The bullwhip effect under different information-sharing settings: a perspective on price-sensitive demand that incorporates price dynamics

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Information sharing has been shown previously in the literature to be effective in reducing the magnitude of the bullwhip effect. Most of these studies have focused on a particular information-sharing setting that assumes demand follows an autoregressive process. In this paper, we contribute to the literature by presenting a price-sensitive demand model and a first-order autoregressive pricing process that is coupled to the optimal order-up-to inventory policy and the optimal minimum mean-squared error forecasting technique. We compare a no information-sharing setting – in which only the first stage of the supply chain observes end-customer demands and market prices, and upstream echelons must base their forecasts on downstream incoming orders – with two information-sharing settings, end-demand and order information and end-demand information. In the case of end-demand and order information, upstream echelons develop their forecasts and plan their inventories based on the end-customer demand, price information, and downstream orders. With end-demand information, upstream echelons use only end-customer demands and market prices to conduct their forecasting and planning. We derive the analytical expressions of the bullwhip effect with and without information sharing, quantify the impact of information sharing on the reduction of the bullwhip effect associated with end-demand and order information and end-demand information, and explore the optimal information setting that could most significantly restrain the bullwhip effect. Our analysis suggests that the value of these two information-sharing settings can be high, especially when the pricing process is highly correlated over time or when the product price sensitivity coefficient is small. Moreover, we find that the value of adopting end-demand and order information is always greater than that of end-demand information.

Keywords: information sharing; bullwhip effect; order-up-to inventory policy; minimum mean-squared error forecasting technique

1. Introduction

Information asymmetry is one of the most powerful sources of the bullwhip effect. However, sharing information between supply chain partners can be viewed as a major means for improving the performance of the supply chain (Lee, So, and Tang 2000). For example, Wal-Mart's unprecedented, high inventory turnover has been attributed to its successful implementation of electronic data exchange (EDI). Information sharing involves the sharing of downstream retailer demand information with its upstream businesses. An active stream of research has been performed on the value of information sharing in the presence of the bullwhip effect. This stream of research is often based on an autoregressive demand process, such as the first-order autoregressive demand (AR (1)) that was published in Lee, So, and Tang (2000), the autoregressive and moving average (ARMA) demand of (1, 1) that was published in Graves (1999), and the more general ARMA demand of \((p, q)\) that was published in Gaur, Giloni, and Seshadri (2005). In addition, the majority of this research focused on a particular information-sharing setting, such as the end-demand and order information setting that was published in Lee, So, and Tang (2000) and the end-demand information setting that was published in Chen et al. (2000).

Two interesting questions arise in the literature regarding information sharing. First, what value can be obtained in information sharing when demands are not AR (1), ARMA (1, 1), or ARMA \((p, q)\)? In particular, in previous studies, demand followed an autoregressive process, and the demand correlation parameter on the bullwhip effect was examined. However, the managerial insights of this parameter are difficult to explain in practice. A price-sensitive demand model will allow us to focus on a different perspective when explaining the impact of demand process characteristics, such as...
the market demand scale and the price sensitivity coefficient, on the value of information sharing, which could provide us with more managerial insights. Second, most studies indicated that upstream businesses would benefit from a given information-sharing setting, such as end-demand and order information or end-demand information. However, what information-sharing setting is best for the supply chain to use? In other words, in what way should individual enterprises in the supply chain share demand information?

To address these questions, we extend the work of Ma et al. (2013), who considered a two-level supply chain in which the demand was price-sensitive by quantifying the bullwhip effect on product orders (i.e., the increase in product-order variability) and inventory (i.e., the increase in inventory variability). We present an extension of that work that allows us to quantify the value of the observed demand and price information on reducing the bullwhip effect. In particular, we consider a three-level supply chain, which consists of a manufacturer, wholesaler, and retailer, where the demand that is faced by the retailer is price sensitive. The price follows dynamics with an AR (1) pricing process, and different demand process characteristics are considered, such as the market demand scale, the price sensitivity coefficient, and the price correlation coefficient, which have not been analysed in previous studies by Chen et al. (2000), Chen, Ryan, and Simchi-Levi (2000), Lee, So, and Tang (2000), or Chen and Lee (2009). Assuming that the retailer and wholesaler use an optimal order-up-to inventory policy and an optimal minimum mean-squared error (MMSE) forecasting technique, we derive the analytical expressions of the bullwhip effect with three information-sharing settings, i.e., no information sharing, end-demand and order information, and end-demand information, and deduce the conditions by which the retailer chooses an information setting to significantly restrain the bullwhip effect.

This paper is organised as follows. Section 2 is devoted to a review of the literature. Section 3 establishes a price-sensitive demand function in which the price follows an AR (1) pricing process. Section 4 introduces the inventory policy and the forecasting technique. Section 5 analyses the retailer’s and wholesaler’s order quantity by treating the retailer’s order quantity as the demand for the wholesaler. Section 6 derives the analytical expressions of the bullwhip effect for each echelon and compares the order oscillations for the three information settings (i.e., no information sharing, end-demand and order information, and end-demand information). Section 7 provides numerical analyses that illustrate the value of information sharing with end-demand and order information and with end-demand information. Finally, Section 8 presents the conclusions from our analyses and suggests follow-up research directions.

2. Literature review

There is a vast body of literature on the bullwhip effect and information sharing. Our research is built on two lines of this literature: the papers on the bullwhip effect and those on information sharing.

2.1 Bullwhip effect

The bullwhip effect is the phenomenon of information distortion as ordering information percolates upstream, which means that a downstream demand fluctuation will lead to larger fluctuations in the variance of upstream ordering (Lee, Padmanabhan, and Whang 1997a, 1997b). This distorted information can lead to tremendous inefficiencies, such as excessive inventory investment, poor customer service, lost revenues, misguided capacity plans, ineffective transportation, and missed production schedules (Lee, Padmanabhan, and Whang 1997b). Therefore, the bullwhip effect is one of the most widely investigated phenomena in supply chain management.

Over the past few decades, the bullwhip effect has become a popular topic for researchers and practitioners. Early studies have attempted to demonstrate the existence of the bullwhip effect and identify the causes of such an effect (Forrester 1958, 1961; Sterman 1989). Currently, theoretical studies focus on quantifying and searching for remedies for this effect. Lee, Padmanabhan, and Whang (1997a) provided a formal definition of the bullwhip effect and systematically analysed its four main causes: demand signal processing, shortage games, order batching, and price adjustment. In addition, they proposed countermeasures, such as avoiding multiple demand-forecast updates, breaking order batches, stabilising prices, and eliminating gaming in shortage. Chen et al. (2000) and Chen, Ryan, and Simchi-Levi (2000) made an important contribution by recognising the role of demand forecasting as a filter for the bullwhip effect. Chen et al. (2000) quantified the bullwhip effect for a two-level supply chain in which the retailer used the moving average (MA) forecasting technique and extended those results to multiple-stage supply chains. Additionally, Chen, Ryan, and Simchi-Levi (2000) demonstrated that the use of exponential smoothing (ES) technology by the retailer could also cause the bullwhip effect. However, although MA and ES are the most commonly used forecasting techniques, these methods are not optimal. Alwan, Liu, and Yao (2003) studied the bullwhip effect when MMSE forecasting was employed and found that it is possible to reduce or even eliminate this effect by using an MMSE-optimal forecasting scheme. Zhang (2004), Hosoda and Disney (2006), Agrawal, Sengupta, and Shanker (2009), and Sodhi and Tang (2011) have conducted similar work. However, Wang, Jia,
and Takahashi (2005) proposed the new term “extended-bullwhip effect” to describe information distortion other than the bullwhip effect, and they quantified the negative impact of this new term for a two-level supply chain. The above studies have analytically examined the bullwhip effect under the assumption of an AR (1) demand model and an order-up-to inventory policy. Furthermore, the discrete control theory was implemented by Disney and Towill (2003) and Disney, Towill, and Van De Velde (2004) to measure the bullwhip effect and to evaluate the inventory variance produced by an ordering policy. Likewise, by using the control theory, Disney et al. (2006) quantified the bullwhip effect, inventory variance, and customer service levels that the inventory variance generates for a generalised order-up-to policy for independent and identically distributed (i.i.d.), AR (1), first-order moving average, and ARMA demand processes. Similar or more advanced demand models have also been adopted by Graves (1999), Aviv (2003), Gaur, Giloni, and Seshadri (2005), Gilbert (2005), Croson and Donohue (2006), Hsiao and Shieh (2006), Kim et al. (2006), Dhahri and Chabchoub (2007), Duc, Luong, and Kim (2008), Chen and Lee (2009), Zhang and Zhao (2010), Zhang and Burke (2011), Ma et al. (2013), and Wei, Wang, and Qi (2013).

In addition to the theoretical efforts for determining mathematical representations of the bullwhip effect, attempts have also been made to validate its existence in empirical studies. Lee, Padmanabhan, and Whang (1997b) used examples such as Procter & Gamble (P&G) and Hewlett-Packard (HP) to exemplify the existence of and remedies for the bullwhip effect. Wu and Katok (2006) used a controlled laboratory simulation of the beer game to investigate the effect of learning and communication on the bullwhip effect, and this group found that the bullwhip effect is, at least in part, caused by insufficient coordination between supply chain partners. Hence, information sharing can be a potentially valuable and effective method by which to secure a competitive advantage and improve organisational performance in supply chain management (Li et al. 2005, 2006). However, although price stability is frequently proposed to counter the bullwhip effect (Lee, Padmanabhan, and Whang 1997a, 1997b), Hamister and Suresh (2008) used data from a supermarket scanner to show that utilising fixed instead of dynamic pricing may result in a higher sales variance, order variance, and the bullwhip effect. Furthermore, Niranjan, Wagner, and Aggarwal (2011) proposed a framework to more comprehensively capture the underlying information distortion through a case study of a real-life automotive supply chain. Klug (2013) examined the variance amplification of orders in a car manufacturing context with the help of system dynamics modelling. The above studies investigate the bullwhip effect using firm-level data. Additionally, macroeconomic industry-level data have been collected by Cachon, Randall, and Schmidt (2007) to search for the bullwhip effect. This group found that wholesale industries exhibit a bullwhip effect, but retail and manufacturing industries generally do not exhibit this effect.

2.2 Information sharing

As mentioned above, information sharing has been empirically shown to be an effective way to improve organisational performance (Kulp, Lee, and Ofek 2004; Li et al. 2005, 2006; Zhou and Benton 2007; Prajogo and Olhager 2012). In addition, analytical models have been established to investigate the impact of various information-sharing models (i.e., product information sharing, process information sharing, resource information sharing, inventory information sharing, planning information sharing, and demand/order information sharing) on the dynamics index model of supply chain performance. For example, inventory performance was used by Lee, So, and Tang (2000), process indices were used by Tsung (2000), customer service indices were used by Chen (1998), financial indices were used by Cachon and Fisher (2000), and the bullwhip effect index was used by Dejonckheere et al. (2004). For a more detailed discussion on the various sharing models and the dynamics performance index model, we refer readers to review the work of Huang, Lau, and Mak (2003). Because the purpose of this paper is to investigate the impact of demand/order information sharing on the bullwhip effect, we restrict our attention to the literature review of demand/order information sharing. The impact that other information sharing models, such as product information sharing, have on the bullwhip effect is beyond the scope of this paper.

Partners along the traditional supply chain communicate demand information exclusively in the form of orders. However, because order data often distort the true dynamics of market demand, the bullwhip effect and larger inventory costs become unavoidable (Lee, Padmanabhan, and Whang 1997a, 1997b; Lee, So, and Tang 2000). To counter these negative impacts, Lee and Whang (2000) described the types of information that are shared and discussed how and why this information is shared by using industrial examples. Lee, So, and Tang (2000) devised an analytical method by which the benefits of demand information sharing could be quantified and the drivers of the magnitudes of these benefits could be identified for a two-level supply chain with an AR (1) demand process. Their analysis suggested that the value of demand information sharing could be high, especially when demands were significantly correlated over time. However, Lee, So, and Tang (2000) assumed that the upstream manufacturer did not infer demand information from the retailer’s orders. Raghunathan (2001) relaxed this assumption by showing that the value of obtaining information on actual demand from the retailer is insignificant if the demand information is inferable. Gaur, Giloni, and Seshadri (2005) extended the
results of Raghunathan (2001) to cases in which demand was generated by a more general ARMA process. In the case of ARMA (1, 1) demand, this group found that sharing or inferring retail demand led to a 16.0% average reduction in the manufacturer’s safety-stock requirement. Chen and Lee (2009) investigated the value of information sharing and order variability control by using a generalised demand model, i.e., the Martingale model of forecast evolution (MMFE). A similar information-sharing setting has also been adopted by Gavirneni, Kapuscinski, and Tayur (1999), Hosoda and Disney (2006, 2012), Hsiao and Shieh (2006), Agrawal, Sengupta, and Shanker (2009), and Ali and Boylan (2011).

