University of Bath

Alternative formats

If you require this document in an alternative format, please contact:
openaccess@bath.ac.uk

Raising Juveniles*

Clive Bell

South-Asia Institute (SAI)

University of Heidelberg

INF 330, D-69120 Heidelberg

Germany

clive.bell@urz.uni-heidelberg.de

Hans Gersbach[†]

CER-ETH—Center of Economic Research

at ETH Zurich

ZUE D7

8092 Zurich, Switzerland

hgersbach@ethz.ch

Maik T. Schneider

CER-ETH—Center of Economic Research

at ETH Zurich

ZUE D15

8092 Zurich, Switzerland

schneider@mip.mtec.ethz.ch

First Version: October 2005
This Version: June 2011

Abstract

This paper investigates how families decide how juveniles use their time. The problem is analyzed in three variations: (i) a 'decentralized' scheme, in which parents control the main budget, but their children dispose of their time as they see fit, together with any earnings from work on their own account; (ii) 'hierarchy', in which parents can enforce, at some cost, particular levels of schooling and supervised work contributing to the main budget; and (iii) the cooperative solution, in which resources are pooled and the threat point is one of the non-cooperative outcomes. Adults choose which game is played. While the subgame perfect equilibrium of the overall game is pareto-efficient, it may yield less education than 'hierarchy'. Restrictions on child labor and compulsory schooling generally affect both the threat point and the feasible set of bargaining outcomes. Families may choose more schooling than the legal minimum.

1 Introduction

The importance of educating juveniles well is beyond dispute, and here, the family plays a central role. For individuals' educational attainments on reaching adulthood are not the result of decisions made by a collection of rational Robinson Crusoes; rather these attainments reflect their families' choices as they were growing up. This fact is well-recognized in the literature that examines investment in education as the outcome of family decision-making and its implications for economic development. In the usual formulation, how much education children receive basically depends on how much their parents are able and willing to finance. Children have the same preferences as their parents, or their parents can force them to go to school against their will without exerting any effort. In reality, however, the family faces a more complicated problem, in that juveniles may have quite different preferences from their parents, and how they spend their time depends on what inducements their parents provide and their efforts at enforcement.[1] Although conflicts between parents and their children due to differences in preferences have been analyzed in the literature on the 'rotten kid theorem' and the 'samaritan's dilemma' (Becker, 1974; Bergstrom, 1989), there are no direct inferences about the long-run consequences of different ways of resolving the conflict.

The object of this paper is to combine these two strands of the literature in order to examine how family decision-making affects investment in education, and hence the economy's long-run growth, when juveniles may also devote their time to work or the pursuit of leisure. As formulated here, parents care about current consumption and their children's future human capital, but the latter are also keen on 'fun'. Thus, juveniles may wish to play long and hard, even though their parents are willing to finance a fuller education. It is also possible that parents are keen to consume, in which case juveniles' ability to earn, and so contribute to the family's main budget, constitutes a reason for their parents to deny them all the education they desire. As in the literature on family decision-making and economic development, the trade-off between education and work has a prominent place, but we go a step further by allowing juveniles to allocate their time in ways that may run counter to their parents' wishes. This step towards realism enables us to arrive at results on the connection between the static resolution of the family's conflict and its long-term consequences.

The decision problem is analyzed in three variations. First, there is a 'decentralized' scheme, in which parents control the purse-strings to the family's 'main' budget, but

---

[1]Broadly related, though different, bargaining problems arise between spouses. Pollak (2007) provides an extensive account, with an emphasis on joint taxation.

the children dispose of their time as they see fit, subject to their parents' willingness to finance their choice of education. Should they work, they may also allocate their earnings between a voluntary contribution to the main budget and pocket money to pursue fun. Second, there is a 'hierarchical' scheme, in which parents can enforce, at some cost, both more schooling than their children desire and compulsory work to contribute to the main budget. Third, there is a cooperative solution, in which resources are pooled and the threat point is one of the two noncooperative outcomes. Parents choose which game is played. We prove that the sub-game perfect equilibrium almost always involves the cooperative solution. The main result is that for some constellations of preferences and technologies, there exists a tradeoff between static efficiency, which always holds in a cooperative solution of the family's current conflict, and long-run growth. The reason is that in the hierarchical scheme, whose outcome is always pareto-inefficient, parents possess the means to force their children to attend school; but they cannot control how the children allocate their residual time or any associated earnings, which together produce fun. Hence, if parents have preferences for schooling that are sufficiently stronger than their children's, but not too strong, and the marginal costs of enforcement are not too high, they would choose extensive schooling under hierarchy. Yet they would do still better in the bargaining outcome. For the savings in enforcement costs would be enjoyed by both parties, partly in the form of more consumption, with the juveniles contributing to the household's budget by working more in exchange for less schooling and perhaps some pocket money to pursue fun, even if there were less time for it.

It is natural to ask how legal restrictions on child labor and compulsory schooling will affect families' decisions. If the main aim is to foster the formation of human capital, the principle of targeting indicates that enforcing extensive schooling is the right way to intervene. Our analysis reveals, however, that there are subtle effects of restrictions on child labor when juveniles can while away their time instead of attending school: these restrictions can induce high levels of education, even when the compulsory minimum does not bind. Joint regulation may, therefore, be advantageous.

There is a large literature on the economics of the family (for an overview, see Neuwirth and Haider, 2004). The family's role in educational decisions is usually emphasized when there are borrowing constraints (Banerjee, 2004), especially in developing countries. As for the motives for such investments and who has the final say, whilst the assumption that parents are in charge of financial decisions is almost universal, there are various formulations of the parties' utility functions. They range from the parents and children sharing a single, unified utility function (Becker, 1981; Loury, 1981) to

that in which parents' only incentive to lend to their children is because the latter will care for them in old age (Barham et al., 1995; Cox, 1987; Cox and Jakubson, 1995; Cremer et al., 1992). There are also formulations in which parents have some altruism towards their children, expressed by the former putting some weight on the latter's income, consumption or human capital.

In the non-cooperative setting of this paper, the fundamental family conflict is also related to the literature on the 'rotten kid theorem' and the 'samaritan's dilemma'. In a sequential game with altruistic parents and a selfish child, the parents can achieve their first-best by moving first (samaritan's dilemma) or last (rotten kid theorem), whereby the right choice depends on the specific form of the utility functions (Dijkstra, 2007). These problems are usually two-dimensional, with each party deciding over one dimension. In most settings, the parents choose the level of an income transfer to the children, who control a variable that is often interpreted as work effort. The setting employed here, in which the parties dispose of a variety of alternatives, is correspondingly richer in its implications.

The paper is organized as follows. Section 2 describes the family's structure, endowments, and activities. Its decision problem is set out in Section 3, in the three variations described above, and the sub-game perfect equilibrium is characterized. The effects of legal restrictions on work and schooling are examined in Section 4, followed by a numerical, illustrative example in Section 5. Some variations and extensions of the model are treated in Section 6. Concluding remarks are drawn together in Section 7.

2   The Structure

A family comprises one juvenile[2] (denoted by the superscript $i = 1$) and one adult ($i = 2$), each of whom is endowed with one unit of time and human capital λ^i, respectively. The mother works full time; her son splits his time among education, e, work, w, and leisure, l. His time budget therefore satisfies

$$e + l + w = 1, \quad (e, l, w) \geq \mathbf{0}, \tag{1}$$

there being no other way to use time. Both members of the family consume an aggregate, private good directly, and both place some, but not necessarily identical, value on education. The juvenile can also produce a good that will be called 'fun', by combining

---

[2]According to the U.N.'s definition, 'youth' are those aged 12 to 24.

inputs of leisure and the aggregate good, that is, pocket money.

2.1 Production and preferences

Several technologies are involved.

Assumption 1
The aggregate good is produced under constant returns to scale by means of human capital alone, with the factor of proportionality normalized to one.

Assumption 2
The technology for producing fun is represented by

$$\zeta = \zeta(l, m), \tag{2}$$

where m denotes the complementary expenditure of pocket money, and $\zeta(\cdot)$ is increasing, concave and differentiable in both arguments. In contrast to pocket money, leisure is essential in the production of fun: $\zeta(0, m) = 0 \ \forall m$ and $\zeta(l, 0) > 0 \ \forall l > 0$.

The level of human capital attained by the juvenile on reaching full adulthood, λ^2_{+1}, depends on his parent's human capital and the time spent in schooling.

Assumption 3

$$\lambda^2_{+1} = \Gamma(\lambda^2, e), \tag{3}$$

where Γ is an increasing and differentiable function, and is concave in e.[3]

Let c^1 and c^2 denote the parties' respective levels of consumption of the aggregate good. We begin by assuming that whereas the juvenile has preferences over c^1, ζ and λ^2_{+1}, the adult has preferences over c^2 and λ^2_{+1} alone. (We relax this assumption in section 6.3 by allowing the adult to view her son's fun as either good or a source of dismay.) Their respective preferences are represented by increasing, strictly concave and differentiable utility functions U^i. All goods are normal, with the exception of fun for the adult, and the aggregate good is essential in consumption.

---

[3]A more general formulation of equation (3) would permit juveniles to vary in their abilities, as represented by λ^1, both naturally and through nurture in early childhood. The more able should profit more from education and able parents, which can be represented by introducing λ^1 as an argument into $\Gamma(\cdot)$. For every value of λ^1, our analysis can be conducted in the same way. Moreover, we would obtain a one-to-one mapping of the distribution of abilities into the distribution of human capital among adults in the next period.

2.2 Family interaction, monitoring and enforcement

We outline how decisions are taken and who controls the output yielded by his or her own work, deferring a precise description of the form of the overall game to Section 3. Whether the parties behave cooperatively or not, it is assumed that a social rule governs the distribution of the total consumption of the aggregate good within the family, c, with the juvenile receiving the fixed fraction β of the adult's consumption level: $c \equiv c^1 + c^2$, $c^1 = \beta c^2$. In the non-cooperative settings, c is financed out of the adult's output and any transfers, voluntary or otherwise, she obtains from her son. The direct costs of education are likewise financed, so that the juvenile cannot finance any education directly from his earnings, if any.

In the 'decentralized' scheme, the adult makes no attempt to control how much the juvenile works or the level of his schooling, beyond vetoing any level not to her liking. The juvenile decides how he will spend his time, subject to his mother's willingness to finance his choice of education (e^1), and how much he will voluntarily contribute to the main budget (η) out of his earnings from working on his own account (w^1), if any, reserving the rest as pocket money (m^1). An addition to m^1, the adult may grant her son some pocket money (m^2) out of the main family budget.

The 'hierarchical' decision scheme differs from the decentralized scheme only in that the adult has some means of directly imposing her choice of education (e^2) and the level of his supervised work (w^2), whose proceeds go wholly to the main budget, if she regards the juvenile's choice of e^1 and the voluntary transfer η as too low, though the associated costs of monitoring and enforcement will be charged to the main budget (see Section 2.3). In these non-cooperative settings, work and pocket money, like c, comprise two elements: $w \equiv w^1 + w^2$ and $m \equiv m^1 + m^2$. Where l and η are concerned, no ambiguity arises, so the superscript is omitted.

Nash bargaining occurs only if both parties are willing to pool their resources and negotiate. Henceforth, these three schemes, namely, decentralized, hierarchical and Nash bargaining, are denoted by D, H and B, respectively.

Monitoring and enforcement are intrinsic to H, and the associated costs weigh heavily in the adult's choice between resorting to such compulsion and leaving her son largely to his own devices in D, should they fail to reach agreement. In fact, parents face great challenges in getting their adolescent children to participate in some of the activities the parents favor, like studying or, perhaps, working. Controlling the children's use of television and the internet is just as challenging (Wight et al., 2009). For juveniles

typically have a quite different view of how to spend time well. Studies show that teenagers are happiest when they are partying, or just spending time together at leisure (Larson, 1998).

