Design of a New Nonlinear Fuzzy State Feedback Controller for HVDC Systems

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Abstract

This paper deals with design and stability of a new nonlinear fuzzy state feedback controller for HVDC systems. A novel simplified nonlinear dynamic model is developed for HVDC system that can be used to design the controller. The proposed nonlinear dynamic model decomposes into several linear systems around its important equilibrium points. These local linear models describe the plant dynamical behavior at its different operating points. The proposed controller is used in the control loops of the HVDC system. The simulation is carried out based on the Cigré benchmark model. Simulation results show improvement of overall AC/DC/AC system performance when severe faults occur, compared to the conventional controller. Also, the stable behavior of a very weak AC/DC system (SCR<2) with proposed controller is very significant when a sudden change in current order and/or a switched reduction in ESCR is applied. The same situation with conventional control brings the system into unstable region.

1 Introduction

The AC/DC interaction becomes more sensitive to disturbances as the effective short-circuit ratio (ESCR) of the AC system interface falls lower and lower [1], and the correct adjustment of control constants for good overall performance becomes much more difficult. To circumvent the above problem, extensive research has been carried out in the area of HVDC control. However, the advanced techniques available in DC adaptive control literature are difficult to apply in practical applications because of the absence of insight into performance with large disturbances. Also, the adaptive control may be not only ineffective but may degrade the performance rather than enhance it [2],[3]. A gain scheduling adaptive control strategy has been tried in [4] where the effect of large disturbances has been taken into account. In [5], the advantages of automatic continuous fine tuning are combined with predetermined gain scheduling in order to achieve robustness during large disturbances. A robust coordinated control scheme for a parallel AC/DC system is proposed in [6]. The paper describes the derivation and validation of a coordinated controller employing on-line identification of the AC/DC system. Fuzzy-logic-based tuning of the controller parameters for the rectifier side current regulator, and inverter side gamma controller in a HVDC system is introduced in [7-9]. In these, error signals and their derivatives are used as inputs to the fuzzy system, and give optimum system performance under various normal and abnormal conditions. To obtain good performance under various disturbance conditions, the fuzzy system parameters (number of fuzzy IF-THEN rule and their membership functions) need to be adjusted in a trial and error manner. In this paper, a different approach to designing a suitable controller for HVDC systems is first considered. As shown later, the new controller only uses output variables. Next, HVDC system modeling is presented which is based on the Cigré benchmark model. In third section, a suitable nonlinear dynamic model is developed for the HVDC system that can be decomposed into linear systems around its equilibrium points. Section four considers the method of robust nonlinear fuzzy control design using the Takagi-Sugeno fuzzy model. Stability conditions of both fuzzy models and fuzzy control systems are given. The simulation results are shown in section five. Finally, concluding remarks are drawn in sixth section.

2 HVDC System Modeling

To date, a wide variety of HVDC converter control strategies have been tested and optimized with the help of various digital programs. Great interest in HVDC-system simulation has led to the establishment of a Cigré benchmark model [10], [11] which is used here as a test system [11]. The selected short-circuit ratio (SCR) and the effective short-circuit ratio (ESCR) for the Cigré benchmark model are typical of a weak system. The combination of the weak inverter system, the DC-side resonance approaching fundamental frequency, and the AC-side resonance near the second harmonic make this system particularly onerous for DC control operation. The proposed dynamic model is derived from the basic system configuration shown in Figure (1). In this figure, the rectifier and inverter AC systems are represented by Thevenin AC equivalents, i.e. a constant AC voltage source behind a short-circuit impedance. This representation is most commonly assumed for the investigation of large-disturbance and small-disturbance voltage instability in weak AC/DC systems [12-14]. In [15] the time frame for model validity is given as approximately several hundred milliseconds.
2.1 Power and Voltage Equations for HVDC System