The above studies investigated the no information sharing and the end-demand and order information settings to develop insights into the value of information sharing. However, because the order quantity of the retailer often distorts the true dynamics of the marketplace and the manufacturer has complete knowledge of the end customer demand history through information sharing, many authors have assumed that upstream businesses only used the actual customer demands, i.e., the end-demand information, for their future planning. Chen et al. (2000) investigated a multiple-stage supply chain under an AR (1) demand process with and without end-demand information and demonstrated that the bullwhip effect could be reduced but not completely eliminated by centralising demand information. A comprehensive survey on the benefits of information sharing on a supply chain can be found in Chen (2003). Additionally, Kim and Ryan (2003) presented an extension of the work conducted by Chen et al. (2000) and quantified the value of the observed demand data and the impact of suboptimal forecasting on the expected costs of the retailer. Dejonckheere et al. (2004) compared a traditional supply chain, in which there was no information sharing, with an information-enriched supply chain, in which customer demand data were shared throughout the chain, for two types of replenishment rules that are based on control systems engineering. This study showed that information sharing helped to significantly reduce the bullwhip effect at higher levels of a chain with an order-up-to policy and that information sharing was necessary to reduce order variance at higher levels of a chain with the smoothing policy. Chatfield et al. (2004) tested the accuracy of the simulation by verifying the results in the papers by Chen et al. (2000) and Dejonckheere et al. (2004) and found that lead-time variability exacerbates the amplification of variance in a supply chain and that information sharing and information quality are highly significant. Moyaux, Chaib-Draa, and D’Amours (2007) studied how to separate demand into original demand and adjustments and described two principles that explained how to use the shared information to reduce the bullwhip effect. Ouyang (2007) analysed the effect of information sharing on supply chain stability and the bullwhip effect in multi-stage supply chains that operated with linear and time-invariant inventory management policies. Zhang and Zhao (2010) analysed two parallel supply chains that had interacting demand streams and investigated the value of acquiring information on the opposing demand stream. Barlas and Gunduz (2011) investigated some of the structural sources of the bullwhip effect and explored the effectiveness of information sharing in eliminating undesirable fluctuations by using a system dynamics simulation. The value of end-demand information sharing can also be found in the following publications: Chen (1998), Fiala (2005), Wang, Jia, and Takahashi (2005), Kim et al. (2006), Viswanathan, Widiarta, and Piplani (2007), Hwang and Xie (2008), Kelepouri, Miliatos, and Pramatari (2008), Sohn and Lim (2008), Bottani and Montanari (2010), Ouyang and Li (2010), Zhang and Cheung (2011), and Chatfield (2013).

The contributions of this paper are twofold. First, in previous research, demand was assumed to follow an autoregressive process, and the demand correlation parameter on the value of information sharing was examined (Lee, So, and Tang 2000; Kim and Ryan 2003). However, the managerial insights of this parameter are difficult to explain in practice. Our research will consider a price-sensitive demand function in which the price follows an AR (1) pricing process. This method will allow us to focus on a different perspective to explain the impact of demand process characteristics such as the market demand scale on the value of information sharing. Second, previous research about the impact of information sharing has focused on end-demand and order information or on end-demand information. In contrast to previous studies, this paper is the first to quantify the value of end-demand and order information and end-demand information simultaneously. By comparing the bullwhip effect under the two information-sharing settings, we show how individual enterprises in the supply chain should share demand information to more significantly restrain the bullwhip effect.

3. Demand model

If a simple three-level supply chain, which consists of a manufacturer, a wholesaler, and a retailer, is considered, the external demand for a single product occurs at the retailer, where the demand that is faced by that retailer is price sensitive. If \( d_t \) and \( p_t \) are the customer demand and market price in period \( t \), respectively, we obtain the following basic linear demand function model:

\[
    d_t = a - bp_t + e_t, \tag{1}
\]

where \( a \) refers to the market demand scale; \( b \) is the price sensitivity coefficient; and \( e_t \) is an i.i.d. variable, which is a normally distributed error term across time that has a mean of zero and a variance of \( \sigma^2 \). We interpret the error term \( e_t \)
to be the exogenous demand shock that is specific to the retailer and has no relation to the market price. Therefore, the covariance structure between the error term and the market price is as follows: \( \text{Cov}(p_t, \epsilon_t') = 0 \) for any \( t \) or \( t' \).

We consider a market setting in which the retailer sells in a perfectly competitive market and exerts no control over the market clearing price. When we incorporate price dynamics into our demand model, the market price evolution is determined by the overall market demand and supply. If the market price \( p_t \) in Equation (1) is an AR (1) pricing process that describes price dynamics:

\[
p_t = \mu + \rho p_{t-1} + \eta_t, \tag{2}
\]

where \( \mu \) is a nonnegative constant that determines the mean of the price; \( \rho \) is the price correlation coefficient and \( \rho \in (0, 1) \); and \( \eta_t \) is an i.i.d. variable, which is a normally distributed error term with a mean of zero and a variance of \( \sigma^2 \). We interpret the error term \( \eta_t \) to be the effect of overall market shocks on the price and assume that \( \eta_t \) and the market price have the following covariance structure:

\[
\text{Cov}(p_t, \eta_t') = 0 \text{ if } t < t'.
\]

We can derive from Equations (1) and (2) that \( d_t = \mu t + \lambda d_{t-1} + \omega_t \), where \( \mu t = a(1 - \rho) - b\lambda, \lambda = \rho, \) and \( \omega_t = \epsilon_t - \rho \epsilon_{t-1} - b\eta_t \). Furthermore, the model that describes the demand in Equation (1) and the price dynamics in Equation (2) can be reduced to an autoregressive demand process. Based on their experience with a major national producer and wholesaler of consumer products, Erkik, Hausman, and Nahmias (1990) have observed high correlations between successive monthly demands (approximately 0.7). Additionally, Lee, So, and Tang (2000) reported that it is common to have a positive demand correlation coefficient \( \lambda \) in a high-tech industry or for the sales pattern of most products. This group found that \( \lambda \) varies from 0.26 to 0.89 for 150 stock-keeping units (SKUs), and because \( \lambda = \rho \), we can also deduce that the price correlation coefficient \( \rho > 0 \) is common. The assumption that \( \rho \in (0, 1) \) ensures that the AR (1) pricing process is stationary (Box and Jenkins 1994), and a similar assumption has been adopted by Ma et al. (2013). When the coefficient \( \rho \) is positive, the process is reflected by a wandering or meandering sequence of observations. In particular, if \( \rho \) has a large positive value, neighbouring values in the process are similar and the process exhibits marked trends. Therefore, by utilising different values for \( \rho \), one can represent a wide variety of pricing process behaviours, and it can easily be shown from Equation (2) that \( \mu_p = \text{E}(p_t) = \mu/(1 - \rho) \) and \( \sigma^2_p = \text{Var}(p_t) = \delta^2/(1 - \rho^2) \). Furthermore, it can be shown from Equation (1) that \( \mu_d = \text{E}(d_t) = a - b\mu/(1 - \rho) \) and \( \sigma^2_d = \text{Var}(d_t) = \sigma^2 + b^2\delta^2/(1 - \rho^2) \). Notably, we have assumed that \( \text{Cov}(p_t, \epsilon_t') = \text{Cov}(\mu + \rho p_{t-1} + \eta_t, \epsilon_t') = \text{Cov}(\eta_t, \epsilon_t') = 0 \) for any \( t \) or \( t' \). Thus, the error terms are independent across time and are not contemporaneously correlated.

In previous studies, most researchers, such as Lee, Padmanabhan, and Whang (1997a), Chen et al. (2000), and Chen, Ryan, and Simchi-Levi (2000), adopted an AR (1) model to describe the demand process. These groups investigated the bullwhip effect as a function of the demand correlation parameter. However, it is difficult to explain the managerial insights of this parameter in practice. Our work analyses a price-sensitive demand function whereby the price is an AR (1) pricing process. Therefore, we focus on a different perspective to explain the impact of demand process characteristics, which include the market demand scale \( a \), the price sensitivity coefficient \( b \), the price correlation coefficient \( \rho \), the error term variances \( \sigma^2 \) and \( \delta^2 \), and information sharing on the bullwhip effect. This analysis provides us with more managerial insights into our research.

Similar to Lee, So, and Tang (2000), we consider a periodic review system in which each stage of the supply chain reviews its inventory level and replenishes its inventory ordering from the upstream site at every period. All of the results are consistent within each adopted review period (e.g., day or week), and we will introduce the ordering process in the next section.

4. Ordering process

The sequence of events during the replenishment period of our model is similar to those in the traditional beer game (Sterman 1989). First, the retailer’s ordering process is described. At the end of period \( t - 1 \), the retailer, or stage 1, observes the consumer demand \( d_{t-1} \), calculates its order-up-to level \( y^1_{t-1} \) for period \( t \), and places an order of quantity \( q^1_t \) to the wholesaler at the beginning of period \( t \) to raise its current inventory to level \( y^1_t \). After the lead time and at the beginning of period \( t + L_1 \), the retailer receives the product from the wholesaler and the excess demand is backordered. Second, the wholesaler handles its ordering process. At the beginning of period \( t \), the wholesaler, or stage 2, receives and ships the required order quantity \( q^1_t \) to the retailer, and backorders are allowed when the wholesaler does not possess enough stock to fill this order. The wholesaler calculates its order-up-to level \( y^2_t \) for period \( t \) and immediately orders \( q^2_t \) from the manufacturer at the beginning of period \( t \) according to its current inventory level. The wholesaler receives the shipment of the order \( q^2_t \) at the beginning of period \( t + L_2 \).
Note that the retailer (or the wholesaler) must utilise certain forecasting techniques to calculate its order-up-to level \( y_i^1 \) (or \( y_i^2 \)). We will introduce the order-up-to inventory policy and the MMSE forecasting technique in this section.

### 4.1 Order-up-to policy

The order-up-to policy is one of the most studied policies of the supply chain model (Lee, Padmanabhan, and Whang 1997a; Chen et al. 2000; Chen, Ryan, and Simchi-Levi 2000). When we assume that the retailer and the wholesaler will adopt the order-up-to inventory policy, the ordering decision in an order-up-to system is as follows:

\[
q_i^1 = y_i^1 - (y_{i-1}^1 - d_{i-1}),
\]

and

\[
q_i^2 = y_i^2 - (y_{i-1}^2 - q_i^1).
\]

Therefore, the order quantity of the retailer (or the wholesaler) at the beginning of period \( t \) is the order-up-to level that is used in period \( t \) minus its inventory position at the end of period \( t-1 \). Notice from Equations (3) and (4) that the product order quantity \( q_i^2 \) \((i = 1, 2)\) may be negative, and if so, we assume that this excess inventory is returned without cost. We discuss the impact of this assumption on our results in Appendix A. Additionally, we assume that backorders are allowed when the retailer has excess demand and the wholesaler does not have enough stock to fill the retailer’s order, i.e., the inventory position of the retailer and the wholesaler at the end of any period, \( y_{i-1}^1 - d_{i-1} \) and \( y_{i-1}^2 - q_i^1 \), may be negative. This assumption may not be realistic in a retail setting, therefore we also consider the impact of this assumption on our results in Appendix A.

The order-up-to level consists of an anticipation stock that is retained to meet the expected lead-time demand and a safety stock for hedging against unexpected demand. Therefore, the order-up-to level is updated every period according to the following:

\[
y_i^1 = \bar{D}_i^1 + z_i \sigma_i^1, \quad i = 1, 2,
\]

where \( \bar{D}_i^1 \) is an estimate of the mean lead-time demand of stage \( i \), \( z_i \) is a constant that has been set to meet a desired service level and is often referred to as the safety factor (Chen, Ryan, and Simchi-Levi 2000), and \( \sigma_i^1 \) is an estimate of the standard deviation of the forecasting error of the \( L_i \) period. To simplify our analysis, we set \( z_i \) to zero in this paper. When a policy of this form is used, an inflated value of \( L_i \) with the excess inventory that represents the safety stock is often used. For example, a retailer that faces a lead time of two weeks may choose to keep inventory that is equal to four weeks of forecast demand, and the extra inventory represents its safety stock. These types of policies have often been used in previous research, such as in Ryan (1997), Chen et al. (2000), and Kim and Ryan (2003).

When the demand is normally distributed, the order-up-to policy minimises the total expected holding and shortage costs of the retailer and is considered to be the optimal inventory policy (Lee, Padmanabhan, and Whang 1997a; Lee, So, and Tang 2000; Zhang 2004). We have shown that our demand model in Equation (1) and price dynamics model in Equation (2) can be reduced to an autoregressive demand process, i.e., \( d_t = \mu + \lambda d_{t-1} + \omega_t \), where \( \lambda = a(1 - \rho) - b \mu \), \( \dot{\lambda} = \rho \), and \( \omega_t = \varepsilon_t - \rho \omega_{t-1} - b \eta_t \). Because the errors \( \varepsilon_t \) and \( \eta_t \) are i.i.d., normally distributed across time, and are not contemporaneously correlated, the demand is also normally distributed. Therefore, in this research, the retailer uses the optimal order-up-to inventory policy.

### 4.2 MMSE forecasting technique

To calculate the retailer’s (or wholesaler’s) order-up-to level \( y_i^1 \) (or \( y_i^2 \)), the retailer (or wholesaler) should use certain forecasting techniques to estimate the mean lead-time demand \( \bar{D}_i^1 \) (or \( \bar{D}_i^2 \)). Most researchers and practitioners focus on three basic techniques to conduct forecasting: the MA, ES, and MMSE techniques. MA is a forecasting technique that uses the average of actual observations from a specified number of prior periods, ES is a forecasting technique that uses a weighted, moving average of past data as the basis for a forecast, and MMSE is provided by the conditional expectation that is given to previous observations (Box and Jenkins 1994). Additionally, MMSE has been considered to be an optimal forecasting procedure that minimises the mean-squared forecasting error. In the area of forecasting, an optimal forecasting model traditionally implies that the forecasting model has minimal mean-squared forecasting errors (Alwan,
Liu, and Yao 2003). However, the MA and ES forecasting techniques do not generally share this optimal property for a time series process (Zhang 2004). This paper examines the value of the information on the bullwhip effect and assumes that the retailer and wholesaler use the optimal MMSE technique to conduct forecasting.5

We assumed that the retailer and the wholesaler adopted the optimal inventory policy, i.e., the order-up-to policy and the optimal forecasting technique, which in this paper, is the MMSE forecasting technique. The assumption that all stages in the supply chain use the same inventory policy and forecasting technique allows us to determine the impact of only demand forecasting without considering the impact of different inventory policies or forecasting techniques between stages.

5. Three-level supply chain model

Our approach for evaluating the impact of information sharing on the bullwhip effect is as follows. For the given order-up-to inventory policy and MMSE forecasting technique, we first analyze the retailer’s order quantity. Then, by treating the retailer’s order quantity as the demand for the wholesaler, we analyze the wholesaler’s order quantity for three information settings (i.e., no information sharing, end-demand and order information, and end-demand information). In Section 6, we derive the bullwhip effect expressions under the three information settings and evaluate the reduction in the bullwhip effect that is associated with information sharing.

The expressions for the ordering decisions of the retailer and wholesaler will be developed. These expressions will allow us to evaluate the value of information sharing on bullwhip reductions.

