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Stochastic Cournot Model for Wind Power Trading in Electricity Markets

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Abstract— **In evolving electricity markets, wind generators would submit bids to the system operator, with an aim to maximize their profits. Generation offered by wind firms is highly random, which may result into heavy imbalance charges. In markets dominated by wind generators, they would optimize their offered bids, considering rival behavior. In oligopolistic electricity markets, this strategic behavior can be represented as a Stochastic Cournot model. Wind uncertainty is represented by scenarios generated using Auto Regressive Moving Average (ARMA) model. With a consideration of wind power uncertainty and imbalance cost, the expected profit of generators is calculated for a practical case study of wind firms located at Massachusetts, USA. Nash equilibrium is obtained using payoff matrix approach. This bidding strategy mechanism offers quantum increase in profit for wind firms, when their behavior is modeled in a game theoretic framework. Flexibility of approach offers opportunities for its extension to associated challenges.**

Index Terms— **Electricity Markets, Nash Equilibrium, Stochastic Cournot Model, Wind Power Uncertainty.**

I. INTRODUCTION

Power sector is being restructured worldwide, with an aim to improve system efficiency and offer economic solutions. At the same time, uncertainties in fossil fuel prices and environmental concerns are enhancing the quantum of wind power generation [1]. Over the last few decades, governments world over are trying to increase the contribution of green energy in electricity supply, by providing subsides and support schemes [2].

Evolving deregulated electricity markets are primarily designed for conventional or fossil fuel generators. These markets operate on a day-ahead timeline, where participants commit their generated power several hours before actual power delivery. Eventual power delivered by wind generators differs from their initial commitment due to intermittent nature of wind. Participants deviating from their committed schedule face penalties. Small capacities and random generation restrict the wind firms to act as strategic players. They participate in the market as 'price takers', and are not able to affect the market clearing price (MCP). Due to high capital cost and imbalance penalties, they cannot operate profitably in pool-based markets. Therefore, they are forced to sell their power through bilateral contracts.

In pool-based markets, wind generators focus on imbalance cost minimization to increase their profits. This can be obtained using Stochastic Programming. With a consideration of forecasting window length and market closure delay, Markov Probability based stochastic model can determine the optimal contracted energy level [3, 4]. Probabilistic forecasting based methodology can help price-taker wind generators to formulate optimal offers [5]. Multistage stochastic programming approaches suggest various trading floors to derive the best offering strategy for a wind generator considering wind and price uncertainties [6]. However, wind generators are still assumed to be price-takers. In addition, focus is on increasing the wind generator's profit by bid selection, with minimum imbalance cost. Strategic gaming by wind generators for bid selection in pool based electricity markets has generally been neglected.

With the present thrust and growth, the wind firms would increasingly supply power to an extent of 20% or more of the market demand in future [7]. They would participate in pool based electricity markets strategically, without any regulatory support and benefits. They would tend to increase their profit by gaming in the market [8].

This paper focuses on formulation of optimal offering strategy for strategic wind firms, in a market dominated by intermittent wind generation. Strategic behavior of wind generators in oligopolistic electricity markets, considering wind uncertainty, is modeled using Stochastic Cournot model. In this model, wind generators aim to maximize profit by offering optimal bids, considering rival behavior and perfect information. Wind generators are allowed to offer zero generation bid. Solution of the proposed model is Nash equilibrium, obtained by payoff matrix approach.

Rest of the paper is organized as follows. In Section II, the market structure, uncertainty characterization, and stochastic Cournot model are described. Section III provides mathematical modeling of the problem and the simulation procedure. Section IV includes numerical and graphical results of testing the proposed model through a comprehensive

analysis on three wind firms located at different locations. In Section V, relevant conclusions are drawn.

II. PROBLEM DESCRIPTION

A. Market Structure

Wind generators participate in pool based day-ahead electricity market, cleared several hours before the actual power delivery using uniform price auction. Real-time balance between supply and demand is maintained by the balancing market, few minutes before power delivery. Imbalance charges resulting from this balancing market are charged to generators causing that system imbalance. In this work, dual imbalance price mechanism used for imbalance charging and adopted in European markets such as NETA, Nord Pool, Iberian Peninsula, *etc.* has been considered [4-6].

