GenCo's Integrated Trading Decision Making to Manage Multimarket Uncertainties

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Abstract—Fossil fuel GenCos trade in multiple uncertain energy markets: fuel and carbon markets on upstream side while electricity market on downstream side. Global economic and environmental benefits lead these markets to pursue overlapping goals, making them highly interactive. GenCos may identify optimal trading strategies for upstream and downstream trading in an integrated framework, to manage an overall secure and profitable position. Further, severe unpredictability of energy market prices may necessitate a GenCo to make trading plans which perform better meeting its goals. Under severe uncertainty of involved markets, this paper proposes Information Gap Decision Theory (IGDT) based approach to select three interrelated trading portfolios, in an integrated framework. Results from a realistic case study provide a comprehensive decision insight to address risk-averse and risk-seeking behavior of GenCo, explicitly highlighting importance of co-variation in prices of interactive markets.

Index Terms—information gap decision theory, portfolio optimization, fuel, emission permit, pool, congestion uncertainty.

I. NOMENCLATURE

A. Indices
i Index of the trading interval
l,m Index of the trading location

B. Parameters
\( a, b, c \) Quadratic, Linear and No-load heat-rate coefficients of generator
\( e_f \) Emission factor in tCO2/MBtu
n Number of locations
t Time for each trading interval in hours
I Considered planning period
\( CO_{2\text{Min}} \), \( CO_{2\text{Max}} \) Minimum and maximum trading limit on contracts for emission in tCO2
\( C_i \) Uncertainty Shape matrix during \( i^{th} \) trading interval
\( \text{Fuel}_{\text{Min}} \), \( \text{Fuel}_{\text{Max}} \) Minimum and maximum trading limit on contracts for fuel in MBtu
\( p^B_{\text{Min}}, p^B_{\text{Max}} \) Minimum and maximum trading limit for electricity bilateral contract with consumer of \( i^{th} \) zone in MWh
\( p_i^G \) Total electricity generation during \( i^{th} \) trading interval in MWh

\( \lambda_i \) Set of uncertain prices in different markets for \( i^{th} \) trading interval
\( \lambda_i^S \) Electricity price of \( i^{th} \) trading zone, during \( i^{th} \) trading interval, in €/MWh
\( \lambda_i^B \) Effective price for electricity bilateral contract with consumer of \( i^{th} \) zone, during \( i^{th} \) trading interval, in €/MWh
\( \lambda_i^C \) Bilaterally agreed price for electricity bilateral contract with consumer of \( i^{th} \) zone, during \( i^{th} \) trading interval, in €/MWh
\( \lambda_i^{F,B} \), \( \lambda_i^{F,S} \) Fuel price for contract and spot market, during \( i^{th} \) trading interval, in €/MBtu
\( \lambda_i^{E,B} \), \( \lambda_i^{E,S} \) Emission permit price for contract and spot market during \( i^{th} \) trading interval in €/tCO2
\( \pi_c \) Critical profit target for robustness function
\( \pi_w \) Windfall profit target for opportuneness function
\( \mu \) Lagrange Coefficient
\( \phi(p) \) Fossil fuel generation heat rate for generating power, in MBtu/h
\( \gamma \) Congestion charge factor, varying from 0 to 1

C. Decision Variables
\( CO_{2j} \) Quantum of emission for which permits are traded through contracts and spot market during \( j^{th} \) trading interval in tCO2
\( CO_{2s} \) Quantum of fuel traded in contracts and spot market, during \( s^{th} \) trading interval, in MBtu
\( \text{Fuel}^B_i \), \( \text{Fuel}^S_i \) Electricity traded in spot market during \( i^{th} \) trading interval in MWh
\( p^S_i \) Electricity traded in bilateral contract with consumer of \( i^{th} \) zone, during \( i^{th} \) trading interval, in MWh
\( Q \) Set of traded quantum of different commodities in various trading options
\( a \) Uncertainty parameter or horizon of uncertainty
\( u_{ij}, v_i \) Binary variables representing selection state of contracts for electricity, emission and fuel, during \( i^{th} \) trading interval

D. Functions
\( U(\alpha, \lambda) \) Uncertainty function
\( \alpha(Q, \pi_c) \) Robustness function
\( \beta(Q, \pi_w) \) Opportuneness function
\( \pi(Q, \lambda) \) Profit function for decision variable \( Q \) and
II. INTRODUCTION

In power sector, fossil fuel fired GenCos are dominant electricity producers and key contributors to emission. Under existing carbon limiting policies, they require securing emission permits in addition to fuel, to produce electricity. GenCos procure these production resources from emission and fuel markets and sell generated outcome in electricity markets [1]. These three markets are interrelated and GenCos manage trading in these interactive markets [2]-[3].

Growing volatility and competitiveness in energy markets force GenCos to strategically plan their trading, to maximize profit. They may go for derivative instruments like spark-spread contacts, to hedge the risk of volatile prices, but market of such contracts is limited, requires additional payment and restricts opportunities for higher profit. Portfolio optimization helps GenCos to identify their optimal hedging strategies, considering their profit-risk trade-off and risk preference [4]-[7]. Portfolio optimization approach can help selecting optimal generation mix and in trading decision making, to deal with various uncertainties such as pool market [4]-[5], transmission congestion charges, environmental compliance costs and fuel prices [6]-[7]. Portfolio diversification in electricity market is affected by external market uncertainties [6]-[7]. Uncertain prices of different market commodities have a mutual effect on prices. GenCo’s combined trading decision making problem does consider the mutual effect of different involved markets, albeit without involved uncertainty [3]. However, while identifying optimal trading strategies to secure profit in a true sense, mutual effect of market uncertainties needs to be considered.

