Traditionally, energy transaction between a microgrid (MG) and a distribution network (DN) is based on contract price or directly wholesale electricity market price, and economic operation of a MG is independent with that of a DN. In order to coordinate the benefits of DNs and MGs, based on the theory of bilevel programming problem (BLPP), a bilevel economic operation model is proposed in this paper. On the upper-level, a DN seeks to meet its load power demand with the least operation cost by optimizing energy exchange price. On the lower-level, quantity and direction of electric power exchange are control variables, a MG minimizes its hourly operation cost based on the energy exchange price got from the upper-level and the operation costs of dispatchable distributed generations (DGs) in it. Furthermore, energy exchange price constraint is considered in the BLPP model, and a method to dynamically determine the limit values of energy exchange price is proposed. The theory of this method is based on the differences among the wholesale electricity market price, operation costs of dispatchable DGs and the load power demand profile in the DN. In addition, real time pricing program (RTP) of residential consumers is considered into the economic operation of MGs. The model of RTP is built based on comprehensive impacts of self-price and cross-price elasticity. The proposed BLPP model is solved by particles swarm optimization based on bilevel iterative algorithm (PSO-BIA). The demonstration is performed on a modified IEEE 33-node radial distribution system.

I. INTRODUCTION

A microgrid (MG) is a modern and small-scale version of the centralized electricity system, comprising various distributed generations (DGs), storage devices and loads [1]. It is an effective way to overcome the stochasticity of renewable DGs and allows consumers to participate in the electricity enterprise, and it also can contribute to carbon emission reduction, diversification of energy sources and cost reduction [2]. A MG has two operating modes: grid-connected mode and off-grid mode. It is generally considered that economic operation and energy management are necessary for MGs to meet

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1) Hong Zhang. Electronic mail: gongchanyudie@126.com.
local power demand, improving renewable energy penetration level, controlling power exchanged with the utility grid [3-4]. Power flow optimization and energy cost management in the MG are researched deeply in [5-6]. A decentralized architecture of multi-agent system for economic dispatch of MGs is presented in [7]. However, bidirectional energy exchanges between MGs and the utility grid make the operation of DNs more complicated than ever before [8]. Various uncertain factors are considered into the economic operation of MGs based on the assumption that low-voltage DNs just only sell energy to MGs at RTP tariff in [9]. Apart from these research efforts only on the economic operation of MGs, there is also some research work on exploring the benefits for MGs and DNs. In order to minimize the total operation costs of a DN with multiple MGs, an integrated solution that considers both MG load dispatch and DN reconfiguration is proposed in [10]. A microgrid-based co-optimization planning model was proposed by considering the power system reliability and economic criteria [11], where generation, transmission system and MGs together were considered. These co-optimization problems cannot be resolved directly by traditional optimal algorithms directly, so some references propose bilevel programming problem (BLPP), which is a Stackelberg game, to resolve it. In a Stackelberg game, there are the leader and the followers. The leader makes the move first by anticipating the reaction of the followers, and then the followers move sequentially by knowing the move of the leader [12]. This strategy has been used to determine the optimal contract price of dispatchable DGs in DNs, in which the distribution corporation operator (DISCO) is the leader and the owners of DGs are followers. However, at present there are rare research efforts on economic operation of both the DISCO and MGs at the same time, in which how to coordinated the energy exchange price and quantity to keep benefits both of them is a difficult problem.

To this end, in this paper, the DISCO is considered to be the sole owner and operator of a DN, which can purchase energy from the wholesale market and MGs integrated in the DN. Furthermore, the DISCO
can also sell energy to these MGs. Based on these considerations, an economic operation model of a DN with MG integration based on BLPP is proposed. The economic operation of the DISCO is on the upper-level and that of MGs is on the lower-level. The hourly energy exchange price is the leader and energy exchange quantity is the follower. Furthermore, the energy exchange price constraints are considered in the model. In addition, the method to dynamically determine the constraints is proposed as well. According to the features of BLPP, particles swarm optimization based on bilevel iterative algorithm (PSO-BIA) is adopted to resolve the model.

The contributions of this paper are to: i) build a bilevel economic operation model in a DN with MGs, where the DISCO sends the optimal hourly energy exchange price signal to MGs to resolve the energy exchange quantity; ii) research effects of real-time pricing demand response on economic operation of MGs; iii) propose and dynamically determine the energy exchange price constraints in the bilevel economic operation.

The remainder of this paper is organized as follows. Section 2 introduces the proposed economic operation problem of DNs with MG integration briefly. Section 3 proposes the formulations of BLPP and the economic operation model in the DISCO with MG integration. Section 4 describes the solution method of this BLPP model and the approach to dynamically determine the energy exchange price constraints. In Section 5, numerical results illustrate the performance of the proposed model and algorithms. Relevant conclusions are drawn in Section 6.

