Interest-Group Size and Legislative Lobbying*

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Abstract

We develop a model of legislative decision making in which lobbying and public policy are jointly determined. We examine how policy outcomes depend on the sizes of the interest groups. While a larger size typically involves favorable effects on policy, we also identify threshold levels of interest-group size where a lobby will be harmed if it becomes larger. This may provide another rationale as to why some interests do not or not fully organize. Spending limits can remove adverse policy effects of interest-group size. However, this is not necessarily welfare improving. Moreover, we find that endogenous proposal making may turn a second-mover advantage in standard legislative lobbying models into a second-mover disadvantage.

Keywords: interest groups, legislative lobbying, political economy, vote buying

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1 Introduction

Interest groups continuously try to influence legislative decision making regarding public policy. Typically proposal making and lobbying are both endogenous and interact with each other. Once a proposal has been made, interest groups will either support or target the proposal. In anticipation of such efforts the politicians will choose their policies. Traditionally legislative lobbying models treat policy proposals as exogenously given. This paper incorporates endogenous policy proposals into a legislative lobbying model where interest groups move once and sequentially and examines how policy outcomes depend on the interest groups’ sizes.¹

The basic version of the model considers only two types of individuals that differ in their preferences regarding policy and are organized into two interest groups of different sizes. Legislators are also of either of the two types. The interest groups’ sizes determine which policies will be approved by the legislature (if introduced for a vote) and how large the associated payments to the legislators will be. We characterize the optimal policy proposal of the agenda-setter² and show that the policy outcomes do not depend monotonically on the sizes of the lobby groups. While typically an increase in size benefits a lobby in terms of policy, we identify threshold values of interest-group size which trigger adverse policy changes for the growing lobby.

The intuition behind these adverse effects of interest-group size on policy outcomes runs as follows. Suppose the lobby making payment offers first supports a policy change that the second-mover interest group opposes. A stronger lobby opposing the policy change involves higher payments to the legislators by the pro-change lobby to secure a majority in the legislature. Anticipating the additional payments associated with a proposal for policy change may lead an agenda-setter who initially favored the status quo to propose the change. In the opposite case where the lobby supporting the status quo moves first and the one in favor of a policy change moves second, a larger pro-change lobby increases the set of policy proposals that cannot be blocked, but also increases the payments to the legislators when the defender of the status quo is able to form a majority against a proposed policy change. If the second effect dominates, an increase in the size of the pro-change lobby can lead an agenda-setter with preferences for a policy change to introduce a bill that is not implementable – i.e., that will be defeated by

¹In the present paper, a larger size of a lobby means a larger budget. We do not consider negative effects of size such as increasing difficulties in preference aggregation or magnified public good problems within the lobby.
²Throughout the paper, we use the terms policy proposer and agenda-setter interchangeably.
the legislative vote, even though implementable (more moderate) policy changes could instead be proposed.

Our analysis reveals that when the interest groups move once and sequentially, only the interest group moving second can be harmed by an increase in its size but never the interest group moving first. An interesting implication of our results is that we can identify situations wherein the second mover possesses a disadvantage in the sense that he would have been better off moving first. By contrast, in standard legislative lobbying models where the policy proposal is exogenously given, the second mover possesses a clear advantage. Hence, the second-mover advantage may become a second-mover disadvantage when the policy proposal becomes endogenous.

We extend our model to allow for greater heterogeneity of policy preferences. Unlike in the basic set-up, we allow interest-group size to vary endogenously. That is, the policy proposal determines the composition of the interest groups, one of which is lobbying for the proposal and the other against it. Considering different proposals, it is the politically moderate individuals who are ‘swinging’ between supporting a policy change or the status quo. We show that it is precisely this ‘swinging’ between the interest groups depending on the policy proposal that can lead to extreme policy outcomes that are detrimental to the politically moderate. In fact, there are situations wherein the politically moderate will be better off if they do not organize into interest groups. This result may offer a new rationale why some interests do not organize.³

Our results are consistent with recent observations by Baumgartner et al. (2009), who followed 98 randomly selected policy issues in the U.S. Congress in which interest groups were involved over the years from 1999 - 2002. They found that a larger interest-group size will not always lead to more favorable policy outcomes, reporting that policy change can become more likely when there is a larger lobby opposing the policy change.⁴ Hence, it may sometimes be beneficial for an interest group defending the status quo to remain small or inactive as the following observation suggests:

“A surprisingly large number of issues consist of a single side attempting to

³ Usually the reason for some interests not being organized is either a collective action problem (Olson, 1965) or fixed costs of organization (Mitra, 1999). An interesting recent paper (Damania and Fredriksson, 2000) relates incentives of firms to form lobby groups to the degree of collusion in the industry.

⁴ They argue that “although uncertainty no doubt increases when advocates face greater active opposition, it would be premature to conclude that policy success is less likely when there is greater opposition. Just as resources are not clear predictors of policy success, the presence of active opposition is likely to be a similarly inadequate predictor.” Baumgartner et al. (2009, p. 76)
achieve a goal to which no one objects or in response to which no one bothers to mobilize. Ironically, the lack of counter-mobilization is a good predictor of failure.[...]. One might think that with no opposition, those lobbyists working on behalf of the issues with only one side would rule the day in Washington. Reality is far from this, even when the “lobbyist” in question is the Defense Department.\(^5\)

In our model the adverse policy effects of interest-group size originate from the payments associated with some policy proposals becoming sufficiently large that the agenda-setter decides to go for the money rather than his own policy preferences when making the proposal. Consequently, if properly enforced, contribution limits that are sufficiently low can avoid adverse policy effects for a growing lobby. However, we show that this may not necessarily improve overall welfare. The welfare effects depend on the degree to which the different interests in society are organized. In fact, in the situation where the degrees of organization of the opposing interests are not too different, adverse policy effects of interest-group size, if they occur, will be welfare improving.

The paper is organized as follows. In the next section, we relate the paper to the existing literature. Section 3 introduces the model. We characterize the equilibrium of the lobbying subgame for a given proposal and the subgame-perfect equilibrium of the entire political game in Section 4. In Section 5, we examine the role of interest-group size on policy outcomes. There we establish our main results and provide a numerical example. We discuss several extensions and the robustness of our results in Section 6. Section 7 concludes. The proofs are relegated to the appendix.

2 Relation to the Literature

The present paper relates to the branch of the lobbying literature where interest groups offer politically valuable resources or campaign contributions in exchange for legislative favors.\(^6\) In particular, our paper augments the sequential legislative lobbying model, pio-


\(^6\)An overview over lobbying models using a common agency framework can be found in Grossman and Helpman (2001). These models assume that policy is set by a bureaucrat or a well disciplined party that is lobbied by one or several interest groups. By contrast, our paper and the related legislative lobbying literature assumes that decisions are made by elected representatives acting as a legislative body. In reality, even if the legislature is controlled by a single party, the instructions of the party leaders are not always honored.

Other papers on sequential legislative lobbying with exogenous proposal building on Groseclose and Snyder (1996) include Breton and Zaporozhets (2009), which shows the connection of the first-mover lobby’s problem in the legislative lobbying model with the combinatorics of sets and notions of cooperative game theory. Diermeier and Myerson (1999) use the sequential legislative lobbying framework with exogenously given policy proposals to examine the internal organization of legislatures. They study a game between different chambers of the legislature that strategically choose their internal organization to maximize the payments they receive from the interest groups. Rather than allowing each lobby to move once as the papers introduced before do, Dekel et al. (2008, 2009) suggest a sequential vote-buying game that ends only after two consecutive offers go by without any change in who would win. The papers study different variants of this type of game and discuss the resulting distribution of payments and voting outcomes.

The model presented in this paper differs from the literature on sequential legislative lobbying and, in particular, from Groseclose and Snyder (1996) in three main respects: First, as already emphasized, the policy proposal is endogenous. Second, legislators have preferences over policy outcomes rather than over how they cast their vote. Third, we consider only two opposed types of preferences over policy. The main results of our paper, the existence of adverse policy effects of interest-group size and the possibility of a second-mover disadvantage, originate from endogenizing the policy proposal. The other two main model differences to the Groseclose and Snyder (1996) model allow us to carve out the forces behind our results in a simple way. However, the second and third main difference prevent a deeper discussion on how different distributions of preferences of the legislators affect the distribution of payments in the legislature, and how they

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7In a continuous legislature, as we assume, this implies that legislators vote for the alternative with the highest bribes.

8An intuition why we obtain similar qualitative results in a set-up where legislators draw utility from how they vote is provided in Footnote 28 in Section 5.1. A formal argument will be provided upon request.
interact with the conditions under which adverse policy effects of lobby size occur.

Other legislative lobbying models with endogenous policy proposals include Snyder (1991), Baron (2006), and Helpman and Persson (2001). Snyder (1991) considers only one lobbyist who crafts the proposal and then buys a majority of votes for it in the legislature. He finds that the equilibrium policy lies between the lobbyist’s ideal point and the median of the legislators’ ideal points. This is not necessarily the case in our model, which includes an additional competing lobby and where policy is proposed by a legislator. These additional components drive our main results and also account for a potentially positive effect of opposition for the lobby seeking policy change. Baron (2006) presents a model of competitive lobbying in a majority-rule legislature with endogenous agenda-setting under complete information. His focus is different from this paper’s in that he considers only two possible proposals and examines the conditions under which both lobbies are active in equilibrium and those leading to minimal winning coalitions or supermajorities. Helpman and Persson (2001) combine a common agency approach with vote-buying in the legislature. However, they do not model direct competition between the lobbies and focus on how the variations of the political system affect the distribution of policy benefits.

In our model situations occur where the agenda-setter proposes public policy to extract rents from the interest groups. This links the paper to the literature on lobbying contests where proposals by rent-seeking politicians can be even more extreme than the ones that would be chosen by the lobby groups themselves (e.g., Epstein and Nitzan, 2006, Münster, 2006).9 When there are several legislators who decide on the implementation of a policy (e.g., via voting) and lobbies move simultaneously, we enter the realm of the so-called Colonel Blotto games.10 These games have proven notoriously difficult to solve and substantial progress has recently been made by Roberson (2006) and Kovenock and Roberson (2010). In these models, the policy proposal is exogenously given. It would be interesting to know how the major innovation of this paper, the introduction of an endogenous policy proposal, would play out in simultaneous move games or in games where lobbying is captured by different contest success functions. Recently, Cotton (2012) found that being stronger may make interest groups worse off in a model of

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9See Konrad (2009) for a comprehensive survey of the literature on contests.

