Monitoring individual wave characteristics in the inner surf with a 2-Dimensional laser scanner (LiDAR)

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Abstract

This paper presents an investigation into the use of a 2-dimensional laser scanner (LiDAR) to obtain measurements of wave processes in the inner surf and swash zones of a microtidal beach (Rousty, Camargue, France). The bed is extracted at the wave-by-wave timescale using a variance threshold method on the time series. Individual wave properties were then retrieved from a local extrema analysis. Finally, individual and averaged wave celerities, are obtained using a crest-tracking method and cross-correlation technique respectively, and compared with common wave celerity predictors. Very good agreement was found between the individual wave properties and the wave spectrum analysis, showing the great potential of the scanner to be used in the surf and swash zone for studies of nearshore waves at the wave-by-wave timescale.

1. Introduction

1.1. LiDAR in coastal engineering

The use of remote sensing techniques in coastal engineering has become increasingly popular during the past 3 decades. These instruments can provide measurements at temporal and spatial scales that are not reached by common in-situ instruments. As an example, video imagery has been used for a wide range of applications: from bathymetric inversion (Stockdon and Holman, 2000) to alongshore swash motion variability (Guedes et al., 2012).

Since remote sensors are non-intrusive instruments, they have the advantage of being easily and safely deployed on existing beachfront structures or specifically installed towers. Furthermore, instruments like the terrestrial LiDAR scanner (TLS) directly measure the wave profile and the wave properties (e.g. wave height and period) can subsequently be extracted. This represents an important advantage over other remote sensing techniques (e.g. video or radar) which are able to cover large domains but cannot directly obtain wave properties. Additionally, the ability of a single TLS to obtain data at multiple locations provides significant advantages over in-situ sensors like pressure transducers, which are commonly used in surf zone studies but provide only point measurements.

The first reported experiment using a TLS to study wave processes is that of Irish et al. (2006), who mounted a 4-rangefinder laser on a pier. A directional wave spectrum obtained with the scanner was compared to that from a submerged wave gauge, showing good agreement.

Recently, a few attempts were made to study the wave propagation or measure wave breaker heights. Harry et al. (2010) investigated the potential of a 3D TLS to capture the water surface of a surf zone. Despite capturing the wave profile successfully, the time spent by the scanner to scan on the three dimensions was a major drawback since it introduced an alongshore time shift on the wave crest propagation. Their conclusion was that a 2D TLS might be a better alternative. Park et al. (2011) also used a 3D TLS to measure breaker heights. They compared the scanner data with visual measurements against a vertical staff, and obtained a relatively good agreement over the 26 measured waves, with a Root Mean Square Error (RMSE) of 5 cm. Individual wave height and celerity measurement was also made possible by combining the use of video camera and a 3D TLS, fixed on an automated robot, in Wübbold et al. (2012). Interestingly, this technique enabled the measurement of several alongshore points of the wave crest, allowing a 2-dimensional description of the wave propagation.

Swash zone data have been obtained using fixed 2D TLS instruments by Blenkinsopp et al. (2010), Brodie et al. (2012) and Almeida et al. (2015), who demonstrated the ability of the instrument to measure swash hydro and morphodynamics with high accuracy. The approach of Wübbold et al. (2012) was also used by Vousdoukas et al. (2014) in laboratory conditions to measure wave-by-wave events in the swash zone. Overall, it was found that the precision of such instruments was lower than that of ultrasonic altimeters which had previously been used to make such swash measurements, however the ability to capture small scale features due to the high spatial resolution and small measurement footprint compared to other remote sensors make this instrument a powerful tool for coastal studies.

1.2. Known drawbacks of the 2D-LiDAR for wave processes studies

Previous studies (Blenkinsopp et al., 2010; Evans, 2010) have shown that an aerated and turbulent water
surface is required for the laser to be sufficiently scattered to enable detection by the instrument. While in the laboratory, this can be achieved by adding particulates to increase the water turbidity (Allis et al., 2011), this is not feasible in the field.

Fortunately, when the wave conditions are sufficiently energetic (wave breaking occurring), the surf and swash zones are very dynamic and are characterised by high levels of turbulence and aeration, which cause sufficient scattering for the consistent detection of the free surface elevation.

