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Title:

Polar-cap Plasma Patch Primary Linear Instability Growth-Rates Compared

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Key Points.

Growth rates for plasma linear instability processes are investigated

Turbulence, Gradient Drift and Current Convective Instabilities can all be important

Abstract

Four primary plasma instability processes have been proposed in the literature to explain the generation of phase scintillation associated with polar-cap plasma patches. These are the Gradient Drift, Current Convective and Kelvin-Helmholtz instabilities and a small-scale "Turbulence" process. In this paper the range of possible values of the linear growth-rates for each of these processes is explored using Dynamics Explorer 2 satellite observations. It is found that the inertial Turbulence instability is the dominant process, followed by inertial Gradient Drift, collisional Turbulence and collisional shortwave Current Convective instabilities. The other processes, such as Kelvin-Helmhotz, collisional Gradient Drift and inertial shortwave Current Convective instabilities very rarely (<1% of the time) give rise to a growth rate exceeding 1/60, that is deemed to be significant (in publications) to give rise to GPS scintillation.

Index terms: 2439, 2471, 2772, 6929

1. Introduction

Large regions of enhanced electron concentration drifting according to an $\mathbf{E} \times \mathbf{B}$ force from the auroral dayside ionosphere into the polar-cap region of the ionosphere were theorised by *Hill* [1963] as an explanation for "sporadic F" observations via f_oF2 measurements in these areas. This theory was confirmed experimentally by *Buchau et al.* [1983] using optical all-sky camera images and the structures (100s – 1000s km in scale) became generally known as polar-cap plasma patches. They were soon observed to cause radio scintillation on radio signals traversing them [e.g., *Weber et al.*, 1984]. Polar-cap plasma patches are also strongly correlated with backscatter of High Frequency (HF) Coherent Scatter Radar (CSR) signals, which occurs when the signals meet electron concentration irregularities [*Oksavik et al.*, 2006].

Irregularities cause scintillation through diffractive scattering [*Hargreaves*, 1992] or ionospheric lensing [*Booker and Majidiahi*, 1981] of the signal. Scintillation inducing irregularities are considered to form through a cascade of energy from longer wavelengths to shorter ones in a manner analogous to that which occurs in the generation of turbulence in neutral fluids [*Kintner and Seyler*, 1985]. This requires some initial process to create wavelike structures in the plasma where there are none to start with – a primary plasma instability [*Kelley*, 2009].

A plasma instability is any wavelike structure that can grow exponentially in amplitude from some initial perturbation that is assumed to pre-exist. In order for this to happen, some source of free energy – energy available to do work – is required [*Mikhailovskii*, 1992]. Such energy derives from a combination of the Earth’s geo-magnetic field and electric field mapped from the magnetosphere to the high-latitude ionosphere, (day to night) electron density gradients and atmospheric gravity wave (TID) ‘seeding’ [e.g. *Hysell et al.*, 2014]. In this paper we consider only the linear regime, where the instability (consisting of alternating regions of higher and lower electron concentration) is assumed to grow exponentially in amplitude, at a particular growth rate. This linear regime is most likely to occur on the bottomside of the F-region, with non-linear processes dominating in the topside, particularly in the Equatorial region [*Hysell et al.*, 2014].

2. Theoretical background

In linear instability theory, the amplitude, A, of the unstable wave can be described as $A = A_0 e^{\gamma t}$ where A_0 is the amplitude of the initial perturbation, t is the elapsed time since the start of the instability process and γ is the linear growth-rate of the instability. If there is more than one possible mode in a given set of physical circumstances, the relative magnitudes of γ provide a measure of which process will dominate the formation of irregularities within the plasma patch. It should, therefore be possible to calculate γ for each process and determine which instability is ultimately responsible for radio scintillation associated with plasma patches [e.g., *Burston et al.*, 2009; 2010].

Doing this is complicated by the natural variation in the parameters that appear in the growth-rate equations from one patch to the next or even for a single patch, both spatially and temporally. For example, the electric field (which appears in all the growth-rate equations treated here) can be viewed as fluctuating in magnitude and direction around a quasi-D.C. component that in itself varies in magnitude and direction on a time scale of 10s of minutes and spatial scale of hundreds of kilometers [*Hanson et al.*, 1993] and the orientation of the field with respect to a gradient in electron concentration is always important. Hence, for any given instability the growth-rate can take a range of values, depending on the range of values possible for the parameters involved. For this reason different instabilities may dominate in different circumstances that depend on the frequencies of occurrence and ranges of values of various solar-terrestrial parameters that appear in growth-rate equations.

