On the Global Supply of Basic Research*

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Abstract

In a two-country Schumpeterian growth model, we study the incentives for basic research investments by governments in a globalized world. We find that a country’s basic research investments increase with the country’s level of human capital and decline with its own market size. This may explain why some smaller countries invest so much in basic research. Compared with the optimal investments achievable when countries coordinate their basic research policies, a single country may over-invest in basic research. However, the total amount of decentralized basic research investments is always below the socially optimal investment level, which justifies policy coordination in this area.

Keywords: basic research, public goods, economic growth, coordination of governments

JEL: O31, O38

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1 Introduction

Basic research investments aim at acquiring new knowledge without any particular (commercial) application in view and are arguably a core driver of economic growth in industrialized countries. Traditionally, basic research investments are a matter of national policy-making. In some areas, however, international cooperation and coordination are playing an increasingly important role. This is most pronounced in the European Union, where large research programs are funded by member states and designed and operated at Union level in Brussels. Moreover, the basic research undertaken at several major institutes such as CERN in Geneva or by other high-technology ventures such as ARIANE are the result of joint efforts and agreements between several countries. Whether international coordination on basic research investment is considered necessary depends both on the way we conceptualize basic research and on the way how investments in one country affect growth and welfare in other countries. There are arguments for and against the coordination of basic research across countries.

When basic research is viewed as a global public good whose output is freely available and whose consumption is non-rivalrous and non-excludable (Arrow, 1962, Nelson, 1959), the standard “free-rider argument” suggests that uncoordinated investment decisions will entail considerable under-investment.

Basic research may also be viewed as a regional good with international spillovers. The ideas created by basic research are non-rival goods in the country where these ideas have been generated. As a consequence, basic research may induce and increase prospects of success for regional firms’ innovation efforts.\(^1\) Moreover, firms with successful innovations may be able to increase the rents generated by these innovations through exports or foreign direct investments. The possibility of capturing rents in foreign markets by taking away business from established firms suggests that basic research investments have negative externalities on other countries, which would cause over-investment.\(^2\)

When the benefits of basic research are embodied in new products and services, and

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\(^1\)The positive side-effects occur through various channels whose outputs are: supply of trained scientists and problem-solvers, new scientific instrumentation, network for knowledge diffusion, enhancement of problem-solving capacities, start-ups and spin-offs from universities, prototypes of new products and processes (e.g., Salter and Martin, 2001, Brooks, 1994, Moverry and Sampat, 2005, and Gersbach et al., 2009).

\(^2\)The negative and positive externalities described in this paragraph are well documented in the literature (Baily and Gersbach, 1995, Keller and Yeaple, 2003, Alfarre et al., 2006)
if a country is open to foreign direct investments, this country could benefit from the basic research of other countries. Foreign direct investments by leading-edge firms directly contribute to higher levels of productivity by transferring the best production techniques and products to the host country, thereby raising wages and consumer surplus. These positive externalities suggest that countries tend to under-invest in basic research.

This paper develops a framework to study the direction of externalities of basic research investments and examine whether there is an under- or overprovision of such investments when each country acts on its own. Two countries select their basic research investments in each period. Such investments foster the innovation prospects of domestic intermediate firms. Firms that develop leading-edge technologies in one country obtain patents and can enter foreign markets through foreign direct investments to earn monopoly profits. When another country invests more in basic research, a country will experience positive and negative externalities of the kind described above. Moreover, if both countries invest in basic research, this increases the risk that innovation efforts may be duplicated in the world. Decentralized basic research investments are studied in a setting with governments maximizing the consumption of the current generation, and the long-term consequences of such decisions are explored. The basic research levels attained when countries coordinate their decisions are determined. Finally, the path of uncoordinated and coordinated basic research investments when governments maximize the welfare of all generations will be studied.

Our main insights are as follows: First, it is shown that the countries’ basic research investments act as strategic substitutes. Further, a country’s basic research investments will increase with its level of human capital, but decline with its relative population size. The reason for the latter is that a small country can earn large profits from gaining a monopoly position in a larger foreign country without sustaining the corresponding deadweight losses accruing abroad. This result may explain why some small open economies such as Korea or Switzerland invest a lot in basic research.

Second, comparing the decentralized basic research investments with the optimal ones when countries coordinate to maximize aggregate consumption, our finding is that both countries under-invest in the decentralized equilibrium if they are similar with

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3 Hence, basic research is a non-rival good in countries at the technology frontier where new ideas are created. See Jones and Romer (2010) for systematic reasoning on why ideas in research should be viewed as partially excludable non-rival goods.
respect to human capital levels and population sizes. Under asymmetry concerning either of these characteristics, one of the countries may over-invest in the decentralized equilibrium relative to the coordination optimum. From the cooperative perspective, however, the aggregate decentralized basic research investments are too low, even if one of the countries over-invests in basic research.

Third, our robustness discussion reveals that our results do not change qualitatively when a one-period or an infinite planning horizon of governments is considered. Of course, investments in basic research increase quantitatively when longer time horizons are considered. The appendix also discusses the implications of different welfare objectives under coordination and different assumptions on the costs of basic research.

The paper is structured as follows: The next section discusses the significance of basic research and relate our paper to the relevant literature in Section 3. Section 4 introduces the model set-up, and Section 5 discusses the households’ and the governments’ optimization problems. The decentralized equilibrium is derived in Section 6, while the dynamics of the model are described in Section 7. Section 8 compares the decentralized basic research investments with the ones optimal when countries coordinate to maximize aggregate consumption. Finally, conclusions are drawn in Section 9. The proofs and the robustness of our results with respect to infinite planning horizons are relegated to the appendix.4

2 Significance of basic research

It is useful to put the significance of basic research in perspective. The empirical pattern of basic research is shown in Table 1. In this table, basic research is defined by the OECD (2002) as “experimental or theoretical work undertaken primarily to acquire new knowledge of the underlying foundation of phenomena and observable facts, without any particular application or use in view”. According to this definition, basic research does not generally provide (commercializable) solutions for specific practical problems, but rather provides the ideas, methods, prototypes, and materials needed to tackle these problems. In the US, basic research is mainly conducted by the federal government and by universities and colleges, and about 80% of it is publicly funded (Cozzi and Galli, 2009 and NSB, 2012).

Two observations from Table 1 are worth emphasizing. First, basic research is mainly

4Further robustness discussions can be found in the appendix.
undertaken by industrialized countries that are at, or close to, the world technological frontier. Some of the emerging countries, such as Korea or Singapore, have considerably stepped up their basic research efforts. Second, large industrial countries such as the U.S. or France spend about 0.5% of their GDP on basic research. By contrast, Switzerland invests a substantially higher share of about 0.8%.

### 3 Relation to the literature

The theme and the model of our paper are influenced by two lines of research. First, there is a large body of literature on the importance of basic research in the innovation process and on the strength of international spillovers. Some major articles have already been referred to. Second, our paper is related to the theoretical literature that incorporates basic research into R&D-driven growth models (e.g. Arnold, 1997, Cozzi and Galli, 2011a, 2009, 2011b, Gersbach et al., 2009). Most of these contributions focus on the optimal level of basic research in closed economies. There are two papers that also investigate open economies. In a two-country model, Park (1998) analyzes
how cross-country knowledge spillovers affect the optimal level of public basic research, while the degree of openness determines how large the spillovers are. However, regardless of the degree of openness, the knowledge spillovers come free of charge. In our model, knowledge spillovers occur via foreign direct investments by technologically-advanced firms, so the cost is the drain of monopoly profits going abroad. Moreover, the strength of spillovers can be influenced by basic research investments. Accordingly, the governments face the trade-off of capturing rents in the foreign country and keeping profits in the country versus realizing technology spillovers from abroad but forgoing profits in the respective sectors. Neither this trade-off, nor the way two countries will play the ensuing basic research investment game have been addressed in the previous literature.5

4 The model

We build on the Schumpeterian growth model with a basic research sector. Two countries, denoted by $H$ and $F$, decide about their investment in basic research. The countries are called $H$ and $F$. When referring to an unspecified country, the indices $j$ and $k$, with $j, k \in \{H, F\}$ are used. If both indices are used, it is always assumed that $j \neq k$. In each country and each period $t$ ($t = 1, 2, ...$) there is a continuum of identical households of measure $L_j$, $j \in \{H, F\}$ that enjoy strictly increasing utility in consumption $u(c)$, inelastically supply one unit of labor, and receive an equal share of the profits made by the final-good firm and from intermediate goods production. More precisely, the standard assumptions $u'(c) > 0, u''(c) < 0$ are made. For each country and each period, a government maximizes the well-being of its citizens by publicly providing basic research, financed by an income tax. Accordingly, a non-overlapping generations model is considered, in which each generation elects a government to provide public goods (here basic research) to maximize its welfare. This is equivalent to maximizing the consumption of the current generation. First, the production side of the economy is described, and the equilibrium for a given level of basic research for each country is derived. Then the basic research game played by the countries will be studied.

5Gersbach et al. (2013) study how openness affects the incentives to invest in basic research in a single-country model with a given world technology frontier. This paper studies how two countries strategically interact with their basic research investments, thereby determining the technology frontier.
4.1 Production

This section describes the production side of the economy for a particular country $j$ in a typical period $t$.

4.1.1 Final-good sector

In the final-good sector, a continuum of competitive firms produces the homogeneous consumption good $Y$ according to

$$Y_j = L_j^{1-\alpha} \int_0^1 [A(i)x_j(i)]^\alpha di.$$  \hfill (1)

There is a continuum of varieties $[0, 1]$, $x_j(i)$ stands for the amount of intermediate input of variety $i$, and $A(i)$ is this variety’s productivity factor. The parameter $\alpha \in (0, 1)$ determines the output elasticity of the intermediate goods. The price of the final consumption good is normalized to one. In the following, one representative final-good firm in each country $j$ maximizes its profit, denoted by $\pi^y_j$,

$$\max \{x_j(i)\}_{i=0}^1 \{ \pi^y_j = Y_j - \int_0^1 p_j(i)x_j(i) di - w_j L_j^d \}.$$ \hfill (2)

where $p_j(i)$ is the price of good $i$, $w_j$ is the wage level, and $L_j^d$ labor demand. Maximizing $\pi^y$ with respect to $x_j(i)$ and taking $p_j(i)$ as given yields the demand functions for the intermediate goods

$$x_j(i) = \left( \frac{\alpha A(i)^\alpha}{p_j(i)} \right)^{\frac{1}{1-\alpha}} L_j^d$$ \hfill (3)

and the inverse demand function of labor

$$w_j = (1 - \alpha) \left( L_j^d \right)^{-\alpha} \int_0^1 [A(i)x_j(i)]^\alpha di.$$ \hfill (4)

Market clearing in the labor market implies $L_j^d = L_j$, and $L_j$ will be used in the following.