II. INTRODUCTION OF THE PROPOSED ECONOMIC OPERATION PROBLEM

A. Construction of DNs with MG integration

The construction of a DN with MGs integration is shown as Figure 1. There is one substation connecting the DN to the transmission system and every MG connects to the DN from a single point of common coupling (PCC). The power flow between the DN and MGs is bidirectional. MGs can not only
buy energy from the DISCO but also can sell energy to it. The DISCO is considered to be the sole owner and operator of a DN. Its responsibilities on economic operation of a DN are as follows:

1. The DISCO decides the amount of energy a DN needs to purchase from the wholesale electricity market;

2. The DISCO is in charge of energy transaction with MGs owned by independent operators[12], including the energy price and the amount of energy transacted with MGs;

3. The DISCO needs weigh the potential benefits obtained from the transaction with MGs. For example, if the energy bought from MGs has a positive impact regulating the peak load power demand and/or contributes to reduce power losses, then, even if the MG energy price is a little higher than the wholesale market price, the DISCO might buy energy from MGs.

![Diagram of a distribution network with MGs](image)

**FIG.1.** Construction of a distribution network with MGs.

The MGs in Figure 1 are built for residential community at low voltage networks and supposed to keep operating in a grid-connected mode to research the energy transaction between the DISCO and MGs. The configuration of the MGs is shown in Figure 2. They are mainly composed of DGs, local consumers and storage facility in a limited geographical area [13]. DGs with conventional energy sources, such as oil, gas, coal, are dispatchable DGs. Correspondingly, DGs with renewable energy sources (RES) which are driven by weather, not by system loads, are non-dispatchable DGs, such as wind turbines.
(WTs) and solar photovoltaics (PVs). In the economic operation problem, these non-dispatchable DGs will be treated as negative loads [1].

![Diagram of a MicroGrid configuration](image)

**FIG.2.** The configuration of a MG.

### B. Bilevel economic operation of distribution networks with MG

The bilevel economic operation of DNs with MG is a kind of economic operation that achieves benefits for both DNs and MGs. As the sole owner and operator of a DN, the DISCO incentives MGs to supply cheaper energy to it at the optimal energy exchange price and MGs operate themselves economically and safely following the price signals. The framework of this bilevel economic operation is shown as Figure.3.

![Diagram of the proposed bilevel economic operation framework](image)

**FIG.3.** The proposed bilevel economic operation framework.

In Figure.3, on the upper level, the DISCO optimizes the day-ahead hourly energy exchange price, scheduled in order to minimize its operation costs. The lower level comprises of MGs, which decides how much energy MGs can exchange with the DISCO according to the optimal energy exchange price. There are two kinds of information communication between the two levels. One is energy exchange price...
sent from the upper level to the lower level, and the other is quantity of energy that MGs could exchange with the DISCO in an hour, supposing that the electric power keeps constant in one hour. The positive direction of the electric power (indicated as $P_{ex}$) is set to be from MGs to DNs (Table I). It is beneficial for the DISCO that MGs can behave as generators in its on-peak load periods and as load power demand in its off-peak load periods.

<table>
<thead>
<tr>
<th>sign</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{ex} = 0$</td>
<td>There is no energy exchanged between MG and the DISCO</td>
</tr>
<tr>
<td>$P_{ex} &gt; 0$</td>
<td>MG sells energy to the DISCO as a generator</td>
</tr>
<tr>
<td>$P_{ex} &lt; 0$</td>
<td>MG buys energy from the DISCO as load power demand</td>
</tr>
</tbody>
</table>

### III. FORMULATION OF THE BILEVEL OPTIMISATION PROBLEM

#### A. General formulation of BLPP

There are two correlative levels in BLPP, upper-level (U) and lower-level (L). Its general formulation is [14]:

\[
\begin{align*}
(U) \quad & \min_{x,y} F(x,y) , \\
& \text{s.t. } G(x,y) \leq 0, \\
(L) \quad & \min_{y} f(x,y) , \\
& \text{s.t. } g(x,y) \leq 0.
\end{align*}
\]

where, $x,y$ are upper-level and lower-level variables respectively; $F(x,y),f(x,y)$ are upper-level and lower-level objective functions respectively; $G$ and $g$ are called upper-level and lower-level constraints respectively. The decisions of the two levels affect and constraint each other, therefore the BLPP can capture the hierarchical relationship dynamically between the DISCO and MGs.

In bilevel optimization problem of the DISCO with MG integration, shown as Figure 4, the DISCO problem is on the upper-level and the MG problem is on the lower-level: i) the optimal energy exchange
price is scheduled on the upper level (DISCO problem); ii) the hourly quantity of energy exchange and generation in MGs is resolved based on the price schedule on the lower level (MG problem).

