Using Control Methods to Model Energy Hub Systems

Dongmin Yu
University of Bath
Dy253@bath.ac.uk

Bo Lian
University of Bath
Bl361@bath.ac.uk

Roderick Dunn
University of Bath
R.W.Dunn@bath.ac.uk

Simon Le Blond
University of Bath
S.P.leBlond@bath.ac.uk

***Abstract*-In this paper, multi-carrier energy system (MCES) optimization problem is solved by using Similar Decoupled Form (SDF) energy hub modeling. Compared with the traditional energy hub modeling method, the Similar Decoupled Form modelling method improves flexibility and automation of the energy hub system. More importantly, energy hub does not need to be linear when using this method. In other words, the limitations of SDF are greatly reduced and it can be used in a more general energy system with energy storage equipment. This method can be used to analyze power flow from both sides of the energy hub (input side and output side) and the stability of the energy hub system. Finally, SDF shows the dynamic change of the energy hub system rather than only the result of mathematical optimization.**

***Index Terms*—Energy Hub Modelling, Decoupled Form, Double-side Energy Flow, Stability, Dynamic System, Optimization**

I. INTRODUCTION

In recent years, the production and utilisation of energy is becoming a hot topic over the world. People not only pay attention to the primary energy sources, but also pay attention to renewable sources. However, there are two major problems in the treatment of energy sources in today's research. These may be identified as the *unbalanced* approach and the *separated* approach.[1, 2]

In the *unbalanced* approach, too many engineers are mainly showing interest in electric power research. Less attention is being given to other kinds of energy sources. For that reason, it is the time for people to pay more attention to other kinds of energy systems.

In the separated approach, engineers usually split the energy system into sub-systems and analyse them separately. However, the overall energy efficiency would be greatly improved if multi-energy sources were analysed in a more unified way.

So for those reasons, Geidl developed a steady-state framework for multi energy carriers system to overcome the limitations in 2007[3]. The great achievements of this model can be summarized as: it describes how power flow can be coupled between different energy carries in steady-state. A second important energy hub model was built by Enrico Fabrizio in 2008[4]. The main difference of two models is the formulation of the coupling matrix. Geidl's model expresses to the hub outputs for given inputs whereas Fabrizio's model expresses the inputs for given outputs. However, in both cases, the most important thing is to find the coupling matrix between input vectors and output vectors.

After establishing the energy hub models, many optimization algorithms were built to get the minimum solution of energy hub system. However, the majority of these optimization algorithms are based on mathematical calculation and pay less attention to system variation. By applying mathematical calculation methods to optimize the energy hub, the original energy system needs to be idealized and this adds some limitations. For example, one limitation is the energy hub system needs to be linear. If not, the mathematical results may not be a global minimum. Another limitation is the system must have already reached steady-state, because a simple mathematical function cannot reflect the dynamic changes of the energy system.

It is therefore necessary to find a method to optimize dynamic and non-linear energy hub systems. Control theory can be used to model a dynamic system; however no literature exists using control laws to design energy hub model. The main reason is the system may be uncontrollable or unobservable by using control laws in an improper way. In this paper, Similar Decoupled Form is proposed to solve this problem.

The rest of paper is organized as follows: Section II introduces Geidl's Energy Hub modelling (Mode1) method and Optimization. Section III describes Fabrizio's Energy Hub modelling (Mode 2) method and Optimization. Section IV shows Similar Decoupled Form (SDF) modelling method. Section V is the comparison of three methods. Further work is shown in the Section VI.

II. ENERGY HUB MODE 1 AND OPTIMIZATION

Geidl's energy hub modelling is easy to understand and implement. Before introducing this theory, four assumptions need to be made. First, within energy hubs, besides conversion losses and storage losses, there is no other form of losses. Second, before modeling an energy hub, this hub has already reached a steady state. Third, energy always flows from inputs to outputs. Last, the only efficiency penalty comes when power flows through converters.

A. Modelling

The input vectors of the energy hub shown in Fig.1 contain electricity (from Grid), natural gas and heat (normally district heat) and the output vectors electricity and heat. Within the energy hub, a CHP plant (combined heat and power) is used to convert gas to heat and power. Some storage equipment can also be used to store heat, electricity or natural gas. Fig. 1

978-1-4799-6557-1/14/$31.00 ©2014 IEEE

shows an example energy hub with CHP, gas tank, heat exchanger and hot water storage equipment.

Matrices and vectors are usually used to represent the power flow within the energy hubs. The basic relationship between inputs and outputs of an energy hub is:

$$L=CP-SE=CP-M_E \tag{1}$$

Where C represents the coupling matrix whose individual terms are normally the energy conversion efficiency of the converters. L and P are output power vectors and input power vectors of the hubs respectively. S and E represent the storage coupling matrix and steady-state energy storage vector respectively. M_E is an equivalent vector of energy stored in the storage elements. Normally, for simplicity the energy storage facilities are modelled at one side of the converters, although in reality they maybe at either the input or output ports of the hub.