A parent's ability to enforce a certain level of 'good' behavior depends on at least three factors. First, there is the time spent on monitoring – seeing that the children do their homework, attending school events, holding regular discussions with their teachers, and simply taking care that they go to bed at a reasonable time. Second, successful enforcement involves discipline, which makes demands on the parent's emotional energies. These exertions to enforce the desired level and quality of schooling are costly. Third, the same applies, in general, to supervised work (w^2); but these costs depend heavily on whether the family runs a business to which juveniles can make a useful contribution. In a farming family, for instance, juveniles can certainly do so, and monitoring their efforts and appropriating the resulting output are practically costless. If they work outside the business, however, these costs are likely to be substantial, except in a rather closed village community.

These considerations underlie the assumptions that enforcing education and work is costly and that the juvenile's earnings from supervised work can be appropriated in full, but that he may allocate his residual time, $1 - e - w^2$, between leisure and work on his own account (w^1), and spend any resulting earnings, as he sees fit.

2.3 Budgets

When the parties behave non-cooperatively, there are two separate expenditure-income identities. The juvenile's is

$$\eta + m^1 = w^1 \lambda^1. \tag{4}$$

The adult's is

$$c + \sigma e + m^2 + k(e^2, w^2) = \lambda^2 + w^2 \lambda^1 + \eta, \tag{5}$$

where σ is the direct cost of a full-time education, e is the actual level thereof, however chosen, and $k(e^2, w^2)$ are the combined costs of enforcing (e^2, w^2) against the juvenile's will, should the adult resort to compulsion. In writing these budget constraints as strict equalities, we have ruled out free disposal of income, for example, by making grants to other families. To do otherwise would introduce another dimension into the parties' action spaces, further complicating the analysis of the non-cooperative games.

In a cooperative outcome, no enforcement is necessary, all resources are pooled, and

lump-sum transfers are effected wholly through m. Hence, there is a single family budget constraint where the aggregate good is concerned:

$$c + \sigma e + m \leq w\lambda^1 + \lambda^2 \equiv y(w). \tag{6}$$

3   The Household's Problem

The parties have to choose an allocation $(c^1, c^2, e, l, w^1, w^2, m^1, m^2, \eta)$. This means a tussle, for their interests are partly opposed. The outcome must respect the associated budget constraint(s) and the juvenile's time budget. As these hold with equality and $c^1 = \beta c^2$ under the social sharing rule, there are effectively five degrees of freedom in the non-cooperative schemes.

We assume that the adult is free to choose between D and H, since she can decide whether she wants to incur the costs of monitoring and enforcing the level of the juvenile's schooling and contribution to the main budget.[4] The household's entire decision process can therefore be depicted by the following overall game structure.

Stage 1: The adult decides among {D, H, B}.

Stage 2: If she chooses B, bargaining will take place. Otherwise her choice of either D or H will be executed.

Stage 3: If bargaining fails to yield agreement, she chooses between D and H and her choice will be executed.

In principle, D, H, and B are themselves games. Accordingly, the overall game can be viewed as the decision process in which the parties choose the particular game they want to play in order to solve the household's allocation problem.

By backward induction, the overall game is solved by both the parent and the juvenile calculating the outcomes of D and H. The outcome that the adult prefers is the threat point in the bargaining game, as she is free to choose between the non-cooperative games in the event that bargaining fails. Support for such a structure is provided by theories of disagreement, as discussed in section 6.1. It is then possible to determine the outcome of the cooperative game. Once the solutions to D, H, and B have been determined, one can characterize the subgame perfect equilibria of the overall game. For simplicity, we add

---

[4]Section 6.1 sketches the converse case, wherein the choice lies with the juvenile.

Assumption 4

If the adult is indifferent between B and one of D and H, she will go with the latter.

The reason for this assumption is that, in principle, the adult could always choose B at stage 1 even if she already knew that bargaining would fail. For she can still decide between D or H at the last stage and bargaining is assumed not to involve any costs. Introducing a minimal cost of bargaining would have the same effect as assumption 4. We now have the following lemma:

Lemma 1

Under assumption 4, the adult's first-stage choice will be executed in every subgame perfect equilibrium.

Proof. By assumption 4, the adult will choose B at the first stage only if she prefers B to D or H and the juvenile accepts that outcome. In this case, B will be implemented. In all other cases, the adult's first-stage choice will be executed by definition. □

Before describing and solving the sub-games D, H and B, we state the following result, which is an immediate consequence of the assumption that the adult does not value 'fun'.

Lemma 2

If the adult has unfettered control over the expenditures in (5), but the juvenile chooses how he will spend his time and the level of the voluntary transfer η no later than the adult decides how to spend the budget, then in all non-cooperative arrangements with no repetitions of play, the adult will grant no allowances.

The consequences for the juvenile's allocation of time depend on the forms of $U^i(\cdot)$ and $\zeta(\cdot)$. Suppose, for example, that although the juvenile is keen on fun, he spends little time having any. The reason must lie in a sufficiently strong degree of complementarity between pocket money, which must come out his own earnings, and leisure in the production of fun. With substantial substitutability, however, he may find it attractive to spend time in leisure without much pocket money. Whether, and if so how much, the juvenile works depends on, *inter alia*, how the adult will respond to her resulting larger budget, which will allocated to consumption (distributed according to the social rule) and education.

3.1 The 'decentralized' scheme

The adult chooses $w^2 = 0$ and has full control over (c, m^2), with a veto over e^1, which she will exercise if the juvenile's choice thereof would involve her in an expense that she is not prepared to bear. The juvenile is free to manage his own time (e^1, l, w^1), subject to the veto over e^1, and he may allocate any earnings between pocket money and the voluntary transfer as he sees fit. This is not quite complete as a description of the parties' respective action spaces. The voluntary transfer increases the total amount that the adult can allocate to all forms of expenditure, whereby her veto may come into play. Thus, the actual outcome where e is concerned will be the minimum of what the juvenile wants and what the adult is prepared to finance. The adult's desired level of e depends, moreover, on the size of the transfer η. Thus, each party's set of feasible actions depends on the other's actual action.

This may lead to a problem. Suppose both players move simultaneously: the adult chooses $w^2 = 0$ and maximizes over (c, m^2) given (e^1, l, w^1, m^1, η), and the juvenile maximizes over (e^1, l, w^1, m^1, η) given $(c, m^2, w^2 = 0)$. Scrutiny of the juvenile's decision problem reveals that, if $(c, m^2, 0)$ is given, making a transfer will bring him nothing, so that he would spend his earnings only on having fun. As the game is one of common knowledge, however, he knows that η influences the adult's behavior. By Lemma 2, he knows that she will choose $m^2 = 0$, and so will respond to a larger transfer, given $e = e^1$, by increasing c. Under the assumption that both players' rationality is common knowledge, this implies that the juvenile knows that if he holds e^1 constant and makes a larger transfer at the expense of his pocket money (m^1), or by working a bit more, then (c, m^2) would change, which contradicts the hypothesis that the juvenile takes $(c, m^2, 0)$ as given when choosing (e^1, l, w^1, m^1, η).

The problem of determining e is solved as follows. The juvenile moves first, taking into account the possibility of a veto. The adult moves second, imposing a veto if the juvenile's choice of e^1 exceeds her desired level of education given η, which is denoted by $e^2(\eta)$, and accepting e^1 otherwise. These possibilities are recognized by both parties before play commences.

To be precise, $(c(\eta), e^2(\eta))$ solves the problem

$$\max_{(c, e^2 | \eta, m^2 = w^2 = 0)} U^2(c^2, \lambda^2_{+1}(e^2)) \quad \text{s.t. (5)}, c \geq 0, e^2 \in [0, 1]. \tag{7}$$

where $k = 0$. Given the above assumptions, $(c(\eta), e^2(\eta))$ is unique and $c(\eta) > 0$. Inspection of (5) with $m^2 = 0$ reveals that the upper frontier of the feasible set in the

11

space of (c^2, λ^2_{+1}) shifts parallel to the right as η increases. The assumption that all goods are normal then ensures that λ^2_{+1}, and hence also $e^2(\eta)$, is increasing in η so long as $e^2(\eta) < e^1$. At the first stage, therefore, the juvenile's decision problem is

$$
\max_{(e^1, l, w^1, m^1, \eta | m^2 = w^2 = 0)} U^1(c^1, \lambda^2_{+1}(e^1), \zeta) \\
\text{s. t. } (1), (4), c^1 = \beta c(\eta)/(1+\beta), e^1 \leq e^2(\eta), (e^1, l, w) \geq \mathbf{0}. \tag{8}
$$

If $e^1 < e^2(\eta)$, the adult accepts e^1 and chooses

$$
(c, m^2) = (\lambda^2 + \eta - \sigma e^1, 0). \tag{9}
$$

Since $U^1(\cdot), \zeta(\cdot)$ and $\Gamma(\cdot)$ are all concave functions and the constraints define a convex feasible set, this problem has a unique solution, and hence also does this non-cooperative game. To complete the characterization of the solution, let $e^1 < 1, w^1 > 0$ and $\eta > 0$, so that the corresponding first-order condition is

$$
-\frac{\partial U^1}{\partial c^1} \cdot \frac{\beta(\lambda^1 + \sigma)}{1+\beta} + \frac{\partial U^1}{\partial \lambda^2_{+1}} \cdot \frac{\partial \lambda^2_{+1}}{\partial e} \leq 0, \ e^1 \geq 0, \text{ compl.} \tag{10}
$$

This may be expressed more intuitively as

$$
-MRS^1_{1,2} \equiv \frac{\partial U^1/\partial c^1}{\partial U^1/\partial \lambda^2_{+1}} \geq \frac{1+\beta}{\beta} \cdot \frac{\partial \lambda^2_{+1}/\partial e}{\lambda^1 + \sigma}, \ e^1 \geq 0, \text{ compl.} \tag{11}
$$

where $MRS^1_{1,2}$ is the juvenile's marginal rate of substitution between c^1 and λ^2_{+1}. On the RHS of the first weak inequality in equation (11), the marginal yield of schooling (in terms of human capital) is divided by its combined direct and opportunity costs, with an adjustment for the parties' claims on the adult's budget where consumption is concerned.

If $e^2(\eta) < e^1$, where e^1 solves problem (8) without the restriction $e^1 \leq e^2(\eta)$, the adult can implement $e^2(\eta)$; so that $c = c(\eta), e = e^2(\eta)$. With the first stage in mind, we now introduce

Assumption 5
$e^2(\eta)$ is a concave function of $\eta \ \forall e^2(\eta) < e^1$.

Together with the earlier assumptions, assumption 5 ensures that the solution of this problem, and hence the solution of this variant of D, is unique.

In what follows, the solution of D will be denoted by $A^d = (c^d, m^{1d}, m^{2d}, e^d, l^d, w^{1d}, w^{2d}, \eta^d)$, where $m^{2d} = w^{2d} = 0$.

3.2 The hierarchical scheme

If the adult is dissatisfied with A^d, she has the option of resorting to compulsion, which is potentially attractive whenever she desires more education than her son and his voluntary transfer is small. The drawback is that monitoring and enforcing involve charges on her budget.

The scheme H differs from D mainly in that the adult has the possibility of fixing the level of education[5] and compulsory work, w^2, at the first stage, before the juvenile decides on the allocation of the remainder of his time between leisure and voluntary work and the level of η at the second stage. Finally, at the last stage, the adult splits her budget net of direct outlays on education and enforcement between consumption and the allowance. By lemma 2, $m^2 = 0$. Working backwards, we obtain, from (5),

$$(c, m^2) = (w^2\lambda^1 + \lambda^2 + \eta - \sigma e^2 - k(e^2, w^2), 0). \qquad (12)$$

Knowing as much, the juvenile solves the following problem at the second stage:

$$\max_{(l, w^1, m^1, \eta | e^2, w^2, m^2 = 0)} U^1(c^1, \lambda^2_{+1}(e^2), \zeta) \quad \text{s.t. (1), (4), (12), } (l, w^1, m^1, \eta) \geq \mathbf{0}. \qquad (13)$$

The solution of this problem is denoted by leisure $l(e^2, w^2)$ and voluntary work $w^1(e^2, w^2)$, with the proceeds of the latter, if any, split between own pocket money $m^1(e^2, w^2)$ and the transfer $\eta(e^2, w^2)$. The adult's first-stage problem is, noting that she will choose $m^2 = 0$ at the last stage,

$$\max_{(e^2, w^2)} U^2(c^2, \lambda^2_{+1}(e^2)) \quad \text{s.t. } c = \lambda^2 + \eta(e^2, w^2) - (\sigma e^2 + k(e^2, w^2)), \ e^2 + w^2 \leq 1. \qquad (14)$$

We impose restrictions on the juvenile's preferences and ζ such that the following holds:

Assumption 6
$\eta(e^2, w^2)$ *is concave.*[6]

Turning to the monitoring and enforcement technology, in view of the fact that the juvenile goes to school voluntarily under D, we assume, *inter alia*, that total and marginal enforcement costs at $(e^d, 0)$ are zero.