10In Colonel Blotto games players simultaneously allocate forces to several battlefields. Within a battlefield, a player wins if he has allocated a higher level of forces than his opponent. The payoff of the game is a function of the number of battlefields won. In the lobbying context, legislators represent the ‘battlefields’ and the (monetary) resources spent by the interest groups for a vote of the legislator in their favor are the analogues to the forces sent to the particular battlefield. Winning the legislative vote is then equivalent to winning over a sufficient number of legislators.
informational lobbying where interest groups buy access to politicians. We show that a similar result may apply when political contributions buy votes rather than access.

We also note that the endogenous policy proposals in the sequential-moves vote-buying game tend to be extreme because the payments associated with less moderate policy proposals are higher. This is not necessarily the case in other lobbying set-ups such as e.g. in Felli and Merlo (2006). They add a lobbying stage modeled as a bargaining subgame in a citizen-candidate model. Anticipating the bargaining outcome, voters can avoid extreme policies by choosing an appropriate candidate in the election.

3 The Model

The model considers a continuous legislature with a measure of seats $N$ that decides via simple majority rule on a policy $t$. We use $\mathcal{N}$ to denote the set of legislators. The policy will be chosen from a closed interval $\tau \subset \mathbb{R}$. Initially, a status quo policy $s \in \tau$ is in place.

In the basic version of the model, there are only two types of individuals, the $X$-type and the $Y$-type. Each individual of type $i \in \{X,Y\}$ enjoys utility $U_i(t) = u_i(t) + d$, where $u_i(t)$ stands for strictly concave and bounded utility from policy $t$ and $d$ denotes transfers.\footnote{In principle, these transfers may also depend on policy $t$.} The $X$-types' most preferred policy is $x = \min \tau$, and the $Y$-types' ideal policy is $y = \max \tau$. We assume that $x < s < y$.

These assumptions imply that $u_X(t)$ is strictly decreasing and $u_Y(t)$ is strictly increasing on $\tau$. Consequently, the $X$-types prefer all policies $t < s$ and the $Y$-types all policies $t > s$ to the status quo. Legislators are also either of type $X$ or type $Y$. Without loss of generality, we assume that there is a majority of $Y$-type legislators. We keep referring to the abstract notation of $X$- and $Y$-types throughout the paper. However, for concreteness these could be easily substituted e.g., by a business lobby and a labor union lobbying on legislation on minimum wages where one part of the legislators leans towards business and the other towards the union.\footnote{Of course, there is a plenty of other examples. A broader interpretation might also refer to the group of $X$-types as a social elite and the group of $Y$-types as the people. In principle, we can think of the types as members of any two groups that have opposing interests. A policy $t$ then determines how the payoffs are distributed among the two groups.}
3.1 Lobbying

Although there are many channels through which lobbying takes place, we assume that transfers are paid to the legislators.\textsuperscript{13} There are two interest groups. A measure $l_i$ of individuals of type $i$ is organized in an interest group denoted by $i \in \{X, Y\}$.\textsuperscript{14} For simplicity, we assume that legislators are not members of interest groups.

Suppose that a policy $t$ is up for vote against $s$ in the legislature. Let $v_i(t) \equiv u_i(t) - u_i(s)$ denote type $i$’s utility difference between $t$ and the status quo. Then, an individual in favor of the proposal possesses the maximal willingness $v_i(t)$ to support $t$. An individual preferring the status quo is willing to spend $-v_i(t)$ to prevent policy $t$ from being implemented. Accordingly, the maximal willingness to pay of interest group $i$ to influence the legislative vote in its favor if $t$ is voted on against $s$ amounts to $l_i|v_i(t)|$. Abstracting from budget constraints of single individuals, we also refer to $l_i|v_i(t)|$ as lobby $i$’s budget. The lobbying expenses of the interest groups are shared equally among their members.

The interest groups can use their budgets to make payments to the legislators. For each legislator $k \in \mathcal{N}$, we use $b_i(k, t)$ to denote the offer of interest group $i$ given policy proposal $t$ for a vote of legislator $k$ according to the preferences of lobby $i$. That is, $b_X(k, t)$ is offered for a vote in favor of $t$ if $t < s$ and against it otherwise. The offer $b_Y(k, t)$ involves a vote for the proposal if and only if $t > s$. Each lobby’s offers must respect the lobby’s budget – i.e., $\int_{\mathcal{N}} b_i(k, t) \, dk \leq l_i|v_i(t)|$.

3.2 Voting behavior of the legislators

We assume that legislators have preferences regarding policy outcomes rather than regarding the act of voting itself. The legislators take the interest groups’ offer functions $b_i(\cdot, t)$ as given and vote for the alternative that yields the greatest expected utility. If no payments are made, they vote for the policy alternative that yields the highest direct utility from policy.\textsuperscript{15} Since the legislature comprises a continuum of legislators, no single legislator is pivotal. This means that each legislator votes in favor of the lobby group that makes the highest offer – i.e., a legislator $k$ who has received at least one

\textsuperscript{13}The transfers can be generally interpreted as something which is beneficial for the receiver and costly for the donor. They can range from explicit bribery over providing lucrative positions for politicians to donations to the policy proposer’s party.

\textsuperscript{14}More broadly, $l_i$ can also be interpreted as the interest groups’ level of organization. The idea is that not only may size (in terms of official members) matter with regard to the budget available for lobbying but also, how efficiently resources can be collected from non-members may have an impact.

\textsuperscript{15}A legislator of type $i$ votes for proposal $t$ if $v_i(t) \geq 0$ and against it otherwise.
positive payment offer supports policy proposal $t$ if and only if $b_i(k, t) \geq b_j(k, t)$, where $i, j \in \{X, Y\}, i \neq j$, and $i$ denotes the lobby in favor of the policy proposal, whereas $j$ prefers the status quo. Note that the weak inequality sign implies that when positive payments are offered and legislators are indifferent, legislators vote against the status quo.

While there is empirical support for the assumption that campaign contributions and other favors by lobbies influence the legislators’ voting behavior (e.g. Stratman, 1998, 2005), the assumption that legislators vote in favor of the lobby offering the highest payments is without doubt a stark simplification. However, it allows a strong focus on the interaction between interest-group sizes and policy outcomes in an analytically appealing way that transparently carves out the major drivers of our results. As indicated in Section 2, the cost of this assumption is that we cannot analyze how the size of the bribed coalition and the amount of bribes per legislator depend on the legislators’ preferences. A formal extension of our set-up where legislators draw utility from voting for their preferred alternative, as assumed e.g. in Groseclose and Snyder (1996), yields similar qualitative results regarding the relation between interest-group size and policy outcomes.\footnote{An intuition why this is the case is given in Footnote 28 in Section 5.1. A formal argument can be provided upon request.}

### 3.3 The political game

Now the entire political game can be described:

1. The policy proposer is randomly drawn among the majority type legislators and decides on a policy proposal $t^*$ to put up for a vote against $s$.

2. Interest group $X$ offers a payment schedule $\{b_X(k, t^*)\}_{k \in N}$ to the legislators for a vote pro $t^*$ if $t^* < s$ and for a vote in favor of the status quo if $t^* > s$.

3. Interest group $Y$ offers $\{b_Y(k, t^*)\}_{k \in N}$ for a vote pro $t^*$ if $t^* > s$ and for a vote in favor of the status quo if $t^* < s$.

4. Each legislator $k$ who receives at least one positive payment offer votes for $t^* < s$ if and only if $b_X(k, t^*) \geq b_Y(k, t^*)$ and for $t^* > s$ if and only if $b_X(k, t^*) \leq b_Y(k, t^*)$. When he obtains no positive payment offer, he votes for $t^* < s$ if and only if he is of type $X$ and for $t^* > s$ if and only if he is of type $Y$. The policy proposal will
be implemented when the majority of the legislators vote in favor of it. Otherwise the status quo remains in place.

It is a common feature of most democracies that the policy proposer is also a member of the legislature. For example, in parliamentary democracies, the party or coalition of parties in government introduce proposals to the legislature for a vote. In the United States’ presidential democracy, only a member of Congress possesses the right to make a proposal. With respect to the U.S. Congress, where the composition of the committees usually reflects the seat shares of the parties in the respective chamber, the $Y$-type individual proposing the policy can be interpreted as the median legislator in the respective committee. With regard to most parliamentary democracies, our specification is equivalent to the assumption that there is a ‘$Y$ party’ that forms the government and proposes a new policy to the legislature. The assumption that the $Y$-type legislators are in the majority or the relative size of this majority have no effect on our results. The crucial assumption is that the preferences of the policy proposer are aligned with those of the second-mover lobby. We discuss the role of this assumption for our results in detail in Section 6.1, where we examine the case where a legislator of (minority) type $X$ is the agenda-setter.\footnote{The size of the relative majority of $Y$-type legislators affects the equilibrium payments when legislators have preferences over voting for their preferred alternative. However, qualitatively our results remain unchanged. The qualitative results will also be similar when assuming a supermajority rule for the approval of policy proposals. A supermajority rule would increase the payments necessary to win the lobbying subgame and tends to make adverse policy changes less extreme, but will not prevent them in general (except in a continuous legislature with unanimity rule). A formal treatment of these arguments is available upon request.}

The idea behind the sequence of the lobbies’ moves is as follows: Because the majority of legislators is of type $Y$, the legislative vote will be in favor of lobby $Y$ without any payments. Hence, interest group $Y$ needs to take action only if $X$ has made an offer. Therefore, lobby $X$ is the natural first mover in the lobbying subgame after the proposal has been made.\footnote{Note that this justification for the order of moves of the lobbies corresponds to the one advanced in Groseclose and Snyder (1996) on p. 306 and Footnote 7. We further discuss the justification of this assumption in light of our results in Section 6.2.} We discuss the role of the order of the moves for our results in Sections 6.1 and 6.2.
4 Equilibrium

We begin the equilibrium analysis by characterizing the equilibrium of the lobbying subgame – i.e., the subgame that starts once policy proposal $t^*$ has been introduced.

4.1 The lobbying subgame

To determine the equilibrium in the lobbying subgame, we need to know how large a budget is necessary for the first mover $X$ to outcompete the second mover $Y$ in the lobbying subgame. The structure of the lobbying subgame allows us to infer from Proposition 1 in Groseclose and Snyder (1996) that it is optimal for $X$ to follow a leveling strategy when making its offers. A strategy is leveling if $b_X(k, t)$ is the same for almost all bribed legislators. The intention behind a leveling strategy is to leave no ‘soft spots’ to the second mover of the lobbying subgame.

We can now determine how expensive it is for $X$ to ensure a majority of votes in the legislature for its preferred policy alternative. We use $m$ to denote the measure of legislators who receive payments additional to those necessary for a minimal majority. This implies that the size of the super-majority that votes for the preferred policy of interest group $X$ is $N/2 + m$ legislators. $m$ is the measure of legislators that interest group $Y$ needs to buy back to ensure the approval of its preferred alternative. Hence, given proposal $t$, for $X$ to win the lobbying subgame, each of the bribed legislators must receive payments of at least $l_Y|v_Y(t)|/m$. Because a leveling strategy is cheapest for $X$, the total payments to establish a super-majority of size $N/2 + m$ accrue to $(N/2 + m) l_Y|v_Y(t)|/m$. Since the objective is declining in $m$, it is optimal to make payments to the entire legislature – i.e., $m^* = N/2$. Consequently, the minimal amount of payments by $X$ necessary to win the lobbying game is $2l_Y|v_Y(t)|$.

