Abstract

Recent evidence questions the nature of the re-employment spike as unemployment insurance (UI) payments expire. Unemployed agents do not appear to devote more time to search and are observed leaving the UI scheme early without necessarily entering employment. We show that benefit fraud is consistent with both observations. Over time, UI recipients become increasingly willing to accept short-term cash-in-hand work. This takes them away from job search. Immediately before UI expiry, the risk of punishment for fraud exceeds the value of remaining payments. Recipients may voluntarily leave the scheme to accept cash-in-hand opportunities.

Keywords: Cash-in-hand, Benefit fraud, Unemployment insurance, Re-employment spike

1 Introduction

The existence of an expiry date for unemployment insurance (UI) payments has given rise to a long-standing theoretical prediction. As an unemployed agent nears the date that they lose their benefits, their incentive to search increases. We would therefore expect to see a re-employment spike as expiry approaches (Mortensen 1977).

Empirical work has recently called this into question. Time use surveys suggest little increase in time devoted to search over the duration of unemployment (Krueger and Mueller 2010, 2011, 2012). Moreover, many studies assume that when agents leaves the
scheme, it is due to re-employment. This is not necessarily the case (Card et al. 2007b). Several theories have been put forward to explain this new-found empirical puzzle, from despondency at the lack of job offers (Krueger and Mueller 2010, 2011, 2012) and social stigma (Contini and Richiardi 2012) to shoe leather costs associated with collecting benefits (Card et al. 2007b) or liquidity constraints (Card et al. 2007a; Chetty 2008). Undoubtedly, all play important roles, but none explain the lack of search and leaving early at the same time.

The novel contribution of this paper is to show that the incentives created by cash-in-hand work and benefit fraud are consistent with both observations. We illustrate this result using a dynamic model of job search where an unemployed agent faces a sequence of one-off cash-in-hand income opportunities whilst receiving UI payments. She may be offered a week’s work on a building site, or bar work on the day of a big sporting event. The agent can either reject the opportunity and remain on benefits, accept it and leave the UI scheme, or commit fraud: accept the opportunity, but hide the income from the authorities. In the latter case, the agent continues to claim benefits, but faces the possibility of being punished if caught.

During the early stages of unemployment, fear of losing future benefits ensures that the agent rejects all but the most lucrative opportunities. Over time, with fewer benefit payments to look forward to, the agent engages in increasing amounts of benefit fraud. It becomes less likely that they search intensively for a job. In the last few weeks of UI receipt, the risk of punishment becomes sufficient for the agent to leave the scheme, foregoing her few remaining benefit payments in order to accept cash-in-hand opportunities legally.

UI schemes in many countries allow individuals to declare casual income earned while receiving unemployment benefit. The agent suffers a reduction in her current UI payment, but remains in the scheme and will not be punished. We find that our results hold even in this more general setting. The agent still engages in an increasing amount of fraud over time. As before, she has less to lose as she approaches UI expiry. In the final period, she leaves the scheme with positive probability in order to avoid the administrative costs associated with declaring income.

The paper proceeds as follows. In the next section, we outline some common features of UI schemes, discuss evidence on benefit fraud and relate our contribution to the literature. Section 3 describes the economic environment. Section 4 derives conditions for fraud and leaving the UI scheme, before Section 5 considers their evolution over the lifetime of the UI scheme. Section 6 extends the analysis by allowing the agent to declare her cash-in-hand income. Section 7 summarises the main findings and reflects on some
of their policy implications. Proofs are provided in the appendix.

2 Unemployment Insurance and Benefit Fraud

Although the design of UI schemes varies across countries, many share the same basic features (see OECD 2013 or Tatsiramos and van Ours 2014 for recent surveys). Payments are only made if certain eligibility criteria are met. Aside from the obvious – unemployment – an agent’s employment history, wealth and previous contributions may also be taken into account, as well as any previous fraudulent behaviour. Many schemes then provide constant weekly payments for a fixed length of time. For example, the United Kingdom will pay up to £73.10 per week for up to six months (UK Government 2015b). Should an agent remain unemployed after this point, payments are stopped (they are said to have expired). Following a long literature (c.f. Mortensen 1977), our model adopts this form of UI payments.

