Anti-symmetric Laminates for Improved Consolidation and Reduced Warp of Tapered C-sections

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Abstract
This paper challenges the conventional use of symmetric stacking sequences by proposing fully uncoupled anti-symmetric laminates that simultaneously improve consolidation and reduce warp of tapered laminates. A demonstrator C-section laminate with an anti-symmetric stacking sequence has been designed to eliminate coupling between in-plane and out-of-plane deformation. Results are compared with a conventional, symmetric baseline. The two laminate designs were deposited on a male tool using Automated Fibre Placement (AFP) and final de-bulk and cure were performed in a single autoclave operation. It is postulated that long continuous plies spanning a tapered region can inhibit consolidation in unidirectional (UD) material when they are not equidistant from the tool surface. The ply termination strategy of the anti-symmetric laminate mitigates this effect. It has been found that the anti-symmetric demonstrator achieved better consolidation compared with the baseline symmetric demonstrator. The results also show for the first time, that the level of warping exhibited by the anti-symmetric laminate is approximately 50% lower than the symmetric baseline.

Keywords
Anti-symmetric; Stacking sequence; Stiffness coupling; Uncoupled; Consolidation; Warping; Automated fibre placement.
Nomenclature

Abbreviations
AFP  Automated Fibre Placement.
CLT  Classical Laminate Theory
HTCS Hygro-thermally curvature-stable.
UD   Unidirectional material.

Symbols
A    In-plane (membrane) stiffness matrix.
B    Coupling matrix.
b_k  Stiffness matrix of layer k in global coordinate system.
D    Out-of-plane (bending) stiffness matrix.
M_{xy} Twisting moment.
N    Total number of layers in a laminate stacking sequence.
t_k  Thickness of layer k.
\bar{z}_k Distance between reference plane and the mid-plane of layer k.
\kappa_x Curvature in x-direction.
\kappa_y Curvature in y-direction.
\kappa_{xy} Twisting curvature.
o   0° ply in stacking sequence notation.
\bullet 90° ply in stacking sequence notation.
+   +45° ply in stacking sequence notation.
-   -45° ply in stacking sequence notation.

Subscripts
(...)_+ A particular operation for positive angle plies only.
(...)_- A particular operation for negative angle plies only.
[...],s The stacking sequence within the brackets repeated symmetrically.
[...],\lambda The stacking sequence within the brackets repeated anti-symmetrically.
1 Introduction

The aerospace industry is using composite materials extensively in order to gain benefit from their weight and maintenance saving properties. As composites are used for a broader variety of components, so there is a need for laminates to form complex structural shapes. The manufacture of these is highly complex and poses many challenges that do not exist when processing other materials. Local thickness increases, commonly known as “pad-ups”, are frequently used in order to accommodate local stress concentrations, for example where engines and landing gear attach to the airframe. The stress distribution is highly complex around the tapered regions where laminates are incrementally thickened by adding layers of material. There has been extensive research into the strength, damage tolerance and delamination characteristics of these tapered regions [1-3]. Design guidelines have been developed for ply drop-offs, which occur in tapered regions [4]. While these works focus on the performance of tapered laminates in-service, this paper is concerned with the manufacturability of such laminates.

At present, industry uses symmetric stacking sequences extensively; however these can be very restricting, typically following a set of design rules [5], which also prescribe the manner in which plies should be terminated. This paper investigates the potential benefit anti-symmetric sequences offer in terms of consolidation and warping during manufacture. During the curing process, laminates de-bulk and reduce thickness, consolidating towards the tool surface, as modelled by Min et al. [6]. The action of consolidation over a tapered section under the influence of long fibres is considered in this paper.