![Bilevel economic operation of DNs with MG integration.](Image)

**FIG. 4.** Bilevel economic operation of DNs with MG integration.

### B. The DISCO problem

#### 1. Objective function

The objective function of the DISCO is to minimize its operation costs based on the price difference between the wholesale market and MGs. The operation costs of the DISCO consists of three parts: i) energy exchange cost with MGs; ii) power purchasing cost from the wholesale market; and iii) energy distribution cost, in which power loss on lines is neglected here considering the electric power transmission distance between MGs and end consumers is not very long in distribution networks. The formulation is:

\[
\min_{t} \text{Cost}_{\text{DN}}(t) = \rho_{E}(t)P_{\text{sub}}(t) + \sum_{n=1}^{N}\rho_{m}(t)P_{\text{ex,n}}(t) + DP_{d}(t).
\]  

(5)

where, \(\text{Cost}_{\text{DN}}(t)\) is the DISCO’s operation cost function at time \(t\), in $/h; \rho_{E}(t)\) is day-ahead clearing price in wholesale market at time \(t\), in $/kWh; \(P_{\text{sub}}(t)\) is electric power from wholesale electricity market at time \(t\), in kW; \(\rho_{m}(t)\) is day-ahead energy exchange price announced by the DISCO for MGs at time \(t\), in $/kWh, \(\rho_{m}(t)\) and \(P_{\text{sub}}(t)\) are decision variables on the upper level; \(P_{\text{ex,n}}(t)\) is electric power exchanged between the DISCO and MG \(n\) at time \(t\), in kW; \(N\) is the amount of MGs integrating on the distribution network; \(D\) is distribution charge, which is assumed to be 5.0cents/kWh [15]; and \(P_{d}(t)\) is load power demand in the distribution network at time \(t\), in kW.

#### 2. Constraints
(1) Power balance constraint

Economic operation must be based on power balance, which is

$$\sum_{n=1}^{N} P_{ex,n}(t) + P_{sub}(t) = P_{d}(t) + P_{loss}(t).$$

(6)

(2) Energy exchange price constraint

In order to coordinate benefits of the DISCO and MGs more properly and keep them operate safely, energy exchange price constraint is imposed. Its formulation is:

$$\rho_{min}(t) \leq \rho_{m}(t) \leq \rho_{max}(t).$$

(7)

where, $\rho_{min}$ and $\rho_{max}$ are minimum and maximum values of energy exchange price respectively, in $$/kWh.

C. The MG problem

Suppose that MGs keep in a grid-connected mode, and a transaction mechanism with the DISCO is in place, where MGs can buy (sell) energy from (to) the DISCO at the optimal energy exchange price. Besides critical load, there is RTP demand response in MGs. As known, PM2.5 particles are fine particles in the (ambient) air 2.5 micrometers or less in size (known as PM2.5), which can lead to respiratory and cardiovascular illness. Fuel coal plants are a main source of PM2.5. Therefore, in order to reduce PM2.5 pollution, MGs accept all electric power generated from renewable DGs such as WTs and PVs based on meeting technical constraints such as network thermal limits, and adopt micro turbine (MT) with combined heat and power system (CHP) as dispatchable DGs. By properly transacting energy with the DISCO, MGs could minimize its operation cost on condition of meeting its own load power demand.

1. Formulation of micro turbine

MT is always used in conjunction with CHP. As one typical example, the Capstone C200 micro turbine (C200) produced by Capstone Turbine Corporation is utilized in this paper, and its fuel cost can be formulated as following [16-17]:
\[ C_{MT} = C_{nl} P_e / L \eta_e . \] (8)

where, \( C_{MT} \) is fuel cost of the C200, in $; \( C_{nl} \) is the price of natural gas, in $/m^3; L \) is low heating value of natural gas, assumed to be 9.7kWh/m^3; \( P_e \) is the electric power generation of the MT, in kW; \( \eta_e \) is electrical efficiency of the C200, assumed to be 33% [16].

2. Characteristics and formulation of RTP

RTP is a kind of demand response in which customers are charged at hourly fluctuating prices reflecting the real costs of energy in wholesale markets [18]. Compared with other kinds of demand response, RTP is fully flexible, because i) it does not need baseline price; ii) it changes based on wholesale real time prices. Economists believe that RTP programs are the most efficient and direct for enabling demand response programs in competitive markets [19].

The prices are informed to RTP customers on a day-ahead or hour-ahead basis. The day-ahead RTP customers are given one-day notice of the prices for the following day’s 24 hours. This gives customers enough time to plan their responses, such as shifting use (often by shifting load to off-peak hours or by using onsite generation) or hedging day-ahead prices with other products if they cannot curtail their demand[20]. Therefore, day-ahead RTP is more suitable for day-ahead optimization of MGs.

Price elasticity can reflect the price sensitivity of demand to energy price. It is composed of self-elasticity and cross elasticity [21], they can be formulated as follows:

\[
\epsilon(i,i)(t) = \frac{P(t) - P_0}{E(t) - E_0} \]  

\[
\epsilon(i,j)(t) = \frac{P(t_i)^{1,2} / P_0}{E(t_j) / E_0}, \quad (i, j = 1,2,...24, i \neq j) \] (9)

where, \( \epsilon(i,i) \), named self elasticity, is a measure of load power curtailment by the consumer at hour \( t_i \); \( \epsilon(i,j) \) is cross elasticity, which is a measure of load power shifting considering energy price
in the other hours; $\Delta P, P_0$ are varied and initial load power demand respectively, in kW; $\Delta E, E_0$ are varied and initial energy price respectively, in $/kWh.