UI represents a contract between the agent and the state. As such, conditions are placed on her activities. All systems expect individuals to accept a permanent job if it is offered. They also require the agent to exert effort in job-search. Some systems explicitly prohibit work whilst in receipt of UI payments (e.g. Italy or Japan [OECD 2013]). Others require that earnings are declared (for example the UK or the US), leading to temporary reductions or removal of payments. Failure to meet any of these criteria constitutes benefit fraud (Becker 1953). We focus on the latter form: the decision to secretly accept work whilst receiving UI payments. Our stylised model mimics the case in which the agent is prohibited from working. She either rejects opportunities to earn cash-in-hand income (as specified by her UI contract) and continues to search, commits fraud by secretly accepting the work, or she leaves the scheme. We then show our the results are robust when she is also able to declare the income but remain in the scheme.

The extent of UI fraud is a matter of some debate. Its clandestine nature makes it difficult to measure. Official statistics are not collected in many countries (van Stolk et al. 2006), and represent a significant underestimate where they are (Kingston et al. 1986; Burgess 1992; Levi and Burrows 2008). For example, the UK suggests that only 0.7 per cent its UI payments are claimed fraudulently\(^1\). However, surveys of UI claimants from similar EU countries suggest that somewhere between one-in-three and one-in-five

\(^1\) The UK is viewed as the world leader in the measurement of UI fraud (van Stolk et al. 2006). The Department of Work and Pensions (DWP) annually samples 0.3 per cent of UI claimants. They review the claimant’s paperwork and conduct an interview. Fraud is determined to have occurred if irregularities are revealed in the claimant’s file, or if the interviewer has cause to become suspicious (because, for example, the interviewee fails to attend).
engage in fraud, mostly failure to declare income (van Stolk et al. 2006).

Should the agent be caught committing fraud, punishments vary considerably. It includes, but is not limited to, withdrawal of eligibility for future benefits, fines and prison terms (in extreme cases; see UK Government 2015a). The size of the fine may be related to the income earned, or the number of counts of fraud that the individual is convicted of (Yaniv 1986). Given the variety of penalties on offer, our model takes a very reduced-form approach to punishment: we assume that it provides a deterrent. Upon being convicted, the agent always regrets committing fraud. Her discounted utility is lower than it would have been if she had left the scheme instead.

Early theoretical work on UI fraud viewed the decision to engage in undeclared work as a time allocation problem (Yaniv 1986). Like our model, agents trade-off the income they receive against the risk of punishment. We extend this in two main directions. First, we consider a dynamic environment. This results in a theoretical framework where the behaviour of unemployed agents is reconciled with the empirical evidence on job search mentioned above. Second, we also endogenise search effort as in Shavell and Weiss (1979). This implies that undeclared income goes hand-in-hand with another form of benefit fraud: inadequate job-search.

Our work is closely related to another recent paper on UI fraud (Fuller et al. 2015). The authors use a dynamic model of job search to investigate when a recently employed agent should inform the authorities of their change of circumstances. They focus on the optimal UI scheme and monitoring. Instead we investigate the incentives generated by the existing institutional frameworks. We also incorporate stochastic cash-in-hand opportunities, as in Burdett et al. (2003), which allows for the possibility that the agent commits fraud on a sporadic basis. Receiving identical opportunities every week is a special case.

The intuition behind our results relies on the agent’s response to dynamic incentives. Evidence on the link between UI payments and crime suggests this is well-founded. A recent study investigated how criminal activity varied in the time between UI payments (Foley 2011). UI recipients were found to consume a large share of their legitimate income soon after they received it, supplementing consumption with increasing amounts of crime as they got closer to their next payment. As in our model, agents’ behaviour is found to be non-stationary. Consistent with this, we also find that the incentive to accept cash-in-hand increases as the expiry date of UI approaches.
3 Economic Environment

We consider a recently unemployed agent, who is entitled to weekly payments under a widely used UI scheme. The agent is liquidity constrained: she has no cash-on-hand and cannot borrow. For the first $T$ weeks of unemployment, she receives $b$. Should she remain jobless thereafter, she receives nothing:

$$b_t = \begin{cases} b, & t \leq T \\ 0, & t > T. \end{cases}$$

Every week, the agent also receives a private i.i.d. cash-in-hand opportunity, paying $c_t \geq 0$ and drawn from some distribution $F(c)$, which she either accepts ($a_t = 1$) or rejects ($a_t = 0$). Accepting cash-in-hand work whilst receiving UI constitutes benefit fraud, and so the agent also chooses whether to remain in the scheme or leave. This is, of course, unrealistic in many countries. We relax the assumption later, but the simplification is useful to outline the intuition. If she leaves, she foregoes her remaining benefits but faces no risk of sanctions. If she commits fraud, she continues to receive benefits, but risks being caught with probability $\phi$. Punishment provides her with discounted utility $V^{\text{pun}}$.