The sequence in which the layers are stacked can be tailored to control stiffness coupling, which has been investigated by a number of authors in order to produce laminates that do not warp during manufacture [7-9]. Mechanical coupling can lead to performance characteristics post-cure, which may or may not be desirable depending on the application. This can give rise to structural instability [10] and bend-twist coupling can have a significant impact on the buckling resistance of laminates [11-12]. It is also essential to consider the coupling behaviour of a laminate under thermal load in order to ensure high quality manufacture. It is particularly important that there is no tendency for the laminate to deform after cure. Special cases of laminates exhibit mechanical coupling but will not warp under
thermal load during manufacture. These are referred to as hygro-thermally curvature-stable (HTCS) [13]. Laminates exhibiting no elastic coupling do not (in theory) warp during manufacture; they are always HTCS, although not usually referred to in this manner. As well as warping, laminates can deform as a result of spring-in. This occurs near corner or angle features as investigated experimentally by [14] for symmetrical lay-ups. Process induced deformations caused by autoclave manufacture of laminates have previously been modelled using finite element simulation [15].

The aim of this paper is to analyse the manufacturing benefit of a new laminate design strategy that has been developed. There are many factors that contribute to the manufacturability of laminates but the scope of the new design strategy is to consider elastic stiffness coupling and geometrical effects caused by tapering thickness. With these in mind, the stacking sequence of an anti-symmetric laminate is tailored to improve consolidation and minimise warp during manufacture.

2 Laminate Design Considerations

There are two considerations driving the design of the anti-symmetric laminate in this paper: the implications of the way in which tapered laminates consolidate and the tendency of a laminate to warp during manufacture as a result of stacking sequence and stiffness coupling.

2.1 Consolidation of Tapered Laminates

The behaviour of tapered laminates during consolidation is postulated, based upon an argument using simplified mechanics and trigonometry. Consolidation is the removal of bulk, principally trapped air, from a laminate. This de-bulking process is highly complex, however in general causes a thickness reduction, typically 10-15%. The complexity is compounded in the presence of tapered sections. Consider the example postulated in Fig. 1(a). This shows how the surface (solid line) and a continuous layer within the laminate (dashed line) move as a result of consolidation. It is assumed that the laminate surface is flat pre-consolidation. Many laminates are designed to be flat post-consolidation or have sacrificial plies added and machined to allow attachment to another flat component. The mechanics in these cases will be similar as for the case illustrated in Fig. 1(a). In Fig. 1(a), it is assumed that a layer exists in the same proportionate position through-thickness in both the (thin) far-field and (thick) pad-up laminates. This is representative of the result of scarfing the far-
field and pad-up laminates together evenly through their entire thickness, as per a conventional ‘diamond’ ply drop-off pattern, illustrated in Fig. 4.

By trigonometry, the dashed line in Fig. 1(a) is longer in the 0° fibre direction post-consolidation. If the layer represented by this dashed line resists this length change, it will in turn hold back consolidation. The length increase can either be accommodated by the layer stretching, drawing in length from the far-field (slipping), or by some combination of these. If the length is assumed to be accommodated entirely by the layer stretching in the tapered section, the strain can be calculated from three dimensionless parameters: ramp rate, consolidation percentage and through-thickness position as a percentage of overall thickness. Fig. 1(b) shows how the strain varies through-thickness assuming a 1:10 ramp rate and 12% consolidation. 0° fibres will offer most resistance, since they have the greatest stiffness in the direction of required length increase. They also run the full length of the component and so drawing in length will also not be readily achieved as that would require the slipping of a large amount of material along the length of the component, assuming the component is longest in the 0° fibre direction.

Now consider if the dashed line remains equidistant from the tool surface throughout the pad-up ramp, as illustrated in Fig. 2(a). This would require there to be no ply drops between the dashed line and the tool surface in the ramp, with the taper achieved by ply drops above the dashed line only. The result would be that, during consolidation, the dashed line would move down equally in the far-field, ramp and pad-up sections. In contrast to Fig. 1(a), the dashed line in Fig. 2(a) therefore does not have to span a greater length in the 0° fibre direction post-consolidation.

Figure 2(b) shows the benefit of the alternative design, with the layers in the lower portion of the laminate (below the dashed line of Fig. 2(a)) not required to accommodate any length change, giving rise to the zero mechanical strain region in Fig. 2(b). The peak strain is the same as in Fig. 1(b), showing that creating a zone of zero strain has not increased the maximum strain required elsewhere in the laminate.