5.1 Retailer’s ordering decision

When considering the retailer’s ordering decision, substituting Equation (5) into Equation (3) with $z_i = 0$ when $i = 1$, the retailer’s order quantity $q_t^i$ at the beginning of period $t$ can be rewritten as follows:

$$q_t^i = \hat{D}_t^{i-1} - \hat{D}_{t-1}^{i-1} + d_{t-1}. \tag{6}$$

We now derive the expression for the retailer’s order-up-to level $\hat{D}_t^{i-1}$. Using the MMSE technique, it has been shown that the MMSE forecast is the conditional expectation that is given to previous observations (Box and Jenkins 1994). If $\hat{d}_{t+i}$ is the demand forecast of period $t+i$ ($i = 0, 1, 2, \ldots$) that is made at the end of period $t-1$, then for the AR (1) demand process, the MMSE forecast of $\hat{d}_{t+i}$ is represented as $E(d_{t+i} | d_{t-1})$ (Lee, So, and Tang 2000; Alwan, Liu, and Yao 2003; Zhang 2004; Agrawal, Sengupta, and Shanker 2009; Sodhi and Tang 2011). However, this paper considers a price-sensitive demand function in which the price follows an AR (1) process. If $p_{t+i}$ is the market price forecast of period $t+i$ that is made at the end of period $t-1$, then for the AR (1) pricing process, $\hat{p}_{t+i}$ is the future price that is conditional upon the actual price that is observed up to period $t-1$, i.e., $E(p_{t+i} | p_{t-1})$. By recursively applying Equation (2), it is simple to show that the following equation is true:

$$p_{t+i} = \mu + \rho p_{t+i-1} + \eta_{t+i} = (1 + \rho)\mu + \rho^2 p_{t+i-2} + (\rho \eta_{t+i-1} + \eta_{t+i})$$
$$= \cdots = \frac{1-\rho^{i+1}}{1-\rho} \mu + \rho^{i+1} p_{t-1} + \sum_{j=0}^{i} \rho^j \eta_{t+j}. \tag{7}$$

Thus,

$$\hat{p}_{t+i} = E(p_{t+i} | p_{t-1}) = \frac{1-\rho^{i+1}}{1-\rho} \mu + \rho^{i+1} p_{t-1}. \tag{8}$$

Then, we can derive the demand forecast of period $t+i$ as follows:

$$\hat{d}_{t+i} = a - b\hat{p}_{t+i} = a - b \left( \frac{1-\rho^{i+1}}{1-\rho} \mu + \rho^{i+1} p_{t-1} \right). \tag{9}$$

Thus, the expression for the order-up-to level, $\hat{D}_t^{i-1}$, can be given as follows:
\[ \hat{D}_{t+1} = \sum_{j=0}^{t-1} \hat{d}_{t+j} = L_t \mu_d + \frac{b \rho}{1 - \rho} \Lambda_t \mu - b \rho \Lambda_t p_{t-1}, \quad (10) \]

where \( \Lambda_t = \frac{1 - \rho^t}{1 - \rho} \). Then from Equation (6), we can achieve the following equation:

\[ q_t^1 = -b \rho \Lambda_t (p_{t-1} - p_{t-2}) + d_{t-1}. \quad (11) \]

We assumed that the retailer and wholesaler used the MMSE technique to conduct forecasting. It is well known that the MMSE forecast is provided by the conditional expectation (Box and Jenkins 1994). To determine the conditional expectation of the retailer’s order quantity \( q_{t+1} \) (\( i = 1, 2, \ldots \)) at the beginning of period \( t + i \) and given the retailer’s observed order \( q_t^1 \), an expression of \( q_{t+1}^1 \) in terms of \( q_t^1 \) can be developed. By using Equations (1), (2), and (11), we determine the retailer’s order quantity for the period \( t+1 \) as follows:

\[ q_{t+1}^1 = (1 - \rho) \mu_d + \rho q_t^1 + \epsilon_t - \rho \epsilon_{t-1} - b \Lambda_{t+1} \eta_t + b \rho \Lambda_t \eta_{t-1}. \quad (12) \]

The repeated use of Equation (12) yields the following equation:

\[ q_{t+i}^1 = (1 - \rho^i) \mu_d + \rho^i q_t^1 + \epsilon_{t+i} - \rho^i \epsilon_{t+i-1} - b \Lambda_{t+i+1} \eta_{t+i} - b \rho^i \Lambda_{t+i} \eta_{t+i-1} \]

\[-b \rho (\Lambda_{t+1} - \Lambda_t) \eta_{t+i-2} - b \rho^2 (\Lambda_{t+2} - \Lambda_t) \eta_{t+i-3} - \cdots - b \rho^{i-1} (\Lambda_{t+i+1} - \Lambda_t) \eta_1 + b \rho^i \Lambda_t \eta_{t-1}, \quad i = 1, 2, \ldots. \quad (13) \]

The expression of \( q_{t+i}^1 \) in terms of \( q_t^1 \) given in Equation (13) allows us to determine the conditional expectation of the retailer’s order quantity \( q_{t+i}^1 \), which is useful in analysing the wholesaler’s order quantities. Additionally, because the retailer’s order quantity corresponds to the wholesaler’s demand and the errors \( \epsilon_t \) and \( \eta_t \) in Equation (12) are i.i.d. normally distributed and not contemporaneously correlated, it can be shown that the wholesaler’s demand is also normally distributed. Therefore, the wholesaler also adopts the optimal inventory policy that minimises its total expected holding and shortage costs.

### 5.2 Wholesaler’s ordering decision

After the wholesaler receives and ships the retailer’s order \( q_t^1 \) at the beginning of period \( t \), the wholesaler immediately places an order \( q_t^2 \) with the manufacturer at the beginning of period \( t \) to bring its inventory position to an order-up-to level of \( y_t^2 \). Thus, from Equations (4) and (5) and with \( z_i = 0 \) when \( i = 2 \), the order \( q_t^2 \) that is placed by the wholesaler at the beginning of period \( t \) can be expressed as:

\[ q_t^2 = \hat{D}_{t+1}^2 - \hat{D}_{t+1}^2 + q_t^1, \quad (14) \]

where \( \hat{D}_{t+1}^2 \) is an estimate of the wholesaler’s mean lead-time demand.

The demands seen by the wholesaler are the orders placed by the retailer. To determine the wholesaler’s order quantity \( q_t^2 \), the wholesaler must estimate the mean lead-time demand \( \hat{D}_{t+1}^2 \). To characterise the demand information flow through the supply chain, we consider the following three information settings: no information sharing, end-demand and order information, and end-demand information. We assume that the parameters of the demand process, i.e., \( \alpha, \beta, \mu, \rho, \sigma^2, \) and \( \delta^2 \), are common knowledge to the retailer and wholesaler, but demand and price realisations are the private knowledge of the retailer.\(^7\) When no information sharing occurs, the wholesaler bases its forecast lead-time demand solely on the order quantity \( q_t^1 \) that is placed by the retailer without knowing the customer demand and market price information. When information is shared throughout the supply chain, two possible, additional methods exist for the wholesaler to estimate the lead-time demand. One method is based on the retailer’s order quantity \( q_t^1 \) and the end customer demand and price information. We refer to this information-sharing setting as end-demand and order information. The other possible method for forecasting is to use only the history of the end customer demand and market price. We refer to this information-sharing setting as end-demand information.

We can compare the bullwhip effect under the two information-sharing settings with that under no information sharing and evaluate the reduction in the bullwhip effect that is associated with information sharing. Furthermore, by using
the reduction in the bullwhip effect that is associated with information sharing, we deduce which of the two information-sharing settings more significantly eliminates the increase in variability. This method allows the wholesaler to choose a better information-sharing setting.

5.2.1 No information sharing
When no information sharing occurs, the wholesaler only receives information about the retailer’s order quantity \( q^1_t \). Moreover, the error terms \( \varepsilon_{t-1} \) and \( \eta_{t-1} \) are realised at the beginning of period \( t \) but are unknown to the wholesaler when she determines her order-up-to level \( q^1_t \). The wholesaler treats the error terms \( \varepsilon_{t-1} \) and \( \eta_{t-1} \) in Equation (13) as variables and determines its forecasting lead-time demand \( \hat{D}^{L2,NIS}_t \) using the MMSE technique, which is based on \( q^1_t \), without knowing the demand and price information. Furthermore, when no information is shared, \( \hat{q}^{1,NIS}_{t+i} \) represents the retailer’s ordering forecast of period \( t + i \) (\( i = 1, 2, \cdots \)); thus, from Equation (13), \( \hat{q}^{1,NIS}_{t+i} \) can be given as follows:

\[
\hat{q}^{1,NIS}_{t+i} = E(q^1_{t+i}|q^1_t) = (1 - \rho')\mu_d + \rho' q^1_t. \tag{15}
\]

Because the retailer’s order quantity corresponds to the wholesaler’s demand, the total shipment quantity over the wholesaler lead time is equal to the total orders that are placed by the retailer over the lead-time period \( t + 1, \ldots, t + L_2 \). Thus,

\[
\hat{D}^{L2,NIS}_t = \sum_{i=1}^{L_2} \hat{q}^{1,NIS}_{t+i} = (L_2 - \rho L_2)\mu_d + \rho L_2 q^1_t, \tag{16}
\]

where \( \Lambda_{L_2} = \frac{1 - \rho^2}{1 - \rho} \). Lastly, when no information is shared and from Equation (14), the wholesaler’s order quantity \( q^{2,NIS}_t \) at the beginning of period \( t \) can be expressed as follows:

\[
q^{2,NIS}_t = \hat{D}^{L2,NIS}_t - \hat{D}^{L1,NIS}_{t-1} + q^1_t = \Lambda_{L_2} q^1_t - \rho \Lambda_{L_2} q^1_{t-1}. \tag{17}
\]

5.2.2 End-demand and order information
In the case of end-demand and order information, the wholesaler knows the retailer’s order quantity \( q^1_t \) and the error terms \( \varepsilon_{t-1} \) and \( \eta_{t-1} \) through the sharing of information about the previous observations \( d_{t-1}, d_{t-2}, \cdots \) and \( p_{t-1}, p_{t-2}, \cdots \). Thus, the wholesaler determines its forecasting lead-time demand \( \hat{D}^{L2,ISI}_t \) using an MMSE technique based on the retailer’s ordering quantity \( q^1_t \) and the end customer demand and price information. If \( \hat{q}^{1,ISI}_{t+i} \) is the retailer’s ordering forecast of period \( t + i \) (\( i = 1, 2, \cdots \)), the error terms \( \varepsilon_{t-1} \) and \( \eta_{t-1} \) in Equation (13) become constants through the sharing of the customer demand and price information. Thus, from Equation (13), \( \hat{q}^{1,ISI}_{t+i} \) can be given as follows:

\[
\hat{q}^{1,ISI}_{t+i} = E(q^1_{t+i}|q^1_t) = (1 - \rho')\mu_d + \rho' q^1_t - \rho' \varepsilon_{t-1} + b \rho' \Lambda_{L_2} \eta_{t-1}. \tag{18}
\]

Thus,

\[
\hat{D}^{L2,ISI}_t = \sum_{i=1}^{L_2} \hat{q}^{1,ISI}_{t+i} = (L_2 - \rho L_2)\mu_d + \rho L_2 q^1_t - \rho L_2 \varepsilon_{t-1} + b \rho L_2 \Lambda_{L_2} \eta_{t-1}. \tag{19}
\]

Then, from Equation (14), the wholesaler’s order quantity \( q^{2,ISI}_t \) at the beginning of period \( t \) can be expressed as follows:

\[
q^{2,ISI}_t = \hat{D}^{L2,ISI}_t - \hat{D}^{L1,ISI}_{t-1} + q^1_t = \Lambda_{L_2} q^1_t - \rho \Lambda_{L_2} q^1_{t-1} - \rho \Lambda_{L_2} (\varepsilon_{t-1} - \varepsilon_{t-2}) + b \rho \Lambda_{L_2} \Lambda_{L_2} (\eta_{t-1} - \eta_{t-2}). \tag{20}
\]
5.2.3 End-demand information

In the case of end-demand information, the wholesaler has complete knowledge of the end customer demands and prices seen by the retailer through information sharing. Importantly, this information-sharing setting is different from end-demand and order information. Because the information transferred in the form of orders tends to distort the true dynamics of the market place, we assume that the wholesaler only uses the actual customer demands and prices to estimate the mean lead-time demand $\hat{D}_{t-IS}$. Thus, we have:

$$\hat{D}_{t-IS} = \sum_{i=0}^{L_z-1} \hat{d}_{t+i} = L_z \mu_d + \frac{b \rho}{1 - \rho} \Lambda_{L_z} \mu - b \rho \Lambda_{L_z} \rho_{t-1},$$  \hspace{1cm} (21)

where $\hat{d}_{t+i}$ is given by Equation (9).

Then, from Equation (14), the wholesaler’s order quantity $q_{t-IS}^2$ at the beginning of period $t$ can be expressed as follows:

$$q_{t-IS}^2 = \hat{D}_{t-IS} - \hat{D}_{t-IS}^1 + q_{t}^1 = -b \rho (\Lambda_{L_z} + \Lambda_{L_z}) (p_{t-1} - p_{t-2}) + d_{t-1}.$$  \hspace{1cm} (22)

6. The value of information sharing

We have analysed the order quantities of the retailer and the wholesaler with and without information sharing and, in this section, we compute the bullwhip effect, which is the ratio of the order variance of each stage to the variance of the end customer demand, under these three information settings. If this ratio is larger than one, then the bullwhip effect is present. In this section, we show the value of information sharing, which is based on the analytical expressions of the bullwhip effect, on reducing the bullwhip effect.

6.1 Bullwhip effect at the retailer

Using Equation (11), the measure of the bullwhip effect at the retailer $BWE_1$ is calculated as the ratio of the variance of the retailer’s order quantity $q_t^1$ and the customer demand $d_t$, which is given in Theorem 1.

