per the proof of Proposition 4, $g_n/f_n < f_n$ implies $k^* = 1$. Moreover, by Lemma 1, $g_r/f_r \geq g_n/f_n$ implies $d^*_{r2} = 0$. Thus $g_n/f_n < \min[f_n, g_r/f_r]$ implies that $\{k^* = 1, d^*_{r2} = 0\}$ is true. Next, to establish that $g_n/f_n \geq \min[f_n, g_r/f_r]$ implies that $\{k^* = 1, d^*_{r2} = 0\}$ is not true, consider two cases: If $g_n/f_n > g_r/f_r$, then either $g_n/f_n < f_n$, in which case $d^*_{r2} \neq 0$ by Proposition 4, or $g_n/f_n \geq f_n$, in which case $k^* \neq 1$ by Proposition 1. Thus, $g_n/f_n > g_r/f_r$ implies that $\{k^* = 1, d^*_{r2} = 0\}$ is not true. Similarly, if $g_r/f_r \geq g_n/f_n \geq f_n$, then either $d^*_{r2} > 0$ in which case $d^*_{r2} \neq 0$, or $d^*_{r2} = 0$, in which case $k^* = 0$ by Proposition 1. Thus, again, $\{k^* = 1, d^*_{r2} = 0\}$ is not true. $\qquad\square$

Proof of Proposition 7: If $g_r \leq g_{rC}$ and $\frac{g_n}{f_n} < \min[f_n, \frac{f_n(2f_n^2 + 3f_nf_r - 3f_r^2)}{(2f_n - f_r)^2(f_n + f_r)^2}((2f_n - f_r) + \frac{e_2}{e_1}f_n)]$, then $d^*_{r2} = d^*_{n1}$ by Proposition 5. Thus, to complete the proof, it suffices to show that (1) $\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1})$ decreases in g_r, in which case $\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1}) \geq \Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1})|_{g_r=g_{rC}}$; and (2) $\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1})|_{g_r=g_{rC}} > \Delta(q^B)$. To prove (1), we establish that $\frac{d\Delta(q^*)}{dg_r} = \frac{\partial\Delta(q^*)}{\partial g_r} + \frac{\partial\Delta(q^*)}{\partial q^*}\frac{dq^*}{dg_r} < 0$ by demonstrating that $\frac{\partial\Delta(q^*)}{\partial g_r} < 0$ and $\frac{\partial\Delta(q^*)}{\partial q^*}\frac{dq^*}{dg_r} < 0$. First, to prove that $\frac{\partial\Delta(q^*)}{\partial g_r} < 0$, note that $g_{rS} < g_r \leq g_{rC} \Rightarrow \frac{\partial\Delta(q^*)}{\partial g_r} = -\frac{e_1(f_n - f_r) + e_2f_n}{2(f_n^2 + f_r(f_n - f_r))}q^* < 0$ and $g_r \leq g_{rS} \Rightarrow \frac{\partial\Delta(q^*)}{\partial g_r} = -\frac{(e_1 + e_2)q^*}{2(f_n + f_r)} < 0$. Next, we prove that $\frac{\partial\Delta(q^*)}{\partial q^*}\frac{dq^*}{dg_r} < 0$ by establishing that $\frac{\partial\Delta(q^*)}{\partial q^*} > 0$ and $\frac{dq^*}{dg_r} < 0$. If $g_r \leq g_{rS}$, then $\frac{\partial\Delta(q^*)}{\partial q^*} = (e_1 + e_2)\frac{(f_n + f_r) - 2(g_n + g_r)q^*}{2(f_n + f_r)} = \frac{(e_1 + e_2)}{6} > 0$ and $\frac{\partial q^*}{\partial g_r} = -\frac{f_n + f_r}{3(g_n + g_r)^2} < 0$. If $g_{rS} < g_r \leq g_{rC}$, then recall from the proof of Lemma 3 that $q^* = q_C = \frac{f_n}{g_n}\frac{b_0}{2b_1 + H}$, where $H = \sqrt{4b_1^2 - 3b_0b_2}$. Thus,

$$\frac{\partial H}{\partial g_r} = \frac{(b_0 - 1)(b_1 + 3(\gamma - \beta))}{g_nH} > 0 \Rightarrow \text{sign}\left\{\frac{\partial q^*}{\partial g_r}\right\} = \text{sign}\left\{-\left(2\frac{db_1}{dg_r} + \frac{\partial H}{\partial g_r}\right)\right\} < 0$$