### 2.2 Coupling of Laminate Stiffness

The anti-symmetric stacking sequences for manufacture in this paper are designed to be fully uncoupled. The level of coupling is assessed using Classical Laminate
Theory and *ABD* matrix notation. The constituent parts of the *ABD* matrix are calculated, according to ESDU 94003 [16], from

\[
A = \sum_{k=1}^{N} t_k b_k , \quad B = \sum_{k=1}^{N} t_k \bar{z}_k b_k , \quad D = \sum_{k=1}^{N} \left( t_k \bar{z}_k^2 + \frac{t_k^3}{12} \right) b_k , \tag{1}
\]

where \( N \) is the number of plies in the laminate, \( t_k \) the thickness of layer \( k \), \( \bar{z}_k \) the distance between the reference plane (usually the mid-plane of the laminate) and the mid-plane of the layer \( k \). \( b_k \) is the stiffness matrix of layer \( k \) in the global laminate co-ordinate system and is different for different layer orientations, which are commonly limited to 0°, 90° and ±45° within industry. This formulation of the *ABD* matrix is convenient when developing criteria for fully uncoupled laminates. The current industry standard is to use balanced, symmetric laminates. By definition a balanced, symmetric laminate cannot exhibit any coupling other than bend-twist. This can be removed, making the laminate fully uncoupled, if

\[
\sum_{k=1}^{N} \left( \bar{z}_k^2 \right)_+ = \sum_{k=1}^{N} \left( \bar{z}_k^2 \right)_- , \tag{2}
\]

where \( \left( \bar{z}_k^2 \right)_+ \) and \( \left( \bar{z}_k^2 \right)_- \) are the square of lever arm terms for positive and negative angle plies, respectively. Anti-symmetric laminates are defined as the opposite of symmetric, i.e. where a positive angle-ply exists one side of the mid-plane, it is mirrored by a negative angle-ply on the other side. Anti-symmetric laminates cannot exhibit any coupling other than extension-twist (together with shear-bending). This can be removed to produce a fully uncoupled laminate if

\[
\sum_{k=1}^{N} \left( \bar{z}_k \right)_+ = \sum_{k=1}^{N} \left( \bar{z}_k \right)_- . \tag{3}
\]

An important observation is that equating the sum of the lever arm terms, Eq. (3), is much more readily achieved than equating the sum of lever arm terms squared, Eq. (2). The result is that there are many more fully uncoupled anti-symmetric laminates than there are symmetric ones [7].
Another major advantage of anti-symmetric laminates is that it is possible to drop a single ± angle-ply pair and maintain a balanced, anti-symmetric laminate. This is the case since dropping a single positive angle ply requires dropping the corresponding negative angle ply on the other side of the mid-plane, which maintains balance and anti-symmetry simultaneously. In contrast, for a symmetric laminate, dropping a positive angle ply requires dropping the corresponding positive angle ply on the other side of the mid-plane, in order to maintain symmetry. This therefore requires two negative angle plies to also be dropped in order to maintain balance, such that a minimum of four angle-plies must be dropped, which is significantly limiting where small thickness changes are required.

A fully uncoupled anti-symmetric laminate will not exhibit bend-twist coupling regardless of the position of the reference plane (about which $\tau_x$ is measured). This is significant where the neutral plane shifts, such as in the corner between the web and flange of a C-section.

### 2.3 Anti-symmetric Laminate Design Strategy

Based upon the consolidation of tapered laminates and laminate stiffness coupling, covered in the previous two sections, a new design strategy is developed. Rule 1 of the design strategy is that continuous 0° plies (i.e. those that run the full length of the laminate, not just the pad-up regions) remain equidistant from the tool surface throughout the ramp, as per the dashed line in Fig. 2(a). Rule 2 is that stiffness coupling is minimised or eliminated throughout all sections of the laminate.

The fact that continuous 0° plies must remain equidistant from the tool surface implies that all layers between them and the tool are also continuous, forming a “protected zone”. This protected zone would exist between the dashed line and the tool surface in Fig. 2(a). In order to minimise the extent of this protected zone and allow as much ply interleaving as possible, 0° plies should therefore be positioned close to the mid-plane of the far-field laminate. Thereafter, the 90° and ±45° plies are positioned to eliminating all stiffness coupling. This has been achieved by performing an exhaustive search of possible stacking sequences. From the candidate fully uncoupled sequences, the one with the least ply blocking is selected.