---

[5]It is assumed that parents are unable to take out loans in order to finance education.
[6]The details of the associated conditions on U^1 and ζ are available upon request.

Assumption 7

Let $k(e^2, w^2) = k_e(e^2) + k_w(w^2)$, where k_e and k_w are increasing, convex and twice-differentiable functions $\forall e^2 > e^d$ and $\forall w^2 > 0$, respectively, with $k_e(e^d) = k_w(0) = 0$. Also, $k'_e(e^d) = k'_w(0) = 0$.[7]

It follows from assumption 6 and the convexity of $k(e^2, w^2)$ that problem (14) possesses a unique solution. The associated allocation is $A^h = (c^h, m^{1h}, m^{2h}, e^h, l^h, w^{1h}, w^{2h}, \eta^h)$. Since the aggregate good is essential in consumption, $c^h > 0$; and by lemma 2, $m^{2h} = 0$. Observe that if $\Gamma(\lambda^2, e)$ satisfies the lower Inada condition with respect to e, then $e^h > 0$ is assured, but not $e^h > e^d$.

Of particular interest is whether the adult chooses $e^h > e^d$ or $w^{2h} > 0$; for otherwise she opts, in effect, for D, in which enforcement is unnecessary. The first-order conditions associated with problem (14), which are sufficient as well as necessary, are

$$\frac{\partial U^2}{\partial c^2} \cdot \left(\frac{\partial \eta / \partial e^2 - \sigma - k'_e}{1 + \beta} \right) + \frac{\partial U^2}{\partial \lambda^2_+} \cdot \frac{\partial \lambda^2_+}{\partial e} - \nu^2 \leq 0, \ e^2 \geq 0, \ \text{compl.} \tag{15}$$

$$\frac{\partial U^2}{\partial c^2} \cdot \left(\frac{\lambda^1 + \partial \eta / \partial w^2 - k'_w}{1 + \beta} \right) - \nu^2 \leq 0, \ w^2 \geq 0, \ \text{compl.} \tag{16}$$

where ν^2 is the multiplier associated with $e^2 + w^2 \leq 1$. If, at the optimum, the juvenile's time is completely controlled ($e^{2h} + w^{2h} = 1$), the adult will prefer H to D, except in the unlikely event that $e^d = 1$ and $w^{2h} = 0$, when the allocations are the same ($A^h = A^d$). Suppose, therefore, that $e^2 + w^2 < 1$, so that $\nu^2 = 0$. Since the adult can choose D ($e^2 = e^d, w^2 = 0$), she will prefer H if

$$\frac{\partial U^2}{\partial c^2} \cdot \left(\frac{\partial \eta / \partial e^2 - \sigma}{1 + \beta} \right) + \frac{\partial U^2}{\partial \lambda^2_+} \cdot \frac{\partial \lambda^2_+}{\partial e} > 0,$$

where all terms are evaluated at ($e^2 = e^d, w^2 = 0$). Whether this holds depends not only on her preferences and the education and enforcement technologies, but also on the juvenile's preferences, which influence $\eta(e^2, w^2)$. Analogously to the juvenile's decision problem (8) under D, this condition may be expressed as

$$-MRS^2_{1,2} > (1 + \beta) \cdot \frac{\partial \lambda^2_{+1} / \partial e}{-\partial \eta / \partial e^2 + \sigma}, \tag{17}$$

---

[7]Additive separability is not unduly restrictive at $(e^d, 0)$, whereby $k'_e(e^d) = k'_w(0) = 0$ is a convenient simplification in what follows. This is seen by taking a Taylor expansion at $(e^d, 0)$ of the unrestricted function $k(e^2, w^2)$. It is somewhat restrictive when a good deal of compulsion is chosen; for then there may be complementarities in controlling what is happening in both spheres. So long as $k(e^2, w^2)$ is an increasing function of both arguments, however, no difficulties will arise.

a comparison of which with (11) reveals that, for any given e, the adjusted ratio of marginal pay-off to marginal cost is not necessarily more favorable here than under D. The adult may, of course, have much stronger tastes for her son's future capital than he does, which makes H potentially attractive. Even if condition (17) does not hold, the adult still has the option of compelling him to work ($w^2 > 0$). From (16), noting that $k'_w(0) = 0$, it is seen that she will indeed exercise this option if

$$\lambda^1 + \partial\eta(e^d, 0)/\partial w^2 > 0. \tag{18}$$

Since the choice ($e^2 \leq e^d, w^2 = 0$) involves no costs of enforcement, it follows that if neither condition (17) nor condition (18) holds, then the adult will prefer D to H. Thus we have established

Proposition 1

The adult will prefer H to D if either (17) or (18) holds. Otherwise, she will prefer D.

3.3 The pareto-inefficiency of D and H

Non-cooperative games often yield pareto-inefficient outcomes, and since the threat point in the cooperative game arises from D or H as alternatives, it is important to establish whether the latter are inefficient in this sense. That H is indeed so follows at once from the fact that the adult could allocate the expenditures on control to pocket money or consumption instead, given that e^h stays constant. That D is, in general, likewise follows from the fact that education produces a public good within the family, and as a mechanism for deciding on its provision, D is almost certain not to yield allocations that satisfy the Samuelson condition; for each party's private calculus does not fully reflect the other's valuation. Under the social rule, moreover, c has a quasi-public element, too. Stated formally, all pareto-efficient allocations arise as solutions of the following problem:

$$\max_{(c, e, l, m, w)} U^1 \quad \text{s.t. (1), (6)}, \ U^2 \geq \bar{U}^2, \tag{19}$$

where $\bar{U}^2$ can vary parametrically over some feasible interval. The terms involving the derivatives of U^2 in the associated first-order conditions make no appearance in their counterparts in D. There are, however, some exceptional cases, in which efficiency can hold when there are corner solutions in connection with the allocation of the juvenile's time. There can then be unanimity, despite non-cooperative behavior. To summarize:

Proposition 2

All allocations under D are pareto-inefficient under the assumptions set out above, with the following exceptions:

(i) *Unanimity on full-time child labor $(w^{1d} = 1, e^2(\lambda^1) = 0)$ is pareto-optimal.*

(ii) *Unanimity on full-time education $(e^d = e^2(0) = 1)$ is pareto-optimal unless leisure and pocket money are sufficiently poor substitutes in producing fun.*

(iii) *If the juvenile just lounges about $(l^d = 1)$ and the adult desires some education $(e^2(0) > 0)$, then the additional assumption that the $|MRTS|$ between leisure and pocket money in producing fun at $(l, w) = (1, 0)$ is smaller than λ^1 is strongly sufficient to ensure that this outcome is pareto-efficient.*

All allocations under H, which necessarily involves compulsion, are pareto-inefficient.

Proof: see Appendix.

3.4 The Nash bargaining solution

The third possibility is that the parties bargain. The distinction between the direct costs of education and the time actually spent in the classroom is a potentially important one where the description of the game is concerned. If agreement can be reached on e, the adult commits herself to pay the corresponding fees and the juvenile not to play truant once the fees have been paid, with each taking the other's commitment as credible. No monitoring is necessary, so that the set of feasible allocations is $S = \{s \in \mathbb{R}^5_+ \,|\, s \text{ satisfies } (1), (6)\}$, with the typical element $s = (c, m, e, l, w)$. Then

$$X = \{(x^1, x^2) \in \mathbb{R}^2 \,|\, (x^1, x^2) = (U^1(s), U^2(s)), s \in S\}$$

is the set of feasible utility pairs.[8]

Since the adult may choose between D and H in the event that the negotiations fail, her choice will determine the threat point. Rational as she is, she will choose that which yields the higher level of utility; for in the absence of any means to pre-commit herself to one or the other for the purposes of negotiation, this will be her best course of action should there be no agreement. Denote by $U^{2d} \equiv U^2(A^d)$ and $U^{2h} \equiv U^2(A^h)$

---

[8]Since the utility and production functions are concave, X is convex.

her corresponding utility levels. Then the threat point is

$$\xi = (x^{1\xi}, x^{2\xi}) = \begin{cases} (U^1(A^d), U^2(A^d)), & U^{2d} \geq U^{2h} \\ (U^1(A^h), U^2(A^h)), & U^{2d} < U^{2h}. \end{cases}$$

Definition 1

(X, ξ) *represents the parties' bargaining problem.*

The Nash-solution to (X, ξ) is $(U^1(s^n), U^2(s^n))$, where

$$s^n = \arg\max_{s \in S} \, (U^1(s) - x^{1\xi})(U^2(s) - x^{2\xi}), \quad \text{s.t.} \ (U^1(s), U^2(s)) \geq \xi.$$

Let the parties' respective gains from cooperation be denoted by $U^{i+} \equiv U^i(s) - x^{i\xi}$. With the exceptions noted in proposition 2, both will be strictly positive.

We now turn to characterization. The first-order conditions, whose solution is s^n, are

$$\left(\beta \cdot \frac{\partial U^1}{\partial c^1} \cdot U^{2+} + \frac{\partial U^2}{\partial c^2} \cdot U^{1+} \right) - \mu(1 + \beta) \leq 0, \ c \geq 0 \tag{20}$$

$$\frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial m} \cdot U^{2+} - \mu \leq 0, \ m \geq 0 \tag{21}$$

$$\left(\frac{\partial U^1}{\partial \lambda_{+1}^2} \cdot U^{2+} + \frac{\partial U^2}{\partial \lambda_{+1}^2} \cdot U^{1+} \right) \cdot \frac{\partial \lambda_{+1}^2}{\partial e} - \mu\sigma - \nu \leq 0, \ e \geq 0 \tag{22}$$

$$\frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial l} \cdot U^{2+} - \nu \leq 0, \ l \geq 0 \tag{23}$$

$$\mu\lambda^1 - \nu \leq 0, \ w \geq 0 \tag{24}$$

where μ and ν are the Lagrange multipliers associated with (6) and (1), respectively.[9] Since U^i $(i = 1, 2)$ is increasing in c, it follows that $\mu > 0$ at s^n; for an increase in $y(w)$ can always be allocated to consumption of the aggregate good. For the same reason, $\nu > 0$ at s^n, since an increase in the time available to the juvenile increases full income $\lambda^1 + \lambda^2$, and so expands X. That the aggregate good is essential in consumption implies that $c^n > 0$, so that the first part of (20) holds as an equality at s^n. Hence, the shadow price of family income is

$$\mu = \left(\beta \cdot \frac{\partial U^1}{\partial c^1} \cdot U^{2+} + \frac{\partial U^2}{\partial c^2} \cdot U^{1+} \right) / (1 + \beta).$$

---

[9]There are also pairs of complementary inequalities corresponding to the corner solutions involving $e = 1, l = 1$ and $w = 1$, whose form is evident, and so omitted to save space.

It is the weighted sum of the parties' marginal utilities of consumption at s^n, where their respective weights are the products of the share of each in c and the other party's gain from cooperation.