The agent simultaneously exerts effort searching for a permanent job. For simplicity we assume that all jobs provide identical discounted utility $V^e$, and that offers are always accepted. This allows us to abstract from discussions of reservation wages. The effort cost of generating an offer with probability $p_t$ is $e(a_t, p_t)$; it is increasing and convex in $p_t$. Accepting cash-in-hand work makes searching more difficult, in the sense that it (weakly) raises the marginal cost of $p_t$.

The agent’s week $t$ payoffs can be represented by the following system of Bellman equations:

$$V^{\text{in}}_t(c_t) = \max_{a_t, p_t} \left\{ u(b + a_t c_t) - e(a_t, p_t) + \beta a_t \phi V^{\text{pun}} + \beta (1 - a_t \phi) [p_t V^e + (1 - p_t) \mathbb{E} V_{t+1}] \right\},$$

(1)

$$V^{\text{out}}_t(c_t) = \max_{a_t, p_t} \left\{ u(a_t c_t) - e(a_t, p_t) + \beta [p_t V^e + (1 - p_t) \mathbb{E} V^{\text{out}}_{t+1}] \right\},$$

(2)

where:

$$\mathbb{E} V_{t+1} = \begin{cases} \mathbb{E} \left[ \max \left\{ V^{\text{in}}_{t+1}(c_{t+1}), V^{\text{out}}_{t+1}(c_{t+1}) \right\} \right], & t + 1 \leq T \\ \mathbb{E} V^{\text{out}}_{t+1}, & t + 1 > T. \end{cases}$$

If she is eligible for, and chooses to remain in, the scheme then she receives $[1]$. She gains
utility from consuming income (satisfying positive diminishing marginal utility and the Inada conditions), and exerts effort searching for a job. If she commits fraud, then she is caught with probability $\phi$ and receives $V^{\text{pun}}$ in the following week. Otherwise, she receives a job offer with probability $p_t$. With probability $1 - p_t$ she remains unemployed and draws a new opportunity. Whilst we assume that job offers are conditional on not being caught committing fraud, our results are robust to alternative formulations.

If she either leaves the scheme in week $t$, or is no longer eligible for benefits (because they expired or she has already left), her payoff is given by (2). Her only income comes from any accepted cash-in-hand work. She exerts search effort $e(a_t, p_t)$. If she receives a job offer, she earns discounted utility $V^e$ in the following week. Otherwise, she draws a new cash-in-hand opportunity and continues to search. We assume that, if the agent is caught committing fraud, she would always have preferred to have left the UI scheme: $V^{\text{pun}} < \mathbb{E}V^{\text{out}}$. In this sense, punishment always provides a deterrent.

Suppose that the agent remained in the UI scheme in week $t - 1$ and $t \leq T$. She learns $c_t$ and then decides whether to leave, choosing between $V^{\text{in}}(c_t)$ and $V^{\text{out}}(c_t)$. However, upon leaving the scheme she is unable to rejoin. She receives $V^{\text{out}}$ until she finds a job.

### 4 When is the Agent Fraudulent?

We now outline the choices that the agent makes in each week. When she is no longer eligible for UI payments, she only has two decisions: her re-employment probability and whether she accepts her cash-in-hand opportunity. Since she no longer faces benefits expiry in the future, the problem is stationary. Her optimal $p_t$, conditional on her cash-in-hand work, satisfies:

$$\beta(V^e - \mathbb{E}V^{\text{out}}) = e_p[a_t, p^{\text{out}}(a_t)].$$

The marginal benefit of search derives from the expected utility gain when a job is offered, starting in the following week. The marginal cost comes from the effort exerted. If the agent accepts cash-in-hand work then her marginal cost is higher, and it becomes less likely that she will find work.

