Inequality and Rules in the Governance of Water Resources

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1 Introduction

The collective management of water and other natural resources is increasingly being recognised as a key determinant of economic performance, especially in the rural sector of developing economies (Platteau, 1991; Balland and Platteau, 1996; Ostrom, 2003; Bardhan et al., 2006). By its nature, collective action involves interdependency among individuals.[1] This, combined with the non-excludable and rival nature of many natural resources, poses significant challenges and raises the question of whether individuals are capable to successfully manage resources held in common.

Over the past decades, significant advancements have been made in the collective action literature and the earlier conventional wisdom that the users of a common resource are inevitably trapped in a process leading to overuse and degradation (Hardin 1968) is no longer regarded as the only relevant view. Using multiple methods of analysis, scholars from different disciplines have shown that the tragedy of the commons is not inevitable.[2] Importantly, they have made considerable progress in identifying the conditions that are most likely to influence the success of collective action and collective good provision. These include: (i) users group characteristics, such as group size and heterogeneity; (ii) institutional arrangements; and (iii) physical attributes of common-pool resources (Sandler, 1992; Agrawal, 2007; Ostrom, 2007). Yet, as suggested by Ostrom and colleagues, advancing our understanding of collective action problems requires

[1]For example, the maintenance of an irrigation network requires the stabilization of the rims and the cleaning of minor channels across farmers' land. In this context, the effort of one farmer is likely to influence the activity of other farmers along the network, thus implying strategic interactions among individual users.

[2]Examples of collective behaviour have been identified in a wide range of contexts. These include the management of fisheries (e.g., Acheson, 2003; Singleton, 1999), forests (e.g., Mckean, 1986, 2000; Schoonmaker Freudnberger 1993), pastures (e.g., Gilles et al. 1992; Netting, 1981; Nugent and Sanchez, 1999), and groundwater resources (e.g., Blomquist 1992; Trawick, 2003; Marchiori et al., 2012).

further investigation of the relationships between these key dimensions, as well as of broader contextual variables (Poteete et al., 2010).

This paper focuses on the mechanisms linking heterogeneity, institutions and incentives within the context of water resources. Specifically, it investigates whether and how land inequality – which is taken here as an exogenous source of heterogeneity – affects the allocation rule that maximises the amount of water collectively provided.[3] In order to trace the fundamental trade-offs that relate initial inequality to the optimal water allocation rule, we introduce a stylised model in which two types of farmer, with unequal land endowments, can voluntarily contribute to a joint project for the maintenance of an irrigation network. Maintenance activity increases the amount of water effectively available. The collective output (water) is then distributed according to some allocation rule and used by each farmer in combination with land to produce a final good.

We find that the initial degree of inequality does affect the optimal allocation rule, and that the nature of such relationship depends on technological features such as the complementarity between agents' efforts in the realization of the collective good. More precisely, we identify two key forces, which affect the distribution of water in opposite directions. The first force, which is referred to as 'effort-augmenting', seeks to maximise the *aggregate* level of effort by pushing the distribution of water towards the agent with the higher marginal return to water. Due to the assumed complementarity between land and water in the production

[3] The paper approaches the problem from a non-cooperative perspective, by studying how inequality and rules affect agents' incentives to contribute in a Nash equilibrium. This is generally regarded as the natural starting point in this kind of analyses. A possible extension for future research is to study the problem from a cooperative perspective. In a cooperative setting, considerations of bargaining power become particularly important. This may require a more explicit account of possible relationships between inequality and power. Other factors we abstract from here, but may affect cooperative decision-making include reciprocity and social norms. The importance of such factors for the emergence of cooperative behaviours have been shown, for example, by Bicchieri (2006) and, within an evolutionary-game-theoretic framework, by Sethi and Somanathan (1996; 2003) and Noailly et al. (2007).

of the final good, this is the agent with the larger endowment of land. This force is the prominent force when efforts are highly substitute. Typically, however, the production technology for the collective good displays some degree of complementarity between agents' efforts. In such cases, the effort *mix*, alongside with aggregate effort, becomes critical for the level of collective good provision. Hence, a second force kicks in, which seeks to correct the effort-augmenting effect by distributing water so as to reach the optimal mix of effort. As we will show, this 'effort-mix' force calls for more egalitarian or even progressive water allocation rules.