B. Optimization

In this mode, the system optimization problem is formulated as a mathematical function. To optimize for energy the hub model one must find the maximum or the minimum value of the functions. Normally multipliers are applied to energy to calculate monetary cost and emissions to give two simultaneous optimization criteria.

The (nonlinear) mathematical optimization functions of the energy hub can be written as:

$$\text{Minimize} \quad F(x) \tag{2}$$

$$\text{Subject to} \quad g(x)=0 \tag{3.1}$$

$$h(x)\leq 0 \tag{3.2}$$

Where $F(x)$ is a scalar-valued objective functions, $g(x)$ normally comes from the conservation of power and $h(x)$ are the limits of energy system, e.g. maximum output power, or number of converters.

For a time-invariant system, all functions always only have one variable. However, by adding energy storage equipment, the energy hub becomes a time-variant system. In this situation, many functions may introduce second variable t (time).

After solving the mathematical problem, many results may be acquired. The reason is $F(x)$ may have many local minima, however not all of them are global minimum. In order to get a reasonable result, it is important to double check the solutions acquired by mathematical functions.

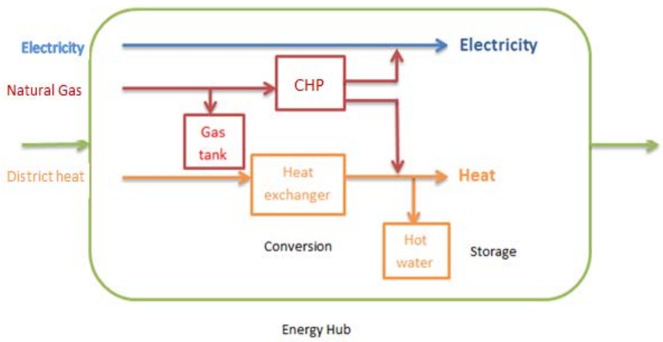

Fig. 1. An example of energy hub system

III. ENERGY HUB MODE 2 AND OPTIMIZATION

Compared with the Geidl's method, Fabrizio's method hinges on finding the minimum input vectors for given output vectors. Although the mathematical expression of mode 2 looks simple, the calculation is actually non-trivial. Fig. 2 shows the topology diagram of Fabrizio's model.

A. Modelling

Normally, input power can be expressed as the sum of the output plus the sum of the storage power. Without energy storage, input power (for a multi-energy carrier system) can be expressed as:

$$P_{in}^{\alpha} = \frac{\varepsilon_{k1}^{a}}{\eta_{K1}} P_{out}^{a} + \frac{\varepsilon_{k2}^{b}}{\eta_{K2}} P_{out}^{b} + \cdots \tag{4}$$

Where η_{Ki} is the conversion efficiency of the generic converter i and η_{Ki} is greater than zero and less than one. $\varepsilon_{ki}{}^{a}$ is the ratio between the load a covered by the converter K_i and the load a. $P_{in}{}^{i}$ and $P_{out}{}^{i}$ are the inputs and outputs of energy hub system, respectively.

With energy storage, the mathematical function of power flow would be complex. Equation (5) shows the relationship between input vectors and output vectors.

$$P_{in}^{\alpha} = \frac{\varepsilon_{K1}^{a}(1+\Omega^{a})}{\eta_{K1}(1-\Omega^{\alpha})} P_{out}^{a} + \frac{\varepsilon_{K2}^{b}(1+\Omega^{b})}{\eta_{K2}(1-\Omega^{\beta})} P_{out}^{b} + \cdots \tag{5}$$

A new parameter Ω^{i} is defined as the ratio of entering or leaving the storage device to the power flow at the input or output of energy hub. For $\Omega>0$, the storage facilities are charged and vice versa.

Equation (6) is matrix form of Equation (5). The matrix representation is straightforward.

$$P_{in} = D * P_{out} \tag{6}$$

Matrix D represents for superposition of the conversion, distribution and storage matrices in energy hub system. Compared with the first method, the relationship between input and output can be seen easily.

