Financial Innovations, Money Demand, and the Welfare Cost of Inflation*

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Abstract

In the 1990s, the empirical relationship between money demand and interest rates began to fall apart. We analyze to what extent financial innovations can explain this breakdown. For this purpose, we construct a microfounded monetary model with a money market which provides insurance against liquidity shocks by offering short-term loans and by paying interest on money market deposits. We calibrate the model to U.S. data and find that the introduction of the sweep technology at the beginning of the 1990s, which improved access to money markets, can explain the behavior of money demand very well. Furthermore, by allowing a more efficient allocation of money, the welfare cost of inflation decreased substantially.

JEL classification: E4, E5, D9.

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1 INTRODUCTION

The behavior of M1 money demand, defined to be the ratio of M1 to GDP, began to change substantially at the beginning of the 1990s. Up until the 1990s, money demand and nominal interest rates had remained in a stable negative relationship. Since then, the empirical relationship between M1 and the movements in interest rates began to fall apart and has not been restored since (Lucas and Nicolini 2013).

— INSERT FIGURE 1 —

In Figure 1, we plot the relationship between M1 money demand and the AAA interest rate in the United States from 1950 until 2013. The black curve displays this relationship from 1950 until 1989, while the blue curve displays it from 1990 until 2013. The green curve shows M1 money demand adjusted for retail sweeps (M1S, hereafter) from 1990 to 2013.¹ The elasticity of money demand with respect to the AAA interest rate is denoted by \( \xi \) in the legend of Figure 1. While the elasticity is \( \xi = -0.62 \) in the period between 1950 and 1989, it decreased rapidly thereafter. In the post-1990 data, the elasticity of M1 is \( \xi = 0.16 \) and the elasticity of M1S is \( \xi = -0.31 \).

What accounts for this shift and the lower interest rate elasticity of money demand? It is well documented that changes in regulations and advances in information technology in the 1980s and 1990s allowed for new financial products that affected the demand for money (Teles and Zhou 2005).² A case in point for this advance in information technology is the sweep technology, which “essentially consists of software used by banks that automatically moves funds from checking accounts to MMDAs” (Lucas and Nicolini 2013, p. 5).³ In the retail sector, retail sweep accounts were introduced in 1993 around the time when the empirical relationship between money demand and the movements in interest rates began to fall apart. Thus, the emergence of retail sweep accounts can be viewed as a prototypical technical innovation in the financial sector that might explain the break in money demand.

In order to explain the behavior of money demand, we first construct a microfounded monetary model with a money market.⁴ In the model, agents face idiosyncratic liquidity shocks which generate an ex-post inefficient allocation of the medium of exchange: some agents will hold cash but have no current need for it, while other agents will hold insufficient cash for their liquidity needs. The money market provides insurance against these liquidity shocks by offering short-term loans and by paying interest on money market deposits. This improves the allocation and affects the shape of the money demand function.
We then explore what changes in financial intermediation in the model are needed to replicate the behavior of money demand as observed in the data. From a theoretical point of view, innovations in financial markets can affect money demand via two channels. First, innovations may allow agents to earn a higher interest rate on their transaction balances. In doing so, such innovations make holding the existing money stock more attractive. Second, financial innovations may allocate the stock of money more efficiently. In the paper, we show that the first channel is responsible for the reduction in the elasticity of money demand, while the second channel is responsible for the observed downward shift. The combination of both effects shifts the money demand curve downwards and makes it less elastic. In the model, an exogenous increase in money market participation generates both effects since an increase in money market participation allows more agents to earn interest on their idle money holdings and it improves the allocation of the medium of exchange.

To study to what extent financial innovation can account for the observed behavior of money demand, we calibrate the model by using U.S. data from 1950 to 1989. In doing so, we assume that during this period no agent participates in the money market ($\pi_0 = 0$), where market participation is captured by the money market access probability $\pi$. We then perform the following experiment: We search numerically for the value of $\pi$ that minimizes the squared error between the model-implied money demand and the data. We find that under competitive pricing a value of $\pi = 0.60$ replicates the observed break in money demand best.$^5$

--- INSERT FIGURE 2 ---

Figure 2 displays the main result of the paper. In this figure, we plot the empirical money demand and the model-implied money demand, by assuming an increase in the money market access from $\pi_0 = 0$ to $\pi_1 = 0.60$ in 1990. The model’s money demand, which is plotted against the interest rate, shifts downwards and becomes less elastic after the 1990s. In particular, in the period spanning 1990 to 2013, the model generates the same average money demand as observed in the data, while we obtain an elasticity of money demand of $\xi = -0.40$ as compared to the empirical elasticity of $\xi = -0.31$.

In a further experiment, we set the initial value of the access probability to $\pi_0 = 0.2$, recalibrate the model, and then increase the access probability to $\pi_1 = 1$. In this case, the model is also able to replicate the empirical velocity of money, and the model’s elasticity is statistically not different from the empirical counterpart at the 95% confidence level. Thus, by changing a single parameter, the probability that any individual agent gains access to
the money market, we are able to explain two separate facts: first, that the average velocity increases and, second, that the interest elasticity of money demand decreases when the data are compared, before and after 1990.

In the paper, we focus on money demand adjusted for sweeps (M1S) for the following reason. Banks use the sweep technology to automatically move funds from checking accounts (included in the definition of M1) into MMDAs (not included in the definition of M1). Such sweeps reduce banks’ required reserves “while leaving unchanged its customers’ perceived holdings of transaction deposits“ (Anderson and Rasche 2001, p. 51). Thus, for a customer, these funds are as liquid as money in a deposit account, since they are automatically swept back in case they are required for payments. Accordingly, M1S represents the available transaction media in the economy more accurately than M1. Since the stock of money in our model is equal to the model’s stock of transaction media, we map the model’s money demand to the empirical money demand adjusted for sweeps.

Deposit-sweeping software was introduced by banks to lower reserve requirements. Van-Hoose and Humphrey (2001) find that the introduction of retail sweep accounts reduced required reserves by more than 70 percent between 1995 and 2000. Anderson and Rasche (2001) find that the sweep technology made the economic burden of reserve requirements zero. The funds released by the reduction of required reserves is invested in consumer and business loans (Anderson and Rasche 2001 p. 57). This means that the sweep technology made more funds available for lending. We model the same mechanism, with an exogenous increase in money market participation. Under this increase, agents that had previously no access to the money market, can suddenly use it either for investing their idle money or for taking out short-term loans.

In practice, banks did not pass on most of the interest earnings to customers, as the customers did not fully understand the sweeping process (see Anderson and Rasche 2001 p. 56). In contrast, in our model, we assume that the money market is perfectly competitive. Consequently, banks pass on the entire earnings to depositors. To address the issue that in practice depositors earned less than under perfect competition, we also consider the case where agents do not earn interest in the money market, but they continue to receive loans against interest. We show that this change mainly affects the elasticity of money demand, while the model still does a good job in matching the observed downward shift in money demand.

We also find that the welfare cost of inflation is considerably lower when we calculate it with our new theoretical money demand function as opposed to traditional models that do
not take into account the recent changes in money demand. In fact, for any pricing protocol, we find that the welfare cost of inflation is considerably smaller after 1990 than before. Finally, our paper also makes a theoretical contribution by introducing limited participation into BCW. As mentioned above, limited participation affects the money demand function in an interesting way.

**Literature** The behavior of money demand is very well documented in Lucas and Nicolini (2013) and Teles and Zhou (2005), who also discuss the regulatory changes that occurred in the 1980s and 1990s. Lucas and Nicolini (2013) carefully think about what objects serve as means of payment and need to be included into M1. They then define a new monetary aggregate called NewM1. Similar to M1S, this aggregate adds MMDAs to the traditional components of M1. They then show that there is a stable long-run relationship between the NewM1 and the relevant opportunity cost of holding the various components of NewM1. We will discuss this paper in more detail in Section 7.

In the course of our research, we reviewed papers that study money demand and, in particular, those that explore the break in money demand that occurred in the 1990s. They often involve Baumol-Tobin style inventory-theoretic models of money (e.g., Attanasio, Guiso and Jappelli 2002, and Alvarez and Lippi 2009). Lucas and Nicolini (2013), Ireland (2009), Teles and Zhou (2005) and Reynard (2004) are more recent attempts to explain the behavior of money demand. Papers that use the search approach to monetary economics are Faig and Jerez (2007), and Berentsen, Menzio and Wright (2011). In Section 7, we discuss the above-mentioned papers in more detail.

Another related branch of the literature are papers that study the welfare cost of inflation in monetary models with trading frictions; see, e.g., Lagos and Wright (2005), Aruoba, Rocheteau and Waller (2007), Craig and Rocheteau (2008), and Chiu and Molico (2010), among many others. Some other related papers study issues such as credit card use (Telyukova and Wright 2008, and Rojas-Breu 2013) and its effect on money demand (Telyukova 2013), and the impact of aggregate and idiosyncratic shocks on money demand over the business cycle (Telyukova and Visschers 2013). The main focus of our work is to investigate the quantitative effects of financial innovation on steady state money demand and velocity.


2 ENVIRONMENT

There is a \([0, 1]\) continuum of infinitely-lived agents\(^8\). Time is discrete, and in each period there are three markets that open sequentially: a money market, where agents can borrow and deposit money; a goods market, where production and consumption of a specialized good take place; and a centralized market, where credit contracts are settled and a general good is produced and consumed. All goods are nonstorable, which means that they cannot be carried from one market to the next.

At the beginning of each period, agents receive two i.i.d. shocks: a preference shock and an entry shock. The preference shock determines whether an agent can consume or produce the specialized good in the goods market: with probability \(n\), he can produce but not consume, while with probability \(1 - n\), he can consume but not produce. We refer to producers as sellers and to consumers as buyers. The entry shock determines whether an agent has access to a frictionless money market. Agents who have access to the money market are called active (probability \(\pi\)), while agents who have no access are called passive (probability \(1 - \pi\)).

In the goods market, buyers and sellers meet at random and bargain over the terms of trade. The matching process is described according to a reduced-form matching function, \(M(n, 1 - n)\), where \(M\) is the number of trade matches in a period. We assume that the matching function has constant returns to scale, and is continuous and increasing with respect to each of its arguments. Let \(\delta(n) = M(n, 1 - n)(1 - n)^{-1}\) be the probability that a buyer meets a seller, and \(\delta^*(n) = \delta(n)(1 - n)n^{-1}\) be the probability that a seller meets a buyer. In what follows, we suppress the argument \(n\) and refer to \(\delta(n)\) and \(\delta^*(n)\) as \(\delta\) and \(\delta^*\), respectively.