The role of inequality has been much debated in the collective action literature, with theoretical works suggesting that inequality can have either positive (Olson, 1965; Alix-Garcia, 2007), negative (Ostrom, 1990), non-linear (Dayton-Johnson and Bardhan, 2002; Baland et al., 2007), or ambiguous (Baland and Platteau, 1997; Bardhan et al., 2006) effects on collective action. Much like the theoretical work in this area, the results from econometric and experimental studies are rather mixed with authors finding that inequality tends to reduce public good provision (Bergstrom et al., 1986; Anderson et al., 2003), while others report higher contributions (Chan et al., 1996; Cardenas, 2002; Cherry et al, 2003). A closer look at this wide range of results suggests that inequality often interact with other factors - e.g., technological properties (Baland et al. 2007), and the degree of publicness of the collective good in question (Bardhan et al., 2006) - that may affect the 'sign' of its impact.

One aspect that has emerged as critical from recent empirical analyses is the relationship between inequality and institutions such as the rules that distribute collective outcomes. Institutions may influence the success with which a community undertakes collective action by shaping agents' returns from cooperation.

The nature of the relationship between inequality and rules, however, is not straightforward: in some studies (e.g., Dayton-Johnson 1999, 2000), allocation rules that favour the rich are more frequently observed in communities characterized by relatively high degrees of inequality, while in others relatively fairer rules are observed in more unequal communities (Bardhan, 2000, and Khwaja 2001).

The forces identified in this paper and the way they depend on technological features contribute to shed some light on the mechanisms linking inequality, rules and incentives. The remaining of the paper is organised as follows. Section 2 illustrates the features of the model. Section 3.1 derives and discusses the main results. Further discussion is provided in section 3.2, where a special case for the production technology of the collective good is considered. Section 4 concludes.

2 Model setup

2.1 Definitions and assumptions

Consider two types of farmer: 1 and 2. Each type is endowed with an amount of irrigable land l_i, with $l_i > 0$ and $i = \{1, 2\}$. Let $l \equiv l_1 + l_2$ denote the total amount of land in the economy. Farmers' endowments can then be defined as: $l_1 = \lambda \times l$ and $l_2 = (1 - \lambda) \times l$, with $\lambda \in (0, 1)$. In the remainder of the paper, we normalize l to one and assume $\lambda > 0.5$. The two types can, therefore, be interpreted as the representatives of two different farmer groups: large landowners (type 1), and small landowner (type 2).

Farmers can voluntarily engage in a joint project for the maintenance of a network of irrigation channels. Collective-maintenance activity increases the supply of water available for irrigation. Better maintenance, for example, leads to lower losses from filtration, leakage and sedimentation. The output of the project, Z, is represented by the average water flow delivered through the system and is a function of farmers' efforts: e_1, e_2. Specifically, we parametrize the production technology for Z by using a CES production function:[4]

$$Z = F(e_1, e_2) = [e_1^\sigma + e_2^\sigma]^{\frac{1}{\sigma}} \tag{1}$$

where $\sigma < 1$ measures the degree of complementarity between individual efforts. Agents' efforts are assumed to be unobservable (or not enforceable). The collective output, Z, is divided among farmers according to some allocation rule $\Gamma = (\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2)$, where $\boldsymbol{\gamma}_1$ and $\boldsymbol{\gamma}_2$ are farmers' shares in Z, with $\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2 \geq 0$ and $\boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2 = 1$. When convenient, we will simplify the notation as follows: $\boldsymbol{\gamma}_1 = \gamma$, $\boldsymbol{\gamma}_2 = 1 - \gamma$.

The amount of water allocated to a farmer according to the allocation rule Γ is given by $z_i = \gamma_i Z$ with $i = \{1, 2\}$. Each agent uses two inputs, land and water, to produce a final good. Agent i's payoff is defined as:

$$\Pi_i = f(l_i, z_i) - e_i$$

where $f(l_i, z_i)$ is the individual production function for the final good and e_i is i's contribution for the maintenance of the irrigation network.