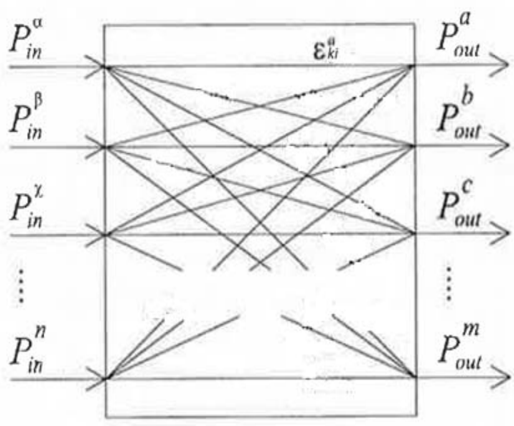

Fig. 2. Topology diagram of Fabrizio's energy hub model

B. Optimization

Optimizing Fabrizio's model is also heavily based on mathematical calculation. Because many algorithms were proposed at early stage to deal with demand side management problem, more methods can be used to solve the mathematical problem for this model. In [5], dynamic programming is introduced to solve power generating unit commitment problem. Demand side management also is used to solve energy hub modeling problem in [6]. Moreover, priority list method is used more frequently to find reasonable input at given output in [7]. For complex systems or time-variant systems, genetic algorithms could be used.

IV. Similar Decoupled Form Modelling Method

Before introducing this method, some control theory must be introduced.

A. Control Theory

A control system can be represented as many forms, i.e. Control Canonical form (CC form), physical form, decoupled form and so on. Different state space forms may share similar features.

Fig. 3. is the block diagram of Decoupled form. The second column is the controller of the energy system. The number of rows in the block diagram represents the number of controllers in the system. The first column of blocks represents the controllability of the system. Any block in this column equal to zero will cause the controller to be uncontrollable. The last column can be used for indicating the observability of system. Similarly, any block in this column equal to zero would cause the controller to be unobservable. Any uncontrollable or unobservable may lead to an unstable system.

B. Similar Decoupled Form Modelling Method

Based on control theory mentioned above, Fig. 4 is an overview of energy hub modelling based on Decoupled Form control theory. To be specific, Similar Decoupled Form of energy hub modeling can work in two states: charging or discharging. Fig. 5 shows SDF energy hub modeling system working on charging state and Fig. 6 shows SDF energy hub modeling system working on discharging state.

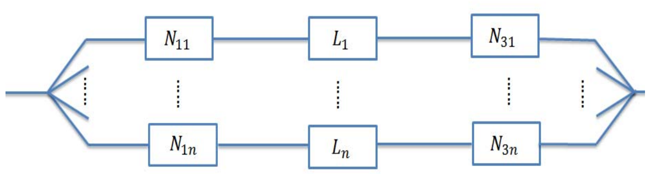

Fig. 3. Decoupled form Block Diagram (Where N_{1i} are numbers and represent for controllability of this row; N_{3i} are numbers as well and represent for observability of this row; L_i is Laplace scalar and represent for controllers)

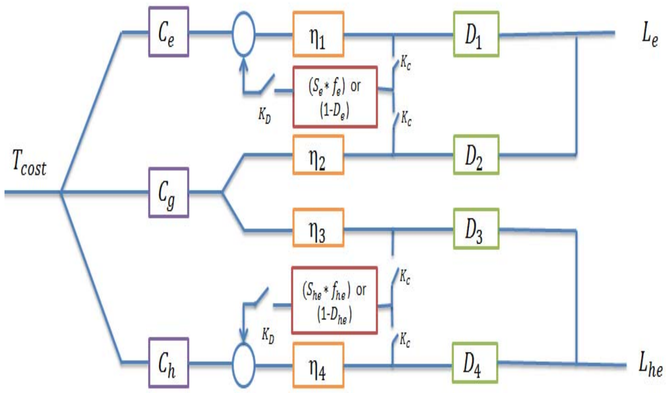

Fig. 4. SDF control energy hub modeling overview

From Fig. 4, it is easy to distinguish that this model considers more the relationship between total cost (T_{cost}) and output (L_e and L_{he}). The purple blocks represent price vectors for electricity, gas and district heat system. The orange blocks show the conversion efficiency of hub's converters. The red blocks are used to illustrate the storage equipment in the energy hub. Finally, the green blocks represent distribution factors. Kc and K_D are electrical switches. On charging model, Kc would be turned on to charge the storage equipment and on discharging model, K_D would be switched on.

Charging mode: the relationship between input power and output power can be written as:

$$\begin{bmatrix} \eta_1 D_1 & \eta_2 D_2 & 0 & 0 \\ 0 & 0 & \eta_3 D_3 & \eta_4 D_4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} L_e \\ L_{he} \end{bmatrix} \quad (7)$$

In Equation (7), P_i is used to represent input power. η_i and D_i are energy conversion efficiency and distribution factor, both of them less than 1 and greater than 0. A memory should be used to record energy stored in the storage equipment and when fully charged, more energy cannot be stored

Discharging mode: storage equipment plays a role as another kind of input power. When discharging, it contributes to the original energy sources. Equation (8) relates to input vectors and storage vectors to output vectors.

$$\begin{bmatrix} \eta_1 & \eta_2 & 0 & 0 \\ 0 & 0 & \eta_3 & \eta_4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} + \begin{bmatrix} \eta_1 S_e & 0 \\ 0 & \eta_4 S_{he} \end{bmatrix} = \begin{bmatrix} L_e \\ L_{he} \end{bmatrix} \quad (8)$$