Environmental conditions (luminosity, air humidity, wind) also have an impact on the scanner measurements. While the influence of humidity or water drops, characterised by noise or spikes in data can be corrected, under high wind conditions the TLS can become too unstable for the data to be used. Indeed, while instrument accuracies are typically of the order of millimetres, the error induced by small oscillations of the instrument increases with distance from the instrument and can lead to measurement errors of the order of centimetres.

2. Experimental Setup

2.1. Site Location - Rousty

The experiment described in this paper was completed at Rousty beach, Camarge, which is located in the South of France on the Mediterranean Sea, from November 2014 until February 2015. The overall aim of the experiment was to study the coupling between the wave field, groundwater table dynamics and the beach morphodynamics. It was organised in two different phases: a 10-day short-term and high-frequency phase within a 3-month long period of low-frequency measurements.

The site presents morphodynamic characteristics typical of the beaches in the National park of Camargue (Sabatier, 2008; Sabatier et al., 2009b). Despite the microtidal environment (tidal range ~0.4 m), this part of the coastline presents very dynamic beach/dune morphologies. This region is subject to seasonal storms accompanied by storm surges that flood the low-lying area of the Camargue beaches (Sabatier, 2008). This region is also exposed to very strong onshore wind episodes (mistral), which cause huge losses of sand due to aeolian transport (Sabatier et al., 2009a).

The high-frequency part of the experiments took place from the 8th to the 18th of December 2014 (10 days). During this period, 15 buried pressure sensors were deployed on the berm located at approximately 60 m from the dune system in addition to a laser scanner fixed on top of a 4.8m-high tower erected at the shoreline, see Figure 1. Both sets of instruments were logged by a computer placed on a scaffold structure, 16 m landward of the scanner.

2.2. Instrumentation

In this section, only the scanner instrumentation will be described since this paper focusses on the capacity of a commercial 2D scanner for inner surf and swash zones studies. During the Rousty experiments, the TLS used was a commercial LMS511 Laser Measurement System manufactured by SICK. This ranging device uses the time of flight method: the distance between two objects is calculated using the time required for an eye-safe pulsed beam ($\lambda = 905 \text{ nm}$) to be detected after reflection from the target. This instrument is similar to that used by Blenkinsopp et al. (2010) in terms of its function and specification.

The TLS has a range of 65 m, a 190° field-of-view with an angular resolution of 0.1667°, and can be sampled at the sample rate of 25 Hz (SICK, 2015). With this sampling rate, each spatial measurement location is measured 25 times per second; the instrument thus providing a total of 28500 measured points per second. During the experiment, a 4.8m-high tower was erected around the shoreline position for mounting the scanner and from this position it was possible to obtain measurements across the whole beach profile and into the inner surf zone (approximately 30% of the surf zone was covered in the present dataset). A schematic of the high-frequency experimental setup can be observed in Figure 2.

For the experimental setup at Rousty and using an angular resolution of 0.1667°, the distance between measurement points varied from 0.014 m at the Nadir point (zero grazing angle) to 0.25 m at the most seaward valid measurement location (Figure 3). This spatial resolution allows the detection of the instantaneous shape of small wave features, something that most conventional, point-measurement instruments such as pressure transducers or wave gauges are unable to do. The systematic error and spot diameter provided by the manufacturer (SICK, 2015) are also shown in the same figure. The systematic error naturally increases with increasing spot diameter and evolves from ±0.025 m in 1 to 10 m from the scanner to ±0.035 m between 10 and 20 m.

As the grazing angle between the laser beam and the target decreases ($\alpha$, Figure 2), the signal reflected by the water surface and returning to the scanner gets weaker. While bore fronts can still be captured due to a more normal-oriented surface relative to the instrument, a signal is not always returned from a more horizontal surface (e.g. wave troughs), resulting in increasing gaps in the dataset as we move offshore. As a result, a cross-shore position of ~20 m relative to the TLS was set as the seaward extent of the dataset for the extraction of wave properties. If we consider a plane surface, the minimum incident angle allowing good quality data with this specific scanner model was found to be around 13.5°. It is noted however that, since wave crests could still be followed from further offshore, the bore celerities were calculated from -22
m relative to the TLS, as discussed in Section 4.

3. Methodology

3.1. Pre-processing

Before analysing the dataset to study wave characteristics in the inner surf and swash zones, pre-processing is required. As in Almeida et al. (2015), a beach survey carried out the same day as the dataset presented in this study (18 December 2014) was used to find the instrument orientation relative to the cross-shore profile. Data transformation from the scanner-centroid coordinate system to the cross-shore coordinate system is then possible from this analysis. This results in two arrays X and Z containing the cross-shore position and height relative to the scanner.