We are now in the position to characterize the equilibrium in the lobbying subgame. Two situations can arise: (1) the willingness to pay of lobby $X$, $l_X|v_X(t)|$, is (weakly) higher than $2l_Y|v_Y(t)|$, or (2) it is lower than $2l_Y|v_Y(t)|$. In the first case, $X$ will spend $2l_Y|v_Y(t)|$ to ensure a majority of legislative votes in its favor. In the second case, it

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19‘Almost all’ means all bribed legislators except a set of measure zero.

20A more detailed discussion of leveling strategies can be found in Groseclose and Snyder (1996).

21Groseclose and Snyder (1996) showed that it can be less expensive for the first mover, in our case interest group $X$, to form a super-majority in the legislature rather than a minimal winning coalition.

22The factor by which lobby $X$’s budget needs to exceed that of $Y$ to win the lobbying subgame has been called the hurdle factor by Diermeier and Myerson (1999). In our set-up, we obtain a hurdle factor of 2.
will abstain from offering payments.\footnote{The reason is that the second mover only needs to secure a minimal majority and will buy back the “cheapest” legislators. By this, the first mover cannot make a positive offer without incurring some costs for itself. Thus, when knowing that it will lose the lobbying subgame, a positive payment offer is not optimal.} Interest group \(Y\) will not make a positive offer in the first scenario because it has no chance of influencing the legislative vote. In the second situation lobby \(Y\) offers no payments because the majority of the legislators is of type \(Y\) and votes in its favor anyway. We summarize our observations in the following proposition:

**Proposition 1 (Equilibrium in the lobbying subgame)**

For any policy proposal \(t \in \tau\), there exists a unique equilibrium in the lobbying subgame which implies that

(i) if \(l_X|v_X(t)| \geq 2l_Y|v_Y(t)|\)

Stage 2 \(X\) offers payments \(b_X(k, t) = \frac{2l_Y|v_Y(t)|}{N}\) to all of the legislators for a vote in its favor.

Stage 3 \(Y\) does not make any payment offer.

Stage 4 All legislators vote in favor of \(X\). Hence, if \(t < s\), \(t\) will be implemented, otherwise the status quo prevails.

(ii) if \(l_X|v_X(t)| < 2l_Y|v_Y(t)|\)

Stage 2 \(X\) makes no payment offers.

Stage 3 \(Y\) makes no payment offers.

Stage 4 All legislators of type \(X\) vote in favor of \(X\) and all legislators of type \(Y\) vote in favor of \(Y\). If \(t > s\), \(t\) will be implemented, otherwise the status quo prevails.

According to Proposition 1, only the first-mover lobby \(X\) will make payments to the legislators in equilibrium. Furthermore, we know that \(X\) offers the same amount of payments to all legislators. Hence, we can drop indices and write \(b(t)\) instead of \(b_X(k, t)\). When bribes are paid in equilibrium, we have

\[
b(t) = \frac{2l_Y}{N}|v_Y(t)|.\tag{1}
\]

Two points are important for our analysis. First, the equilibrium amount of bribes per legislator paid by the first mover increases with the size of the second mover \(l_Y\). Second,
$b(t)$ is higher the farther is the proposal $t$ from the status quo policy – i.e, the larger is the willingness to pay of an individual of type $Y$ to either support or avoid a policy change towards policy $t$.\footnote{The result that the first mover makes payments to the entire legislature is a consequence of the assumption that the legislators only care about payments when voting for or against a proposed policy change. The first mover will form different coalitions when legislators also care about voting for their preferred alternative. While such coalitions are more realistic, the main forces driving the results can be carved out in a simpler and more transparent way with the assumptions on voting behavior made in Section 3.2.}

### 4.2 Partitions of the policy space

The equilibrium of the lobbying subgame allows us to characterize which policy proposals will be approved by the legislature. This is important information for the policy proposer when considering his proposal. To identify the policies that can be implemented, it is convenient to use the function

$$F(t) \equiv l_X v_X(t) + 2 l_Y v_Y(t),$$

which indicates for each policy $t$ whether the budget of $X$ exceeds the amount necessary to outcompete $Y$ in the lobbying subgame. A policy $t$ is implementable if and only if $F(t) \geq 0$. The function $F(t)$ possesses two important properties. First, it is a strictly concave function on $\tau$ as both $v_X(t)$ and $v_Y(t)$ are strictly concave. Second, $F(s) = 0$, which follows from $v_X(s) = v_Y(s) = 0$. The two properties imply that $F(t)$ possesses at most two roots in the interval $\tau$ with one of them being $s$. We introduce $q$ to denote the second root of $F(t)$, if it exists within $\tau$. Otherwise $q = x$ if $F(x) > 0$, and $q = y$ if $F(y) > 0$. In this way, $q$ represents the boundary of the set of implementable policies: if $q < s$ then all policies in the interval $[q, s]$ are implementable while all policies $t \notin [q, s]$ will not be approved by the legislature. Analogously, if $q > s$ the set of implementable policies is $[s, q]$. This case is illustrated in Figure 1. Note that if $F(t)$ reaches its maximum at $s$, we obtain $q = s$ implying that the status quo is the only implementable policy.

![Figure 1: Illustration of the partition of the policy space if $q > s$.](image-url)
policy. Further it follows immediately that the implementable set will comprise policies favoring either $X$ (if $q < s$) or $Y$ (if $q > s$) but not both. Note that the reason why policies are implementable depends on whether $q < s$ or $q > s$. In the former case, the policies $t \in [q, s)$ are implementable because $X$ is sufficiently strong to pay large enough bribes to secure a majority. In the latter case, the policies $t \in (s, q]$ are implementable as $X$ cannot outcompete $Y$.

### 4.3 Subgame-perfect equilibrium of the political game

To complete the equilibrium analysis, we determine the policy proposal at the first stage of the political game. Facing a certain partition of the policy space induced by the equilibrium of the lobbying subgame, the agenda-setter maximizes his utility considering for each potential proposal whether it will be implemented and how large the bribes associated with it will be. Formally, he chooses the policy proposal so as to maximize\(^{25}\)

$$V_Y(t) \equiv U_Y(t) - U_Y(s) = \begin{cases} 1_{t \in [q, s]} v_Y(t) + 1_{t \geq q} b(t), & \text{if } q < s, \\ 1_{t \in [s, q]} v_Y(t) + 1_{t > q} b(t), & \text{if } q \geq s. \end{cases} \quad (3)$$

In the proposer’s objective function, $1_C$ is the indicator function that returns a 1 if condition $C$ is satisfied and a zero otherwise. The first term of the objective function represents the utility gain derived directly from the new policy. This utility gain is only obtained when introducing an implementable policy. The second summand represents the payments associated with the policy proposal. The corresponding indicator function captures the fact that bribes are paid only for a vote in support of implementable policy proposals favoring the first mover $X$ or a vote against non-implementable policy proposals in favor of the second mover $Y$.\(^{26}\) No payments will be made when an implementable policy favoring $Y$ or a non-implementable policy in favor of $X$ is proposed as in these cases the first mover is not strong enough to win the lobbying subgame and consequently abstains from offering bribes. It follows directly that proposing a non-implementable policy favoring $X$ will not involve a utility gain relative to the status quo as it entails neither payments nor a policy change.

\(^{25}\)Here we assume that the agenda-setter cares only about his own utility when crafting the proposal. It might be more realistic that he is also concerned about the utility of his fellow party members in the legislature because the party determines his committee membership. An extension interpreting the $Y$-type legislators as the $Y$-party could easily be incorporated into the agenda-setter’s objective and would not affect the qualitative results.

\(^{26}\)If $q < s$ the implementable policy set in favor of $X$ is $[q, s)$ and the non-implementable policies favoring $Y$ are $(s, q]$. If $q > s$, no policy favoring $X$ is implementable and the set of non-implementable policies favoring the second mover is $(q, s]$. 
The maximization problem of the policy proposer can be solved by using a two-step procedure. First, within the implementable and non-implementable policy set, the most preferred policy is identified. Then, in a second step, the policy is chosen that yields the highest utility level of these two. Using the tie-breaking rule that in case of indifference between an implementable and a non-implementable policy, the agenda-setter proposes the implementable one, we can characterize the subgame-perfect equilibrium of the political game as follows:

**Proposition 2 (Subgame-perfect equilibrium of the political game)**

The political game possesses a unique subgame-perfect equilibrium where at the first stage the agenda-setter proposes policy \( t^* = q \) if and only if

\[
V_Y(q) \geq V_Y(y)
\]

and \( t^* = y \) else. Then the equilibrium of the lobbying subgame is played as specified in Proposition 1.

A formal proof is provided in Appendix A.3. Proposition 2 states that if the agenda-setter proposes an implementable policy, it will be the boundary of the implementable set \( q \). Otherwise the best non-implementable policy will be proposed, which is \( y \). In the following, we focus on the case where \( q < s \). We discuss the case where \( q > s \) in Appendix A.1. If \( q < s \), the intuition behind the result in Proposition 2 can be summarized as follows. Policy \( y \) is not implementable, and \( y \) is the agenda-setter’s most preferred choice among the non-implementable policies because no payments are associated with a non-implementable policy favoring the first-mover lobby \( X \), and the payments to prevent a policy favoring the second mover \( Y \) increase with \( t > s \). Consequently, any implementable policy must yield at least the utility level of \( y \) to be proposed. As detailed in Appendix A.3, the agenda-setter’s objective is either monotonically increasing or decreasing over the implementable policy set.\(^{27}\) Hence, the most preferred implementable proposal is either \( s \) or \( q \). The status quo will never be proposed as it involves no utility gain for the proposer (in contrast to proposing the non-implementable policy \( y \)). Therefore, if an implementable policy is introduced for a vote it must be \( q \).