4.1.2 Intermediate-goods sectors

The intermediate goods $x(i)$ are produced via a one-to-one technology from the final good. The intermediate firms compete à la Bertrand in their intermediate sector. The productivity leader is able to establish a monopoly position, and if there is no technological leader perfect competition prevails. Accordingly, the intermediate firms are either monopolistic or fully competitive. Their prices are denoted by $p^c(i)$ and
\( p^m(i) \), respectively. A competitive intermediate firm sets prices equal to the marginal costs. As the price of the final good has been normalized to 1, it follows that \( p^c(i) = 1 \), and profits vanish. The monopolistic intermediate producer chooses \( p^m(i) = \frac{1}{\alpha} \). For the monopolist this leads to profits of \( \pi^m_j(i) = nL_jA(i)\frac{\alpha}{1-\alpha} \) where \( n = \frac{1}{\alpha}(\frac{2}{\alpha}-1) \).

### 4.2 Technological state, innovation, and foreign entry

It is assumed that the world technological frontier is determined by two industrial countries, e.g. the U.S. and Europe/Japan. The productivity levels of a variety \( i \) produced in the countries \( H \) and \( F \) in period \( t \) are denoted by \( A^H_t(i) \) and \( A^F_t(i) \), respectively. At the end of period \( t-1 \), a sector \( i \) in country \( H \) has achieved the technological level \( A^H_{t-1}(i) \). For each type of intermediate, an innovation may take place at the beginning of each period. The probability that an innovation in a variety in country \( j \) takes place is denoted by \( \rho_{jt} \). If an innovation takes place in sector \( i \) in period \( t \), productivity increases according to \( A_t(i) = \gamma A_{t-1}(i) \) with \( \gamma > 1 \).

### 4.3 Major assumptions

We make three major assumptions. Our first assumption is that the basic research investment of a government increases the innovation probability of domestic firms. Second, other countries are also affected by technological diffusion (or in the extended model in Section 13.1 by access to the newly-created knowledge). Third, successful innovation by domestic firms can be protected by patents (or by complexity and difficulty of imitation). The assumptions are justified in three steps.

First, there is rich empirical evidence that basic research, as documented in Table 1, has strong local/regional effects and fosters innovation and growth of firms located in the same region or country. These positive effects include the supply of scientists and problem-solvers, joint research projects by universities, private companies, and spin-offs, and the establishment of scientific networks.\(^6\) For domestic firms, basic research is often the first step in the innovation process (see Grossman and Shapiro, 1987, Aghion and Howitt, 1996, Cozzi and Galli, 2009 and Aghion et al., 2008). These local/regional effects are the reason why, as reported in Table 1, even small countries (such as Iceland, Korea, and Switzerland) invest approximately the same percentage of GDP in basic research as large industrialized countries such as France or the US. A recent example

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\(^6\)See, e.g. Jaffe et al. (1993), Anselin et al. (1997), Audretsch and Lehmann (2005), Monjon and Waelbroeck (2003), and Williams (2013).
is provided by Williams (2013), documenting how the public Human Genome Project has triggered innovations in life science companies headquartered in the US.

Second, basic research investment by one country can also be beneficial to other countries, either indirectly in the form of technological spillovers from the entry of successfully innovative firms or directly, as outlined by Nelson (1959), Arrow (1962) and Cohen et al. (2002), by access of firms to increases in the knowledge base in the other country.

The basic version of the model abstracts from the direct effects of basic research on the innovation success of firms located in other countries and model the global effects of basic research through the entry of successfully innovative firms in other countries. The reason for this is twofold. The empirical evidence for direct effects in the literature referred to above is weak. Furthermore, incorporating these effects would reinforce our result. In particular, under-provision of total basic research investments would become more pronounced, and the need for coordination among countries on basic research would be reinforced.\(^7\)

Third, an intermediate firm with a new innovation will be able to protect its innovation through patents (or by other means that make imitations costly). Accordingly, such a firm will achieve a monopoly on this intermediate product. We stress that though basic research itself is not patentable, it can be turned into a patent through commercialization by domestic firms.

### 4.4 Formalization

We assume diminishing returns on basic research investments of the kind listed in Table 1 in enhancing the innovation prospects of domestic firms. Formally, a particular level of innovation probability of \(\rho_{jt}\) in country \(j\) for each variety requires basic research investments in period \(t\) of

\[
R_{jt}(\rho_{jt}) = \rho_{jt}^2 \frac{L_j A_{t-1}}{2\theta_j}. \tag{5}
\]

Equivalently, the innovation probability can be written as

\[
\rho_{jt} = \min \left\{ \sqrt{2\theta_j R_{jt}} \frac{L_j A_{t-1}}{L_j A_{t-1}, 1} \right\}. \tag{6}
\]

\(^7\)This is addressed in a suitable extension of our model in Section 13.1.
where $\theta_j$ is a parameter that captures the efficiency of basic research investments in country $j$. In our standard set-up, $\theta_j$ is interpreted as the average (per-capita) level of human capital in country $j$. This specification implies that the costs of basic research decline with a country’s per-capita level of human capital. $\bar{A}_{t-1} = \int_0^1 [A_{t-1}(i)]^{\frac{1}{1-\alpha}} di$ is an index of the average level of technology in the world. With respect to the cost function of research, two issues are worth noting. First, multiplying by the size of the country’s population avoids a strong scale effect in the countries’ growth rates. Second, the higher the knowledge stock is, the more difficult it becomes to innovate because the costs of basic research will increase with the average technology level $\bar{A}_{t-1}$. In this sense, the model features negative intertemporal externalities of knowledge production. A discussion on the foundation for the specification of the basic research costs is provided in Section 11 in the appendix.

### 4.5 Market structure

It has been assumed until now that a new innovation obtains a patent that expires after one period. As a period it is plausible to think of roughly 20 calendar years representing the period of one generation.\(^8\) Further, foreign intermediate firms are assumed to enter a domestic market if they have higher productivity than domestic producers.\(^9\) It immediately follows that $A^H_t(i) = A^F_t(i) \forall i, \forall t$. Consequently, in each period there are four possible constellations for the market structure in the market for variety $i$: (I) domestic monopoly in $H$, domestic monopoly in $F$; (II) domestic monopoly in $H$, foreign monopoly in $F$; (III) foreign monopoly in $H$, domestic monopoly in $F$; and

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\(^8\)Section 13.4 discusses the case where the costs of basic research decline with the absolute level of human capital.

\(^9\)We chose a period length of 20 years as Article 33 of the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPs), which all developed countries and most developing countries signed, provides that ‘the term of protection available for patents shall not end before the expiration of a period of twenty years counted from the filing date’. As a consequence, in most countries’ patent laws, the term of patent is 20 years from the filing date of the application. Detailed information on patent laws in different countries is provided by the World Intellectual Property Organisation (WIPO) under www.wipo.int.

\(^{10}\)It is excluded that foreign firms contest domestic markets if they have the same level of productivity. Alternatively we could assume that there is always one firm that is slightly faster in filing a patent for the entire world market. Suppose that the probability for the domestic firm to be faster is $p_d$. This would add a term $\rho_k \rho_k (p_d \bar{\pi}_k - (1 - p_d) \bar{\pi}_k)$ to government $j$’s objective function. If this term is positive it would additionally increase the incentives for basic research investments in country $j$ and decrease the basic research investment incentives in the other country. It seems plausible to assume that firms in both countries have the same probability of obtaining the patent, i.e. $p_d = 0.5$. Then a small country’s basic research incentives will be even higher whereas those of the large country will become smaller than in our basic model set-up.
(IV) perfect competition in $H$, perfect competition in $F$. Domestic monopoly means that an innovator in country $j$ possesses a patent on the highest-quality intermediate good $i$ in country $j$, whereas a foreign monopoly would exist if the patent were held by an innovator headquartered in country $k$. As patents expire after one period, a sector is characterized by perfect competition in period $t$ when neither in country $j$ nor in country $k$ an innovation in this sector occurred in this period.

5 The households’ and the governments’ problems

Our basic model intentionally keeps the households’ problem very simple. In fact, each household is assumed to offer one unit of labor inelastically to the labor market. They receive income from working and profits as owners of firms in intermediate sectors and from final-good producers headquartered in their country.

This allows us to move immediately to the government’s problem. To establish the latter, total consumption in a country $j$ in a period $t$ will be derived next.

Let us first reconsider expected final-good production, which writes

$$Y_{jt} = L_j^{1-\alpha} \left[ \int_0^1 [1 - q_t] [\bar{A}_{t-1}(i)]^{\alpha} \gamma^\alpha (x_j^n(i))^\alpha \, di + \int_0^1 q_t [\bar{A}_{t-1}(i)]^{\alpha} (x_j^c(i))^\alpha \, di \right],$$

(7)

where the abbreviation $q_t \equiv (1 - \rho_{jt})(1 - \rho_{kt})$ is used. The first integral represents the part of final-good production resulting from the sectors where an innovation has taken place. In these sectors either a foreign innovator has entered the intermediate-good market or a domestic innovator has offered a technologically advanced product. The probability of an innovation in sector $i$ in period $t$ is $[1 - q_t]$. With complementary probability $q_t$ no innovator is successful in sector $i$, and the technological level remains.