Corner solutions for the remaining variables cannot be ruled out without further assumptions. If a small investment in formal education at $e = 0$ produces a large enough improvement in the juvenile's human capital, then $e^n > 0$ and the first part of (22) will hold as an equality at s^n when $e^n < 1$. Now suppose that $l^n > 0$, so that ν follows from the first part of (23). Recalling from assumption 2 that pocket money is of no use unless $l > 0$, consider $(l^n, m^n) > \mathbf{0}$. It then follows from (21) and (23) that $|MRTS_{lm}| = \nu/\mu$, so that if $(\partial\lambda^2_{+1}/\partial e)_{e=0}$ is sufficiently large, the first part of (22) may be written

$$\left(\frac{\partial U^1}{\partial \lambda^2_{+1}} \cdot U^{2+} + \frac{\partial U^2}{\partial \lambda^2_{+1}} \cdot U^{1+}\right) \cdot \frac{\partial\lambda^2_{+1}}{\partial e} - \mu(\sigma + |MRTS_{lm}|) = 0.$$

As a final step, let $w^n > 0$, so that there is a full interior solution, $s^n > \mathbf{0}$. It follows from (24) that $\nu = \mu\lambda^1$ and hence that $|MRTS_{lm}| = \lambda^1$, the opportunity cost of the juvenile's time in terms of the aggregate good. Substituting for $|MRTS_{lm}|$, we obtain

$$\left(\frac{\partial U^1}{\partial \lambda^2_{+1}} \cdot U^{2+} + \frac{\partial U^2}{\partial \lambda^2_{+1}} \cdot U^{1+}\right) \cdot \frac{\partial\lambda^2_{+1}}{\partial e} - \mu(\sigma + \lambda^1) = 0.$$

This condition states that the full cost of a (small) unit of education, when multiplied by the shadow price of family income, be equal to the weighted sum of the marginal utilities of human capital induced by that unit of education.

To complete the characterization of a full interior solution, (23) and (24) yield

$$\frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial l} \cdot U^{2+} = \lambda^1\mu = \nu.$$

This reveals that the change in the juvenile's utility brought about by a little more leisure time in the production of fun, when multiplied by the adult's gain from cooperation, be equal to the shadow price of family income times the amount of the aggregate good forgone by working that much less.

3.5 Solution of the overall game

From Lemma 1, the adult's first-stage choice will rule in a subgame perfect equilibrium. To determine this choice, one must solve the overall game by backward induction. The first step is to determine the adult's threat point in B. Proposition 1 supplies the answer. If she chooses H, the juvenile will be worse off ($U^{1h} < U^{1d}$). For in D, he is free to choose e^1, may keep all of his earnings, and does not have to share in the costs of enforcing $e^h > e^1$ or $w^2 > 0$. That $U^{2h} > U^{2d}$ implies $U^{1h} < U^{1d}$ has implications for the distribution of the gains from cooperation. If the adult chooses H as her threat in the event of a disagreement, she will also do better in the cooperative game than if she were to choose D – a threat that, in any event, is not credible in view of $U^{2h} > U^{2d}$.

As both players possess perfect information, they know the outcomes of D, H and B; in particular, the adult knows whether the juvenile would accept B. We can now state our main result:

Proposition 3
(i) The subgame perfect equilibrium of the overall game is statically pareto-efficient.
(ii) H will never be implemented, D almost never.
(iii) The subgame perfect equilibrium of the overall game may yield a lower level of human capital formation than H.

Proof: see appendix. The intuition runs as follows. Parts (i) and (ii) follow from the fact that if H and D are inefficient, the adult will choose B; for this will yield gains to both parties. It is also pareto-efficient. As emphasized in part (iii), however, static efficiency does not necessarily yield the greatest possible long-term gains, which are generated by education. This result stems from the fact that while the adult can force the juvenile to go to school or to work, while appropriating the resulting earnings, the juvenile can use any residual time as he sees fit. He can, moreover, keep all his earnings from 'voluntary' work as pocket money. In B, he can offer to work and contribute more at the expense of his schooling. If the adult's preferences for consumption relative to his future human capital are sufficiently strong, and the costs of enforcing work are sufficiently high, she will accept this offer.

4   Regulation

Thus far, all decisions have been made free of any legal restrictions on how much juveniles may work or the levels of their schooling. In affluent countries, such work

is fairly strictly regulated, and compulsory education is likewise strongly enforced. In developing countries, there are moves in this direction, so it is interesting to examine the effects of such restrictions, severally and jointly. In view of part (ii) of proposition 3, we confine the discussion to the case where B is the outcome of the overall game.

Three general points should be made at the outset. First, if a regulation binds in the subgame perfect equilibrium of the overall game, its imposition will have caused a contraction of the feasible set S in some neighborhood containing the allocation s^n, with attendant effects on X. There will also be substitution, in the form of a reallocation of the remainder of the juvenile's time, again in equilibrium. Second, if the regulation binds in equilibrium, it will almost certainly do so in the non-cooperative games. The regulation may, indeed, cause the adult to switch from one to the other. Third, even if the regulation does not bind in equilibrium, it may still do so in the non-cooperative games, and hence affect the outcome in equilibrium. To sum up, a binding regulation will affect the final outcome indirectly as well as directly, and a regulation that does not bind in equilibrium may still affect the allocation in equilibrium.

4.1 The Labor Market

Denote by $\bar{w}$ the legal upper limit of a juvenile's working time. This will affect the outcome under D if and only if $e^d + l^d > 1 - \bar{w}$. Since $w^{1d} > \bar{w}$, the juvenile must be worse off; for he has the option of choosing $\bar{w}$ in the absence of regulation.[10] With lower earnings at his disposal, he may reduce the contribution η, if any, which will make the adult worse off. With more time on his hands, however, he may also desire more schooling, which the adult will welcome. If leisure and pocket money are good substitutes in producing fun, he is likely to spend more time lounging about, while reducing η only modestly. If they are rather poor substitutes, he will cut back heavily on fun in favor of schooling. These considerations indicate that the regulation will damage him more than the adult, with corresponding effects on the outcome of the overall game when D without a parental veto is the threat point.

If the adult does exercise her veto in the absence of regulation, its introduction will make her worse off, *cet. par.*, if the juvenile reduces η, which is certain unless l and m are perfect substitutes. The adult will also continue to impose a veto; for the *income effect* of the regulation induces a reduction in $e^2(\eta)$, so as to moderate the fall in c that would otherwise result from the reduction in η, and the adult has no direct control

---

[10]This is proved formally by introducing the constraint $w^1 \leq \bar{w}$ into problem (8) and applying the envelope theorem. The same applies to related claims made below.

over the juvenile's earnings. His leisure is simply the residual $1 - e^2(\eta) - \bar{w}$, which compounds the damage wrought by the regulation, in that he has no other margin at which to make an adjustment. In this variant of D also, therefore, the adult probably comes out relatively better.

Turning to H, let the authorities be in a position to observe both w^1 and w^2, and suppose that if the regulation binds ($w^1 + w^2 = \bar{w}$), the adult has first claim on $\bar{w}$, so that problem (14) must be augmented by the restriction $w^2 \leq \bar{w}$. If the latter binds at the optimum, then $w^1 = 0$ and hence η also. The adult is worse off, though the lower costs of enforcing w^2 will ameliorate matters somewhat. The juvenile is now certainly worse off, *cet. par.*, if $w^{1h} > 0$; but the adult may respond by easing up where schooling is concerned, which the juvenile would favor. As before, the tightening of the adult's budget will work through the *income effect* to reduce e^2; for both goods are normal and a reduction in e^2 will further lower outlays on enforcement, and so moderate the fall in c. A regulation such that $w^2 \leq \bar{w}$ binds also produces a *substitution effect*: the juvenile spends the rest of his time $(1 - e^2 - \bar{w})$ just hanging about without pocket money, whereas without the restriction, he may have spent some time working on his own account. Thirdly, there is a direct *enforcement cost effect* with respect to education. This effect results from changes in the juvenile's voluntary school attendance in response to the restriction on working. This effect can go either way.

There remains the possibility that the regulation directly affects only the juvenile: $w^2 < \bar{w} < w^1 + w^2$. The adult is also worse off if η falls, and will then respond by changing (e^2, w^2) – where it will be recalled that problem (14) is augmented by $w^2 \leq \bar{w}$. Hence, whether education increases or decreases under H as a result of labor regulation depends on the particular strength of the said three effects.

So much for the effect of the regulation on the threat point. In B, the set of feasible allocations in the presence of the regulation is denoted by

$$S(\bar{w}) = \{s \in \mathbb{R}^5_+ | \ s \text{ satisfies } (1), (6), w \leq \bar{w}\},$$

and the associated outcome by $s^n(\bar{w})$. If the regulation binds in the overall game, whose outcome almost always involves $s^n(\bar{w})$, the juvenile's combined time at school and play, $e^n(\bar{w}) + l^n(\bar{w})$, must exceed its level when there is no regulation. The income effect points to a reduction in education, and hence to an increase in leisure; but the changes in the disagreement pay-offs imply substitution effects, which may pull in the opposite direction, especially if the regulation weakens the juvenile's bargaining position. To examine this more closely, observe that (24) now reads $\mu\lambda^1 - \nu \geq 0$, $w \leq \bar{w}$; so that,

21

at a full interior solution with respect to e and l,

$$\left(\frac{\partial U^1}{\partial \lambda^2_{+1}} \cdot U^{2+} + \frac{\partial U^2}{\partial \lambda^2_{+1}} \cdot U^{1+} \right) \cdot \frac{\partial \lambda^2_{+1}}{\partial e} \leq \mu(\sigma + \lambda^1),$$

which holds with equality only when the regulation does not bind. Although Γ is concave, $e^n(\bar{w}) > e^n$ cannot be ruled out because the product of the parties' gains from cooperation must be smaller when the regulation binds.

Let D constitute the threat point in B, regulation or no. It has been argued above that, in D, a binding regulation will be more damaging to the juvenile than the adult, so that in B, she will be in a position to extract some concessions. Since she is keener on education when she imposes no veto, and he will have more time at his disposal, the concession should take the form of more education $(e^n(\bar{w}) > e^n)$ and less pocket money to help finance it. If, on the contrary, she vetoes his choice, he must value education more; and since she is likely to be damaged less by the regulation, he will have to forgo some education or pocket money by way of concessions – more likely, pocket money if it is a good substitute for leisure.

If, instead, the threat point is H, and the regulation is not so stringent as to limit the adult's choice of w^2 directly, the juvenile will certainly get the short end of the stick, relatively speaking. Here, too, the changes in the bargaining outcome will involve more education and less pocket money. The same conclusion will hold when the regulation is so strong as to limit the adult's choice of w^2, but not too heavily. A complete ban on juvenile labor $(\bar{w} = 0)$ will bind in all subgames. If $e^h < 1$, $e^n(\bar{w})$ may exceed e^h; for with $w^h = w^n = 0$ and given the fixed sharing rule for consumption, the only other adjustments in the adult's favor after the distribution of the savings in enforcement costs is for the juvenile to accept more schooling and receive less pocket money.

4.2   Schooling

The regulation is interesting only if the juvenile has some spare time to devote to leisure or work. For otherwise the family would have no latitude of any kind, family income being fixed, at $y(0)$, none of it allocated to pocket money, there being no leisure, and the residual after meeting the direct costs of education and enforcement, if any, going to consumption. Hence, let the compulsory level of education be fixed at $\underline{e} < 1$, with families enjoying the discretion to choose more if they wish.