Trading decisions are planned far ahead of real time, relying on estimates/forecasts of market prices based on historical data. For such medium term planning, price forecasting is a complex task due to long forecasting horizon [8]. Prices of fuel, electricity and carbon markets may severely differ from forecasted/estimated ones, as these are affected by several unpredictable real time factors like weather, policy and supply demand forces [9]. The traditional uncertainty based decision making approaches, such as mean-variance theory, stochastic programming or fuzzy theory, depend upon forecast and use probability distribution or membership function for parameter estimation, to obtain individual’s optimal choice [4]-[7]. However, with gap between estimated and true parameters, treating estimation as a true value may lead to imprudent decisions.

Information Gap Decision Theory (IGDT) quantifies information gap between forecasted and actual values of parameter of interest and makes necessary assumptions for the structure of uncertainty, to provide strategies which are robust against losses and opportunistic to windfall benefits, without sacrificing performance requirements [10]. In addition to existing approaches, IGDT considers opportunistic behavior to benefit from favorable situations. This theory has recently been adopted in electricity market as an attractive option to solve a variety of market issues, viz. electricity bidding and scheduling of large consumers and GenCos [11]-[12].

Considering interactive nature of fuel, emission and electricity markets and weak ability of precise price forecasts, this paper proposes an integrated portfolio selection approach for a GenCo, based on IGDT framework, involving uncertainties of upstream and downstream trading sides, along with their inter-dependencies. Price uncertainties of congestion charges, electricity, fuel and emission permit markets have been modeled using ellipsoid bound info-gap uncertainty model. The work highlights importance of correlation between different uncertain trades in decision making. Results from a practical case study illustrate that selected portfolios of three involved markets provides wide range of decisions which are robust towards losses and capable to capture windfall gains.

III. GENCO’S INVOLVEMENT IN MULTIPLE MARKETS

Despite the presence of other generation types, fossil fuel GenCos generally govern electricity market prices. They are involved in two trading sides: procurement of production resources from fuel and carbon markets, and selling their production outcome in electricity markets. In competitive markets, prices of production cost in ‘upstream’ (i.e. fuel, carbon) and revenue in ‘downstream’ (i.e. electricity) markets are uncertain, and GenCos have to manage the risks associated with each.

GenCos fulfill their fuel requirements primarily through certain fuel contracts and remaining through spot market purchases. They decide the proportion of required fuel to be procure from either option, to secure minimum fuel cost and optimize their fuel portfolio [7].

World over, current climate policy proposals involve ‘cap and trade’ mechanism, with increasingly tight caps on carbon emission. Among them, European Union Emission Trading Scheme (EUETS) is the largest multi-national greenhouse gas emission trading scheme [13]. With continuously increasing stress on emission reduction, upcoming phase of the scheme from 2013 puts an end to free allocation of emission allowances and shifts to full auction mechanism for the power industry [13, 14]. This would boost demand for emission permits and consequently increase volatility in their prices [15]. GenCos have to procure required emission permits from carbon market via contracts and spot trading [14]. Thus, a GenCo has to additionally consider uncertainty of carbon market and optimize its emission portfolio.

In electricity trading, GenCos are mostly affected by price fluctuations caused by pool and transmission congestion [1], [5]. Under considered zonal pricing system, prices of all zones are uniform during normal operating conditions. Congestion causes the power system to split into two separate pricing areas connected by congested lines and each area having its own MCP, called zonal price. GenCos selling electricity through contracts with different pricing areas face price volatility of bilateral contracts if affected by congestion, in addition to pool price uncertainties.

Given the size of power sector in carbon markets and its dependence on fossil fuel generation, prices of three markets represent strong correlations [2]. Volatility and correlation of
energy and power markets has secured little attention, despite their significance for portfolio selection. In present competitive scenario, it is prudent to coordinate trading decisions for all three interactive portfolios in an integrated framework [16]. Under existing competitive and interrelated market scenario, influenced by physical and environmental constraints, the three markets and trading options comprehensively reflect a general trading decision making problem of GenCo.

IV. PROBLEM DESCRIPTION AND FORMULATION

A considered price taker fossil-fuel based GenCo wishes to secure maximum net profit from trading in all involved markets, by coordinating three portfolios of interrelated markets over a specified time period. A presumed generation considering operational, fuel and emission constraints is allocated to spot market and bilateral contracts of various zones. GenCos’ involvement in a larger variety of trading contracts and markets can be modelled as an extension to the model proposed here. The required fuel and emission permits are procured from their respective spot markets and bilateral contracts. For this medium-term planning, it is assumed that markets are completely efficient and competitive.

A. Cost from Fuel and Carbon Markets

Cost of electricity generation is generally calculated based on fuel usage of plant and expressed in terms of plant heat rate in MBtu/hr, as

$$\phi(p) = a p^2 + b p + c$$

(1)

With the introduction of emission trading schemes, emission cost is considered as a component of generation cost. Quantum of emission depends upon quantum of fuel consumed and the two can be calculated using generation heat rate equation for \(p_i\) generation, at \(i^{th}\) trading interval, as

$$Fuel_i = t \phi(p_i)$$

(2)

$$CO_{2i} = t e_f \phi(p_i)$$

(3)

where \(p_i = P_i^f / t\). Emission factor \(e_f\) calculates the quantum of CO2 emission for certain fuel type and plant design parameters for per unit heat rate [17]. A single unit heat rate curve is assumed for single fuel type plant, maintaining the focus on trading price uncertainty of multiple markets.