Several primary instabilities have been proposed as the cause of scintillation inducing irregularities associated with plasma patches. These are the Gradient Drift Instability (GDI), which has received the most attention by researchers [e.g., *Burston et al.*, 2009; 2010; *Gondarenko, and Guzdar*, 1999; 2001; 2004a; 2004b; 2006a; 2006b; *Gondarenko et al.*, 2003; *Guzdar et al.*, 1998; *Sojka et al.*, 1998], the Current Convective Instability (CCI) which is a modification of the GDI [*Kelley*, 2009], a “turbulent” process [*Burston et al., 2010; Earle and Kelley, 1993; Kelley, 2009*] and the Kelvin-Helmholtz Instability (KHI) [*Basu et al.*, 1990; *Carlson et al.*, 2007; *Gondarenko, and Guzdar*, 2006a; *Oksavik et al.*, 2012].

With four instabilities under examination it would be expected that there would be four growth-rate equations but for a single instability, there can in fact be more than one equation. This is because the growth-rates may differ in the collisional and inertial plasma regimes and in the long and short perturbation wavelength assumptions. In the inertial regime only the bulk electric fields are important. In the collisional regime, the effects of individual particles’ electric fields must be taken into account. The peak of the F-region (at ~300km) of the ionosphere represents a transitional region from collisional to inertial regimes (delineated by where the collisional growth rate and ion-neutral collision rate become equal). Thus, it is necessary to consider both situations [e.g. *Sojka et al*., 1998]. Each instability is outlined below in its own section, with the appropriate growth-rate equations given.

2.1 The Gradient Drift Instability

The Gradient Drift Instability (GDI) requires a horizontal gradient in electron concentration, parallel to the electric field and a vertical magnetic field. Polar-cap patches potentially meet all these requirements as the geo-magnetic field is approximately vertical in the polar-cap, the electric field has a large horizontal component and the patch itself has gradients in all horizontal directions [e.g., *Basu et al.*, 1990; *Burston et al.*, 2009; *Chaturvedi et al.*, 1994; *Gondarenko,* *and Guzdar*, 1999; 2001; 2004a; 2004b; 2006a; 2006b; *Gondarenko and Guzdar*, 2004a; *Gondarenko et al.*, 2003; *Gondarenko and Guzdar*, 1999; 2001; 2004b; 2006; *Guzdar et al.*, 1998]. The growth-rate in the collisional regime is given by

$$\gamma_{cGDI} = \frac{\mathbf{E}\times\mathbf{B}}{B^2}\left(\frac{\nabla n}{n}\right) \tag{1}$$

where **E** is the electric field vector, **B** is the magnetic field vector and *n* is the electron concentration (Figure 1) and ∇n is the gradient of electron concentration per metre (Figure 2). In the inertial regime the growth rate is given by

$$\gamma_{iGDI} = \left[v_{in}\frac{\mathbf{E}\times\mathbf{B}}{B^2}\left(\frac{\nabla n}{n}\right)\right]^{\frac{1}{2}} \tag{2}$$

[*Sojka et al.*, 1998] where v_{in} is the ion-neutral collision frequency, given by

$$v_{in} = \pi n_n r^2\left(\frac{4kT}{\pi M}\right)^{\frac{1}{2}} \tag{3}$$

where, in turn, *r* is the colliding particles' radius, assumed the same for all species and *M* is the colliding species' mass, also assumed the same for all species. *T* is the temperature, assumed the same for all species and n_n is the neutral concentration. The constant, *k*, is Boltzmann's constant [*Kauzman*, 1966]. This formula for the ion-neutral collision frequency is in fact that for the collision of two neutral species and a more accurate version would take into account the small correction required for a singly ionized species colliding with a neutral species. The assumption that *T* is the same for all species is valid in quiet geomagnetic conditions only ($K_p<2$) otherwise the collision rate will be larger than those used in this study. In the following, n_n is taken as the density of monatomic oxygen (figure 3) and *T* is the O^+ ion temperature (figure 4) as these are the main constituents of the neutral and ionized atmosphere at F-region heights.