Who bears the costs of enforcing $\underline{e}$, should this be necessary? At one extreme, the

authorities could charge parents with this task, with the threat of severe sanctions if they fail. D is then no longer an option if $e^d < \underline{e}$, in which event, the adult must make (unwilling) outlays of $k_e(\underline{e})$ on enforcing schooling. The regulation has no effect on the allocation under H if $e^h \geq \underline{e}$; but if $e^h < \underline{e}$, complying with it will entail additional outlays on enforcement. Hence, if $e^d < \underline{e}$ or $e^h < \underline{e}$, the juvenile now solves

$$\max_{(l,\, w^1,\, m^1,\, \eta\, |\, e\, =\, \underline{e},\, w^2)} U^1(c^1, \lambda^2_{+1}(e^1), \zeta) \quad \text{s.t. (1), (4), (5), } m^2 = 0,\ (l, w^1) \geq \mathbf{0}.$$

The following marginal condition will hold at an interior optimum:

$$-MRS^1_{1,3} \equiv \frac{\partial U^1/\partial c^1}{\partial U^1/\partial \zeta} = \frac{(1 + \beta) \cdot \partial \zeta/\partial l}{\lambda^1}\,. \tag{25}$$

At the other extreme, the school authorities could enforce at least $\underline{e}$,[11] with the family bearing any additional costs under H if the adult chooses $e^h > \underline{e}$. In its unrestricted form, D is no longer an option if $e^d < \underline{e}$; and in that event, D is played out with $e = \underline{e}$, the costs of its enforcement falling on the public purse. As for H, the fact that the authorities bear enforcement costs in the amount $k_e(\underline{e})$ makes education yet more attractive to the adult, so that e^h must be derived accordingly. Problem (14) is augmented by the restriction $e \geq \underline{e}$, with the charge on the family's main budget now amounting to $k_e(e) - k_e(\underline{e})$. Let the adult's optimal choice be denoted by $e^h(\underline{e})$. Since the public expenditure $k_e(\underline{e})$ is effectively a lump-sum subsidy, the income effect may result over-compliance ($e^h(\underline{e}) > \underline{e}$), even if, in the absence of the regulation, $e^h < \underline{e}$. The remaining possibility is bare compliance ($e^h(\underline{e}) = \underline{e}$). If $e^d < e^h$ in the absence of regulation, it follows that the non-cooperative game may be either effectively D ($w^2 = 0$) or H ($w^2 > 0$), but with $e = \underline{e}$ and no enforcement costs where education is concerned. In all variations, the juvenile allocates his remaining time, if any, between work on his own account and leisure, and chooses η. Consumption is then determined as a residual after meeting the direct costs of education, which are non-discretionary. The remainder of the solution is characterized by (25).

If, in the absence of regulation, the adult chooses D and desires $e^2(\eta) > e^1$, she will welcome the school authorities' efforts to enforce $\underline{e}$, even if the family has to bear all the costs, provided $\underline{e}$ does not go too far beyond $e^2(\eta)$. The juvenile will certainly be worse off. Thus, the adult's bargaining position will be strengthened, and this will normally lead to a higher level of education than that ruling in the absence of regulation, even if $e^n > \underline{e}$. The same holds if, in the absence of regulation, the adult chooses D and

---

[11]For simplicity, we ignore the fact that such public expenditures must be financed by taxation.

imposes a veto. For the regulation will now very likely damage the adult more than the juvenile, who, in this configuration, places a higher value on education, and so strengthens his hand in B. If, instead, the adult chooses H in the absence of regulation, with $e^h < \underline{e}$, and the authorities bear the costs of enforcing $\underline{e}$, the adult still may be better off with the regulation. The same holds for the juvenile, who will also benefit in the form of higher consumption, but the net effect is still likely to be a strengthening of the adult's bargaining position in B. These effects on the threat point are can be at work even when the regulation does not bind in B itself.

It should be remarked that compulsory education will not overturn proposition 2. If D is chosen in the absence of regulation, with $e^2(\eta) > e^1$, and the school authorities bear the costs of enforcing $\underline{e}$ $(> e^1)$, D with $e^d = \underline{e}$ will hold under the regulation. If, however, H is chosen in the absence of regulation, with $e^h < \underline{e}$, the regulation will not necessarily result in D with $e = \underline{e}$; for $e^h(\underline{e}) > \underline{e}$ is compatible with $e^h < \underline{e}$. Compulsory education rules out full-time work and is irrelevant if there is unanimity on full-time education. The only remaining possibility involves $l^d = 1$, but $\underline{e} > 0$ rules that out.

4.3   Joint regulation

The level of compulsory education may be such that, despite the restrictions on juvenile labour, both bind in one or both of D and H. In that event, the juvenile's leisure time will be the residual $1 - \underline{e} - \bar{w}$, and his only remaining choice will be the allocation of his earnings $w^1 \lambda^1$, if any, between m^1 and η. If the adult chooses $w^2 = \bar{w}$, even that choice will vanish, and with enforcement costs of $k_e(\underline{e}) + k_w(\bar{w})$, consumption will also be a residual. What is there left to bargain over in B when both regulations bind? The answer is m, and if $l = 1 - \underline{e} - \bar{w} > 0$ and the threat point is pareto-inefficient, the juvenile will indeed enjoy some pocket money in the subgame perfect equilibrium; but if $\underline{e} + \bar{w} = 1$, pocket money will be useless.

4.4   Summary and discussion

A novel feature of our analysis is that juveniles desire time in order to have 'fun'.[12]

---

[12]In section 6 we take up the possibility that there is "bad" as well as "good" fun, depending on how fun activities affect the utility of parents. As long as good fun and bad fun balance out in the population, the qualitative conclusions concerning regulation still hold. Recalling footnote (3), good and bad fun will induce heterogeneity of human capital in the next generation which we do not explore in this paper.

Where regulation is concerned, this adds effects that are not present in the previous literature on family decision-making and economic development, in which it is often emphasized that the negative income effect of an upper limit on child labor may lead to less schooling. By introducing the juvenile's decision problem, we identify additional effects of limiting child labor, in particular, the substitution effect and the enforcement-cost effect. As established above, and as the example in the next section illustrates, these effects are important when evaluating a ban on child labor and compulsory schooling as measures to promote human capital formation.

The principle of targeting tells us that it is almost always optimal to attack a distortion as close to its source as possible. If private decisions yield levels of schooling that are deemed socially to be too low and, as here, the capital market plays no role, it is therefore tempting to conclude that the right form of intervention is to enforce the desired level of compulsory education. Yet it is important to note that although families almost always arrive at a cooperative solution, changes in regulation can also affect the threat point. Longer compulsory schooling will increase actual schooling if families are currently choosing less; but by influencing the threat point in parents' favor, the more stringent regulation may increase schooling even more strongly. Indeed, it may lead to an increase in schooling even when families' earlier choices exceeded the new minimum.

What role is left for child labor laws? None, if leisure is ruled out. In practice, however, adolescents find various ways of spending time in leisure, so that families have another margin to work with in response to changes in regulation. We have established that tighter restrictions on child labor alone will have ambiguous effects on the level of schooling in D and H. What counts, however, is the finding that in B, which almost always rules in the overall game and in which the juvenile normally enjoys some leisure, such restrictions may lead to more schooling, whereby their indirect effects on the threat point play an important role. As for their direct effects, any contraction in the set of feasible allocations that favors the adult will result in the juvenile sacrificing some fun by forgoing some leisure – this being the only margin left to him when he may not work as much as he would like – and some pocket money.

The above arguments ignore possible differences in the state's costs of enforcing the two forms of regulation, which must be financed by taxation in some form or other. The most effective way to foster human capital formation may therefore involve the joint regulation of child labor and education, whereby the costs of administration, subsidies and the taxes to pay for them must be integrated into the analysis.

5 A Numerical Example

To reduce the burden of computation, we employ a special form of the model, in which the adult can easily appropriate *all* the juvenile's earnings, but finds it prohibitively costly to control how he uses *any* time that remains after schooling, so that he has at least one degree of freedom in the non-cooperative games. Under these assumptions, therefore, $w = w^1$ and $\eta = w^1\lambda^1$. The juvenile has full control over all his time in D, subject to the possibility of a veto, and full control over his discretionary time $1 - e^2$ in H. This corresponds to settings like closely knit communities in which families know a great deal about one another's doings, but adolescents also enjoy some autonomy in recognition of their contributions and approaching adulthood.

The functional forms for the production of human capital, the costs of enforcing education and the production of fun are as follows:

$$
\begin{aligned}
\lambda_{+1}^2 &= z\lambda^2 e^\phi + 1, \\
k_e(e^2) &= \begin{cases} A(e^2 - e^d)^\kappa, & \text{if } e^2 > e^d, \\ 0, & \text{otherwise,} \end{cases} \\
\zeta(l, m) &= B\left[ql^{-\rho} + (1-q)m^{-\rho} \right]^{-\frac{1}{\rho}}.
\end{aligned}
$$

The parties' preferences are given by

$$
U^1 = (c^1)^{a^1} \left\{ \left[\omega^1 \zeta^{-\gamma} + (1-\omega^1)(\lambda_{+1}^2)^{-\gamma} \right]^{-\frac{1}{\gamma}} \right\}^{(1-a^1)} \quad \text{and} \quad U^2 = (c^2)^{a^2}(\lambda_{+1}^2)^{(1-a^2)}.
$$

The following parameter values characterize our 'standard scenario'. In particular, note that the juvenile has stronger tastes for consumption than the adult ($a^1 > a^2$).[13]

a^1	a^2	ω^1	γ	β	z	ϕ	A	κ	q	ρ	B	σ	λ^1	λ^2
$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0.6	0.6	$\frac{1}{2}$	1	1.2	0.6	$-\frac{1}{2}$	2	1	1	4

Table 1: Parameter values in the standard scenario

The allocations in the three decision schemes are given in Table 2. Enjoying his freedom in D, the juvenile spends about two-thirds of his time in leisure and splits the rest almost equally between education and work. The adult, however, prefers H, in which, at close to one-half, e is much higher. This comes at the expense of the juvenile's time at play

---

[13]The form of U^1 violates assumption 2, $\zeta(0, m) > 0 \; \forall m > 0$. In this section, however, all allocations involve $l > 0$, so that the said condition plays no role.

and, to a lesser extent, at work. Enforcing this level of schooling claims 0.2301 of the aggregate good; together with the partial loss of the juvenile's earnings, the sacrifice of consumption is substantial. In B, the juvenile agrees to a sharp reduction in his leisure time and an even bigger increase in time at work in exchange for some pocket money and a little less schooling than the rigors of e^h. He is still worse off than in D, were that available.

Scheme	e	$k_e(e^2)$	w	l	m	c^2	U^2	U^1
D	0.1664	0	0.1749	0.6587	0	2.5053	2.2267	1.3537
H	0.4603	0.2301	0.1006	0.4391	0	2.1314	2.3669	1.2492
B	0.4069	0	0.4253	0.1679	0.0745	2.4649	2.4976	1.3380

Table 2: Outcomes in the standard scenario

Labor regulation takes the form of a complete ban ($\bar{w} = 0$). In D, the juvenile devotes the time he would otherwise have spent working almost wholly to leisure, and the adult does not veto the minimal increase in e^1. She, however, continues to prefer H, despite the loss of his earnings. Indeed, she steps up e^h, with somewhat higher associated enforcement costs of 0.2794. Recalling section 4, this increase is the net outcome of changes in the opportunity costs of enforcement and the direct income effect, which by itself would lower e^h, but is more than offset by the former. That the substitution effect is very large – under D, the juvenile would spend almost all of his additional time in leisure – is particularly relevant in this example. Higher enforcement costs and stronger tastes for consumption on the adult's part may result in a dominant income effect and thus reduce e^h. To illustrate, let $A = 1.5$, $\kappa = 1.1$, and $a^2 = \frac{2}{3}$: these yield $e^h = 0.2232$ without regulation and $e^h = 0.2213$ with $\bar{w} = 0$.

Scheme	e	$k_e(e^2)$	w	l	m	c^2	U^2	U^1
D	0.1691	0	0	0.8309	0	2.3943	2.1811	1.3497
H	0.5147	0.2794	0	0.4854	0	2.0037	2.3353	1.2197
B	0.5513	0	0	0.4487	0.0340	2.1342	2.4366	1.3096

Table 3: Outcomes in the standard scenario with a ban on child labor

The outcome of the overall game is, as before, the allocation in B. Unable to work, the only way for the juvenile to make his mother better off is to agree to more schooling: e^n now exceeds e^h, and both are larger than their counterparts in the absence of regulation. The reduction in consumption is substantial, and with so much leisure time at his disposal, the juvenile makes do with less pocket money than he would receive without the ban. Both parties are worse off under the regulation.