Her decision to take cash-in-hand work similarly depends upon her likelihood of
receiving a job offer. She will accept an opportunity if and only if:

\[
\begin{align*}
    u(c_t) - e[1, p^{\text{out}}(1)] + \beta\{p^{\text{out}}(1)V^e + [1 - p^{\text{out}}(1)]EV^{\text{out}}\} \\
    \geq -e[0, p^{\text{out}}(0)] + \beta\{p^{\text{out}}(0)V^e + [1 - p^{\text{out}}(0)]EV^{\text{out}}\} \\
    \iff c_t \geq c^{\text{out}}.
\end{align*}
\] (4) (5)

The utility she receives by accepting cash-in-hand work (the left-hand side of (4)) is strictly increasing in the value of her current opportunity, \(c_t\). Conversely her utility from rejecting the opportunity (the right-hand side of (4)) is independent of \(c_t\). There consequently exists a threshold, \(c^{\text{out}}\) such that she gains higher utility by accepting if and only if \(c_t \geq c^{\text{out}}\). Intuitively, whilst accepting cash-in-hand work enables the agent to enjoy instantaneous utility from consumption, it increases her marginal cost of effort. She generates a job offer with a lower probability, reducing her expected future payoff. She therefore only accepts opportunities that provide a sufficiently large increase in instantaneous consumption to compensate for the effects of lower search effort.

If, instead, the agent is in receipt of UI, she must decide whether to remain in the scheme as well as choosing \(p_t\) and \(a_t\). If she leaves, her optimal choices solve (3) and (5). Otherwise, she chooses a probability of finding a job, \(p^{\text{in}}(a_t)\), satisfying:

\[
\beta(1 - a_t\phi)(V^e - EV_{t+1}) = c_p[a_t, p^{\text{in}}(a_t)].
\] (6)

The interpretation is almost identical to (3). However, the marginal benefit of search is lower. She may be caught committing fraud and lose her job offer. In this sense, two forms of benefit fraud go hand-in-hand: accepting cash-in-hand work leads to lower search effort than would be the case if she conformed to the requirements of the UI scheme. Since she still receives benefits, her continuation payoff from remaining unemployed is also higher, reducing the marginal benefit of search further. The solution is non-stationary, as \(EV_{t+1}\) depends on the number of remaining UI payments.

If she accepts \(c_t\), she commits benefit fraud. Although her instantaneous income is higher, she could be caught and punished. Assuming that she does not leave the UI scheme, she will commit fraud if and only if:

\[
\begin{align*}
    u(b + c_t) - e[1, p^{\text{in}}(1)] + \beta\phi V^{\text{pun}} + \beta(1 - \phi)\{p^{\text{in}}(1)V^e + [1 - p^{\text{in}}(1)]EV_{t+1}\} \\
    \geq u(b) - e[0, p^{\text{in}}(0)] + \beta\{p^{\text{in}}(0)V^e + [1 - p^{\text{in}}(0)]EV_{t+1}\} \\
    \iff c_t \geq c^{\text{in}}.
\end{align*}
\] (7)
This has a similar interpretation to (5).

Finally, the agent chooses to leave only if $V_{\text{out}}^t(c_t) \geq V_{\text{in}}^t(c_t)$. Comparing (1) and (2), the agent can always guarantee greater instantaneous income by committing fraud. If she chooses not to (i.e. $c_t < c_{\text{in}}^t$), then her payoff must be higher still. A necessary condition to leave is:

\[
-e[1, p_{\text{in}}^t(1)] + \beta \phi V_{\text{pun}}^t + \beta (1 - \phi) \{p_{\text{in}}^t(1)V^e + [1 - p_{\text{in}}^t(1)]E_{V_{t+1}}\} \\
< -e[1, p_{\text{out}}^t(1)] + \beta \{p_{\text{out}}^t(1)V^e + [1 - p_{\text{out}}^t(1)]E_{V_{\text{out}}}\}. \tag{8}
\]

The left-hand side of (8) is increasing in $E_{V_{t+1}}$. So long as the continuation payoff from remaining in the UI scheme is sufficiently large, the agent prefers to remain in the scheme – committing benefit fraud when it is profitable – to leaving. If (8) holds, then she will leave if the current UI payment is not sufficient to compensate for lower future utility.