Theorem 1: If the retailer uses the order-up-to inventory policy and the MMSE forecasting technique, the expression of the bullwhip effect at the retailer is the following:

$$BWE_1 = \frac{Var(q_t^1)}{Var(d_t)} = 1 + 2 b^2 \rho (1 - \rho) \Lambda_{L_z} \Lambda_{L_z+1} \frac{\delta^2}{(1 - \rho^2) \sigma^2 + b^2 \delta^2}.$$  \hspace{1cm} (23)

Proof: See Appendix C.

From the expression of the bullwhip effect at the retailer in Theorem 1, we know that $BWE_1$ depends on the following five parameters: the price sensitivity coefficient $b$, the price correlation coefficient $\rho$, the retailer lead time $L_z$, and the error term variances $\sigma^2$ and $\delta^2$. However, the market demand scale $a$ has no effect on $BWE_1$. Note that because the retailer could directly observe the end customer demand and price information, information sharing does not change the retailer’s ordering decision, and therefore does not affect the bullwhip effect at the retailer. Thus, we shall focus on the impact of information sharing on the bullwhip effect at the wholesaler.

6.2 Bullwhip effect at the wholesaler

We now develop the expressions for the bullwhip effect at the wholesale level with and without information sharing. We consider three information settings in this work: no information sharing, end-demand and order information, and end-demand information.
6.2.1 Expressions for the bullwhip effect under different information-sharing settings

Using Equations (17), (20), and (22), the measures of the bullwhip effect at the wholesaler $BWE_{2}^{NIS}$, $BWE_{2}^{S1}$, and $BWE_{2}^{IS2}$ under the three information settings are given in Theorems 2, 3, and 4, respectively.

**Theorem 2:** If the wholesaler uses the order-up-to inventory policy and the MMSE forecasting technique, the expression of the bullwhip effect at the wholesaler without information sharing is the following:

$$BWE_{2}^{NIS} = \frac{\text{Var}(d_{t})}{\text{Var}(d_{t})} = 1 + 2\rho A_{L_{1}} A_{L_{2}+1} + 2b^{2}\rho(1-\rho)\frac{\delta^{2}}{1-\rho^{2}\sigma^{2}+b^{2}\delta^{2}}.$$  \hfill (24)

**Proof:** See Appendix D.

**Theorem 3:** If the wholesaler uses the order-up-to inventory policy and the MMSE forecasting technique, the expression of the bullwhip effect at the wholesaler with end-demand and order information is the following:

$$BWE_{2}^{S1} = \frac{\text{Var}(d_{t}^{IS})}{\text{Var}(d_{t})} = 1 + 2b^{2}\rho(1-\rho)\left((A_{L_{1}} A_{L_{2}+1} + A_{L_{1}} A_{L_{2}}) + (1+\rho)\rho^{L_{1}+L_{2}+1} A_{L_{1}} A_{L_{2}} - (1-\rho)^{2} A_{L_{1}} A_{L_{2}+1} A_{L_{2}+2}\right)\frac{\delta^{2}}{1-\rho^{2}\sigma^{2}+b^{2}\delta^{2}}.$$  \hfill (25)

**Proof:** See Appendix E.

**Theorem 4:** If the wholesaler uses the order-up-to inventory policy and the MMSE forecasting technique, the expression of the bullwhip effect at the wholesaler with end-demand information is the following:

$$BWE_{2}^{IS2} = \frac{\text{Var}(q_{t}^{IS2})}{\text{Var}(q_{t})} = 1 + 2b^{2}\rho(1-\rho)(A_{L_{1}} + A_{L_{2}})(A_{L_{1}+1} + \rho A_{L_{2}})\frac{\delta^{2}}{1-\rho^{2}\sigma^{2}+b^{2}\delta^{2}}.$$  \hfill (26)

**Proof:** See Appendix F.

From Theorems 2, 3, and 4, we know that the bullwhip effect at the wholesaler has no relation to the market demand scale $a$. However, this effect depends on the price sensitivity coefficient $b$, the price correlation coefficient $\rho$, the retailer lead time $L_{1}$, the wholesaler lead time $L_{2}$, and the variances $\sigma^{2}$ and $\delta^{2}$. We are interested in comparing the increase in variability at each stage of the supply chain under the three information settings. Because the bullwhip effect at the retailer is not affected by information sharing, we focus on the impact of parameters $b$, $\rho$, $L_{1}$, $L_{2}$, $\sigma^{2}$, and $\delta^{2}$ on the bullwhip effect reduction at the wholesale level when evaluating the value of information sharing.

First, we will perform an analytical analysis on the value of information sharing with end-demand and order information and end-demand information to understand the impact of model parameters, such as the price sensitivity coefficient, on reducing the bullwhip effect. Then, we will compare the bullwhip effect under the two information-sharing settings to gain insights into choosing an appropriate information-sharing setting to restrain the bullwhip effect. In Section 7, we provide a numerical study to explain the value of information sharing and a comparison between the two information-sharing settings.

6.2.2 Bullwhip effect reduction under end-demand and order information

We define the value of information sharing $V_{NIS-IS1}$ as the percentage of decrease in the bullwhip effect at the wholesale level due to end-demand and order information, as follows:

$$V_{NIS-IS1} = \frac{BWE_{2}^{NIS} - BWE_{2}^{S1}}{BWE_{2}^{NIS}} \times 100\%,$$  \hfill (27)

where $BWE_{2}^{NIS}$ and $BWE_{2}^{S1}$ are given by Equations (24) and (25), respectively.

Equation (27) can be illustrated by comparing a strategy where the wholesaler uses its previous period order quantity and customer demand and price information, such as shared point of sale (POS) data to conduct forecasting on a benchmark case when no information is shared. To facilitate this analysis, we assume the retailer lead time $L_{1}$ is zero.\textsuperscript{9} If
$L_1 = 0$, Proposition 1 describes the influence of the model parameters $b$, $L_2$, $\sigma^2$, and $\delta^2$ on the value of the end-demand and order information $V_{\text{VNIS-IS1}}$.

**Proposition 1:** For $L_1 = 0$, it follows that:

1. $\frac{\partial (V_{\text{VNIS-IS1}})}{\partial b} \leq 0$.
2. $\frac{\partial (V_{\text{VNIS-IS1}})}{\partial L_2} \geq 0$.
3. $\frac{\partial (V_{\text{VNIS-IS1}})}{\partial \sigma^2} \geq 0$.
4. $\frac{\partial (V_{\text{VNIS-IS1}})}{\partial \delta^2} \leq 0$.

**Proof:** See Appendix G.

Relations (1) and (4) in Proposition 1 indicate that the value of end-demand and order information, $V_{\text{VNIS-IS1}}$, decreases with an increase in the price sensitivity coefficient $b$ and an increase in the overall market shocks $\delta^2$. However, Relations (2) and (3) in Proposition 1 show that $V_{\text{VNIS-IS1}}$ increases with an increase in the wholesaler lead time $L_2$ and an increase in the demand shocks $\sigma^2$. Because the bullwhip effect at the wholesaler level makes the manufacturer’s large inventory costs unavoidable, our theoretical analysis implies that the end-demand and order information sharing is beneficial to the manufacturer, especially when $b$ is small, $L_2$ is long, $\sigma^2$ is large, or $\delta^2$ is small. These benefits are in the form of a reduction in the bullwhip effect at the wholesale level. To have the retailer share its demand and price information with the wholesaler, the manufacturer must provide incentives to the retailer, such as financial incentives that include price reduction and a better return policy, and operational schemes, which include the vendor managed inventory (VMI) program, EDI platform, and POS system. However, we note that the manufacturer has incentives to increase the wholesaler lead time to gain more benefits that are associated with information sharing, which may trigger a non-cooperative behaviour from the wholesaler. We showed in Section 6.1 that information sharing does not change the retailer’s ordering decision and has no impact on the bullwhip effect at the retailer, while the bullwhip effect at the retailer mainly provides potential costs for its upstream wholesaler. As such, information sharing does not prove to have potential benefits to the wholesaler. It is counterintuitive that the wholesaler should estimate its lead-time demand through information sharing while the manufacturer increases the wholesaler lead time $L_2$. However, this occurrence may be a means for the wholesaler to entice the manufacturer to reduce $L_2$, which benefits the wholesaler. Therefore, the manufacturer and wholesaler may obtain benefits when information sharing and lead time reduction are implemented together, which has been previously reported by Lee, So, and Tang (2000).

### 6.2.3 Bullwhip effect reduction under end-demand information

We have conducted a theoretical analysis for the value of information sharing with end-demand and order information. Similarly, we analyse the value of information sharing with end-demand information. If $V_{\text{VNIS-IS2}}$ is the value of information sharing because of end-demand information, then the following equation is true:

$$V_{\text{VNIS-IS2}} = \frac{BWE_{\text{VN}}^{\text{IS2}} - BWE_{\text{IS2}}^{\text{IS2}}}{BWE_{\text{IS2}}^{\text{IS2}}} \times 100\%,$$  

(28)

where $BWE_{\text{IS2}}^{\text{IS2}}$ is given by Equation (26) in Theorem 4.

Proposition 2 describes the influence of parameters $b$, $L_2$, $\sigma^2$, and $\delta^2$ on the value of the end-demand information $V_{\text{VNIS-IS2}}$ when $L_1 = 0$.

**Proposition 2:** For $L_1 = 0$, it follows that:

1. $\frac{\partial (V_{\text{VNIS-IS2}})}{\partial b} \leq 0$.
2. $\frac{\partial (V_{\text{VNIS-IS2}})}{\partial L_2} \geq 0$.
3. $\frac{\partial (V_{\text{VNIS-IS2}})}{\partial \sigma^2} \geq 0$.
4. $\frac{\partial (V_{\text{VNIS-IS2}})}{\partial \delta^2} \leq 0$.

**Proof:** Based on Theorems 3 and 4, $BWE_{\text{IS2}}^{\text{IS2}} = BWE_{\text{IS2}}^{\text{IS2}}$ when $L_1 = 0$, and $V_{\text{VNIS-IS1}} = V_{\text{VNIS-IS2}}$. Thus, we can prove Proposition 2 using the same approach as Proposition 1, but the proof is omitted here.

Similarly, Proposition 2 shows that $V_{\text{VNIS-IS2}}$ decreases with an increase in $b$ and $\delta^2$, and increases with an increase in $L_2$ and $\sigma^2$. Thus, end-demand information sharing results in a higher percentage of bullwhip reduction at the wholesaler level when $b$ is small, $L_2$ is long, $\sigma^2$ is large, or $\delta^2$ is small. If $L_1 = 0$, the value of end-demand and order information $V_{\text{VNIS-IS1}}$ is equal to that of end-demand information $V_{\text{VNIS-IS2}}$, i.e., $V_{\text{VNIS-IS1}} = V_{\text{VNIS-IS2}}$. An objective of this paper is to develop insights for choosing an appropriate information-sharing setting to more significantly restrain the bullwhip effect. If $L_1 = 0$, then no difference exists between the two information-sharing settings. A natural question then arises: which information-sharing setting should the wholesaler use that will result in a greater benefit when $L_1 \neq 0$? To
answer this question, we compare the bullwhip effect under end-demand and order information $BWE_{1}^{IS2}$ with that under end-demand information $BWE_{2}^{IS2}$. Based on Equations (27) and (28), if $BWE_{2}^{IS1} \leq BWE_{2}^{IS2}$, the value of end-demand and order information $\Delta BWE_{2}^{IS}$ is no less than that of end-demand information $\Delta BWE_{2}^{IS1}$. Thus, the wholesaler should adopt the end-demand and order information setting. Likewise, the converse could be analysed in the same way.

6.2.4 Comparison of the bullwhip effect under end-demand and order information with end-demand information

Compared with the bullwhip effect under end-demand information in Theorem 4, the result obtained under end-demand and order information in Theorem 3 can be interpreted as the amount of bullwhip effect that remains when the wholesaler uses its previous period order quantity as additional information. Let $\Delta BWE = BWE_{2}^{IS1} - BWE_{2}^{IS2}$. Proposition 3 below shows the influence of the model parameters $b$, $L_1$, $L_2$, $\sigma^2$, and $\delta^2$ on $\Delta BWE$.

**Proposition 3:** It follows that:

1. $\Delta BWE \leq 0$.
2. $\frac{\partial (\Delta BWE)}{\partial b} \leq 0$, $\frac{\partial (\Delta BWE)}{\partial L_1} \leq 0$, $\frac{\partial (\Delta BWE)}{\partial L_2} \leq 0$, $\frac{\partial (\Delta BWE)}{\partial \sigma^2} \geq 0$, $\frac{\partial (\Delta BWE)}{\partial \delta^2} \leq 0$.

**Proof:** See Appendix H.

Relation (1) in Proposition 3 implies that the difference $\Delta BWE$ is non-positive. Thus, $BWE_{2}^{IS1} \leq BWE_{2}^{IS2}$, which indicates that $\Delta BWE_{2}^{IS1} \geq \Delta BWE_{2}^{IS2}$. The value of end-demand and order information is no less than that of end-demand information, and the wholesaler should always adopt the end-demand and order information setting. This relationship can be explained as follows. The wholesaler’s order quantity $q_{t}^{2,IS1}$ under end-demand and order information, which is shown by Equation (20), can also be given as $q_{t}^{2,IS1} = A_{t+1}A_{t+1}d_{t-1} - \rho (A_{t}A_{t+1} + A_{t-1}A_{t})d_{t-2} + \rho^2 A_{t}A_{t-1}d_{t-3} - \rho (A_{t}A_{t+1} + A_{t}) (e_{t-1} - e_{t-2}) + \rho^2 A_{t}A_{t-2} (e_{t-2} - e_{t-3}) + b \rho A_{t}A_{t-1} (e_{t-1} - e_{t-2})$ using the relationship $q_{t}^{1} = A_{t+1}d_{t-1} - \rho A_{t}d_{t-2} - \rho A_{t} (e_{t-1} - e_{t-2})$, and $q_{t}^{2,IS2}$, when under end-demand information shown by Equation (22), can be given as $q_{t}^{2,IS2} = (1 + \rho (A_{t} + A_{t-1}))d_{t-1} - \rho (A_{t} + A_{t})d_{t-2} - \rho (A_{t-1} + A_{t}) (e_{t-1} - e_{t-2})$ using Equation (1). Because the retailer’s order history also contains information about demand and price (despite not reflecting the true dynamics of the marketplace), when the wholesaler uses its previous-period order quantity as additional demand and price information, the benefits the manufacturer obtains under end-demand and order information are greater than those under end-demand information. Relation (2) in Proposition 3 shows that $\Delta BWE$ decreases with an increase in the price sensitivity coefficient $b$, the lead times $L_1$ and $L_2$, and the overall market shocks $\delta^2$, and increases with an increase in the demand shocks $\sigma^2$. Therefore, compared to the bullwhip effect under end-demand information, the bullwhip effect savings from adopting the end-demand and order information setting can be very substantial, especially when $b$ is large, $L_1$ is long, $L_2$ is long, $\sigma^2$ is small, or $\delta^2$ is large.