\(^{27}\)The intuition is that both the disutility from implementing a policy \( t < s \) as well as the payments associated with such a proposal are multiples of \( v_Y(t) \). This is the case because the policy proposer is of type \( Y \) and the payments by the first-mover lobby are a multiple of the willingness to pay of the second-mover lobby. Hence, the utility gain from proposing a pro-\( X \) policy increases monotonically if the payments overcompensate the proposer’s utility loss from implementing the policy, and it decreases monotonically otherwise.
5 The Role of Interest-Group Size

In this section, we examine how interest-group size affects the policy outcome and derive the paper’s main results. We focus on the size of the second-mover lobby and identify thresholds where an increase in its size may lead to adverse policy outcomes for this lobby. In particular, we provide the results and intuition for the case when the first mover is very large and only pro-\(X\) policies are implementable. Then a larger size of \(Y\) may induce the policy proposer to introduce a policy change in favor of \(X\) rather than maintaining the status quo. The case where the implementable policy set contains pro-\(Y\) policies, and an increase in the size of lobby \(Y\) may lead the proposer to introduce a non-implementable pro-\(Y\) policy rather than an implementable one, has a similar intuition and is discussed in Appendix A.1. In Appendix A.2, we explain why the first mover will always (weakly) benefit from an increase in its size in terms of policy outcomes.

5.1 Second-mover lobby defends the status quo

Consider the case where the first-mover lobby \(X\) is substantially larger than the second mover \(Y\) such that \(q < s\). Then lobby \(X\) seeks policy change that \(Y\) opposes. The trade-off for the agenda-setter is as follows. On the one hand, he can propose \(y\) in expectation of payments from lobby \(X\) while effectively maintaining the status quo. On the other hand, he can propose \(q\), which involves payments from interest group \(X\) but additionally entails the implementation of a pro-\(X\) policy change. Clearly, the \(Y\)-type policy proposer will introduce a policy change in favor of \(X\) if the expected payments associated with it are sufficiently larger than those associated with the non-implementable pro-\(Y\) policy to compensate the utility loss incurred by the implementation of the pro-\(X\) policy change. The intuition for why an increase in size may harm a lobby can be summarized as follows.

When a proposal has been made, the first-mover lobby pays just the amount necessary to outcompete the second mover. Since the first mover will pay higher bribes as the second-mover lobby grows, the agenda-setter may switch from proposing an extreme policy, \(y\), that in effect maintains the status quo in favor of the second mover, to proposing a policy in favor of the first-mover lobby, \(q\), because the first mover now pays more for it.\(^{28}\)

\(^{28}\)The bribes by the first mover are paid to preempt a counter offer by the second-mover lobby. If the legislators had utility from voting for their preferred alternative, the first mover might have to make additional payments to compensate them for voting in its favor. However, this part of the bribe for a legislator is independent of the size of the second mover, while the part to preempt a counter-attack will increase when the second-mover lobby becomes larger. Hence, this preemptive part of the bribes drives
To derive our results, we draw on Condition (4) in Proposition 2 stating that the agenda-setter will introduce the pro-X policy change if and only if

\[ v_Y(q) + b(q) - b(y) = v_Y(q) - \frac{2l_Y}{N}[v_Y(q) + v_Y(y)] \geq 0. \]  

(5)

Condition (5) reveals the crucial role of the size of the second-mover lobby \( l_Y \) for whether the policy change \( q \) will be introduced. If the second mover is very small (i.e. \( l_Y \approx 0 \)), the expected bribes will be negligible and cannot compensate for the utility loss associated with a policy change.\(^{29}\) This mirrors the observations by Baumgartner et al. (2009) where policy change did not occur even without opposition. Instead, they reported that stronger opposition may well benefit the proponents of a policy change. In our model, an increase in the size of the defender of the status quo, \( l_Y \), has two opposing effects on the incentives of the policy proposer to introduce the policy change \( q \). On the one hand, given \( -v_Y(q) > v_Y(y) \), the payments associated with \( q \) increase stronger than those expected from proposing the non-implementable proposal \( y \). On the other hand, the boundary of the set of implementable policies \( q \) also depends on the lobbies' sizes. If the second-mover lobby becomes larger, \( q \) increases which reduces the utility gain achievable by the agenda-setter from proposing an implementable pro-X policy.\(^{30}\)

When the interval of \( l_Y \) where the first effect dominates is sufficiently large, i.e. the payments associated with the pro-X proposal \( q \) increase sufficiently strongly, we obtain the result that a policy change in favor of X can be induced by an increase in the size of lobby \( Y \). Hence, although an increase in the size of \( Y \) increases the costs for \( X \) to ensure a majority in the legislature, it may still benefit \( X \) as the policy proposer’s incentive to propose a policy tilted towards \( X \) increases. In this case, stronger opposition benefits the pro-change lobby.\(^{31}\)

We now aim at formally characterizing the intervals of second-mover interest-group our results. By neglecting utility from voting, we simplify the analysis substantially while retaining the main driving force behind our results.

\(^{29}\)Note that \( v_Y(q) \) is negative, i.e. the policy proposer suffers from the pro-X policy change \( q \).

\(^{30}\)According to the agenda-setter’s objective (3) and as detailed in Appendix A.3, the utility gain from introducing \( q < s \) is \( v_Y(q)[1 - 2l_Y/N] \). Hence, in the relevant case where the utility gain is positive (\( 2l_Y > N \)), it declines with \( q \). Intuitively, a higher value of \( q \) reduces the utility loss from the policy change for the agenda-setter. However, a larger \( q \) also implies lower payments in equilibrium as lobby \( Y \)’s willingness to pay to prevent the policy change declines. If \( 2l_Y > N \) the second effect dominates implying that the policy proposer’s utility from introducing policy change \( q \) declines with \( q \).

\(^{31}\)Let us state it explicitly: if the second-mover lobby grows larger and the policy proposer’s most preferred proposal thereby changes from the non-implementable policy \( y \) to the implementable policy \( q \), the utility of the members of \( Y \) declines and that of the members of \( X \) increases as a result. The utility loss of lobby \( Y \) results from the worse policy’s being implemented when it grows larger. By contrast, interest group \( X \) saves payments \( 2l_Y v_Y(y) \) and additionally realizes a utility gain of \( l_X v_X(q) + 2l_Y v_Y(q) \geq 0 \).
sizes where the policy proposer will introduce a policy change in favor of the first mover. Recalling that the boundary of the implementable set \( q \) depends on lobby sizes, we express \( q \) as a function of \( l_Y \), \( q(l_Y)^{32} \). Slightly abusing notation, we will use \( q(l_Y) \) when emphasis is on the dependence of \( q \) on the size of lobby \( Y \), and we will use simply \( q \) otherwise. Using this definition and Condition (5), we can represent our discussion formally in the next proposition.

**Proposition 3 (Policy change)**

The policy proposer will introduce the implementable pro-\( X \) policy change \( q(l_Y) < s \) if and only if

\[
\begin{align*}
(i) \quad & -v_Y(q(l_Y)) > v_Y(y) \\
(ii) \quad & l_Y \geq \frac{N}{2} \frac{v_Y(x)}{v_Y(x)+v_Y(y)} \equiv \chi^c(l_Y).
\end{align*}
\]

The first condition in Proposition 3 ensures that the payments associated with proposal \( q \) are higher than those associated with proposal \( y \).\(^{33} \) The second condition indicates whether the difference in bribes for \( q \) and \( y \) is sufficiently large to compensate the agenda-setter for the utility loss from implementing \( q \). This is the case when \( l_Y \) is larger than the threshold \( \chi^c(l_Y) \). A possible shape of \( \chi^c(l_Y) \) is depicted in Figure 2.

![Figure 2: Illustration of Proposition 3.](attachment:image.png)

Three issues are important. First, as for very small sizes of the second mover the entire set of pro-\( X \) policies will be implementable, we have \( q(l_Y) = x \). Consequently, the

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\(^{32}\)Details on the definition of \( q(l_Y) \) are provided in Appendix A.4.

\(^{33}\)A necessary and sufficient condition that there exist interest-group sizes of \( Y \) for which (i) holds is \(-v_Y(x) > v_Y(y)\).
function $\chi^c(l_Y)$ is equal to a strictly positive constant for very small second-mover lobby sizes. Second, for larger sizes of the second mover, an increase in $l_Y$ reduces the set of implementable pro-$X$ policies. That is, $q(l_Y)$ is strictly increasing in $l_Y$ and so is $\chi^c(l_Y)$. Third, the function $\chi^c(l_Y)$ possesses a pole at the second-mover lobby size at which the boundary of the implementable set is such that $-v_Y(q(l_Y)) = v_Y(y)$. For larger second-mover interest group sizes, Condition (i) in Proposition 3 will be violated implying that the bribes will be higher when proposing $y$ rather than $q$.\footnote{These claims are verified in Appendix A.4.}

The strictly positive ordinate intercept possesses two implications. The one is, as discussed previously, that there will be no policy change favoring lobby $X$ for very small sizes of lobby $Y$. The other implication is that policy changes in favor of $X$ are only possible if $\chi^c(l_Y)$ possesses fixed points. In this case, there exists a threshold value for $l_Y$ where the status quo will remain if $l_Y$ is below the threshold, while a pro-$X$ policy change will occur if $l_Y$ marginally exceeds the threshold. This can be observed in Figure 2.\footnote{While $\chi^c(l_Y)$ possesses these three properties, we note that $\chi^c(l_Y)$ is not necessarily convex over the entire interval as depicted in Figure 2. In general there may be more than one threshold level where an increase in the size of the status-quo defender induces a policy change in favor of the pro-change lobby.}

We are now interested in when an interval of interest-group sizes $l_Y$ leading the agenda-setter to propose a policy change favoring the opposed lobby exists. Or more formally whether there are parameter values for which $\chi^c(l_Y)$ possesses fixed points. The result that we obtain and state in the next proposition is that with strictly concave utility functions for the $X$- and $Y$-types, the set of parameter values for $s, N, \text{and} \ l_X$ for which $\chi^c(l_Y)$ possesses fixed points is not empty.

**Proposition 4 (Parameter values allowing for policy change)**

Suppose the types have strictly concave utility functions $u_i(t)$ with the highest utility at policy $x$ for the $X$-type and at policy $y$ for the $Y$-type. Then, there are triples $(s, N, l_X)$ such that an interval of second-mover interest-group sizes leading to policy changes favoring the first-mover lobby exists.

A formal proof can be found in Appendix A.5. For an intuition of this result, recall that the size of the bribes per legislator motivate the $Y$-type agenda-setter to propose the pro-$X$ policy. As described previously, an increase in second-mover lobby size increases the payments per legislator necessary to win the lobbying subgame. But by this, $q$ increases as well, thereby reducing the set of implementable policies and reducing the equilibrium.
payments associated with a pro-X policy change \( q \). Policy switches in response to a larger second mover lobby occur if the increase in payments per legislator is sufficiently stronger than the decline in equilibrium payments originating from the increase in \( q \). The increase in bribes when the second mover becomes larger will be high in settings where \( N \) is small and \( s \) is close to the ideal policy of the second mover.\(^{36}\) The increase in \( q \), and hence the decline in equilibrium payments due to an increase in \( Y \), will be small, if the first mover is large. Hence, adverse policy changes in response to an increase in second mover size may occur in situations where the first mover is large, the legislature is rather small, and the status-quo is not too far from the ideal policy of the second-mover lobby.