GenCos’ required fuel \(Fuel_i\) and emission permits \(CO_{2i}\) are purchased from contracts as \(Fuel_i^b\), \(CO_{2i}^b\) and from spot market as \(Fuel_i^s\), \(CO_{2i}^s\). Total fuel cost (FC) and emission cost (EC) for purchasing fuel and emission permits from contracts at prices \(\lambda_i^F,B\), \(\lambda_i^{E,B}\) and spot trading at market clearing prices \(\lambda_i^{F,S}, \lambda_i^{E,S}\) respectively, can be expressed as

$$FC = \sum_{i=1}^{I} Fuel_i^b \lambda_i^{F,B} + \sum_{i=1}^{I} Fuel_i^s \lambda_i^{F,S}$$

(4)

$$EC = \sum_{i=1}^{I} CO_{2i}^b \lambda_i^{E,B} + \sum_{i=1}^{I} CO_{2i}^s \lambda_i^{E,S}$$

(5)

B. Revenue from Electricity Market

For considered \(n\) zones, GenCo located at zone \((l=1)\), can have three types of electricity trading contracts under considered zonal pricing mechanism: i) bilateral contract within same zone ii) bilateral contract with other zone \((l=2\sim n)\) and iii) spot market contract. Where \(l\) is area index. Considering a single spot market and only one bilateral contract with consumer of a certain zone, revenue from spot market \(R^S\) and bilateral contracts \(R^B\) for respective traded quantity \(P^S_l, P^B_{ij}\) for planning period \(I\) is

$$R^S = \sum_{i=1}^{I} P^S_i \lambda_i^S$$

(6)

$$R^B = \sum_{i=1}^{I} \sum_{j=1}^{I} P^B_{ij} \lambda_{ij}^B$$

(7)

where \(\lambda_{ij}^B\) represents zonal prices of area \(l\). For spot market trading, GenCo would receive prices of its own area as spot market price. Difference between prices of two zones (where generator and load are connected), are applicable congestion charges for underlying contract, which are fully or partly paid by supplier based on \(\gamma (0 \leq \gamma \leq 1)\), that depend upon market rule. So, effective bilateral contract prices \(\lambda_{ij}^C\) for zone \(l\) at \(i^{th}\) trading interval are

$$\lambda_{ij}^C = \lambda_{ij}^S - \gamma (\lambda_{ij}^S - \lambda_{ij}^B)$$

(8)

For intra-zonal trading, GenCo pays bilaterally agreed contract price, assuming intra-zonal congestion to be negligible.

$$\lambda_{ij}^C = \lambda_{ij}^S$$

(9)

C. Total Profit

Net profit \(\pi\), of GenCo is calculated as the difference of total revenue generated and involved production cost, as

$$\pi = R^S + R^B – FC – EC$$

(10)

$$\pi = \sum_{i=1}^{I} \lambda_i^S P_i^S + \sum_{i=1}^{I} \sum_{j=1}^{I} \lambda_{ij}^B P_{ij} + \sum_{i=1}^{I} \left( Fuel_i^B \lambda_{ij}^{F,B} + Fuel_i^S \lambda_i^{F,S} \right) + CO_{2i}^B \lambda_{ij}^{E,B} + \sum_{i=1}^{I} \left( CO_{2i}^S \lambda_i^{E,S} \right)$$

(11)

All spot market prices and bilateral contract prices of different zone \(\lambda_{ij}^B\) \((l=2\sim n)\) are not known during planning. This work concentrates on securing optimal trading position of a GenCo in all involved markets, with the given price information for emission permits, fuel and electricity. The problem has been formulated under IGDT framework considering severe price uncertainty of different trades.

V. IGDT BASED DECISION MAKING

IGDT quantifies size of unknown gap between nominal estimates and true value of parameter of interest, with a free uncertainty parameter \(\alpha\), for decision making. This evaluates decisions based on specified performance requirements, i.e. doing well enough in worst case, for robustness to failure and allowing minimum error to achieve windfall profit, for opportunity of windfall gains [10].
A. Uncertain Parameters

Spot trading prices of electricity, fuel and carbon markets and inter-zonal bilateral contract prices in electricity market are uncertain. All these prices depend upon real time conditions and are uncertain input parameters for considered problem. True value of these uncertain parameters may vary from nominal estimate $\lambda$ with an error $\Delta \lambda$.

\[ \lambda_{ij}^S = \lambda_{ij} + \Delta \lambda_{ij}^S \quad \forall i \]  
(12)

\[ \lambda_{ij}^B = \lambda_{ij} + \Delta \lambda_{ij}^B \quad (l = 2 \sim n) \quad \forall i \]  
(13)

\[ \lambda_{ij}^{F,S} = \lambda_{ij}^{F} + \Delta \lambda_{ij}^{F,S} \quad \forall i \]  
(14)

\[ \lambda_{ij}^{E,FS} = \lambda_{ij}^{E,S} + \Delta \lambda_{ij}^{E,FS} \quad \forall i \]  
(15)

These uncertain prices of different markets are considered as a set, $\lambda = [\lambda_{ij}^S, \lambda_{ij}^B, \lambda_{ij}^{F,S}, \lambda_{ij}^{E,FS}]$ \quad \forall i \]  
(16)

\[ \lambda = \lambda_{ij} + \Delta \lambda_{ij} \]  
(17)

where

\[ \lambda_{ij} = [\lambda_{ij}^S, \lambda_{ij}^B, \lambda_{ij}^{F,S}, \lambda_{ij}^{E,FS}] \quad \forall i \]  
(18)

\[ \Delta \lambda_{ij} = [\Delta \lambda_{ij}^S, \Delta \lambda_{ij}^B, \Delta \lambda_{ij}^{F,S}, \Delta \lambda_{ij}^{E,FS}] \quad \forall i \]  
(19)

B. Decision Variable

Traded quantity via different contracts is the strategy or trading decision for the GenCo. On downstream electricity market, power traded in various uncertain electricity market contracts and on upstream side, quantum of fuel supply and emission permits purchased from their respective spot markets are decision variables of the problem. Upstream variables are considered with a negative sign signifying purchase. All these variables are decided in an integrated way, thus are represented as a single set.

\[ Q = \begin{bmatrix} P^S_{i,j} & P^B_{i,j} & -Fuel^S_{i} & -CO^S_{i} \end{bmatrix} \quad \forall i \]  
(20)

IGDT evaluates decisions at many points, as uncertainty varies from estimation in an unbounded manner and compares different trading decisions satisfying system performance criteria. Three components needed for an info-gap analysis are: i) System model ii) Uncertainty model and iii) Performance requirements.