5 Evolution of Benefit Fraud

Consider now how the agent’s decisions change during the first $T$ weeks of unemployment. Suppose that the UI scheme provides sufficient insurance that (8) does not hold in week one. She would never choose to leave UI, but may commit benefit fraud. Although the agent receives constant payments during this time, she understands that she is gradually approaching her benefits expiry date. The continuation payoff from remaining unemployed and receiving UI, $E_{V_{t+1}}$, is falling. This changes her incentive to accept cash-in-hand work:

**Proposition 1 (Fraud over time)** If the probability of punishment is sufficiently large, then the agent accepts a greater range of opportunities over time:

\[
\frac{\partial c_{\text{in}}^t}{\partial t} < 0.
\]

If the probability of punishment is sufficiently large, then the agent risks a lot by committing fraud during the first few weeks of unemployment. She has a lot of benefits to look forward to, and would lose them if she were caught. As such, she only accepts very profitable opportunities. As her duration of unemployment increases, she has less to lose by committing fraud. She therefore becomes willing to accept lower $c_t$s.
This result is consistent with the aforementioned evidence from time use surveys. The expected probability of finding a job, conditional on not being caught, is:

\[ F(c^i_t)p^i_t(0) + [1 - F(c^i_t)]p^i_t(1). \]

Although she applies more effort conditional on \( a_t \) as the expiry date approaches, she also becomes more likely to commit fraud. This lowers the expected probability of receiving a job offer.

As the length of time that the agent has been unemployed increases, the benefit from remaining in the UI scheme declines. She runs the risk of being caught committing fraud, but has fewer UI payments to look forward to in return. Eventually, she may prefer to leave the scheme:

**Proposition 2 (Leaving early)** The agent would never leave the UI scheme when she first becomes unemployed. As the duration of her unemployment increases, the probability that she leaves the scheme weakly increases, and is strictly positive by week \( T \).

To illustrate, consider the agent’s week-\( T \) decision. She knows that she will not receive future payments: \( EV_{T+1} = EV^\text{out} \). Since \( V^\text{pun} < EV^\text{out} \) and \( p^i_T(1) < p^\text{out}(1) \), \( (8) \) is satisfied. If she accepts her current opportunity, then her payoff from remaining in the scheme and leaving are:

\[
\begin{align*}
&u(b + c_T) - e[1, p^i_T(1)] + \beta \phi V^\text{pun} + \beta (1 - \phi)\{p^i_T(1)V^e + [1 - p^i_T(1)]EV^\text{out}\}, \\
&u(c_T) - e[1, p^\text{out}(1)] + \beta \{p^\text{out}(1)V^e + [1 - p^\text{out}(1)]EV^\text{out}\}.
\end{align*}
\]

Her only benefit from remaining in the UI scheme is \( b \) today. However, she runs the risk of being prosecuted for fraud. It is straightforward to show that, excluding instantaneous utility from consumption, she is better off leaving. If \( c_T \) is sufficiently large, then her marginal utility of income is small enough that the extra consumption from receiving \( b \) does not compensate her for this risk. She would prefer to leave the scheme early.

This is consistent with the empirically-observed ‘re-employment’ spike at benefits exhaustion. In the final few weeks of unemployment, the size of the cash-in-hand opportunity required to cause the agent to leave declines.

### 6 Declared Cash-in-Hand Earnings

Suppose that the UI scheme allows the agent to declare her cash-in-hand income (as, for example, in the UK or US). She faces no sanctions for fraud and is allowed to remain in...
the scheme, but suffers a reduction in her current benefits proportional to her earnings. For simplicity, we assume that she cannot underreport the income that she receives. Upon accepting and declaring an opportunity $c_t$, she receives:

$$b^{dec}(c_t) = \max\{b - \alpha c_t, 0\}.$$ 

For every £1 earned, her UI payment is cut by £$\alpha$, where $\alpha \in (0, 1)$, up to the point where she is no longer eligible for any payment in the current week. Her total income is $\max\{b + (1 - \alpha)c_t, c_t\} \geq c_t$, so she is always able to enjoy weakly higher consumption by declaring $c_t$ than by leaving the scheme. However, declaring accepted cash-in-hand income involves completing paperwork. This generates a small effort cost, $\tau > 0$, similar to the shoe-leather costs discussed by [Card et al. (2007b)]. Whilst not essential for our results, introducing $\tau$ prevents us from relying on indifference arguments later on.