As discussed earlier, in sectors where no innovation occurs there is no patent protection and hence perfect competition prevails. The part of final output attributed to these sectors is reflected by the second integral. Since the innovation probabilities are not sector-specific, inserting (3) and making some minor mathematical manipulations yields

$$Y_{jt} = L_j \left[ (1 - q_t) \gamma^{1-\alpha} \alpha \int_0^1 [\bar{A}_{t-1}(i)]^{\frac{\alpha}{1-\alpha}} \, di + q_t \alpha^{1-\alpha} \int_0^1 [\bar{A}_{t-1}(i)]^{\frac{\alpha}{1-\alpha}} \, di \right].$$

(8)

Using the index of the average technological level in the world $\bar{A}_{t-1} = \int_0^1 [\bar{A}_{t-1}(i)]^{\frac{\alpha}{1-\alpha}} \, di$
yields

\[ Y_{jt} = L_j \tilde{A}_{t-1} \alpha^{\frac{1}{1-\alpha}} \left( q_t + (1 - q_t) \gamma^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \right), \] (9)

Increasing the number of innovations in the aggregate means reducing \( q_t \). Consequently, additional innovations have a positive effect on output if and only if \( \alpha^{\frac{\alpha}{1-\alpha}} \gamma^{\frac{\alpha}{1-\alpha}} > 1 \), which is equivalent to \( \gamma > 1/\alpha \). This illustrates the trade-off associated with innovations concerning final-good production. On the one hand, higher quality of an intermediate good involves higher productivity in final-good production reflected by \( \gamma \). On the other, it induces monopoly distortions in the intermediate-good market that lead to a mark-up on the price of intermediates of \( 1/\alpha \) and consequently have a negative effect on final output. If \( \gamma > 1/\alpha \), the effect of higher productivity dominates, and innovations in period \( t \) have a positive effect on final output in \( t \). However, if \( \gamma < 1/\alpha \), output in \( t \) declines as a consequence of an innovation because the monopoly distortions dominate. In the following, it is assumed that \( \gamma > 1/\alpha \), i.e. that innovations in \( t \) positively affect output in the same period.\(^{11}\)

Let us now turn to the expected costs of producing the intermediates used in final-good production. Making use of (3), the aggregate production costs of intermediate goods are referred to by \( X_{jt} \) and can be written as

\[ X_{jt} = \int_0^1 (1 - q_t)x_{jt}^m(i)di + \int_0^1 q_tx_{jt}^c(i)di, \] (10)

where \( x_{jt}^m(i) = L_j (\alpha^2 \gamma^\alpha A_{t-1}^\alpha)^{\frac{1}{1-\alpha}} \) and \( x_{jt}^c(i) = L_j (\alpha A_{t-1}^\alpha)^{\frac{1}{1-\alpha}} \). Inserting the latter yields

\[ X_{jt} = \int_0^1 (1 - q_t) \gamma^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} L_j [A_{t-1}(i)]^{\frac{1}{1-\alpha}} + \int_0^1 q_t \alpha^{\frac{1}{1-\alpha}} L_j [A_{t-1}(i)]^{\frac{1}{1-\alpha}} \] (11)

which can be rewritten as

\[ X_{jt} = L_j \tilde{A}_{t-1} \alpha^{\frac{1}{1-\alpha}} \left( q_t + (1 - q_t) \gamma^{\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \right). \] (12)

\(^{11}\)In the case of \( \gamma < \frac{1}{\alpha} \), the incentives to perform basic research are reduced. Within our framework, basic research investments would then be driven mainly by the rents the countries earn in foreign countries. The extent to which incentives for basic research investments are reduced by the negative output effect also depends on the planning horizons of the governments, as the effect of innovations on output will turn positive once the patent has expired. To sum up, if \( \gamma < \frac{1}{\alpha} \), equilibrium basic research levels would be lower, but not necessarily zero. If \( \gamma > \frac{1}{\alpha} \) in some industries and \( \gamma < \frac{1}{\alpha} \) in others, equilibrium basic research levels will be between the case analyzed in our paper and the polar case described here.
The last line describes a trade-off associated with innovations concerning the number of intermediates. This trade-off is similar to the one identified with respect to final output. On the one hand, a higher quality intermediate good attracts higher demand (reflected by \( \gamma \alpha^{1-\alpha} \)). On the other, it is protected by a patent, so supply decreases relative to the competitive situation (represented by \( \alpha^{1-\alpha} \)). Consequently, if and only if \( \gamma^{1-\alpha} \alpha^{1-\alpha} > 1 \) the demand effect is dominant, and the amount of an innovative intermediate used in final-good production increases.

Total expected profits accruing in the intermediate sectors in country \( j \) read

\[
\pi_{jt} = \int_{0}^{1} (1 - q_t)nL_j[A_{t-1}(i)]^{\alpha^{1-\alpha}} \gamma^{1-\alpha} di = (1 - q_t)nL_j\tilde{A}_{t-1}\gamma^{1-\alpha}. \tag{13}
\]

The profits in an innovative sector are \( \bar{\pi}_{jt} = nL_j\tilde{A}_{t-1}\gamma^{1-\alpha} \). Note that up to this point nothing has been said about the distribution of the profits to domestic or foreign innovators. This however will play a key role for the total level of consumption in a country. Given \( \rho_{kt} \), expected aggregate consumption in country \( j \) amounts in period \( t \) to

\[
C_{jt} = Y_{jt} - X_{jt} + \rho_{jt}(1 - \rho_{kt})\bar{\pi}_{kt} - \rho_{kt}(1 - \rho_{jt})\bar{\pi}_{jt} - R_{jt}. \tag{14}
\]

The first two terms reflect net output of the final good. The third term represents the profits innovators of country \( j \) earn in country \( k \), while the fourth term captures the profits that innovators of country \( k \) earn in country \( j \). Finally, the government has to finance basic research. We assume that basic research is financed by an income tax. For simplicity, the tax was not written explicitly into the formula for total consumption. Using the expressions above, expected aggregate consumption in country \( j \) in period \( t \) can be written as

\[
C_{jt} = L_j\tilde{A}_{t-1}\left[y_n(q_t) + \gamma^{1-\alpha}n(\rho_{jt}(1 - \rho_{kt})L_k/L_j - \rho_{kt}(1 - \rho_{jt})) - \frac{\rho_{jt}^2}{2\theta_j}\right], \tag{15}
\]

where \( y_n(q_t) \equiv \alpha^{1-\alpha}y(q_t) - \alpha^{1-\alpha}x(q_t) \) represents expected net final-good production – i.e., expected total production net of the costs for the intermediate products.

The government’s objective function displays the sources of the effects of basic research described in the introduction.\(^{12}\) First, expected net final-good production depends on
the probability that an innovation will occur in either of the two countries. Basic research investments increase this probability and consequently have a positive external effect on the expected net final-good production of the foreign country via technological spillovers associated with foreign direct investments. This will lead to under-investment in decentralized basic research investments. Second, basic research investments determine the distribution of profits as reflected in the second and third summand. Basic research enables more domestic firms to obtain profits from the foreign country in sectors where foreign firms have not been successful in creating innovation. Moreover, basic research investments enable more domestic firms to compete in sectors where foreign firms have successfully innovated and hence to keep profits in these sectors in the home country. This tends to prompt countries to over-invest in basic research.

6 Decentralized basic research investment

In this section, the static game of two governments maximizing current domestic consumption by choosing the level of basic research investments and taking the investments of the other country as given will be considered. This can be interpreted as maximizing $C_{jt}$ via control $\rho_{jt}$ given $\rho_{kt}$ rather than via $R_{jt}$. Considering the change in aggregate consumption in country $j$, $C_{jt}$, in response to a marginal increase in the innovation probability in country $j$, $\rho_{jt}$, yields

$$
\frac{dy_n(q_t)}{dq_t} \frac{dq_t}{d\rho_{jt}} + \gamma^{1-\alpha} n((1 - \rho_k) L_k + \rho_k) - \frac{\rho_j}{\theta_j}.
$$

(16)

It is now possible to simplify this expression in the following way: First, let us recall the definition of $y_n(q) = \alpha^{\alpha-\gamma} y(q) - \alpha^{1-\alpha} x(q)$, where $y(q)$ and $x(q)$ are linear functions of $q$. Consequently, the derivative of the expected net output with respect to the probability that no innovation will occur is a constant, which can be denoted by $y'_n$.\(^{13}\)

\(^{13}\)The derivatives of $x(q)$ and $y(q)$ are

$$
\frac{dx(q)}{dq} = 1 - \gamma^{\alpha-\gamma} \alpha^{1-\gamma},
$$

(17)

$$
\frac{dy(q)}{dq} = 1 - \gamma^{\alpha-\gamma} \alpha^{\alpha-\gamma},
$$

(18)

and thus $y'_n = \alpha^{\alpha-\gamma} (1 - \gamma^{\alpha-\gamma} \alpha^{\alpha-\gamma}) - \alpha^{1-\gamma} (1 - \gamma^{\alpha-\gamma} \alpha^{1-\gamma})$. 

13
Note that $y_n'$ is negative as an increase in the probability that no innovation will occur in either country decreases country $j$’s net output in expectation. On the other hand, a marginal increase in the probability of an innovation taking place will increase net output in expectation by $-y_n'$, which is referred to in the following as $y_p^j$. It will also be convenient to use the abbreviations $L \equiv \frac{L_k}{L_j}$ to denote relative population size and $\gamma \equiv \gamma_1$. As the governments’ problems considered in this section are static, time indices are neglected. With these simplifications, the first-order condition can be written as

$$y_n' \frac{dq}{d\rho_j} + \gamma n \{(1 - \rho_k)L + \rho_k\} - \frac{\rho_j}{\theta_j} = 0.$$  \hspace{1cm} (19)

Accordingly, the reaction function of country $j$ is

$$\rho_j^*(\rho_k) = \theta_j \left[ (1 - \rho_k)y_n^j + \gamma n ((1 - \rho_k)L + \rho_k) \right].$$  \hspace{1cm} (20)

In the reaction function, the first term in brackets reflects the effect of a marginal increase in basic research in country $k$ on country $j$’s output, while the second term represents the change in expected net profit flows from technology exchange. The derivative of $\rho_j^*(\rho_k)$ with respect to $\rho_k$ writes as

$$\rho_j' \equiv \frac{\partial \rho_j^*(\rho_k)}{\partial \rho_k} = -\theta_j \left[ y_n^j + \gamma n (L - 1) \right].$$  \hspace{1cm} (21)

An intuitive interpretation of (21) is that a marginal increase in $j$’s basic research investment is less valuable in increasing total output in country $j$, the higher basic research investment in country $k$ is due to increased research duplication. In other words, the higher the basic research investments of the foreign country, the smaller the number of sectors where no innovation occurs and where domestic basic research investment could contribute to increasing expected net output. Further, the effect of a marginal increase in basic research on profit flows obtained from the foreign country declines with the foreign country’s basic research efforts (reflected by $-\theta_j \gamma n L$). However, the profits prevented from flowing into the foreign country increase with the foreign country’s basic research investment (reflected by $\theta_j \gamma n$). This implies that only very large countries (which implies a low value of $L$), where the motive of keeping profits in the country dominates, will potentially increase basic research investments in response to an increase in basic research investments by the other country. It will now be shown that this will not occur in our framework as basic research investments are
strategic substitutes for both countries.\footnote{\text{As Lemma 1 stresses, this depends on our assumption that innovations are output-increasing rather than output-decreasing. In the latter case, the governments’ motives for investing in basic research are mainly based on capturing profits from the other country. Then there might occur a situation where the basic research investments of the large country are strategic complements to those of the small country.}} It can also be observed in (21) that the level of human capital of a country $\theta$ affects both the reaction functions’ ordinate intercepts and their slopes. The same is true of relative population size $L$. However, the abscissa intercepts of the reaction functions are independent of $\theta$. Before examining the slopes of the reaction functions, let us state

Lemma 1
If $\gamma > \frac{1}{\alpha}$, then $y_n^p > \tilde{\gamma} n$.