Based on the per-unit method presented in [16], and by selecting the nominal DC power \( P_{dN} \) and ideal open circuit DC voltage \( V_{d0N} \) as base power and base voltage, respectively, the following equations may be derived:

\[
\begin{align*}
V_{dr} &= \frac{a_r}{a_N} V_r \cos \alpha_r - \frac{3}{\pi} x_{cr} I_{dr} \\
V_{di} &= \frac{a_i}{a_N} V_i \cos \gamma_i - \frac{3}{\pi} x_{ci} I_{di} \\
P_{dr} &= \frac{a_r}{a_N} V_r I_{dr} \cos \alpha_r - \frac{3}{\pi} x_{cr} I_{dr}^2 \\
P_{di} &= \frac{a_i}{a_N} V_i I_{di} \cos \gamma_i - \frac{3}{\pi} x_{ci} I_{di}^2 \\
Q_{dr} &= \frac{3}{\pi} I_{dr} \sqrt{V_r^2 \sin^2 \alpha_r + T_r I_{dr} V_r \cos \alpha_r - x_{cr} I_{dr}^2} \\
Q_{di} &= \frac{3}{\pi} I_{di} \sqrt{V_i^2 \sin^2 \gamma_i + T_i I_{di} V_i \cos \gamma_i - x_{ci} I_{di}^2}
\end{align*}
\]

where \( a, x_c, \alpha, \gamma, V, V_i \) and \( I \) are converter transformer turns ratio, commutation reactance, firing angle, extinction angle, AC voltage, DC voltage and DC current, respectively. The subscripts \( r, i \) and \( N \) denote rectifier, inverter and nominal values, respectively.

Also, \( T_m = \frac{\pi^2 a_m^2}{2 \sqrt{3} 2 a_m N} \) and \( T_{mx} = \frac{2 \pi a_m}{3 \sqrt{2} 2 a_m N} \), in which \( m = r, i \).

2.2 Power Flow Equations at AC Buses

By assuming that \( Z_{rs} = z_{rs} \angle 90^\circ \) and \( Z_{is} = z_{is} \angle 90^\circ \), the power flow equations may be given by:

\[
\begin{align*}
\frac{V_r E_{r}}{z_{rs}} \sin \delta_r + P_{dr} &= 0 \\
\frac{V_i E_{i}}{z_{is}} \cos \delta_i - V_r^2 b_{cr} + Q_{dr} &= 0 \\
\frac{V_i E_{i}}{z_{is}} \sin \delta_i - P_{di} &= 0
\end{align*}
\]

\[
\frac{V_i^2}{z_{is}} - \frac{V_r E_{r}}{z_{rs}} \cos \delta_i + Q_{di} - V_i^2 b_{ci} = 0
\]

(10)

2.3 Converters Control System Equations

The proposed converters control-system block-diagrams are depicted schematically in Figure (2). The converter control-system equations (11) and (12) are derived from these diagrams.

\[
\begin{align*}
\dot{\alpha}_r &= \frac{K_r}{T_r} (I_{ord} - I_{dr} + I_{ur}) - \frac{1}{T_r} \alpha_r + \frac{\alpha_0}{T_r} \\
\dot{\beta}_i &= \frac{K_i}{T_i} (\gamma_{ref} - \gamma_i + \gamma_{di}) - \frac{1}{T_i} \beta_i + \frac{\beta_0}{T_i}
\end{align*}
\]

where \( I_{ord}, \gamma_{ref}, \gamma_i, \gamma_{di}, K, \beta_0 \) and \( T \) are current order, reference extinction angle, advance firing angle, current control signal, extinction angle control signal, gain and time constant, respectively.

2.4 DC Transmission Line Equations

The DC transmission line is represented by an equivalent \( T \) network with the lumped charging capacitance at the midpoint of the DC link, thereby dividing the series impedance into two parts. From this model equation (13–15) may be derived:

\[
\begin{align*}
I_{\omega} &= \frac{\omega_0}{X_r} V_r \cos \alpha_r - \frac{\omega_0}{X_i} V_i - \left( \frac{3 \omega_0 X_{cr}}{\pi X_r} + R \frac{\omega_0}{X_r} \right) I_{\omega} \\
i_{\omega} &= -\frac{\omega_0}{X_i} V_r \cos \gamma_i + \left( \frac{3 \omega_0 X_{ci}}{\pi X_i} + R \frac{\omega_0}{X_i} \right) i_{\omega} \\
V_r &= \omega_0 X_{\omega} - \omega_0 X_{i}\omega
\end{align*}
\]

where \( X = \frac{1}{\omega_0 C} \), \( X_r = \omega_0 L_r \) and \( X_i = \omega_0 L_i \) are appropriate reactances.