5.2 Numerical example

Now we give an example to illustrate the effects discussed in the previous section. Let us consider the policy interval \([-1, 1]\) with the status quo at \( s = 0 \) and quadratic utility functions \( u_X(t) = -\frac{1}{2}(t - x)^2 \) and \( u_Y(t) = -\frac{1}{2}(t - y)^2 \) where \( x = -1 \) and \( y = 1 \). In this case \( \chi^c(l_Y) \) is a convex function leading to a graph as illustrated in Figure 2.

We consider the case where \( l_X/N \) is high. For example with \( l_X/N = 4.4 \), \( \chi^c(l_Y) \) possesses fixed points at \( l_Y = 0.17l_X \) and \( l_Y = 0.24l_X \). Hence, if \( l_Y \) is smaller than about a sixth of the size of the pro-change interest group \( X \), the status quo will remain, while if the second-mover lobby size increases to a value between \( 0.17l_X \) and \( 0.24l_X \), a pro-X policy change towards \( q \) will occur. In terms of policies, this implies that policy changes towards pro-X policies within the interval \([-0.96, -0.7] \) are possible. The higher \( l_X/N \) the larger is the set of \( l_Y \)-values where pro-X policy change ensues. For example with a relation \( l_X/N = 5 \), we obtain the fixed points \( l_Y = 0.15l_X \) and \( l_Y = 0.27l_X \) with possible policy changes in the interval \([-1, -0.61]\).

6 Discussion

In this section, we first establish that an increase in lobby size can only have negative effects on policy outcomes if the second-mover’s preferences are aligned with the agenda-setter’s. Further we discuss how the results are affected by different sequences of moves in the lobbying subgame. Next, we explain why a second-mover advantage

\(^{36}\)Regarding the effect of the status quo \( s \), note that the farther away the pro-X policy from the status-quo, the higher the willingness to pay by the second-mover to defend the status quo and the higher the payments necessary for the first mover to win a majority for policy change.
in the sequential legislative lobbying model with exogenous proposal may turn into a
second-mover disadvantage when the policy proposal is endogenous. Then we extend
the model to allow for greater heterogeneity of preferences among the members of the
interest groups. There we show how the 'swinging’ behavior of the politically moderate
described in the introduction can lead to extreme policy outcomes. Finally, we argue
that contribution limits may remove adverse policy effects of interest-group size but that
this is not necessarily welfare-improving.

6.1 Agenda-setter’s preferences and different lobbying subgames

We start by pointing out the role of the sequential nature of the lobbying subgame for our
results. The sequential structure of the game implies that in equilibrium the first mover
makes payments to preempt a counter-attack by the second-mover, which increase when
the second mover becomes stronger. The particular sequence of the moves is relevant in
so far as it specifies whether the second mover preferences are aligned with those of the
agenda-setter. Only if the preferences of the policy proposer and the second-mover lobby
are aligned will the agenda-setter switch from proposing a pro-second-mover policy to a
pro-first-mover policy induced by the first mover’s higher preemptive bribes in response
to a stronger second-mover lobby. Otherwise, negative effects of interest-group size
cannot occur, as we explain next.

Let us consider the situation where the preferences of the policy proposer are not aligned
with those of the second-mover lobby $Y$, but instead the agenda-setter is of type $X$.
Staying with the previous set-up of the lobbying subgame where lobby $Y$ is the second
mover, the partition of the policy space remains as described in Section 4.2. The only
difference from our previous set-up is that the policy proposer values implementable
policies differently.

Now an increase in the strength of the second mover does not lead to adverse ef-
fects for this lobby for the following reason: A larger second-mover lobby increases the
bribes associated with implementable policies favoring the first-mover’s type and non-
implementable policies favoring the second-mover’s type. When the policy proposer’s
preferences are aligned with the first mover’s, these policies are the most attractive for
him to propose even without payments. Hence, higher payments induced by a larger
second-mover lobby will not lead to an adverse change in policy for the second mover or
to status-quo persistence because the $X$-type policy proposer will not choose a policy
favoring $Y$ when bribes are low initially. The formal argument can be found in Appendix
A.6. Here, we summarize:

**Proposition 5 (Agenda-setter’s preferences differ from second-mover’s)**

An increase in the size of the second mover $Y$ will not lead to adverse effects on policy for this lobby if the policy proposer is of the opposite type $X$.

In the basic model, the timing of the game reflects the idea that for any policy proposal, the lobby that would like to change the majority decision occurring without payments is the first mover of the lobbying subgame. Another specification of the sequential lobbying game could be that for any given policy proposal, the interest group defending the status quo is the second mover (see e.g. Diermeier and Myerson (1999), Polborn (2006), Breton and Zaporozhets (2009)). Following similar steps as in the analysis of the basic model one can show that in such a set-up only adverse policy changes induced by an increase in strength of the second-mover lobby can occur, but not status-quo persistence as described in Appendix A.1. By contrast, if we specify the timing such that, given a proposal, the defender of the status quo had to move first, status-quo persistence may result from an increased strength of the second-mover interest group, but no adverse policy changes will occur.\(^{37}\)

### 6.2 Second-mover disadvantage

In the literature, the asymmetry in the bidding process of the Groseclose and Snyder (1996) model gives the second mover an advantage as the first mover needs to preempt concentrated counter-offers by the second mover. This advantage is captured by the *hurdle factor* indicating the factor by which the first-mover lobby’s budget needs to exceed the second mover’s to win the lobbying subgame.\(^{38}\) Since the hurdle factor is typically greater than one (in our set-up it is 2), lobby $Y$ may avert the approval of a given pro-$X$ policy proposal when moving second but not when moving first. In the case where $X$ is sufficiently strong to support the given pro-$X$ policy proposal as a first mover, $X$ would have to make sufficiently large payments to the legislators to outcompete the second mover $Y$. By contrast, as a second-mover, lobby $X$ could win the lobbying subgame following the pro-$X$ policy proposal with negligible costs since $Y$

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\(^{37}\)The intuition is that if the defender of the status-quo always moves last, the $Y$-type agenda-setter’s preferences will be aligned with the second mover’s only if $q < s$, but not if $q > s$. When the defender of the status quo always moves first in the lobbying subgame, the $Y$-type policy proposer’s preferences will be aligned with the second mover’s if $q > s$, but not if $q < s$.

\(^{38}\)The expression ‘hurdle factor’ was first used by Diermeier and Myerson (1999).
as the first mover would have no hope of securing a majority against the proposal and hence would abstain from offering bribes.

When the policy proposal is endogenous, the equilibrium payments after a proposal has been introduced additionally influence the agenda-setter’s decision whether to propose such a policy in the first place. Consider again the situation where $X$ is sufficiently large to support a pro-$X$ policy proposal as the first-mover lobby. Then the equilibrium payments to ensure the approval of the proposal will become negligibly small if $X$ is the second mover and lobby $Y$ the first mover. If the payments associated with the pro-$X$ policy change induced the (Y-type) agenda-setter to propose the pro-$X$ policy change, this incentive will vanish when $Y$ becomes the first mover instead and the policy proposer will effectively maintain the status quo by proposing $y$ instead of $q$ ($< s$). Consequently, both lobbies will be worse off when moving second. If $Y$ is the second mover, the pro-$X$ policy change will be implemented while the status quo would be sustained if it were the first mover. For the pro-change lobby $X$ being the first mover is more favorable, as then the desired policy change would occur rather than a persistence of the status quo when being the second mover.\(^{39}\)

This might call into question the assumptions regarding the order of the moves of the lobbies in the lobbying subgame, where we assumed that lobby $Y$ always moves last. However, it is necessary to make this assumption initially to see whether there really is a second-mover advantage when endogenous proposals are considered.

As in Section 3.3, the order of the lobbies’ moves is often justified by the argument that – given a proposal has been made – the lobby which opposes the majority preference in the legislature has to move first. We emphasize that the second-mover disadvantage originates from the endogeneity of the policy proposals. Ex ante, before a policy proposal has been introduced, there are situations where the (majority) lobby, $Y$, would like to commit to move first. However, once the policy proposal has been made, it is beneficial to move second (independent of the particular bill that has been proposed). Consequently, even when an ex-ante second-mover disadvantage exists, the justification regarding the lobbies’ sequence of moves will still be valid if – as we assume – there is no possibility to commit to a sequence of moves before a proposal is made.

\(^{39}\)A simple more formal argument can be crafted for the case where $q = x$ and $l_Y$ is sufficiently large such that $x$ will be implemented. In general, the partition of the policy space changes when changing the sequence of the lobbies’ moves. In this example, $q$ remains at $x$ because if $F(x) = l_Xv_X(x) + 2l_Yv_Y(x) \geq 0$, then $F_n(x) = l_Yv_Y(x) + 2l_Xv_X(x) > 0$, where $F_n(t) \geq 0$ characterizes the implementable policies when $Y$ is the first mover.
6.3 Endogenous interest-group size

So far, we have considered two different types of individuals. Allowing for more than two types of individuals who may join either the interest group supporting policy change or the one defending the status quo is interesting because then the size of the lobbies is endogenous. The moderate types with bliss points around the center of the policy interval switch interest-group membership depending on the policy proposal. In fact, it may be exactly this ‘swinging’ behavior of the politically moderate that can cause policy outcomes to their disadvantage. The intuition is that the interest-group sizes directly depend on the policy proposal $t$ in such a way that the farther away the policy proposal is from the center of the political spectrum, the larger is the lobby opposing this proposal. This increases the bribes associated with extreme proposals thereby making them more attractive for the policy proposer. As a consequence, the politically moderate individuals would be better off if they did not organize.

We illustrate the effects by adding a type $Z$ with measure $l_Z$ and utility $u_Z = -\frac{1}{2}(t-z)^2$, where $z = s = 0$, to the example in Section 5.2. For $l_X/N = 4.4$, $l_Y/l_X = 0.15$, and $l_Z = 0$, we know from Section 5.2 that the policy proposer will introduce $y$ and in effect, the status quo – which is the best policy for type $Z$ – will remain. However, if $l_Z/l_X = l_Y/l_X = 0.15$, the bribes associated with pro-$X$ proposals increase resulting in a policy change towards $t = -0.875$. Hence a larger size of lobby $Z$ with moderate policy preferences leads to a more extreme policy outcome which is to $Z$’s disadvantage.