The agent’s payoff from accepting a cash-in-hand opportunity and declaring it is represented by the following Bellman equation:

$$V^{dec}_t(c_t) = \max_{p_t} \left\{ u[b^{dec}(c_t) + c_t] - \tau - e(1, p_t) + \beta [p_t V^e + (1 - p_t) \mathbb{E}V_{t+1}] \right\},$$  

(9)

where:

$$\mathbb{E}V_{t+1} = \begin{cases} \mathbb{E} \left[ \max \{V^{in}_{t+1}(c_{t+1}), V^{out}_{t+1}(c_{t+1}), V^{dec}_{t+1}(c_{t+1})\} \right], & t + 1 \leq T \\ \mathbb{E}V^{out}, & t + 1 > T, \end{cases}$$

and $V^{in}_t(c_t)$ and $V^{out}_t(c_t)$ are still defined by (1) and (2) respectively. By declaring an accepted opportunity ($a_t = 1$ for her to have income to declare), the agent receives a reduced UI payment and pays the administrative cost, $\tau$. She exerts effort searching, but faces no risk of sanctions.

Each week, the agent receives her cash-in-hand opportunity. She can decline it, accept it and commit fraud, leave the UI scheme, or accept the opportunity and declare the income. In the first three scenarios, her search effort is described in Section 4. In the latter case, her optimal probability of an offer satisfies:

$$\beta (V^e - \mathbb{E}V_{t+1}) = e_p(1, p^{dec}_t).$$  

(10)

The agent will commit fraud if and only if $c_t \geq c^{in}_t$ and:

$$V^{in}_t(c_t) \geq \max\{V^{out}_t(c_t), V^{dec}_t(c_t)\}.$$
The set of opportunities that will induce the agent to act fraudulently is lower than previously. In addition to the previous conditions, it must be the case that the agent prefers not to declare her income. Declaring gives her a strictly higher continuation payoff, as she faces no risk of sanctions. However, she receives less income in the current week, and faces the administrative burden of informing the UI authorities. In order to commit fraud, these combined costs must outweigh the gains from avoiding punishment.

How do the agent’s incentives change over time? Once again, it is straightforward to show that $EV_{t+1}$ is falling. This has a greater impact on her payoff when she does not commit fraud:

**Proposition 3 (Fraud over time with declared income)** If the probability of punishment is sufficiently large, then the agent accepts a greater range of opportunities over time.

The intuition echoes that of Proposition 1. The instantaneous gain from committing fraud by accepting a given cash-in-hand opportunity does not vary over time relative to declaring income or leaving the scheme. However the cost – losing claims on future UI payments – declines. As such, the agent becomes increasingly willing to engage in benefit fraud, accepting opportunities that she would previously either have rejected or declared.

As before, the agent may opt to leave the UI scheme early. The intuition, however, has changed:

**Proposition 4 (Leaving early with declared income)** The agent would never leave the UI scheme when she first becomes unemployed. As the duration of her unemployment increases, the probability that she leaves the scheme weakly increases, and is strictly positive by week $T$.

The agent must now compare the payoff from leaving with that of simply declaring her income. As before, consider the choice in the final period:

$$u[b_{dec}(c_T) + c_T] - \tau - e(1, p_{T}^{dec}) + \beta[p_{T}^{dec}V^e + (1 - p_{T}^{dec})EV^{out}],$$
$$u(c_T) - e[1, p_{out}(1)] + \beta[p_{out}(1)V^e + [1 - p_{out}(1)]EV^{out}].$$

Comparing (3) and (10), it is clear that $p_{T}^{dec} = p_{out}(1)$. The effort cost and continuation payoff are identical. The agent will strictly prefer leaving to declaring income in the final period if and only if:

$$u[b_{dec}(c_T) + c_T] - \tau < u(c_T).$$
For sufficiently large $c_T$, the above condition holds. The agent would rather forego any UI payment she is entitled to (if any) in order to avoid the administrative cost of declaring her income. Combined with Proposition 2, the agent would still prefer to leave early when she receives a particularly profitable cash-in-hand opportunity.

7 Conclusions

Recent empirical evidence has questioned the nature of the re-employment spike around the time that an unemployed agent’s UI payments are stopped. In contrast to traditional theories, agents do not appear to search more intensively (Krueger and Mueller 2010, 2011, 2012). If they do leave the scheme early, it is not necessarily because they have secured a permanent job (Card et al. 2007b).

We describe a new dynamic model of unemployment insurance where cash-in-hand opportunities alter the incentive to look for a job. Our framework, whilst highly stylised, generates theoretical predictions consistent with both recent observations.