The proof can be found in the Appendix.

The lemma implies that cases where $y_n^p < \tilde{\gamma} n$ can be neglected because of the assumption that $\gamma > \frac{1}{\alpha}$. This allows us to determine the signs of the slopes in the countries’ reaction functions:

Proposition 1
Basic research expenditures in the two countries are strategic substitutes – i.e., $\rho_H' < 0$ and $\rho_F' < 0$.

In the remainder of the paper, the parameter set of our analysis is restricted as follows:

Assumption 1
$\theta_j(\tilde{\gamma} n L + y_n^p) < 1$, which is equivalent to $\rho_j'(0) < 1$.

Assumption 1 requires that a country chooses basic research spending to obtain $\rho < 1$ (instead of the corner solution $\rho = 1$) when there are no basic research investments by the other country. In other words, Assumption 1 says that the basic research investments of a country cannot with certainty lead to innovation in each intermediate sector, which seems very realistic.

6.1 Equilibrium

The equilibrium analysis shows that there is a unique equilibrium $(\rho_H^e, \rho_F^e) \in (0, 1)^2$. 

\footnotetext[14]{}
Proposition 2 (existence of unique equilibrium)

Given Assumption 1, there exists a unique equilibrium \((\rho^e_H, \rho^e_F) \in (0, 1)^2\) that is characterized by

\[
\rho^e_j = \frac{1}{\theta^e_j \theta^e_k} \left( y^e_n + \gamma^e n L \right) + \frac{1}{\theta^e_j \theta^e_k} \left( y^e_n + \gamma^e n / L \right) \left( \gamma^e n (1 - L) - y^e_n \right) - \frac{1}{\theta^e_j \theta^e_k} \left( \gamma^e n (1 - L) - y^e_n \right) \left( \gamma^e n (1 - 1 / L) - y^e_n \right) .
\] (22)

The proof is given in the Appendix.

6.2 The role of human capital

This section examines how basic research investments are affected by a change in a country’s research capacity \(\theta\), which is also interpreted as a country’s human capital level. The following proposition gives the results:

Proposition 3 (comparative statics with respect to \(\theta\))

(i) If \(L = 1\), then the country with the higher research capacity will invest more in basic research – i.e., \(\rho^e_j > \rho^e_k\) if and only if \(\theta_j > \theta_k\).

(ii) In equilibrium, basic research investments of country \(j\) will increase with \(\theta_j\) and decrease with \(\theta_k\), i.e.

\[
\frac{d\rho^e_j}{d\theta_j} > 0, \quad \frac{d\rho^e_k}{d\theta_j} < 0 .
\] (23)

The proof can be found in the Appendix.

Intuitively, if the research capacity of one country, say \(\theta_j\), increases, its basic research investments will become more productive. Hence, country \(j\) will increase its basic research efforts. The reaction function of country \(k\) is not directly affected by a change in \(\theta_j\) but indirectly via the induced change in \(\rho_j\). As according to Proposition 1 basic research investments are strategic substitutes, \(k\) will decrease \(\rho_k\) in response to the increase in \(\rho_j\). This result is illustrated with the following example, where both countries are of equal size \(L = 1\), \(\alpha = 0.5\), \(\gamma = 2.1\), and \(\theta_k = 0.03\).

\[\text{15}\] In this setting, \(\theta_j\) is varied from 0.005 to 0.07. The result is shown in Figure 1.

\[\text{15}\] Note that the assumption \(\gamma > 1 / \alpha\) defines a critical value of \(\gamma\) that depends negatively on \(\alpha\). The values \(\alpha = 0.5\) and \(\gamma = 2.1\) were chosen. The latter value is larger by 0.1 than the critical value. For general innovations, this value of \(\gamma\) may seem relatively high, but it is justified for basic research, which if successful typically implies large technological improvements. The level of human capital \(\theta_k = 0.03\) has been specified to obtain realistic growth rates.
6.3 The role of relative population size

Now how countries of different sizes interact with respect to basic research investments will be studied.

Proposition 4 (comparative statics with respect to relative population sizes)

(i) Let $\theta_j = \theta_k$. Then the smaller country will invest more in basic research than the larger one – i.e., $\rho_j^e > \rho_k^e$ if and only if $L_k > L_j$.

(ii) $\rho_j^e$ increases and $\rho_k^e$ decreases with $L$.

The proof can be found in the Appendix.

Recall that the relative populations of the countries are defined by $L \equiv \frac{L_k}{L_j}$. Inspection of the first-order condition of the government, (19), reveals that a relatively larger foreign country $k$ will increase the ratio of profits received from abroad and the profits paid to the foreign country. By contrast, the relation of market sizes does not play a direct role for a country’s final output and research costs. Consequently, a relatively larger market abroad makes innovation and thus basic research investment more attractive. There is no comprehensive study on the relationship between the level of basic research and the size of population, after controlling for human capital and other
country characteristics. Nevertheless, two observations from Table 1 are intriguing. Not only Switzerland has comparatively high basic research expenditures. Other small countries like Korea, Austria, and Iceland have roughly the same – if not higher – basic research expenditures as a percentage of GDP as the US. Moreover, with increasing human capital over the last two decades, several small countries such as Ireland, Korea, and Singapore have considerably increased their basic research investments. Figure 2 uses the parameter values of the previous example, but here, instead of varying \( \theta_j \), it is assumed that \( \theta_j = \theta_k = 0.03 \) and vary the relative market size \( L \).

![Figure 2: Basic research investments in country \( j \), \( \rho_j \) (blue), and in country \( k \), \( \rho_k \) (red, dashed), depending on relative population sizes \( L \).](image)

7 Dynamics

Let us now turn to the dynamics of the model. Note that the optimal decisions by each government is independent of the level of technology \( A_{t-1}(i) \). Hence, the governments will not change their basic research investments over time. It can be inferred from Equation (15) that this implies that consumption grows at the rate of the average technological level. Further, the economy does not exhibit transitional dynamics. To determine the economies’ growth rate, the world’s average technological level can be written as

\[
\bar{A}_t = \int_0^1 [1 - q|\hat{\gamma}[A_{t-1}(i)]^{\alpha} - q[A_{t-1}(i)]^{\alpha}] \, di + \int_0^1 q[A_{t-1}(i)]^{\omega} \, di
\]

\[
= \bar{A}_{t-1}[\hat{\gamma} + q(1 - \hat{\gamma})].
\]

\[24\]

\[25\]
Consequently we obtain

**Proposition 5**

The growth rate of the two economies is given by

\[ g = (\tilde{\gamma} - 1)(1 - q). \]  

(26)

It will now be interesting to establish how the growth rate reacts to changes in \( \theta_j \) and \( L \). The expression \( \tilde{\gamma} - 1 \), which is positive since \( \gamma > 1 \), reflects the innovation steps of a successful invention and is independent of \( \theta_j \) and \( L \). As a consequence, the focus is on the term \( 1 - q \), which can be rewritten as \( \rho_j + \rho_k - \rho_j \rho_k \). This expression reveals nicely that the growth rate increases with the sum of basic research investment \( \rho_j + \rho_k \) but declines with the amount of research duplication \( \rho_j \rho_k \). In general, let us state

**Proposition 6**

A higher level of human capital in one country will lead to higher growth if and only if

\[ -\frac{d\rho_j^e}{d\theta_j} / \frac{d\rho_k^e}{d\theta_j} > \frac{1 - \rho_j^e}{1 - \rho_k^e}. \]  

(27)

An increase in \( L \) will involve higher growth if and only if

\[ -\frac{d\rho_j^e}{dL} / \frac{d\rho_k^e}{dL} > \frac{1 - \rho_j^e}{1 - \rho_k^e}. \]  

(28)

A proof of Proposition 6 follows directly from taking the derivative of the growth rate \( g \) with respect to \( \theta_j \) and \( L \) respectively. Conditions (27) and (28) simply state that the effect of a change in \( \theta_j \) and \( L \) on aggregate basic research investments is larger than the effect on duplication. With respect to a change in \( \theta_j \), the following corollary is obtained:

**Corollary 1**

Given Assumption 1, total basic research expenditures will increase if \( \theta_j \) becomes larger, i.e. \( -\frac{d\rho_j^e}{d\theta_j} / \frac{d\rho_k^e}{d\theta_j} > 1 \).

The proof can be found in the Appendix.

The intuition is that if \( \theta_j \) increases, \( \rho_j \) will become larger, while \( \rho_k \) decreases. The latter effect results from the fact that the countries’ basic research investments are strategic substitutes (cf. Proposition 1). The decline in \( \rho_k^e \) induced by an increase in \( \theta_j \) is a second-order effect that cannot neutralize the increase in \( \rho_j^e \) with respect to aggregate innovation probability.
Further, it follows directly from Proposition 6 that the total effect of an increase in $\theta_j$ on the growth rate is positive if $\rho_j > \rho_k$. Intuitively, in this case an increase in $\theta_j$ will not only increase aggregate basic research investments but also lead to a more unequal distribution of investments across countries, thereby reducing the duplication effect (which is largest for $\rho_j = \rho_k$). As a consequence, an increase of $\theta_j$ will positively affect the growth rate. This is different if $\rho_j < \rho_k$. Then there exists a trade-off between the effect on aggregate basic research and duplication. Though analytically not excludable, in none of our numerical simulations could we find a case where the duplication effect dominates and leads to negative effects of an increase in $\theta_j$ on growth. The left panel of Figure 3 shows the typical situation where the growth rate increases with $\theta_j$.

Figure 3: Growth rates depending on human capital in country $j$, $\theta_j$, (left) and for different relative population sizes $L$ (right).