We assume that the cost of e_i units of effort is simply e_i and that the pro-

[4] CES production functions cover the whole spectrum of substitution and complementarity among efforts. For example, when the parameter σ in Equation (1) tends to one, the production technology for Z approximates a linear production function; as σ approaches zero, the isoquants of the CES looks like the isoquants of the Cobb-Douglas production function; while in the limit case for σ that approches $(-\infty)$, the CES function approximates a Leontiev technology where efforts are perfect complements. Hence, although they impose a regularity in the shape of isoquants, CES production functions allow considering a wide range of collective action relevant to water resources - from small dam construction to channel maintenance and pollution reduction activities, where the degree of complementarity among individual efforts is progressively increasing.

duction technology for the final good is well represented by the following Cobb-Douglas production function[5]

$$f(l_i, z_i) = (z_i)^{\alpha} (l_i)^{1-\alpha}, \text{ with } \alpha \in (0,1)$$

From the complementarity between l_i and z_i in (2), it follows that the marginal return to water is an increasing function of land.

Although the paper focuses on land inequality as the only source of heterogeneity, an alternative interpretation is possible, which views the parameter λ as capturing some characteristic of an agent, such as skills or locational differences. As long as these characteristics affect the marginal productivity of water, this alternative interpretation is consistent with the analysis.

2.2 Individual optimization problem

Each agent chooses the level of effort that maximizes her own payoff, given the contribution made by the other. Specifically, for any given expectation $\overline{e}_2$ about the level of effort exerted by agent 2, type 1 solves the following problem

$$\max_{e_1 \geq 0} \Pi_1 = f(l_1, z_1(e_1, \overline{e}_2)) - e_1 = \left[\gamma(e_1^{\sigma} + \overline{e}_2^{\sigma})^{\frac{1}{\sigma}}\right]^{\alpha} (\lambda)^{1-\alpha} - e_1$$

The first-order condition is:

$$\frac{\partial \Pi_1}{\partial e_1} = 0 \Rightarrow \left[\alpha(\gamma)^{\alpha}(\lambda)^{1-\alpha}(Z)^{\alpha-1}\right] \frac{\partial Z}{\partial e_1} - 1 = 0 \tag{3}$$

From (1), the derivative of Z with respect to e_1 can be written as:

[5] Although specific, the Cobb-Douglas form has been widely used in economics because it generally fits the data well. Moreover, it displays complementarity between land and water as inputs of production, which seems a realistic feature of the production process for most agricultural products.

$$\frac{\partial Z}{\partial e_1} = (Z)^{1-\sigma}(e_1)^{\sigma-1}. \tag{4}$$

By substituting (4) into (3) and upon some calculation, we have:[6]

$$(e_1)^{\sigma} = (\alpha)^{\frac{\sigma}{1-\sigma}} \times \left[(\gamma)^{\alpha}(\lambda)^{1-\alpha}\right]^{\frac{\sigma}{1-\sigma}} \times (Z)^{\frac{\sigma(\alpha-\sigma)}{1-\sigma}} \tag{5}$$

Similarly, one can define type 2's optimization problem and obtain:

$$(e_2)^{\sigma} = (\alpha)^{\frac{\sigma}{1-\sigma}} \times \left[(1-\gamma)^{\alpha}(1-\lambda)^{1-\alpha}\right]^{\frac{\sigma}{1-\sigma}} \times (Z)^{\frac{\sigma(\alpha-\sigma)}{1-\sigma}} \tag{6}$$

By substituting (5) and (6) into equation (1) and rearranging the terms, the following expression for Z can be derived:

$$Z^* = \phi \times \left\{\left[(\gamma)^{\alpha}(\lambda)^{1-\alpha}\right]^{\beta} + \left[(1-\gamma)^{\alpha}(1-\lambda)^{1-\alpha}\right]^{\beta}\right\}^{\mu} \tag{7}$$

where $\phi \equiv (\alpha)^{\frac{1}{1-\alpha}}$, $\beta \equiv \frac{\sigma}{1-\sigma}$ and $\mu \equiv \frac{1-\sigma}{\sigma(1-\alpha)}$.[7] Equation (7) represents the amount of collective output produced in equilibrium.[8]

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[6] Using (4), we can write equation (3) as follows:

$$\alpha(\gamma)^{\alpha}(\lambda)^{1-\alpha}(Z)^{\alpha-1}(Z)^{1-\sigma}(e_1)^{\sigma-1} - 1 = 0$$