During the early stages of unemployment, an agent has many future payments to look forward to if she does not find work. She does not want to risk losing them by committing fraud, and rejects all but the most profitable cash-in-hand opportunities. If she remains unemployed as her expiry date nears, she becomes increasingly willing to accept opportunities, reducing her job search effort. Immediately before expiry, she may decide to leave the UI scheme voluntarily. She loses little by way of future payments, and no longer runs the risk of being prosecuted. We find that these results hold even when the UI scheme allows individuals to declare casual income earned while searching for a job and receiving unemployment benefits, as in the US and the UK.

Our analysis assumed that both the probability of being caught and the punishment associated with committing fraud were constant. This is because we were interested in a framework that predicts individual behaviour consistent with the empirical evidence. Yet, our results suggest that both should be time-dependent. A profile of increasingly harsh punishment should be adopted when the UI scheme features an expiry date, independent of the severity of the offence. As the gains from turning down cash-in-hand opportunities decline over time, so must the expected gains from committing fraud in order to provide an effective deterrent.
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A Proofs

A.1 Proof of Proposition 1

Implicitly differentiating (7) with respect to $E_{V_{t+1}}$ yields:

$$\frac{\partial c^\prime_t}{\partial E_{V_{t+1}}} = \frac{[1 - p_{t}^{in}(0)] - (1 - \phi)[1 - p_{t}^{in}(1)]}{u'(b_t + c^\prime_t)}$$

So long as $\phi \geq 1 - \min \left[ \frac{1 - p_{t}^{in}(0)}{1 - p_{t}^{in}(1)} \right]$, $\frac{\partial c^\prime_t}{\partial E_{V_{t+1}}} > 0$. Since $E_{V_{t+1}}$ is declining, $c_{t}^{\prime in}$ decreases over time. The agent accepts more cash-in-hand opportunities over time. This completes the proof. ■

A.2 Proof of Proposition 2

We need to show that, in period $T$, condition (8) holds. Compare:

$$-e[1, p_T^{in}(1)] + \beta \phi V_{pun} + \beta(1 - \phi)[p_T^{in}(1)V_e + [1 - p_T^{in}(1)]E_{V_{out}}}$$

$$-e[1, p_T^{out}(1)] + \beta[p_T^{out}(1)V_e + [1 - p_T^{out}(1)]E_{V_{out}}}$$

Note that if $\phi = 0$, (3) and (6) are equivalent and so (11) and (12) are equal.

Now, consider the effect of increasing $\phi$ on (11):

$$\frac{d(11)}{d\phi} = \frac{\partial (11)}{\partial p_T^{in}(1)} \frac{\partial p_T^{in}(1)}{\partial \phi} + \frac{\partial (11)}{\partial \phi}$$

$$= \beta \{V_{pun} - p_T^{in}(1)V_e - [1 - p_T^{in}(1)]E_{V_{out}}\}$$

$$< 0,$$

since $V_{pun} < E_{V_{out}} < V_e$. So, for any $\phi > 0$, (11) < (12). Call the difference, (12) − (11) $\equiv \Delta$.

It remains to show that, for some range of $c_T$, $V_{out}(c_T) > V_T^{in}(c_T)$. We have:

$$V_{out}(c_T) - V_T^{in}(c_T) = u(c_T) - u(b + c_T) + \Delta.$$
The agent will prefer to leave if and only if:

\[ u(b + c_T) - u(c_T) < \Delta. \]

Since \( u(y_t) \) satisfies diminishing marginal utility and the Inada conditions, \( u(b + c_T) - u(c_T) \) is strictly decreasing with \( u(b + c_T) - u(c_T) \to 0 \) as \( c_T \to \infty \). There exists some threshold, \( \tilde{c}_T \) such that the agent prefers to leave whenever she receives an opportunity \( c_T > \tilde{c}_T \). The probability that she leaves the scheme in period \( T \) is \( 1 - F(\tilde{c}_T) \).

In weeks prior to \( T \), a similar argument holds. Since \( \mathbb{E}V^t_{t+1} \) is declining over time, \( \tilde{c}_t \geq \tilde{c}_{t+1} \). The probability that the agent leaves is increasing from one week to the next. This completes the proof.

\[ \blacksquare \]

A.3 Proof of Proposition 3

Proposition 1 demonstrates that the range of opportunities that induces the agent to commit fraud rather than leave the scheme is increasing over time. It remains to show that the range of opportunities that induces the agent to commit fraud rather than declare income is also increasing.