According to our simulations, the growth rate exhibits a U-shaped form when the relative population size is varied. Numerically, we also find that the aggregate basic research expenditures are U-shaped in $L$. Intuitively, this means that the effect of a relative increase in the population size on aggregate basic research investments tends to be larger, the more unequal the population shares are. This originates from the incentives associated with the distribution of profits as described earlier. With very unequal country sizes, the small country has a high incentive to invest in basic research to capture profits in the foreign market. This, however, also increases the large countries’ basic research incentives, since it is now more valuable to invest in basic research to keep the profits in the country. Additionally, with more unequal relative basic research investments, the negative duplication effect is lower, thus reinforcing the U-shape of the growth rate in $L$. The right hand panel of Figure 3 illustrates the typical situation.
8 Coordinated basic research investments

Now let us ask which levels of basic research the countries should choose, if they coordinate to maximize current aggregate consumption $C_t = C_{jt} + C_{kt}$. Using (15), the objective can be written as

$$C_t = Y_t - X_t - R_t = \bar{A}_{t-1} \left[ (L_j + L_k) y_n(q_t) - L_j \frac{\rho^2_{jt}}{2\theta_j} - L_k \frac{\rho^2_{kt}}{2\theta_k} \right],$$

(29)

where the variables without country indices denote world values, i.e., $Y_t = Y_{jt} + Y_{kt}$ etc.

Equation (29) reveals that when consumption is aggregated, the profit flows between the two countries drop out, and aggregate consumption equals net production minus total basic research expenditures.

The necessary optimality conditions for coordinated basic research investments are

$$\rho_{jt} = \theta_{jt} y_n^p (1 - \rho_{kt})(1 + L),$$

(30)

recalling that $y_n^p = -y'_n$. Comparing condition (30) with the reaction function of the government’s problem in the non-cooperative setting (19) reveals that the cooperative solution additionally considers the basic research investments’ effects on the other country’s output and thus attaches weight $(L_j + L_k)$ to the increase in final-good production resulting from higher quality intermediates. The denominator $L_j$ in (30) represents the fact that in absolute terms basic research is less costly in the smaller country.

It is instructive to rewrite the reaction function of a country in the decentralized basic research investment game as follows:

$$\rho'_{jt}(\rho_{kt}) = \theta_{jt} y_n^p (1 - \rho_{kt})(1 + L) - (y_n^p - \bar{\gamma} n)(1 - \rho_k)L + \bar{\gamma} n \rho_k.$$

(31)

While the first term reflects the country’s socially optimal level of basic research expenditures, the second and third term illustrate the sources for the country’s sub-optimal basic research investment. The second summand represents the consumer surplus in the foreign country that is not taken into account by the domestic country in the decentralized game, as the latter cares only about the profits that flow in from the foreign country. For this reason, countries under-invest in basic research. The third term reflects the country’s incentive to prevent profits from flowing out by making domestic firms more competitive and, in effect, fostering socially wasteful duplication of innovations. This force leads to over-investment in basic research. Whether a country over- or under-invests in basic research relative to the social optimum depends on the
relative market sizes, that is, the relative population sizes of the countries. A relatively small country, i.e., a country facing a large $L$, will likely invest a lot in basic research (recall that ceteris paribus the reaction function $\rho^r_{jt}$ increases in $L$) driven by prospects of reaping profits in the large foreign country. In this case, the second term in the reaction function dominates the third, and the country is likely to invest too little in basic research due to its neglect of the consumer surplus in the foreign country. On the other hand, a large country (with small $L$) has relatively small profit prospects in the small foreign market, but its basic research investments are mainly driven by protecting the profits from flowing to the small foreign country investing aggressively in basic research, and thus being highly innovative. In this case, the third term may dominate the second, prompting the country to over-invest in basic research relative to the social optimum. In summary, driven by the prospects of making large profits in foreign countries, very small countries invest heavily in basic research but from a global social-welfare perspective most likely still invest too little. The major motive for large countries to invest in basic research is to keep profits in the country. Therefore they may over-invest in basic research relative to the social optimum. From Equation 31 it can be directly inferred that, when both countries are of the same size, there will be under-investment in basic research. Consequently, for over-investment to occur, $L$ must be sufficiently smaller than one, implying that only one country – the larger one – may over-invest in basic research. Hence there will never be a situation where both countries over-invest in basic research.

The optimality conditions (30) yield the optimal coordinated basic research investments as

$$\rho^\text{eff}_j = \frac{\theta_j \gamma_m (1 + L) y_n^p - (1 + L)(1 + 1/L)(y_n^p)^2}{\gamma_m - (1 + L)(1 + 1/L)(y_n^p)^2}.$$  \hspace{1cm} (32)

An analytical comparison of the cooperative solution with the equilibrium values of basic research in the decentralized setting given in Proposition 2 only yields interpretable conditions for the symmetric case.

**Proposition 7**

If $L_j = L_k$ and $\theta_j = \theta_k < \frac{1}{2} \left( \frac{1}{\gamma_m} - \frac{1}{y_n} \right)$, then $\rho^\text{eff}_j > \rho_j$ and $\rho^\text{eff}_k > \rho_k$.

The proof follows directly from a comparison of (32) and (22). The condition with respect to the levels of human capital $\theta$ in Proposition 7 is satisfied for reasonable parameter values. For further comparisons of the cooperative solution and the market
equilibrium, numerical simulations are used. Our results are derived in the standard scenario, as specified previously (i.e., $\alpha = 0.5$, $\gamma = 2.1$, $\theta_k = 0.03$), and it is shown in Section 13 that our results are very robust. Let us start by holding $\theta_j = \theta_k = 0.03$ fixed and examining the effect of relative population size. The simulation shows that if the population sizes are relatively equal, both countries will invest too little in the decentralized equilibrium relative to the coordination optimum. However, for very different population sizes, the smaller country, despite having a strong incentive due to large profit flows from the large country, will invest too little in basic research in the decentralized equilibrium, while the large country will invest too much. A similar result is obtained when considering countries with equally large population sizes but different research capacities $\theta$.\textsuperscript{16} Under-investment by both countries if the human capital levels are not too different and over-investment by the country with substantially smaller levels of human capital in the case of pronounced asymmetry with respect to $\theta$.

However, although there is over-investment by one country when population sizes or research capacities are very asymmetric, our simulations show that the total level of basic research in the cooperative solution is always higher than the one realized in the market solution. As a consequence, optimal coordination of basic research will always produce a higher growth rate.

Our results could be interpreted as follows: First, consider Switzerland and the European Union. They have symmetric levels of human capital but very different market sizes. As argued earlier, the incentive to gain foreign profit flows may lead to the high basic research investments by Switzerland. The results in this section suggest that from a cooperative perspective aiming to maximize aggregate consumption in both Switzerland and the EU, the already large Swiss basic research investments will still be too small, while those in the EU tend to be too high. A second interesting case is a comparison between the EU and the US. Here both market sizes and human capital levels are approximately equal. Accordingly, our welfare analysis suggests that both regions invest too little in basic research in the decentralized equilibrium.

As detailed in Section 12 in the appendix, the results remain qualitatively unchanged when the governments and the social planner have infinite planning horizons.

\textsuperscript{16}Specifically, $\theta_k = 0.03$ and $\theta_j$ is varied from 0 to 0.4.
9  Conclusions

Basic research investments in open economies have been studied in a two-country framework, where each country faces the following trade-off: Via basic research investments, domestic output increases and, additionally, profits can be generated by domestic firms in the foreign country. On the other hand, basic research is costly and leading-edge technology could be imported by free-riding on the other country's basic research efforts. We examine (a) the decentralized game and (b) the cooperative solution where countries coordinate their basic research investments.

Our findings are that in the decentralized game, basic research investments are strategic substitutes. A country's basic research investments increase with its average level of human capital and decrease with the human capital of the foreign country. Moreover, all else being equal, a small country has higher incentives to invest in basic research than a large country, because the relation between profit inflows and profit outflows is greater. This may explain that even comparatively small countries such as Switzerland, Korea or Singapore invest a lot in basic research which may even exceed in some cases the share of investments in larger industrialized countries. Compared with the optimal basic research investments when countries coordinate, there may be cases where one country will invest too much in basic research in the decentralized setting if the countries' human capital levels or market sizes are very asymmetric. However, compared to the coordination optimum the total investments in basic research are always too low in the decentralized setting. This directly implies that the global rate of economic growth will be too low if governments pursue national basic research strategies.

The paper opens up several avenues for future research. First, a suitable extension would include costs/barriers to foreign direct investment. The existence of barriers is well-documented in the literature (see, e.g., Evans, 2003, Daude and Fratzscher, 2008, and Keller and Yeaple, 2013).\(^\text{17}\) These costs can be incorporated into our model in two ways. On the one hand, if there are costs to foreign direct investments across all sectors, country-specific benefits from basic research investments will tend to decline, as fewer rents can be captured in foreign markets and fewer rents need to be protected at home. The relative ranking of basic research investments in the decentralized case and in the coordination case remain unaffected.\(^\text{18}\) Accordingly, policy coordination

\(^{17}\)Considering openness as a governmental choice, it would be interesting to further explore how openness and basic research investments interact.

\(^{18}\)Details are available upon request.
continues to be justified. On the other hand, only some sectors may face high barriers to foreign direct investments, so that these investments never occur. If foreign direct investment is only possible in some sectors, the country-specific consumption gains will again decline for a specific level of basic research, and the same conclusion as before can be drawn.

Second, the model can be extended with respect to its micro-economic foundations. For example, explicitly modeling firms’ decisions on applied research would enrich the model to capture interactions between basic and applied research. This is particularly interesting from the perspective of a global labor market with firms and governments competing for the best applied and basic researchers. Moreover, our model could be extended to explore the firms’ location and off-shoring decisions and to examine how these decisions affect the governments’ incentives to invest in basic research.

The model and the perspective of our paper can be broadened further by incorporating decisions on human capital investments, thereby endogenizing the human capital variable in the model. Such human capital investments can be left to individuals or supported by public funding and public institutions. The former case has been dealt with in subsection 11.1. In the latter case, public funds devoted to education may especially compete for resources invested in basic research.\textsuperscript{19} Such an extension increases the opportunity costs of basic research investments and can lower basic research investments in the decentralized case and when basic research investments are coordinated. However, the qualitative results, and hence the conclusions in our paper, remain unchanged in simple extensions.\textsuperscript{20}

Extending the model to a multi-country setting would be another important extension. For this purpose, one can ideally draw on the work of Peretto (2003), who developed a multi-country Schumpeterian innovation model to study the impact of policies on growth and welfare. Such a framework would enable us to examine the interplay of intra-national and international technological spillovers when basic research investments are made. As basic research investments are strategic substitutes – this is a robust result in our model –, increasing gains from the coordination of basic research investments in a multi-country framework should be expected. How precisely a multi-

\textsuperscript{19}Basic research may have an additional positive impact on human capital, which can be added in our model, and reinforces the results.