B. Uncertainty Characterization

Stochastic wind speed is considered as a continuous random variable, represented by scenarios. Scenarios are possible outcomes of the random input, with corresponding occurrence probability. To generate wind speed scenarios, statistical time series based ARMA model is used. A typical ARMA (p, q) model is expressed as

$$Z_t = \sum_{j=1}^{p} \phi_j Z_{t-1} + \varepsilon_t - \sum_{j=1}^{q} \theta_j \varepsilon_{t-1} \tag{1}$$

Here, Z_t is wind speed scenario at time t with p autoregressive parameters $\phi_1, \phi_2, ..., \phi_p$ and q moving average parameters $\theta_1, \theta_2, ..., \theta_q$. The term ε_t is a normal distributed random number with zero mean and σ standard deviation, referred as a white noise or error.

Generated wind speed scenarios are converted into power scenarios, using power curve of wind turbines installed at the wind farms. For accurate representation of any stochastic process, a large number of scenarios are required. Due to computational complexity and time limitations, generated scenarios need to be reduced. These reduced scenarios reflect expected power generated by the wind firms [9].

C. Stochastic Cournot Model

Cournot game theory is a general approach to represent strategic behavior of firms in oligopolistic electricity markets. Firms make decisions independently and simultaneously, without cooperating with each other. With an aim to maximize profit, each firm chooses quantity bids to be offered, considering rival behavior. Nash equilibrium is a solution of Cournot model; this is a standoff condition where no firm can unilaterally increase its profit by changing its production level. In a deterministic Cournot model, input variables are scalar and independent, while in a Stochastic Cournot model, input variables are stochastic in nature or dependent on other stochastic variables [10-12].

III. MATHEMATICAL FORMULATION

A. Model

Consider $i \in N_i$ wind firms participating strategically in an oligopolistic electricity market. Each firm aims to maximize its own profit by offering a certain quantity bid. The profit maximization problem of ith wind firm in a day-ahead electricity market is formulated as follows:

$$\underset{Pof_{it}}{Maximize} \quad U(.) = \lambda_t Pof_{it} + \lambda_t^+ \left(P_{it}(\omega) - Pof_{it} \right) x_{it}$$
$$+ \lambda_t^- \left(P_{it}(\omega) - Pof_{it} \right) (1 - x_{it}), \quad \forall i, \forall t, \forall \omega \tag{2}$$

Subject to

$$0 \le Pof_{it} \le \bar{P}_i, \quad \forall i, \forall t \tag{3}$$

$$x_{it} \in \{0,1\}, \quad \forall i, \forall t \tag{4}$$

Where, Pof_{it}, $\bar{P}_i$ is an offer quantity bid and maximum capacity of ith wind firm respectively. $P_{it}(\omega)$ is an expected power output in scenario ω at time t. λ_t, λ_t^+, λ_t^- are MCP, positive imbalance price (PIP) and negative imbalance price (NIP) respectively. x_{it} is a binary variable ensure positive and negative imbalance for strategic wind firms.

The objective function (2) shows the profit of ith wind producers, under the assumption that wind power generation cost is zero; therefore, expected profit is equal to expected revenue. It is assumed that wind firms individually participate in the market without any control strategy. Each firm selects contracted power Pof_{it}, which maximizes its expected profit, considering imbalance cost. MCP at time t is determined by inverse linear demand curve (5). Demand is the sum of power contracted by the wind firms,

$$\lambda_t = \lambda_{max} - K P_{dt} \tag{5}$$

Where, K is the ratio of maximum value of MCP λ_{max} to demand P_{dmax}. The maximum power is equal to the installed capacity of firms, while the minimum power production is considered to be zero. Wind firms do not generate any power when wind speed is below cut-in or above cut-out speed of the installed turbines. This decision-making problem is formulated as a Cournot model, where all firms try to maximize their profit by optimizing their offered quantities. In mathematical terms, Cournot Nash equilibrium is a vector, which solves a collection of profit maximization problems of the form

$$U\left(Pof_1^*, Pof_{i-1}^*, Pof_i^*, ... Pof_{N_i}^*, \omega_i \right)$$
$$\ge U\left(Pof_1^*, Pof_{i-1}^*, Pof_i, ... Pof_{N_i}^*, \omega_i \right) \tag{6}$$

Cournot Nash equilibrium provides optimal offered bids, considering behavior of rival wind generators.