The requirement for a protected zone dictates much of the pad-up stacking sequence, with the remainder designed to produce a fully uncoupled sequence, again using an exhaustive search technique. The protected zone leads to non-
symmetric sequences in the ramp as layers are dropped-off. It is difficult or impossible to ensure all of these are fully uncoupled if the requirement is to drop a single layer at a time [17]. This is also the case for the conventional symmetric intermediate sequences in the ramp. However, minimising the coupling in the ramp, and hence its effect on warp, is readily achievable by assessing the possible intermediate stacking sequences for each ply termination and selecting the one that produces the least coupling. Determining which sequence represents the least coupling is not entirely trivial since there are many stiffness coupling terms. It is therefore necessary to either priorities certain coupling terms or to create a method for combining them to produce an overall measure of coupling. For the demonstrator laminates in this paper the coupling terms were prioritised, with in-plane to in-plane coupling terms ($A_{16}$ and $A_{26}$) deemed most important, followed by in-plane to out-of-plane coupling ($B$ matrix), with out-of-plane to out-of-plane coupling ($D_{16}$ and $D_{26}$) considered least important.

3 Manufacture of Example C-section Laminates

3.1 Demonstrator Designs

In order to experimentally validate the potential benefit of the anti-symmetric design strategy, two laminates were manufactured; one using the anti-symmetric design strategy and the other using balanced, symmetric stacking sequences, typical of many industrial applications. The symmetric laminate forms a baseline to compare the anti-symmetric laminate against. Apart from the stacking sequence, the two laminates are geometrically identical, forming two C-section components with 3 thickness pad-ups along their length. Both laminates are designed to have the same $A$ matrix, fulfilling the requirement to carry the same in-plane load. The tool onto which the laminates were laid, using an AFP machine, is illustrated in Fig. 3. The C-sections themselves did not span the full length of the tool, such that their overall length was approximately 1.6 m.

Figure 3 highlights the 3 pad-up regions. These pad-up regions consist of a total of 44 layers, while the thinner sections consist of a total of 24. Each layer is approximately 0.2 mm thick pre-cure, resulting in a total pad-up thickness increase of 4 mm from the 20 additional layers. This thickness increase is accommodated by the 3 pad-up ramps in the tool surface (inside the C-section), meaning once the lay-up process was complete, the laminate surface (outside the C-section) was
approximately flat. The stacking sequences of the baseline symmetric laminate in the thin and thick (pad-up) sections are \([+/-\circ/+/-/\circ/+/-/\circ/+/-/\circ/+/-/\circ/+]_S\) and \([+/-\circ/+/-/\circ/+/-/\circ/+/-/\circ/+/-/\circ/+]_S\) respectively. The corresponding anti-symmetric sequences are \([+/-\circ/+/-\circ/+/-/\circ/+/-/\circ/+]_A\) and \([+/-\circ/+/-\circ/+/-/\circ/+/-/\circ/+]_A\). The stiffness matrices for these 4 laminates is recorded in the Appendix. In both demonstrators the percentages of 0°, 90° and ±45° plies are, respectively, \([17/67/17]\) for thin and \([18/64/18]\) for thick sections, to the nearest percentage. The cross-section A-A from Fig. 3 is illustrated in Fig. 4 and Fig. 5, showing ply terminations and the resulting stacking sequences over the tapered region for the baseline and anti-symmetric laminates respectively. The baseline incorporates a diamond ply termination sequence as per conventional industrial practice.