To understand the above point, consider the following example. Consider the two three-level supply chains that were described in Section 3, where each supply chain distributes the same single product. We assume that the customer demand and price information can be seen by both wholesalers after the information about the POS date is shared. The first wholesaler, i.e., the wholesaler in the first supply chain, uses the retailer’s previous order quantity and the demand and price information to conduct forecasting, while the second wholesaler only uses the history demands and prices to conduct forecasting. In this case, the orders that are placed by the first wholesaler are less variable than those placed by the second wholesaler, although both supply chains face the same demand process. Therefore, when compared to the second manufacturer, the first manufacturer benefits more from bullwhip effect reduction at the wholesale level and, consequently, has a greater incentive to invest in information sharing, especially when $b$ is large, $L_1$ is long, $L_2$ is long, $\sigma^2$ is small, or $\delta^2$ is large. However, we have shown that the value of information sharing under the two information settings is significant when $b$ is small, $L_2$ is long, $\sigma^2$ is large, or $\delta^2$ is small. Therefore, if the two supply chains have been selling products with a small price sensitivity coefficient $b$, large demand shocks $\sigma^2$, or small overall market shocks $\delta^2$, the first manufacturer is not superior to the second manufacturer when evaluating the information-sharing settings that are adopted by the two wholesalers, although both manufacturers benefit from information sharing. However, if the product lead times $L_1$ or $L_2$ are long, both manufacturers benefit from information sharing, but the first manufacturer benefits more than the second when the two wholesalers adopt different information-sharing settings. Notably, the manufacturer has incentives to increase the wholesaler lead time, $L_2$, and motivate the wholesaler to increase the retailer lead time $L_1$ to gain more benefits. However, doing so may trigger a non-cooperative behaviour between the wholesaler and
retailer. Therefore, to entice the retailer to share its demand and price information, the manufacturer may need to motivate the wholesaler to respond quickly to the retailer’s order and reduce the wholesaler lead time $L_2$, which benefits all partners in the supply chain.

Table 1. The values of $V_{\text{VNIS-1}}$, $V_{\text{VNIS-2}}$, and $\Delta BWE$ for $L_1 = L_2 = 2$ when $\sigma^2 = \delta^2 = 1$.

<table>
<thead>
<tr>
<th>$V_{\text{VNIS-1}}$ (%)</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho$</td>
<td>19.5313</td>
<td>36.6306</td>
<td>50.0821</td>
<td>59.8493</td>
<td>66.5138</td>
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<td>$\rho_{\text{max}}$</td>
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<tr>
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<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho$</td>
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<td>36.2625</td>
<td>49.2312</td>
<td>58.5078</td>
<td>64.7822</td>
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<td>$\rho_{\text{max}}$</td>
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<td>70.8020</td>
<td>71.6105</td>
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<td>0.801</td>
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Table 2. The values of $V_{\text{VNIS-1}}$, $V_{\text{VNIS-2}}$, and $\Delta BWE$ for $L_1 = 2L_2 = 4$ when $\sigma^2 = \delta^2 = 1$.

<table>
<thead>
<tr>
<th>$V_{\text{VNIS-1}}$ (%)</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
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<td>0.2</td>
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<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho$</td>
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<td>37.2704</td>
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<td>70.7575</td>
</tr>
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<td>$V_{\text{max}}$</td>
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<tr>
<td>$\rho$</td>
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<td>70.7575</td>
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<tr>
<td>$\rho_{\text{max}}$</td>
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<td>81.2636</td>
<td>81.5277</td>
<td>0.867</td>
</tr>
<tr>
<td>$V_{\text{max}}$</td>
<td>81.6590</td>
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<tr>
<td>$\rho$</td>
<td>-0.1311</td>
<td>-0.3253</td>
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<td>-1.1049</td>
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<td>$\rho_{\text{min}}$</td>
<td>-1.2664</td>
<td>-1.2427</td>
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<th>9</th>
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<tbody>
<tr>
<td>$\rho$</td>
<td>-0.2578</td>
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<td>-0.4872</td>
<td>0.597</td>
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<tr>
<td>$\Delta BWE_{\text{min}}$</td>
<td>-2.0608</td>
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</table>
7. Numerical analysis

We have conducted a theoretical analysis on the impact of the price sensitivity coefficient \( b \), the lead time \( L_2 \), and the error term variances \( \sigma^2 \) and \( \delta^2 \), on the value of information sharing \( V_{\text{VNIS-IS1}} \) and \( V_{\text{VNIS-IS2}} \) when \( L_1 = 0 \). To choose an information-sharing setting that more significantly restrains the bullwhip effect, we have also compared the bullwhip

\[ \Delta B W E = \frac{V_{\text{VNIS-IS1}} - V_{\text{VNIS-IS2}}}{V_{\text{VNIS-IS1}}} \]

\[ V_{\text{max}} = \max(V_{\text{VNIS-IS1}}, V_{\text{VNIS-IS2}}) \]

\[ V_{\text{min}} = \min(V_{\text{VNIS-IS1}}, V_{\text{VNIS-IS2}}) \]

Table 3. The values of \( V_{\text{VNIS-IS1}}, V_{\text{VNIS-IS2}}, \) and \( \Delta B W E \) for \( L_1 = L_2/2 = 2 \) when \( \sigma^2 = \delta^2 = 1 \).

<table>
<thead>
<tr>
<th>( \Delta B W E )</th>
<th>( b )</th>
<th>( \sigma^2 )</th>
<th>( \delta^2 )</th>
<th>( \rho )</th>
<th>( \rho_{\text{max}} )</th>
<th>( V_{\text{max}} )</th>
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<tr>
<td>1</td>
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<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
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<td>0.5</td>
</tr>
<tr>
<td>0.95</td>
<td>1.28</td>
<td>0.99</td>
<td>0.91</td>
<td>0.87</td>
<td>0.83</td>
<td>0.79</td>
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<tr>
<td>0.90</td>
<td>1.38</td>
<td>1.00</td>
<td>0.92</td>
<td>0.88</td>
<td>0.84</td>
<td>0.80</td>
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<td>0.85</td>
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<td>0.90</td>
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<td>0.82</td>
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<tr>
<td>0.80</td>
<td>1.58</td>
<td>1.02</td>
<td>0.94</td>
<td>0.91</td>
<td>0.87</td>
<td>0.83</td>
</tr>
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</table>

Table 4. The values of \( V_{\text{VNIS-IS1}}, V_{\text{VNIS-IS2}}, \) and \( \Delta B W E \) for \( L_1 = L_2 = 2 \) when \( \sigma^2 = 2\delta^2 = 2 \).

\[ \Delta B W E = \frac{V_{\text{VNIS-IS1}} - V_{\text{VNIS-IS2}}}{V_{\text{VNIS-IS1}}} \]

\[ V_{\text{max}} = \max(V_{\text{VNIS-IS1}}, V_{\text{VNIS-IS2}}) \]

\[ V_{\text{min}} = \min(V_{\text{VNIS-IS1}}, V_{\text{VNIS-IS2}}) \]

<table>
<thead>
<tr>
<th>( \Delta B W E )</th>
<th>( b )</th>
<th>( \sigma^2 )</th>
<th>( \delta^2 )</th>
<th>( \rho )</th>
<th>( \rho_{\text{max}} )</th>
<th>( V_{\text{max}} )</th>
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<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
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<tr>
<td>0.95</td>
<td>1.28</td>
<td>0.99</td>
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<td>0.87</td>
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<td>0.79</td>
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<tr>
<td>0.90</td>
<td>1.38</td>
<td>1.00</td>
<td>0.92</td>
<td>0.88</td>
<td>0.84</td>
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<tr>
<td>0.85</td>
<td>1.48</td>
<td>1.01</td>
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<td>0.82</td>
</tr>
<tr>
<td>0.80</td>
<td>1.58</td>
<td>1.02</td>
<td>0.94</td>
<td>0.91</td>
<td>0.87</td>
<td>0.83</td>
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</table>

International Journal of Production Research
effect under end-demand and order information $BWE_{I1}^{S1}$ with that under end-demand information $BWE_{I2}^{S2}$ and conducted a theoretical analysis on the influence of parameters $b$, $L_1$, $L_2$, $\sigma^2$, and $\delta^2$ on the difference between these two bullwhip effects $\Delta BWE$. In this section, we provide a numerical example to illustrate the impacts of $b$, $L_1$, $L_2$, $\sigma^2$, and $\delta^2$ on

Table 5. The values of $V_{NIS-I1}$, $V_{NIS-I2}$, and $\Delta BWE$ for $L_1 = 2L_2 = 4$ when $\sigma^2 = 2\delta^2 = 2$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\rho_{max}$</th>
<th>$V_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>19.6264</td>
<td>37.2924</td>
</tr>
<tr>
<td>0.2</td>
<td>37.2730</td>
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</tr>
<tr>
<td>0.3</td>
<td>51.7740</td>
<td>62.8407</td>
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<tr>
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<td>62.8299</td>
<td>70.7452</td>
</tr>
<tr>
<td>0.5</td>
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</tr>
<tr>
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<td>62.5370</td>
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</tr>
<tr>
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<td>68.7975</td>
<td>71.1796</td>
</tr>
<tr>
<td>0.8</td>
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</tr>
<tr>
<td>0.9</td>
<td>82.2181</td>
<td>81.3291</td>
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</table>

Table 6. The values of $V_{NIS-I1}$, $V_{NIS-I2}$, and $\Delta BWE$ for $L_1 = L_2/2 = 2$ when $\sigma^2 = 2\delta^2 = 2$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\rho_{max}$</th>
<th>$V_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>19.7346</td>
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<tr>
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$\Delta BWE = -0.8885 -0.2284 -0.4308 -0.7021 -1.0327 -1.5329 -1.5329 -0.8542 -0.734 -1.6396$
$-0.2162 -0.5497 -1.0106 -1.5856 -2.2130 -2.9347 -2.9347 -1.3130 -0.685 -2.9419$
$-0.2444 -0.6194 -1.1325 -1.7631 -2.4356 -2.9860 -2.9860 -1.3130 -0.677 -3.1569$
$-0.2535 -0.6418 -1.1714 -1.8192 -2.5348 -3.0590 -3.0590 -2.007 -1.1697 -0.675 -3.2226$
$-0.2575 -0.6515 -1.1883 -1.8433 -2.5348 -3.0900 -3.0900 -2.2267 -1.6313 -0.674 -3.2506$
Table 7. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and $\Delta BWE$ for $L_1 = L_2 = 2$ when $\sigma^2 = \delta^2/2 = 1$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\rho_{\text{max}}$</th>
<th>$V_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>$V_{NIS-IS1}$ (%)</td>
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</tr>
<tr>
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</tr>
<tr>
<td>3</td>
<td>19.4563</td>
<td>36.2010</td>
</tr>
<tr>
<td>4</td>
<td>19.4549</td>
<td>36.1939</td>
</tr>
<tr>
<td>5</td>
<td>19.4544</td>
<td>36.1910</td>
</tr>
</tbody>
</table>

Table 8. The values of $V_{NIS-IS1}$, $V_{NIS-IS2}$, and $\Delta BWE$ for $L_1 = 2L_2 = 4$ when $\sigma^2 = \delta^2/2 = 1$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\rho_{\text{max}}$</th>
<th>$V_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>$\Delta BWE$</td>
<td>1</td>
<td>1.7275</td>
</tr>
<tr>
<td>2</td>
<td>3.0075</td>
<td>10.0321</td>
</tr>
<tr>
<td>3</td>
<td>2.5354</td>
<td>9.4079</td>
</tr>
<tr>
<td>4</td>
<td>2.4007</td>
<td>9.2309</td>
</tr>
<tr>
<td>5</td>
<td>2.3446</td>
<td>9.1574</td>
</tr>
</tbody>
</table>

$V_{NIS-IS1}$ and $V_{NIS-IS2}$ when $L_1 \neq 0$. Additionally, the impact of the price correlation coefficient $\rho$ on $V_{NIS-IS1}$, $V_{NIS-IS2}$, and $\Delta BWE$ has also been investigated through numerical analysis.