C. System Model

System model is the objective function for which the decision is applied. GenCo wishes to maximize profit (11) based on allocation in available trading alternatives of different markets, which can be rewritten in terms of $\lambda_{ij}$ (16) and $Q_i$ (20) as

\[ \pi(Q, \lambda) = \sum_{i=1}^{l} \left( Q_i^T \lambda_{ij} + \Delta \lambda_{ij}^T P_{i,j}^B - \lambda_{ij}^{F,B} Fuel_{i} - \lambda_{ij}^{E,B} CO_{i} \right) \]  
(21)

Using (17), it can be written as

\[ \pi(Q, \lambda) = \sum_{i=1}^{l} \left( Q_i^T \lambda_{ij} + \Delta \lambda_{ij}^T P_{i,j}^B \right) - \lambda_{ij}^{F,B} Fuel_{i} - \lambda_{ij}^{E,B} CO_{i} \]  
(22)

\[ \Rightarrow \pi(Q, \lambda) = \sum_{i=1}^{l} Q_i^T \lambda_{ij} + \Delta \lambda_{ij}^T P_{i,j}^B \]  
(23)

It is to be noted here that purchase from contracts in case of fuel and carbon markets and selling electricity via intra-zone bilateral contract are considered deterministic and known at the time of decision making.

D. Uncertainty model

Uncertainty model consists of nominal values of unknowns and a horizon of uncertainty $\alpha$. It is defined to best represent uncertainty, depending upon the information available. Uncertainty in parameter of interest is modeled by minor assumptions on the uncertainty structure [8]. Historical data provides the estimated prices, individual uncertainties and correlation between prices of uncertain trades. Considering that, uncertainty in all trades is price uncertainty, a single uncertainty horizon $\alpha$ is used to handle it. The distinguished information associated with each uncertain trade and their correlations have been modelled with Ellipsoid Bound Info-gap Model. The considered uncertainty model formulates uncertainty in prices of different trades $\lambda_i$, as an unbounded family of nested sets $\Upsilon$, nested by uncertainty parameter $\alpha$, around estimate $\lambda_i$. This model represents that all $\lambda_i$, with possible deviations $\Delta \lambda_i$ in estimated prices $\lambda_i$, would lie within the region defined by $\Upsilon$, for a particular $\alpha$, and is mathematically represented as

\[ \Upsilon(\alpha, \lambda_i) = \{ \lambda_i : \lambda_i + \Delta \lambda_i : \Delta \lambda_i^T C_i^{-1} \Delta \lambda_i \leq \alpha^2 \}, \alpha \geq 0 \quad \forall i \]  
(24)

Here, $^T$ represents transpose and $C_i$ denotes uncertainty shape matrix, which is symmetric and positive definite. It represents the degree of variability and co-variability between prices of different markets and is shown as

\[ C_i = \begin{bmatrix} \text{Var}(\lambda_{ij}^S) & \text{Cov}(\lambda_{ij}^S, \lambda_{ij}^B) & \text{Cov}(\lambda_{ij}^S, \lambda_{ij}^{F,B}) & \text{Cov}(\lambda_{ij}^S, \lambda_{ij}^{E,B}) \\ \text{Cov}(\lambda_{ij}^B, \lambda_{ij}^S) & \text{Var}(\lambda_{ij}^B) & \text{Cov}(\lambda_{ij}^B, \lambda_{ij}^{F,B}) & \text{Cov}(\lambda_{ij}^B, \lambda_{ij}^{E,B}) \\ \text{Cov}(\lambda_{ij}^{F,B}, \lambda_{ij}^S) & \text{Cov}(\lambda_{ij}^{F,B}, \lambda_{ij}^B) & \text{Var}(\lambda_{ij}^{F,B}) & \text{Cov}(\lambda_{ij}^{F,B}, \lambda_{ij}^{E,B}) \\ \text{Cov}(\lambda_{ij}^{E,B}, \lambda_{ij}^S) & \text{Cov}(\lambda_{ij}^{E,B}, \lambda_{ij}^B) & \text{Cov}(\lambda_{ij}^{E,B}, \lambda_{ij}^{F,B}) & \text{Var}(\lambda_{ij}^{E,B}) \end{bmatrix} \]  
(25)

where $l,m$ indexes of two different areas address prices of two different inter-zonal contracts. Matrix elements can be calculated from historical prices of different markets by statistical calculations for each trading interval. Diagonal elements represent variability by variance while off-diagonal elements represent co-variability by covariance between different contract prices.

E. Performance Requirements

Performance requirements for selecting a decision are evaluated on the basis of robustness and opportunity functions [8]. Decision maker’s anticipation from uncertain market prices varies and it considers both pernicious and propitious faces of uncertainty. A robustness function guarantees a certain profit expectation under adverse future conditions that deviate from the best estimate. Info-gap also examines beneficial opportunity arising out of uncertainty, to obtain windfall profit. Both functions optimize uncertainty parameter $\alpha$ such as

\[ \Rightarrow \pi(Q, \lambda) = \sum_{i=1}^{l} Q_i^T \lambda_{ij} + \Delta \lambda_{ij}^T P_{i,j}^B \]
\[ \alpha(Q, \pi_c) = \max \{ \alpha : \min_{\pi(Q, \lambda)} > \pi_c \} \]

(26)

\[ \beta(Q, \pi_w) = \min \{ \alpha : \max_{\pi(Q, \lambda)} > \pi_w \} \]

(27)

Robustness function \( \alpha(Q, \lambda) \) expresses the maximum level of uncertainty (\( \alpha \)) at which critical performance \( \pi_c \) must be achieved. Robustness represents immunity against losses, thus a large value is desirable. It addresses conservative nature of decision maker and expresses the level of protection for the selected decision under unfavorable price movement.