The dataset was de-spiked to reduce noise in the measurements and environmental effects such as splashes or people passing within the TLS field-of-view. De-spiking the time series was achieved using gradient thresholds between two consecutive points. Then to reduce random noise, the dataset was time-averaged using a moving-window method (0.2 s), and spatially interpolated onto a regular cross-shore grid (δx = 0.1 m).

3.2. Bed extraction

Since the instrument simply measures the distance to the closest target, no distinction on the medium is made, e.g. water or sand. Due to the scanner’s location in the swash zone which is alternatively dry and submerged, an important step in the data processing is to separate the water signal from the bed. The methodology used in this study to extract the bed follows the work of Almeida et al. (2015).

Almeida et al. (2015) calculated the time series variance over 4-second windows at every point on the regular grid. This methodology relies on the fact that the time series variance when the target is the exposed bed is much smaller than that from a moving water surface. Therefore, by defining empirical thresholds at every cross-shore location, one can extract data corresponding to stationary, dry bed. By defining a water-depth criterion (0.015 m in this study) one can separate the original time series into separate ‘bed’ and ‘wet’ time series. This water-depth criterion ensures that the noise in the measurements (of the order O(mm)) is not interpreted as ‘wet’ data.

By interpolating in time the extracted bed points, a beach profile can be obtained at each time step. This enables the monitoring of bed morphology at several hundred points and at the time scale of individual waves. An example of the result from this extraction is shown in Figure 4, where both accretionary and erosive swash events can be observed at x = −10 m.

3.3. Wave properties extraction

In order to obtain the individual wave characteristics at each point on the grid, a local maxima analysis was carried out on the surface elevation time series to detect the wave crests. This technique has been used in previous surf zone studies by Power et al. (2010) or Postacchini and Brocchini (2014) because it is insensitive to low-frequency motions, unlike most common methods such as zero-down crossing which define waves relative to intersection between the instantaneous free-surface elevation and mean sea level. When studying the surf zone, and especially the inner surf where low-frequency motions can be predom-
3.4 Wave celerities

To calculate the wave celerities, two different approaches have been used. The first one was developed in the scope of this study and is based on a simple crest-tracking technique, allowing the estimation of individual wave celerities. The second uses a cross-correlation between two time series to calculate the averaged wave celerities. The process described above is enabled the detection of 275 waves and thus more than 3000 individual wave celerities. The process described above is illustrated for a 4-minute-period in Figure 5a, where the selected waves for this time window are shown in orange.

Averaged wave celerities were calculated following the method of Tissier et al. (2011). The cross-correlation was calculated between two 10-minute time series from two cross-shore locations (separated by 2 m). The maximum correlation found between the two time series is the averaged time delay between the surface elevation features. Physically, it represents an estimation of the averaged wave celerity over the time series.

Using these two different methods to estimate the wave celerity is interesting in several aspects. The TLS data opens up the possibility to detect wave celerity and geometry in shallow water right up to the shoreline without any mathematical transformation on the measure-
ments (e.g. Radon transform in Almar et al. (2013)). The present dataset corresponds to shallower water than investigated by Tissier et al. (2011), thus the relationship between wave properties and celerity can be studied closer to the shoreline. Furthermore, the estimation of individual celerities will provide more insight into the dispersion of these values.

4. Results
4.1. Bed Monitoring

Following the methodology presented in Section 3.1, the bed morphology has been monitored using the bed time series. By subtracting the initial beach face profile from the measured profile at each time step, erosion/accretion patterns over the measurement period can be observed. An example is presented in Figure 6 where the erosion/accretion patterns are shown every minute, after window-averaging the extracted bed (15-second window), for the period of the 13th to the 14th of December (30 continuous hours). This corresponded to the most energetic period of the 10-day experiments (energy peak around 13pm on the 13th of December).