Turning to compulsory schooling, let this stipulate the minimum level $\underline{e} = 0.5$. Table 4 reports the outcomes both when there is direct enforcement of $\underline{e}$ by the state and when parents have to pay all enforcement costs. Note that when $\underline{e}$ exceeds the level chosen in D and enforcement has to be provided by the adult, the imposition of the regulation on the latter is identical to its imposition on H when $e^h < \underline{e}$. In this particular constellation, e exceeds the required minimum only in H when the state enforces $\underline{e}$. This accords with intuition, since part of the adult's enforcement costs are then fully subsidized. Comparing these outcomes with those without regulation, compulsory education favors the adult, whose ranking of D and H is left unchanged. The juvenile is worse off, except under H when $\underline{e}$ is enforced by the state. The adult's tastes for consumption are sufficiently strong that the income effect of the subsidy dampens the attractions of education so much that the juvenile, whose tastes for consumption are stronger still, is more than compensated. The actual outcome of the overall game is B under both arrangements for bearing the costs of enforcement. In this constellation, this particular regulation is welcomed by the adult, but not by the juvenile.

Scheme	e	$k_e(e^2)$	w	l	m	c^2	U^2	U^1
	$\underline{e}$ enforced by the state							
D	0.5	0	0.0409	0.4591	0	2.2131	2.4431	1.2940
H	0.5370	0.0191	0.0243	0.4378	0	2.1666	2.4455	1.2780
B	0.5	0	0.3301	0.1699	0.0755	2.3466	2.5157	1.3172
	$\underline{e}$ enforced by parents							
H	0.5	0.2678	0.0857	0.4144	0	2.0752	2.3658	1.2290
B	0.5	0	0.3336	0.1664	0.0730	2.3498	2.5174	1.3163

Table 4: Outcomes in the standard scenario with compulsory schooling ($e \geq \underline{e}$)

As argued in Section 4.4, no comparison of the two forms of regulation is complete without a full specification of the associated costs of implementing them. For ignoring such costs, one could simply set $\underline{e} = 1$, if the aim be to promote human capital formation to the fullest – though the old saying that 'all work and no play makes Jack a dull boy' suggests that a slightly milder regime is desirable. It is still interesting, however, to examine the outcomes when the above regulations are jointly implemented and the state enforces $\underline{e} = 0.5$ directly, which are set out in Table 5. Joint regulation promotes education still further, as the juvenile cannot evade education by working. He is, morever, even worse off; and his mother would prefer to have compulsory education alone. Note that if parents have to bear all enforcement costs, the outcomes would be the same as in H and B in the example with a complete ban ($\bar{w} = 0$) only.[14] The

---

[14]Again, D effectively drops out, because parents have to enforce $\underline{e}$.

reason is that under the ban, the adult will enforce education in excess of $\underline{e}$ voluntarily.

scheme	e	$k_e(e^2)$	w	l	m	c^2	U^2	U^1
D	0.5	0	0	0.5	0	2.1875	2.4180	1.2937
H	0.6105	0.0711	0	0.3895	0	2.0730	2.4410	1.2422
B	0.6360	0	0	0.3630	0.0201	2.0899	2.4678	1.2761

Table 5: Outcomes in the standard scenario with joint regulation ($\bar{w} = 0$, $\underline{e} = 0.5$)

In developing economies, regulation may affect fertility. The present structure can, in principle, be extended to incorporate such effects when parents rationally anticipate all the problems that will arise in the course of rearing their children to adulthood, which we discuss briefly in Section 6.2. There is no space to pursue this extension here, but one can vary the number of juveniles (n^1) parametrically to gain some insight into how changes in fertility affect decisions later in the life-cycle.[15] Let $U^2(c^2, \lambda_{+1}^2; n^1) = (c^2)^{a^2} \cdot (n^1 \lambda_{+1}^2)^{(1-a^2)}$, so that the factor n^1 merely scales the adult's utility function, and so does not affect the values of the maximizers.

The findings are as follows.[16] The adult's utility increases with n^1, implying maximal fertility. The converse holds if n^1 does not enter directly into U^2, since each extra juvenile then makes a claim on the main budget without increasing the weight on λ_{+1}^2. At some intermediate value of the elasticity of substitution between n^1 and λ_{+1}^2 (the quantity-quality trade-off), therefore, there will be an interior optimum where n^1 is concerned. In all settings, a ban on child labor and compulsory schooling of $\underline{e} \geq 0.5$ to be enforced by parents make the adult worse off, but do not alter the behavior of U^2 in relation to n^1. Whether this finding is robust to more flexible formulations of the adult's preferences over quantity and quality remains to be determined.

6 Discussion and Extensions

Certain aspects of our model merit further discussion. We also take up some extensions and examine how they modify the findings obtained thus far.

---

[15]The model in our working paper (Bell et al., 2010) has two adults and n^1 juveniles.

[16]Detailed results are available upon request.

6.1 The treatment of disagreement

Ours is a structure in which the players' relationship continues whatever happens, but they can choose further actions if they fail to agree. Hence, the threats they make when bargaining are endogenous.[17] Our model is therefore related to the general theory of cooperative bargaining, in particular to the recent contribution of Miller and Watson (2010), who outline a theory of repeated games with two dimensions in which relationships continue even if there is disagreement.

That parents rather than juveniles choose which non-cooperative game is played in the event of a disagreement is arguably the more natural arrangement of the two, but several of the main results also hold under the alternative. For the choice of the public good λ_{+1}^2 is still left to one party. If, in fact, the juvenile has the choice between D and H, intuition suggests that he will always prefer the former, since he is then subjected to no compulsion. This claim is proved as follows. Noting the outcome in H, namely, A^h, the juvenile may certainly choose $e^1 = e^h, w^1 = w^{1h} + w^{2h}$ and $\eta^1 = \eta^h + w^{2h}\lambda^1$ in D. These choices yield the same levels of λ_{+1}^2 and ζ as in H, but more consumption, since there are no costs of enforcement. Hence, given the option, the juvenile strictly prefers D to H. Recalling propositions 2 and 3, it is seen that the fact that H never comes into play in this variant somewhat improves the chances that there will be unanimity, but also rules out part (iii) of proposition 3.

6.2 Dynamics

An interesting extension would be to split the single period into two sub-periods, namely, 'early childhood' and 'school-age'. In keeping with the literature on early childhood development (e.g., Heckman, 2008), let parents invest in the cognitive and non-cognitive skills of their young children, for example, by spending more time with them in diverse activities. For simplicity, such early childhood parenting can be represented as forgone earnings, which constitute a charge on the adult's budget. In the second sub-period, the children become autonomous agents and the game would then proceed as described above. The literature suggests that there is a dynamic complementarity in human capital formation: the return to education increases with the child's cognitive and non-cognitive skills on entering school. If consumption in one sub-period is a fairly good substitute for the other, investment in early parenting will

---

[17]The abstract theory of how to incorporate players' actions in the event of a disagreement into the Nash bargaining solution is presented and surveyed in chapter 7.3 of Houba and Ball (2002). Applications in the field of innovation and licensing can be found in Scotchmer (2004).

make the juvenile's subsequent contributions, voluntary or otherwise, to the adult's budget more valuable. The fact that motivation, sociability, the ability to concentrate on tasks, and self-discipline are fostered by good parenting in the early years of life also makes H more attractive, to the extent that enforcement costs are lower for a given level of education. Whether, in this extension, the final outcome involves more education or, which is a separate thing, the juvenile attaining more human capital in adulthood depends, therefore, on a complicated interplay of preferences and 'technology'.

The division of time into two sub-periods gives parents an additional degree of freedom to react to policy, even before its implementation. If, for example, the government announces in the first sub-period that parents must enforce a (binding) level of schooling in the next, they will have a stronger incentive to increase parenting in early childhood in order to lower enforcement costs; for this will also improve their position in the cooperative game. A ban on child labor can induce different reactions. If the income effect dominates, parents know that they will invest less in schooling in the next sub-period, which lowers their incentive to invest in enhancing their young children's skills in the present. The opposite holds if the substitution effect dominates and there is strong complementarity between parenting in early childhood and the returns to education.

To extend the time horizon still further, our model can be interpreted as a reduced form of an overlapping generations model in which individuals live for three periods, namely, youth, adulthood and retirement, whereby an individual's human capital in adulthood is the main determinant of the family's opportunity set. Suppose that in retirement, individuals rely on support from their children. How much support the children will extend depends on their capacity (human capital) and their willingness to do so. The latter may well depend on how generous their parents were with letting and enabling them to have fun in adolescence.[18]

6.3 Parents care about fun

Thus far, parents have viewed fun neutrally, as neither 'good' nor 'bad'. We now relax this assumption, so that the juvenile's fun becomes the second pure public good within the family. The adult may value it, perhaps in the expectation of higher transfers from her son in old age or simply out of pure altruism; but she would surely take a dim view of 'fun' if it involved risky behavior, such as taking drugs. The only formal differences

---

[18]Extending the model in this direction will leave the main insights intact. It should be added, however, that the additional complications will make the drawing of conclusions about the impact of policies more uncertain.

from the basic structure are that ζ now appears as an argument of U^2 and that the juvenile only produces fun, once $m\,(= m^1 + m^2)$ has been determined by the adult's choice of m^2, which then fixes the level of ζ. What follows is largely a summary of how this extension affects the results obtained under the assumption that adults view fun neutrally, most technical details being consigned to Appendix C.

An immediate implication is that lemma 2 does not hold, in general, if the adult views fun as 'good'; for granting pocket money then yields her a pay-off, even in the non-cooperative games. In both D and H, the adult chooses c and m^2 in the last stage, now with the restriction $m^2 \geq 0$. Since m^1 is given at this stage, the first-order conditions reveal that the optimal amount of m^2 is governed by the condition

$$\frac{\partial U^2}{\partial \zeta}\frac{\partial \zeta}{\partial m} \leq \frac{1}{1+\beta}\frac{\partial U^2}{\partial c^2}\,,\ m^2 \geq 0,\ \text{compl.} \tag{26}$$

or, more intuitively,

$$-MRS^2_{1,3} \equiv \frac{\partial U^2/\partial c^2}{\partial U^2/\partial \zeta} \leq (1+\beta)\frac{\partial \zeta}{\partial m}\,,\ m^2 \geq 0,\ \text{compl.} \tag{27}$$

Granting a unit of pocket money costs the adult $1/(1+\beta)$ units of consumption under the social rule and yields $\partial \zeta/\partial m$ units of fun. If, at $m^2 = 0$, this relative price exceeds the adult's corresponding $|MRS|$, she will indeed grant no pocket money. For her to do so, she must place a sufficiently high value on fun. If, on the contrary, she views fun with dismay ($\partial U^2/\partial \zeta < 0$), she will certainly try to limit it by setting $m^2 = 0$.

A more fundamental point concerns the adult's ranking of D and H, and if she chooses H, with what rigor. She is more likely, *cet. par.*, to prefer D to H when she values her son's fun, and conversely when she shudders to think of it. For whereas D involves no enforcement costs, these may seem well worth bearing when limiting his discretionary time may keep keep him on the straight and narrow. Introducing fun into her preferences therefore has potentially important effects on the threat point in B, and hence on the outcome of the overall game. No substantial modifications to proposition 2 are entailed. H is inherently inefficient, owing to the associated deadweight loss; and D may be efficient when there is unanimity. There are now, however, two pure public goods, λ^2_{+1} and ζ, as opposed to one, which will lower the chances of attaining an efficient allocation when their provision is the outcome of non-cooperative behavior.

This brings us, once more, to B, whose outcome will almost surely rule in the overall game. The juvenile will welcome a switch from the straitjacket of H to the comparative

freedom of D; but that does not necessarily translate into a stronger bargaining position, because the switch, if it occurs, stems from the fact that his mother now values his fun – as opposed to viewing it neutrally. Conversely, he will fret if he is subjected to the rigors of H because she now takes a dim view of his fun; but she will have to bear much of the cost of imposing H. An examination of the counterparts to the first-order conditions (21) and (23) reveals that, in B, fun may well be greater at the expense of education and, perhaps, work when the adult also values fun.