2.2 The Current Convective Instability

The Current Convective Instability may occur when the criteria for the GDI are met and current flows parallel to the magnetic field (z-axis). Additionally, the initial perturbation must have a finite component along the direction of ∇n (y-axis). For the CCI, there is a wavelength dependency as well as a difference between the collisional and inertial regimes, leading to four equations [*Chaturvedi and Ossakow*, 1979; 1981; *Huba*, 1984; *Kelley*, 2009; *Ossakow and Chaturvedi*, 1979]:

The growth rate for the CCI, collisional long wavelength regime is given [e.g. *Huba*, 1984] by

$$\gamma_{lwcCCI} = -k_z \left(\frac{qB}{m_i v_{in}}\right)\left(\frac{n_1 - n_2}{n_1 + n_2}\right)\left(\frac{\mathbf{E}\times\mathbf{B}}{B^2}\right)\left\{1 + \left(\frac{k_z}{k_y}\right)^2 \left(\frac{q^2 B^2}{m_i m_e v_{in} v_{ei}}\right)\right\}^{-\frac{1}{2}} \tag{4}$$

where q is the charge on the electron, k_z and k_y are the wavenumbers in the z and y directions respectively, n_1 and n_2 are the electron concentrations either side of a step-boundary perturbation (with $n_2 > n_1$), m_e is the mass of an electron, m_i is the mass of an ion and v_{ei} is the electron-ion collision frequency. The latter, in S.I. units, is given [e.g. *Ichimaru*, 1973] by

$$v_{ei} = \frac{nq^4}{4\pi\varepsilon_0^2 m_e^{1/2} (3kT)^{3/2}} \ln\left[12\pi n \left(\frac{nq^2}{\varepsilon_0 kT}\right)^{-3/2}\right] \tag{5}$$

The growth rate for the CCI collisional short wavelength regime is given [e.g. *Huba*, 1984] by

$$\gamma_{swcCCI} = -\frac{k_z}{k_y}\left(\frac{qB}{m_i \nu_{in}}\right)\left(\frac{\nabla n}{n}\right)\left(\frac{\mathbf{E}\times\mathbf{B}}{B^2}\right)\left\{1+\left(\frac{k_z}{k_y}\right)^2\left(\frac{q^2B^2}{m_i m_e \nu_{in}\nu_{ei}}\right)\right\}^{-1} \qquad (6)$$

The growth rate for CCI inertial long wavelength regime is given [e.g. *Huba*, 1984] by

$$\gamma_{lwiCCI} = \left(\frac{\nu_{ei} m_e B^2}{m_i \mathbf{E}\times\mathbf{B}}\right)^{1/3}\left[k_y\left(\frac{n_1-n_2}{n_1+n_2}\right)\right]^{2/3}\frac{\mathbf{E}\times\mathbf{B}}{B^2} \qquad (7)$$

CCI, inertial short wavelength:

$$\gamma_{swiCCI} = \left(\frac{\nu_{ei} m_e n B^2}{4 m_i \mathbf{E}\times\mathbf{B}\,\nabla n}\right)^{1/3}\frac{\mathbf{E}\times\mathbf{B}}{B^2}\frac{\nabla n}{n} \qquad (8)$$

The short wavelength regime applies when kL << 1 and the long wavelength regime applies when kL >> 1, where k is the perturbation wavenumber and $L = n/\nabla n$ is the electron concentration gradient scale length. L is given by *Huba and Ossakow* [1980] as 10-100km. The perturbation wavelength on a plasma patch will be of the order of several tens of km or less, making the short wavelength equations the more applicable.

2.3 Turbulence

The Turbulence process was first put forward by *Kelley and Kintner* [1978] and further examined in *Burston et al.* [2010] and *Kelley* [2009]. It takes into account the fact that the electric field fluctuates around the quasi-D.C. component indicated by **E** in the preceding equations. If it is assumed that these fluctuations are random in direction then there is always a direction in which the instability can occur. The growth-rate equation replaces the velocity (given by $(\mathbf{E} \times \mathbf{B})/B^2$) with its equivalent integrated over all relevant wavenumbers of the fluctuations of the electric field.

$$\left[\int_{k_L}^{\infty} E(k)^2 dk\right]^{\frac{1}{2}} /B \tag{9}$$

where k_L is the smallest relevant wavenumber, in this case 2π/(scale of patch), or of order 2π/100,000m. Substituting Eq.9 into Eq.1 gives the growth rate for the turbulent collisional regime as

$$\gamma_{cT} = \left[\int_{k_L}^{\infty} E(k)^2 dk\right]^{\frac{1}{2}} \left(\frac{\nabla n}{nB}\right). \tag{10}$$

Substituting Eq.9 into Eq.2 gives the growth for the turbulent inertial regime as

$$\gamma_{iT} = \left\{v_{in} \left[\int_{k_L}^{\infty} E(k)^2 dk\right]^{\frac{1}{2}} \left(\frac{\nabla n}{nB}\right)\right\}^{\frac{1}{2}} \tag{11}$$