Offshore wave conditions were measured by a buoy located 40 km west of Rousty beach, moored in a water depth of 30 m. Measured significant wave height and peak and mean spectral periods are shown in Figure 6a and 6b respectively. Mean water levels were obtained by a tidal gauge located at Fos-sur-Mer port (20 km west of Rousty). Interestingly, we can observe the influence of the tide even in this microtidal environment (high tides at 12:55pm on the 13th, 1:25am and 1:35pm on the 14th). In addition to the direct influence on the mean sea level, a significant reason for these oscillations is thought to be the weaker energy dissipation during high tides on this low-sloping barred beach (Guedes et al., 2011). During the first part of this storm event (9am to 6pm on the 13th of December), the swash zone profile flattened and experienced the strongest erosion (-0.15 m) between x = -10 and -5 m. When the conditions became milder, there is evidence of berm building centered around x = -10 m at a rate of approximately 10 mm/hr. This berm remained present until the end of the experiment, with evolving steepness depending on the offshore conditions.

4.2. Validation of the extracted wave-by-wave properties

The methodology to extract wave properties based on the extremum analysis was compared to a classic spectral analysis (Figure 7). Significant wave height $H_s$ was calculated by means of a Fast-Fourier Transform on a 15-minute time series, between cutoff frequencies of 0.05 Hz and 0.5 Hz. $H_s$ was compared to the averaged extracted wave height of the 1/3 highest waves $H_{1/3}$ from the wave-by-wave analysis described in section 3.3, over the same period. The mean extracted individual wave period $T_{i,m}$ was compared to the mean wave period $T_{01} = \bar{H}_0/m_1$, which is the inverse of the centroidal frequency, where $m_1$ is the n^th^ spectral moment is defined as:

$$m_n = \int_{-\infty}^{\infty} f^n E(f) df$$

with $E(f)$ the power density spectrum.

Figure 6: Bed extraction results: a) Measured mean water levels above Chart Datum at Fos-sur-Mer port and offshore significant wave height measured by a buoy close to Sète; b) Measured wave periods ($T_{01}$ and $T_{02}$) by the same buoy; c) Beach morphology evolution for the 13th and 14th of December (30 continuous hours of measurement). Erosion and accretion patterns were calculated by subtracting the initial beach profile to that of the actual moment. Red color corresponds to accretion while blue corresponds to erosion. The significant runup limit $R_{1/3}$ is shown as a circled black line.

Figure 7: Comparison of a) significant wave heights and b) mean wave period calculated with two different methods: using the centroidal frequency inverse from spectral analysis (black dots) and averaged values over the same period of time, from an extremum analysis (red dots).

Plotted against the mean water depth over the same time period $\bar{h}$, $H_s$ and $H_{1/3}$ show very good agreement at all water depths (Figure 7a), validating the extraction method based on the local extremum analysis. Both statistical ($H_{1/3}$) and spectral ($H_s$) significant wave height were found to show little scatter and to linearly decrease with averaged water depth ($r^2 = 0.86$). Though such depth-dependence is generally observed when saturated conditions are found in the inner surf (Sallenger and Holman, 1985), the relatively short dataset (2h30) and the consistent offshore conditions do not allow for such statement. Furthermore, waves were found to stop breaking and reform between the two beach bars, consistent with unsaturated conditions (Thornton and Guza, 1982).

In contrast to averaged values, measured individual wave heights showed considerably more scatter, see Figure 8a. This scatter is explained by two main factors: the influence of infra-gravity motions and the presence of high-frequency waves increasing or lowering the wave trough height. Naturally, it is also visible in the individual wave height to water depth ratio $\gamma_w = H/\bar{h}_w$ (Figure 8b), which shows increasing values as waves approach...
the shoreline, something previously observed by Sénéchal et al. (2004) and Power et al. (2010). In particular, the wide range of observed individual $\gamma_w$ values show the inappropriateness of choosing constant values for this parameter in numerical models. Finally, the individual $\gamma_w$ values, obtained closer to the shoreline than these two previous studies, seem to be in agreement with the line fit obtained with averaged $\gamma_w$ values by Power et al. (2010).

The comparison between $T_{i,m}$ and $T_{01}$ (Figure 7b) also shows interesting results. While for the deepest waters considered ($h \geq 0.2$ m), the mean extracted individual wave periods are consistent with $T_{01}$, as we get closer to the shoreline, the difference between the two values increases with decreasing water depth. This analysis gives some support to the idea of using the centroidal frequency to define a characteristic period in the inner surf, which represents the fractional change in water depth over a wavelength. In this expression, $\beta$ represents the bed slope, $k$ the wave number calculated from the averaged estimated celerities and a characteristic period and $h$ the averaged water depth over the same period.