In the goods market, a buyer receives utility \(u(q)\) from consuming \(q\) units of the specialized good, where \(u(q)\) satisfies \(u'(q) > 0, u''(q) < 0, u'(0) = +\infty,\) and \(u'(\infty) = 0\). A seller incurs a utility cost \(c(q) = q\) from producing \(q\) units. Furthermore, agents are anonymous, and agents’ actions are not publicly observed. These assumptions mean that an agent’s promise to pay in the future is not credible, and sellers require immediate compensation for their production. Therefore, a means of exchange is needed for transactions.

The general good can be produced and consumed by all agents and is traded in a frictionless, centralized market. Agents receive utility \(U(x)\) from consuming \(x\) units, where \(U'(x), -U''(x) > 0, U'(0) = \infty,\) and \(U'(\infty) = 0\). They produce the general good with a linear technology, such that one unit of \(x\) is produced with one unit of labor, which generates one unit of disutility \(h\). This assumption eliminates the wealth effect, which makes
the end-of-period distribution of money degenerate (see Lagos and Wright 2005). Agents discount between, but not within, periods. Let $\beta \in (0, 1)$ be the discount factor between two consecutive periods.

There exists an object, called money, that serves as a medium of exchange. It is perfectly storable and divisible, and has no intrinsic value. The supply of money evolves according to the law of motion $M_{t+1} = \gamma M_t$, where $\gamma \geq \beta$ denotes the gross growth rate of money and $M_t$ the stock of money in $t$. In the centralized market, each agent receives a lump-sum transfer $T_t = M_{t+1} - M_t = (\gamma - 1)M_t$. To economize on notation, next-period variables are indexed by $+1$, and previous-period variables are indexed by $-1$.

The money market is modeled similar to the one in BCW. In the money market, perfectly competitive financial intermediaries take deposits and make loans, which allows agents to adjust their money balances before entering the goods market. In particular, an agent with high liquidity needs can borrow money, while an agent with low liquidity needs can deposit money and earn interest. All credit contracts are one-period contracts and are redeemed in the centralized market. Financial intermediaries operate a record-keeping technology that keeps track of all agents’ past credit transactions at zero cost. Perfect competition among financial intermediaries in the money market implies that the deposit rate, $i_d$, is equal to the loan rate, $i_l$. Throughout the paper, the common nominal interest rate is denoted by $i$.

Following BCW, we make the following assumptions about commitment. Firstly, we assume limited commitment in the goods market, which rules out bilateral trade credit among agents. Secondly, we assume full commitment of borrowers via banks. These assumptions allow for the coexistence of fiat money and credit. In this paper, we generalize BCW by assuming that only a fraction, $\pi \leq 1$, of agents have access to the money market in each period.$^9$

3 AGENTS’S DECISIONS

In what follows, we present the agents’ decision problems within a representative period, $t$. We proceed backwards, moving from the last to the first market. All proofs are relegated to the Appendix.

**The centralized market.** In the centralized market, agents can consume and produce the centralized market good $x$. Furthermore, they receive money for their deposits plus interest payments. Additionally, they have to pay back their loans plus interest. An agent entering
the centralized market with \( m \) units of money, \( \ell \) units of loans, and \( d \) units of deposits has the value function \( V_3(m, \ell, d) \). He solves the following decision problem

\[
V_3(m, \ell, d) = \max_{x, h, m+1} U(x) - h + \beta V_1(m+1),
\]

subject to the budget constraint

\[
x + \phi m + 1 = h + \phi T + \phi (1 + i) d - \phi (1 + i) \ell,
\]

where \( h \) denotes hours worked and \( \phi \) denotes the price of money in terms of the general good. As in Lagos and Wright (2005), we show in the Appendix that the choice of \( m+1 \) is independent of \( m \). As a result, each agent exits the centralized market with the same amount of money, and, thus, the distribution of money holdings is degenerate at the beginning of a period.

**The goods market.** In the goods market, the terms of trade are described by the pair \((q, z)\), where \( q \) is the amount of goods produced by the seller and \( z \) is the amount of money exchanged. Here, we present the generalized Nash bargaining solution. In the Appendix, we also consider Kalai bargaining and competitive pricing. The Nash bargaining problem is given by

\[
(q, z) = \arg \max [u(q) - \phi z] \theta (-q + \phi z)^{1-\theta} \text{ s.t. } z \leq m.
\]

If the buyer’s constraint is binding, the solution is given by \( z = m \) and

\[
\phi m = g(q) \equiv \frac{\theta q u'(q) + (1 - \theta) u(q)}{\theta u'(q) + 1 - \theta}.
\]

If the buyer’s constraint is not binding, then \( u'(q) = 1 \) or \( q = q^* \), and \( z = m^* = \frac{g(q^*)}{\phi}. \)

**The money market.** At the beginning of each period, an agent learns his type; that is, whether he is a buyer or seller and his participation status in the money market (active or passive). Let \( V^b_1(m) \) and \( V^s_1(m) \) be the value functions of an active buyer and an active seller, respectively, in the money market. Furthermore, \( V^b_2(m + \ell, \ell) \) denotes the value function of a buyer at the beginning of the goods market with \( m + \ell \) units of money and \( \ell \) units of loans, and \( V^s_2(m - d, d) \) denotes the value function of a seller at the beginning of the goods market with \( m - d \) units of money and \( d \) units of deposits. Accordingly, the value function of an
agent at the beginning of each period is

\[ V_1(m) = \pi \left[ (1 - n) V_{1b}(m) + nV_{1s}(m) \right] + (1 - \pi) \left[ (1 - n) V_{2b}(m, 0) + nV_{2s}(m, 0) \right]. \] (5)

An agent in the money market is an active buyer with probability \(\pi (1 - n)\), an active seller with probability \(\pi n\), a passive buyer with probability \((1 - \pi) (1 - n)\), and a passive seller with probability \((1 - \pi) n\). Passive agents in the money market just wait for the goods market to open, so their value functions in the money market are \(V_{2b}(m, 0)\) and \(V_{2s}(m, 0)\), respectively.

An active buyer’s optimization problem in the money market is

\[ V_{1b}(m) = \max_{\ell} V_{2b}(m + \ell, \ell), \] (6)

and an active seller’s optimization problem in the money market is

\[ V_{1s}(m) = \max_d V_{2s}(m - d, d) \quad s.t. \quad m - d \geq 0. \] (7)

The constraint in (7) means that a seller cannot deposit more money than the amount he has. Let \(\lambda_s\) be the Lagrange multiplier on this constraint. As we will see below, the nature of the equilibrium will depend on whether this constraint is binding or not.

In an economy with full commitment of borrowers via banks, there are two types of equilibria: an equilibrium where active sellers do not deposit all their money (i.e., \(\lambda_s = 0\)), and another equilibrium where active sellers deposit all their money (i.e., \(\lambda_s > 0\)). We refer to these equilibria as the type-A and type-B equilibrium, respectively.

### 3.1 Type-A equilibrium

In the type-A equilibrium, active sellers do not deposit all their money. For this to hold, sellers must be indifferent between depositing their money and not depositing it. This can be the case if, and only if, \(i = 0\). In what follows, we denote the consumption quantity of an active buyer by \(\hat{q}\) and the consumption quantity of a passive buyer by \(q\).
Proposition 1  A type-A equilibrium is a list \( \{ i, \hat{q}, q, \phi \ell \} \) satisfying

\[
\begin{align*}
    g(\hat{q}) &= g(q) + \phi \ell, \\
i &= \delta \left[ \frac{u'(\hat{q})}{g'(\hat{q})} - 1 \right], \\
i &= 0, \\
\frac{\gamma - \beta}{\beta} &= (1 - \pi)(1 - n) \delta \left[ \frac{u'(q)}{g'(q)} - 1 \right].
\end{align*}
\]  

According to Proposition 1, in a type-A equilibrium, the following holds. From (8), the real amount of money an active buyer spends in the goods market, \( g(\hat{q}) \), is equal to the real amount of money spent as a passive buyer, \( g(q) \), plus the real loan an active buyer gets from the bank, \( \phi \ell \). Equation (8) is derived from the active buyer’s budget constraint and immediately shows that in this equilibrium \( \hat{q} > q \). An active buyer’s consumption satisfies equation (9), which is derived from the first-order condition for the choice of loans, \( \ell \). Equation (10) is derived from the seller’s deposit choice in the money market. In the proof of Proposition 1, we show that the first-order condition is \( \phi i = \lambda_s \), and since \( \lambda_s = 0 \), we have \( i = 0 \); together with (9), this implies \( u'(\hat{q}) = g'(\hat{q}) \). From (11), a passive buyer consumes an inefficiently low quantity of goods in the goods market unless \( \gamma = \beta \). This last equation is derived from the choice of money holdings in the centralized market.

As in BCW, to obtain the first-best allocation \( \hat{q} = q = q^* \), the central bank needs to set \( \gamma = \beta \). Note further that as \( \pi \to 1 \), \( q \to 0 \). The reason for this is the following: if the chance that agents have no access is small, then the value of money is small as well. However, note that as \( \pi \to 1 \), the economy does not remain in the type-A equilibrium. Rather, it switches to the type-B equilibrium as explained below.

### 3.2 Type-B equilibrium

In the type-B equilibrium, active sellers deposit all their money at the bank, and so the deposit constraint is binding; i.e., \( \lambda_s > 0 \). For this to hold, the nominal interest rate must be strictly positive. In this case, we have:
Proposition 2 A type-B equilibrium is a list \( \{ i, \hat{q}, q, \phi \ell \} \) satisfying

\[
\begin{align*}
g(\hat{q}) &= g(q) + \phi \ell, \\
i &= \delta \left[ \frac{u'(\hat{q})}{g'(\hat{q})} - 1 \right], \\
g(q) &= (1 - n) g(\hat{q}), \\
\frac{\gamma - s}{\beta} &= \pi \delta \left[ \frac{u'(\hat{q})}{g'(\hat{q})} - 1 \right] + (1 - n)(1 - \delta) \left[ \frac{u'(q)}{g'(q)} - 1 \right].
\end{align*}
\] (12) (13) (14) (15)

Equations (12), (13), and (15) in Proposition 2, have the same meaning as their counterparts in Proposition 1. In contrast, equation (10) must be replaced by the market clearing condition in the money market (14).

Let \( \bar{\gamma} \) be the value of \( \gamma \) such that equations (11) and (15) hold simultaneously; i.e., \( u'(\hat{q}) = g'(\hat{q}) \). Then, the following holds: (i) for any \( \beta < \gamma \leq \bar{\gamma} \), then \( \lambda_s = 0 \); (ii) for any \( \gamma > \bar{\gamma} \), then \( \lambda_s > 0 \).