In our numerical example, we set the parameters $b \in \{1, 3, 5, 7, 9\}$ and $\rho \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ when different combinations of lead times are considered, $L_1 = L_2 = 2$, $L_1 = 2L_2 = 4$, and $L_1 = L_2/2 = 2$, and we
considered three simulation scenarios \( \sigma^2 = \delta^2 = 1 \), \( \sigma^2 = 2\delta^2 = 2 \), and \( \sigma^2 = \delta^2/2 = 1 \). Given these parameters, we computed the value of end-demand and order information \( V_{\text{VNIS-IS1}} \) using Equation (27), the value of end-demand information \( V_{\text{VNIS-IS2}} \) using Equation (28), and the difference between the two bullwhip effects \( \Delta BWE = BWE_{\text{IS2}} - BWE_{\text{IS1}} \), in which \( BWE_{\text{IS1}} \) is given by Equation (25) and \( BWE_{\text{IS2}} \) is given by Equation (26). The results are presented in Tables 1–9. Tables 1–3 show the impact of \( \rho \) on \( V_{\text{VNIS-IS1}}, V_{\text{VNIS-IS2}}, \) and \( \Delta BWE \) for \( L_1 = L_2 = 2 \), \( L_1 = 2L_2 = 4 \), and \( L_1 = L_2/2 = 2 \), respectively, when \( b = 1, 3, 5, 7, \) and \( 9 \) for the scenario \( \sigma^2 = \delta^2 = 1 \). Tables 4–6 show the corresponding results for the scenario \( \sigma^2 = 2\delta^2 = 2 \), and Tables 7–9 show the corresponding results for the scenario \( \sigma^2 = \delta^2/2 = 1 \). These tables show that for the given values of \( L_1, L_2, \) and \( b \) under different scenarios, the values of information sharing \( V_{\text{VNIS-IS1}}, \) and \( V_{\text{VNIS-IS2}} \) reach their maximum value (denoted as \( V_{\text{max}} \)) at a certain value \( \rho_{\text{max}} \), whereas the difference \( \Delta BWE \) reaches its minimum value \( \Delta BWE_{\text{min}} \) at \( \rho_{\text{min}} \). In our numerical example, as the value of \( \rho \) increases from zero to one, \( V_{\text{VNIS-IS1}} \) and \( V_{\text{VNIS-IS2}} \) increase with an increase in \( \rho \) from zero to \( \rho_{\text{max}} \).

![Figure 1. The impact of the lead times](image-url)
and these values decrease with an increase in $\rho$ from $\rho_{\text{max}}$ to one. In addition, $\DeltaBWE$ decreases with an increase in $\rho$ from zero to $\rho_{\text{min}}$ and increases with an increase in $\rho$ from $\rho_{\text{min}}$ to one. Given a particular scenario, such as $L_1 = L_2 = 2$ when $\sigma^2 = \delta^2 = 1$, $V_{\text{max}}$ decreases as $b$ increases when investigating the value of end-demand and order information $V_{\text{NIS-IS1}}$. Additionally, when we investigate the value of end-demand information $V_{\text{NIS-IS2}}$, although $V_{\text{max}}$ decreases as $b$ increases, $\rho_{\text{max}}$ increases as $b$ increases. In contrast, we observe that $\rho_{\text{min}}$ and $\DeltaBWE_{\text{min}}$ decrease as $b$ increases when we investigate the difference, $\DeltaBWE$.

As derived in Propositions 1 and 2, if $L_1 = 0$, $V_{\text{NIS-IS1}}$ and $V_{\text{NIS-IS2}}$ decrease with respect to $b$ and $\delta^2$ and increase with respect to $L_2$ and $\sigma^2$. It can be clearly shown in our numerical example that $V_{\text{NIS-IS1}}$ and $V_{\text{NIS-IS2}}$ still decrease with an increase in $b$ if $L_1 \neq 0$; see, for example, the scenario $\sigma^2 = \delta^2 = 1$. If we compare the scenario $\sigma^2 = \delta^2 = 1$ with that of $\sigma^2 = 2\delta^2 = 2$, such as in Tables 1 and 4, we can see that $V_{\text{NIS-IS1}}$ and $V_{\text{NIS-IS2}}$ increase if $\sigma^2$ increases from 1 to 2 when $\delta^2 = 1$. Likewise, when comparing the scenario $\sigma^2 = \delta^2 = 1$ with that of $\sigma^2 = \delta^2/2 = 1$, we observe that $V_{\text{NIS-IS1}}$ and $V_{\text{NIS-IS2}}$ decrease if $\delta^2$ increases from 1 to 2 when $\sigma^2 = 1$. Given a particular scenario, such as $\sigma^2 = \delta^2 = 1$, if we compare the value in Table 1 for $L_1 = L_2 = 2$ with that in Table 3 for $L_1 = L_2/2 = 2$, we can see that $V_{\text{NIS-IS1}}$ and $V_{\text{NIS-IS2}}$ increase if $L_2$ increases from 2 to 4. Additionally, Figure 1 shows the impact of $L_1$ and $L_2$ on the value of information sharing $V_{\text{NIS-IS}}$ (i.e., $V_{\text{NIS-IS1}}$ and $V_{\text{NIS-IS2}}$) when $b = 5$, $\rho = 0.5$, and $\sigma^2 = \delta^2$. It can be shown that $V_{\text{NIS-IS1}}$ and $V_{\text{NIS-IS2}}$ increase with an increase in $L_1$ and $L_2$,\textsuperscript{11} Therefore, our numerical analysis when $L_1 \neq 0$ is consistent with the theoretical findings that are presented in Propositions 1 and 2 when $L_1 = 0$. In addition, we observe in Tables 1–9 that a negative $\DeltaBWE$ decreases as $b$ increases, decreases as $L_1$ increases, decreases as $L_2$ increases, increases as $\sigma^2$ increases, and decreases as $\delta^2$ increases. Figure 2 shows a similar observation for the impact of $L_1$ and $L_2$ on $\DeltaBWE$ when $b = 5$, $\rho = 0.5$, and $\sigma^2 = \delta^2$. These observations also confirm our analytical findings that are presented in Proposition 3.

We used numerical experiments to analyse the impact of the price correlation coefficient $\rho$ on the values of two information-sharing settings $V_{\text{NIS-IS1}}$ and $V_{\text{NIS-IS2}}$, and on the bullwhip effect difference $\DeltaBWE$ under the two settings. Our numerical analysis indicates that the value of information sharing is significant for products with a highly correlated pricing process, especially when the product price sensitivity coefficient $b$ is small, the retailer (or wholesaler) lead time $L_1$ (or $L_2$) is long, the demand shocks $\sigma^2$ are high, or the overall market shocks $\delta^2$ are low. For example, when $b = 1$, $L_1 = L_2 = 2$, and $\sigma^2 = \delta^2 = 1$, the value of end-demand and order information $V_{\text{NIS-IS1}}$ reaches its maximum value of 74.1844% when $\rho_{\text{max}} = 0.808$, while the value of end-demand information $V_{\text{NIS-IS2}}$ reaches its maximum value of 69.5429% when $\rho_{\text{max}} = 0.907$.\textsuperscript{12} Additionally, we analysed the bullwhip effect difference $\DeltaBWE$ that is associated with end-demand and order information and end-demand information. This numerical analysis indicates that the savings from using end-demand and order information can be very substantial for a medium, larger price correlation coefficient value.
For example, when $b = 1$, $L_1 = L_2 = 2$, and $\sigma^2 = \delta^2 = 1$, the savings reaches its highest value of 1.2845 when $\rho_{\min} = 0.641$. Therefore, the wholesaler should adopt end-demand and order information, i.e., use the retailer’s previous order history and customer demand and price information to conduct forecasting for products with a medial, more highly correlated pricing process, especially when $b$ is large, $L_1$ is long, $L_2$ is long, $\sigma^2$ is small, or $\delta^2$ is large. In this situation, the manufacturer benefits more from using end-demand and order information than from using end-demand information.

From the above analyses, several important managerial insights are revealed. When the underlying, overall market product pricing process is medial (or even highly) correlated over time and the overall market shocks are small, benefits from information sharing will occur. In addition, the wholesaler should adopt end-demand and order information, especially when the product price sensitivity coefficient is large or the demand shocks are low. In contrast, for products with a medial (or even highly) correlated pricing process and high overall market shocks, if the product price sensitivity coefficient is small or the demand shocks are high, a need for information sharing will exist, and the wholesaler should adopt end-demand and order information. Furthermore, we have showed that if the lead times $L_1$ or $L_2$ are long, the manufacturer would have a greater incentive to invest in information sharing and, therefore, adopt end-demand and order information.

8. Conclusions

Information sharing is frequently suggested to reduce the bullwhip effect in a supply chain. In this paper, we have considered three information settings: no information, end-demand and order information, and end-demand information sharing. We derived the analytical expressions of the bullwhip effect under the three information settings and performed a theoretical analysis to determine the value of the two information-sharing settings (i.e., end-demand and order information and end-demand information) in respect to the percentage of reduction in the bullwhip effect. We also compared the bullwhip effect under the two information-sharing settings to gain insights into choosing an appropriate information setting to restrain this effect. The results showed that: (1) because the market demand scale has no effect on the bullwhip effect, it does not influence the value of information sharing; (2) the value of information sharing is significant when the underlying overall market pricing process is highly correlated over time, the overall market shocks are low, the product price sensitivity coefficient is small, the demand shocks specific to the retailer are high, or when the retailer (or wholesaler) lead time is long; (3) the value of adopting end-demand and order information is always greater than when adopting end-demand information. Thus, the wholesaler should use the retailer’s previous order history and customer demand and price information to conduct forecasting, especially when the underlying overall market pricing process is medically (or even highly) correlated over time, the overall market shocks are high, the product price sensitivity coefficient is large, the demand shocks are low, or when the retailer (or wholesaler) lead time is long.

The key implication of our findings is that, if the overall market shocks for products with a medically (or even highly) correlated pricing process are small, great benefits from information sharing will occur. Thus, the retailer should share its customer demand and price information with its upstream businesses. In addition, the wholesaler should adopt end-demand and order information, especially when the product price sensitivity coefficient is large or the demand shocks are low. However, if the overall market shocks are high when the product price sensitivity coefficient is small or the demand shocks are high, information sharing is needed and the wholesaler should adopt end-demand and order information. Additionally, if the retailer (or wholesaler) lead time is long, the manufacturer will have a greater incentive to invest in information sharing and adopt end-demand and order information. These findings provide valuable insights to the partners along a supply chain when evaluating information-sharing programs.

The research presented here can lead to several future works that focus on the empirical validation of our analytical results or theoretical extensions of our model. Empirically, firm-level demand data, order data, and macroeconomic industry-level pricing data can be collected to estimate the key parameters of our model, which can be used to validate our findings on the impact of information sharing on the bullwhip effect. Theoretically, our model considers only the order-up-to inventory policy and the MMSE forecasting technique, and other inventory policies and forecasting techniques still require further study. Moreover, because the bullwhip effect may lead to misguided inventory levels and make upstream, large inventory costs unavoidable, the methods for quantifying the impact of information sharing on inventory and expected costs in a supply chain is another future direction of study.

Acknowledgements

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Notes
1. Zhang and Burke (2011) considered an AR (1) pricing process to investigate compound causes of the bullwhip effect by analysing an inventory system with multiple price-sensitive demand streams. However, this paper uses the AR (1) pricing process to study the impact of information sharing on the bullwhip effect.
2. Note that \(\eta_t\) is the function of two types of error terms, the demand shocks that are specific to the retailer, \(e_t\), and overall market shocks, \(\tilde{e}_t\). The reduced demand model is not an AR (1) or more general ARMA demand process.
3. We use the stationary AR (1) pricing process to simplify our exposition. However, when the pricing process is nonstationary due to its increasing (or decreasing) trend or business cycle, the mean price, \(\mu_t\), may vary over time. However, if the nonstationarity is as simple as the mean price varying in a known way (\(\mu_t = \text{constant}\), e.g., because of the business cycle, then we can use the same approach to analyse when the pricing process is nonstationary, i.e., \(p_t - \mu_t = \rho(p_{t-1} - \mu_{t-1}) + \tilde{\eta}_t\). The results presented in this paper remain unchanged. The demand model with the AR (1) demand process also used this approach to deal with a non-stationary situation (Sodhi and Tang 2011).
4. Our model can be extended to when \(z_t \neq 0\). However, it can be shown that the estimation of the standard deviation of the \(L_t\) period forecasting error is independent of time, and the results in this paper remain unchanged. For a better understanding, we refer readers to read through this paper and then see Appendix B for a more detailed discussion of these contents.
5. The assumption that is presented here can be extended to analyse different forecasting techniques, such as the MA or ES techniques. However, because our intent is to analyse the value of information sharing on the bullwhip effect, we shall restrict our attention to only the optimal forecasting technique, i.e., the MMSE technique. A similar assumption has also been made by Lee, So, and Tang (2000), Hosoda and Disney (2006), and Sodhi and Tang (2011).
6. The retailer’s order quantity can also be written as \(q_{t+1} = l_{t+1}d_{t+1} - \rho l_t d_t - \rho l_t (e_{t-1} - e_t)\). Therefore, the wholesaler can utilise this equation to estimate the actual value of \(d_t\) and then utilise Equation (1) to estimate the actual value of \(p_t\). However, because it is complicated to conduct a theoretical analysis on the value of information sharing when the wholesaler utilises historical order quantities to estimate the actual demand and price, we shall limit the scope of our paper by assuming that the wholesaler would not utilise these equations to estimate the actual value of \(d_t\) and \(p_t\). A similar assumption has also been adopted by Lee, So, and Tang (2000) and Ali and Boylan (2011).
7. In reality, neither the retailer nor the wholesaler knows the exact values of the parameters of the demand process. However, the retailer can use the statistical software and the historical demand and price data to estimate the parameters of the demand process with sufficient accuracy. In addition, as shown in Lee, So, and Tang (2000), it is also reasonable that the wholesaler knows the demand process parameters, as information about the underlying demand process can be communicated to the wholesaler by discussing periodically with the retailer, or the wholesaler can be provided with historic demand and price data from which the demand process parameters can be readily deduced. A similar assumption has also been adopted by Gaur, Giloni, and Seshadri (2005).
8. We can rewrite Equations (1) and (2) as \(d_{t+1} = a - bp_{t+1} + e_{t+1}\) and \(p_{t+1} = \mu + \rho p_{t-2} + \eta_{t+1}\). Thus, \(e_{t+1}\) can be given as \(d_{t+1} - (a - bp_{t+1})\) and \(\eta_{t+1}\) can be given as \(p_{t+1} - (\mu + \rho p_{t-2})\).
9. The assumption presented here can be extended to analyse when \(L_1 \neq 0\); however, the analysis would become more complex. Because our intent is to obtain basic managerial insight, we shall restrict our attention to the assumption that \(L_1 = 0\). We will analyse the influence of \(b, L_1, L_2, \sigma^2, \text{and } \delta^2\) on the value of information sharing using the numerical analysis in Section 7, when \(L_1 \neq 0\).
10. We did not conduct a theoretical analysis on the impact of the price correlation coefficient, \(\rho\), on the value of information sharing. However, it can be shown that the value of information sharing reaches a maximum value at a certain \(\rho\) value, and we will conduct a numerical analysis in Section 7 to understand this point.
11. Note, there is one special case in our numerical example where \(V_{\text{NIS-SIS}}\) decreases when \(L_1\) increases from 2 to 4; see the scenario \(\sigma^2 = 2\delta^2 = 2\) for \(b = 1\) and for \(\rho = 0.1\) when comparing the values in Table 4 with those in Table 5. However, for other cases, \(V_{\text{NIS-SIS}}\) increases with \(L_1\).
12. For large values of \(L_1\) and \(L_2\), \(\rho_{\text{max}}\) is close to one under the two information-sharing settings. Numerical results for these cases are not given in this paper as tabular forms. For example, \(V_{\text{NIS-SIS}}\) reaches its maximum when \(\rho_{\text{max}} = 0.970\), and \(V_{\text{NIS-SIS}}\) reaches its maximum when \(\rho_{\text{max}} = 0.990\) when \(b = 1, L_1 = L_2 = 10\), and \(\sigma^2 = \delta^2 = 1\). Also note that \(\rho_{\text{max}}\) increases when the lead times \(L_1\) and \(L_2\) are increased. For example, \(\Delta BWE_{\text{min}}\) reaches its minimum value, i.e., the savings reach their highest value when \(\rho_{\text{max}} = 0.875\) when \(b = 1, L_1 = L_2 = 10\), and \(\sigma^2 = \delta^2 = 1\).