B. Simulation Procedure

This section describes the procedure used for obtaining the solution of proposed Stochastic Cournot model.

Step 1: Time Counter Initialization: Initialize time counter to obtain optimal hourly offers of firms. Time counter starts with $t = 1$.

Step 2: Scenario Generation and Reduction: Initialize the strategic firms' expected outcome by generation of scenarios. For scenario generation and reduction, the algorithms proposed in [9] are used.

Step 3: Nash Equilibrium: Each firm has discrete set of possible offering outputs. They select only one offer among possible offers, which maximizes their expected profit. To obtain Nash equilibrium, resulted payoff matrix constructed having probabilistic information about each scenario. For each combination in payoff matrix, MCP is calculated using (5). For resultant payoff matrix, Nash equilibrium is obtained by payoff matrix approach [13]. This equilibrium gives optimal power output that can be offered by the firms.

Step 4: Update Time Counter: For each considered hour, offer for each firm is obtained. In the next step, update time counter by $t+1$ and go step 2.

Step 5: End

IV. CASE STUDY

The present section considers a pool-based market, where three wind firms interact strategically. The results illustrate effectiveness of the proposed model for bidding strategy formulation of wind firms.

A. Data

The present study considers three wind firms, with an installed capacity of 100 MW each. These firms are situated at three different locations, Barnstable, Savoy and Kingston, of Massachusetts State, USA. Each firm has 40 wind turbines, with commercial 2.5 MW, VENSYS100 turbine installed at 100 m hub height. Air density and temperature conditions are assumed same for each installed wind turbine. The used turbine model and its power curve are detailed in manufacturer database [14]. For all these firms, the actual wind speed data of August 2005 is taken, publically available at Wind Energy Center, University of Massachusetts, USA [15].

B. Results

Wind uncertainty of each wind firm is characterized by scenarios. The estimated parameters time series based ARMA model that is used for scenario generation is shown in Table I. For accurate modeling of wind power uncertainty, 1000 scenarios are generated and then reduced to 10 scenarios for each firm. From these reduced scenarios, in every hour, each firm can formulate their resultant payoff matrix.

TABLE I
FIRMS' TIME SERIES MODEL PARAMETERS

Parameters	Firm 1	Firm 2	Firm 3
Order	ARMA (1,0)	ARMA (1,1)	ARMA (2,0)
ϕ_1	0.8693	0.8933	0.6313
ϕ_2	-	-	0.9711
θ_1	-	0.0654	-
δ	0.5141	0.6048	0.6213

In this work, two cases are considered. First is the base case, where each firm offers their forecasted generation. In Case II, the firms behave strategically, and offer output power, which gives maximum expected profit, as obtained by Nash equilibrium. Market operation and imbalance price mechanism are same for both the cases.

C. Case I: Base Case

In the base case, firms offer power corresponding to forecasted outputs. Rival behavior is not considered for offer selection. For each firm, the hourly offered bids are shown in Fig. 1. At the first hour, power offered by Firm 1, Firm 2 and Firm 3 is 47.1143 MW, 51.0163 MW and 48.8089 MW, respectively.
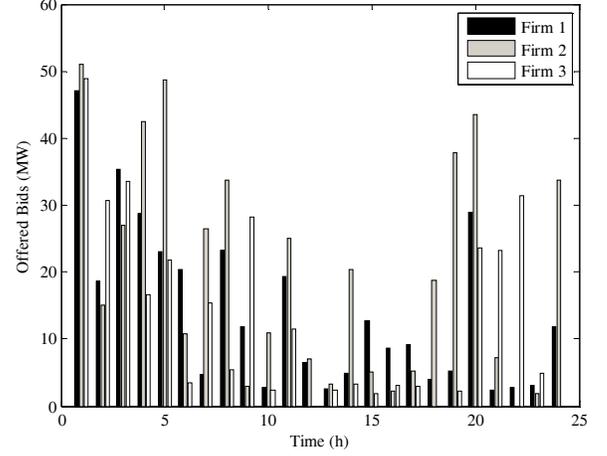