3.3 Discussion

With partial access to the money market, the quantities of goods consumed by active and by passive buyers are represented by the two loci drawn on the right-hand side graph of Figure 3. To draw this figure, we assume \( \theta = 1 \) and a linear cost function \( c(q) = q \).\(^{11}\) The dotted (solid) line denotes the quantity consumed by an active (passive) buyer as a function of \( \gamma \).

— INSERT FIGURE 3 —

In the type-A equilibrium \( (\gamma \leq \bar{\gamma}) \), an active buyer’s consumption is independent of \( \gamma \) and equal to \( q^* \), while a passive buyer’s consumption is decreasing in \( \gamma \) and smaller than \( q^* \) unless \( \beta = \gamma \). In the type-B equilibrium \( (\gamma > \bar{\gamma}) \), both the active and passive buyers’ consumption is decreasing in \( \gamma \). The dotted vertical line that separates the two equilibria intersects the horizontal axis at \( \gamma = \bar{\gamma} \). How does \( \bar{\gamma} \) change in the rate of participation \( \pi \)? Our numerical examples show that \( \bar{\gamma} \) is decreasing in \( \pi \) with \( \bar{\gamma} \to \beta \) as \( \pi \to 1 \). Hence, with full participation, the type-A equilibrium exists under the Friedman rule only, while the type-B equilibrium exists for any \( \gamma > \beta \). The graph on the left-hand side of Figure 3 shows the consumed quantities for the full participation case (i.e., \( \pi = 1 \)). In this case, all agents are active, and the first-best consumption is achieved at the Friedman rule.
4 QUANTITATIVE ANALYSIS

We choose a model period of one year. The functions $u(q)$, $U(x)$, and $c(q)$ have the forms $u(q) = q^{1-\alpha}/(1-\alpha)$, $U(x) = A\log(x)$, and $c(q) = q$, respectively. Regarding the matching function, we follow Kiyotaki and Wright (1993) and choose $\mathcal{M}(B, S) = BS/(B+S)$, where $B = 1 - n$ is the measure of buyers, and $S = n$ is the measure of sellers. Therefore, the matching probability of a buyer in the goods market, $\delta$, is equal to measure of sellers; i.e., $\delta = n$.

The parameters to be identified are the following: (i) the preference parameters $\beta$, $A$, and $\alpha$; (ii) the technology parameters $n$ and $\pi$; (iii) the bargaining weight $\theta$; and (iv) the policy parameter $i_b$. To identify these parameters, we use quarterly U.S. data from the first quarter of 1950 to the fourth quarter of 1989. All data sources are provided in the Appendix.

--- INSERT TABLE 1 ---

The three parameters $\beta$, $n$, and $i_b$ can be set equal to their equivalent targets. The nominal interest rate in the settlement market, $i_b = \gamma/\beta - 1 = 0.07$, matches the average yield on AAA corporate bonds. We set $\beta = (1 + r)^{-1} = 0.974$ so that the model’s real interest rate matches the empirical counterpart, $r = 0.027$, where $r$ is measured as the difference between the AAA corporate bonds yield and the change in the consumer price index. In order to maximize the number of matches, we set $n = 0.5$.

The three parameters $A$, $\alpha$, and $\theta$ are obtained by matching the average velocity of money, the elasticity of money demand, and the goods sector mark-up simultaneously. We do this by minimizing the sum of squared differences between the target values and the respective model-generated moments. With this calibration strategy we are able to hit the three targets exactly. The average velocity of money before 1990 is $v = 5.22$. The elasticity of money demand with respect to the AAA corporate bond is $\xi = -0.619$. We estimate $\xi$ by using ordinary least squares and a log-log specification. The markup in the goods market is $\mu = 0.3$, which represents an average value used in related studies.

The model’s velocity of money is

$$v = \frac{Y}{\phi M_{-1}} = \frac{A + (1-n)\delta [\pi g(\hat{q}) + (1-\pi) g(q)]}{g(q)}$$

which depends on $i$ via $q$ and $\hat{q}$, and on $\alpha$ via the function $g(q)$ and $g(\hat{q})$. As for the empirical elasticity, the model’s elasticity of money demand is estimated by ordinary least
squares and a log-log specification. The model’s markup in the goods market is given by the real amount of money exchanged in a bilateral match divided by the production cost; i.e.,

$$\pi \frac{g(\hat{q})}{c(\hat{q})} + (1 - \pi) \frac{g(q)}{c(q)} - 1.$$

With probability $\pi$, a buyer has access to the money market and buys $\hat{q}$ units of goods for $\hat{m}$ units of money in the goods market. The real value of money he exchanges with the seller is $\phi \hat{m} = g(\hat{q})$. The seller has real costs $c(\hat{q})$ to produce $\hat{q}$. So in this case, the markup is $g(\hat{q})/c(\hat{q})$. With probability $1 - \pi$, a buyer has no access to the money market and gets $q$ units of goods for $m$ units of money, in which case the markup is $g(q)/c(q)$.

Our targets discussed above, and summarized in Table 1, are sufficient to calibrate all but one parameter: the money market access probability $\pi$. For the baseline calibration, we calibrate the above-specified parameters for the period from 1950 to 1989 under the assumption that $\pi_0 = 0$, where the index 0 indicates the initial value of $\pi$. Further below, we also consider various other initial values for $\pi_0$. In each case, we set up the following test for the model: by changing the single parameter, $\pi$, from its initial value $\pi_0$ to some new value $\pi_1$, we want to be able to explain two separate facts: that the average velocity of money increases, but that the interest elasticity of money demand decreases when the data are compared, before and after 1990.

Table 2 presents the calibration results for Nash bargaining, Kalai bargaining, and competitive pricing. Under Kalai bargaining, $g(q)$ in (4) is replaced by $g^K(q) \equiv \theta q + (1 - \theta) u(q)$. For competitive pricing, we set $\theta = 1$.

Table 2 also displays the welfare cost of inflation, $1 - \Delta$, which is the percentage of total consumption agents would be willing to give up in order to be in a steady state with a nominal interest rate of 3 percent instead of 13 percent. Under competitive pricing, the welfare cost of inflation is roughly 1.23 percent, which is in line with the estimates in Craig and Rocheteau (2008), and Rocheteau and Wright (2005, 2009). For the other trading mechanisms, the welfare cost of inflation is higher, due to the holdup problem under bargaining. In particular, we obtain the highest estimate under Nash bargaining, with a number equal to 1.96 percent of the steady state level of total consumption. In all cases, the goods-market share of total output, $s_{GM}$, is equal to 4.8 percent, which is in line with the estimates in Aruoba, Waller and Wright (2011), and Lagos and Wright (2005).
4.1 One-time increase in $\pi$ in 1990

We now investigate the extent to which the improved liquidity provision in the 1990s accounts for the observed behavior of money demand. For this, we consider how a one-time increase in $\pi$ in 1990 affects the money demand and the welfare cost of inflation. We assume that in 1990 the access probability to the money market increased from $\pi_0 = 0$ to $\pi_1 = 1$, while keeping all other parameters at their calibrated values. Then, we feed in the actual path of the nominal interest rate to simulate the model. This allows us to calculate the model-implied money demand properties and the welfare cost of inflation for the period from the first quarter of 1990 to the fourth quarter of 2013. The simulation results are provided in Table 3 below.

— INSERT TABLE 3 —

Table 3 shows that the increase in the access probability to the money market results in a substantial reduction in the welfare cost of inflation. For example, under competitive pricing, the welfare cost of inflation decreases from 1.23 percent to 0.53 percent. Furthermore, the model proves competent in replicating the higher velocity of money and the lower elasticity of money demand with respect to the AAA interest rate. An increase from $\pi_0 = 0$ to $\pi_1 = 1$ reduces the elasticity of money demand from -0.62 to -0.33 under competitive pricing, while it increases the velocity from 5.22 to 7.11. For comparison, in the post-1990 data, we obtain an empirical velocity of $v = 6.41$ and an empirical elasticity of $\xi = -0.31$ with a 95% confidence interval of $[-0.355, -0.270]$.

To illustrate the implications of the model, we show the simulated money demand under competitive pricing in Figure 4.

— INSERT FIGURE 4 —

Figure 4 shows that the model works well in replicating the lower and less elastic money demand that occurred in the 1990s by increasing the access probability from $\pi_0 = 0$ to $\pi_1 = 1$. In particular, the model-implied elasticity is not different from the post-1990 data at the 95% confidence level, but the model-implied money demand is too low.

4.2 Optimal value of $\pi_1$

A one-time increase in $\pi$ from $\pi_0 = 0$ to $\pi_1 = 1$ results in a model-implied money demand which is too low compared to the data. In what follows, we identify the value of $\pi_1$, labelled $\pi_1^*$, that best fits the data. For this purpose, we search numerically for the value of $\pi$ that
minimizes the squared error between the model-implied money demand and the data. As before, we assume that there was one-time increase in $\pi$ from $\pi_0 = 0$ to $\pi_1^*$ in 1990, while keeping all other parameters at their calibrated values. The simulation results are shown in Table 4.

— INSERT TABLE 4 —

The estimated velocity comes closer to its observed value when considering the optimal increase rather than the zero-to-one increase in $\pi$, while the gap between the model’s and the observed money demand elasticity increases. In particular, the elasticity of money demand in the data and in the model are now different at the 95% confidence level, which was not the case before. Furthermore, the welfare cost of inflation is higher under the optimal market access shift than it is under the zero-to-one shift. For example, under competitive pricing, the welfare cost of inflation increases from 0.53 percent with $\pi_1 = 1$ to 0.69 percent with $\pi_1^* = 0.60$. Table 4 also shows the critical interest rate, $\tilde{i}$, that separates the type-A equilibrium from the type-B equilibrium. For all the trading protocols, we find that $\tilde{i}$ is close to 2.5 percent, and thus our estimates of the welfare cost of inflation are not affected by the type-A equilibrium. The simulated money demand properties under competitive pricing are shown in Figure 5.

— INSERT FIGURE 5 —

Our numerical results indicate that the improved liquidity provision by financial intermediaries ($\pi_1 > 0$) can replicate the observed shift in money demand and the lower elasticity of money demand to a large extent.

4.3 Initial value $\pi_0 > 0$

In the previous section, the key question is whether there is a value $\pi_1$ that can be set to match the post-1990 statistics of 6.41 for the average velocity and $-0.31$ for the money demand elasticity. In the previous section, we match the number for the average velocity exactly. But while the interest elasticity of money demand falls in absolute value, it only drops to $-0.40$ (for competitive pricing), which is statistically different from $-0.31$ at the 95% confidence level. In this section, we show that we can improve the model’s performance by assuming that the initial value $\pi_0$ is larger than 0 in the period from 1950 to 1989. In particular, we calibrate the model with the initial values of $\pi_0 = 0.2, 0.4, 0.6$ and 0.8.
Thereafter, we assume that in 1990 there was a one-time shift in $\pi$ from the initial value $\pi_0$ to $\pi_1 = 1$. For this experiment, we assume competitive pricing in the goods market. The calibration and simulation results are shown in Table 5 below.