If having fun involves risky behavior, the government could undertake counter-measures, such as stronger policing, tougher laws, and employing social workers, which would increase the costs of having such fun. An increase in expenditures on such measures, g, would plausibly reduce the marginal productivities of leisure and pocket money, so inducing juveniles to substitute work or education for leisure. If, in D, no veto is exercised and e^1 increases, the adult will be better off; and in H, enforcement costs for any given e^2 will fall. Hence, these measures will increase the adult's bargaining power in B, and e^n and w^n, which the adult favors, will increase with g.

It is also plausible that risky fun has damaging effects on the formation of human capital. Prime examples are having to spend some time in jail or attempting to do school assignments under the influence of drugs. Such effects will manifest themselves as the appearance of ζ in the function $\Gamma(\cdot)$. If the adult's tastes for her son's human capital are stronger than his, she will welcome increases in g all the more, with the same effects on e^n and w^n as above.[19]

6.4 Influencing the private costs of enforcing schooling

Governments can attempt to to promote education by lowering parents' enforcement costs. One way of doing so is to encourage better parenting in early childhood. There are also interventions aimed at school-age children, such as supporting parent-teacher associations, improving school management, providing educational programs for parents, and raising teaching standards. Such policies are not, in general, equivalent to a stringent level of compulsory education, because their effectiveness depends on parents' willingness to invest in education. Parents with strong preferences for consumption relative to education may make only scant additional use of the resulting

---

[19]If in the population some juveniles incline to good fun and others to bad fun, the design of public policies depends on which type of juveniles dominates. If the government has limited knowledge about the distribution of types, the corresponding policy problem is more difficult and the outcome more prone to error.

enforcement possibilities under H, and even if they do respond, it does not follow that a high minimum level will be attained in B. As emphasized above, lower enforcement costs influence the threat point; but this does not ensure that $e^n > e^h$.

This point can be illustrated using the example in Section 5. An exogenous reduction in enforcement costs can be effected by reducing the parameter A, from 1 to 0.5, say. In the standard scenario, this yields $e^h = 0.545$ instead of $e^h = 0.460$. This level also exceeds that in the standard scenario when the state enforces $\underline{e} = 0.5$, namely, $e^h = 0.537$. In B with $A = 0.5$, however, $e^n = 0.428$, which is higher than the level realized with $A = 1$ ($e^n = 0.407$), but falls far short of the stipulated minimum of 0.5. The postulated decrease in enforcement costs is even less effective if the adult's tastes for consumption are stronger. Let $a^2 = a^1 = 2/3$. Then the drop in A from 1 to 0.5 will increase enforced education in H from $e^h = 0.267$ to $e^h = 0.311$; and in B, the outcome will be $e^n = 0.241$ instead of $e^n = 0.236$. These results suggest that there may be strict limits to what can be accomplished by indirect measures, even if parents have fairly strong preferences for education. Hence, if a certain minimum level of education is desired, it should be enforced directly, in keeping with the principle of targeting.[20]

One further possibility involves inter-generational effects. Lundberg et al. (2009) find that the children of college-educated mothers are more likely to make shared decisions with their parents than to go their own way. (This could be interpreted as λ^2 entering $k_e(\cdot)$.) The upshot is that better educated parents will exert tighter control over their children's behavior because they face lower enforcement costs. In particular, they are more able to help their children with their school tasks, to keep them up to the mark, and to assess their progress. Better-educated parents are also presumably wealthier and so can better afford after-school activities or tutorials. A direct implication is that, *cet. par.*, children in families with well educated parents are more likely to receive more education than the compulsory minimum, making due allowance for the fact that the final outcome will arise from bargaining. Over the long run, one result will be increasing inequality in the distribution of human capital and incomes.[21]

---

[20]See e.g. Bell and Gersbach (2009), Dessy (2000), and Hazan and Berdugo (2002) on the social desirability of compulsory schooling in developing countries.

[21]On the relationship between initial wealth distribution, human capital accumulation and ensuing income inequalities see Galor and Zeira (1993) and the discussion of this literature in Gersbach and Siemers (2010).

7 Concluding remarks

How juveniles split their time among schooling, work and play involves a clash of interests with their parents; for the parties almost surely have different preferences, whatever be the common bonds of family. The structure developed and analyzed here places this clash at the center of the family's allocation problem, which includes choosing how much to produce of the household good we have called 'fun'. Subject only to the social 'rule' that juveniles receive a fixed fraction of an adult's consumption of the aggregate good, the cooperative solution is always pareto-efficient viewed statically; but under the 'hierarchical' arrangement, in which parents devote resources to enforce their desired level of schooling, juveniles may receive more education.

The hierarchical scheme is always pareto-inefficient, and if the decentralized scheme is likewise, the outcome will be the bargaining solution. A change in the enforcement technologies affects the threat points of the cooperative game, but not the upper frontier of the bargaining set. Regulation in the form of restrictions on child labor or compulsory schooling, in contrast, can affect both. A ban on child labor may reduce schooling in the non-cooperative schemes, but as the only means then left to the juveniles is to agree to more education at the expense of their leisure, the ban may lead to an increase in education in the bargaining solution, which will actually rule. Indeed, such a ban may well be more effective in promoting education than a modest level of compulsory schooling. For if juveniles are not effectively restricted in how much they may work, they can bargain for less schooling by offering to work more, with an outcome that may still comply with the minimum level of schooling. This indicates that, in general, joint regulation is called for in order to foster fuller education. Establishing the best form of regulation, however, requires that the costs of administration, subsidies and the taxes to pay for them must be integrated into the analysis. That is a task for another paper.

Appendix

A   Proof of Proposition 2

That H is inefficient is established in the text. Where parts (i)-(iii) are concerned, we begin with allocations in which no veto is exercised.

Case (a): $0 < e^1 < e^2(\eta)$. If, at the juvenile's optimum, $l > 0$, he can reduce it slightly in favor of e^1. This has only a second-order effect on U^1, but yields a first-order improvement in U^2. If $l = 0$, then $w^1 = \eta/\lambda^1 > 0$. A small increase in e^1 at the expense of w^1 will have only a second-order effect on U^1, but it will yield a first-order improvement in U^2 by virtue of $e^2(\eta) > e^1$, which implies that the adult is willing to sacrifice some current consumption in favor of education at $e = e^1$.

Case (b): $0 < e^1 = e^2 < 1$. Despite the agreement about e, a small increase in w^1 at the expense of l will have a second-order effect on U^1, while yielding a first-order improvement in U^2.

Case (c): $0 = e^1 < e^2$. If, at the juvenile's optimum, $0 < l < 1$, a small increase in w^1 with some part of the earnings used to increase η will have only a second-order effect on U^1, but will yield a first-order improvement in U^2. If $l = 1$, it suffices to show that $e^1 = w^1 = m^1 = \eta = 0$ does not solve problem (19) when $\bar{U}^2 = U^2\left(\lambda^2/(1+\beta), \lambda^2_{+1}(0)\right)$. Writing down the first-order conditions in the form of complementary inequalities and employing the hypothesis $l = 1, m^1 = w^1 = 0$, a little manipulation yields

$$\frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta(1,0)}{\partial l} > \nu > \mu\lambda^1 > \lambda^1 \cdot \frac{\partial U^1}{\partial \zeta} \cdot \frac{\partial \zeta(1,0)}{\partial m}.$$

Hence, if the |MRTS| between leisure and pocket money in producing fun at (1,0) is, in fact, smaller than λ^1, we have a contradiction. Since lounging about the whole day without any pocket money is not especially attractive, the condition $|\text{MRTS}|_{l=1,m=0} < \lambda^1$ does not seem very restrictive. Be that as it may, we have established part (iii).

Case (d): $e^1 = e^2 = 1$. With $l = w^1 = m^1 = \eta = 0$ and both parties preferring λ^2_{+1} to consumption at the margin defined by $e = 1$, there is no way of improving one party's position without worsening the other's unless the juvenile enjoys some fun, which involves $l > 0$, and the adult enjoys more consumption, which involves $w^1 > 0$ to finance $\eta > 0$, both to compensate for the corresponding reduction in e^1. Since, by hypothesis, the juvenile chooses $l = w^1 = m^1 = 0$, the solution to problem (19) when $\bar{U}^2 = U^2\left(\lambda^2/(1+\beta), \lambda^2_{+1}(1)\right)$ involves $m > 0$ only if l and m are sufficiently poor substitutes. Otherwise, $e^1 = e^2 = 1$ is pareto-optimal. This establishes part (ii).

We turn to the remaining allocations, in which the adult imposes $e = e^2(\eta)$ at the second stage and hence a (binding) constraint on the juvenile's choices at the first.

Case (e): $e^1 > e^2(\eta) > 0$. A small increase in e^2 at the expense of c will have only a second-order effect on U^2; but, by the envelope theorem, it will yield a first-order improvement in U^1, with corresponding optimal adjustments in (l, w^1, m^1, η).

Case (f): $e^2(\eta) = 0$. If $1 > l > 0$, a small reduction in l in favor of w^1 with some part of the earnings used to increase η will have a second-order effect on U^1, while yielding a first-order improvement in U^2. If $l = 1$ and $e^1 > 0$, devoting a little time to work under the condition that the whole of the proceeds go to financing education would make the juvenile better off at no cost to the adult. Such a transfer is ruled out in D, however, since the adult decides on the allocation of her budget. Finally, in the extreme case $w^1 = 1$, which implies $\eta = \lambda^1$, the allocation is pareto-optimal. For $l = 0$ yields the largest feasible set from the adult's point of view, and if she desires $e^2(\lambda^1) = 0$, both cannot possibly do better under any other arrangement; so that $e^1 = e^2 = 0$ solves problem (19) when $\bar{U}^2 = U^2\left((\lambda^1 + \lambda^2)/(1 + \beta), \lambda^2_{+1}(0)\right)$. This establishes part (i). $\qquad\square$

B   Proof of Proposition 3

(i) If the threat point is efficient, it will be implemented; for there is no possibility of making one party better off without making the other worse off. If, on the other hand, the threat point is inefficient, both parties will gain from cooperation; for transfers of the aggregate good are possible and perfectly divisible. Hence, B will always be implemented in such a situation. As the Nash solution is pareto-optimal by definition and will fail to be implemented only in situations in which the alternative non-cooperative game yields an efficient outcome, the subgame perfect equilibrium of the overall game must be pareto-optimal from a static perspective.

(ii) Since H is not pareto-optimal, the adult never chooses it at the first stage of the overall game, as she will be better off in B. The same applies, almost always, to D.

(iii) Let the constellation of preferences and technologies be such that the first part of (15) holds as an equality with $e^h = 1$, where $\partial\eta/\partial e^2$ is evaluated as $e^2 \to 1$ from below. If the juvenile has strong tastes for consumption, he will also cut back on leisure (without pocket money), if he had chosen any, as he is forced to undergo more schooling when e^h is close to 1. Hence, $\partial\eta/\partial e^2 \geq -\lambda^1$; and if $m^1 = 0$, $\partial\eta/\partial e^2 = 0$. In such an allocation, the adult would also accept a very small increase in consumption, financed by the juvenile working a little bit on his own account at the expense of his education.