2.4 Kelvin-Helmholtz Instability

The idea that primary ordinary Kelvin-Helmholtz Waves structure plasma patches was examined observationally by *Basu et al.* [1990] and modelled by *Gondarenko, and Guzdar* [2006a] who concluded that primary shear slows the development of the GDI. The idea was advanced again by *Carlson et al.* [2007] who, in contradiction of this earlier work suggested it would accelerate the generation of irregularities. The Carlson hypothesis was further tested in a case-study by *Oksavik et al.* [2012] where it was found not to be dominant over the GDI except in a narrow band of wavelengths. The growth-rate used in these KHI studies of $2\Delta U/L$ (where ΔU is the velocity change across a shear zone and L is the distance scale length) disguises the electric and geomagnetic field dependence and is therefore not appropriate for comparison with the equations already given in this paper. Instead, the equation used here is derived as a special case of the dispersion relation equation given in *Mikhailovskii* [1992] for ordinary Kelvin-Helmholtz Waves.

$$\frac{(\omega - kV_1)^2}{C_{A1}^2} + \frac{(\omega - kV_2)^2}{C_{A2}^2} - 2k^2 = 0 \qquad (12)$$

where C_1 is the Alfven speed in zone 1, C_2 is the Alfven speed in zone 2, V_1 is the plasma velocity in zone 1 and V_2 is the plasma velocity in zone 2 and k the perturbation wavenumber. If n_1 and n_2 are the electron densities in zone 1 and 2 respectively, then the Alfven speed is given by $C_j^2 = \frac{B^2}{\mu_0 (m_i + m_e) n_j}$ with $j = 1$ or $j = 2$ for the two different zones and m_i and m_e are the ion and electron masses respectively.

Assuming that the patch (taken as zone 1) is moving and its surroundings (taken as zone 2) is stationary then $V_1 = V = |(\mathbf{E} \times \mathbf{B})/B^2|$ and $V_2 = 0$. Hence, Eq.12 becomes

$$\frac{(\omega - kV)^2}{C_1^2} + \frac{\omega^2}{C_2^2} - 2k^2 = 0 \qquad (13)$$

Solving for ω gives:

$$\omega = \left(\frac{kVC_2^2}{C_1^2 + C_2^2}\right) \pm \frac{\left\{4k^2V^2C_2^4 - 4\left(C_1^2 + C_2^2\right)\left(k^2V^2C_2^2 - 2k^2C_1^2C_2^2\right)\right\}^{\frac{1}{2}}}{2\left(C_1^2 + C_2^2\right)} \qquad (14)$$

and a growth-rate (rightmost term) given by

$$\gamma_{KH} = \frac{k\{V^2C_2^4 - (C_1^2+C_2^2)(V^2C_2^2 - 2C_1^2C_2^2)\}^{\frac{1}{2}}}{(C_1^2+C_2^2)} \tag{15}$$

which simplifies to

$$\gamma_{KH} = \frac{kC_1C_2\{2(C_1^2+C_2^2) - V^2\}^{\frac{1}{2}}}{(C_1^2+C_2^2)} \tag{16}$$

with instability condition

$$V^2 > 2(C_1^2 + C_2^2) \tag{17}$$

For unstable growth, there must be an imaginary component to ω, which implies a negative square root in Eq.16.

Substituting V, C_1^2 and C_2^2 into Eq.16 gives

$$\gamma_{KH} = \frac{k(n_1n_2)^{1/2}}{(n_1+n_2)}\left[\frac{2B^2}{\mu_0(m_i+m_e)}\left(\frac{1}{n_1}+\frac{1}{n_2}\right) - \left(\left|\frac{\mathbf{E}\times\mathbf{B}}{B^2}\right|\right)^2\right]^{1/2} \tag{18}$$

The instability condition becomes

$$\left|\frac{\mathbf{E}\times\mathbf{B}}{B^2}\right|^2 > \frac{2B^2}{\mu_0(m_i+m_e)}\left(\frac{1}{n_1}+\frac{1}{n_2}\right) \tag{19}$$

It is notable that the instability condition is independent of wavenumber but larger wavenumbers (shorter wavelengths) will grow faster.

The above four processes can be divided into two categories; those that are dependent on a specific geometrical relationship between the electric field and the electron concentration gradient and those that are not. The GDI, whether collisional or inertial, falls into the former category, whereas the others fall into the latter. Specifically, the GDI requires that the electric field be parallel to the gradient, which in turn means it will only operate on the trailing edge of a patch. (On the leading edge, the electric field would be anti-parallel to the gradient.) At first glance it might be thought that this should apply to the Turbulence process, too, but in fact, because it is assumed that the A.C. fluctuations in the electric field have no directional bias, all slopes are unstable approximately half the time.