— INSERT TABLE 5 —

With an initial value of $\pi_0 = 0.2$, the model is able to replicate the behavior of money demand in the post-1990 period. In particular, the model-implied elasticity with $\pi_1 = 1$ is not different from the elasticity in the data at the 95% confidence level, and the model-implied level of velocity is only slightly too low. The simulated money demand properties with an initial value of $\pi_0 = 0.2$ are shown in Figure 6.

— INSERT FIGURE 6 —

Increasing the initial value of $\pi_0$ results in a lower calibrated value of $\alpha$ and $A$. This tends to increase the welfare cost of inflation slightly. The simulation results highlight that a higher initial value of $\pi_0$ results in a less pronounced shift, and the elasticity of money demand remains higher. Intuitively, a higher initial value of $\pi_0$ results in a higher welfare cost of inflation after the one-time shift to $\pi_1 = 1$.

4.4 Discussion

As discussed in several instances throughout the paper, the introduction of the sweep technology in the 1990s had two effects. First, it effectively allowed agents to earn interest on their transaction balances. Second, it allowed for a more efficient allocation of money in the economy. Our money market also displays these two effects. It is, therefore, straightforward to study these two effects in isolation in our model. We first study the counterfactual experiment of paying interest on money without reallocating liquidity with regard to the relationship between M1S money demand and the triple AAA interest rate. We, then, perform the counterfactual experiment in which the money market reallocates liquidity without paying interest on money. Finally, in the last subsection, we also show that our model can reasonably replicate the behavior of M1.

4.4.1 The effects of paying interest on money

In this subsection, we assume that agents earn interest on their idle money holdings, but that no credit is available. This means that no cash is reallocated. To avoid introducing any
additional distortion, the interest rate on idle money is financed via lump-sum taxes. With these assumptions, equation (15) can be rewritten as follows

$$\frac{\gamma - \beta}{\beta} = (1 - n) \delta \left[ \frac{u'(q)}{g'(q)} - 1 \right] + \pi n i,$$

where $i$ is assumed to be exogenous and set equal to the endogenous interest rate that we obtain in the unrestricted model. That is, we allow agents to earn the same interest rates as they do under the experiment described in Figure 5. We assume that there was a one-time shift in $\pi$ from $\pi_0 = 0$ to $\pi_1 = 0.60$ in 1990. The simulation results are displayed in Figure 7 below.

--- INSERT FIGURE 7 ---

Intuitively, the possibility to earn interest payments on idle money holdings increases the demand for money, as it reduces the opportunity cost of holding money. This shifts the money demand function up and reduces its elasticity (see the curve labeled No Credit: $\pi_1 = 0.60, \xi = -0.39$). In particular, with no credit and $\pi_1 = 0.60$, the velocity of money decreases to 4.42 and the elasticity of money demand decreases to $-0.39$ as compared to the money demand properties with the initial value of $\pi_0 = 0$, where we obtain $v = 5.22$ and $\xi = -0.62$.

### 4.4.2 The effects of reallocating the medium of exchange

Here, we continue to assume the existence of a money market that reallocates money, but we artificially hold the interest rate for depositors at zero. By doing so, the market clearing condition in the money market continues to hold; i.e., $g(q) = (1 - n) g(\hat{q})$. Furthermore, equation (15) can be rewritten as follows

$$\frac{\gamma - \beta}{\beta} = (1 - n) \delta \left\{ \pi \left[ \frac{u'(\hat{q})}{g'(\hat{q})} - 1 \right] + (1 - \pi) \left[ \frac{u'(q)}{g'(q)} - 1 \right] \right\}.$$

Using the calibrated parameter values and assuming that there was a one-time shift in $\pi$ from $\pi_0 = 0$ to $\pi_1 = 0.60$ in 1990, we obtain the simulation results shown in Figure 8.

--- INSERT FIGURE 8 ---

Figure 8 shows that in this case the velocity increases significantly to 7.16 (see the curve labeled No Interest: $\pi_1 = 0.60, \xi = -0.61$), while the elasticity of money demand remains essentially at the initial value with $\pi_0 = 0$, where we obtain $\xi = -0.62$. 

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To summarize, the money market’s more efficient allocation of money is the driving force behind the downward shift in money demand, while the possibility to earn interest on idle money is the driving force behind the reduction in the elasticity of money demand.

4.4.3 Difference between M1 and M1S

As mentioned in the introduction, we focus on money demand adjusted for retail sweeps (M1S), because it represents the available transaction media in the economy more accurately than M1. In what follows, we show that our model can also reasonably replicate the behavior of M1. In order to show this, we need to identify which fraction of the stock of money in the model is M1.

In practice, M1S includes the funds from demand deposit accounts that are automatically transferred into MMDAs by means of the sweep technology. In our model, the funds allocated to MMDAs are equal to the amount that active sellers deposit. Hence, M1 in the model is equal to M1S minus these deposits. The real value of M1S in the model is equal to the real stock of money \( \phi M = g(q) \). The real value of all deposits is \( n\pi \phi M = n\pi g(q) \). Consequently, the real value of M1 in the model is \( \phi M1 = \phi M - n\pi \phi M = (1 - n\pi)g(q) \).

To study the properties of M1 money demand in the model, we again assume that there was a one-time increase in \( \pi \) from \( \pi_0 = 0 \) to \( \pi_1 = 0.6 \) in 1990. The simulation results are shown in Figure 9, where we use competitive pricing in the goods market and the parameter values calibrated to the 1950 to 1989 period.

— INSERT FIGURE 9 —

Figure 9 shows that our model does a good job in matching M1 as well. The fit of M1 money demand is so good that the reader might be tempted to believe that we experimented a lot to attain it. In fact, we just run one experiment since the difference between M1S money demand and M1 money demand in the model only depends on \( \pi \) and \( n \). Throughout the paper we have held \( n \) constant at \( n = 0.5 \) and \( \pi \) is equal to the optimal \( \pi \), namely \( \pi_1 = 0.6 \). Thus, in the model M1 money demand is 70% of M1S money demand; i.e., \( 1 - 0.5 \times 0.6 = 0.7 \). This fraction corresponds to what we see in the data if we take into account a transition period.

— INSERT TABLE 6 —

If we compare the model M1 moments to the empirical counterparts for the period from 1990 to 2013, we find that the model-implied M1 elasticity and model’s M1 velocity are too
high. However, one has to bear in mind that our experiment does not take into account transition periods that in practice can last for several years. If we take into account a transition period and calculate the elasticities after the transition only, we get the elasticities shown in Table 6. In particular, for the period from 1998 to 2013 we find that the model-implied elasticity of M1 is not different from the data at the 95% confidence level and the model-implied M1 velocity is very close to the empirical velocity.

5 ROBUSTNESS

In this section, we perform two robustness checks.

5.1 Money demand shift in 1980 instead of 1990

Lucas and Nicolini (2013) argue that the break in money demand took place in 1980, because money market deposit accounts were introduced at this time. As a robustness check, we follow Lucas and Nicolini (2013) and assume that the technological one-time shift in \( \pi \) was realized in 1980 and that \( \pi \) remained constant thereafter. For this purpose, we recalibrate the model with \( \pi_0 = 0 \) for the period from 1950 to 1979. We obtain a target value of the nominal interest rate of \( i = 0.055 \) and the real interest rate of \( r = 0.017 \), which corresponds to a value of \( \beta = (1 + r)^{-1} = 0.983 \). Furthermore, the average velocity of money is \( v = 4.56 \), and the elasticity of money demand with respect to the AAA corporate bond yield is \( \xi = -0.65 \). The markup-target under Kalai and Nash bargaining remains unchanged at \( \mu = 0.30 \). The calibration results are presented in Table 7 below.

— INSERT TABLE 7 —

Calibrating the model to the period from 1950 to 1979 results in a higher welfare cost of inflation. For instance, under competitive pricing we obtain a welfare cost of inflation of 1.40 percent as compared to 1.23 percent when we calibrate the model to the 1950 - 1989 period.\(^{19}\) Furthermore, the goods-market share of total output, \( s_{GM} \), increases by roughly one percentage point to 5.7 percent.

As before, we search numerically for the value of \( \pi \) that minimizes the squared error between the model-implied money demand and the data. The simulation results are shown in Table 8.

— INSERT TABLE 8 —
Table 8 shows that the model is able to replicate the higher velocity of money for the post-1980 data. However, it fails to replicate the lower elasticity of money demand, since for the post-1980 data the empirical elasticity is $-0.24$, while the model’s elasticity is $-0.58$. Below, we argue that this is mainly because of a period of exceptionally high interest rates in the 1980s. The simulated money demand properties under competitive pricing are shown in Figure 10.

— INSERT FIGURE 10 —

5.2 Taking out the 1980s

The 1980s characterize a period of exceptionally high interest rates. From 1980 to 1989, the elasticity of money demand is only $-0.23$, which strongly affects the elasticity for the entire post-1980 data (where we obtain a value of -0.24). Without the 1980s, the elasticity increases to $-0.31$. In what follows, we treat the period from 1980 to 1989 as outliers by using the parameters calibrated to the period from 1950 to 1979, and then search numerically for the value of $\pi$ that minimizes the squared error between the model-implied money demand and the data in period from 1990 to 2013. The simulation results are shown in Table 9.

— INSERT TABLE 9 —

The above table shows that the model is again able to replicate the average velocity in the post-1990 data. In addition, the model’s elasticity of $-0.45$ is much closer to the empirical elasticity of $-0.31$ when we exclude the 1980s. We conclude that if we take out 1980s and assume that the one-time shift in $\pi$ occurred in 1980, our model does a reasonable job in replicating the observed break in money demand.

5.3 Discussion

The last two sections have shown that our model is better able to replicate the money demand behavior if we assume that the "innovation" occurred at the beginning of the 1990s rather than at the beginning of the 1980s. The initial introduction of money market deposit accounts occurred in the 1980s and the introduction of the sweep technology in the retail sector at the beginning of the 1990s. In what follows, we argue that the introduction of money market deposit accounts played a lesser role. The reason is that from 1980 to 1989 the average
share of money market mutual funds relative to M1S increased from zero to only 39 percent. However, after the introduction of the sweep technology in the early 1990s the average share of money market mutual funds relative to M1S increased further and averaged 93 percent in the period spanning 1990 to 2013. The development of the share of money market mutual funds relative to M1S is shown in Figure 11 below.