Two different comparisons were made (using the same typology as in Section 4.2): 1. Comparison shown in Figure 9a using $H_{i/3}$ for $\gamma_s$ and $T_{i,m}$ and $T_{01}$ to derive $k$. 2. Comparison shown in Figure 9b using $H_i$ for $\gamma_s$ and $T_{01}$ to derive $k$. For both comparisons, a strong linear dependence was found between $\gamma_s$ and $\beta/kh$. For deeper water and using two different frequency cutoffs, Raubenheimer et al. (1996) and Sénéchal et al. (2001) found similar linear relationship, but with different coefficients. For the present dataset and for both derived $\gamma_s$, a good match is found with the linear fit obtained by Sénéchal et al. (2001) when $0 \geq \beta/kh \geq 0.5$. For greater values of $\beta/kh$, lower values compared to Sénéchal et al. (2001) are obtained when using the mean extracted wave period $T_m$, while that using $T_{01}$ still matches the linear fit. This limit value of $\beta/kh$ corresponds to the critical depth where $T_{01}$ does not match to $T_{i,m}$ any more (Figure 7b).

It is noted that the three compared datasets use different frequency cutoffs ($0.05$ Hz $\leq f \leq 0.18$ Hz for Raubenheimer et al. (1996), $0.09$ Hz $\leq f \leq 0.3$ Hz for Sénéchal et al. (2001) and $0.05$ Hz $\leq f \leq 0.18$ Hz for the present study). Except for the influence of the much lower high frequency cutoff used by Raubenheimer et al. (1996), it is unclear why the present dataset shows higher values than in Raubenheimer et al. (1996) but matches that of Sénéchal et al. (2001). Finally, it has to be noted that the dataset presented in this study contains much shallower depths than that considered in the two previous studies. For instance, the highest value of $\beta/kh$ considered by the previous studies was 0.25 while it is approximately 1.75 in the current work.

4.4. Wave celerities

Individual wave celerities were compared to a range of previously developed predictors summarized in Table 1. In the different formulations, $h$, $h_c$, $h_t$ are respectively the mean water depths, the crest height and the trough height. A more complete introduction to these predictors is given by Catálan and Haller (2008) who compared a wider range of celerity predictors against measurements obtained using video imagery from laboratory experiments.

Prior to this work, only a few studies have been published on the measurement of individual broken-wave celerities in the surf zone. Radon transform on video camera data have been used by Yoo et al. (2011) and Almar et al. (2014) to track wave crests, while Tissier et al. (2013) used a large array of wave gauges for this purpose. Additionally, Postacchini and Broccolini (2014) calculated individual broken-wave celerities by correcting the averaged celerities obtained by a cross-correlation method (Tissier et al., 2011) for each detected wave. While Tissier et al. (2011) found better agreement with Bonneton (2004) predictor using averaged celerities, individual celerities from Postacchini and Broccolini (2014) and this study were found to better match the solitary wave theory celerity, see Figure 10a.

In contrast to the study of Tissier et al. (2011) whose data was concentrated in the outer and mid-surf zone, the present study uses data from the inner surf to the swash zone. In particular, this enables one to look more closely at the boundary between the two zones in terms of wave celerities using the cross-correlation method. This is illustrated in Figure 10b, where the 10-minute averaged celerities are plotted against the corresponding averaged water-depth.

Between water depth of 0.2 and 0.4 m, the averaged celerities show good agreement with the modified shallow water wave predictor, though they are slightly underestimated. This is in agreement with the results found in Figure 10a. Indeed, the modified shallow water wave predictor corresponds to the solitary wave predictor with a constant wave height to water depth ratio of 0.78. Hence, despite a not-insignificant scatter when using the individual celerities (shown by Postacchini and Broccolini (2014), not shown in this study), the modified shallow water predictor provides good estimates of the averaged wave celerities seaward of $h \geq 0.2$ m, corresponding to $\gamma = 0.5$ in this study, see Figure 8b. Interestingly landward of this depth, averaged celerities remain quite constant, slightly decreasing, to finally present a much broader value range.
at the shoreline position (1.3 m.s\(^{-1}\) < \(c_b\) < 2.2 m.s\(^{-1}\)). This scatter of averaged values implies a wider range of individual celerities at the surf-swash boundary, which could be explained by the interaction between surf and swash processes.

