Implementation of closed loop signal shaping in a hydraulic system

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Abstract— The effectiveness of ZVD signal shaping in a closed loop for a hydraulic system is demonstrated. This is done by implementing a control loop first in a simple simulation of a single hydraulic actuator with a simple mass as loading. The control loop is then validated experimentally on a 2-link articulated hydraulic robot leg. Placing a ZVD filter inside a closed-loop proportional controller improves performance without the use of high-speed controllers or acceleration feedback which may be otherwise needed in order to achieve good position control performance of hydraulic actuators.

I. INTRODUCTION

Electric motors are often used with high gear-ratios which eliminate non-linear dynamics. This then allows the use of a simple PID (proportional + integral + derivative) controller to eliminate disturbance and control position. Hydraulic actuation is often desirable when greater power density is required but the non-linear behaviour of control valves cylinders can introduce resonant vibrations which limit dynamic performance [1], [2]. The work done in this paper forms part of a larger project towards controlling a hydraulically actuated robot leg [3]. The non-linear behaviour of the electro-hydraulic system means satisfactory position control performance of the foot isn’t achievable using a simple PD (proportional + derivative) controller in the air while hopping. During each flight phase of a hop, it is required that the foot be repositioned before the next touch-down without inducing vibrations.

Command shaping is a well established method for eliminating vibrations from a control system [4] because it is effective and easy to implement. The most common form this takes is open-loop input shaping. Input shaping is a control technique where the input reference signal to a system is modified in order to reduce oscillations in the response. Input shaping has been demonstrated to be useful in preventing oscillations when positioning cranes and flexible beams [5–7]. The application of shaping algorithms within a closed-loop has been labelled ‘closed loop signal shaping’ (CLSS). Placing a shaper within a closed loop controller allows for oscillations due to disturbances as well as demand changes to be removed. This paper presents simulation and experimental results demonstrating the implementation of CLSS to control hydraulic actuators. It is believed that in the context of electro-hydraulic control the application of closed-loop signal shaping is a novelty.

II. NOMENCLATURE

\[ a_\alpha \quad \text{Signal shaper gains.} \]
\[ \Delta t \quad \text{Signal shaper delay.} \]
\[ \zeta \quad \text{Damping ratio.} \]
\[ \omega_d \quad \text{Damped frequency.} \]
\[ \omega_n \quad \text{Natural frequency.} \]

III. BACKGROUND

A. ZV signal shaping

In many systems undesired transient oscillations may be induced when the input is varied. One solution is to use signal shaping algorithms which modify the command signal to reduce or remove transient oscillations. For second order systems below critical damping, the ZV (zero-vibration) signal shaper can remove transient vibrations fully.

The ZV shaper functions by delaying a portion of the command signal by half the period of the system’s oscillation frequency. The delayed part generates transient oscillations in anti-phase. The superposition of the responses to the immediate and delayed command signals results in the removal of transient oscillations. Figure 1 illustrates a signal shaper in block diagram form. For a simple ZV signal shaper only one delay is required so \[ a_\alpha = a_\Delta = 0 \]. For a second-order system the values \[ a_0, a_\zeta, \] and \[ \Delta t \] can then be determined through mathematical analysis [4] to be:
\[
a_0 = \frac{1}{1 + k}
\]
\[
a_1 = \frac{k}{1 + k}
\]
\[
\Delta t = \frac{\pi}{\omega_d}
\]
\[
k = e^{\left(\frac{\zeta \pi}{\sqrt{1 - \zeta^2}}\right)}
\]

As an example consider the second order response to an impulse as illustrated in Figure 2. By applying a second impulse of an appropriate magnitude and at an appropriate time, it is possible to remove all oscillations except half of the first. The ZV shaper would transform a single pulse into the two shown.

Signal shapers with multiple delays such as ZVD (2 delays) and ZVDD (3 delays) work similarly but can reduce vibrations over a greater bandwidth of frequencies at the cost of response speed due to the extra delays. Further details can be found in [4].

![ZV signal shaper block diagram.](image)

![Second order system response to impulses.](image)

**B. Hydraulic behaviour**

The relationship between the input current to a DC electric motor and the resulting positional change can be modeled as a second order system. Such a second order system can be represented by the following equation:

\[
a_0 = \frac{1}{1 + k}
\]
\[
a_1 = \frac{k}{1 + k}
\]
\[
G_1 = \frac{K \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

When a PD (proportional \(K_p\) + derivative \(K_d\)) controller is applied to this system the closed loop poles are given by the following characteristic equation:

\[
s^2 + (K_p K^2 + 2\zeta \omega_n) s + (K_p K^2 + \omega_n^2) = 0
\]

It can be seen that increasing the differential gain \(K_d\), the gain on the velocity error in the case of an electric motor, has a similar effect on the dynamics of the system as increasing the damping ratio \(\zeta\). A proportional valve controlled hydraulic cylinder can be approximately modeled as a 2nd order system between the input spool position and output cylinder velocity [1], [2]. In terms of position, the open loop transfer function would be given by:

\[
G_2 = \frac{K \omega_n^2}{s(s^2 + 2\zeta \omega_n s + \omega_n^2)}
\]