It should be noted here that another possibility has been suggested for the structuring of plasma patches; particle precipitation during formation. Anisotropic precipitation of energetic particles from above the ionosphere would lead to localized regions of increased ionization and hence gradients in electron concentration within the plasma patch. By modifying n and ∇n in the patch and causing localized heating, this process would affect the subsequent growth-rate of all plasma instabilities discussed here. This process would only occur during the formation of the patch and would cease once the patch left the region of the cusp [*Oksavik et al.*, 2012; *Walker et al.*, 1999]. Since it is not itself an instability process it cannot be directly investigated by the method outlined below.

3.0 Method

The possible magnitudes of the growth-rate for the four processes discussed above (for both inertial and collisional regimes) are assessed below using some 550 days of data from the Dynamics Explorer 2 (DE2) satellite covering August 1981 to February 1983. Thus, the DE2 data are concentrated at the peak of solar-cycle (#21) and therefore do not represent a whole solar cycle. Similarly, only data recorded by the satellite when at 300 – 400 km altitude and greater than 65° magnetic latitude (north and south) are used in order to constraint our study to the F2 peak and polar cap regions. For illustration we do not separate the two hemispheres. However, it should be noted that there will be differences between hemispheres because of measurement times and the offset of the geographic from the geomagnetic and dip poles [e.g. *Coley and Heelis,* 1998]. The DE2 data were chosen as all of the parameters in the growth rates equations used are either directly observed by, or can be calculated from, DE2 observations, with a minimum in the way of approximation or assumption, except with regard to the electric field because of a partial instrument failure (discussed below) and the wavenumbers of initial perturbations, the range of values of which are assumed. Full details of the instruments aboard the satellite can be found in [*Hoffman et al.*,1981]. In the following, the relative magnitudes of growth-rates for all the relevant processes are compared. This done by randomly sampling the data for each parameter, which are then use to calculate all possible values of growth-rate for each growth-rate equation.

The values of v_{in} and v_{ei} used in the following were calculated from DE2 data by using Eq.3 and Eq.5 respectively. The values for T are taken as the (O^+) ion temperatures (whose histogram of occurrence is displayed in Figure 4), n_n is the neutral monatomic oxygen concentration (histogram of occurrence displayed in Figure 3) and $n = n_i$ (i.e. the ion and electron concentrations are assumed equal and histogram of occurrence is displayed in Figure 1). The full field strength B (histogram of occurrence displayed in Figure 5) is also approximated by the vertical magnetic field component (B_z). In the following only the magnitude of the growth rate is calculated (vector quantities are replaced by scalar ones in all cases) simplifying the equations slightly.

The Electric field data comes from the Vector Electric Field Instrument (VEFI) on board DE2. This consisted of three mutually perpendicular instruments measuring orthogonal electric field strength components, allowing recovery of the full electric field strength vector $\mathbf{E}$ [*Heppner et al.*, 1978a; 1978b]**.** Unfortunately the z-axis instrument did not deploy and no z-axis data were recorded. Because the axes were relative to the spacecraft, it is not possible to determine the components of the field in the Earth centred co-ordinate system without the z-axis data. Hence a method of approximating values of $E = |\mathbf{E}|$ had to be applied. Examination of the relevant data recorded in the x-y plane (spacecraft co-ordinates) showed that 50% of the time the values of each component E_x and E_y were the same order of magnitude and over 90% of the time E_x and E_y differed by no more than one order of magnitude (See Table 1). In the case of the A.C. field (see also Table 1) the E_x and E_y components also differed from each other by no more than one order of magnitude over 90% of the time. Since the spacecraft co-ordinate system continuously varied in relation to the Earth centred co-ordinate system, it was assumed that E_z would also rarely differ from E_x and E_y by more than this. Hence the approximation

$E_z^2 \approx \left(E_x^2 + E_y^2\right)/2$ was adopted, leading to

$$E \approx \sqrt{\tfrac{3}{2}\left(E_x^2 + E_y^2\right)}. \tag{20}$$

Given all of the above, the equation used for the collisional GDI growth rate, Eq.1, becomes