References


**Appendix A**

It is necessary to obtain analytical results based on the assumptions that (1) excess inventory can be freely returned, i.e., \( q_i \) can be negative, and (2) backorders are allowed. However, because these assumptions will not be appropriate in many retail settings, we are interested in determining whether these assumptions significantly affect the variance of order quantities. We use simulation to estimate the value of the order variance when excess inventory cannot be returned and backorders are not allowed, and we show the simulation results of the retailer’s ordering process as an example to simplify our exposition. The results of the wholesaler’s ordering process are not reported here.

In our simulation example, the demand function model is specified by \( a = 1000 \) and \( b = 1, 2, 3, 4, 5, 6, 7, 8, \) and 9. The pricing process is specified by \( \mu = 10 \) and \( \rho = 0.1, 0.3, 0.5, 0.7, \) and 0.9; and we fixed \( L_1 = 2 \) and \( \sigma^2 = \delta^2 = 25 \). Given these parameters, we first generated the random price and the corresponding demand for 1000 consecutive time periods, then we computed simulated estimates of the amplified value of the variance of the retailer’s order quantity, which were assigned models B1, B2, B3, and B4 for the following four scenarios

1. Orders can be negative and backorders are allowed, i.e., \( q_i = y_i – (y_{i-1} - d_{i-1}) \).
2. Orders cannot be negative and backorders are allowed, i.e., \( q_i = \max\{y_i – (y_{i-1} - d_{i-1}), 0\} \).
3. Orders can be negative and no backorders are allowed, i.e., \( q_i = y_i - \max\{y_{i-1} - d_{i-1}, 0\} \).
4. Orders cannot be negative and no backorders are allowed, i.e., \( q_i = \max\{y_i – \max\{y_{i-1} - d_{i-1}, 0\}, 0\} \).

Note that the first model is the model that was analysed in this paper.

Figure 3 compares the variance amplification of the orders in these four models for various values of \( b \) when \( \rho = 0.1, 0.3, 0.5, 0.7, \) and 0.9. According to Figures 3(a), (b), (c), and (d), we can see that there is no difference among these four models if we compare the variance amplification for all given values of \( b \) in our simulation setting. In addition, we note from Figure 3(e) that for small values of \( b \), the four lines are also indistinguishable, which indicates that very little difference occurs between the variance amplification of the orders in each case. However, we see that for large values of \( b \), the models in which orders cannot be negative (i.e., models B2 and B4) have slightly lower variance amplification than the models in which orders can be negative (i.e., models B1 and B3). Additionally, for large values of \( b \), the models with no backorders (i.e., models B3 and B4) have a slightly lower variance amplification than the models with backorders (i.e., models B1 and B2). Based on additional (unreported) simulations given different values of \( L_1, \sigma^2, \) and \( \delta^2 \), we can derive similar tendencies. We conclude that, in most cases, very little difference occurs between the variance amplification of the retailer’s orders in these four models. This finding implies that the assumptions that excess inventory can be returned and backorders are allowed do not significantly affect the variance of the order quantities when compared to models in which (1) excess inventory cannot be returned or (2) no backorders are allowed.

**Appendix B**

The estimation of the standard deviation of the \( L_1 \) period forecasting error can be given as follows:

\[
\delta_i^{L_1} = \sqrt{\text{Var}(D_i^{L_1} - \hat{D}_i^{L_1})}, \quad i = 1, 2. \tag{B.1}
\]

Proof of the retailer’s order quantity when \( z_1 \neq 0 \) remains the same as that when \( z_1 = 0 \).

When \( i = 1 \), Ma et al. (2013) demonstrated that the variance of the lead-time demand forecasting error is independent of time and can be expressed as:

\[
(\delta_i^{L_1})^2 = \text{Var}(D_i^{L_1} - \hat{D}_i^{L_1}) = L_1 \sigma^2 + \frac{b^2}{(1 - \rho)^2} \left( L_1 + \frac{\rho(1 - \rho^{L_1})(\rho^{L_1+1} - \rho - 2)}{1 - \rho^2} \right) \delta^2. \tag{B.2}
\]

Thus, \( \delta_i^{L_1} \) is also independent of time and can be expressed as:
Figure 3. \(\text{Var}(q_t)/\text{Var}(d_t)\) for \(a = 1000, \mu = 10, L_1 = 2,\) and \(\sigma^2 = \delta^2 = 25\) when \(\rho = 0.1, 0.3, 0.5, 0.7,\) and 0.9.
\[
\sigma_i^{t+1} = \sqrt{L_1 \sigma^2 + \frac{b^2}{(1-\rho)^2} \left( L_1 + \frac{\rho(1-\rho^3)(\rho^3 - \rho - 2)}{1-\rho^2} \right) \delta^2}.
\]

(B.3)

Therefore, if we substitute Equation (5) into Equation (3), the retailer’s order quantity when \( z_1 \neq 0 \) remains the same as that when \( z_1 = 0 \), and the results in this paper remain unchanged. \( \square \)

Proof of the wholesaler’s order quantity when \( z_2 \neq 0 \) remains the same as that when \( z_2 = 0 \).

When \( i = 2 \), three information settings are considered in this paper: no information sharing, end-demand and order information, and end-demand information.

The wholesaler’s demand corresponds to the retailer’s order quantity. Using Equation (13), thus,

\[
D_i^{t+2} = \sum_{i=1}^{L_2} q_{i+1}^t = (L_2 - \rho A_{L_2})\mu_d + \rho A_{L_2}q_{i+1}^t + \sum_{i=1}^{L_2} e_{i+1} - \rho A_{L_2}e_{i-1}
- b \sum_{i=1}^{L_2} (A_{L_2+1} \eta_{t+i-1} + \rho(A_{L_2+1} - A_{L_1}) \eta_{t+i-2} + \rho^2(A_{L_2+1} - A_{L_1}) \eta_{t+i-3} + \cdots + \rho^{i-1}(A_{L_2+1} - A_{L_1}) \eta_0) + b\rho A_{L_2}A_{L_2} \eta_{i-1}
- b\sum_{i=1}^{L_2} (A_{L_2+1} \eta_{t+i-1} + \cdots + A_{L_2+2} \eta_{t+i-2} + A_{L_2+1} \eta_{t+i-3}) + b\rho A_{L_2}A_{L_2} \eta_{i-1}
= (L_2 - \rho A_{L_2})\mu_d + \rho A_{L_2}q_{i+1}^t + \sum_{i=1}^{L_2} e_{i+1} - \rho A_{L_2}e_{i-1} - b \sum_{i=1}^{L_2} A_{L_2+1} \eta_{t+i-1} + b b \rho A_{L_2}A_{L_2} \eta_{i-1}.
\]

(B.4)

where \( A_{L_1} = (1-\rho^L)/(1-\rho) \) and \( A_{L_2} = (1-\rho^{l_2})/(1-\rho) \).

(1) If there is no information sharing, the forecasting lead-time demand \( D_i^{t+2,NS} \) can be given as \( (L_2 - \rho A_{L_2})\mu_d + \rho A_{L_2}q_{i+1}^t \). See Equation (16). Thus, Equation (B.1) can be expressed as:

\[
\sigma_i^{t+2,NS} = \sqrt{\Var(D_i^{t+2} - D_i^{t+2,NS})}
= \sqrt{\Var \left( \sum_{i=1}^{L_2} e_{i+1} - \rho A_{L_2}e_{i-1} - b \sum_{i=1}^{L_2} A_{L_2+1} \eta_{t+i-1} + b \rho A_{L_2}A_{L_2} \eta_{i-1} \right)}
= \sqrt{(L_2 + \rho^2(A_{L_2})^2) \sigma^2 + b^2 \sum_{i=1}^{L_2} (A_{L_2+1} \eta_{t+i-1})^2 + \rho^2(A_{L_2})^2(A_{L_2})^2 \delta^2}.
\]

(B.5)

(2) If there is end-demand and order information, the forecasting lead-time demand \( D_i^{t+2,JSI} \) can be given as \( (L_2 - \rho A_{L_2})\mu_d + \rho A_{L_2}q_{i+1}^t - \rho A_{L_2}e_{i-1} + b \rho A_{L_2}A_{L_2} \eta_{i-1} \). See Equation (19). Thus, Equation (B.1) can be expressed as:

\[
\sigma_i^{t+2,JSI} = \sqrt{\Var(D_i^{t+2} - D_i^{t+2,JSI})}
= \sqrt{\Var \left( \sum_{i=1}^{L_2} e_{i+1} - b \sum_{i=1}^{L_2} A_{L_2+1} \eta_{t+i-1} \right)}
= \sqrt{L_2 \sigma^2 + b^2 \sum_{i=1}^{L_2} (A_{L_2+1} \eta_{t+i-1})^2}.
\]

(B.6)

(3) If there is end-demand information, the forecasting lead-time demand \( D_i^{t+2,JSI} \) can be given as \( L_2 \mu_d + \frac{b \rho A_{L_2} \mu}{1-\rho^L} - bp A_{L_2}p_{i-1} \). See Equation (21). Thus, Equation (B.1) can be expressed as:
\[ \sigma^2_{t,\text{NS}} = \sqrt{\text{Var}(D_l^2 - \hat{D}_l^2)} \]

\[
= \sqrt{\text{Var} \left( \rho A_{l,t} q_t^l + b \rho A_{l,t} p_{t-1} + \left( \sum_{i=1}^{L_2} \varepsilon_{i,t+i-1} - \rho A_{l,t} \varepsilon_{t-1} - b \sum_{i=1}^{L_2} A_{l,i+L_2-i+i} \eta_{t+i-1} + b \rho A_{l,t} A_{l,t} \eta_{t-1} \right) \right)}
\]

\[
= \frac{\rho^2 (A_{l,t}^2 \text{Var}(q_t^l) + b^2 \rho^2 (A_{l,t}^2 \text{Var}(p_t) + (\sigma^2_{t,\text{NS}})^2 + 2b \rho^2 (A_{l,t})^2 \text{Cov}(q_t^l, p_{t-1})}
+ 2b \rho A_{l,t} \text{Cov} \left( q_t^l, \sum_{i=1}^{L_2} \varepsilon_{i,t+i-1} - \rho A_{l,t} \varepsilon_{t-1} - b \sum_{i=1}^{L_2} A_{l,i+L_2-i+i} \eta_{t+i-1} + b \rho A_{l,t} A_{l,t} \eta_{t-1} \right)
\]

\[
\left( + 2b \rho A_{l,t} \text{Cov} \left( p_{t-1}, \sum_{i=1}^{L_2} \varepsilon_{i,t+i-1} - \rho A_{l,t} \varepsilon_{t-1} - b \sum_{i=1}^{L_2} A_{l,i+L_2-i+i} \eta_{t+i-1} + b \rho A_{l,t} A_{l,t} \eta_{t-1} \right) \right) \right)
\]

\[
= \frac{\rho^2 (A_{l,t}^2 \text{Var}(q_t^l) + b^2 \rho^2 (A_{l,t}^2 \text{Var}(p_t) + (\sigma^2_{t,\text{NS}})^2 + 2b \rho^2 (A_{l,t})^2 \text{Cov}(q_t^l, p_{t-1})}
+ 2b^2 \rho^2 (\rho^2 A_{l,t} - A_{l,t+1})(A_{l,t})^2 \sigma_p^2 - 2b^2 (A_{l,t}^2)^2 (\sigma^2 + b^2 A_{l,t}^2 \eta_{t+1} \delta^2) + 2b^2 \rho^2 (A_{l,t})^2 \delta^2}
\]

\[
= \frac{\rho^2 (A_{l,t}^2 \text{Var}(q_t^l) + b^2 \rho^2 (A_{l,t}^2 \text{Var}(p_t) + (\sigma^2_{t,\text{NS}})^2 + 2b \rho^2 (A_{l,t})^2 \text{Cov}(q_t^l, p_{t-1})}
+ 2b^2 \rho^2 (\rho^2 A_{l,t} - A_{l,t+1})(A_{l,t})^2 \sigma_p^2 - 2b^2 (A_{l,t}^2)^2 (\sigma^2 + b^2 A_{l,t}^2 \eta_{t+1} \delta^2) + 2b^2 \rho^2 (A_{l,t})^2 \delta^2}
\]

where

\[ \text{Var}(q_t^l) = \left( 1 + 2b^2 \rho (1 - \rho) A_{l,t} A_{l,t+1} \frac{\delta^2}{(1 - \rho^2) \sigma^2 + b^2 \delta^2} \right) \sigma_p^2 \] (see Equation (23)),

\[ \text{Cov}(q_t^l, p_{t-1}) = \text{Cov}(-b \rho A_{l,t} (p_{t-1} - p_{t-2}) + d_{t-1}, p_{t-1}) \]