Opportunity function models the risk seeking nature of decision maker to benefit from opportunity arising out of favorable change in market prices. Opportuneness function \( \beta(Q, \lambda) \) represents the minimum uncertainty which has to be tolerated to enable the possibility of windfall gains as large as \( \pi_w \). This is immunity against windfall benefit. Thus, a small value is desirable.

1) Robustness Function

Robustness of portfolio selection strategy \( Q \) to achieve critical profit \( \pi_c \) is the largest value of uncertainty parameter \( \alpha \), such that any price within the region \( U(\alpha, \lambda_i) \) would give profit \( \pi(Q, \lambda) \) which is at least \( \pi_c \). For performance requirement (26) to be satisfied for all \( \lambda_i \in U(\alpha, \lambda_i) \), minimum profit for GenCo would be

\[ \min_{\pi(Q, \lambda)} = \sum_{i=1}^{I} \left( Q_i \lambda_i^T + \lambda_{\pi} R_i^T C_i \right) + \min_{\lambda} \sum_{i=1}^{I} Q_i \Delta \lambda_i^T \]

s.t. \( \Delta \lambda_i C_i^{-1} \Delta \lambda_i \leq \alpha^2 \)

(28)

Applying Lagrange Relaxation method to the convex optimization problem would give first order optimality condition as

\[ V_{\lambda, \mu} \left( Q_i \lambda_i^T + \mu \left( \alpha^2 - \Delta \lambda_i C_i^{-1} \Delta \lambda_i \right) \right) = 0 \]

where \( \mu \) is Lagrange multiplier. Taking derivatives as

\[ \left( Q_i^T - 2 \mu C_i^{-1} \Delta \lambda_i, \alpha^2 - \Delta \lambda_i C_i^{-1} \Delta \lambda_i \right) = (0, 0) \]

(30)

Hence,

\[ \Delta \lambda_i^T = \frac{Q_i^T C_i}{2 \mu} \text{ and } \alpha^2 = \Delta \lambda_i^T C_i^{-1} \Delta \lambda_i \]

(31)

Considering \( C_i \) as symmetrical matrix, after substituting value of \( \Delta \lambda_i \)

\[ \alpha^2 = \frac{Q_i^T C_i}{2 \mu} \frac{Q_i C_i}{4 \mu^2} = \frac{1}{4 \mu^2} Q_i^T C_i Q_i \]

(32)

\[ \Rightarrow \frac{1}{2 \mu} = \pm \frac{\alpha}{\sqrt{Q_i^T C_i Q_i}} \]

(33)

\[ \Rightarrow \Delta \lambda_i = \pm \alpha \frac{Q_i C_i}{\sqrt{Q_i^T C_i Q_i}} \]

(34)

\[ \Rightarrow Q_i \Delta \lambda_i^T = \pm \alpha \sqrt{Q_i^T C_i Q_i} \]

(35)

Selecting a negative value to attain minimum profit, (28) can be written as

\[ \min_{\pi(Q, \lambda)} = \sum_{i=1}^{I} \left( Q_i \lambda_i^T + \lambda_{\pi} R_i^T C_i \right) - \alpha \sum_{i=1}^{I} \sqrt{Q_i^T C_i Q_i} \]

(37)

From (26), minimum profit should be at least equal to \( \pi_c \), so

\[ \sum_{i=1}^{I} \left( Q_i \lambda_i^T + \lambda_{\pi} R_i^T C_i \right) - \alpha \sum_{i=1}^{I} \sqrt{Q_i^T C_i Q_i} = \pi_c \]

\[ \Rightarrow \alpha(\pi_c) = \sum_{i=1}^{I} \left( Q_i \lambda_i^T + \lambda_{\pi} R_i^T C_i \right) - \pi_c \sum_{i=1}^{I} \sqrt{Q_i^T C_i Q_i} \]

(38)

For critical profit \( \pi_c \), largest value of \( \alpha \) is robustness

\[ \alpha(Q, \pi_c) = \max_{\alpha} \alpha(\pi_c) \]

\[ = \max_{\alpha} \sum_{i=1}^{I} \left( Q_i \lambda_i^T + \lambda_{\pi} R_i^T C_i \right) - \pi_c \sum_{i=1}^{I} \sqrt{Q_i^T C_i Q_i} \]

(40)

2) Opportunity Function

An optimistic decision maker, positively anticipated about market, wishes to benefit from favorable price movements. It has to bear certain uncertainty to enable this possibility. Opportunity \( \beta(Q, \pi_c) \) is the least level of uncertainty which must be tolerated in order to enable the possibility of attaining profit as large as \( \pi_w \). Maximum possible profit up to uncertainty \( \alpha \), when all \( \lambda_i \in U(\alpha, \lambda_i) \), subject to (29) for \( \alpha > 0 \), can be calculated using Lagrange method, same as in case of robustness, considering positive value of \( Q_i \Delta \lambda_i^T \) from (36), as

\[ \max_{\pi(Q, \lambda)} = \sum_{i=1}^{I} \left( Q_i \lambda_i^T + \lambda_{\pi} R_i^T C_i \right) + \alpha \sum_{i=1}^{I} \sqrt{Q_i^T C_i Q_i} \]

(41)