From (22) and (24), it is seen that the corresponding first-order condition under B is

$$\Omega \equiv \frac{\beta \cdot U^{2+} \cdot \partial U^1/\partial c^1 + U^{1+} \cdot \partial U^2/\partial c^2}{U^{2+} \cdot \partial U^1/\partial \lambda_{+1}^2 + U^{1+} \cdot \partial U^2/\partial \lambda_{+1}^2} \leq (1 + \beta) \cdot \frac{\partial \lambda_{+1}^2/\partial e}{\lambda^1 + \sigma} \, , \ e \leq 1, \ \text{compl.}$$

Observe that $\Omega > -MRS_{1,2}^2$ iff $-\beta \cdot MRS_{1,2}^1 > -MRS_{1,2}^2$. Substituting from (15) under the said hypothesis about the allocation under H, we have

$$\Omega \leq -MRS_{1,2}^2(A^h(e^h = 1)) \cdot \frac{(-\partial \eta/\partial e^2 + \sigma + k')_{e=1}}{\lambda^1 + \sigma} \cdot \frac{\partial \lambda_{+1}^2/\partial e}{(\partial \lambda_{+1}^2/\partial e)_{e=1}} \, , e \leq 1, \ \text{compl.}$$

Now suppose also that $e^n = 1$, so that $l^n = w^n = m^n = 0$. Since $k_e(1) > 0$, $c^n > c^h$ and hence $-MRS_{1,2}^2(s^n(e^n = 1)) < -MRS_{1,2}^2(A^h(e^h = 1))$, though the difference will be small if $k(1)$ is sufficiently small. If the juvenile has sufficiently strong tastes for consumption $(-\beta \cdot MRS_{1,2}^1 > -MRS_{1,2}^2)$, $\Omega|_{e=1} > -MRS_{1,2}^2(s^n(e^n = 1))$. Inspection of the RHS of the above weak inequality reveals, however, that if $k(1)$ and $k'(1)$ are sufficiently small, and $\partial \eta/\partial e^2$ exceeds $-\lambda^1$ by a sufficient margin, this will contradict $\Omega|_{e=1} > -MRS_{1,2}^2(s^n(e^n = 1))$.

By continuity, the argument will also go through if e^h is sufficiently close to 1, or if, at $e^h = 1$, (15) holds as an inequality in reverse, provided the RHS exceeds $-MRS_{1,2}^2$ by a sufficiently small margin. $\qquad \square$

C Parents care about fun

In D, let $(c(e^1, l, m^1, \eta), m^2(e^1, l, m^1, \eta))$, which is implicitly defined by condition (26) and the adult's budget constraint (5), solve problem (7) so amended when $e^1 < e^2(\cdot)$. The juvenile notes this at the first stage when choosing (e^1, l, w^1, m^1, η), whereby $m^2(\cdot)$ can be interpreted as a sort of reward function that depends on his behavior. Since (e^1, m^1, w^1) is given at the last stage, the adult can respond to any change in m^1 by making an offsetting change in m^2, so preserving m and c – unless m^2 is zero and she desires to reduce it. Hence, if $m^2 > 0$, then $\eta = 0$ and $\partial m^2/\partial m^1 = -1$, in contrast to lemma 2 $(m^2 = 0)$. While fun now appears in U^2, the mechanism that determines the level of its provision under D does not, in general, satisfy the Samuelson condition.

That m^2 may be positive affects the juvenile's decision problem in various ways. If, rationally anticipating $m^2 > 0$, he also chooses to work, then $m^1 > 0$. The need for leisure implies $w^1 < 1$. The associated first-order conditions yield $\mu = \nu \lambda^1$, as before, and, together with $\partial m^2/\partial m^1 = -1$, $\nu = (\beta/(1 + \beta)) \cdot \partial U^1/\partial c^1$. Substituting into the

first-order condition with respect to education, this reads

$$\frac{\partial U^1}{\partial \lambda^2_{+1}} \frac{\partial \lambda^2_{+1}}{\partial e} \leq \frac{\beta(\sigma + \lambda^1)}{1 + \beta} \frac{\partial U^1}{\partial c^1} - \Delta U^1_{mc} \frac{\partial m^2}{\partial e^1} \ , \ e^1 \geq 0, \text{ compl. },$$ (28)

where the term

$$\Delta U^1_{mc} \equiv \frac{\partial U^1}{\partial \zeta} \frac{\partial \zeta}{\partial m} - \frac{\beta}{1 + \beta} \frac{\partial U^1}{\partial c^1}$$

is his net valuation of extra pocket money over the associated change in consumption.

Whether there will be less education when the adult values fun sufficiently to choose $m^2 > 0$ depends on three factors. First, there is the response of $|MRS^1_{1,2}|$ to changes in the arguments of U^1. Since $\eta = 0$ and m^1 and m^2 are perfect substitutes in the strategic sense when $m^2 > 0$, c will be smaller given that e^1 stays unchanged; and if consumption is not strongly complementary with human capital and fun in the juvenile's preferences, $|MRS^1_{1,2}|$ will rise. Second, if the juvenile is keen on fun, ΔU^1_{mc} will be positive; for he has only limited and indirect control over c, given that $\eta = 0$ is optimal, and none over $m \ (\equiv m^1 + m^2)$. Third, is $m^2(\cdot)$ decreasing in e^1? The answer depends on whether leisure and pocket money are good substitutes in producing fun. If they are, a reduction in leisure in favor of education can be compensated at low cost by an increase in pocket money. Otherwise, an increase in e^1 at the expense of l is likely to induce a fall in m^2. Inspection of (28) indicates that, on balance, these factors make for less education – and more fun – under D when the adult values fun.

For the parent, the attraction of H is getting closer control over her son's doings. What, then, is the point of choosing $m^2 > 0$ when m can be increased by the simple expedient of choosing a lower level of w^2 so as to leave him with more discretionary time, some or all of which can be devoted to earning on his own account, while also saving on enforcement costs? Relaxing w^2 will allow the juvenile to substitute leisure for work at the second stage, and so induce a corresponding rise in m^2 at the expense of c at the last stage. It is not clear, therefore, that $m^2 > 0$ can be ruled out under H.[22]

How the adult views fun affects the threat point, and hence, almost always, the outcome in the overall game. Suppose she prefers D to H when she takes a neutral view of fun. If, in a second, otherwise identical family, the adult values fun, differences between the two allocations under D can arise only if the adult exercises a veto in the first family but not in the second. Even so, the adult in the second family will be better off in relation to her son; for she places some value on his fun, and the two juveniles have the same preferences and consume the same bundle. If, on the contrary, the adult in

---

[22]If the adult could pre-commit to $m^2 > 0$ at the first stage, that would suffice.

the first family prefers H to D, the adult in the second will opt for a less stringent regime in H, or even choose D. Whichever holds, the juvenile in the second family will be better off than his counterpart in the first. Thus, the parties' bargaining strengths in the second family could go either way relative to their counterparts' in the first.

Where the characterization of the bargaining solution is concerned, the introduction of ζ into U^2 results in modifications to the first-order conditions with respect to leisure and pocket money. Thus, (21) and (23) become, respectively,

$$\left(\frac{\partial U^1}{\partial \zeta} \cdot U^{2+} + \frac{\partial U^2}{\partial \zeta} \cdot U^{1+} \right) \cdot \frac{\partial \zeta}{\partial m} - \mu \; \leq \; 0, \; m \geq 0, \text{ compl.} \tag{29}$$

$$\left(\frac{\partial U^1}{\partial \zeta} \cdot U^{2+} + \frac{\partial U^2}{\partial \zeta} \cdot U^{1+} \right) \cdot \frac{\partial \zeta}{\partial l} - \nu \; \leq \; 0, \; l \geq 0, \text{ compl.} \tag{30}$$

If the adult values fun, the additional terms are positive, so that the bargaining outcome likely entails more leisure and pocket money than when she views fun neutrally, and conversely, less of both when she thinks it damaging.

A binding restriction on child labor involves an income effect, which reduces the adult's willingness to finance education in D, and her chosen level in H. If she values fun, the substitution effect will not act as strongly in favor of an increase in e as it does when she views fun neutrally, because the opportunity cost of time in school is higher. Taken together, these effects make for less education than in the basic setting. The opposite holds if the adult is averse to fun activities.[23] When $\bar{w}$ binds in B, conditions (29) and (30) imply that the effect of the regulation on education will be less pronounced when the adult values fun and more so if she takes a dim view of it.

Compulsory schooling tends to damage the adult less than the juvenile in both D and H. In B, however, the level of education will exceed the prescribed minimum by a smaller margin if she values fun and the level agreed when she views fun neutrally exceeds the minimum. The converse will hold if she is averse to fun. The reason lies in the opportunity cost of time in school, which is higher and lower, respectively.

As for public measures to make risky forms of fun less attractive, inspection of conditions (29) and (30) reveals that, at the optimum, the weighted marginal benefit from fun increases with expenditures on such measures, provided the multipliers μ and ν fall, if at all, at most modestly, as is probable.

---

[23]Note that the enforcement-cost effect remains the same as when she views fun neutrally, except that now the choice under D may differ due to the possibility that $m^2 > 0$.

References

Barham, V., Boadway, R., Marchand, M., Pestieau, P., 1995. Education and the poverty trap. Europ. Econ. Rev. 39, 1257-1275.

Banerjee, A., 2004. Educational policy and the economics of the family. J. Dev. Econ. 74, 3-32.

Becker, G.S., 1974. A theory of social interactions. J. Polit. Econ. 82, 1095-1117.

Becker, G.S., 1981. A treatise on the family. Harvard University Press, Cambridge.

Bell, C., Gersbach, H., Schneider, M.T., 2010. Raising juveniles. IZA Discussion Paper No. 5036, Bonn.

Bell, C., Gersbach, H., 2009. Child labor and the education of a society', Macroecon. Dyn. 13, 220-249.

Bergstrom, T.C., 1989. A fresh look at the rotten kid theorem – and other household mysteries. J. Polit. Econ. 97, 1138-1159.

Cox, D., 1987. Motives for private income transfers. J. Polit. Econ. 95, 508-546.

Cox, D., Jakubson, G., 1995. The connection between public transfers and private interfamily transfers. J. Public Econ. 57, 129-167.

Cremer, H., Kessler, D., Pestieau, P., 1992. Intergenerational transfers and private interfamily transfers. Europ. Econ. Rev. 36, 1-16.

Dessy, S.E., 2000. A defense of compulsive measures against child labor. J. Dev. Econ. 62, 261-275.

Dijkstra, B.R., 2007. Samaritan vs. rotten kid: another look. J. Econ. Behav. Organ. 64, 91-110.

Galor, O., Zeira, J., 1993. Income distribution and macroeconomics. Rev. Econ. Stud. 60, 35-52.

Gersbach, H., Siemers, L., 2010. Land reforms and economic development. Macroecon. Dyn. 14, 527-547.

Heckman, J.J., 2008. Schools, skills, and synapses. Econ. Inq. 46, 289-324.

Houba, H., Ball, W., 2002. Credible threats in negotiations – a game-theoretic approach, in: Theory and Decision Library, Series C – Game Theory, Mathematical Programming and Operations Research 32. Kluwer, Boston.

Larson, R.W., 1998. Getting adolescents, families, and communities in sync: implications for policy and practice, in: Crouter, A. C., Larson, R. (Eds.), Temporal Rhythms in Adolescence: Clocks, Calendars and the Coordination of Daily Life. New Directions for Child and Adolescent Development 82. Jossey-Bass, San Francisco, pp. 83-88.

Loury, G.C., 1981. Intergenerational transfers and the distribution of earnings. Econometrica 49, 843-867.

Hazan, M., Berdugo, B., 2002. Child labor, fertility, and economic growth. Econ. J. 112, 810-828.

Lundberg, S., Romrich, J.L., Tsang, K.P., 2009. Decision making by children. Rev. Econ. of the Househ. 7, 1-30.

Miller, D.A., Watson, J., 2010. A theory of disagreement in repeated games with renegotiation. Mimeo, University of California, San Diego.

Neuwirth, N., and Haider, A., 2004. The economics of the family – a review of the development and a bibliography of recent papers. Austrian Institute for Family Studies Discussion Paper 45.

Pollak, R.A., 2007. Family bargaining and taxes: prolegomenon to the analysis of joint taxation. IZA Discussion Paper No. 3109. Forthcoming in, Taxation and the Family, M.I.T Press.

Scotchmer, S., 2004. Innovation and Incentives. M.I.T. Press, Cambridge.

Wight, V.R., Price, J., Bianchi, S.M., Hunt, B.R., 2009. The time use of teenagers. Soc. Sci. Res. 38, 792-809.