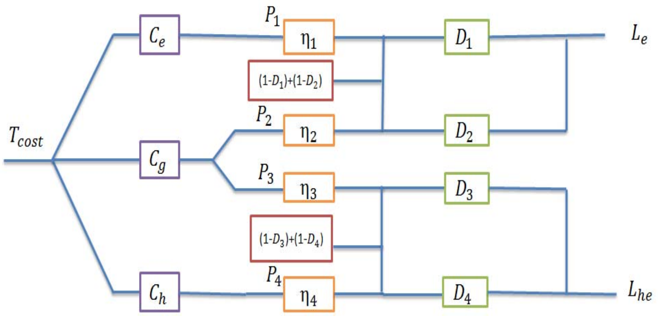

Fig. 5.Charging Model of Energy Hub

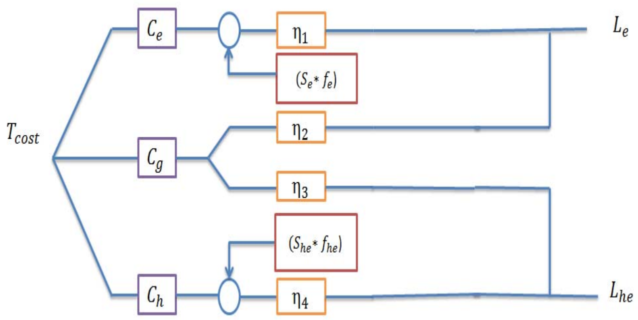

Fig. 6. Discharging Model of Energy Hub

For the same kind of energy sources, charging and discharging cannot be achieved at the same time, so the distribution constant is 1 when the system is discharging. In Equation (8), S_e and S_{he} are electricity discharging efficiency and heat discharging efficiency. f_e and f_{he} are rated output power of energy storage equipment.

Now, the expression for T_{cost} becomes:

$$T_{cost} = \sum_{t=1}^{1440} C_e P_1 t + C_g(P_2 + P_3)t + C_h P_4 t \quad (9)$$

This system is measured every minute in a 24 hour time window. T_{cost} is the cost of function.

V. Discussion and Conclusion

Compared with the existing two methods, SDF energy hub modelling is easy to realize. There are two methods to optimize this model.

The first method is to optimize the objective function $T_{cost.}$ The Objective function is relatively easy compared with other models. With the increment of input vectors, mathematical computation would be obviously easier compared with others. SDF modelling shows two clear states of energy hub, charging or discharging. The matrix expression of both cases is clear.

The second method is through simulation software such as MATLAB Simulink™ . Matlab already provides Simulation tools to build the structure of the control system. After building the block diagram in Matlab, the steady-state result can be automatically determined by the software. In this case, no extra mathematical algorithm needs to be devised.

The second advantage of this SDF is it can be used to deal with a dynamic system. It is well known that heat, gas and electricity systems are all dynamic systems. However, in Geidl and Fabrizio's models, both treat heat, gas and electricity systems as steady state systems and this introduces error. In SDF energy hub model, a Laplace scalar can be used in energy conversion block to represent dynamic behaviour of heat or gas flows.

When this system is modelled with Simulink, both input vectors and output vectors can be shown, if scopes are added in the block diagram. So this model can be used to analyse double-sided energy flow in energy hub system.

So this is an effective method to model energy hub systems. It provides a way to analyse energy flow in both sides of the energy hub and moreover the calculation of this method is also simple. Finally, the model can be highly automated and it can be used to analyse dynamic behaviour of the system.

VI. Further Work

Case studies should be used to illustrate how the system works in the real world, particularly to show how the dynamic behaviour of the energy hub is modelled.

References

[1] Chu, S. and A. Majumdar, *Opportunities and challenges for a sustainable energy future.* Nature, 2012. **488**(7411): p. 294-303.

[2] Dincer, I. and M.A. Rosen, *Energy, environment and sustainable development.* Applied Energy, 1999. **64**(1-4): p. 427-440.

[3] Geidl, M., *Integrated modeling and optimization of multi-carrier energy systems* 2007.

[4] FABRIZO, E., *Modelling of multi-energy systems in buildings.* 2008.

[5] Singhal, P.K. and R.N. Sharma. *Dynamic programming approach for solving power generating unit commitment problem.* in *Computer and Communication Technology (ICCCT), 2011 2nd International Conference on*. 2011.

[6] Ahc, et al. *Simulating demand response and energy storage in energy distribution systems*. in *Power System Technology (POWERCON), 2010 International Conference on*. 2010.

[7] Senjyu, T., et al. *A fast technique for unit commitment problem by extended priority list*. in *Power Engineering Society General Meeting, 2003, IEEE*. 2003.