Moreover,

$$\frac{\partial\Delta(q^*)}{\partial q^*} = e_1\frac{f_n(f_n - 2g_nq^*) + (f_n - f_r)(f_n + f_r - 2(g_n + g_r)q^*)}{2(f_n^2 + f_r(f_n - f_r))}$$
$$+ e_2\frac{f_n^2 + 2(f_rg_n - f_ng_r - f_ng_n)q}{2(f_n^2 + f_r(f_n - f_r))}$$
$$\propto e_1\frac{(2 - \gamma^2)H - 2\gamma(\gamma - \beta)}{2b_1 + H} + e_2\frac{H + 2(\gamma - \beta)}{2b_1 + H} \geq 0$$

where the inequality follows because $g_r > g_{rS}$ implies that $\beta \geq \left(\frac{3\gamma - 2}{3 - 2\gamma}\right)^+$, which in turn implies that $(2 - \gamma^2)H - 2\gamma(\gamma - \beta) \geq \frac{2((3\gamma - 2) + \gamma^2(1 - \gamma)(4 - \gamma))}{(3 - 2\gamma)(1 + \gamma - \gamma^2)} \geq 0$ if $\gamma \geq 2/3$, and $(2 - \gamma^2)H - 2\gamma(\gamma - \beta) \geq \frac{2\gamma^2(1 - \gamma)}{(1 + \gamma - \gamma^2)} \geq 0$ if $\gamma < 2/3$. Thus, we conclude that $\frac{d\Delta(q^*)}{dg_r} < 0$ for $g_r \leq g_{rC}$, which implies $\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1}) \geq \Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1})|_{g_r=g_{rC}}$.

Finally, to complete the proof, we establish that $\Delta(q^*|k^* = 1, d^*_{r2} = d^*_{n1})|_{g_r=g_{rC}} > \Delta(q^B)$ as follows. At $g_r = g_{rC}$, $\frac{g_n}{f_n} < \frac{f_n(2f_n^2 + 3f_nf_r - 3f_r^2)}{(2f_n - f_r)^2(f_n + f_r)^2}((2f_n - f_r) + \frac{e_2}{e_1}f_n)$ implies that $\Delta(q^*)|_{g_r=g_{rC}} = \frac{2f_n^2(2f_n^2 + 3f_nf_r - 3f_r^2)}{9(2f_n - f_r)^2(f_n + f_r)^2g_n}(e_1(2f_n - f_r) + e_2f_n) > \frac{2e_1}{9} = \Delta(q^B)$. $\qquad\square$

Proof of Proposition 8: To prove that $\Delta^*(q) = e_1(d_{n1}(q) + d_{n2}(q))q + e_2d_{r2}(q)q > e_1(1 - q)q = \Delta^B(q)$, it suffices to show that $e_1(d_{n1}(q) + d_{n2}(q)) + e_2d_{r2}(q) > e_1(1 - q)$. If $g_n < f_n$, then, from (4), $\Pi^{NR}(q, k = 1) = \frac{q(f_n - g_nq)^2}{2f_n} > \frac{q(1-q)^2}{2} = \Pi^{NR}(q, k = 0)$. Moreover, If, on one hand, $g_r \geq \max[\frac{g_nf_r}{f_n}, \frac{g_nf_r}{f_n} - \frac{2e_1(f_n - f_r)f_r(f_n - g_n)}{f_n(e_1f_r - e_2f_n)}]$, then $\Pi^{NR}(q, k = 1) \geq \Pi^R(q)$ by Lemma 1. Thus, in this case, the

optimal remanufacturing strategy for given q is $k(q) = 1$ and $d_{r2}(q) = 0$, and the corresponding optimal pricing strategy implies, from (3), that $e_1(d_{n1}(q)+d_{n2}(q))+e_2 d_{r2}(q) = e_1(1-\frac{g_n q}{f_n})q > e_1(1-q)q$. If, on the other hand, $\frac{g_n f_r}{f_n} > g_r > \frac{g_n f_r}{f_n} - \frac{2e_1(f_n-f_r)f_r(f_n-g_n)}{f_n(e_1 f_r - e_2 f_n)}$, then $\Pi^R(q) > \Pi^{NR}(q, k=1)$ by Lemma 1. Thus, in this case, the optimal remanufacturing strategy for a given q is $k(q) = 1$ and $d_{r2}(q) > 0$, and the corresponding optimal pricing strategy implies, from Lemma 2(i), that $e_1(d_{n1}(q)+d_{n2}(q))+e_2 d_{r2}(q) > e_1(1-q)$. $\qquad \square$