— INSERT FIGURE 11 —

In any case, money market deposit accounts and the sweep technology simultaneously affected money demand. In particular, a sweep technology without MMDAs would have no effects on money demand, since this technology requires that funds deposited in checking accounts can be swept to MMDAs. Our model suggests, however, that only after the sweep technology was introduced did the large changes to money demand occur.

6 LITERATURE

In this section, we discuss several papers in more detail and relate their results to ours.

Lucas and Nicolini (2013). A closely related paper is Lucas and Nicolini (2013). They first document the empirical breakdown of the previously stable relationship between M1 and interest rates that occurred in the 1980s. They, then, discuss the possible factors, such as financial deregulation, that could explain the observed breakdown. Finally, they construct a novel monetary aggregate called NewM1 and show that there is stable negative relationship between NewM1 and the relevant opportunity cost of holding the various components of NewM1. In the theory part of their paper, the monetary aggregate is derived endogenously according to the different role played by currency, reserves, and commercial bank deposits as means of payment. They assume two means of exchange (cash and checks). Consumption goods are of different sizes and the use of checks in transactions is profitable only if the size of the consumption good is sufficiently large. They find that the new monetary aggregate performs as well on low and medium frequencies during the period 1915-2008, as was the case with M1 for the period 1915-1990.

At the time of our writing, we had no access to their new monetary aggregate NewM1.

Faig and Jerez (2007). Our paper is also closely related to Faig and Jerez (2007), who study money demand and money velocity in a search model with villages, where money
is necessary for goods transactions. They assume that buyers are subject to idiosyncratic preference shocks, and only a fraction of them \((1 - \theta)\) can readjust their money holdings before trading in the goods market. This generates a role for the precautionary demand for money. Using United States data from 1892 to 2003, Faig and Jerez (2007) show that the demand for money and the welfare cost of inflation decreased dramatically at the end of the sample. In Table 3, they show that the welfare cost of inflation was 0.15 percent in 2003 as opposed to 1 percent for most of the 20th century.\(^{21}\) Their estimates of \(\theta\) are decreasing over time with \(\theta\) being equal to 1 in 1892, and 0.139 in 2003. They also document that the demand for precautionary balances almost halved in the last part of the sample, being 47 percent in 2003 as opposed to over 80 percent for all the preceding years, while the velocity of money showed an upward trend over the second half of the past century.

**Berentsen, Menzio and Wright (2011, BMW hereafter).** In the extension section, BMW introduce bilateral trade credit into the Lagos and Wright (2005) framework to investigate whether this modification can account for the observed downward shift in money demand. They find that it can account for this to a large extent, but that the elasticity of money demand moves in the wrong direction. To replicate the effects of bilateral trade credit, we perform the same experiment as BMW. That is, we assume that until 1990 the probability that a bilateral meeting between a buyer and a seller is non-anonymous is zero; note that bilateral credit is feasible in non-anonymous meetings. After 1990, the probability that such a meeting is non-anonymous is \(\pi_1 = 0.30\) (see Figure 12). The value of \(\pi_1 = 0.30\) is chosen numerically, such that the squared error between the model-implied money demand and the data is minimized.

--- INSERT FIGURE 12 ---

Introducing bilateral trade credit results in a downward shift in money demand and an increase in the elasticity of money demand, which conflicts with the data. In Table 10, we compare the effects of a one-time increase in \(\pi\) in BMW (i.e., from \(\pi_0 = 0\) to \(\pi_1 = 0.30\)) with the optimal shift suggested by our model (i.e., from \(\pi_0 = 0\) to \(\pi_1 = 0.60\)).

--- INSERT TABLE 10 ---

As can be seen from Table 10, modeling financial intermediation through a money market as compared to trade credit in BMW, allows us to replicate the change in the elasticity of money demand more accurately.
Reynard (2004). Reynard (2004) studies the stability of money demand in the United States using cross-sectional data. In particular, he relates the evolution of financial market participation to the downward shift in the money demand and its higher interest rate elasticity observed in 1970s. Agents who participate in the financial market can hold both money and non-monetary assets (NMAs), whereas the latter are assets that can be converted into money by paying a transaction cost —examples of NMAs are certificates of deposits, stocks, and bonds. Agents who do not participate in the financial market can only hold money. Reynard (2004) shows that an important component of financial market participation is the household’s real financial wealth, which increased steadily during the 1960s and 1970s, and that the probability of holding NMAs is positively related to the household’s wealth. He then uses the measure of asset market participation to estimate the stability of money demand. He finds that, as real wealth and the opportunity cost of holding money increased during the 1970s, a higher portion of the population decided to participate in the financial market and hold part of their wealth in NMAs. This led to an increase in the interest rate elasticity of money demand, since only agents who participate in the financial market can adjust their portfolio of money and NMAs when interest rates change. Thus, using cross-sectional data, Reynard (2004) concludes that the money demand remains stable during the post war period, while previous time-series studies "inappropriately" suggest instability, and their estimates of the interest rate elasticity are "flawed".

Teles and Zhou (2005). Teles and Zhou (2005) also observe that the stable relationship between M1 and the interest rate broke down at the end of the 1970s. Their view is that M1 ceased to be a good measure of the transaction demand for money after 1980. Before 1980, there was a clear distinction between M1 and M2: M1 could be used for transactions and did not yield any rate of return, while M2 offered a positive rate of return, but could not be used for transactions. Since 1980, this distinction vanished due to changes in regulation, the development of electronic payments, and the introduction of retail sweep programs. Teles and Zhou (2005) show that an appropriate measure of the transaction demand for money after 1980 is provided by the money zero maturity aggregate (MZM). This money aggregate includes financial instruments that can be used for transaction immediately at zero cost. They show that the long-run relationship between the money demand and the interest rate is restored when M1 is used for the period 1900-1979 and MZM for the period 1980-2003.
Ireland (2009). Lucas’s (2000) quantifies the welfare costs of inflation. Ireland (2009) revisits the results of Lucas (2000) by using newly available data for the period from 1995 to 2006. This is the period during which sweep retail programs were introduced, thereby severely distorting the role of M1 as a measure of the transaction demand for money. Owing to the change in the nature of M1, a new aggregate, M1RS, was used in Ireland’s (2009) estimations; M1RS is computed by adding the value of sweep funds into M1. To isolate the recent behavior of the money demand, Ireland (2009) focuses on two subperiods, 1980-2006 and 1900-1979. He shows that the relationship between M1RS and the interest rate remains stable in the period after 1980, but that the relationship is different from the one highlighted by Lucas (2000). He finds that the modest growth of M1RS observed in earlier data can be better explained by a semi-log specification of the money demand, as opposed to the log-log specification proposed by Lucas (2000). Furthermore, the interest rate elasticity of the money demand seems to be much lower in 1980-2006 data than in 1900-1979 data. Both these changes lead to estimates of the welfare cost of inflation which are lower than those presented in Lucas (2000).

VanHoose and Humphrey (2001). VanHoose and Humphrey (2001) study the effect of the introduction of retail sweep accounts on bank reserves and the ability of the Federal Reserve to conduct monetary policy. In particular, they investigate the effect of lower required reserve balances on funds-rate volatility and monetary policy in a model of optimal bank reserve management. They document that the introduction of the retail sweep accounts that began in 1993 reduced the required bank reserves at the FED by 70 percent. Theoretically, VanHoose and Humphrey show that lower bank reserves have an ambiguous effect on the fund-rate volatility. On the one hand, lower reserve requirements reduce the sensitivity of the demand for reserves and funds borrowing to variations in the Fed-funds rate, which makes the funds rate more volatile. On the other hand, lower reserve requirements, increase, for any level of total reserve balances, the portion of reserves the banks can use to cover unexpected payments, which ultimately reduce the overnight funds demand and so the volatility of the overnight funds rate. As a result, the composite effect of lower reserve balances on the funds-rate volatility is ambiguous. Moreover, the higher Fed-funds rate volatility, which may be triggered by the reduced reserve balances, can be transmitted to the yield curve and raise the volatility of the short-term interest rate, thereby affecting the effectiveness of monetary policy. Empirically, VanHoose and Humphrey test for this possibility. They find that lower reserve balances increase the short-term interest rate volatility only before the period where
the Fed publicly announced the target for the funds rate. After the funds-rate target was announced, the effect of lower reserve requirements on the short-term interest rates was not significant.

**Baumol-Tobin cash-management models.** The impact of financial innovation on money demand and money velocity has also been studied using Baumol-Tobin cash-management models (e.g., Attanasio, Guiso and Jappelli 2002, and Alvarez and Lippi 2009). Using micro data from an Italian survey for the period 1989 - 1995, Attanasio, Guiso and Jappelli (2002) study the implications of the Automated Teller Machine (ATM) card adoption on money demand, the interest, and the expenditure elasticity of money demand, and the welfare cost of inflation. They show that the interest-rate elasticity for households with an ATM card is twice as large as it is for households which do not possess one, i.e., $0.59$ as opposed to $0.27$ (Table 3, p.331). Overall, they estimate a welfare cost of inflation that equates to 0.06 percent of nondurable consumption. They also show that the welfare cost of inflation is higher for households with an ATM card (0.09 percent) than for households without one (0.05 percent), and that it is declining over time for each household’s type (Table 4, p.339).

Using the same data set from 1993 to 2004, Alvarez and Lippi (2009) estimate the effect of ATM card use on money demand in a model with random withdrawal arrival rates. They estimate an interest rate elasticity of money demand equal to 0.43 for households with ATM cards, and 0.48 for households without them (p.391). They also show that, as a result of the financial innovation, the welfare loss of inflation in 2004 is approximately 40 percent smaller than it was in 1993 (Table VII, p.394).

## 7 CONCLUSION

At the beginning of the 1990s, the empirical relationship between M1 and interest rates began to fall apart. In this paper, we ask what accounts for this shift and the lower interest-rate elasticity of money demand. To answer this question, we construct a microfounded monetary model with a money market. Agents face idiosyncratic liquidity shocks which generate an ex-post inefficient allocation of the medium of exchange: some agents will hold cash, but have no current need for it, while other agents will hold insufficient cash for their liquidity needs. We find that the money market affects money demand via two channels. First, it allows agents who hold cash, but have no current need for it, to earn interest. Second, it allows agents who hold insufficient cash to borrow and, by doing so, reallocates the existing...
stock of cash more efficiently.