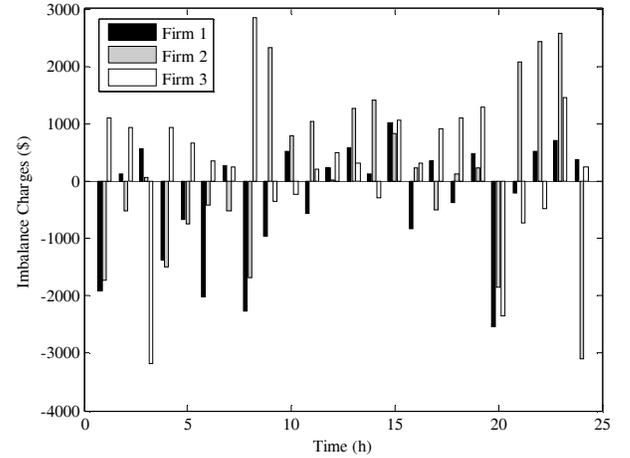Fig. 1. Bid offered by wind firms.

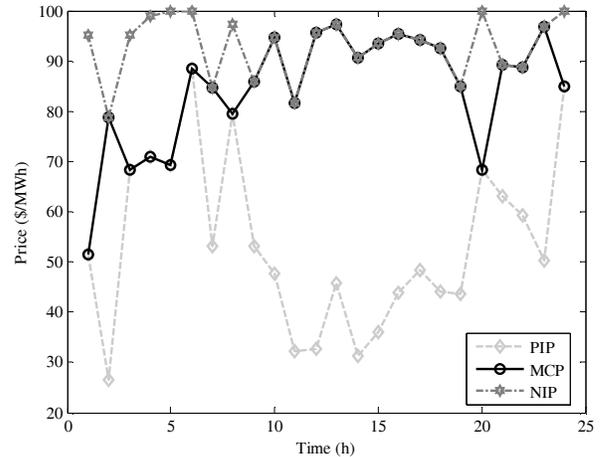
Fig. 2. Imbalance charges for each wind firms.

Fig. 3. MCP and imbalance prices.

Hourly imbalance charges for each firm are shown in Fig. 2. Firm 1 and 2 face negative imbalance charges while Firm 3

receives positive imbalance charges because of power offered by Firms 1 and 2 exceeds the generated power, while that by Firm 3 falls short. Firms 2 and 3 offer zero generation bids at 22 Hours and at 12, 18, 24 Hours, respectively. At these hours, imbalance cost is zero and firms earn revenue for any surplus generation.
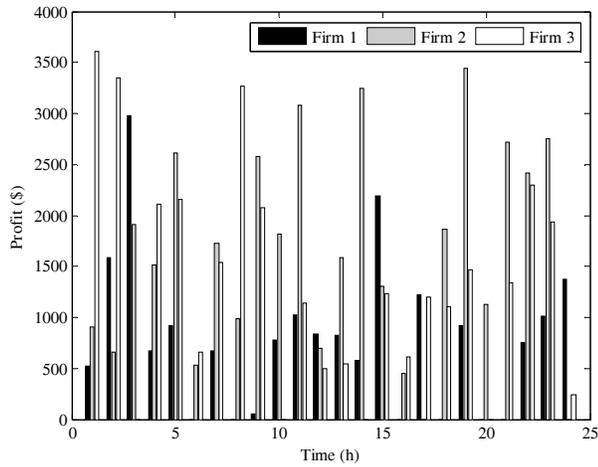

Fig. 4. Expected profit of firms.

For bids offered with zero generation, imbalance charges would either be positive or zero. MCP determined by inverse demand curve and imbalance price obtained from imbalance mechanism are shown in Fig. 3. Maximum demand is considered to be 300 MW and maximum MCP is considered as $100/MWh. Demand at any particular hour is equal to the sum of power offered by the firms. From these figures, it is observed that PIP is less than MCP while NIP is equal to MCP during system has surplus generation, and vice versa. The expected profit obtained by each firm is shown in Fig. 4. For the first hour, profit of Firms 1, 2 and 3 are $523.82, $903.53 and $3609.96, respectively.