That is

$$\alpha(\gamma)^{\alpha}(\lambda)^{1-\alpha}(Z)^{\alpha-\sigma}(e_1)^{\sigma-1} = 1$$

which leads to

$${e_1}^{\sigma-1} = \left[\alpha(\gamma)^{\alpha}(\lambda)^{1-\alpha}(Z)^{\alpha-\sigma}\right]^{-1}$$

Raising both side to the power of $\sigma/(\sigma-1)$, we have

$${e_1}^{\sigma} = \left[\alpha(\gamma)^{\alpha}(\lambda)^{1-\alpha}(Z)^{\alpha-\sigma}\right]^{\frac{\sigma}{1-\sigma}}$$

which is equivalent to equation (5). The same step-by-step derivation applies to equation (6) once we have changed the index of the player from 1 to 2 in the maximisation problem.

[7] Notice that ϕ, β and μ are only well defined if $\alpha \neq 1$ and $\sigma \neq 1$, which will be assumed in the reminder of the paper.

[8] It can be shown that there exists another solution which involves $e_i = 0$ for all i. This, however, will be disregarded, as it implies $Z^* = 0$. The analysis will, instead, focus on the non-trivial equilibrium in which the collective output is positive.

3 Results

3.1 Inequality and Rules

In this section, we start by identifying the distribution of water that maximises the collective output produced in equilibrium, and then proceed to analyse how that is affected by inequality in initial conditions - as represented by $\lambda > 0.5$. The problem can be expressed as follows:

$$\max_{0\leq\gamma\leq1} Z^* = \phi \times \left\{ \left[(\gamma)^{\alpha}(\lambda)^{1-\alpha}\right]^{\beta} + \left[(1-\gamma)^{\alpha}(1-\lambda)^{1-\alpha}\right]^{\beta} \right\}^{\mu} . \tag{8}$$

If a solution interior to the interval [0,1] exists, then the following FOC must hold:

$$\begin{aligned}\frac{\partial Z^*}{\partial \gamma} &= \frac{\alpha}{1-\alpha} \times \left\{ \left[(\gamma)^{\alpha}(\lambda)^{1-\alpha}\right]^{\beta} + \left[(1-\gamma)^{\alpha}(1-\lambda)^{1-\alpha}\right]^{\beta} \right\}^{\mu-1} \times \\ &\quad \times \left\{ \left[(\gamma)^{(\alpha\beta-1)}(\lambda)^{(1-\alpha)\beta}\right] - \left[(1-\gamma)^{(\alpha\beta-1)}(1-\lambda)^{(1-\alpha)\beta}\right] \right\} \\ &= 0\end{aligned} \tag{9}$$

Notice that, for $\alpha \in (0,1)$ and $\lambda \in (0,1)$ the first two terms in (9) are strictly positive.

Condition (9) can, therefore, be simplified as follows:

$$\left[(\gamma)^{(\alpha\beta-1)}(\lambda)^{(1-\alpha)\beta}\right] - \left[(1-\gamma)^{(\alpha\beta-1)}(1-\lambda)^{(1-\alpha)\beta}\right] = 0 \tag{10}$$

By substituting for $\beta = \frac{\sigma}{1-\sigma}$ and solving with respect to γ, we obtain:

$$\gamma^* = \frac{\lambda^{\theta}}{\lambda^{\theta} + (1-\lambda)^{\theta}} \tag{11}$$

where: $\theta \equiv \frac{(1-\alpha)\sigma}{(1-\sigma-\alpha\sigma)}$.

It can be shown that for $\sigma < \frac{1}{1+\alpha}$ the maximization problem in (8) admits the interior solution derived above. Equation (11) can be interpreted as a 'weighted' index of the degree of inequality characterizing the economy. More precisely, inequality in land distribution is weighted by the parameter θ, which is a function of two elements: (i) the strategic importance of agents' efforts in the realization of the collective good – as measured by σ; and (ii) the relative importance of water compared to land in the production of the final good – as measured by α.

From (11), the derivative of γ^* with respect to λ is:

$$\frac{\partial \gamma^*}{\partial \lambda} = \frac{\theta \times [\lambda(1-\lambda)]^{\theta-1}}{\left[\lambda^\theta + (1-\lambda)^\theta\right]^2} \tag{12}$$

with $\theta \equiv \frac{(1-\alpha)\sigma}{(1-\sigma-\alpha\sigma)}$.