Define \( C_t \) to be the set of cash-in-hand opportunities for which the agent prefers committing fraud to declaring income in week \( t \). We show that \( C_0 \subseteq C_1 \subseteq ... \subseteq C_{T-1} \subseteq C_T \).

Take a particular cash-in-hand opportunity, \( \hat{c} \) as given. The payoffs from accepting it and committing fraud, versus accepting and declaring income in week \( t \) are:

\[
V^\text{in}_t(\hat{c}) = u(b + \hat{c}) - e[p^\text{in}_t(1)] + \beta \phi V^\text{pun}_t + \beta(1 - \phi)\{p^\text{in}_t(1)V^e + [1 - p^\text{in}_t(1)]\mathbb{E}V^{t+1}_t\},
\]
\[
V^\text{dec}_t(\hat{c}) = u[b^\text{dec}(\hat{c}) + \hat{c}] - \tau - e(1, p^\text{dec}_t) + \beta[p^\text{dec}_t V^e + (1 - p^\text{dec}_t)\mathbb{E}V^{t+1}_t].
\]

We have that:

\[
\frac{dV^\text{in}_t(\hat{c})}{d\mathbb{E}V^{t+1}_t} = \beta(1 - \phi)[1 - p^\text{in}_t(1)],
\]
\[
\frac{dV^\text{dec}_t(\hat{c})}{d\mathbb{E}V^{t+1}_t} = \beta(1 - p^\text{dec}_t).
\]

If \( \phi \geq 1 - \min_t \left[ \frac{1 - p^\text{dec}_t}{1 - p^\text{in}_t(1)} \right] \), then a decline in \( \mathbb{E}V^{t+1}_t \) reduces \( V^\text{dec}_t(\hat{c}) \) by a larger amount than \( V^\text{in}_t(\hat{c}) \).

If \( \hat{c} \in C_t \) then \( V^\text{in}_t(\hat{c}) \geq V^\text{dec}_t(\hat{c}) \). In week \( t + 1 \), since \( \mathbb{E}V^{t+2}_t < \mathbb{E}V^{t+1}_t \), it must be that \( V^\text{in}_{t+1}(\hat{c}) > V^\text{dec}_{t+1}(\hat{c}) \), so \( \hat{c} \in C_{t+1} \). So \( C_t \subseteq C_{t+1} \). The range of opportunities that the agent
accepts but does not declare is also increasing over time. This completes the proof. ■

A.4 Proof of Proposition 4

The proof is almost identical to that of Proposition 2. Consider the final week. If \( c_T \geq \tilde{c}_T \), as defined in Proof A.2, then the agent prefers to leave the UI scheme than commit fraud. It remains to show that there exist opportunities for which the agent would prefer to leave rather than declare the income. Comparing payoffs:

\[
\begin{align*}
    u[b_{\text{dec}}(c_T) + c_T] &- \tau - e(1, p_{T}^{\text{dec}}) + \beta[p_{T}^{\text{dec}} V_{e} + (1 - p_{T}^{\text{dec}}) E V_{\text{out}}], \\
u(c_T) &- e[1, p_{\text{out}}(1)] + \beta[p_{\text{out}}(1) V_{e} + (1 - p_{\text{out}}(1)) E V_{\text{out}}].
\end{align*}
\]

Comparing (3) and (10), it is immediately clear that \( p_{\text{out}}(1) = p_{T}^{\text{dec}} \). So:

\[
- e(1, p_{T}^{\text{dec}}) + \beta[p_{T}^{\text{dec}} V_{e} + (1 - p_{T}^{\text{dec}}) E V_{\text{out}}] = - e[1, p_{\text{out}}(1)] + \beta[p_{\text{out}}(1) V_{e} + (1 - p_{\text{out}}(1)) E V_{\text{out}}].
\]

The agent will this choose to leave the scheme if and only if:

\[
u[b_{\text{dec}}(c_T) + c_T] - u(c_T) < \tau.
\]

If \( c_T > \frac{b}{\alpha} \) then \( b_{\text{dec}}(c_T) = 0 \). The left-hand side of the above inequality is zero. The agent prefers to leave. The probability that the agent will leave the scheme is strictly larger than \( 1 - F \left( \frac{b}{\alpha} \right) > 0 \).

A similar argument holds in previous weeks, noting of course that \( E V_{t+1} > E V_{\text{out}} \). This completes the proof. ■

References