D. Case II: Strategic Firms

In this case, the firms behave strategically and consider rival behavior for their offer selection. They offer power as per Nash equilibrium solution of the proposed Stochastic Cournot model.
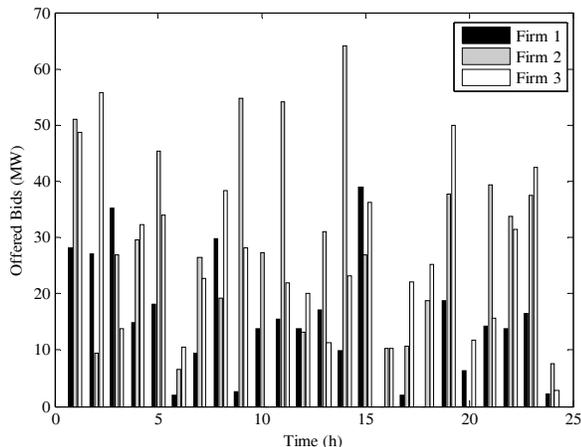

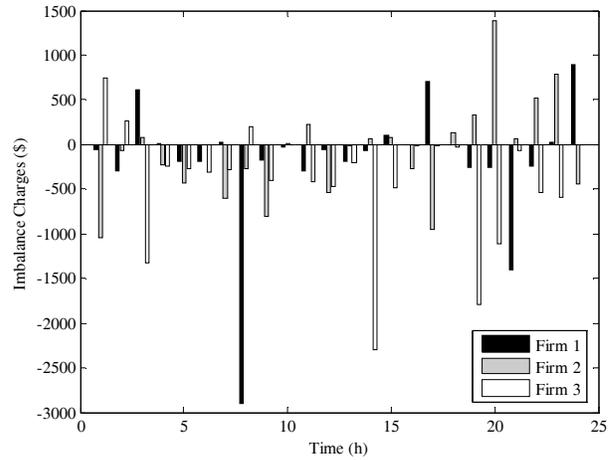Fig. 5. Bids offered by strategic wind firms.

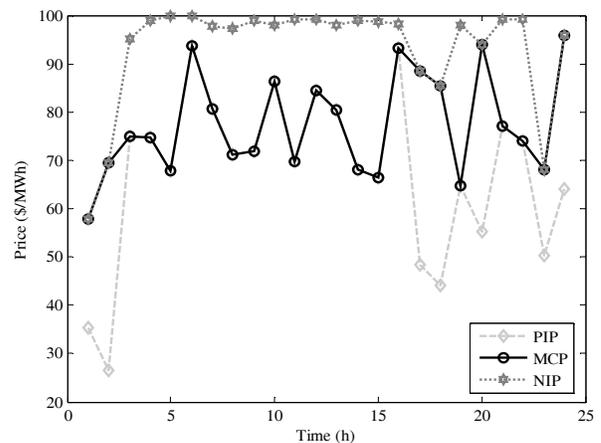Fig. 6. Imbalance charges for each strategic firm.

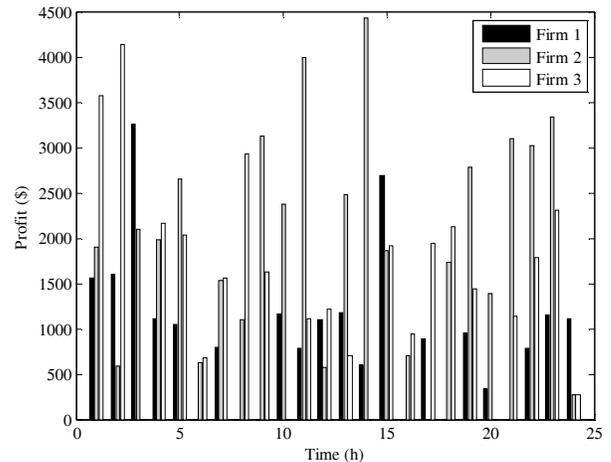Fig. 7. MCP and imbalance prices.

Fig. 8. Expected profit profile of strategic firms.