Fig. 1 Basic dynamic model
3 Dynamic System Model

In this section a mathematical model of the dynamic system in Figure (1) is derived. The DC transmission line differential equations and load flow equations result in the set of differential-algebraic (DA) equations (16).

\[ \dot{x} = f(x, y, \mu, u) = f(x, y, \mu) + h_f(x, y, \mu)u \]

\[ 0 = g(x, y, \mu) + h_g(x, y, \mu)u \]

where, \( x = [i_{dc}, i_{di}, v_c, \alpha_r, \beta_i]^T \)

\( y = [v_r, \delta_r, \delta_i, \gamma_i]^T, \mu \) and \( u \) are vectors of state variables, vector of algebraic variables, vectors of AC/DC system parameters and input vector, respectively. \( f \) and \( g \) are vectors of functions of these DA variables. The set of algebraic equations may be viewed as a manifold over which the dynamics of the differential system are constrained to occur [17]. Under equilibrium conditions, the system of equations (16) is described by:

\[ 0 = f(x_0, y_0, \mu_0, u_0), 0 = g(x_0, y_0, \mu_0) \]

where, \( (x_0, y_0, \mu_0) \) is an equilibrium or fixed point. An operating point is an equilibrium point. Hence at each operating point, the system may be described by a linearized form of equation (16). The general linear system model is given by (18).

\[ \Delta \dot{x} = f_s(x_0, y_0, \mu_0) \Delta x + f_y(x_0, y_0, \mu_0) \Delta y + h_f \Delta u \]

\[ 0 = g_s(x_0, y_0, \mu_0) \Delta x + g_y(x_0, y_0, \mu_0) \Delta y \]

where \( f_s, f_y, g_s, \) and \( g_y \) are the jacobian sub-matrices comprising partial derivatives of \( f \) and \( g \) with respect to \( x \) or \( y \), as indicated by the subscript label. Substituting \( \Delta y \) from the second equation of (18) into the first one gives equation (19).

\[ \Delta \dot{x} = [f_s - f_y g_s^{-1} g_y] \Delta x + h_f \Delta u \]

where \( A \) is the dynamic state matrix describing local dynamic behavior of the nonlinear system, as given by Equation (20), assuming that \( g_y \) remains nonsingular along system trajectories as the system parameters vary.

\[ A(x_0, y_0, \mu_0) = f_s - f_y g_y^{-1} g_s \]

The per-unit values of the proposed dynamic model are calculated based on the method presented in [16]. Then, the nonlinear system equation (17) is solved for different operating conditions and several operating points are obtained.

4 Nonlinear Fuzzy State Feedback Controller

In the proposed controller design, the nonlinear system model decomposes into linear system models in accordance with the cases for which linear models are suitable, and then individual linear models are aggregated into a single nonlinear model in terms of membership functions. This is a non-local approach which is conceptually simple and straightforward. The nominal model of a nonlinear system is:

\[ \dot{x}(t) = f(x(t)) + g(x(t))u(t) \]

\[ y(t) = Cx(t) \]

where \( x(t), u(t), y \) and \( C \) are vectors of state variables, vector of control inputs, vector of outputs and constant matrix, respectively. By considering the Takagi-Sugeno fuzzy model [18, 19], the in nonlinear system (21) can be modeled as the following fuzzy system:

**Plant Rule i**: \( \theta_i \) is \( \mu_{i1} \) and... and \( \theta_i \) is \( \mu_{ir} \)

**THEN** \( \dot{x}(t) = A_i x(t) + B_i u(t) \) \( y(t) = C x(t) \)

where \( \theta_i (j = 1, ..., p) \) are the premise variables, which are functions of state variables \( x, \mu_i (t = 1, ..., r) \) are fuzzy sets, \( r \) is the number of fuzzy rules, and \( p \) is the number of the premise variables. Fuzzy blending of each individual model yields the overall fuzzy model as follows [16]:

\[ \dot{x}(t) = \sum_{i=1}^{r} \alpha_i(\theta) [A_i x(t) + B_i u(t)] \]

\[ y(t) = \sum_{i=1}^{r} \alpha_i(\theta) \]

where \( \alpha_i(\theta) = \frac{\omega_i}{\sum_{j=1}^{r} \omega_j(\theta)} \) with constraints \( \alpha_i(\theta) \geq 0 \)

\[ \sum_{i=1}^{r} \alpha_i(\theta) = 1, \omega_i \ (i = 1, ..., r) \] are the membership function of the systems belonging to plant rule \( i \). The output equation is also added into the overall model for convenience, and it is assumed that all the triples \( (A_i, B_i, C) \), \( i = 1, ..., r \), are controllable and observable.