$$\gamma_{cGDI} = \frac{E}{B}\frac{|\nabla n|}{n} \tag{21}$$

∇n was derived from DE2 Langmuir probe observations of electron concentration n (as displayed in Figure 1) in the following manner: First, only data recorded at 300 – 400 km altitude were admitted, in order to retain only F-region altitudes. Second, only data at > 65° magnetic latitude were admitted (in either hemisphere), in order to retain only geographical regions where patches plausibly occur. The 65º magnetic latitude was chosen to capture patches during storms equatorward of the polar cap, as patches are more prevalent during such storm time conditions. Third, only gradients in the remaining data, indicative of a plasma patch, were admitted. Such gradients were defined as being at least 40% slopes over a distance of 140 ($\pm 5\%$) km, following [*Coley and Heelis*, 1995]. We believe that using this distance should exclude the smaller non-patch structures within the auroral oval. Such gradients were calculated by dividing the electron concentration data into sequences covering 140 ($\pm 5\%$) km, performing a linear regression on them and taking the resulting straight-line slope as $|\nabla n|$. Note that such sequences may overlap each other and that these slopes do not necessarily represent the steepest gradients possible on a given structure as they are only the slope in the direction of motion of the satellite. The histogram of $|\nabla n|$ occurrence, as derived from DE2 is displayed in Figure 2.

The values of all other parameters used in this and all subsequent cases were those observed during the time periods when admissible gradients were present. Hence the subset of the DE2 data used plausibly represents conditions within patches. No attempt has been made to independently verify the existence of patches at these times and places.

For the inertial GDI case, Eq.2 becomes

$$\gamma_{iGDI} = \left(v_{in}\frac{E|\nabla n|}{Bn}\right)^{1/2} \tag{22}$$

For the collisional, short wavelength CCI, Eq.6 becomes

$$\gamma_{swcCCI} = \frac{k_z}{k_y}\left(\frac{qE|\nabla n|}{nm_i v_{in}}\right)\left\{1+\left(\frac{k_z}{k_y}\right)^2\left(\frac{q^2B^2}{m_i m_e v_{in} v_{ei}}\right)\right\}^{-1} \tag{23}$$

For the inertial, short wavelength CCI, Eq.8 becomes:

$$\gamma_{swiCCI} = \left(\frac{v_{ei} m_e |\nabla \mathrm{n}|^2 E^2}{4 m_i n^2 B^2}\right)^{1/3} \tag{24}$$

For the KHI, Eq.18 becomes

$$\gamma_{KH} = \frac{k(n_1 n_2)^{1/2}}{(n_1+n_2)}\left[\frac{2B^2}{\mu_0(m_i+m_e)}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)-\left(\frac{E}{B}\right)^2\right]^{1/2} \tag{25}$$

with n_1 and n_2 taken as the maximum and minimum figures in the electron concentration used to calculate $|\nabla n|$.

In Eqs.21-25, inclusive, the parameter, E, refers to the quasi-D.C. measurements of the electric field. For the two Turbulence equations, the A.C. component is required. This was measured by DE2 in the range 4Hz-1024Hz across eight channels, so the integral over the measured region becomes $\sum_{k=2\pi/1024}^{2\pi/4} E(k)^2$ taking Eq.20 into account for each $E(k)$ before summation.

Eq.10, for the collisional turbulent growth rate becomes

$$\gamma_{cT} = \left(\sum_{k=2\pi/1024}^{2\pi/4} E(k)^2\right)^{1/2} \left(\frac{|\nabla \mathrm{n}|}{nB}\right) \qquad (26)$$

For the inertial turbulent regime, Eq.11 becomes

$$\gamma_{iT} = \left\{v_{in} \left(\sum_{k=2\pi/1024}^{2\pi/4} E(k)^2\right)^{1/2} \left(\frac{|\nabla \mathrm{n}|}{nB}\right)\right\}^{1/2} \qquad (27)$$

The ideal approach to exploring the range of values of the growth rates for each process is to calculate a value of γ for every possible combination of the necessary input measurements (e.g. the Magnetic and Electric field data as summarized in Figures 5-7). However, given the subset of DE2 data selected this would become unmanageably large. Hence, in order to handle the number of output values of γ in a sensible time it was necessary to take representative samples of the full data sets of each input parameter. The method of simple random sampling was adopted, using the following equation to determine the minimum sample size, S, required:

$$S = p/[1 + p/P] \qquad (28)$$

Where $p = Z^2/(4M^2)$, M is the desired margin of error expressed as a fractional percentage, P is the size of the population being sampled and Z is a factor chosen dependent on the required confidence limit. The practical memory constraint led to the best possible sampling regime of a

95% confidence limit, corresponding to $Z = 1.95$, and a margin of error of 9% ($M = 0.09$) leading to a sample size of ~116 for each parameter (as given in Table 2).