3.2 Manufacturing Process Description

The UD material used for both demonstrator laminates was 8552/AS4, supplied by Hexcel, with a nominal pre-cure ply thickness of 0.2 mm. The two demonstrator laminates were manufactured one after another using the same equipment and process. Bagging material was laid over the tool and a picture frame of glass fibre was applied outside edge-of-part to aid adhesion of the first carbon fibre layer. Deposition was performed using a Coriolis AFP machine at room temperature with a single room temperature de-bulk after the first layer was laid to facilitate better adhesion to the tool. All other layers were then laid down before the final de-bulk took place. Interim de-bulks would reduce the movement of each ply but add significant time and cost so are ideally avoided. The final de-bulk was carried out in the autoclave at 55°C, with full vacuum applied for 15 minutes. The cure cycle was then applied in the autoclave. 7 Bar pressure was applied and the vacuum reduced to 0.2 Bar. It was then heated at 1-3°C/min up to 180°C. The temperature was held at 180 ±5°C for 120 minutes. Cool down then took place at 2-5°C/min until 50°C was reached, at which point the autoclave pressure was vented and the vacuum removed.

4 Comparison of Results for C-section Laminates

4.1 Consolidation

Laminate thicknesses were measured using a CMM (co-ordinate measuring machine) before and after they were de-bulked and cured in order to assess the overall change in thickness and hence the amount of consolidation. This was
performed for the 3 pad-up sections and the 2 thinner sections in between the pad-ups. See Fig. 3 for an illustration of the 3 pad-up design. The amount of consolidation in the web and a flange is recorded in Table 1 for each of the 5 flat regions of both C-sections. Each recorded consolidation figure is generated from approximately 100 CMM data points pre- and post-consolidation. The accuracy of the measurement of these points is ±8 μm, which corresponds to an error in the consolidation figures tabulated of up to ±0.3%.

<table>
<thead>
<tr>
<th>Section</th>
<th>Thickness</th>
<th>Consolidation (%)</th>
<th>Proportional Increase (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Anti.</td>
<td>Sym.</td>
</tr>
<tr>
<td>Web</td>
<td>1 Pad-up</td>
<td>11.5</td>
<td>11.8</td>
</tr>
<tr>
<td></td>
<td>2 Thin</td>
<td>14.5</td>
<td>11.8</td>
</tr>
<tr>
<td></td>
<td>3 Pad-up</td>
<td>11.9</td>
<td>11.2</td>
</tr>
<tr>
<td></td>
<td>4 Thin</td>
<td>14.7</td>
<td>13.9</td>
</tr>
<tr>
<td></td>
<td>5 Pad-up</td>
<td>11.3</td>
<td>12.4</td>
</tr>
<tr>
<td>Flange</td>
<td>1 Pad-up</td>
<td>15.6</td>
<td>14.3</td>
</tr>
<tr>
<td></td>
<td>2 Thin</td>
<td>12.7</td>
<td>14.1</td>
</tr>
<tr>
<td></td>
<td>3 Pad-up</td>
<td>14.9</td>
<td>13.5</td>
</tr>
<tr>
<td></td>
<td>4 Thin</td>
<td>13.0</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>5 Pad-up</td>
<td>14.9</td>
<td>10.9</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>13.5</td>
<td>12.4</td>
</tr>
</tbody>
</table>

Table 1 Comparison of consolidation data for anti-symmetric and symmetric C-sections. The proportional increase indicates how much more (or less if negative) the anti-symmetric consolidated relative to the baseline symmetric.

The anti-symmetric laminate increased consolidation by 8.7%, compared proportionately to the baseline symmetric laminate. It is believed this difference would be exacerbated for longer parts, where the effect of long fibres inhibiting consolidation would be greater. The demonstrator C-sections of this study were only 1.6 m in length, whereas those for industrial applications can be over 10 m.

Analysing Table 1, the anti-symmetric laminate consolidated more in the majority of locations. For the few locations that the anti-symmetric consolidated less, it is observed that in the corresponding web/flange section consolidation was vastly higher. For example, referring to Table 1, consolidation decreased by 8.6% in section 5 of the web. However in section 5 of the flange it increased by 37.5%. It can be said of all 5 sections that overall, across the web and flange, the anti-symmetric laminate showed increased consolidation. It is thought that high consolidation in a web/flange section is at the expense of that in the adjacent web/flange of that same section. This is because edge bars were not used during the manufacture of the demonstrators. Edge bars can be positioned along the free
edge of the material in the flanges to alleviate bag pressure. These help facilitate the movement of material around the C-section during consolidation by shedding excess at the free edge. Since the material was highly constrained during manufacture of the demonstrators, excess material may have been trapped. If a web or flange section consolidated particularly highly, this might be an indication that the excess material bulk was taken up in the adjacent web/flange section, causing poor consolidation in that region.