We calibrate the model to U.S. data and find that a one-time increase in the access probability to the money market at the beginning of the 1990s replicates the behavior of money demand well. This result suggests that the introduction of the sweep technology in the 1990s is to a large extent responsible for the observed empirical changes in money demand.
8 APPENDIX A: PROOFS

Proof of Proposition 1. In order to derive equations (8)-(11), we first characterize the solutions to the agent’s decision problems stated in the text.

The first-order conditions of the agent’s problem (1) are

\[ U'(x) = 1, \quad \text{and} \quad \frac{\beta V_1}{\partial m_{+1}} = \phi. \] (16)

The term \( \frac{\beta V_1}{\partial m_{+1}} \) reflects the marginal value of taking one additional unit of money into the next period, and \( \phi \) is the marginal cost of doing so. As in Lagos and Wright (2005), the choice of \( m_{+1} \) is independent of \( m \). As a result, each agent exits the centralized market with the same amount of money, and so the distribution of money holdings is degenerate at the beginning of a period. The envelope conditions are

\[ \frac{\partial V_3}{\partial m} = \phi, \quad \frac{\partial V_3}{\partial d} = \phi (1 + i), \quad \text{and} \quad \frac{\partial V_3}{\partial \ell} = -\phi (1 + i). \] (17)

The marginal value of money at the beginning of the centralized market is equal to the price of money in terms of centralized market goods. This implies that the value function \( V_3 \) is linear in \( m \). The value function for a buyer in the goods market is

\[ V^b_2(m, \ell, 0) = \delta [u(q) + V_3(m - z, \ell, 0)] + (1 - \delta) V_3(m, \ell, 0). \]

The buyer’s envelope conditions are

\[ \frac{\partial V^b_2}{\partial m} = \delta \left[ u'(q) \frac{\partial q}{\partial m} + \phi \left( 1 - \frac{\partial z}{\partial m} \right) \right] + (1 - \delta) \phi, \quad \text{and} \quad \frac{\partial V^b_2}{\partial \ell} = -\phi (1 + i). \]

If the buyer’s constraint (3) is not binding, then \( \frac{\partial q}{\partial m} = 0 \) and \( \frac{\partial z}{\partial m} = 0 \). In this case, the buyer’s first envelope condition reduces to \( \frac{\partial V^b_2}{\partial m} = \frac{\partial V_3}{\partial m} = \phi \). If the constraint is binding, then \( \frac{\partial q}{\partial m} = \frac{\phi}{g'(q)} \) and \( \frac{\partial z}{\partial m} = 1 \). In this case, the buyer’s envelope conditions in the goods market become

\[ \frac{\partial V^b_2}{\partial m} = \delta \phi \frac{u'(q)}{g'(q)} + \phi (1 - \delta), \quad \text{and} \quad \frac{\partial V^b_2}{\partial \ell} = -\phi (1 + i). \] (18)

The value function for a seller in the goods market is

\[ V^s_2(m, 0, d) = \delta [-q + V_3(m + z, 0, d)] + (1 - \delta) V_3(m, 0, d), \]
and envelope conditions are
\[
\frac{\partial V^s_2}{\partial m} = \phi, \quad \text{and} \quad \frac{\partial V^s_2}{\partial d} = \phi (1 + i).
\]

(19)

The first-order condition of the buyer’s problem (6) is
\[
\frac{\partial V^b_2}{\partial m} + \frac{\partial V^b_2}{\partial \ell} = 0.
\]

(20)

The first-order condition of the seller’s problem (7) in the money market is
\[
-\frac{\partial V^s_2}{\partial m} + \frac{\partial V^s_2}{\partial d} = \lambda_s.
\]

(21)

The envelope condition of (5) is
\[
\frac{\partial V_1}{\partial m} = \pi \left[ (1 - n) \frac{\partial V^b_1}{\partial m} + n \left( \frac{\partial V^s_2}{\partial m} + \lambda_s \right) \right] + (1 - \pi) \left[ (1 - n) \frac{\partial V^b_2}{\partial m} + n \frac{\partial V^s_2}{\partial m} \right].
\]

Applying the envelope theorem to (6) and (7), the above envelope condition can be rewritten as
\[
\frac{\partial V_1}{\partial m} = \pi \left[ (1 - n) \frac{\partial V^b_2}{\partial m} + n \left( \frac{\partial V^s_2}{\partial m} + \lambda_s \right) \right] + (1 - \pi) \left[ (1 - n) \frac{\partial V^b_2}{\partial m} + n \frac{\partial V^s_2}{\partial m} \right].
\]

(22)

We can now derive the type-A equilibrium equations (8)-(11).

**Derivation of (8).** The real amount of money an active buyer spends in the goods market, \( g(\hat{q}) \), is equal to the real amount of money spent as a passive buyer, \( g(q) \), plus the real loan an active buyer receives from the bank, \( \phi \ell \).

**Derivation of (9) and (10).** Assuming \( \lambda_s = 0 \), (21) becomes
\[
-\frac{\partial V^s_2}{\partial m} + \frac{\partial V^s_2}{\partial d} = 0.
\]

(23)

Substituting \( \frac{\partial V^b_1}{\partial m}, \frac{\partial V^b_1}{\partial \ell}, \frac{\partial V^s_2}{\partial m}, \) and \( \frac{\partial V^s_2}{\partial d} \) from (18) and (19), the first-order conditions in the money market, (20) and (23), can be written as (9) and (10), respectively.

**Derivation of (11).** From (9) and (10), \( u'(\hat{q}) = g'(\hat{q}) \). Use \( \lambda_s = 0 \), (18), (19), and \( u'(\hat{q}) = g'(\hat{q}) \), to rewrite (22) as follows:
\[
\frac{\partial V_1}{\partial m} = \phi + (1 - \pi) (1 - n) \phi \delta \left[ \frac{u'(q)}{g'(q)} - 1 \right].
\]
Update this expression by one period and replace $\frac{\partial V_{b}}{\partial m}$ using (16), to obtain (11). ■

**Proof of Proposition 2.** Equations (12)-(15) hold in a type-B equilibrium. Equation (12) is equal to (8), so we refer to the proof of Proposition 1 for its derivation.

**Derivation of (13).** Substituting $\frac{\partial V_{b}}{\partial m}$, $\frac{\partial V_{b}}{\partial \ell}$, $\frac{\partial V_{s}}{\partial m}$, and $\frac{\partial V_{s}}{\partial d}$ from (18) and (19), the first-order conditions in the money market, (20) and (21), become

$$
(13), \quad \text{and} \quad \phi i = \lambda_{s}, \quad (24)
$$

respectively. Unlike the type-A equilibrium, the deposit constraint is binding here, and therefore the interest rate is strictly greater than zero in a type-B equilibrium. The second equation in (24) gives us the value of the multiplier.

**Derivation of (14).** In a type-B equilibrium, active sellers deposit all their money at the bank; i.e., $d = m$. Moreover, active buyers carry $\hat{m}$ units of money out of the money market, where $\hat{m} = m + \ell$, and the market clearing condition in the money market requires that total deposits must be equal to total loans; i.e., $\pi nd = \pi (1 - n) \ell$. Using $d = m$ and $\hat{m} = m + \ell$, the market clearing condition in the money market can be rewritten as $m = (1 - n) \hat{m}$. Multiplying each side of the last equation by $\phi$, and using (4), we obtain (14).

**Derivation of (15).** Use (24) and the envelope conditions in the goods market, (18) and (19), to rewrite the money market envelope condition (22) as follows

$$
\frac{\partial V_{1}}{\partial m} = \pi \phi \left[ \delta u'(\hat{q}) + 1 - \delta \right] + (1 - \pi) \phi \left[ (1 - n) \left( \delta \frac{u'(q)}{g'(q)} + 1 - \delta \right) + n \right].
$$

Finally, update this expression by one period and replace $\frac{\partial V_{1}}{\partial m}$ using (16), to obtain (15). ■

9 **APPENDIX B: PRICING MECHANISMS**

The derivation of the terms of trade using the Nash bargaining approach is today a standard practice in Lagos-Wright-type models. In recent years, however, other pricing mechanisms such as Kalai bargaining and competitive pricing have received attention.

**Kalai bargaining.** Unlike the Nash bargaining solution, the egalitarian solution proposed by Kalai (1977) is strongly monotonic in the sense that no agent is made worse off from an expansion of the bargaining surplus. Because of this property, the Kalai solution has
been increasingly used in monetary economics. The Kalai bargaining problem is to solve

\[(q, z) = \arg \max u(q) - \phi z \]
\[s.t. \quad u(q) - \phi z = \theta [u(q) - q] \quad \text{and} \quad z \leq m.\]

When the buyer’s cash constraint is binding, i.e., \(m = z\), the solution to this problem is

\[\phi m = g^K(q) \equiv \theta q + (1 - \theta) u(q).\] (25)

If \(m = z\), the Kalai solution is different from the Nash solution, unless \(\theta = 0\) or \(\theta = 1\); if \(z < m\), Nash bargaining and Kalai bargaining yield the same solution. In order to adapt the model to Kalai bargaining, we only need to replace \(g(q)\) with \(g^K(q)\), where the superscript \(K\) stands for Kalai solution.

**Competitive market.** Assume competitive pricing in the goods market. Then, buyers and sellers do not bargain over the terms of trade. Instead, they take the price as given in this market.

Under competitive pricing, it is natural to interpret \(\delta\) and \(\delta^s\) as participation probabilities. In particular, let \(\delta\) (\(\delta^s\)) be the probability that a buyer (seller) participates in the goods market. Then, the value function of a buyer at the opening of the goods market is

\[V^h_2(m, \ell) = \delta \max_q \left[ u(q) + V^3_3(m - pq, \ell) \right] + (1 - \delta) V^3_3(m, \ell),\] (26)

where \(p\) is the price, and \(q\) the quantity of goods he consumes if he enters the goods market. The first-order condition to this problem is

\[u'(q) = p (\phi + \lambda_q),\] (27)

where \(\lambda_q\) denotes the Lagrange multiplier on the cash constraint, \(m \geq pq\).

The seller’s value function at the opening of the goods market is

\[V^s_2(m, d) = \delta^s \max_{qs} [-qs + V^3_3(m + pq, d)] + (1 - \delta^s) V^3_3(m, d).\] (28)

The first-order condition to this problem is

\[p\phi = 1.\] (29)
If \( m > pq \), the buyer consumes the efficient quantity \( q^* \), where \( q^* \) solves \( u'(q^*) = 1 \). If \( m = pq \), he spends all his money and consumes \( q < q^* \). Note that, in equilibrium, an active buyer holds more money than a passive buyer. This means that \( \lambda_q > \hat{\lambda}_q \). It then follows that \( \hat{q} > q \).