In this case, applying a PD control would result in the following characteristic equation:

\[
s^3 + 2\zeta \omega_n s^2 + (K_p K^2 + \omega_n^2) s + K_p K^2 = 0
\]

Whereas applying a proportional plus acceleration feedback \(K_a\) controller would give:

\[
s^3 + (K_a K^2 + 2\zeta \omega_n) s^2 + \omega_n^2 s + K_p K^2 = 0
\]

It can be seen that acceleration feedback applied to a hydraulic cylinder \(G_2\) is analogous to velocity feedback applied to an electric motor \(G_1\) because it has a similar effect to increasing the damping ratio in (9). Acceleration feedback allows proportional gain \(K_p\) to be increased for faster performance. Acceleration feedback however can be noisy to obtain through numerical differentiation or requires the use of accelerometers.

An alternative approach to feeding back acceleration in a servo-hydraulic system is outlined in this paper. Oscillations can be reduced through the application of a command shaping filter. The rest of the paper demonstrates through simulation and experimental validation how the addition of a simple filter allows higher proportional gains \(K_p\) without acceleration feedback being necessary.

![Hydraulic actuator model.](image)
IV. MODEL

This section details the hydraulic actuator model used to compare control techniques (Figure 3). The model, excluding the controller, consists of a set of equations which simulate 4 parts:

- Valve spool
- Valve orifices
- Piston
- Extension force

A. Valve spool

The valve spool adjusts its position in proportion to the control current which in turn is proportional to a signal voltage from the controller. The response of the valve to the control signal is modelled using the following second order equation:

\[ \ddot{x} + 2\zeta\omega_n\dot{x} + x = K_v V \omega_n^2 \]  
(10)

B. Valve orifices

Opening the valve results in flow through the valve orifices forced by pressure differences across the valve. This is modelled by the following set of equations:

For \( x \geq 0 \):

\[ Q_1 = K_v x \text{sign}(P_3 - P_1) \sqrt{|P_3 - P_1|} \]
\[ Q_2 = -K_v x \text{sign}(P_2 - P_R) \sqrt{|P_2 - P_R|} \]  
(11)

For \( x < 0 \):

\[ Q_1 = K_v x \text{sign}(P_1 - P_R) \sqrt{|P_1 - P_R|} \]
\[ Q_2 = -K_v x \text{sign}(P_2 - P_3) \sqrt{|P_2 - P_3|} \]

C. Piston model

Flow of hydraulic fluid results in pressure changes on the two ends of the piston. This is modelled using the following equations:

\[ Q_1 = A_1 \dot{y} + \frac{A_3 y + W_1}{\beta} \dot{p}_1 + C(P_1 - P_2) \]  
(12)

\[ Q_2 = -A_1 \dot{y} + \frac{A_2 \dot{L} - y + W_2}{\beta} \dot{p}_2 + C(P_2 - P_1) \]  
(13)

D. Extension force

The extension force generated by the actuator is the sum of the pressure and friction forces acting on the piston. This is given by:

\[ F = A_1 P_1 - A_2 P_2 - c_f \dot{y} \]  
(14)

E. Load

The force exerted by the piston will act to accelerate the piston and load mass giving:

\[ m\ddot{y} = F \]  
(15)

V. SIMULATION

A. Parameters

The model parameters used in simulation, listed in Table I, are selected to be close to those of the valve and cylinder used to actuate the experimental hydraulic system used for validation. A fixed refresh rate of 200 Hz was imposed on the input voltage to and output position from the model in order to simulate a digital controller running at 200 Hz.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>Piston area</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>Annulus area</td>
</tr>
<tr>
<td>( C )</td>
<td>Leakage coefficient</td>
</tr>
<tr>
<td>( c_f )</td>
<td>Friction coefficient</td>
</tr>
<tr>
<td>( K_i )</td>
<td>Spool gain</td>
</tr>
<tr>
<td>( K_v )</td>
<td>Valve coefficient</td>
</tr>
<tr>
<td>( L )</td>
<td>Full actuator stroke length</td>
</tr>
<tr>
<td>( m )</td>
<td>Load mass</td>
</tr>
<tr>
<td>( P_S )</td>
<td>Supply pressure</td>
</tr>
<tr>
<td>( P_R )</td>
<td>Return pressure</td>
</tr>
<tr>
<td>( W_1 )</td>
<td>Excess volume piston side</td>
</tr>
<tr>
<td>( W_2 )</td>
<td>Excess volume rod side</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Bulk modulus</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Spool damping ratio</td>
</tr>
<tr>
<td>( \omega_n )</td>
<td>Spool natural frequency</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>0 m</td>
</tr>
<tr>
<td>( \dot{x} )</td>
<td>0 m s^{-1}</td>
</tr>
<tr>
<td>( y )</td>
<td>0.04 m</td>
</tr>
<tr>
<td>( \dot{y} )</td>
<td>0 m s^{-1}</td>
</tr>
<tr>
<td>( P_1 )</td>
<td>2.9 MPa</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>4.8 MPa</td>
</tr>
</tbody>
</table>

B. Method

The model constructed in section IV can be represented by a block with: one input, the spool position control voltage \( V \); and one output, the actuator position \( y \). The simplest method of controlling the actuator position is using a proportional control loop, Figure 4. The performance of this is to be compared against a modified control loop in which the input voltage to the valves is shaped as shown in Figure 5. By placing the signal shaper inside the closed loop where it acts
on the error signal as shown, it will act to remove oscillations caused by disturbances as well as by changes in demand.