Then the overall variation in load power demand considering RTP at hour $t_i$ is [21]:

$$P(t_i) = \sum_{j=1}^{24} (i, j) E_j / (E_0 P_0) \quad (i = 1, 2, ..., 24).$$ (10)

With smarter meters on the rise, it is possible for residential consumers to get real-time data and curb energy usage, which is beneficial for both consumers and utilities [22-23]. Therefore, it is supposed that RTP consumers in MGs can get hourly energy price by smart meters, and operators of MGs can get how much variation in load power enabled by RTP before the day-ahead bilevel economic operation begins. Then, at the lower-level, the hourly load power demand MGs need to meet is:

$$P_L(t_i) = P_{L0}(t_i) + P(t_i).$$ (11)

where, $P_L(t_i)$ is load power demand considering RTP at time $t_i$, in kW; $P_{L0}(t_i)$ is initial load power demand in MGs at time $t_i$, in kW.

4. **Principles of the economic operation of MGs**

The basic principles of the proposed economic operation of MGs on the lower level are:

1. A MG operator is in charge of collecting operation data of a MG and the information communication between a MG and the DISCO.

2. Consumers with RTP in a MG can get 24-hour energy prices day ahead by smart meters. Then hourly quantity of RTP on next day is calculated and sent to the MG operator automatically.

3. Considering the environmental protection, renewable DGs such as WTs and PVs in a MG are adopted firstly when the MG operator performs economic operation.

5. **Objective function**

Load power demand can always be met when MGs operate in a grid-connected mode. Therefore, the operation objective of MGs is to minimize their operation and maintenance cost according to DG bids.
and RTP schedule from the DISCO. By considering the environment, all power generated by WTs and PVs is used in MGs. The hourly operation and maintenance cost here comprises of operation and maintenance cost of DGs, energy purchase cost from the DISCO and participation incentive cost of RTP demand response.

\[
\min Cost_{MG} = Cost_{DG} + Cost_{pur} + Cost_{inc} = \sum_{k=1}^{K} P_{Gk}(t) \eta_{Gk} C_{r,Gk} P_{ex,n}(t) + P(t) C_{inc}. \tag{12}
\]

where, \(Cost_{MG}\) is operation and maintenance cost of the MG at time \(t\), in $/h; P_{Gk}(t)\) is electric power bought from DGs by the MG at time \(t\), in kW; \(\eta_{Gk}\) is generating efficiency of DGs, in $/kWh; k\) is type of DGs in a MG; \(C_{inc}\) is participation incentive of RTP demand response, which is constant (1.4cents/kWh[19]); Because the positive direction of \(P_{ex,n}(t)\) is from MGs to the DISCO, there is a minus sign before it.

6. Constraints
   a. Power flow balance constraint considering RTP

\[
P_{Gk}(t) + P_{ex,n}(t) = P_L(t) + P_{loss}(t). \tag{13}
\]

Suppose that the hourly real-time electricity price sent to residential RTP consumers by MGs is based on the hourly clearing price in wholesale electricity market and transmission cost of MGs, then

\[
\rho_r(t) = \rho_E(t) + D (t = 1, 2, \ldots, 24). \tag{14}
\]

Therefore, based on equation (11), the final load demand with RTP in time interval \(t\) is presented by:

\[
P_L(t) = P_{L0}(t) + P(t)
\]

\[
= P_{L0}(t) \left(1 + \sum_{j=1}^{24} \left(1 + \rho_r(j)/\rho_r(t)\right) \right). \tag{15}
\]
where, $\rho_r(t)$ is electricity price for RTP consumers at time $t$, in $$/kWh; \Delta \rho_r(t)$ is electricity price variation for RTP consumers at time $t$, in $$/kWh; \rho_E(t)$ is clearing price in wholesale electricity market at time $t$, in $$/kWh; \text{D}$ is transmission cost of MGs, supposed to be 5.0cents/kWh[15].

b. Considering the influences on the main grid, power exchanged with the MG at PCC

$$p_{ex,n}^{\text{min}} \leq p_{ex,n}(t) \leq p_{ex,n}^{\text{max}}.$$  \hspace{1cm} (16)

c. Load power demand constraint in MG

$$P_L^{\text{min}} \leq P_L(t) \leq P_L^{\text{max}} \quad (t_i=1,2,...,24).$$  \hspace{1cm} (17)

According to equations (11) and (15), the equation (17) can be reformulated as:

$$\frac{P_L^{\text{min}}}{P_L(t)} \leq \sum_{j=1}^{24} \left( \frac{\rho_r_j(t)\rho_r(t)}{\rho_r_0(t)} \right) \leq \frac{P_L^{\text{max}}}{P_L(t)}.$$  \hspace{1cm} (18)

d. Power generation constraint of dispatchable DGs

$$P_{G_k}^{\text{min}} \leq P_{G_k}(t) \leq P_{G_k}^{\text{max}}.$$  \hspace{1cm} (19)

where, $p_{ex,n}^{\text{min}}, p_{ex,n}^{\text{max}}$ are minimum and maximum value of electric power exchanged between the DISCO and MG $n$ respectively, in kW; $P_L^{\text{min}}, P_L^{\text{max}}$ are minimum and maximum load power demand in the MG at time $t$ respectively, in kW.