\textsuperscript{20}The simplest extension is a human capital production function with decreasing returns on investments in human capital – or at most, constant returns. A simple functional form is $L_j\theta_j = f(E_j)$, with $E_j$ denoting aggregate investments in human capital and $L_j\theta_j$ stands for aggregate human capital.
country setting affects the relative magnitude of coordination gains is an issue that must be left to future research.

Finally, other approaches to formulate perpetual growth could be combined with the basic research game of this paper. One especially promising framework in which to study basic research games is the growth model based on the continuing reduction of the importance of non-reproducible factors in production, as substantially developed by Seater (2005) and Zuleta (2008) regarding the social planner solution and Peretto and Seater (2013) for the balanced growth path and the complete transition path of the market equilibrium.\textsuperscript{21}

Overall it appears that the analysis of basic research games offers new insights into the potential and scope of policy-making to spur growth.

\textsuperscript{21}The type of technical change is, however, quite different from other forms of technical change (see Peretto and Seater, 2013, Peretto, 1999) and from the general characterization of Rebelo (1991). Thus, incorporating factor-eliminating technical change will require an entire new analysis.
Appendix

10 Proofs

This section provides the main proofs of our results.

10.1 Proof of Lemma 1

$y^p_n - \tilde{\gamma}n$ can be written as

$$y^p_n - \tilde{\gamma}n = \alpha \frac{\alpha}{\alpha+1} (\tilde{\gamma} \frac{\alpha}{\alpha+1} - 1) - \alpha \frac{1}{\alpha+1} (\tilde{\gamma} \frac{1}{\alpha+1} - 1) - \frac{1}{\alpha} \frac{1}{\alpha+1} \alpha^2.$$  (33)

This expression can be transformed into

$$y^p_n - \tilde{\gamma}n = (\alpha \frac{\alpha}{\alpha+1} - \alpha \frac{1}{\alpha+1}) (\tilde{\gamma} \frac{1}{\alpha+1} - 1).$$  (34)

For $\alpha \in (0, 1)$, we obtain $\alpha \frac{\alpha}{\alpha+1} > \alpha \frac{1}{\alpha+1}$. If additionally $\tilde{\gamma} \frac{1}{\alpha+1} > 1$, which is equivalent to $\gamma > 1/\alpha$, it follows that $y^p_n - \tilde{\gamma}n > 0$. $\blacksquare$

10.2 Lemmata 2 and 3 with proofs

Now let us state and prove two lemmata that are useful for the following proofs. It is useful to define the function

$$\rho^*_j(\rho_k) = \theta_j \left[ (1 - \rho_k) y^p_n + \tilde{\gamma}n \left( (1 - \rho_k) L + \rho_k \right) \right],$$  (35)

which is allowed to assume values in $\mathbb{R}$. Note that on the interval $[0, 1]$ the function (35) is identical to the reaction function given by (20). As there will be no confusion, the function in (35) will also be referred to as country $j$’s reaction function. The inverse function of $\rho^*_j(\rho_k)$ is denoted as $\rho^*_k(\rho_j)$. Hence, $\rho^*_k(0)$ gives the value of $\rho_k$ such that country $j$’s preferred basic research investment is 0, i.e. that $\rho^*_j(\rho_k) = 0$.

Lemma 2

If $y^p_n \geq \tilde{\gamma}n$, then $\rho^*_k(0) > 1$.

Proof. To verify the lemma, it is useful to write

$$\rho^*_k(0) = \frac{\tilde{\gamma}n L + y^p_n}{\tilde{\gamma}n L + y^p_n - \tilde{\gamma}n}.\quad (36)$$

The condition on $y^p_n$ at the beginning of the lemma ensures that the denominator of (36) is positive. Then the claim in Lemma 2 follows immediately. $\blacksquare$
The next lemma examines the slopes of the countries’ reaction functions. For this purpose, let us use

$$\rho_k^j(\rho_j) = \theta_k \left[ (1 - \rho_j)y_n^j + \hat{\gamma}n((1 - \rho_j)(1/L) + \rho_j) \right],$$

representing the reaction function of country \(k\), which is symmetric to \(\rho_j^k(\rho_j)\) as defined in (35). The inverse of \(\rho_k^j(\rho_j)\) is referred to by \(\rho_j^k(\rho_k)\).

**Lemma 3**

If \(y_n^j \geq \hat{\gamma}n\), a unique interior equilibrium \((\rho_j^e, \rho_k^e) \in (0, 1)^2\) implies

$$\frac{d\rho_j^k(\rho_k)}{d\rho_j} > \frac{d\rho_k^j(\rho_k)}{d\rho_k}.$$  \hspace{1cm} (38)

**Proof.** As the reaction functions are linear, it is sufficient to show that \(\rho_j^e(0) < \rho_k^e(0)\). This follows directly from Lemma 2 and Assumption 1. \(\square\)

**10.3 Proof of Proposition 2**

According to Proposition 1, \(\rho_H^j < 0\) and \(\rho_F^j < 0\). It follows directly from Lemma 2 and Assumption 1 that the reaction functions intersect in \((0, 1) \times (0, 1)\). Note that Assumption 1 also prevents the two reaction functions from coinciding, which would involve multiple equilibria.

The particular equilibrium values \((\rho_H^e, \rho_F^e)\) follow directly from calculation of the intersection of the reaction functions. \(\square\)

**10.4 Proof of Proposition 3**

Let us first consider (ii). According to (20), \(\frac{\partial \rho_j^k(\rho_k)}{\partial \theta_j} > 0 \ \forall \rho_k \in (0, 1)\). The inverse of the reaction function of \(k\), \(\rho_j^k(\rho_k)\), remains unchanged. As according to Proposition 1, it holds that \(\rho_j^j < 0\) and \(\rho_k^j < 0\), it follows from Lemma 3 that the new intersection of \(\rho_j^j(\rho_k)\) and \(\rho_k^j(\rho_k)\) involves a higher level \(\rho_j^e\) and a lower value \(\rho_k^e\).

Now consider item (i). Given the symmetry of the two countries’ reaction functions, it can be observed that if \(\theta_j = \theta_k\) and \(L = 1\), then \(\rho_j^j = \rho_k^k\). The claim in (i) can be verified by using (ii) and the following line of reasoning: The associated investment in basic research at any pair \((\hat{\theta}_j, \hat{\theta}_k)\) can be decomposed in two steps. First, let us start from the basic research levels associated with \((\theta_j = \hat{\theta}_k, \hat{\theta}_k)\), which implies symmetric basic research investments. Second, basic research levels can be adjusted by increasing or decreasing \(\theta_j\) to \(\hat{\theta}_j\).  \(\square\)
10.5 Proof of Proposition 4

Let us start with (ii). As $\rho_j' < 0$ and $\rho_k' < 0$, an increase in the relative population size of country $k$ affects the reaction functions in the following way: $\frac{\partial \rho_j'(\rho_k)}{\partial L} > 0 \forall \rho_k \in (0,1)$ and $\frac{\partial \rho_k'(\rho_j)}{\partial L} < 0 \forall \rho_j \in (0,1)$. That is, after the increase in $L$, the reaction function of $j$ lies above the reaction function before the increase, and the new reaction function of $k$ lies below the old one. Since $\rho_j' < 0$ and $\rho_k' < 0$, this immediately implies that the new intersection of the reaction functions involves $\frac{d\rho_j}{dL} > 0$ and $\frac{d\rho_k}{dL} < 0$.

(i) Since $\rho_j = \rho_k$ if $L = 1$ and $\theta_j = \theta_k$, it follows directly from item (ii) of this proposition that $\rho_j > \rho_k$ if $L > 1$.

10.6 Proof of Corollary 1

Consider an increase in $\theta_j$. According to (20), this involves $\frac{\partial \rho_j'(\rho_k)}{\partial \theta_j} > 0 \forall \rho_k \in (0,1)$, while the reaction function of $k$, $\rho_k'(\rho_k)$, remains unchanged. As a consequence, the new equilibrium will still be on $\rho_j'(\rho_k)$ and would involve no change in the sum of basic research investments if $\frac{\partial \rho_j'(\rho_k)}{\partial \rho_k} = -1$, while $\frac{\partial \rho_k'(\rho_k)}{\partial \rho_k} < (>) - 1$ would imply higher (lower) total basic research investment. Due to Assumption 1 and Lemma 2, it holds that $\frac{\partial \rho_j'(\rho_k)}{\partial \rho_k} < -1$.

11 Foundation for specification of basic research costs

Analytical convenience has prompted us to choose the particular functional form of the relation between the innovation success probability of domestic firms and the investments in basic research as displayed in Equation 6. More generally, our results require that the innovation probability be a strictly concave function of basic research investments. Our specification of the relation between the innovation success probability and basic research investments can also be interpreted as a reduced form of a more detailed model formulation where firms additionally invest in applied research to turn the abstract ideas created by basic research into blueprints for new intermediate products. At a general level, suppose that the probability of innovation success for an intermediate firm on variety $i$ in country $j$ is given by $\rho_{jt,i} = \min \{ \chi_t (R_{jt})^{\nu} (R_{jlt})^\nu, 1 \}$, where $R_{jt}^{BR}$ is the investment in basic research by the government in country $j$ in period $t$ and $R_{jlt}^{AR}$...
is the applied research of a firm innovating in sector \( i \). Further parameters such as the average level of human capital, \( \theta \), or the technology frontier, \( \bar{A}_{t-1} \), are subsumed in the parameter \( \chi_t \), and with respect to the elasticities, it is assumed that \( \mu, \nu \in (0,1) \) and \( \mu < 1 - \nu \). This specification exhibits complementarities between basic and applied research. When, given some basic research \( R_{jt}^{BR} \), the firm under consideration chooses the optimal level of applied research, it will invest

\[
R_{jt,i}^{AR} = (\nu \chi_t \frac{1}{1-\nu}) (R_{jt}^{BR})^{\frac{\mu}{1-\nu}}. \tag{39}
\]

This will be the case for all varieties \( i \). Consequently, the probability of successful innovation will be same for all varieties and can be written as

\[
\rho_{jt,i} = \min \left\{ \nu \chi_t \frac{1}{1-\nu} (R_{jt}^{BR})^{\frac{\mu}{1-\nu}}, 1 \right\}. \tag{40}
\]

Under our assumption that \( \mu < 1 - \nu \), the innovation success probability will be a strictly concave function of basic research, as is necessary for our results. In particular, our specification in Equation 6 can be recovered by setting \( \mu = \frac{1}{2} (1 - \nu) \) and \( \chi_t = \nu^{-\nu} \sqrt{\left(\frac{2\theta_j}{L_j \bar{A}_{t-1}}\right)^{1-\nu}} \), that is, by defining

\[
\rho_{jt,i} = \min \left\{ \nu^{-\nu} \sqrt{\left(\frac{2\theta_j}{L_j \bar{A}_{t-1}}\right)^{1-\nu}} (R_{jt}^{BR})^{\frac{1-\nu}{2\nu}}, 1 \right\}. \tag{41}
\]

Moreover, this more general definition of innovation success probabilities allows for different interpretations of the effects of human capital and the technological level on the innovation success of domestic firms. These are outlined in the next subsection.