Implementing the ZV shaper or its variants requires the natural frequency and damping ratio of the system as parameters. These can be obtained by giving a step input in voltage in an open loop to the hydraulic system and analyzing the oscillations in the velocity response \( \dot{y} \) to get a value for natural frequency and damping ratio. Before giving the open loop step input in voltage, closed loop proportional control is used to bring the actuator to a starting position at which it is held for a few seconds in order to allow transient motion to die down.

**C. Results**

The simulation results of implementing proportional control with and without a ZVD filter in the loop are shown in Figure 6 with the initial conditions listed in Table II. Based on the results in Figure 6(a), a natural frequency of 22.3 Hz and damping ratio of 0.1 were determined and used for the signal shaper parameters in (b). It can be clearly seen that the signal shaping filter reduces oscillations allowing for higher gains. The reduced response speed due to use of the filter is compensated by the ability to increase gain.

**VI. EXPERIMENTAL VALIDATION**

Figure 7 shows the system used to experimentally validate the effectiveness of the closed loop signal shaping control loop in a hydraulic context. The apparatus is a 2-DoF articulated robot leg consisting of two links with pivotal joints actuated by double acting hydraulic cylinders controlled by proportional valves. Hydraulic fluid is supplied to the leg by a 5 lpm pump with a relief valve limiting the supply pressure to 16 MPa. The actuators are similar to those modeled for simulation experiments. Sensors include position encoders at the joints.

**A. Method**

The leg is lifted off the ground for all experiments in this paper. Both actuators are commanded to hold a starting position using simple proportional controllers operating at 200 Hz (0.023 m for the upper actuator and 0.057 m). An offset voltage is also applied to correct for steady state position error. From a steady state, the controller is switched off and the signal voltage to the upper actuator is stepped up by 2 V whilst the lower actuator signal voltage remains unchanged. The velocity of the upper actuator is recorded and analyzed to obtain the natural frequency and damping ratio of the oscillations, parameters required for ZVD shaping. The same process is then conducted for the lower actuator, with the upper actuator signal remaining steady, to obtain a second set of natural frequency and damping ratio parameters.
The velocity response frequency and damping ratio results from step voltage inputs are shown in Table III. These values were used to implement the ZVD filters in each actuator’s position control loop. The position response of the actuators to step changes in demand position of 0.005 m is compared in Figure 8 for different proportional gain values $K_p$. A step is
given to the upper actuator while keeping the demand to the lower one steady and vice versa.

It can be clearly seen that for the upper actuator 1 closed-loop ZVD (b) performs better at following the demand by removing oscillations than simple proportional control (a). Without the ZVD filter (a), actuator 1 oscillates at approximately 6.3 Hz. The ZVD filter, tuned to 7.6 Hz, does not exhibit these oscillations.

The lower actuator with proportional control only (c) appears to exhibit oscillations at two separate frequencies at least. There are oscillations at 27 Hz which the ZVD control loop (d) tuned to 20 Hz seems to remove. Oscillations also occur at a frequency of 6.3 Hz for actuator 2 (c). The fact that 6.3 Hz oscillations are present in the response of actuator 2 is likely due to the coupling in the dynamics of the two links of the leg. A potential solution would be to implement a signal shaping filter tuned to remove multiple frequencies.

Based on these experiments alone, it is clear that CLSS has improved the control of the upper actuator which experiences higher inertial loading than the lower actuator. Whether CLSS has improved the performance of the lower actuator is arguable. High frequency oscillations have been removed and this may contribute to greater stability. Oscillations at 6.3 Hz remain unaffected. Additionally, the filter seems to result in a greater overshoot likely due to delays inherent in ZVD.

VII. CONCLUSION

The results of the simulation experiments and the validation experiments demonstrate that the relatively simple addition of a ZVD signal shaping filter into a closed-loop proportional controller can lead to a number of benefits including the following:

- Destabilizing oscillations can be removed allowing for increased servo-hydraulic performance and stability.
- The filter is in a closed loop controller so oscillations due to disturbances are also rejected.
- Implementing the filter only required the frequency and damping response of the system rather than a detailed model.
- The controller runs at a relatively low rate of 200 Hz and only required position feedback. Alternative methods to improving hydraulic position control would require a higher tick rate and/or acceleration feedback. This would increase the cost and complexity of the system.

CLSS potentially offers a relatively easy to method for improving the performance of hydraulic actuators without requiring additional sensors. For this reason, further work is ongoing to:

- Assess robustness when subjected to different loads and bad tuning.
- Compare against existing common control techniques including first order lag, notch filters as well as acceleration feedback.

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REFERENCES