The formulation of the proposed bilevel economic operation of DNs with MG integration is:
(U) minimize \( \text{Cost}_{\text{DN}}(t) = \rho_E(t)P_{\text{sub}}(t) + \sum_{n=1}^{N} \rho_m(t)P_{\text{ex},n}(t) + DP_d(t) \)

with respect to

\[
\begin{align*}
\sum_{n=1}^{N} P_{\text{ex},n}(t) + P_{\text{sub}}(t) &= P_a(t) + P_{\text{loss}}(t) \\
\rho_{\text{min}}(t) &\leq \rho_m(t) \leq \rho_{\text{max}}(t)
\end{align*}
\]

(L) minimize \( \text{Cost}_{\text{MG}} = \sum_{k=1}^{K} P_{Gk}(t)/\eta_{Gk}C_{r,Gk} - P_{\text{ex},n}(t)\rho_m(t) + \Delta P(t)C_{\text{inc}} \)

with respect to

\[
\begin{align*}
\sum_{k} P_{Gk}(t) + P_{\text{ex},n}(t) &= P_L(t) + \Delta P(t) + P_{\text{loss}}(t) \\
p\text{_{min}} &\leq P_{\text{ex},n} \leq p\text{_{max}} \\
p_L(t) - P_L(t) &\leq \sum_{j=1}^{24} e(t,j)(\Delta \rho_r(j)/\rho_r(t)) \leq \frac{p_L(t) - p_{L0}(t)}{P_L(t)} \\
p_{Gk} &\leq P_{Gk}(t) \leq P_{Gk}\text{_{max}} \\
t &= 1, 2, \ldots, 24
\end{align*}
\]

(20)

IV. SOLUTION METHOD

The traditional solution methods of BLPP are changing the bilevel problem into a single level problem by replacing the lower-level problem as constraints of the upper-level problem [14], which are not very efficient. In this paper, the proposed BLPP model is resolved directly by integrating Bilevel Iterative Algorithm (BIA) and Particle Swarm Optimization algorithm (PSO) in this paper. It can get global optimal result and has general applicability.

A. Particle Swarm Optimization based on Bilevel Iterative Algorithm (PSO-BIA)

PSO has been successfully adopted in electric power system until now. Recently, in order to avoid trapping in local optimum [24], Adaptive Mutation PSO (AMPSO) is proposed in many references [25-
BIA is a mathematical procedure that generates a sequence of improving approximate solutions for a class of problems. It can be used to solve the interaction between upper-level optimal problem and lower-level optimal problem. Therefore, the proposed BLPP model is solved by PSO based on BIA. Solve the upper-level problem with AMPSO and the lower-level with traditional PSO, and then iterate between two levels based on BIA. Its flowchart is shown as Figure 5.

(1) Initialization

Initialize parameters in both AMPSO and PSO, including position $X_i$, $Y_i$ and speed $V_{xi}$, $V_{yi}$ of particles on upper level and lower level, $1 \leq i \leq m$; $m$ is population size. $P_{ix}$, $P_{iy}$ are present positions of upper-level and lower-level particles respectively; $P_{gx}$, $P_{gy}$ are best positions of particles on upper level and lower level respectively.

(2) Update all particles on the upper level.

(3) Get the optimal values of $P_{iy}$ and $P_{gy}$ by PSO based on positions of upper-level particles.

(4) Evaluate the upper-level fitness $F(X_i, P_{iy})$, $1 \leq i \leq m$.

(5) Update values of lower-level $P_{iy}$ and $P_{gy}$ according to $P_{ix}$ and $P_{gx}$ correspondingly.

(6) Stopping criteria

The algorithm is continued until the precision of solution or iteration number is reached.

(7) Adaptive mutation if it does not stop in (6), update the value of $P_{gx}$ by following formulation:

$$P_{gx} = P_{gx}(1+\eta).$$
where, $\eta$ is a random variable obeying standard normal distribution, $\eta \sim N(0,1)$.

**B. Dynamic limit values of energy exchange price**
In this proposed BLPP model, the hourly optimal energy exchange price on upper-level plays a leading role and its result affects the value and direction of electric power on lines between the DN and MGs. In order to: i) avoid the DISCO from minimizing its operation cost infinitely; and ii) incentive MGs to curtail the peak-regulation pressure of the DISCO, the energy exchange price constraint is formulated in equation (7). However, because the energy price in wholesale electricity market changes from hour by hour and is affected by many factors, it is hard to give fixed values on the limits $\rho_{\text{min}}$ and $\rho_{\text{max}}$ to satisfy the requirement on every time period. Further, most smart optimal algorithms such as PSO, GA, are sensitive to the limit values of variables. Therefore, in order to incentive MGs to curtail peak load of DISCO and maximize profits of both MGs and DISCO, the method that determines the limit values of energy exchange price dynamically and automatically hourly is proposed.