The buyer’s envelope conditions are

\[
\frac{\partial V^b_2}{\partial m} = \phi [\delta u'(q) + 1 - \delta] \quad \text{and} \quad \frac{\partial V^b_2}{\partial \ell} = -\phi (1 + i),
\]

where we have used (17), (27), and (29). Notice the similarity between (30) and (18). The two expressions are the same if \( \theta = 1 \).

The seller’s envelope conditions are exactly the same as (19); i.e., \( \frac{\partial V^s_1}{\partial m} = \phi \), and \( \frac{\partial V^s_2}{\partial d} = \phi (1 + i) \).

Using the buyer’s budget constraint at equality (i.e., \( pq = m \)) and (29) we obtain

\[
\phi m = g^C(q) \equiv q,
\]

where the superscript \( C \) stands for competitive pricing.

Finally, note that under competitive pricing the goods market clearing condition holds, i.e.,

\[
\delta (1 - n) [\pi \hat{q} + (1 - \pi)q] = \delta^s n q_s,
\]

where \( \hat{q} (q) \) is the quantity consumed by a buyer who has (does not have) access to the money market.

10 APPENDIX C: DATA SOURCES

The data we use for the calibration is provided by the U.S. Department of Commerce: Bureau of Economic Analysis (BEA), the Board of Governors of the Federal Reserve System (BGFRS), the Federal Reserve Bank of St. Louis (FRBL), and the U.S. Department of Labor: Bureau of Labor Statistics (BLS). Table C1 gives an overview about the data sources that we used for the quantitative analysis.

Insert Table C1 around here.

As the time series of M1 adjusted for retail sweeps of the Federal Reserve Bank of St. Louis is only available from 1967:Q1, we use the series M1SL as a measure of the M1 for the period from 1959:Q1 to 1966:Q4 and the series M1SA for the period from 1950:Q1 to
1958:Q4 (downloadable at http://research.stlouisfed.org/aggreg/). The definition that we apply to calculate the quarterly value of M1SA is in line with the Federal Reserve Bank of St. Louis FRED® database and is defined as the average of the monthly data.

References


Notes

1 Throughout the paper, we use M1S. M1S is called M1RS in Cynamon, Dutkowsky, and Jones (2006b) and M1ADJ in the St. Louis FRED® database. Detailed information on M1S is available in Cynamon, Dutkowsky, and Jones (2006b, 2007).

2 We use the term financial innovation for three complementary scenarios: New financial products can originate from advances in information technology and science, from changes in financial regulation, or from both. A case in point is the Glass-Steagall prohibition against paying interest on commercial bank deposits, which was in force until 2011. Relaxation of this regulation in the 1980s and 1990s spurred a range of financial innovations; such as, MMDAs in the 1980s or sweep accounts in the 1990s (see Teles and Zhou 2005, and Lucas and Nicolini 2013). Consequently, allowing for MMDAs and the sweep technology were ultimately policy decisions. When it became clear that the costs of inflation were serious, policy makers allowed these innovations to reduce the cost of inflation.

3 A MMDA (money market deposit account) is a checking account where the holder is only allowed to make a few withdrawals per month.

4 The monetary model is the Lagos and Wright (2005) framework, and the money market is the same as the one introduced in Berentsen, Camera and Waller (2007, BCW hereafter). Our theoretical contribution is that we consider a limited participation version of BCW.

5 The experiment is conducted for three trading protocols: Nash bargaining, Kalai bargaining, and competitive pricing. These different pricing protocols generate different quantitative results, but the results are of an equal qualitative nature.

6 Anderson and Rasche (2001, p. 71) conclude their paper that “the use of deposit-sweeping software has made statutory reserve requirements a ‘voluntary constraint’ for most banks. That is, with adequately intelligent software, many banks seem easily to be able to reduce their transaction deposits by a large enough amount that the level of their required reserves is less than the amount of reserves that they require for day-to-day operation of the bank. For these banks at least, the economic burden of statutory reserve requirements is zero.”

7 The literature on the welfare cost of inflation was initiated by Bailey (1956) and Friedman (1969). Subsequent works include, but are not limited to, Fischer (1981), Lucas (1981), and Cooley and Hansen (1989, 1991). Most of these papers are cash-in-advance or money-in-the-utility-function models.

8 The basic environment is similar to BCW which builds on Lagos and Wright (2005). The Lagos and Wright framework is useful, because it allows us to introduce heterogeneity while still keeping the distribution of money holdings analytically tractable.

9 In an earlier version of this paper, we also consider limited commitment of borrowers via banks as in BCW. Since limited commitment did not affect our calibration results much, we do not present this case here.

10 It is routine to show that the first-best quantities satisfy $U' (x^*) = 1$ and $u'(q^*) = 1$.

11 The shapes of the curves in Figure 4 do not change qualitatively for $\theta < 1$.

12 In the model, $i_b$ is the interest of a riskless bond that is issued in the settlement market of some period $t$ and redeemed in the settlement market of period $t + 1$. Arbitrage guarantees that $i_b = \gamma / \beta - 1$. Since the central bank chooses the growth rate of the money supply $\gamma$ and $\beta$ is a preference parameter, it is convenient to assume that the central bank chooses $i_b$. 
We also experimented with different values of \( n \) and found that for \( n > 0.5 \) the shift in money demand when changing \( \pi \) from \( \pi_0 = 0 \) to \( \pi_1 > 0 \) becomes more pronounced. However, when we search numerically for the value of \( \pi \) that minimizes the squared error between the model-implied money demand and the data, the money demand properties of the model remain essentially unchanged as compared to \( n = 0.5 \).

We use the ratio of nominal GDP to M1 as our target for the velocity of money. Money demand is the inverse of the velocity of money. Sweep accounts were only introduced in the early 1990s, thus there is no difference between M1 and M1S in the pre-1990 data.

Aruoba, Waller and Wright (2011) and Berentsen, Menzio and Wright (2011), also use an average markup of 30 percent. This is the value estimated by Faig and Jerez (2005) for the United States. See also Christopoulou and Vermeulen (2008) for an estimated markup of 32 percent.

The model’s velocity of money is derived as follows: The real output in the goods market is \( Y_{GM} = (1 - n) \delta [\pi \hat{\phi} M + (1 - \pi) \hat{\phi} M] \), where \( \hat{\phi} M = g(\hat{q}) \) and \( \phi M_{-1} = \hat{\phi} M = g(q) \), and the real output in the centralized market is \( Y_{CM} = A \) for \( U(x) = A \log(x) \). Accordingly, the total real output of the economy is \( Y = Y_{GM} + Y_{CM} \), and the model-implied velocity of money is \( v = Y / \phi M_{-1} \).

The markup-target is only used for the calibrations under Nash bargaining and Kalai bargaining, as the markup is by definition zero under competitive pricing.

This is the same measure adopted by Craig and Rocheteau (2008).

The reason behind the higher welfare cost of inflation for the 1950 - 1979 period is mainly attributed to the higher elasticity of money demand. In order to match \( \xi = -0.65 \), a lower value of \( \alpha \) is required, which results in a less concave utility function. In turn, with a less concave utility function, agents are willing to give up a higher percentage of total consumption in order to be in a steady state with a nominal interest rate of 3 percent instead of 13 percent.

MMMFs and MMDAs both invest in short-term fixed income investments. The difference between the two instruments is that MMDAs are insured by the government, while MMMFs are not. We use the development of MMMFs as a proxy for the development of the money market in the United States.

When deriving the welfare cost of inflation, Faig and Jerez (2007) consider an increase of the inflation rate from 0 percent to 10 percent.

In Baumol (1952) and Tobin (1956), agents face a cash-in-advance constraint, and money can be exchanged for other assets at a cost; two well-known extensions of the Baumol-Tobin model are Grossman and Weiss (1983) and Rotemberg (1984). Examples of inventory-theoretic models of money demand with market segmentation are Alvarez, Lucas and Weber (2001), Alvarez, Atkeson and Kehoe (2002), and Alvarez, Atkeson and Edmond (2009). Silber (1983) provides a survey of the financial innovations that occurred in the period from 1950 to 1980. He argues that both financial innovations and technological changes respond to economic incentives, and that both are welfare-improving. In particular, he documents that financial innovation improves protection against risk and reduces transaction costs.

Some recent extensions of Baumol (1952) and Tobin (1956) have studied exogenous versus endogenous market segmentation (Alvarez, Atkeson and Edmond 2009, and Chiu 2014). In these models, agents decide to transfer the money from the goods market to the credit market periodically. As a result, only a fraction of them are able to trade in the credit market at a given point in time. These papers do not investigate the effect of financial innovation on the precautionary demand for money.

Some previous studies on cross-sectional household data report elasticities smaller than 0.25 (e.g., Lippi
and Secchi 2009, and Daniels and Murphy 1994).

25 One of the first papers to use the Kalai approach in Lagos-Wright-type models is Arouba, Rocheteau and Waller, (2007). Other applications that followed are Rocheteau and Wright (2013), Lester, Postlewaite and Wright (2012), He, Wright and Zhu (2012), and Trejos and Wright (2012).
Figure 1: M1 money demand in the United States
Figure 2: Simulated financial innovation
Figure 3: Consumed quantities
Figure 4: Simulated money demand properties
Figure 5: Money demand with optimal market access
Figure 6: Initial value of $\pi_0 = 0.2$
Figure 7: Money demand with interest-bearing money
Figure 8: Money demand with interest-free credit
Figure 9: M1 Money demand with optimal market access
Figure 10: One-time shift in $\pi$ in 1980.
Figure 11: MMMFs relative to M1S
Figure 12: Simulated bilateral trade credit
**Table 1: Calibration targets**

<table>
<thead>
<tr>
<th>Target description</th>
<th>Target value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average real interest rate $r$</td>
<td>0.027</td>
</tr>
<tr>
<td>Average AAA yield</td>
<td>0.070</td>
</tr>
<tr>
<td>Average velocity of money</td>
<td>5.217</td>
</tr>
<tr>
<td>Retail sector markup</td>
<td>0.300</td>
</tr>
<tr>
<td>Elasticity of money demand</td>
<td>-0.619 (0.011)</td>
</tr>
</tbody>
</table>

*a* Table 1 reports the calibration targets and the target values. It also reports the model fit. As explained below we can fit all target values exactly. The number in parentheses refers to the standard error.
Table 2: Baseline Calibration from 1950 to 1989$^a$

<table>
<thead>
<tr>
<th></th>
<th>Nash Bargaining</th>
<th>Kalai Bargaining</th>
<th>Competitive Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A$</td>
<td>2.147</td>
<td>2.638</td>
<td>2.242</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.389</td>
<td>0.377</td>
<td>0.309</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.724</td>
<td>0.760</td>
<td>1.0</td>
</tr>
</tbody>
</table>