Hourly profile of the power offered by different firms is shown in Fig. 5. At the first hour, power offered by Firm 1, Firm 2 and Firm 3 are 28.1260 MW, 51.0613 MW and 48.8089 MW, respectively. Actual generated power for Case I is equal to actual historical generation while in Case II it is equal to reduced scenarios. Imbalance charges for the firms arise due to deviation between offered and generated power, as shown in Fig. 6. For the first hour, power generated by Firm 3 is more than that originally offered, and hence it earns

revenue corresponding to this positive imbalance. However, power generated by Firms 1 and 2 is less than their offered power, and hence have to pay negative imbalance prices.

Fig. 7 shows hourly imbalance charges for each firm. From these figures, it is evident that at the first hour, positive imbalance price is lower than both MCP and negative imbalance price due to system demand being less than generation, Hourly profile of the expected profits, for different strategic firms is shown in Fig. 8. At the first hour, profits earned by Firm 1, Firm 2 and Firm 3 are $1565.01, $1899.12 and $3571.92, respectively.

E. Discussion

Considering the results obtained from the two cases in the first hour, it is evident that the proposed Stochastic Cournot model increases the profits earned by different firms and reduces the imbalance charges significantly. A comparative evaluation of the profits earned by different firms at the first hour, as evident from Figs. 4 and 8, shows that the profit of Firms 1 and 2 increases by $1041.99 and $995.59 respectively, while that of Firm 3 decreases by $38.04. This is because Firm 1 decreased its offered bid by 18.9883 MW in Case II, while the remaining firms did not change their offers, as shown Figs. 1 and 5. The firms behave strategically, and change their offered bids, if they have the opportunity to earn more revenue.
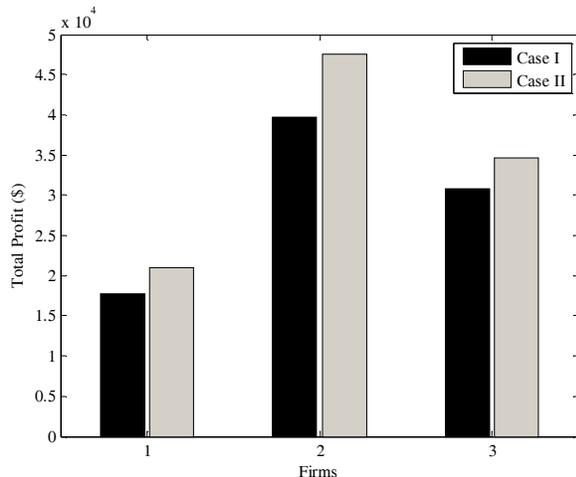

Fig. 9. Comparative total profit earned by each firm in one day.

As Firm 1 reduced its offered power output, the corresponding imbalance charges are also changed. As the system demand is considered to be the sum of offered bids, the assumed system demand also reduces. Therefore, in Case II, system has surplus generation, while in Case I, the system has power shortage. This surplus generation reduces imbalance prices shown in Figs. 3 and 7. Imbalance charges for Firms 2 and 3 are slightly reduced due to imbalance price reduction. Due to this, profit of Firm 3 is slightly reduced.

A consideration of the profit earned by different firms over a period of 24 hours is shown in Fig. 9. The overall profit earned by different firms increase significantly, when the offered bids are selected by the Stochastic Cournot model. A comparative reflection of the daily benefit earned by each firm clearly shows that the increase in profit earned by different firms would be substantial over a longer period of time.

Simulation are performed on MATLAB® platform with a Windows based Personal Computer, 1.73 GHz processor and 1GB RAM.

V. CONCLUSIONS

In a pool-based market, strategic behavior of wind power producers is modeled by a Stochastic Cournot model. Wind uncertainty and imbalance costs are considered for evaluating the expected profit. Wind uncertainty is represented as scenarios generated by ARMA model, which are reduced by Simultaneous Backward Reduction method, so as to reduce the computational burden. Nash equilibrium is obtained with payoff matrix approach. Proposed model is implemented on three wind firms located at Massachusetts, USA. A comparative study of the two different cases shows that a consideration of rival behavior in selecting the bid offers results in a significant increase in the firms' profit. The work considers a near-future scenario, when system demand would predominantly be supplied by wind generators. The proposed model can be improved by considering behavior of conventional generators and modeling demand and price uncertainty.

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