Given $\lambda \in (0,1)$, the sign of $\frac{\partial \gamma^*}{\partial \lambda}$ in (12) is the same as the sign of θ. Moreover, within the range of parameter values $\sigma < \frac{1}{1+\alpha}$, the sign of θ varies as follows: $\theta < 0$ for $\sigma < 0$; and $\theta > 0$ for $\sigma \in \left(0, \frac{1}{1+\alpha}\right)$. We show in the appendix that, for $\sigma \in \left(\frac{1}{1+\alpha}, 1\right)$, Z^* is still increasing in γ at the value $\gamma = 1$. In this case, the supply of irrigation water is maximized by setting $\gamma^* = 1$ for any $\lambda > 0.5$.[9]

Hence, the relationship between inequality and rules can be summarized as follows:

- For $\sigma < 0 \Rightarrow \frac{\partial \gamma^*}{\partial \lambda} < 0$;
- For $\sigma \in \left(0, \frac{1}{1+\alpha}\right) \Rightarrow \frac{\partial \gamma^*}{\partial \lambda} > 0$;
- For $\sigma \in \left(\frac{1}{1+\alpha}, 1\right) \Rightarrow \gamma^* = 1, \forall \lambda > 0.5$.

[9]The appendix also shows that the other possible corner solution, $\gamma = 0$, can never be a global maximum for any $\lambda \geq 0.5$.

For $\sigma < 0$ – that is, as one moves towards relatively high degree of complementarity between agents' efforts – the collective output is maximized by allocating a relatively larger share of water to the small landowner. The opposite holds within the interval $\sigma \in \left(0, \frac{1}{1+\alpha}\right)$ – that is, for lower degrees of complementarity. In this case, assigning more water to the large landowner favours the provision of the collective good and the share of the large landowner increases as inequality in land holding becomes more pronounced. Finally, when agents' contributions display relatively high degrees of substitutability – that is for $\sigma \in \left(\frac{1}{1+\alpha}, 1\right)$ – the supply of irrigation water is maximized by allocating all the water available to the large landowner, independently of the degree of inequality in landholding (i.e., for any $\lambda > 0.5$).

How can these results be interpreted? In the context of the present analysis, it is possible to identify two key forces which affect the distribution of water in opposite directions. We refer to the first force as 'effort-augmenting'. This force pushes the distribution of water towards the agent with the higher marginal return to water in the attempt to maximise the *aggregate* level of effort. Due to the complementarity between land and water in the production of the final good, this is the agent with the larger endowment of land. The effort-augmenting force is the prominent force when the production technology for the collective good displays relatively high degrees of substitutability among agents' efforts (i.e. for strictly positive values of σ). However, collective activities associated with the management of water resources generally display some complementarity in efforts. In the presence of complementarity, aggregate effort is not all that matters; indeed, the effort *mix* is also important. Inequality may hamper the achievement of the optimal effort-mix (with negative consequences on collective output) because it reduces the incentives to contribute of the small landowner.

Hence, a second force kicks in, which seeks to 'correct' for this by distributing water in a more progressive manner so as to reach the optimal mix of effort. For sufficient complementarity ($\sigma < 0$), this 'effort-mix' force tends to dominate.[10]

The use of water as an incentive mechanism has consequences that might seem at first glance counterintuitive in that it implies allocating more water to the agent with lower marginal returns. This, however, may still represent the 'constrained' optimum when other contracting possibilities are not available, as in the context considered here where effort is unobservable. Furthermore, in some institutional settings contracting over output may also be difficult due to lack of commitment on the part of the producers or to limited enforcement capacity on the part of governmental authorities.[11]

3.2 Collective production function: A special case

This section discusses a special case for the production technology of the collective good, which generates an interesting result as far as the interaction between inequality and rules is concerned. Specifically, we assume that the production technology for Z is well represented by the following Cobb-Douglas production function with constant returns to scale:[12]

[10] Applications of a similar idea can be found in the case study literature in relation to other forms of heterogeneity. In some villages in Nepal, for instance, institutional arrangements have been used to moderate the effects of locational heterogeneity on users' incentives to monitor and maintain their resources, by allowing more distant members to pay lower fees in exchange for more time spent in monitoring and maintenance work (Varughese and Ostrom, 2001; Adhikari et al., 2004; Adhikari and Di Falco, 2009).