The detail research concept is introduced in the following part.

a. *Keeping MGs operating in an economic status.*

It is beneficial for MGs to sell electric power to the DISCO when the exchange price is higher and buy energy from the DISCO when the exchange price is lower. From the lower-level objective in the proposed BLPP model, it can be seen that the value and direction of electric power exchange $p_{\text{ex,n}}(t)$ is decided by the difference between $\rho_m(t)$ and operation cost of dispatchable DGs $C_{r,Gk}$ at the same time $t$. This can be shown as follows:

\[
\begin{align*}
\text{when} & \quad \rho_{\text{max}}(t) = \min \{C_{r,Gk}\}, \quad p_{\text{ex,n}}(t) < 0 \\
\text{when} & \quad \rho_{\text{min}}(t) = \max \{C_{r,Gk}\}, \quad p_{\text{ex,n}}(t) > 0
\end{align*}
\]

(22)

b. *Keeping the DISCO operating economically.*
On one hand, from the upper-level objective (equation (5)), when the energy price in wholesale market \( \rho_E(t) \) is higher, it is economic for the DISCO to incentivize MGs to sell energy at lower price. That can be formulated as

\[
\begin{align*}
\text{when } \rho_E(t) > \max \{ C_{r,Gk} \}, \quad & \begin{cases} 
\rho_{\text{max}}(t) = \rho_E(t) & \text{and} \\
\rho_{\text{min}}(t) = \max \{ C_{r,Gk} \} 
\end{cases} \\
\text{when } \rho_E(t) \leq \min \{ C_{r,Gk} \}, \quad & \begin{cases} 
\rho_{\text{max}}(t) = \rho_E(t) & \text{and} \\
\rho_{\text{min}}(t) = D
\end{cases}
\end{align*}
\]

(23)

Where, D is distribution cost of the DISCO, which is farther less than all energy prices in wholesale market and dispatchable DGs operation cost. When the DISCO sells electric power to MGs, the price must be larger than its distribution cost.

On the other hand, it is more economic for DISCO to incentivize MGs to supply energy on its peak-load time periods and buy energy when the load is lower. Therefore, the other constraints can be modeled as follows

\[
\begin{align*}
\text{when } P_d(t) \geq \overline{P}_d, \quad & P_{\text{ex,n}}(t) > 0 \\
\text{when } P_d(t) < \overline{P}_d, \quad & P_{\text{ex,n}}(t) < 0
\end{align*}
\]

(24)

where, \( \overline{P}_d \) is the average value of 24-hour load power demand in the DISCO, in kW.

By taking all these constraints and the value difference between \( C_{r,Gk} \) and \( \max \{ C_{r,Gk} \} \), the procedure of deciding the values of \( \rho_{\text{min}}(t) \) and \( \rho_{\text{max}}(t) \) are detailed in Figure.5.
In Figure 6 it is shown that:

(1) When energy price in wholesale market at time $t$ is not smaller than the maximum operation cost of all kinds of dispatchable DGs in MGs and load power demand in the DN at time $t$ is not less than the average load power demand in the day, the maximum and minimum energy exchange price is equal to the energy price in wholesale market at time $t$ and the maximum operation cost of all kinds of dispatchable DGs in MGs respectively, shown as formulations A in Figure 6. Therefore, MGs can be incentivized to sell energy to the DISCO to share its peak load power demand.

(2) When energy price in wholesale market at time $t$ is not more than the minimum operation cost of all kinds of dispatchable DGs in MGs and load power demand in the DN at time $t$ is less than the average load power demand in the day, the maximum and minimum energy exchange price is equal to the energy price in wholesale market at time $t$ and the distribution cost of the DISCO respectively, shown as
formulations B in Figure.6. Therefore, MGs can be incentivized to buy energy from the DISCO to save its operation cost.

(3) In any other situations, the limits of energy exchange price are set by formulations C in Figure.6. They can coordinate the benefits of the DISCO and MGs properly.

V. NUMERICAL RESULTS AND DISCUSSION

The proposed bilevel economic operation of DNs with MG is applied to the IEEE 33-node radial distribution system [27], shown in Figure.7. It has 33 nodes and 37 branches. The reference voltage at slack bus is 12.66 kV. The maximum active peak demand is 3.7MW. There are five MGs with the same construction integrated on nodes 7, 15, 22, 24, 30 separately through distribution lines. The capacity of each distribution line is assumed to be 0.3MW. It is also assumed that there is no energy exchange between these five MGs.

![Diagram of the 33-node distribution network with MG.](image)

It is supposed that the construction of MGs integrated on the DN is the same. The details of one MG are shown as Figure.8 [28] and Table II.
There are two feeders in each MG. Electric power is generated by WTs, PVs and MTs, whose operation parameters are shown in Table II. There are 10 WTs with the capacity of 0.15MW on node 4, 20 PVs of 0.1MW on node 6 and 10 MTs of 0.2MW on node 7. The natural gas price is 0.4\$/m^3.