4. Results

In general there are two sources of error relating to these results: instrument error and sampling error. The former applies to all the basic input measurements and introduces some level of uncertainty in their values; these are then combined when the calculation is made, generating an over-all level of uncertainty that could be estimated from the individual uncertainties. The second arises since, instead of calculating gamma for every possible combination of input measurements (a practical impossibility) a representative sample of each input population has been used, instead. Again, an overall uncertainty due to this sampling could be calculated and combined with the instrumental error. However there is a third source of error relating specifically to the electric and magnetic fields. These have been taken as their full-field magnitudes when, more rigorously, the horizontal and vertical components, respectively, should be used. This introduces an over-estimating bias in both of these measurements. Finally, the approximation necessitated by the failure of the z-axis Langmuir probe to deploy leads to an error in the electric field magnitude such that the values used can only be considered accurate to order-of-magnitude. This final source of error swamps the others. Hence, all values of γ presented below should be considered to be only order-of-magnitude accurate.

In [*Carlson et al.*, 2007] evidence for a rise-time $(1/\gamma)$ of 60s or less is presented for patches still in the cusp region. Hence, in the following the various growth rate mechanisms are compared to find which can give values for $\gamma > 1/60$ and under what circumstances. The percentage of derived growth rates (using the sample data) exceeding, 1/6, 1/60 and 1/600 for each of the four processes are presented in Table 3. All processes except Kelvin-Helmholtz Instabilities (KHI) are found capable of giving rise-times less than 60s using the DE2 data. For collisional Gradient Drift and shortwave inertial Current Convective instabilities this occurs less than 1% of the time, but occurs for the majority of the time for inertial Turbulence instabilities and around 10% of the time for inertial Gradient Drift and collisional Turbulence and shortwave Current Convective instabilities.

5. Conclusions

The derived distributions of possible linear growth-rates for each mechanism show that only Turbulence, inertial GDI and shortwave collisional CCI would regularly give the rise-times of approximately 60s or less observed in the cusp region during patch formation [e.g. *Carlson et al.*, 2007 and references there-in]. However, the relative importance of the various mechanisms could be significantly different in the non-linear regime [*Gondarenko, and Guzdar*, 2006; *Gondarenko and Guzdar*, 2004a; *Gondarenko et al.*, 2003; *Gondarenko and Guzdar*, 1999; 2004b; *Guzdar et al.*, 1998]. In this regard, the linear regime assumes the instability grows exponentially in amplitude, at the growth rate γ given by the above. Whilst there are various mechanisms that can drive an instability, one possibility is an increasing velocity shear, δV (due to an exponentially increasing perturbation in electric field, $\delta E = \delta E_0 e^{\gamma t}$**)** which over time could become large enough to drive secondary, Kelvin-Helmholtz Instabilities (KHI) that damp the growth of the instability, ending the linear growth regime. Based on Eq. 19, taking $n_1 = n + \delta n$, $n_2 = n - \delta n$ (where n is the background electron density) and making the not unreasonable assumptions that $n^2 \gg \delta n^2$ and $m_i + m_e \approx m_i$, this occurs for the ordinary KHI when $t > t_c$ where

$$t_c \sim \left(\frac{1}{\gamma}\right) \ln\left(\frac{2B^2}{\delta E_0}\sqrt{\frac{1}{\mu_0 n m_i}}\right) \tag{28}$$

Using the sample mean values for B, E and n this would indicate that the linear regime would likely last for a duration of order 10/γ. This would indicate that grow rates greater than 1/6 would imply that the mechanism becomes non-linear within minutes, whereas growth rates less than 1/600 would take several hours for the mechanism to become unstable.

There are two limitations with regard to the method deployed here for comparing growth-rates for candidate irregularity generating mechanisms associated with polar-cap plasma patches that cannot easily be overcome. First, the data are concentrated at the peak of solar-cycle 21 (late 1981 – early 1983) and therefore do not represent a whole solar cycle. Second, there is a tacit assumption that all the input variables in the growth-rate equations are uncorrelated with all of the others. This may not be entirely accurate, especially with regard to the electric and magnetic fields and the electron concentration and its gradient.

Because the output values of growth rate are considered accurate only to order of magnitude, the relatively limited sampling of the populations is not considered a significant draw-back. Nor is the use of full field values of the electric and magnetic fields or the approximation made in order to obtain values of electric field strength in the absence of z-axis data.