### 11.1 Further micro-foundations

More precisely, there may be several reasons why the marginal productivity of basic research expenditures increases with the domestic average level of human capital.\(^{25}\)

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\(^{22}\)Details are available upon request.

\(^{23}\)We could also consider patent races between the incumbent intermediate firm and outside challengers. How many resources the firms will optimally spend on applied research will then depend on the particular specification of the probability of winning the patent given the applied research investments of the other firms. As long as net profits after deducting the applied research investments are positive, the forces in our model will remain at work. However, if all rents are dissipated in the patent races, i.e. the expected net profits are zero, the profit distribution across countries will of course have no effect on basic research investments.

\(^{24}\)Of course, strictly speaking this holds only for the range of basic research investments where \( \rho_{jt} < 1 \). However, this is the case we are concerned with in our analysis, as formally delineated by Assumption 1. This reflects the assumption on the model’s parameters such that endogenously chosen basic research investments will never be sufficient to achieve an innovation probability of 1.

\(^{25}\)More generally, it allows different interpretations with respect to the parameters captured in \( \chi_t \).
Those interpretations become particularly relevant when human capital is endogenized, a matter that is addressed in the concluding section. First, it could be the case that most of the basic researchers are recruited domestically and with better education and experience the expenditures in basic research will have a larger impact on the success probability of domestic firms. Second, rather than the basic researchers, who may be highly mobile and recruited internationally, it could be the applied researchers in the R&D labs of private firms who are mostly recruited locally and can better absorb the knowledge created by basic research when their human capital is higher. This in turn would imply that basic research expenditures are more productive when domestic human capital is higher. A third interpretation could be that high-tech entrepreneurs invest in applied research to commercialize ideas created by basic research. If the average human capital of these domestic entrepreneurs is higher, they may have higher business success (rather than higher absorptive capacity). Again, this would increase the marginal productivity of basic research. These interpretations illustrate that, when basic research and applied research are complements, all factors making applied research more productive will also increase the marginal productivity of basic research.

Our model framework takes the average level of human capital as exogenous. However, education is at least to a certain degree also a choice of the households. As our framework focuses on basic research investments of governments in open economies, the only benefit of human capital is to increase the probability of innovation success. Consequently, human capital could be endogenized in a similar way as applied research expenditures. Given basic research investments by the government, potential entrepreneurs choose how much to spend on education to increase their innovation success probability. We would then again arrive at an equivalent functional form of the relation between innovation success probability and basic research investment as in our basic model, where the parameter $\theta$ would now be substituted by the exogenous parameters reflecting the education technology of how resources spent on education are translated into human capital.

Our specification of the relation between innovation success probabilities and basic research investments also allows for a simple way of incorporating direct knowledge spillovers from domestic basic research investments to foreign firms by replacing $R_j$ by $R_j + bR_k$, where $b$ ($0 \leq b \leq 1$) is a parameter reflecting the strength of basic research spillovers. The implications of such an extension of the model are discussed in the
appendix in Section 13.1.

12 Intertemporal basic research investments

Until now, the governments have aimed at maximizing the current period’s consumption but do not consider future periods. There are good arguments for this assumption. Governments are usually appointed for a restricted period of time and focus on that period in their decisions. Also, in supranational institutions such as the European Commission, it is rare for the decision process to look any further than 20 years into the future (which is our interpretation of the model’s period length). Nevertheless, this section explores whether our results change substantially when the decision-makers consider longer time horizons. Accordingly, the intertemporal optimization problems of the governments in the decentralized setting and the one with coordination of basic research investments to maximize aggregate consumption are depicted. In this section, it is assumed that households enjoy linear utility from consumption, i.e., \( u(c) = c \).

Unfortunately, the optimization problems cannot be solved analytically. Using the same parameter values as before, our simulations suggest (a) that the decentralized equilibrium and the cooperative solution are unique, and (b) that the qualitative results remain unchanged. Of course, in quantitative terms, the intertemporal solutions exhibit generally higher levels of basic research investment.

First, the non-cooperative game is examined. Here the governments choose paths for basic research investment rather than investments in a single period only. The government’s problem is given by

\[
\max_{\{\rho_{jt}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t C_{jt}\]

subject to the evolution of the stock of technological knowledge

\[
\bar{A}_t = \bar{A}_{t-1} [\tilde{\gamma} + \theta_t (1 - \tilde{\gamma})].
\]

It is assumed that the discount factor \( \beta \) is sufficiently small for the objective to converge to a finite value. Standard dynamic programming arguments yield the first-order condition

\[
\bar{A}_{t-1} L_j \left[ y_n \frac{d q_t}{d \rho_{jt}} + \tilde{\gamma} n (\rho_{kt} + (1 - \rho_{kt}) L) \right] - \bar{A}_{t-1} L_j \frac{\rho_{jt}}{\theta_j} + \beta \tilde{c}_{jt+1} \frac{d \bar{A}_t}{d q_t} \frac{d q_t}{d \rho_{jt}} = 0,
\]
where \( \hat{c}_{jt+1} = C_{jt+1}/\bar{A}_t \). Note that

\[
\hat{c}_{jt+1} = L_j \left[ y_n(q_t) - \gamma \frac{1}{\alpha} n (\rho_{kt+1} (1 - \rho_{jt+1}) - \rho_{jt+1} (1 - \rho_{kt+1}) L) - \frac{\rho_{jt+1}^2}{2\theta_j} \right],
\]

(45)

which will be constant in the steady state. Focusing on the steady state, the first-order condition of country \( j \) reads

\[
L_j \left[ y_n(p_t) (1 - \rho_k) + \tilde{\gamma} n (\rho_k + (1 - \rho_k) L) \right] - L_j \frac{\rho_j}{\theta_j} + \beta \hat{c}_j (\rho_j, \rho_k) (\tilde{\gamma} - 1) (1 - \rho_k) = 0,
\]

(46)

where

\[
\hat{c}_j (\rho_j, \rho_k) = L_j \left[ y_n(q_t) - \gamma \frac{1}{\alpha} n (\rho_k (1 - \rho_j) - \rho_j (1 - \rho_k) L) - \frac{\rho_j^2}{2\theta_j} \right].
\]

(47)

As mentioned, it is not possible to solve this problem analytically. Instead, the coordinated investment problem is introduced first and then, the obtained simulation results are discussed.

Cooperative basic research investments maximize discounted aggregate consumption of both countries:

\[
\max_{\{\rho_j, \rho_k\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t (C_{jt} + C_{kt})
\]

subject to (15) and (43). Using standard dynamic programming methods, the first-order conditions are given by

\[
\bar{A}_{t-1} (L_j + L_k) y_n(1 - \rho_k) - \bar{A}_{t-1} L_j \frac{\rho_j}{\theta_j} + \beta \hat{c}_{jt+1} \frac{d\bar{A}_t}{dq_t} \frac{dq_t}{d\rho_{jt}} = 0,
\]

(49)

where \( \hat{c}_{jt+1} = C_{jt+1} / \bar{A}_t \). Note that the two first-order conditions imply

\[
\frac{\theta_{jt}}{\theta_{kt}} L = \frac{\rho_{jt}}{\rho_{kt}} \frac{1 - \rho_{jt}}{1 - \rho_{kt}},
\]

(50)

which can be interpreted as a condition on the cost efficiency of aggregate research expenditures. Again the focus is on the steady state of the economy. The necessary conditions for a maximum can then be rewritten as

\[
(L_j + L_k) y_n (1 - \rho_k) - L_j \frac{\rho_j}{\theta_j} + \beta \hat{c}(\rho_j, \rho_k) (\tilde{\gamma} - 1) (1 - \rho_k) = 0,
\]

(51)

where

\[
\hat{c}(\rho_j, \rho_k) = (L_j + L_k) y_n(q) - L_j \frac{\rho_j^2}{2\theta_j} - L_k \frac{\rho_k^2}{2\theta_k}.
\]

(52)

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26Conceptually, there exists a formula that allows us to solve explicitly for basic research investments. However, the extremely long terms only allow interpretation via simulation.
Two parts can be identified in the optimality conditions for the cooperative solution. The first two summands in (51) reflect static optimality if the influence of a higher knowledge stock on future outcomes is neglected. The latter is represented by the last summand.

Before discussing the numerical simulation results, it is noted that there is the (implicit) assumption that at time \( t = 0 \) both, the governments in the decentralized game and the social planner in the coordinated problem can fully commit to the entire path of basic research investments. However, since all first-order conditions are independent of the technological level, there will be no incentive to re-optimize by a later government based on the evolution of the technology stock, which is the only stock variable in the model. Moreover, time-inconsistency problems are precluded by assuming standard exponential discounting. Consequently, the first-order conditions would remain the same when considering the non-commitment case where governments or a social planner with infinite planning horizons were able to re-optimize in each period.

Let us now turn to numerical simulations using the parameter values of our standard specification. Figures 4 and 5 give the decentralized steady-state equilibrium values of \( \rho_j \) and \( \rho_k \) (blue) compared to the cooperative solution (red) depending on relative population sizes and the level of human capital in country \( j \) given that \( \theta_k = 0.03 \).

![Figure 4](image1.png)

Figure 4: Steady state basic research investments in country \( j \), \( \rho_j \), (left) and in country \( k \), \( \rho_k \), (right) in the decentralized solution (blue) and the coordination optimum (red, dashed) for different relative population sizes \( L \), when the governments and the social planner have infinite planning horizons.

The next Figures 6 and 7 show the sum of basic research investments and the growth rates in the decentralized equilibrium (blue) and the cooperative solution (red, dashed) depending on \( L \) and \( \theta_j \).
Figure 5: Steady state basic research investments in country $j$, $\rho_j$, (left) and in country $k$, $\rho_k$, (right) in the decentralized solution (blue) and the coordination optimum (red, dashed) for different levels of human capital in country $j$, $\theta_j$, given the level of human capital in country $k$, $\theta_k = 0.003$, when the governments and the social planner have infinite planning horizons.