**TABLE II.** Characteristics of DGs in a MG.

<table>
<thead>
<tr>
<th>Type</th>
<th>MT</th>
<th>PV</th>
<th>WT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated capacity(MW)</td>
<td>0.2</td>
<td>0.1</td>
<td>0.15</td>
</tr>
<tr>
<td>Cr,G($/MWh)</td>
<td>41</td>
<td>1.28</td>
<td>1.76</td>
</tr>
<tr>
<td>quantity</td>
<td>10</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

24-hour electric power generation curves of WTs and PVs are shown in Figures 9 and 10 respectively. These data are supplied by Hainan Power Grid Corporation in China. PVs generate electric power during 7:00 and 18:00, and the max power is 1.0MW. WTs generate electric power all the day, and from 21:00 to 23:00 the electric power is more than other time intervals.
Initial 24-hour load power demand profile in MGs is shown as Figure 12. There are RTP consumers on feeder#1 and critical load power demand on feeder#2. The self-price and cross-price elasticity values of RTP are shown in Table III. There are three time intervals (on-peak, mid-peak, off-peak) in a day. The winter price elasticity values are used here.

TABLE III. Price elasticity values.

<table>
<thead>
<tr>
<th>Time intervals</th>
<th>On-peak</th>
<th>Mid-peak</th>
<th>Off-peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>On-peak</td>
<td>-0.15</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>Mid-peak</td>
<td>-0.08</td>
<td>-0.14</td>
<td>0.05</td>
</tr>
<tr>
<td>Off-peak</td>
<td>-0.07</td>
<td>-0.05</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

*winter:on-peak(7-11,17-19);mid-peak(11-17);off-peak(19-7). Summer:on-peak(11-17);mid-peak(7-11,17-19);off-peak(19-7).*

FIG. 9. Power generation curve of PVs.

FIG. 10. Power generation of WTs.
The parameters of PSO-BIA are: study factors \( C_1 = C_2 = 2 \), inertia weight \( \omega_{\text{max}} = 0.9, \omega_{\text{min}} = 0.1 \), population size is 30, maximum number of iteration times is 50. This solution method is encoded on Microsoft Visio Studio 2010.

**A. Different pricing methods**

In order to examine that optimal real time pricing signals sent to MGs are better for economic operation of both the DISCO and MGs, two pricing methods are considered here. Case A shows the impact of directly sending day-ahead wholesale market prices signals to MGs. Case B presents the results of posting optimal day-ahead RTP tariff resolved by the proposed BLPP economic operation model. In these two cases initial data and economic operation model of MG are assumed to be the same.

1. **Case A: sending directly price signals to MGs**

In this case, there is no upper-level economic operation, and MGs are seen as independent active consumers in the DISCO which gives day-ahead wholesale market price signals, shown in Figure 11. MGs decide how much energy they can exchange with the DISCO day ahead by economic operation. Then the DISCO arranges its operation based on these decisions of MGs. The results are shown in Figure 11.

2. **Case B: sending optimal price signals to MGs**

In this case, the construction of the DISCO and MGs are the same as in case A. The difference is that economic operation solution of MGs not only follows but also affects the optimal real time pricing tariffs of DISCO. The optimal results of BLPP are shown in Figure 11.
(a). Comparison of exchange price between DISCO and MGs.

(b). Operation cost comparison in DISCO.
There is only economic operation of MGs in case A. The operation cost of MGs in one day is $3681$, lower than that in case B, $3736$, but the operation cost of DISCO in case A is higher than that in case B. From Figure 11, it can be seen that:

(1) From 0:00 to 6:00, the wholesale market prices are lower, and MGs buy energy from the DISCO in both cases. However, in Case A exchange prices are a little higher than those in Case B, shown as Figure 11.a. Therefore, the operation cost of the DISCO in Case B is a little higher than which in case A, shown as Figure 11.b;

(2) From 7:00 to 21:00 except 16:00, the wholesale market prices are higher than DGs cost in MGs. In these situations it is economic for MGs to sell energy to the DISCO, and therefore energy exchange keeps being positive during this period, shown in Figure 11.c. Because exchange prices are higher in Case A than those in case B, the DISCO needs to pay more to buy energy from MGs, shown in Figure 11.b;
(3) At 16:00 the exchange price in Case A is $126.19/MWh which is a very little higher than MT operation cost in MGs, so the optimal result is MGs buy energy from the DISCO, shown as Figure 11.c. Therefore, compared with direct wholesale market price signals, supplying optimal price signals to MGs in the proposed BLPP model can reduce the DISCO operation cost further.

B. The impact of demand response in MG

In one case, it is supposed that there are RTP demand response consumers in each MG. They can receive hourly wholesale market price signals day ahead. In the other case, consumers can just receive fixed flat tariff, at $190/MWh. Initial 24-hour load power demand curve and RTP tariff are shown in Figure 12. The incentive cost of MG is $14/MWh. The price elasticity values of RTP are shown in Table II. One day in winter is taken as an example. The hourly amount of load power demand based on RTP tariff in the MG is as shown in Figure 12.

![Figure 12: 24-hour load power demand considering RTP tariff.](image)

From Figure 12, it is clear that because of the self and cross price elasticity, initial peak load power demand is not only curtailed but also shifted:

1. On the general trend of load profiles and RTP tariffs, during 6:00am to 20:00pm the energy prices are higher than other periods, and the load power demand in this time period is accordingly...
curtailed largely by its self-price elasticity. For example, load power demand at 7:00am is curtailed from 2.86MW to 2.7MW and at 18:00pm the initial peak load power demand is curtailed down to 2.77MW from 3.13MW.