It is remarkable that under the conditions selected as representative of patches, the A.C. component of the electric field is often larger than the quasi-D.C. component. This implies that the Turbulence process can be a cause of scintillation on its own and that all slopes of a patch can, in the right circumstances, be sufficiently unstable to cause measurable phase scintillation. In the past it has been assumed that patches showing irregularities or scintillation through-out all regions, rather than just on the trailing edge had undergone the GDI to such an extent that the irregularities had penetrated throughout the patch having started forming only on the trailing edge. The present results show that the same situation could be obtained by the action of the Turbulence process on all slopes, with irregularities forming on the entire circumference and working towards the centre. In this situation the maximum irregularity amplitude need only be the radius of the patch, rather than the entire diameter. This mechanism would therefore appear at first glance to take approximately half the time to cause irregularities to form through-out an entire patch. In fact, because the electric field is fluctuating in magnitude and direction, the growth rate in any one particular direction should be halved. The time to reach complete instability is still accelerated, however.

The values of γ_{GDI} and γ_T obtained suggest that irregularities should be observed in association with most patches. This is not backed by the observational record on either count. The explanation arises from the use here of local, linear theory to obtain the growth-rate equations used. Non-local, 3-dimensional, non-linear theory tends to strongly reduce growth-rate values and introduces wavelength dependencies if none were present in the local, linear theory [e.g., *Gondarenko et al.*, 2003; *Gondarenko and Guzdar*, 1999; 2001; 2004a; 2004b; 2006a; 2006b; *Guzdar et al.*, 1998; *Kelley*, 2009]. It is possible, that results may differ if non-linear processes dominate.

This does not affect the main conclusion that the KHI is not important as it was negligible to start with. However, these results are based on the statistical analysis, and thus in this context the KHI is not the main player for the patch structuring. However, these results do not exclude the situation where the KHI dominates under conditions of the strong flow shears. Nor is the conclusion that Turbulence is a process that must be taken into account affected, as its rise-time will be reduced to a similar extent to that for the GDI, but not more so. If, taking the full complexities into account, none of the processes considered here can give a rise-time less than 60s, then the most probable explanation of the appearance of phase scintillation very rapidly, whilst the patch is still in the cusp, is that inhomogeneous precipitation of energetic particles into the patch as it is forming accelerates the action of the GDI and/or Turbulence processes [*Oksavik et al.*, 2012].

To completely understand what is happening, an extension of the work in *Gondarenko et al.* [2003]; *Gondarenko and Guzdar* [1999; 2001; 2004a; 2004b; 2006a; 2006b] and *Guzdar et al.* [1998] must be made so that the effects of particle precipitation during formation are included, along with the effects of the A.C. electric field. Additionally, the patch geometry and initial electron concentration distribution should take account of recent observations in order to be fully realistic. (The assumption that patches are approximately circular through-out their life-time, having been demonstrated by Ionospheric Ray Tomography to be particularly unsound [e.g., *Yin et al.*, 2009].)

Further complications arise when compound processes are considered. For example GDI and Turbulence processes will be acting on the trailing edge of the patch at the same time. For such processes, the ratio γ_{GDI}/γ_T indicates which dominates. This is given by

$$\frac{\gamma_{GDI}}{\gamma_T} = \frac{|E_{\mathrm{D.C.}}|}{\left[\int_{k_L}^{\infty}\left(E_{\mathrm{A.C.}}(k)\right)^2\right]^{1/2}} \tag{29}$$

Hence, this can be obtained whenever simultaneous measurements of both the D.C. and A.C. components are available. Unless one or the other (or both) is negligible, the compound action of both will accelerate irregularity production compared to either one acting alone. This could lead to a situation where radar back-scatter from irregularities is present throughout a patch but stronger from the trailing edge. The relative impact of Turbulence vs. the GDI should also be more thoroughly examined by analysis of observational data based on Eq.29 for each admissible electron concentration gradient. In this regard, it is interesting that comparing the quasi-D.C. and A.C. electric field strengths (Figures 6 and 7) shows that, under the conditions described for the selection of the data above, the summed A.C. field is usually stronger than the quasi-D.C. field and Turbulence dominates (Figure 8).

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Figure Captions

Figure 1: Histogram of DE2 electron concentration data.

Figure 2: Histogram of the gradient of electron concentration as derived from DE2 data.

Figure 3: Histogram of the DE2 neutral monatomic oxygen concentration data.

Figure 4: Histogram of the DE2 O^+ ion temperature data.

Figure 5: Histogram of DE magnetic field strength data.

Figure 6: Histogram of the quasi-D.C. electric field strength data.

Figure 7: Histogram of the A.C. electric field data (as summed over the measured range of 4Hz-1024Hz).

Figure 8: The percentage of derived growth rates exceeding the given growth rate for ‘turbulence’ (Turb) and gradient drift (GDI) in both the inertial and collisional regimes. The inertial turbulent regime is most likely to exceed any given growth rate and is therefore the more important.