4.2 Warping and Spring-in Deformation

After the two laminates were cured a series of measurements were taken from their inner faces, again using a CMM. Multiple planes were created along the length of the C-section, representing the inner faces of the web and flanges. Each plane was generated from approximately 70 data points from the CMM measurements, with a typical RMS error of 5-10 μm. The angle between the web and flange planes could then be calculated. This was done for 10 locations along the length of each C-section in order to determine the average angle of spring in between the web and flanges. Both laminates were found to have sprung in a similar amount; the anti-symmetric (1.41°) marginally less than the baseline symmetric (1.53°).

The level of twist of the two laminates was assessed using the planes generated from the web. Comparing the angle of each plane along the length of the C-sections produces the plot in Fig. 6. The observed section in Fig. 6 is approximately half the length of each C-section, spanning 2 thin and 2 pad-up sections (see Fig. 3). Figure 6 is representative of the level of twist for the full length of each C-section. Here it is shown that the anti-symmetric laminate exhibited approximately half the level of twist as compared with the baseline. Taking the average level of twist from Fig. 6 gives $\kappa_{xy}$ equal to $3.3 \times 10^{-6}$ rad/mm for the anti-symmetric and $6.4 \times 10^{-6}$ rad/mm for the baseline C-sections. This large difference is primarily as a result of bend-twist coupling present in the symmetric laminate, but which is eliminated in the anti-symmetric laminate constant thickness sections.

From the $ABD$ matrix of CLT, where there is a null $B$ matrix (as is the case for both demonstrator laminates), it can be stated that

$$M_{xy} = D_{16} \kappa_x + D_{26} \kappa_y + D_{66} \kappa_{xy}. \quad (4)$$
Since there is no applied twisting moment, $M_{xy}$, and no curvature, $\kappa_x$, this can be reduced and rearranged to find

$$\kappa_{xy} = \frac{D_{26}}{D_{66}} \kappa_y.$$  \hspace{1cm} (5)

The ratio of $D_{26}$ and $D_{66}$ therefore describes how much the laminate will twist as a result of curvature in the y-direction, caused primarily by spring-in for the demonstrator laminates as described previously. Equation 5 describes a general principle that will apply to all laminates and therefore the results of the two demonstrator laminates in this paper are likely to be representative of the general case. The ratio is approximately 9% and 6% respectively for the thin and thick sections of the baseline laminate. Since both thin and thick sections of the anti-symmetric laminate are fully uncoupled and $D_{26}$ is zero, the ratio is also zero, causing significantly reduced twist, as seen in Fig. 6. However, in the ramp sections, where single ply terminations exist, it is not possible to maintain zero $D_{26}$ terms. Moreover, in the presence of the corner features, the shift in neutral plane for the laminates will lead to a non-zero $B$ matrix, leading to twist as a result of thermal load after manufacture. These factors, combined with manufacturing imperfections, are likely to be the cause of the anti-symmetric laminate also exhibiting twist, albeit less than the symmetric. As a result of the relative level of twist for the two demonstrator laminates, significant force was required to remove the baseline laminate from the tool, post-cure. The anti-symmetric laminate, in contrast, was readily removed.

5 Conclusions

The anti-symmetric laminate showed an improvement in consolidation against the baseline (symmetric) laminate. The anti-symmetric laminate was also significantly less warped, allowing it to be removed from the tool without the force required to remove the baseline laminate. This is reinforced by a two-fold reduction in twist and a small reduction in spring-in. These results show that challenging the conventional use of symmetric stacking sequences, using anti-symmetric sequences instead, has the potential to achieve improved quality of manufacture. Moreover it is believed the results of this study would be exacerbated for longer C-sections, which are commonly used for industrial applications. Further research will investigate the
corner unfolding performance of the two laminates by subjecting test coupons to a 4-point bend test.