Figure 6: Total basic research investments in the decentralized solution (blue) and the coordination optimum (red, dashed) depending on relative population sizes $L$ (left) and levels of human capital $\theta_j$ (right), given the level of human capital in country $k$, $\theta_k = 0.003$.

A comparison with the results in Section 6 indicates that basic research investments and growth rates are higher than in the setting where only current consumption is taken into account. However, qualitatively similar functional shapes and comparisons between the decentralized and the coordinated solutions are obtained. Also, our simulations strongly suggest that the above solutions are unique. Deviating reasonably from our standard parameter values does not change the qualitative results.
Figure 7: Growth rates in the decentralized solution (blue) and the coordination optimum (red, dashed) depending on relative population sizes $L$ (left) and levels of human capital $\theta_j$ (right), given the level of human capital in country $k$, $\theta_k = 0.003$.

13 Robustness

The robustness of our results will be discussed with respect to the inclusion of direct knowledge spillovers from basic research, to expanding the range of parameter values in the simulations, to different objectives of coordination, and to other formulations of the costs of basic research.

13.1 Direct knowledge spillovers from basic research

In our basic framework, basic research investments in one country do not directly affect the innovation probability of firms in the foreign country. Here, our definition of the innovation probabilities of firms is extended to capture such knowledge spillovers from basic research. In particular, let us define

$$\rho_{jt} = \min\left\{ \sqrt{\frac{2\theta_j}{L_j A_{t-1}}(R_{jt} + bR_{kt})}, 1 \right\},$$  

(53)

where $b \in [0, 1]$ reflects the strength of direct knowledge spillovers from basic research. With this definition it is possible to write the government’s objective similar to Equation 15

$$C_{jt} = \left( y_n(q_t) + \gamma \nu n \left( \rho_{jt}(1 - \rho_{kt}) \frac{L_k}{L_j} - \rho_{kt}(1 - \rho_{jt}) \right) \right) - R_{jt},$$

(54)
where gross consumption without research costs is referred to by $C^g_{jt}$. Again, neglecting time indices for the static problem, the government’s first order condition now becomes

$$\frac{\partial C^g_{jt}}{\partial \rho_j} \frac{\partial \rho_j}{\partial R_j} - 1 + \frac{\partial C^g_{jt}}{\partial \rho_k} \frac{\partial \rho_k}{\partial R_j} = 0. \quad (55)$$

The first two summands represent the first-order condition without direct knowledge spillovers, while the last summand reflects the effect of a marginal increase in basic research investment on consumption in country $j$ via direct knowledge spillovers to country $k$. As it is assumed that knowledge spillovers increase the innovation success of firms in the foreign country, $\frac{\partial \rho_k}{\partial R_j} > 0$, it depends on the sign of the term $\frac{\partial C^g_{jt}}{\partial \rho_k}$ whether the knowledge spillovers will increase or decrease basic research investments relative to the situation without knowledge spillovers. The effect of an increase in the innovation probability of foreign firms on domestic consumption can be written as follows:

$$\frac{\partial C^g_{jt}}{\partial \rho_k} = y^p_n(1 - \rho_j) - \tilde{\gamma} n(\rho_j L + (1 - \rho_j)). \quad (56)$$

The first term reflects the increase in domestic output per capita via knowledge spillovers from domestic basic research investments. As due to knowledge spillovers foreign firms will innovate with higher probability, there will be a higher chance that an innovation will occur in a sector where no domestic firm has succeeded in innovating. However, this increase in the expected net output of final-good production comes at the cost of forgoing the profits in the respective sector. This cost is captured by the last summand in (56). Moreover, with more innovative firms in the foreign country, it will be harder for domestic firms to reap profits in the foreign country. These two cost components are captured by the second summand. Rewriting Equation 56 as

$$\frac{\partial C^g_{jt}}{\partial \rho_k} = (1 - \rho_j)(y^p_n - \tilde{\gamma} n) - \tilde{\gamma} n \rho_j L, \quad (57)$$

the terms can be summarized to identify two effects: knowledge spillovers from basic research investments increase domestic consumer surplus but reduce the profit flows from abroad. The first effect will increase basic research investments, while the second will reduce them. It can be observed directly that for very small countries, where $L$ is very large, knowledge spillovers have a negative effect on basic research investments, while in large countries, where $L$ is small, the opposite is the case.

Let us now compare the decentralized solution with that of a social planner maximizing aggregate welfare

$$C_t = (L_j + L_k)y_n(q) - R_j - R_k. \quad (58)$$
This involves the following first-order conditions for a maximum:\footnote{The social planner’s objective is strictly concave in \( R_j \) and \( R_k \).}

\[
\frac{\partial C_t}{\partial \rho_j} \frac{\partial \rho_j}{\partial R_j} + \frac{\partial C_t}{\partial \rho_k} \frac{\partial \rho_k}{\partial R_j} = 0.
\]  

(59)

The first term corresponds to the first-order condition in the problem without direct knowledge spillovers from basic research. The effects of knowledge spillovers on socially optimal basic research investments are captured by the second term, in particular by \( \frac{\partial C_t}{\partial \rho_k} \). Dividing the first-order condition by \( L_j \) yields

\[
\frac{\partial C_t}{\partial \rho_k} \frac{1}{L_j} = (1 + L_j)y_n^p(1 - \rho_j).
\]  

(60)

Now, it is interesting to compare this part of the first-order condition of the socially optimal solution representing the effects of knowledge spillovers from basic research with the corresponding part in the first-order conditions of the decentrally acting governments. It can be observed that the social planner accounts for the expected global output increase due to the higher innovativeness of firms in country \( k \) as a consequence of marginally higher basic research investment in country \( j \). However, country \( j \) takes only the increase in its consumer surplus into account. Consequently, in the decentralized solution the inclusion of knowledge spillovers leads to more pronounced under-investment in basic research relative to the social optimum. This under-investment by decentrally acting governments is further reinforced by the additional motive of not reducing the inflowing profits from the foreign country by making the latter’s firms more competitive via direct knowledge spillovers.

In summary, direct knowledge spillovers from basic research investments in one country increasing the innovativeness of firms in the foreign country may increase or decrease decentrally chosen basic research investments. This depends on the relation between the increases in consumer surplus in the domestic country and the reduction in profit inflows from the foreign country. However, relative to the global socially optimal solution, the inclusion of knowledge spillovers constitutes an additional source of under-investment in basic research.

### 13.2 Larger range of parameter values

Section 8 derived our results on the relation between the decentralized equilibrium and the social planner solution by varying either the relative population size while keeping...
both countries’ human capital levels fixed at $\theta_j = \theta_k = 0.03$ or by varying country $j$’s level of human capital while assuming symmetric population sizes. Our simulation results show that the results derived in Section 8 possess broad validity with respect to different combinations of $\theta_j, \theta_k$ and $L$. Moreover, the same qualitative results are obtained for a wide range of other parameter values for $\alpha$, $\gamma$, and $\theta_k$.

13.3 Coordination objective

As already discussed, the coordination of basic research investments aiming to maximize the sum of aggregate consumption in both countries involves higher levels of basic research in the smaller country because in absolute terms the costs of basic research are lower there. This incentive will not be present if coordination is concerned with maximizing net per capita consumption:

$$\bar{c}_t = \bar{A}_{t-1} \left( 2y_n(q_t) - \frac{\rho_j^2}{2\theta_j} - \frac{\rho_k^2}{2\theta_k} \right).$$

(61)

The necessary conditions for an optimum are

$$\rho_j = 2\theta_j y_n^p(1 - \rho_k).$$

(62)

Note that there is no weighting factor reflecting relative population sizes. Instead, the contribution of basic research to each country’s per capita output levels obtains the same weight in the cooperative solution. When comparing (62) with the optimality conditions of the cooperative solution in Section 8, it can be observed that they coincide for equal population sizes. The following figures illustrate how the decentralized equilibrium differs from the coordination optimum maximizing net per capita consumption for different relative population sizes in the standard scenario.

Figure 8 indicates that when cooperation maximizes net per capita consumption, the decentralized equilibrium implies too little basic research investment in large countries and too much in small ones. The contrary was the case in the coordination optimum maximizing aggregate consumption. The conclusions that both objectives share is that there is under-investment in basic research if the countries are symmetric with respect to population size and human capital levels. However, for very asymmetric population sizes, the decentralized equilibrium involves total basic research investments and growth rates that are too high relative to the net per-capita consumption coordination optimum.
Figure 8: Basic research investments in country \( j \), \( \rho_j \), (left) and in country \( k \), \( \rho_k \), (right) in the decentralized solution (blue) and the coordination optimum (red, dashed) for different relative population sizes \( L \), when the social welfare function does not include population weights.

Figure 9: Total basic research investments (left) and growth rates (right) in the decentralized solution (blue) and the coordination optimum (red, dashed) for different relative population sizes \( L \), when the social welfare function does not include population weights.

13.4 Different research costs

Let us now assume that the costs of basic research decline with absolute levels of human capital rather than with per capita human capital levels. Then the research costs can be written as \( R_j = \frac{\rho_j^2}{2\theta_j} \). Consequently, the governments’ objectives in the non-cooperative setting can be written as

\[
C_{jt} = L_j \tilde{A}_{t-1} \left[ y_n(q_t) - \gamma \frac{\alpha}{\gamma_0} n(\rho_{kt}(1 - \rho_{jt}) - \rho_{jt}(1 - \rho_{kt})L) \right] - \frac{\rho_{jt}^2}{2\theta_j}. \tag{63}
\]

The reaction functions are

\[
\rho_j^*(\rho_k) = \theta_j L_j \left[ (1 - \rho_k)y_n^p + \gamma n(1 - \rho_k)L + \rho_k \right]. \tag{64}
\]
We observe that the only difference with respect to the reaction functions in Section 6 is that the optimal level of a country’s basic research investments increases with its population size. Now there are two conflicting motives for basic research investment with respect to relative population size (and given total population). On the one hand, a relatively larger foreign market increases incentives for basic research due to higher net profit flows. On the other hand, a smaller home market reduces the incentives for basic research.

Coordinating basic research to maximize current aggregate consumption \( \hat{C}_j + \hat{C}_k \) yields the following necessary conditions for an optimum: \( \rho_j = \theta_j(L_j + L_k)y_n^p(1 - \rho_k) \). Hence, the optimal levels of the coordinated basic research investments do not depend on relative population sizes but only on the two countries’ total population. The necessary conditions are (up to a constant) equivalent to those derived for the cooperative optimum in the previous Section 13.3.
References