[11] For simplicity, the model proposed in this paper assumes that output is deterministic. However, in the rural sector of developing economies, output tends to be highly sensitive to idiosincratic shocks. This, in turn, gives room to opportunistic behaviour by the parties - who might have an incentive to cheat about the amount of output being produced - thus adding further difficulties to the possibility of contracting over output.

[12] As explained in section 2, this can be thought of as a limit case of the CES production function.

$$Z = F(e_1, e_2) = e_1^{\sigma} e_2^{(1-\sigma)} \tag{13}$$

with $\sigma \in (0,1)$.

Type 1's maximization problem can be written as follows:

$$\max_{e_1 \geq 0} \Pi_1 = (\gamma Z)^{\alpha} (\lambda)^{1-\alpha} - e_1$$

The FOC for the above problem is given by:

$$\frac{\partial \Pi_1}{\partial e_1} = 0 \Rightarrow \alpha \times \left[(\gamma)^{\alpha}(\lambda)^{1-\alpha}\right] \times (Z)^{\alpha-1} \times \frac{\partial Z}{\partial e_1} - 1 = 0 \tag{14}$$

From (13), the derivative of Z with respect to e_1 is:

$$\frac{\partial Z}{\partial e_1} = \sigma {e_1}^{(\sigma-1)} {e_2}^{(1-\sigma)} \tag{15}$$

By substituting (15) into (14) and solving with respect to e_1, the following reaction function can be derived:

$$e_1(e_2) = \left[\alpha\sigma\gamma^{\alpha}\lambda^{(1-\alpha)} e_2^{\alpha(1-\sigma)}\right]^{1/(1-\alpha\sigma)}$$

Similarly, from type 2's maximization problem we have:

$$e_2(e_1) = \left[\alpha(1-\sigma)(1-\gamma)^{\alpha}(1-\lambda)^{(1-\alpha)} e_1^{\alpha\sigma}\right]^{1/[1-\alpha(1-\sigma)]}$$

Solving the system of farmers' reaction functions, the following equilibrium levels of effort can be obtained:

$$\left(e_1^* = g_1^{\left(\frac{1-\alpha(1-\sigma)}{1-\alpha}\right)} \times g_2^{\left(\frac{\alpha(1-\sigma)}{1-\alpha}\right)};\ \ e_2^* = g_1^{\left(\frac{\alpha\sigma}{1-\alpha}\right)} \times g_2^{\left(\frac{1-\alpha\sigma}{1-\alpha}\right)} \right) \tag{16}$$

where: $g_1 \equiv \alpha\sigma\gamma^{\alpha}\lambda^{(1-\alpha)}$, and $g_1 \equiv \alpha(1-\sigma)(1-\gamma)^{\alpha}(1-\lambda)^{(1-\alpha)}$.

The collective output produced in equilibrium is:

$$Z^* = (e_1^*)^{\sigma}\,(e_2^*)^{(1-\sigma)} = g_1^{\left(\frac{\sigma}{1-\alpha}\right)} \times g_2^{\left(\frac{1-\sigma}{1-\alpha}\right)} \tag{16}$$

The allocation scheme that maximizes the provision of the collective good is given by the rule γ that solves the following equation:

$$\frac{\partial Z^*}{\partial \gamma} = 0 \Rightarrow \left(\frac{1}{1-\alpha}\right) \times \left[g_1^{\left(\frac{\sigma}{1-\alpha}\right)} \times g_2^{\left(\frac{1-\sigma}{1-\alpha}\right)}\right] \times \left[\frac{\sigma}{\gamma} - \frac{(1-\sigma)}{1-\gamma}\right] = 0 \tag{17}$$

Note that (17) is not well defined for $\gamma = 0$ and $\gamma = 1$. In what follows, we assume $\gamma \in (0,1)$. Under this assumption, the first term in (17) is strictly positive since the parameters α, σ and λ vary within the interval $(0,1)$. Condition (17) can, therefore, be simplified as follows:

$$\frac{\sigma}{\gamma} - \frac{(1-\sigma)}{1-\gamma} = 0 \tag{18}$$

From (18), we have: $\gamma^* = \sigma$. Thus, collective output is maximised by allocating water according to farmers' marginal productivity of effort (MPE). This would imply equal distribution of water when MPE is identical across farmers. The Cobb-Douglas production technology, hence, leads to a special case in which the two forces identified in section 3.1 perfectly offset one another and the allocation rule γ^* is *independent* of the degree of inequality in land-holding.