(2) Because of cross-price elasticity, initial peak load power demand, at 18:00pm, is shifted from higher price period to lower price period, at 20:00pm. Under RTP tariffs the daily electricity cost of all consumers in this MG is 8258.74 $ which is 3555.36 $ less than that under the fixed flat tariffs.

C. The impact of limits on variable energy exchange price

Here three cases are demonstrated to understand the effects of exchange price constraints on economic operation of DNs with MG. Except the constraints are different, all these cases are based on the same conditions, including construction of DN, initial data and parameters used in the PSO-BIA. It is also supposed that there is no capacity limit on the substation lines.

(1) Case A. Dynamic constraint: In this case the values of exchange price limits are determined in the way as shown in Figure 6. MTs are dispatchable DGs in each MG, their efficiency is 33% and unit operation cost is 41$/MWh. Therefore, the following values are also assumed: \( \max \{ C_{r,Gk} \} = \min \{ C_{r,Gk} \} \times 0.33 = 124.24 \) $/MWh. The average load power demand in the day is 3.19MW. The hourly load power demand and the wholesale market prices are shown in Figure 13.
From Figure 13 it is seen that: because of dynamic values of energy exchange price limits, MGs are incentivized to act as load when load of DN is lower and the wholesale market price is lower, and act as generator in the contrary situation. For example, from 17:00 to 20:00 when both load power and the wholesale market prices are at peak, the exchange prices are also higher in the effect of dynamic energy exchange price limits, and MGs are incented to sell as much energy as possible to the DISCO. From 1:00 to 7:00 when load power demands in the DISCO is lower and wholesale market prices are also lower, MGs act as load buying energy from the DISCO to accomplish minimizing their operation cost at mean time.

(2) Case B. Fixed constraint: the values of exchange price limits keep constant. \(\rho_{\text{max}}(t) = 300\) $/MWh, \(\rho_{\text{min}}(t) = D = 50\) $/MWh, \(t=1,2,\ldots,24\).

(3) Case C. No constraint: delete the energy exchange price constraint from the proposed BLPP model. Then in the procedure of PSO-BIA, it needs not to limit the initial or update values of the \(\rho(t)\). However, the initial value and the speed of \(\rho(t)\) are the same as Case A and B.

The results of these three cases are shown as follows.
Firstly, shown in Figures 14 and 15, the operation costs of DN and MGs in Case B and Case C are higher than those in Case A. and because the initial values and particle update speed of energy exchange price $\rho(t)$ are the same in Case B and Case C, although the optimized operation costs of DN and MGs in Case C are higher than those in Case B, their differences are not large.
Secondly, MGs can not help the DISCO reduce peak load in Case B and Case C, shown in Figure 16. The DISCO still sells energy to MGs when its own load is higher in the DN (from 6:00 to 8:00 and from 17:00 to 20:00). This leads to the DISCO buying more energy from the wholesale market which may cause congestion in the whole transmission system.

Therefore, determining the exchange price limits dynamically is necessary and essential in the bilevel economic operation of a DN with MG.

![Electric power exchange in case B and case C](image)

**FIG.16.** Electric power exchange in case B and case C.

**VI. CONCLUSIONS**

The model and solution methods of a bilevel economic operation of DNs with MG integration are proposed in this paper. In this bilevel optimization problem: the economic operation of the DISCO is at the upper-level and its control variable is the hourly energy exchange price; the economic operation of MGs is at the lower-level and independent with each other, the quantity of hourly energy exchange is the followed variable. Further, the energy exchange price constraint is proposed in this bilevel economic operation and the limits are dynamically determined according to the differences among the wholesale market.
market price, operation costs of dispatchable DGs and the load power demand profile in the DN. In addition, the model and effects of RTP demand response are considered into the economic operation of MGs. Instead of converting the bilevel optimization problem into the single-level, a novel PSO-BIA is introduced to resolve it directly, in which AMPSO is used to resolve the upper-level optimization and BIA is adopted to coordinate the relationships of variables between the two levels.

IEEE 33-node radial distribution system with MGs integration is used to demonstrate the proposed models and algorithm. It is shown that i) the operation cost of both DNs and MGs can be reduced by BLPP; ii) under RTP tariffs, residential consumers can save electricity payment and peak load power demand in MGs can be curtailed and shifted effectively; iii) the method of dynamically determining limit values of energy exchange price can incentivize MGs to share peak load power demand in DNs effectively.

In conclusion, from an economic point of view, the proposed bilevel optimization model efficiently coordinates the economic operation both of the DISCO and MGs. The optimal energy exchange prices incentivize MGs to curtail peak load and power loss in the DN. RTP demand response in MGs can curtail and shift their peak load power demand efficiently. The stochasticity and volatility of renewable DGs is a challenge for MGs to supply stable electricity for DNs, therefore, further work will include stochasticity of renewable DGs, volatility of load power demand and environment factor in economic operation of DNs with MG integration.

REFERENCES