Opportuneness function is obtained by equating maximum profit to windfall profit \( \pi_w \) as

\[ \sum_{i=1}^{I} \left( Q_i \lambda_i^T + \lambda_{\pi} R_i^T C_i \right) + \alpha \sum_{i=1}^{I} \sqrt{Q_i^T C_i Q_i} = \pi_w \]

(42)

Which gives

\[ \alpha(\pi_w) = \frac{\pi_w - \sum_{i=1}^{I} \left( Q_i \lambda_i^T + \lambda_{\pi} R_i^T C_i \right)}{\sum_{i=1}^{I} \sqrt{Q_i^T C_i Q_i}} \]

(43)

Opportuneness is minimum tolerable uncertainty to obtain profit as large as \( \pi_w \), i.e.
\[
\beta(Q, \pi_w) = \min_Q \quad \alpha(\pi_w) \\
\pi_w = \sum_{i=1}^3 \left( Q_i \lambda_i^T + \lambda_i^F P_i^F - \lambda_i^E E_i^E - \lambda_i^C CO_i^C \right) \\
\text{Values of both the functions, robustness (40) and opportuneness (44), cannot be negative which means that the nominal response does not violate the performance requirement. Denominator term in equation (40) and (44) is the standard deviation of overall profit to the GenCo. Robustness } \alpha(Q, \pi_c) \text{ maximizes for low values of denominator term, while opportuneness } \beta(Q, \pi_w) \text{ minimizes for high values of the same denominator term. Thus, robustness and opportunity represent antagonistic behavior, i.e. any change in decision } Q \text{ which leads to increase in one is obtained at the expense of other. For certain values of } \pi_c \text{ and } \pi_w, \text{ robustness (40) and opportunity (44) strategies can be provided based on decision maker’s nature, subject to constraints }
\]

\[
P_i^G = P_i^S + \sum_{i=1}^3 P_{i,i}^B \\
Fuel_i = Fuel_i^S + Fuel_i^F \\
CO_{2i} = CO_{2i}^S + CO_{2i}^E \\
P_{i,M} u_i, l_i, \leq P_{i,i}^B \leq P_{i,M} u_i, l_i, \forall i, \forall i \\
CO_{2i}^{Max} v_i \leq CO_{2i}^S \leq CO_{2i}^{Max} v_i, \forall i \\
Fuel_{i,Max} \leq Fuel_i^B \leq Fuel_{i,Max} w_i, \forall i \\
u, v, w, \in \{0,1\} \forall i
\]

VI. CASE STUDY, RESULTS AND DISCUSSION

To analyze the proposed methodology, a case study for typical Gas fired Generation Company has been considered (specifications shown in Table I). It procures fuel and emission permits from the respective markets, through fixed price contracts (Table II) and spot trading, and sells electricity in day-ahead spot market and through bilateral contracts with customers of three different zones as shown in Table III. GenCo is situated at area NO1, indexed as \( l = 1 \). Zonal price of this area is spot contract price for GenCo. In downstream electricity market, intra-zonal bilateral contract (with NO1) is considered deterministic, while remaining inter-zonal contracts and spot market are uncertain. GenCo wishes to take optimum trading position in all involved markets for some future month considering each day as trading interval. It is assumed that GenCo makes trading plan to allocate its total capacity. Based on fuel type, emission factors are estimated for CO2 emissions [18]. Simulations are performed over several months, and a representative analysis is presented.

### TABLE I

<table>
<thead>
<tr>
<th>Generating Unit Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Type</td>
</tr>
<tr>
<td>Generation capacity</td>
</tr>
<tr>
<td>Quadratic heat-rate coefficient</td>
</tr>
<tr>
<td>Linear heat-rate coefficient</td>
</tr>
<tr>
<td>No-load heat-rate coefficient</td>
</tr>
<tr>
<td>Emission Factor</td>
</tr>
</tbody>
</table>

### TABLE II

<table>
<thead>
<tr>
<th>Specifications of Fuel and Emission Bilateral Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Contract prices</strong></td>
</tr>
<tr>
<td>Gas</td>
</tr>
<tr>
<td>EUA</td>
</tr>
</tbody>
</table>

### TABLE III

<table>
<thead>
<tr>
<th>Specifications of Electricity Bilateral Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Area Index</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

### A. Data

Analysis is based on historical data of August month, for 2008 to 2012, of electricity from NordPool [19], fuel from Nordpool Gas [20] and emission permit (EUA) from Bluenext exchange [21]. Prices for some dates were unavailable for carbon market, and are approximately assumed. Expected values of prices for each market \( \lambda_i^S, \lambda_i^F, \lambda_i^E \) and \( \lambda_i^C \) are calculated as the average of price vectors for each trading interval. Each EUA represents a right to emit a ton of CO2.

### TABLE IV

<table>
<thead>
<tr>
<th>Co-variability Matrix between Uncertain Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Spot electricity</strong></td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>Spot electricity</td>
</tr>
<tr>
<td>Contract 2</td>
</tr>
<tr>
<td>Contract 3</td>
</tr>
<tr>
<td>Spot fuel</td>
</tr>
<tr>
<td>Spot emission</td>
</tr>
</tbody>
</table>

Uncertainty shape matrices for each trading interval are calculated from (25), using variance-covariance between uncertain trades, by appropriate function in MATLAB® [22]. For the considered case, there exist five uncertain contracts, i.e. three of electricity market (one spot market and two inter-zonal bilateral contracts), fuel spot and emission spot market. Thus, 31 number of order 5×5 matrices are formed. All matrices are not shown in the paper due to space constraints. It is observed that in downstream market, electricity trading in spot market and Contract 3 is highly uncertain while Contract 2 represents comparatively less uncertainty. Table IV shows the average correlation matrix between different uncertain trades, reflecting variability and co-variability between different uncertain trades for the entire planning period. Unity values of diagonal elements represent correlation between prices of same trade. Electricity fuel and emission permit prices are usually positively correlated, where emission permit prices represent a strong correlation with electricity market. This is also reflected in considered case, as correlation
between spot electricity and emission permits prices is highest. Also, inter-zonal bilateral contracts have divergent values of correlation with other uncertain trades due to congestion.