4 Conclusions

This paper focused on collective action problems associated with the management of water resources at the local level. Specifically, it considered a situation in which two types of farmer, with unequal land endowments, can voluntarily engage in collective maintenance activities to enhance the amount of water available. Water is distributed according to some allocation rule and used by each farmer as an input of production in combination with land. Within this context, we investigated the relationship between land inequality and water allocation rules, by determining whether and how the former affects the distribution of water that maximises collective good provision.

We found that two opposing forces are at work. The logic behind the first force (effort-augmenting) is to maximise the aggregate level of effort. In the attempt to do so, such force pushes the distribution of water towards the agent with the higher marginal return to water. The complementarity between land and water in the production of the final good implies that the marginal return to water increases with land; consequently, the first force works in favour of the large landowner. The second force (effort-mix) seeks to allocate water so as to reach the optimal mix of effort and calls instead for more egalitarian or even progressive water allocation rules.

The trade-off between these two forces depends on technological features of the problem. Specifically, the first force is the prominent force when the production technology for the collective good is characterised by relatively high degrees of substitutability among efforts. In this case, the allocation rule that maximises collective output is such that a larger share of water is assigned to the large landowner and the share of the large landowner increases with the degree of inequality in land-holding. This result is in line with the Olson's argument - namely, inequality may favour collective good provision by enhancing the interest of the richest agent. However, this is not generally the case when the production technology for the collective good displays (as it typically does) some degree of complementarity. In particular, for sufficient complementarity we found that the effort-mix effect becomes relatively more important and so does the second force; in this case, more egalitaria or even progressive rules perform better in terms of collective good provision.

Although the paper focused on land inequality (measured by the exogenous parameter lambda), an alternative interpretation is possible that views lambda as capturing some characteristic of an agent, such as skills or locational differences. As long as these characteristics affect the marginal productivity of water, this alternative interpretation is consistent with the analysis.

There are several avenues along which to extend the present work. Here, we focused on the class of linear sharing rules; indeed, although the amount of water that each player receives depends on aggregate water output, the shares per se do not. The analysis of more general classes of mechanisms, where the shares may also vary with the level of output, could provide further insights. The paper examines how inequality and rules affect agents' incentives to contribute in a Nash-equilibrium; that is, it concentrates on the non-cooperative case. A second extension could be to study the problem from a cooperative perspective. In a cooperative setting, considerations of bargaining power become particularly important; this may require a more explicit account of possible relationships between inequality and power. Finally, it would be interesting to test the predictions of the model in laboratory or even field experiments; this could enrich the experimental literature on the subject by providing new insights and interpretations. For example, experiments based on linear public good games could be extended to account for the possibility that agents' efforts display varying degrees of complementarity, and heterogeneity affects the marginal benefits from contributing through the relationship between private and collective inputs.

Appendix

In the context of the present analysis, γ^* is the solution to the following maximization problem:

$$\max_{0\leq\gamma\leq1} Z^* = \phi \times \left\{ \left[(\gamma)^{\alpha}(\lambda)^{1-\alpha}\right]^{\beta} + \left[(1-\gamma)^{\alpha}(1-\lambda)^{1-\alpha}\right]^{\beta} \right\}^{\mu} \qquad (a.1)$$

where $\phi \equiv (\alpha)^{\frac{1}{1-\alpha}}$, $\beta \equiv \frac{\sigma}{1-\sigma}$ and $\mu \equiv \frac{1-\sigma}{\sigma(1-\alpha)}$.