**Acknowledgements**

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References


Figures

Figure 1  (a) Example laminate pad-up, showing the movement of a layer (dashed line) as the laminate consolidates. The layer has to cover a longer length in the 0° fibre direction post-consolidation compared with when it is first laid down. All other layers are shown as a shaded region for clarity. (b) Through-thickness variation of axial strain caused by the length change, assuming 12% consolidation and a 1:10 ramp.

Figure 2  (a) Alternative laminate pad-up. In this case, the dashed line remains equidistance from the tool surface throughout the pad-up ramp. As a result, it will not have to cover a longer length in the 0° fibre direction post-consolidation. (b) Through-thickness variation of axial strain, assuming 12% consolidation, a 1:10 ramp and that the layers in the bottom half of the far-field laminate remain equidistant from the tool surface.
Figure 3  Isometric view of demonstrator C-section tool. The 3 thickness pad-ups are shown. Section A-A highlights one of the 1:10 ramps associated with the pad-ups, in which a thickness change of 4 mm occurs over a length of 40 mm. All 3 pad-ups have identical geometry, however the inner two thin sections are shorter than the outer two. All dimensions in mm.

Figure 4  Section A-A from Fig. 3 for the baseline laminate, showing the (thin) far-field and pad-up stacking sequences. Also illustrated is the way in which the 20 additional layers are dropped-off in the ramp to transition between the far-field and pad-up sequences. For the baseline this is done using a conventional ‘diamond’ drop-off pattern. The stacking sequences are largely balanced and symmetric. Colour version of figure available online.
Figure 5  Section A-A from Fig. 3 for the anti-symmetric laminate. The (thin) far-field and pad-up sequences are anti-symmetric and fully uncoupled. The 20 additional layers are dropped-off in the ramp using the anti-symmetric design strategy. The continuous 0° plies remain equidistant from the tool surface. Colour version of figure available online.

Figure 6  Level of twist of the two demonstrator laminates, illustrated by plotting angle of the web vs distance along the length of the cured C-sections (relative to the angle at the starting point) after removal from the tool. In general the anti-symmetric laminate exhibits half the level of twist compared with the baseline. Error bars are shown for each data point.
**Appendix**

The stiffness matrices for the thin and thick sections of the two demonstrator laminates are recorded in Table A1. Note the $B$ matrix is zero for all cases and so is not shown.

<table>
<thead>
<tr>
<th>Laminate</th>
<th>$\mathbf{A}$ matrix (kN/mm)</th>
<th>$\mathbf{D}$ matrix (kNmm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin baseline symmetric</td>
<td>$\begin{pmatrix} 224 &amp; 101 &amp; 0 \ 224 &amp; 0 \ \text{sym} &amp; 105 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 287 &amp; 174 &amp; 16 \ 376 &amp; 16 \ \text{sym} &amp; 180 \end{pmatrix}$</td>
</tr>
<tr>
<td>Thin anti-symmetric</td>
<td>$\begin{pmatrix} 224 &amp; 101 &amp; 0 \ 224 &amp; 0 \ \text{sym} &amp; 105 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 247 &amp; 183 &amp; 0 \ 398 &amp; 0 \ \text{sym} &amp; 190 \end{pmatrix}$</td>
</tr>
<tr>
<td>Thick baseline symmetric</td>
<td>$\begin{pmatrix} 418 &amp; 178 &amp; 0 \ 418 &amp; 0 \ \text{sym} &amp; 186 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 2120 &amp; 978 &amp; 60 \ 2160 &amp; 60 \ \text{sym} &amp; 1020 \end{pmatrix}$</td>
</tr>
<tr>
<td>Thick anti-symmetric</td>
<td>$\begin{pmatrix} 418 &amp; 178 &amp; 0 \ 418 &amp; 0 \ \text{sym} &amp; 186 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 1940 &amp; 1020 &amp; 0 \ 2250 &amp; 0 \ \text{sym} &amp; 1060 \end{pmatrix}$</td>
</tr>
</tbody>
</table>

*Table A1* Stiffness matrices for the two demonstrator spars.