B. Simulations

Profit \((23)\), subject to constraints \((45)\) to \((51)\) is maximized with respect to \(Q\) by managing portfolio of all involved upstream and downstream markets. This is the maximum possible value of profit which a GenCo can have if prices of all uncertain trades remain same as expected and are represented as \(\pi(Q, \lambda)\). This can be considered as risk neutral behavior of GenCo, when its decisions are not affected by any uncertainty in market prices.

Based on this obtained maximum profit, values of targeted critical and windfall profits are assumed in small steps, less than or greater than \(\pi(Q, \lambda)\). Profit values less than the obtained maximum portfolio profit \(\pi(Q, \lambda)\) are considered as critical profits \(\pi_c\) for robustness and profit values higher than the obtained maximum portfolio profit \(\pi(Q, \lambda)\) are considered as windfall gains \(\pi_w\).

For each value of \(\pi_c\) and \(\pi_w\), a particular trading strategy is obtained for appropriate allocation in different available trades by optimizing the two MINLP optimization problems \((40)\) and \((44)\). For the presented analysis, both optimization problems have been solved with 501 real and 248 discrete variables, using SBB-CONOPT© solver in a Core i5, 3.2 GHz processor and 4 GB RAM computer, with an average solution time of 0.342 seconds [23]. SBB offers node selections using standard Branch and Bound algorithm and solution is used by NLP algorithm of CONOPT in loop to optimize NLP problem. SBB finds best bounds/estimates to provide the starting point for NLP sub models which NLP solver uses to select solution approach most suitable for the model in hand based on considerable built-in logic.

C. Scenario Consideration

This work considers uncertainty between different correlated trades. To highlight the impact of co-variability between prices of different uncertain trades, uncertainty shape matrices are considered with and without off-diagonal elements. With off-diagonal elements, the matrix considers variability as well as co-variability between uncertain prices, referred to as Scenario I. In the absence of off-diagonal elements, considered uncertainty model ignores the impact of co-variability between trades and focuses only on their individual uncertainty, as considered in Scenario II.

D. Results

From simulations, maximum obtained value of profit \(\pi(Q, \lambda)\) is 5107501€, for allocation in different trades as shown in Table V. This is the maximum value of GenCo’s profit, when the market prices remain same as expected, representing GenCo’s risk neutral behavior.

From \(\pi(Q, \lambda)\) = 5107501€, values of both \(\pi_c\) and \(\pi_w\) are evaluated. For the present analysis, \(\pi_c\) decreases from 5107501€ to 3700000 € while \(\pi_w\) increases from 5107501€ to 10000000 € in small steps. For these values of \(\pi_c\) and \(\pi_w\), the two optimization problems of robustness \((40)\) and opportuneness \((44)\), subject to constraints \((46)\) to \((51)\), are simulated multiple times. From this, uncertainty \(\alpha\) i.e. gap from expectation is calculated considering both faces of uncertainty. Obtained results are shown in Fig. 1 to 6 for the two considered scenarios, collectively for robustness and opportunity. Fig. 1 represents robustness \(\alpha(Q, \pi_c)\) as the maximum uncertainty that the system can sustain without sacrificing critical profit target \(\pi_c\), and opportunity \(\beta(Q, \pi_w)\)

![Table V](image)

**TABLE V**

<table>
<thead>
<tr>
<th>Downstream Electricity Market (MWh)</th>
<th>Upstream Fuel Market (MBtu)</th>
<th>Upstream Carbon market (tCO₂)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot Market</td>
<td>Contract 1</td>
<td>Spot Market</td>
</tr>
<tr>
<td>206400</td>
<td>4800</td>
<td>1045290</td>
</tr>
<tr>
<td>Long-term Contract</td>
<td>81600</td>
<td>268800</td>
</tr>
<tr>
<td>Contract 2</td>
<td>79200</td>
<td></td>
</tr>
<tr>
<td>Contract 3</td>
<td>1045290</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1 Robustness and opportuneness for different targeted profits representing uncertainty horizon

Fig. 2 Expected profit to GenCo for different targeted profits in both scenarios
as the minimum uncertainty that could potentially improve the performance as large as $\pi_w$. This represents that for error $\alpha$ in market prices, a GenCo may secure profit at least equal to $\pi_c$, when prices change unfavorably, while a similar market fluctuation in favorable direction may provide the opportunity to attain profit as large as $\pi_w$. At targeted profit $\pi_c = \pi_w = \pi(Q, \lambda) = 5107501\€$, robustness $\alpha(Q, \pi_c)$ and opportunity $\beta(Q, \pi_w)$ are zero and both increase for the two scenarios due to variation in profit target from $\pi(Q, \lambda)$.

Results for robustness and opportuneness are discussed individually hence.

1) Immunity to Uncertainty

Robustness $\alpha(Q, \pi_c)$ of a decision increases with reducing values of $\pi_c$, i.e. the decision can sustain higher uncertainty in market prices, as expectation of critical profit $\pi_c$ decreases, as shown in Fig. 1. This happens because contracts with low or no uncertainty are usually accompanied with low profits and for lower values of $\pi_c$, decision maker focuses on reducing uncertainty. Thus, it trades in contracts with low or no uncertainty thereby enhancing robustness of decision. For the present case, with lower values of $\pi_c$ at downstream side, it trades mostly in Contracts 1 and 3 (Fig. 4) and at upstream side reduces purchase from spot market (Fig. 5). Portfolio’s standard deviation rapidly decreases with increasing robustness (Fig. 6). Fig. 2 represents expected value of profit evaluated from expected revenue and cost (Fig. 3). Reducing values of expected profit represent the cost of robustness, i.e. if for a certain decision prices don’t change as per anticipation, the GenCo would get expected profit, which is always less than $\pi(Q, \lambda)$ (Fig. 2).