6.4 Contribution limits and welfare

In the literature, there is an ongoing debate whether contribution limits for lobbies should be introduced. Given that limits on payments of interest-groups to legislators can be enforced, these would clearly benefit the second-mover lobby in our model. The reason is that the first-mover lobby will have to spend twice (the hurdle factor is 2) the second mover’s budget to win the lobbying subgame. Consequently, an effective spending

\[ F(t) = l_Xv_X(t) + 2(l_Yv_Y(t) + l_Zv_Z(t)) \]

and for $q < s$, the objective of the agenda-setter becomes $V_Y(q) - V_Y(y) = v_Y(q) - \frac{1}{2}(l_Yv_Y(q) + l_Zv_Z(q)) - \frac{1}{2}l_Yv_Y(y)$. 

\[ 40 \text{We do not consider greater heterogeneity of preferences of the legislators as they are non-pivotal in our model. Hence, only the preferences of the policy proposer matter. It is possible to show that the results derived in the basic model carry over as long as the preferences of the agenda-setter are sufficiently tilted towards the } Y\text{-type.} \]

\[ 41 \text{A formal treatment is provided upon request. The question of what happens if lobbies can raise money at a cost and thereby determine their budget endogenously would be an interesting issue for future research.} \]

\[ 42 \text{This yields } F(t) = l_Xv_X(t) + 2(l_Yv_Y(t) + l_Zv_Z(t)) \text{ and for } q < s, \text{ the objective of the agenda-setter becomes } V_Y(q) - V_Y(y) = v_Y(q) - \frac{1}{2}(l_Yv_Y(q) + l_Zv_Z(q)) - \frac{1}{2}l_Yv_Y(y). \]
limit will reduce the set of policies for which the first mover can pay this amount to the legislators. Further, for sufficiently low caps on contributions, the threshold values of lobby size leading to adverse policy changes or status-quo persistence would disappear since they crucially depend on sufficiently large lobbying payments.

Would it be welfare improving to avoid adverse policy effects of interest-group size via contribution limits? Naturally welfare properties in our model strongly depend on assumptions about how large a share of the population is organized in lobby groups. Let us consider a society consisting of $X$-type and $Y$-type individuals and denote the total number of $X$- and $Y$-type individuals in the society by $L_X$ and $L_Y$, respectively. Employing a utilitarian welfare function summing all individuals’ utilities implies that ‘monetary’ transfers are neutral for welfare as they enter the individuals’ utilities linearly. Hence, the welfare effect depends on the utility changes induced by the policy change. An adverse policy effect for the second-mover lobby $Y$ involves a welfare improvement if $l_X |v_X(q)| > l_Y |v_Y(q)|$, i.e. if the utility gain of a policy change from $s$ to $q$ ($q < s$) or from $q$ ($q > s$) to $s$ for the $X$-type individuals is larger than the corresponding utility loss for the $Y$-type individuals. Since $q$ is implementable, we know from the discussion in Section 4.2 that $l_X |v_X(q)| \geq 2l_Y |v_Y(q)|$. Consequently, we obtain that the adverse policy change for lobby $Y$ increases social welfare if $l_Y/L_Y > 0.5 l_X/L_X$. To be precise, note that if an adverse policy effect is triggered by an increase in lobby $Y$’s size, $l_Y$ in the previous condition refers to lobby $Y$’s size after the increase if $q < s$ (adverse policy change) while it refers to the size before the increase if $q > s$ (status-quo persistence). Hence, the adverse policy effect for $Y$ will also involve negative welfare effects if the share of $Y$-type individuals who are organized in lobby $Y$ is smaller than half the share of organized $X$-type individuals. In particular this implies that when both interests exhibit about the same degree of organization adverse policy effects increase the welfare level. Consequently, preventing adverse policy effects of lobby size by contribution limits does not necessarily entail welfare improvements.

## 7 Conclusions

This paper examines the role of interest-group size on policy outcomes in a legislative lobbying model with endogenously derived policy proposals. The main insight of the paper is that the relation between policy outcomes and the relative sizes of the interest groups is non-monotone and possesses discontinuities that can lead to counter-intuitive results. In particular, the incentive of the agenda-setter to propose a favorable policy
for one lobby may increase with the relative strength of the opposing lobby. As a consequence, a smaller defender of the status quo is not always good for a lobby seeking policy change. This may shed light on recent observations on lobbying and policy change and may also provide another explanation as to why some interests in society are not fully organized. We argue that contribution limits can prevent adverse policy effects of interest-group size. However, whether this is welfare improving depends on the degree to which the interests in society are organized.

The model offers several avenues for future research. Some interesting extensions of the model, such as allowing for greater heterogeneity of preferences or including different lobbying subgames, have been noted in the previous section. In particular, the paper provides a framework that can be extended to study how different forms of lobbying at the proposal stage interact with vote-buying in the legislature and how this affects policy outcomes. It would also be interesting to examine a dynamic version of the model where the policy chosen in the current period is the status quo in the next. This extension allows one to study which policies are stable in the long term and how this depends on the interest groups’ sizes.
A Appendix

A.1 Second-mover lobby supports policy change

Now we examine the situation where $Y$ is relatively strong such that the implementable policies favor $Y$ while $X$ is only able to prevent the worst policies from its perspective. Formally this means that $s < q < y$ and $q$ is a pro-$Y$ policy change.\(^{43}\) Then the policy proposer faces the trade-off between (1) proposing the implementable pro-$Y$ policy $q$ involving no payments and (2) proposing the non-implementable policy $y$ that is associated with payments of lobby $X$. The agenda-setter proposes the pro-$Y$ policy change $q$ if and only if the associated utility gain outweighs the bribes obtained when effectively maintaining the status quo:

$$v_Y(q) - b(y) = v_Y(q) - \frac{2l_Y}{N}v_Y(y) \geq 0.$$  \hspace{1cm} (A.1)

A larger size of lobby $Y$ increases the upper boundary of the set of implementable policies, $q$, implying that a more favorable policy change from a $Y$-type’s perspective would be possible. However, the payments associated with the non-implementable proposal $y$ increase as well if lobby $Y$ becomes larger. The former positively affects the proposer’s incentive to propose the policy change $q$, whereas the latter exerts a negative influence on this incentive. Depending on which effect dominates, an increase in the size of interest group $Y$ can induce the persistence of the status quo or a policy change to the benefit of $Y$.

Using (A.1) and the function $q(l_Y)$ as introduced in Section 5.1, we capture the effects of $l_Y$ formally in the next proposition.

**Proposition 6 (Status-quo persistence)**

*The policy proposer will introduce the implementable pro-$Y$ policy $q(l_Y) > s$ if and only if*

$$l_Y \leq \frac{N}{2} \frac{v_Y(q(l_Y))}{v_Y(y)} \equiv \chi^p(l_Y)$$  \hspace{1cm} (A.2)

*and $t^* = y$ else.*

The content of Proposition 6 is illustrated in Figure 3. $\chi^p(l_Y)$ is zero at the size of the second mover $l_Y$ where $q(l_Y) = s$. It strictly increases with $l_Y$ and approaches $N/2$ at

\(^{43}\)In the case where $s < q < y$, the intuition for why $y$ is the best non-implementable policy proposal is the same as for the case where $q < s$. Policy $q$ is the best implementable proposal because now implementable policies are not associated with payments and $q$ yields the highest direct utility from policy in the set of implementable policies.

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the level of $l_Y$ where $q(l_Y) = y$\textsuperscript{44}.

As the second-mover lobby size where $q(l_Y) = s$ is strictly positive, but $\chi^p(l_Y)$ is zero at this level of $l_Y$, Condition (A.2) cannot hold for this interest-group size. Hence, the status quo will remain in place. Pro-$Y$ policy changes are possible if the function $\chi^p(l_Y)$ possesses fixed points. As illustrated in Figure 3, the first fixed point determines a threshold level for the size of the second-mover lobby which induces the agenda-setter to propose the pro-$Y$ policy change $q$ instead of the non-implementable policy $y$. In this case, an increase in size benefits the pro-change lobby $Y$. Adverse policy effects where an increase in $l_Y$ induces the agenda-setter to propose the non-implementable policy $y$ rather than the implementable policy $q$ can occur if $\chi^p(l_Y)$ intersects the bisecting line from above. The following proposition states that we can find sizes of the first-mover lobby and the legislature where such a fixed point of $\chi^p(l_Y)$ exists, as long as the relative slopes of the utility functions satisfy a condition (Condition (A.3)) implying that there are intervals in which the slope of $\chi^p(l_Y)$ is smaller than one.

![Figure 3: Illustration of Proposition 6](image)

**Proposition 7 (Status-quo persistence due to larger pro-change lobby)**

There are values for $N$ and $l_X$ such that $\chi^p(l_Y)$ intersects the bisecting line from above

\textsuperscript{44}The derivative of $\chi^p(l_Y)$ writes

$$\frac{d\chi^p(l_Y)}{dl_Y} = \frac{N v'_Y(q) d_q(l_Y)}{2 v_Y(y) dl_Y},$$

which is positive as $\frac{d_q(l_Y)}{dl_Y} > 0$ according to Lemma 2 in Appendix A.4.
if and only if a policy interval exists within \((s, y)\) where

\[
2\frac{v_Y'(t)}{v_Y(t)} < \frac{v_X'(t)}{v_X(t)}.
\]  

(A.3)

Appendix A.7 provides a proof of the proposition. Similar to the case where \(X\) is very large relative to \(Y\), as discussed in Section 5.1, the larger is \(l_X/N\) the larger is the increase in bribes relative to the increase in \(q\) when lobby \(Y\) grows larger. Then the utility from bribes when proposing \(y\) increases stronger than the utility from a more favorable implementable policy \(q\), leading the agenda-setter to propose the non-implementable pro-\(Y\) policy rather than the implementable one.\(^{45}\)

Numerical example

Using the same set-up as in the numerical example given in Section 5.2, but considering a low value of \(l_X/N\) such as 0.4, the function \(\chi^p(l_Y)\) is strictly concave and intersects the bisecting line at \(l_Y = 0.7l_X\) and \(l_Y = 1.22l_X\).\(^{46}\) This implies that \(q\) will be implemented if \(l_Y\) is between these fixed points and that the proposal will change to the non-implementable \(y\) if \(l_Y\) becomes larger than 1.22\(l_X\) (but remains smaller than \(l_Y(y) = 1.5l_X\), in which case \(y\) becomes implementable).

A.2 Policy effects of first-mover lobby size

The previous two subsections focused on the effects of the size of the second-mover lobby on policy outcomes and identified threshold sizes which trigger adverse policy effects for

\(^{45}\)Interestingly, in the case that the policy proposer’s most preferred proposal changes from \(q > s\) to the non-implementable policy \(y\) in response to an increase in \(l_Y\), both lobbies will be worse off. That \(Y\) loses in utility is obvious because with the smaller size \(l_Y\) its members would enjoy \(v_Y(q) > v_Y(s) = 0\). Since the status quo by itself is more favorable than \(q\) for \(X\), the members of lobby \(X\) seem to benefit from the increase in \(l_Y\). However, they have to make payments in the amount of \(2l_Y v_Y (y)\) to prevent the implementation of \(y\) when \(l_Y\) is large. By contrast, if \(l_Y\) is small, each member of \(X\) suffers \(v_X(q)\). Since \(2l_Y v_Y (y) > 2l_Y v_Y (q) \geq -l_X v_X (q)\) (the last inequality holds because \(F(q) = l_X v_X (q) + 2l_Y v_Y (q) = 0\) by definition of \(q\)), the utility loss from preventing \(y\) from implementation is larger than the one suffered from implementing \(q\). Hence lobby \(X\) also loses in utility if \(l_Y\) becomes larger. Of course, the pure benefit of the better policy \(s\) would accrue to \(X\)-types that are not organized in lobby \(X\). They free-ride on the lobbying activities of interest group \(X\).