We utilize the concept of parallel distributed compensation to design fuzzy controllers to stabilize fuzzy system (26). The idea is to design a compensator for each rule of the fuzzy model. The resulting overall fuzzy controller, which is nonlinear in general, is a fuzzy blending of each individual linear controller. The fuzzy controller shares the same fuzzy sets with the fuzzy system (26).
Plant Rule $i$: IF $\theta_i$ is $\mu_\theta$ and...and $\theta_i$ is $\mu_\theta$ THEN $u(t) = -F_i x(t)$ (27)

where $i=1,2,...,r$. Hence, the fuzzy controller is

$$u(t) = -\sum_{i=1}^{r} \alpha_i(\theta) F_i x(t)$$ (28)

where $F_i (i=1,2,...,r)$ are constant matrices (state-feedback gain); such that the closed loop system (29) is asymptotically stable.

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \alpha_i(\theta) \alpha_j(\theta) (A_i - B_j F_j) x(t)$$ (29)

Theorem 1: The equilibrium of a fuzzy control (29) is asymptotically stable in the large if there exists a common positive definite matrix $P$ such that

$$\{A_i - B_j F_j\}^T P (A_i - B_j F_j) - P < 0, \quad i,j=1,2,...,r$$ (30)

The control design problem is to find the state-feedback gains $F_i (i=1,2,...,r)$ such that the closed loop system (29) is (quadratically) stable. This quadratic stabilizability problem can be recast as an LMI (Linear Matrix Inequality) problem. By defining a new variable $Q=P^{-1}$ and $K_i=Q^{-1} A_i F_j$, the quadratic stabilizability of the T-S fuzzy models via a linear state feedback can be cast as the following LMI problem in $Q$ and $K$:

$$\begin{bmatrix} Q & (A_i Q - B K_j)^T \\ (A_j Q - B K_i) & Q \end{bmatrix} < 0, \quad Q > 0, \quad (31)$$

with the state feedback gain $F_i = K_i Q^{-1}$.

5 Simulation

To investigate system performance with the proposed nonlinear fuzzy state-feedback controller, several simulations are performed. The design of HVDC controllers is strongly affected by the effective short circuit ratio (ESCR) at the converter stations. After optimizing these controllers for rated conditions of operation, the problem of operating them under different ESCR levels is critical, especially in the case of weak AC networks. In a weak AC system, and with an unsuitable control design, the AC/DC system may lose its stability under such disturbances. Such conditions can be created in two ways:

- During the operation of the system with a very weak AC network (very low ESCR) and/or operating the system with very small stability margin, changing one of the system reference values suddenly (e.g. step change of current order),
- Reducing the effective short-circuit ratio to a very low level by an AC network switching (switched reduction of ESCR).

In this study, for the switched reduction of ESCR, parallel AC transmission lines with switching capability are used. In order to obtain the different operating points, the nonlinear system equations (17) are solved at five different operating conditions. All of the required val-

ues, e.g. jacobian sub-matrices, dynamic state matrices, feedback matrices $F_i (i=1,2,...,5)$, are calculated off-line based on the previous section’s theories. There are many options to assign membership functions. For the sake of convenience in computation, triangular functions are selected as membership functions; some of which are depicted in Figure (3).

The block diagram of the proposed controller is shown in Figure (4). The $FSF$ block in Figure (4) is a fuzzy system that adjusts the $\alpha_1,...,\alpha_5$ constants based on the input state vector, $X$.

![Fig. 3 Some selected membership functions](image)

![Fig. 4 Block diagram of proposed robust nonlinear fuzzy controller](image)
posed controller. The existence of some oscillations in the waveforms is normal. This is due to operating the system with very weak AC systems. In this situation, the capability of system to retain a sinusoidal AC voltage is reduced and the harmonic distortion is increased.

6 Conclusions

The design and stability of a new nonlinear fuzzy state-feedback controller for HVDC systems has been studied in this paper. A new simplified nonlinear dynamic model has been developed for HVDC systems that can be used to design the controller. The proposed model decomposes into several linear systems around its important equilibrium points. Simulation results show that the transient stability can be improved by using proposed controller when large faults appear in the system. Also, the stable behavior of a very weak AC/DC system (SCR<2) with proposed controller is very significant when a sudden change in current order and/or a switched reduction in ESCR is applied. The same situation with conventional control brings the system into unstable region.

Fig. 5 System performance with conventional controllers during a 10% reduction of ESCR.

Fig. 6 System performance with proposed controller during a 10% reduction of ESCR.

References