By substituting for $\beta = \frac{\sigma}{1-\sigma}$, the FOC for the above problem can be expressed as follows:

$$(\gamma)^{\left(\frac{\alpha\sigma}{1-\sigma}-1\right)} \times (\lambda)^{\frac{(1-\alpha)\sigma}{1-\sigma}} - (1-\gamma)^{\left(\frac{\alpha\sigma}{1-\sigma}-1\right)} \times (1-\lambda)^{\frac{(1-\alpha)\sigma}{1-\sigma}} = 0 \qquad (a.2)$$

Condition $(a.2)$ is implicitly assuming that the solution to $(a.1)$ is *interior* to the interval $[0,1]$. However, the maximum may well be a corner solution. In other words, it may well be: $\gamma^* = 1$ and/or $\gamma^* = 0$. This appendix will show whether and under what conditions those limit values for γ could represent a solution to $(a.1)$.

Consider first $\gamma = 1$. This will be a maximum if $\frac{\partial Z^*}{\partial \gamma} > 0$ when γ approaches one. In that case, Z^* would still be increasing in γ at the value $\gamma = 1$.

Notice that, when the exponent of $(1-\gamma)$ in equation $(a.2)$ is negative, it cannot be true that $\gamma^* = 1$, because:

$$\frac{\alpha\sigma}{1-\sigma} - 1 < 0 \Rightarrow \lim_{\gamma\to1}(1-\gamma)^{\left(\frac{\alpha\sigma}{1-\sigma}-1\right)} = \lim_{\gamma\to1}\frac{1}{(1-\gamma)^{\left(1-\frac{\alpha\sigma}{1-\sigma}\right)}} = \infty$$

Hence,

$$\lim_{\gamma \to 1} \left[\frac{1}{(\gamma)^{\left(1-\frac{\alpha\sigma}{1-\sigma}\right)}} \times (\lambda)^{\frac{(1-\alpha)\sigma}{1-\sigma}} - \frac{1}{(1-\gamma)^{\left(1-\frac{\alpha\sigma}{1-\sigma}\right)}} \times (1-\lambda)^{\frac{(1-\alpha)\sigma}{1-\sigma}} \right] = -\infty$$

violating the FOC.

For $\sigma < \frac{1}{1+\alpha}$, the magnitude $\left(\frac{\alpha\sigma}{1-\sigma} - 1\right)$ is negative, and the solution to $(a.1)$ is, therefore, given by equation (11).

For $\sigma > \frac{1}{1+\alpha}$, we have:

$$\frac{\alpha\sigma}{1-\sigma} - 1 > 0 \Rightarrow \lim_{\gamma \to 1} (1-\gamma)^{\left(\frac{\alpha\sigma}{1-\sigma}-1\right)} = 0$$

Therefore:

$$\lim_{\gamma \to 1} \left[(\gamma)^{\left(1-\frac{\alpha\sigma}{1-\sigma}\right)} \times (\lambda)^{\frac{(1-\alpha)\sigma}{1-\sigma}} - (1-\gamma)^{\left(1-\frac{\alpha\sigma}{1-\sigma}\right)} \times (1-\lambda)^{\frac{(1-\alpha)\sigma}{1-\sigma}} \right] = (\lambda)^{\frac{(1-\alpha)\sigma}{1-\sigma}} > 0$$

which, in turn, implies that for any $\sigma > \frac{1}{1+\alpha}$, Z is maximised by setting $\gamma^* = 1$.

Consider now the limit case $\gamma = 0$. It is easy to prove that this can never be a global maximum within the compact set $[0, 1]$. Let:

$$Z_0 \equiv Z^*(\gamma = 0) = (\alpha)^{\frac{1}{1-\alpha}} \times \left[(1-\lambda)^{\frac{(1-\alpha)\sigma}{1-\sigma}} \right]^{\frac{1-\sigma}{\sigma(1-\alpha)}} = (\alpha)^{\frac{1}{1-\alpha}} \times (1-\lambda)$$

and

$$Z_1 \equiv Z^*(\gamma = 1) = (\alpha)^{\frac{1}{1-\alpha}} \times \left[(\lambda)^{\frac{(1-\alpha)\sigma}{1-\sigma}} \right]^{\frac{1-\sigma}{\sigma(1-\alpha)}} = (\alpha)^{\frac{1}{1-\alpha}} \times \lambda$$

For $\lambda > 0.5$ – as it was assumed in section 3 – it is straightforward to observe that $Z_0 < Z_1$. Then, $\gamma = 0$ cannot be a global maximum for $\gamma \in [0, 1]$, since there exists at least one value of $\gamma \in [0, 1]$ such that $Z^*(\gamma) > Z_0$.

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