\(^{46}\)When interpreting \(N\) literally as the size of the legislature, the condition that \(N > l_X\) may seem unrealistic. This changes however, if \(N\) is interpreted more broadly to also reflect institutional details such as accountability in office. Suppose, e.g., that there is a certain probability \(\mu\) that a legislator will be caught when taking bribes and, if so, that he will lose all of the bribes he has been paid. Then, expected bribes are \((1 - \mu)\frac{2N}{X} [v_Y(t)]\). Defining \(\bar{N} = \frac{N}{1 - \mu}\) the equations in the main text remain valid when substituting \(\bar{N}\) for \(N\). However, with this broader interpretation, if it is very likely that taking bribes becomes public (i.e., \(\mu \rightarrow 1\) \(\bar{N}\) approaches infinity. Hence, the situation wherein \(\bar{N} > l_X\) seems plausible in well developed democracies. A more formal treatment of this argument can be provided upon request.
the interest group when exceeding it. Here we explain why such adverse policy effects of an increase in its size cannot occur for the first-mover lobby: For a given policy proposal, the size of the first-mover interest group determines whether the budget exceeds the amount necessary to win the lobbying subgame. The latter depends on the size of the second-mover lobby. Hence, a larger budget of the first-mover lobby does not change the amount of payments made to win the legislative vote for or against a given policy proposal, but expands the set of policy proposals for which the first-mover lobby can win the lobbying subgame. As a consequence, a larger first-mover lobby does not affect the bribes associated with proposing the non-implementable policy \( y \). However, if the first-mover interest group becomes larger, the boundary of the implementable policy set \( q \) decreases as the first mover is now able to make higher payments and outcompete the second mover on further pro-X policy proposals.

In the situation where the first mover \( X \) is very large relative to the second-mover lobby \( Y \) such that only pro-X policies are implementable, i.e. \( q < s \), a lower value of \( q \) increases the attractiveness to propose the pro-X policy change \( q \).\(^{47}\) Hence, an increase in the size of the first-mover lobby \( X \) can lead to a switch of the policy proposal from \( y \) to \( q \), but not to a change from \( q \) to \( y \). As a consequence, if the policy proposer changes his proposal in response to an increase in the first mover’s size, it will be to the benefit of this lobby. When the second-mover lobby \( Y \) is relatively large such that the implementable policies favor \( Y' \) (\( q > s \)), a lower value of \( q \) resulting from an increase in the first-mover lobby reduces the agenda-setter’s incentive to propose the pro-Y policy change.\(^{48}\) Consequently, an increase in the size of lobby \( X \) either leads to a less extreme policy change in favor of \( Y \), a switch from proposing \( q \) to proposing \( y \) or the status quo is maintained. In either case an increase in its size weakly benefits the first-mover lobby in terms of policy outcomes. We summarize these arguments in the following proposition, which we formally prove in Appendix A.8.

**Proposition 8 (No adverse effects of size for first-mover lobby)**

There are no adverse effects of interest-group size on policy outcomes for the first-mover lobby.

\(^{47}\)Recall from Section 5.1 that by decreasing \( q \), the utility loss for the policy proposer incurred by a pro-X policy change \( q \) increases. However, the payments associated with proposing the policy change \( q \) increase as well. In the relevant case where the utility of the policy proposer from proposing an implementable pro-X policy is positive (\( 2l_Y > N \)) the bribes increase faster than the utility loss when \( q \) decreases, thus, making the proposal \( q \) more attractive. Note that in the case where \( (2l_Y < N) \) proposing an implementable pro-X policy will involve a negative utility change \( V_Y(q) \) for the policy proposer. In this case the agenda-setter always maintains the status quo by introducing \( y \).

\(^{48}\)This can be directly observed in Condition (A.2) which captures the policy proposer’s trade-off.
A.3 Proof of Proposition 2

We follow the two step procedure described in the main text to identify the policy proposer’s best proposal. We verify that this proposal is unique and that it can be characterized as in Proposition 2 via Condition (4). Thus it follows directly that the subgame-perfect equilibrium of the entire game will also be unique, since the lobbying subgame following the proposal announcement possesses a unique equilibrium.

Step 1:

We start by identifying the agenda-setter’s most preferred non-implementable policy. First, as stated in the main text, proposing a non-implementable pro-X policy, \( t < \min\{s, q\} \), involves \( V_Y(t) = 0 \) and is therefore never strictly preferred to the status quo.

If \( q < y \), the set of non-implementable pro-Y policies comprises all policies \( t > \max\{s, q\} \). The agenda-setter’s most preferred non-implementable proposal is \( y \) because lobby \( X \) needs to pay bribes to prevent pro-Y policies from being implemented and – according to (1) – the amount of bribes \( b(t) \) increases with \( t \) over the set of non-implementable pro-Y policies.

If \( q = y \), then all pro-Y policies are implementable and the best choice among the non-implementable policies is either the status quo or any policy favoring \( X \), \( t < s \), which according to the argument above yield zero utility gain.

Let us now consider the most preferred choice among the implementable policies. If \( q \geq s \), the implementable policy set, \([s, q]\), comprises pro-Y policies and the most preferred policy proposal maximizes \( v_Y(t) \). Consequently, the maximizer is the upper boundary of the implementable set, \( q \).

If \( q < s \), the implementable policies benefit \( X \) and the policy proposer faces the following trade-off. On the one hand, he will obtain bribes for a vote in favor of the pro-X policy. On the other hand, being of type \( Y \) and knowing that the policy proposal will be implemented, the agenda-setter suffers himself from a policy change favoring \( X \). For \( q \leq t < s \), the proposer’s objective writes \( V_Y(t) = v_Y(t) \left[ 1 - \frac{2l_Y}{N} \right] \). This reveals that the utility gain of a \( Y \)-type policy proposer from introducing an implementable pro-X policy is positive if and only if \( 2l_Y > N \).\(^{49}\) Moreover, the proposer’s utility increases with \( |v_Y(t)| \), implying that \( q \) is the most preferred implementable proposal if \( 2l_Y > N \). If \( 2l_Y \leq N \), the status quo is the most preferred implementable policy.\(^{50}\)

\(^{49}\)Intuitively, this condition implies that the bribes compensate the agenda-setter’s utility loss from the implementation of \( t \).

\(^{50}\)In case that \( 2l_Y = N \) any policy within the implementable set yields zero utility gain. So the
Step 2:

Since among the most preferred implementable and non-implementable policies the agenda-setter proposes the one that maximizes his utility, the claim of Proposition 2 that \( q \) is proposed if \( V_Y(q) \geq V_Y(y) \) and \( y \) otherwise is obvious in the cases where the unique most preferred implementable proposal is \( q \) and the unique most preferred non-implementable proposal is \( y \). Two situations deserve further attention. First, if \( q < s \) and \( 2l_Y < N \), and second, if \( q = y \).

In the first situation any implementable pro-\( X \) proposal yields less or equal utility than the status quo while the non-implementable \( y \) involves a positive utility gain from bribes. As \( V_Y(q) \leq V_Y(s) = 0 < V_Y(y) \), \( y \) is the unique proposal choice which is in line with Condition (4) of Proposition 2.

In the second situation, the best implementable policy is \( q = y \), which yields positive utility gain while all non-implementable policies involve no utility gain relative to the status quo. Hence, the unique proposal choice is \( q = y \) as predicted by Proposition 2.

A.4 Effects of interest-group size

First we define the function \( q(l_Y) \), then we verify that it increases in \( l_Y \), and finally we show that \( \chi^c(l_Y) \) is an increasing function in \( l_Y \).

For the definition of \( q(l_Y) \), it is convenient to first define the function \( \hat{l}_Y(t) \equiv -l_X \frac{v_X(t)}{2v_Y(t)} \), for \( t \neq s \), and \( \hat{l}_Y(s) \equiv -l_X \frac{v_X'(s)}{2v_Y'(s)} \). The function \( \hat{l}_Y(t) \) is derived from the condition \( F(t) = 0 \). \( \hat{l}_Y(t) \) gives the size of interest group \( Y \) such that \( t \) is the second root of \( F(t) \).

As verified in the next lemma, \( \hat{l}_Y(t) \) is strictly increasing.

Lemma 1

\( \hat{l}_Y(t) \) is increasing in \( t \).

Proof. Consider the case \( t \neq s \). As the function \( \hat{l}_Y(t) \) is defined by \( F(t) = 0 \), we can use the implicit-function theorem to obtain

\[
\frac{d\hat{l}_Y(t)}{dt} = -\frac{\partial F(q)}{\partial l_Y} = -\frac{l_X v'_X(q) + 2l_Y v'_Y(q)}{2v_Y(q)}.
\]

If \( q < s \) the numerator is positive since \( F(t) \) is concave and possesses another root at \( s \). The denominator is negative. If \( q > s \) the opposite is true: the numerator is negative and the denominator is positive.

The proposer is indifferent between choosing \( s \) and any policy \( t < s \).
In the case that \( t = s \), the derivative of \( \hat{l}_Y(t) \) writes
\[
\frac{d\hat{l}_Y(s)}{dt} = -l_X \frac{v''_Y(s)v'_Y(s) - v'_X(s)v''_Y(s)}{2(v'_Y(s))^2} > 0 .
\]
Note that \( v'_Y(s) > 0 \) and \( v'_X(s) < 0 \).

In sum, we conclude that \( \frac{d\hat{l}_Y(t)}{dt} > 0 \). \( \square \)

Using lemma 1, we can define the boundary of the implementable set \( q(l_Y) \) as
\[
q(l_Y) \equiv \begin{cases} 
x , & \text{if } l_Y < \hat{l}_Y(x) , \\
\hat{l}^{-1}_Y(l_Y) , & \text{if } l_Y \in [\hat{l}_Y(x), \hat{l}_Y(y)], \\
y , & \text{if } l_Y > \hat{l}_Y(y) .
\end{cases}
\]

Next, we verify the following lemma.

**Lemma 2**

\( q \) is (weakly) increasing with \( l_Y \) and (weakly) decreasing with \( l_X \).