$1 - \Delta$   | 1.96%           | 1.64%            | 1.23%               |

$^a$Table 2 displays the calibrated values for the key parameters $A$, $\alpha$ and $\theta$ for $\pi_0 = 0$. Table 2 also displays the welfare cost of inflation, $1 - \Delta$, which is the percentage of total consumption that agents would be willing to give up in order to be in a steady state with a nominal interest rate of 3 percent instead of 13 percent.
<table>
<thead>
<tr>
<th></th>
<th>Data (1990-2013)</th>
<th>Nash Bargaining</th>
<th>Kalai Bargaining</th>
<th>Comp. Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_1$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Velocity</td>
<td>6.41</td>
<td>7.05</td>
<td>7.17</td>
<td>7.11</td>
</tr>
<tr>
<td>Elasticity</td>
<td>-0.31 (0.021)</td>
<td>-0.34 (0.004)</td>
<td>-0.31 (0.004)</td>
<td>-0.33 (0.004)</td>
</tr>
<tr>
<td>$1 - \Delta_{\pi=1}$</td>
<td>1.04%</td>
<td>0.68%</td>
<td>0.53%</td>
<td></td>
</tr>
</tbody>
</table>

*Table 3 displays the simulation results of the velocity of money and the elasticity of money demand with respect to the AAA interest rate after a one-time increase in the access probability to the money market from $\pi_0 = 0$ to $\pi_1 = 1$ in 1990. Table 3 also displays the welfare cost of inflation with $\pi_1 = 1$, $1 - \Delta_{\pi=1}$. The numbers in parentheses refer to the standard errors.*
Table 4: Optimal market access

<table>
<thead>
<tr>
<th></th>
<th>Data (1990-2013)</th>
<th>Nash Bargaining</th>
<th>Kalai Bargaining</th>
<th>Comp. Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_1^*$</td>
<td>0.64</td>
<td>0.58</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td>6.41</td>
<td>6.41</td>
<td>6.40</td>
<td>6.39</td>
</tr>
<tr>
<td>Elasticity</td>
<td>-0.31 (0.021)</td>
<td>-0.40 (0.005)</td>
<td>-0.39 (0.005)</td>
<td>-0.40 (0.005)</td>
</tr>
<tr>
<td>$1 - \Delta_{x_1^*}$</td>
<td>1.22%</td>
<td>0.92%</td>
<td>0.69%</td>
<td></td>
</tr>
<tr>
<td>$\tilde{t}$</td>
<td>2.04%</td>
<td>2.62%</td>
<td>2.39%</td>
<td></td>
</tr>
</tbody>
</table>

*aTable 4 displays the simulation results of the velocity of money and the elasticity of money demand with respect to the AAA interest rate after a one-time increase in the access probability to the money market from $\pi_0 = 0$ to the optimal value of $\pi_1^*$ in 1990. Table 4 also displays the welfare cost of inflation with the optimal value of $\pi_1^*$, $1 - \Delta_{x_1^*}$. The table also shows the critical interest rate, $\tilde{t}$, that separates the type-A equilibrium from the type-B equilibrium. The numbers in parentheses refer to the standard errors.*
### Table 5: Competitive pricing - calibration from 1950 to 1989

<table>
<thead>
<tr>
<th>Calibration</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_0 )</td>
<td>1.868</td>
<td>1.513</td>
<td>1.318</td>
<td>1.177</td>
</tr>
<tr>
<td>( A )</td>
<td>0.278</td>
<td>0.231</td>
<td>0.204</td>
<td>0.183</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1.25%</td>
<td>1.25%</td>
<td>1.25%</td>
<td>1.26%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_1 )</td>
<td>1</td>
</tr>
<tr>
<td>Velocity</td>
<td>6.41</td>
</tr>
<tr>
<td>Elasticity</td>
<td>-0.31 (0.021)</td>
</tr>
<tr>
<td>( 1 - \Delta )</td>
<td>0.66%</td>
</tr>
</tbody>
</table>

\( a \)The first part of Table 5 displays the calibrated values for the key parameters \( A \) and \( \alpha \) for \( \pi_0 = 0.2, 0.4, 0.6, \) and 0.8 under competitive pricing. Table 5 also displays the welfare cost of inflation under the initial value of \( \pi_0, 1 - \Delta \). The second part of Table 5 displays the simulation results of the velocity of money and the elasticity of money demand with respect to the AAA interest rate after a one-time increase in the access probability to the money market from the initial value of \( \pi_0 \) to \( \pi_1 = 1 \) in 1990. Table 5 also displays the welfare cost of inflation with \( \pi_1 = 1, 1 - \Delta \pi_1 \). The numbers in parentheses refer to the standard errors.
Table 6: M1 elasticities

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>9.12</td>
<td>8.27</td>
<td>8.66</td>
<td>8.77</td>
<td>8.85</td>
<td>8.90</td>
</tr>
<tr>
<td>Elasticity</td>
<td>-0.40 (0.005)</td>
<td>+0.16 (0.067)</td>
<td>-0.10 (0.075)</td>
<td>-0.20 (0.070)</td>
<td>-0.29 (0.065)</td>
<td>-0.37 (0.064)</td>
</tr>
</tbody>
</table>

*aTable 6 displays the velocity of M1 and the elasticity of M1 money demand with respect to the AAA interest rate for several subperiods in the post-1990 data. For the simulation, we assumed competitive pricing in the goods market. The numbers in parentheses refer to the standard errors.*
Table 7: Calibration from 1950 to 1979

<table>
<thead>
<tr>
<th></th>
<th>Nash Bargaining</th>
<th>Kalai Bargaining</th>
<th>Competitive Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A$</td>
<td>1.881</td>
<td>2.333</td>
<td>1.971</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.341</td>
<td>0.336</td>
<td>0.254</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.675</td>
<td>0.707</td>
<td>1.0</td>
</tr>
<tr>
<td>$1 - \Delta$</td>
<td>2.26%</td>
<td>2.04%</td>
<td>1.40%</td>
</tr>
</tbody>
</table>

$^a$Table 7 displays the calibrated values for the key parameters $A$, $\alpha$ and $\theta$ for $\pi_0 = 0$.

Table 7 also displays the welfare cost of inflation, $1 - \Delta$. 

58
Table 8: Optimal market access\textsuperscript{a}

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_1^* )</td>
<td>0.71</td>
<td>0.68</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td>6.64</td>
<td>6.97</td>
<td>7.03</td>
<td>6.99</td>
</tr>
<tr>
<td>Elasticity</td>
<td>-0.24 (0.012)</td>
<td>-0.58 (0.008)</td>
<td>-0.58 (0.009)</td>
<td>-0.58 (0.009)</td>
</tr>
<tr>
<td>( 1 - \Delta \pi_1^* )</td>
<td>1.44%</td>
<td>1.12%</td>
<td>0.79%</td>
<td></td>
</tr>
<tr>
<td>( \tilde{\pi}_1 )</td>
<td>1.32%</td>
<td>1.62%</td>
<td>1.49%</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{a} Table 8 displays the simulation results of the velocity of money and the elasticity of money demand with respect to the AAA interest rate after a one-time increase in 1980 of the access probability to the money market from \( \pi_0 = 0 \) to the optimal value of \( \pi_1^* \). Table 8 also displays the welfare cost of inflation with the optimal value of \( \pi_1, 1 - \Delta \pi_1^* \). The table also shows the critical interest rate, \( \tilde{\pi}_1 \), that separates the type-A equilibrium from the type-B equilibrium. The numbers in parentheses refer to the standard errors.
Table 9: Optimal market access: 1990 to 2013

<table>
<thead>
<tr>
<th></th>
<th>Data (1990-2013)</th>
<th>Nash Bargaining</th>
<th>Kalai Bargaining</th>
<th>Comp. Pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^*_1$</td>
<td>0.76</td>
<td>0.69</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td>6.41</td>
<td>6.44</td>
<td>6.43</td>
<td>6.43</td>
</tr>
<tr>
<td>Elasticity</td>
<td>-0.31 (0.021)</td>
<td>-0.46 (0.006)</td>
<td>-0.45 (0.006)</td>
<td>-0.45 (0.006)</td>
</tr>
<tr>
<td>$1 - \Delta\pi^*_1$</td>
<td>1.41%</td>
<td>1.11%</td>
<td>0.77%</td>
<td></td>
</tr>
<tr>
<td>$\tilde{I}$</td>
<td>1.09%</td>
<td>1.56%</td>
<td>1.35%</td>
<td></td>
</tr>
</tbody>
</table>

*Table 9 displays the simulation results of the velocity of money and the elasticity of money demand with respect to the AAA interest rate after a one-time increase in the access probability to the money market from $\pi_0 = 0$ to the optimal value of $\pi^*_1$ in 1990. Table 9 also displays the welfare cost of inflation with the optimal value of $\pi^*_1$, $1 - \Delta\pi^*_1$. The table also shows the critical interest rate, $\tilde{I}$, that separates the type-A equilibrium from the type-B equilibrium. The numbers in parentheses refer to the standard errors.*
Table 10: Comparison of results

<table>
<thead>
<tr>
<th></th>
<th>Data 1990-2013</th>
<th>BMW</th>
<th>Our model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity</td>
<td>6.41</td>
<td>6.41</td>
<td>6.39</td>
</tr>
<tr>
<td>Elasticity</td>
<td>-0.31 (0.021)</td>
<td>-0.81 (0.009)</td>
<td>-0.40 (0.005)</td>
</tr>
</tbody>
</table>

*Table 10 displays the velocity of money and the elasticity of money demand with respect to the AAA interest rate for the period from 1990 to 2013 in BMW and in our model. For the simulation, we assumed competitive pricing in the goods market. The numbers in parentheses refer to the standard errors.*
<table>
<thead>
<tr>
<th>Description</th>
<th>Identifier</th>
<th>Source</th>
<th>Period</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer price index</td>
<td>CPIAUCSL</td>
<td>BLS</td>
<td>50:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>M1 money stock</td>
<td>M1SL</td>
<td>BGFRS</td>
<td>59:Q1-66:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>Sweep-adjusted M1</td>
<td>M1ADJ</td>
<td>FRBL</td>
<td>67:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>Nominal GDP</td>
<td>GDP</td>
<td>BEA</td>
<td>50:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>AAA Moody’s corporate bond</td>
<td>AAA</td>
<td>BGFRS</td>
<td>50:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
<tr>
<td>Money market mutual funds</td>
<td>MMMFFAQ027S</td>
<td>BGFRS</td>
<td>50:Q1-13:Q4</td>
<td>quarterly</td>
</tr>
</tbody>
</table>