5. Conclusion

In this study, a methodology for monitoring the beach morphology and individual wave characteristics using a shore-mounted 2-dimensional commercial laser scanner has been presented. The conclusions of this investigation can be summarized with the following points:

- The laser scanner can be used to measure time-varying water surface profiles in the inner surf and swash zones, enabling the study of wave propagation on a wave-by-wave as well as time-averaged basis.
- Individual wave properties (\(H, T\)) can be extracted using an extrema analysis on the measured time series. The extracted wave height was found to compare well with that from spectral analysis. It was also shown that for these conditions, the wave period derived from the centroidal frequency could be chosen as a characteristic wave period for water depths down to 0.2 m. Further investigation is needed on the reason why this changes at the swash/inner surf boundary.

- \(\gamma_s\) was found to be linearly dependent on \(\beta/k\). Furthermore, the present dataset seem to match well that of Sénéchal et al. (2001), for values of \(\beta/k\) lower than 0.5. For higher values, discrepancies are observed and are due to the differences observed between \(T_m\) and \(T_01\).
- Individual wave celerities were estimated using a simple crest-tracking method. Comparisons with various predictors showed that the solitary wave theory gave the best agreement with the present dataset. However, in the shallow water depths investigated here, these values exhibit considerable variability.
- 10-minute averaged wave celerities were also calculated using a cross-correlation technique. These values agree well with the modified shallow-water predictor in depths greater than 0.2 m, becoming almost constant as the water depths decrease landwards. This critical depth also corresponds to that when \(T_m\) and \(T_01\) start to show discrepancies. Since the celerity is a function of the wave period, the two facts could be physically linked. This will be the subject of further investigation, since it could bring new insight into the conditions at the surf-swash boundary.

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Formulation of (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear theory (shallow water assumption)</td>
<td>(c = \sqrt{gh})</td>
</tr>
<tr>
<td>Modified shallow water formulation</td>
<td>(c = 1.3\sqrt{gh})</td>
</tr>
<tr>
<td>Solitary wave theory</td>
<td>(c = \sqrt{gh(1 + H/h)})</td>
</tr>
<tr>
<td>Bore model (Svendsen et al., 1978)</td>
<td>(c = \sqrt{gh h (h_t + h_c)/(2h_t^2)})</td>
</tr>
<tr>
<td>Shock model (Bonneton, 2004)</td>
<td>(c = -2\sqrt{gh} + 2\sqrt{gh_t} + \frac{gh c (h_t + h_c)}{2h_t})</td>
</tr>
</tbody>
</table>

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**Figure 9:** Averaged significant wave height to averaged water depth ratio plotted against \(\beta/k\). a) Ratio calculated with \(k\) based on the mean extracted individual wave period \(T_{i,m}\); b) Ratio calculated with \(k\) based on the mean spectral wave period \(T_{01} = m_0/m_1\), inverse of the centroidal frequency. The present dataset (black dots, and its Q-Q fit shown as red line) is compared to the fit obtained in two previous studies: dashed gray lines for Sénéchal et al. (2001) and gray continuous line for Raubenheimer et al. (1996).
**Figure 10:** Scatter plot of measured wave celerities: a) individual wave celerities against the predictor from the solitary wave theory. Data circles are coloured by their concentration in every 0.05 m/s bins. The wave-period-averaged depth is used for the soliton celerity formulation, following Postacchini and Brocchini (2014). Correlation coefficient $r = 0.65$; b) Averaged wave celerities obtained from the cross-correlation of two 10-minute time series, plotted against water depth. Their standard deviation is plotted as red bars, using 0.025m-wide bins. The modified and original linear wave theories in shallow water are represented in red dashed lines.

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**Appendix A: Statistical parameters**

The different statistical parameters (Root-Mean Square Error, Scatter Index and a correlation coefficient noted $r$) used in this study are defined in this section. If we denote the two compared series as $X = \{x_1, \ldots, x_n\}$ and $Y = \{y_1, \ldots, y_n\}$, they are defined as follows:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - y_i)^2}$$

(2)

$$SI = \frac{\sqrt{\sum_{i=1}^{n} (x_i - y_i - \bar{X})^2}}{\bar{Y}}$$

(3)

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{X})(y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{X})^2 \sum_{i=1}^{n} (y_i - \bar{Y})^2}}$$

(4)

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