**Proof.** The fact that \( \hat{l}_Y(t) \) strictly increases with \( t \) implies that \( q(l_Y) \) is an increasing function for \( l_Y \in (\hat{l}_Y(x), \hat{l}_Y(y)) \). For all other values of \( l_Y \) outside of this interval, \( q(l_Y) \) is constant and hence the derivative is zero.

With respect to \( l_X \), we have for \( q \neq s \)
\[
\frac{dq}{dl_X} = -\frac{\partial F(q)}{\partial l_X} = -\frac{v_X(q)}{\partial F(q)/\partial t} < 0 .
\]

Note that with respect to the sign of the denominator the same arguments as stated above concerning the derivative of \( q \) with respect to \( l_Y \) apply.

In the case \( q = s \), we obtain
\[
\lim_{q \to s} \frac{dq}{dl_X} = \lim_{q \to s} -v_X(q) < 0 .
\]

Finally we show that \( \chi^c(l_Y) \) increases with \( l_Y \).

**Lemma 3**

The function \( \chi^c(l_Y) \) is strictly increasing with \( l_Y \).

**Proof.** The derivative of \( \chi^c(l_Y) \) with respect to \( l_Y \) writes
\[
\frac{d\chi^c(l_Y)}{dl_Y} = \frac{v'_Y(q)v_Y(y)}{(v_Y(q) + v_Y(y))^2} \frac{dq}{dl_Y} .
\]

The first term is positive and for \( q \) being the second root of \( F(t) \) in \( \tau \), \( \frac{dq}{dl_Y} > 0 \) according to Lemma 2. Hence, \( \frac{d\chi^c(l_Y)}{dl_Y} > 0 \). \( \square \)
A.5 Proof of Proposition 4

Analytically Proposition 4 can be verified by inserting $\hat{l}_Y(t)$, as defined in Appendix A.4, into the conditions of Proposition 3. Then, a change towards policy $t$ is proposed by the agenda-setter and approved by the legislature if and only if

\[(i') -v_Y(t) > v_Y(y) \text{ and} \]
\[(ii') \frac{l_X}{N} \geq -\frac{v_Y(t)}{v_X(t)} - \frac{v_Y(t)}{v_Y(t) + v_Y(y)}. \]

Let us define the policy $\bar{t}$ as the policy such that $-v_Y(\bar{t}) = v_Y(y)$. Suppose $s$ is such that $-v_Y(x) > v_Y(y)$ implying that Condition (i') is satisfied and the right-hand side of (ii') is positive for all $t < \bar{t}$. Then there are policies $t \in [x, \bar{t})$ for which Condition (ii') holds if $l_X/N$ is sufficiently large. Hence, a sufficient condition for pro-$X$ policy changes to be possible is that $l_X/N$ exceeds the right-hand side of (ii') evaluated at $x$. This is equivalent to the point $(\hat{l}_Y(x), \chi^c(\hat{l}_Y(x)))$ being below the bisecting line in Figure 2. On the other hand, for given $s$, $N$, and $l_X$, the conditions (i') and (ii') characterize the utility functions that allow for pro-$X$ policy changes in response to increases in the interest-group size of $Y$.

A.6 Proof of Proposition 5

The $X$-type agenda-setter’s proposal maximizes

$$V_X(t) \equiv U_X(t) - U_X(s) = \begin{cases} 1_{t \in [q,s]} v_X(t) + 1_{t \geq q} b(t), & \text{if } q < s, \\ 1_{t \in [s,q]} v_X(t) + 1_{t > q} b(t), & \text{if } q \geq s. \end{cases}$$

Now we proceed in two steps. (1) We show that when $q < y$, the $X$-type agenda-setter proposes $t^* = q$ if and only if $I_X(q) \equiv V_X(q) - V_X(y) \geq 0$ and $t^* = y$, else. In case that $q = y$, any policy $t \leq s$ may be proposed. (2) Using the function $I_X(q)$, we show that an increase in the second mover’s size has no adverse effects on policy for this lobby.

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51 Note that by inserting $\hat{l}_Y(t)$ for $l_Y$, we implicitly assume that $l_Y$ is at least as large as $\hat{l}_Y(x)$. This is not a problem for Condition (i) in Proposition 3 as $q(l_Y)$ cannot become smaller than $x$. With respect to Condition (ii) it does not impose a restriction because if the condition does not hold for $l_Y = \hat{l}_Y(x)$, it can neither be satisfied for $l_Y < \hat{l}_Y(x)$.

52 An intuitive explanation why $l_X/N$ has to be large for policy changes to be possible runs as follows. Consider a policy $t < \hat{t}$. $t$ may be proposed only if it is the boundary of the implementable set. For larger values of $l_X$, $l_Y$ must be larger for $t$ to remain the boundary of the implementable set. A larger second-mover lobby implies higher bribes associated with $t$ and hence proposing $t$ becomes more attractive. With respect to $N$ the attractiveness to propose $t$ is increased by a decline of $N$ implying higher payments per legislator.
(1) Assume that $q < y$. The best non-implementable proposal for the X-type agenda-setter is $y$ as for all $t < \min\{q, s\}$, $V_X(t) = 0$, and for all $t > \max\{q, s\}$, $V_X(t) = b(t) = \frac{2l_Y}{N}v_Y(t) > 0$ which increases with $t$.

The most preferred implementable proposal is $q$ if $q < s$ and $s$ else for the following reasons:

In the case where $q < s$, $V_X(t) = v_X(t) - \frac{2l_Y}{N}v_Y(t)$. The derivative with respect to $t$ reads $\frac{dV_X(t)}{dt} = v'_X(t) - \frac{2l_Y}{N}v'_Y(t) < 0$. Thus, $q$ maximizes $V_X(t)$.

If $s \leq q < y$, then an implementable policy $t$ involves $V_X(t) = v_X(t) \leq 0$. Hence, $s$ maximizes $V_X(t)$.

Now we can compare the best implementable and non-implementable proposal. If $q < s$, then $q$ is proposed if and only if $V_X(q) \geq V_X(y)$ and $y$ else. In case that $s \leq q < y$, $y$ is proposed. Since in the latter situation $V_X(t) < 0, \forall t \in (s, q]$, the proposal choice when $q < y$ can be summarized via $I_X(q)$ as described above.

Given $q = y$, the best non-implementable policy is any policy $t \leq s$ (implying $V_X(t) = 0$) as the entire set of policies favoring $Y$ is implementable.

(2) We concentrate on the case $q < y$, as given $q = y$ the proposer’s best proposal does not change in response to marginal changes in interest-group size.\textsuperscript{53} We show that

(a) if $q < s$, an increase in $l_Y$ does not lead to a change in the sign of $I_X(q)$ from negative to positive (adverse policy change).

(b) if $s < q < y$, an increase in $l_Y$ does not lead to a change in the sign of $I_X(q)$ from positive to negative (status quo persistence).

(a) If $q < s$, we have $I_X(q) = v_X(q) - \frac{2l_Y}{N}[v_Y(q) + v_Y(y)]$. When $-v_Y(q) \geq v_Y(y)$, the term in brackets is either negative or zero, and thus $I_X(q) > 0$ for all $l_Y \geq 0$.

Now consider the case where $-v_Y(q) < v_Y(y)$. The derivative of $I_X(q)$ with respect to $l_Y$ reads

$$\frac{dI_X(q)}{dl_Y} = \left[v'_X(q) - \frac{2l_Y}{N}v'_Y(q)\right] \frac{dq}{dl_Y} - \frac{2}{N}[v_Y(q) + v_Y(y)].$$

The first term is negative as, according to Lemma 2, $\frac{dq}{dl_Y} \geq 0$. Then, $\frac{dI_X(q)}{dl_Y} < 0$ follows from $-v_Y(q) < v_Y(y)$.

\textsuperscript{53}Note that we neglect the non-differentiable point of $q$ with respect to interest-group size at $y$ to avoid tedious case distinctions that do not yield additional insight.
In summary, if $-v_Y(q) < v_Y(y)$, $I_X(q)$ declines in $l_Y$ and consequently will not change its sign from negative to positive. If $-v_Y(q) \geq v_Y(y)$, $I_X(q)$ is positive independent of $Y$’s lobby size and thus does not change its sign in response to a change in $l_Y$.

(b) If $s < q < y$, we have $I_X(q) = v_X(q) - \frac{2l_Y}{N}v_Y(y) < 0$. Since the first term is negative, $I_X(q)$ is unambiguously negative for all $l_Y$ and, hence, will not change its sign in response to a change in $l_Y$. \hfill \Box

A.7 Proof of Proposition 7

Inserting $\hat{l}_Y(t)$, as defined in Appendix A.4, into Condition (A.2), we obtain that policy $t > s$ is proposed if and only if

$$\frac{l_X}{N} \leq \frac{(v_Y(t))^2}{v_X(t)v_Y(y)}$$

and the non-implementable $y$, else. Hence, status-quo persistence induced by a greater pro-change lobby is equivalent to an intersection of the right-hand side (RHS) of (A.4) with $l_X/N$ from above. Such an intersection can only be found if there is an interval of policies where RHS is decreasing with $t$. Taking the derivative with respect to $t$, we obtain that RHS is declining if and only if (A.3) is satisfied. As RHS is positive for all $t > s$, this condition is necessary and sufficient to find values of $l_X/N$ that intersect with the declining part of RHS.

A.8 Proof of Proposition 8

We define $I_Y(q) \equiv V_Y(q) - V_Y(y)$ with the interpretation that $q$ is proposed if and only if $I_Y(q) \geq 0$ and $y$ else. Then we have to show that

(a) if $q < s$, an increase in $l_X$ does not lead to a change in the sign of $I_Y(q)$ from positive to negative (status-quo persistence).

(b) if $s < q < y$, an increase in $l_X$ does not lead to a change in the sign of $I_Y(q)$ from negative to positive (adverse policy change).

(a) If $q < s$, $I_Y(q)$ as given by (5) may only be positive if $-v_Y(q) < v_Y(y)$ and $2l_Y > N$. As only in this case the sign of $I_Y(q)$ could change from positive to negative, we assume in the following that these conditions are satisfied. Then

$$\frac{dI_Y(q)}{dl_X} = v_Y(q) \frac{dq}{dl_X} \left[ 1 - \frac{2l_Y}{N} \right] \geq 0,$$
since \( \frac{\partial v}{\partial l_X} \leq 0 \) according to Lemma 2. Hence, \( I_Y(q) \) will not change its sign from positive to negative in response to an increase in \( l_X \).

(b) In the situation where \( s \leq q < y \), \( I_Y(q) = v_Y(q) \) and consequently \( \frac{dI_Y(q)}{dl_X} = v_Y(q) \frac{\partial v}{\partial l_X} < 0 \). Thus, \( I_Y(q) \) will not change its sign from negative to positive due to an increase of \( l_X \).

\[ \Box \]

References