For the present case, in Scenario I (considering co-variability between trades) for the same value of critical profit targets, obtained robustness $\alpha(Q, \pi_c)$ is higher than that for Scenario II (without considering co-variability). It means that with co-variations, the GenCo may tolerate higher deviation in market prices, without sacrificing critical profit $\pi_c$. This could happen due to two situations i) dominating negative co-variation between prices of available electricity sell options, ii) dominating positive co-variation between prices of revenue and cost side markets. For the considered case, both situations lead to trading increment via spot market and trading reduction via Contract 2 and 3 in downstream market for Scenario I, as visualized by relating Fig. 4 and Table V. For these situations, price fluctuation in one are compensated by the other, i.e. allocation in trades having such correlation
reduces uncertainty, as they hedge each other’s risk, as seen in Fig. 6. Thus, decisions are more robust with co-variability considerations. Opposite situations of co-variability may reduce system robustness.

In Scenario I, due to positive correlation of downstream spot electricity with upstream spot markets of fuel and emission, with reducing values of $\pi_c$, purchase from spot carbon market increases, in sharp contrast to Scenario II results (Fig. 5). This happens because Scenario II considers individual uncertainty of trades, which reduces with decreasing trade in uncertain markets. In Scenario I, due to strong correlation between emission and spot prices, price fluctuations of carbon market would be correlated with price fluctuations in electricity spot market. Thus, combined trading in these two reduces overall uncertainty and improves the robustness of decision.

2) Opportunity Arising from Uncertainty

Opportuneness $\beta(Q, \pi_w)$ is the lowest horizon of uncertainty at which windfall profit is possible. This increases with windfall returns (Fig. 1), because possibility of achieving windfall benefits increases with uncertainty. Contracts with higher variability have higher windfall possibility, so trading in high variability contracts increases with growing windfall profit targets and vice-versa (Figs. 4 and 5). This increases standard deviation of profit (Fig. 6).

Fig. 1 shows that as in the case of robustness, consideration of co-variation offers superior opportuneness $\beta(Q, \pi_w)$ as well. This can be explained as follows. Co-variability situations in the considered case dominantly reduce overall uncertainty, thus the range of windfall possibilities grow slowly with co-variation, than without them. As opportunity requires greater horizon of uncertainty with co-variations, the value of $\beta(Q, \pi_w)$ is larger. Thus, one must accept higher values of uncertainty $\alpha$, to enable possibility of windfalls.

Hence, $\beta(Q, \pi_w)$ curve in Scenario I lies above that for Scenario II, which is without considering co-variability.

Co-varying revenue and cost side markets compensate price fluctuations of each other so a combined trading in these reduces windfall possibilities. In order to enhance opportuneness, trading decisions are selected to have i) dominating positive co-variation among electricity selling contracts and ii) negative co-variation between upstream and downstream trades. Contract 2 and 3 are correlated, however Contract 2 is strongly anti-correlated with both fuel and carbon markets (from Table V). Such situations enhance trading from Contract 2 on downstream side and sharply decrease emission spot trading, as shown in Fig. 4 and 5. This is the reason for inferior trading in spot downstream markets during Scenario I and prominently decreasing spot trading of emission permits (Fig. 4 and 5).

The results highlight that for the considered problem, info-gap model with co-variation considerations is more robust to uncertainty, than without co-variation considerations. However, this leads to higher value of Opportuneness function ($\beta(Q, \pi_w)$) as well. This is because with co-variation, price fluctuations are compensated between different trades, which reduces uncertainty of the selected decision, making it robust. With similar co-variations, the range of possibilities to attain windfall grows slowly, as the opportunity requires greater uncertainty. Thus, co-variation improves robustness but worsens opportunity. This highlights that robustness $\alpha(Q, \pi_c)$ and opportuneness $\beta(Q, \pi_w)$ represent antagonistic behavior in the sense that improvement in one would worsen the other.

VII. Conclusion

This paper proposes an IGDT based analytical and quantitative approach to obtain optimum trading position in upstream and downstream markets for a fossil fuel GenCo. This approach incorporates uncertainty in prices of electricity, congestion, fuel and emission permits in an ellipsoid bound model considering variability and co-variability between different trading options.

The proposed formulation is illustrated using a realistic example. Depending upon performance aspirations, IGDT formulation offers immunity from uncertainty, i.e. robustness to failure and opportunity to benefit from windfall gain. Robustness and opportunity have an associated cost, depending on GenCo’s preference for the quantum of tolerance for uncertainty in expectation and the quantum of large windfall it aims.

Simulations show that the strategy of deciding trading proportions of fuel, emission and electricity among their available options depends on correlation among these markets. Co-variation consideration between available trading options enhances robustness but reduces opportunity for same performance aspiration. With co-variations, for the same targeted profit, GenCo can tolerate higher market price deviations but this makes it less opportunistic. Also, to enable the possibility of same windfall profit, it has to accept higher uncertainty with similar co-variability. This could be reversed with opposite situations in co-variability, i.e. robustness would be worse and opportuneness would be better.

The usual correlation between electricity, fuel and emission market effectively improves robustness of the decision which is helpful for risk-averse GenCos to achieve an optimal trading plan in all involved markets. The proposed approach can be extended to comprehensive operational decision-making for short-term planning, such as self-scheduling and unit-commitment with different types of generation technologies.

VIII. Acknowledgement

Nordpool Exchange is acknowledged for providing access to electricity prices. The authors thank Prof. Yakov Ben-Haim of Israel Inst. of Tech. for his support on IGDT.

IX. References


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