\[ = -b \rho A_{l,t} \sigma_p^2 + b \rho A_{l,t} \text{Cov}(p_{t-1}, p_{t-2}) + \text{Cov}(d_{t-1}, p_{t-1}) \]

\[ = -b \rho A_{l,t} \sigma_p^2 + b \rho^2 A_{l,t} \sigma_p^2 + \text{Cov}(a - b p_{t-1} + \varepsilon_{t-1}, p_{t-1}) \]

\[ = b (\rho^2 A_{l,t} - A_{l,t+1}) \sigma_p^2, \]

\[ \text{Cov} \left( q_t^l, \sum_{i=1}^{L_2} \varepsilon_{i,t+i-1} - \rho A_{l,t} \varepsilon_{t-1} - b \sum_{i=1}^{L_2} A_{l,i+L_2-i+i} \eta_{t+i-1} + b \rho A_{l,t} A_{l,t} \eta_{t-1} \right) \]

\[ = \text{Cov} \left( q_t^l, \sum_{i=1}^{L_2} \varepsilon_{i,t+i-1} - \rho A_{l,t} \text{Cov}(q_t^l, \varepsilon_{t-1}) - b \text{Cov} \left( q_t^l, \sum_{i=1}^{L_2} A_{l,i+L_2-i+i} \eta_{t+i-1} \right) + b \rho A_{l,t} A_{l,t} \text{Cov}(q_t^l, \eta_{t-1}) \right) \]

\[ = -b^2 \rho A_{l,t} A_{l,t+1} \delta^2, \]

\[ \text{Cov} \left( p_{t-1}, \sum_{i=1}^{L_2} \varepsilon_{i,t+i-1} - \rho A_{l,t} \varepsilon_{t-1} - b \sum_{i=1}^{L_2} A_{l,i+L_2-i+i} \eta_{t+i-1} + b \rho A_{l,t} A_{l,t} \eta_{t-1} \right) \]

\[ = b \rho A_{l,t} A_{l,t} \text{Cov}(p_{t-1}, \eta_{t-1}) = b \rho A_{l,t} A_{l,t} \delta^2, \]

where

\[ q_t^l = -b \rho A_{l,t} (p_{t-1} - p_{t-2}) + d_{t-1} \] (see Equation (11)),

\[ \text{Cov}(q_t^l, p_{t-1}) \]

\[ = \text{Cov}(-b \rho A_{l,t} (p_{t-1} - p_{t-2}) + d_{t-1}, p_{t-1}) \]

\[ = -b \rho A_{l,t} \sigma_p^2 + b \rho A_{l,t} \text{Cov}(p_{t-1}, p_{t-2}) + \text{Cov}(d_{t-1}, p_{t-1}) \]

\[ = -b \rho A_{l,t} \sigma_p^2 + b \rho^2 A_{l,t} \sigma_p^2 + \text{Cov}(a - b p_{t-1} + \varepsilon_{t-1}, p_{t-1}) \]

\[ = b (\rho^2 A_{l,t} - A_{l,t+1}) \sigma_p^2, \]


\[ \text{Cov} \left( q_i^1, \sum_{i=1}^{l_2} \eta_{i+1} \right) = \text{Cov} \left( -b p \Lambda_i (p_{t-1} - p_{t-2}) + d_{t-1}, \sum_{i=1}^{l_2} \Lambda_i \eta_{i+1} \right) \]

\[ = \text{Cov} \left( d_{t-1}, \sum_{i=1}^{l_2} \Lambda_i \eta_{i+1} \right) = \text{Cov} \left( a - b p_{t-1} + \epsilon_{t-1}, \sum_{i=1}^{l_2} \Lambda_i \eta_{i+1} \right) = 0, \]

\[ \text{Cov} (q_i^1, \eta_{t-1}) = \text{Cov} (-b p \Lambda_i (p_{t-1} - p_{t-2}) + d_{t-1}, \eta_{t-1}) \]

\[ = \text{Cov} \left( a - b \Lambda_i + p_{t-1} + b \Lambda_i p_{t-2} + \epsilon_{t-1}, \eta_{t-1} \right) \]

\[ = -b \Lambda_i \text{Var} (p_{t-1}, \eta_{t-1}) = -b \Lambda_i \delta^2. \]

It can be shown from Equations (B.5), (B.6), and (B.7) that \( \sigma_{q_i^1}^{TNS}, \sigma_{q_i^1}^{TSL}, \) and \( \sigma_{q_i^1}^{TSL^2} \) are all independent of time. Therefore, if we substitute Equation (5) into Equation (4), the wholesaler’s order quantity when \( z_2 \neq 0 \) remains the same as that when \( z_2 = 0 \), and the results in this paper remain unchanged.

**Appendix C**

**Proof:** The variance of the retailer’s order quantity can be derived from Equation (11) as follows:

\[ \text{Var} (q_i^1) = b^2 \rho^2 (\Lambda_i)^2 \text{Var} (p_{t-1} - p_{t-2}) + \text{Var} (d_{t-1}) - 2 bp \Lambda_i \text{Cov} (d_{t-1}, p_{t-1} - p_{t-2}), \quad (C.1) \]

where

\[ \text{Var} (p_{t-1} - p_{t-2}) = 2 \text{Var} (p_t) - 2 \text{Cov} (p_{t-1}, p_{t-2}) = 2(1 - \rho) \text{Var} (p_t) = 2 \delta^2 / (1 + \rho), \quad (C.2) \]

\[ \text{Cov} (d_{t-1}, p_{t-1} - p_{t-2}) = \text{Cov} (a - b p_{t-1} + \epsilon_{t-1}, p_{t-1} - p_{t-2}) \]

\[ = -b \text{Var} (p_t) + b \text{Cov} (p_{t-1}, p_{t-2}) = -b(1 - \rho) \text{Var} (p_t) = -b \delta^2 / (1 + \rho). \quad (C.3) \]

Substituting Equations (C.2) and (C.3) into Equation (C.1) and dividing both sides of Equation (C.1) by \( \text{Var} (d_t) \), and because \( \text{Var} (d_{t-1}) = \text{Var} (d_t) = \sigma^2 + b^2 \delta^2 / (1 - \rho^2) \), we can prove Theorem 1. This completes the proof. \( \square \)
Appendix D

**Proof:** The variance of the order quantity without information sharing can be derived from Equation (17) as follows:

\[
\text{Var}(q_t^{\text{INS}}) = (\lambda_{t+1})^2 \text{Var}(q_t^1) + \rho^2(\lambda_{t+1})^2 \text{Var}(q_t^{1,1}) - 2\rho \lambda_{t+1} \lambda_{t+2} \text{Cov}(q_t^1, q_t^{1,1})
\]

\[
= ((\lambda_{t+1})^2 + \rho^2(\lambda_{t+1})^2) \text{Var}(q_t^1) - 2\rho \lambda_{t+1} \lambda_{t+2} \text{Cov}(q_t^1, q_t^{1,1}),
\]

where

\[
\text{Cov}(q_t^1, q_t^{1,1}) = \text{Cov}(1 - \rho)\mu_t + p\mu_{t-1} + \epsilon_{t-1} - \rho\epsilon_t - b\lambda_{t+1} \eta_{t-1} + bp\lambda_{t+1} \eta_{t-2}, q_t^{1,1})
\]

\[
= \rho \text{Var}(q_t^1) + \rho \text{Cov}(q_t^{1,1}, \epsilon_{t-1}) - \rho \text{Var}(q_t^{1,1}, \epsilon_t) - b\lambda_{t+1} \text{Cov}(q_t^{1,1}, \eta_{t-1}) + bp\lambda_{t+1} \text{Cov}(q_t^{1,1}, \eta_{t-2})
\]

\[
= \rho \text{Var}(q_t^1) - \rho \sigma^2 - b^2 \rho \lambda_{t+1} \lambda_{t+2} \epsilon_t^2,
\]

where

\[
q_t^1 = (1 - \rho)\mu_t + p\mu_{t-1} + \epsilon_{t-1} - \rho\epsilon_t - b\lambda_{t+1} \eta_{t-1} + bp\lambda_{t+1} \eta_{t-2},
\]

\[
\text{Cov}(q_t^{1,1}, \epsilon_{t-1}) = \text{Cov}(-bp\lambda_t (p_{t-2} - p_{t-3}) + \epsilon_{t-2}, \epsilon_{t-1}) = \text{Cov}(d_{t-2}, \epsilon_{t-1})
\]

\[
= \text{Cov}(a - bp_{t-2} + \epsilon_{t-2}, \epsilon_{t-1}) = \text{Cov}(0),
\]

\[
\text{Cov}(q_t^{1,1}, \epsilon_{t-2}) = \text{Cov}(-bp\lambda_t (p_{t-2} - p_{t-3}) + d_{t-2}, \epsilon_{t-2}) = \text{Cov}(d_{t-2}, \epsilon_{t-2})
\]

\[
= \text{Cov}(a - bp_{t-2} + \epsilon_{t-2}, \epsilon_{t-2}) = \text{Cov}(0, \sigma^2),
\]

\[
\text{Cov}(q_t^{1,1}, \eta_{t-1}) = \text{Cov}(-bp\lambda_t (p_{t-2} - p_{t-3}) + d_{t-2}, \eta_{t-1}) = \text{Cov}(d_{t-2}, \eta_{t-1})
\]

\[
= \text{Cov}(0),
\]

\[
\text{Cov}(q_t^{1,1}, \eta_{t-2}) = \text{Cov}(-bp\lambda_t (p_{t-2} - p_{t-3}) + d_{t-2}, \eta_{t-2})
\]

\[
= -bp\lambda_t \text{Cov}(p_{t-2}, \eta_{t-2}) + \text{Cov}(d_{t-2}, \eta_{t-2})
\]

\[
= -bp\lambda_t \text{Cov}(a - bp_{t-2} + \epsilon_{t-2}, \eta_{t-2})
\]

\[
= -b(1 + \rho \lambda_t) \text{Cov}(p_{t-2}, \eta_{t-2}) = -b\lambda_{t+1} \delta^2.
\]

Substituting Equation (D.2) into Equation (D.1), and dividing both sides of Equation (D.1) by \( \text{Var}(d_t) \), we can prove Theorem 2 using Equation (23). This completes the proof. \( \square \)

Appendix E

**Proof:** The variance of the order quantity under end-demand and order information can be derived from Equation (20) as follows:

\[
\text{Var}(q_t^{\text{INS}}) = (\lambda_{t+1})^2 \text{Var}(q_t^1) + \rho^2(\lambda_{t+1})^2 \text{Var}(q_t^{1,1}) + 2\rho^2(\lambda_{t+1})^2 \text{Var}(\epsilon_{t-1} - \epsilon_t) + b^2 \rho^2(\lambda_{t+1})^2 \text{Var}(\eta_{t-1} - \eta_{t-2})
\]

\[
- 2\rho \lambda_{t+1} \lambda_{t+2} \text{Cov}(q_t^1, q_t^{1,1}) - 2\rho \lambda_{t+1} \lambda_{t+2} \text{Cov}(q_t^1, \epsilon_{t-1} - \epsilon_t) + 2bp\lambda_{t+1} \lambda_{t+2} \text{Cov}(q_t^1, \eta_{t-1} - \eta_{t-2})
\]

\[
+ 2\rho^2(\lambda_{t+1})^2 \text{Cov}(q_t^{1,1}, \epsilon_{t-1} - \epsilon_t) - 2bp^2 \lambda_{t+1} \text{Cov}(q_t^{1,1}, \eta_{t-1} - \eta_{t-2})
\]

\[
= ((\lambda_{t+1})^2 + \rho^2(\lambda_{t+1})^2) \text{Var}(q_t^1) + 2\rho^2(\lambda_{t+1})^2 \sigma^2 + b^2 \rho^2(\lambda_{t+1})^2 \delta^2 - 2\rho \lambda_{t+1} \lambda_{t+2} \text{Cov}(q_t^1, q_t^{1,1})
\]

\[
- 2bp\lambda_{t+1} \lambda_{t+2} \text{Cov}(q_t^1, \epsilon_{t-1} - \epsilon_t) + 2bp\lambda_{t+1} \lambda_{t+2} \text{Cov}(q_t^1, \eta_{t-1} - \eta_{t-2})
\]

\[
+ 2\rho^2(\lambda_{t+1})^2 \text{Cov}(q_t^{1,1}, \epsilon_{t-1} - \epsilon_t) - 2bp^2 \lambda_{t+1} \text{Cov}(q_t^{1,1}, \eta_{t-1} - \eta_{t-2})),
\]

\( \square \)
where

\[ \text{Cov}(q^1_t, q^1_{t-1}) = \rho \text{Var}(q^1_t) - \rho \sigma^2 - b^2 \rho \Delta L_t \Delta L_{t+1} \delta^2, \]

\[ \text{Cov}(q^1_t, \epsilon_{t-1}) = \text{Cov}(-b \rho \Delta L_t (p_{t-1} - p_{t-2}) + d_{t-1}, \epsilon_{t-1}) = \text{Cov}(d_{t-1}, \epsilon_{t-1}) \]
\[ = \text{Cov}(a - b \rho \Delta L_t + \epsilon_{t-1}, \epsilon_{t-1}) = \sigma^2, \]

\[ \text{Cov}(q^1_t, \epsilon_{t-2}) = \text{Cov}(-b \rho \Delta L_t (p_{t-1} - p_{t-2}) + d_{t-1}, \epsilon_{t-2}) = \text{Cov}(d_{t-1}, \epsilon_{t-2}) \]
\[ = \text{Cov}(a - b \rho \Delta L_t + \epsilon_{t-1}, \epsilon_{t-2}) = 0, \]

\[ \text{Cov}(q^1_t, \eta_{t-1}) = \text{Cov}(-b \rho \Delta L_t (p_{t-1} - p_{t-2}) + d_{t-1}, \eta_{t-1}) \]
\[ = -b \rho \Delta L_t \text{Cov}(p_{t-1}, \eta_{t-1}) + \text{Cov}(a - b \rho \Delta L_t + \epsilon_{t-1}, \eta_{t-1}) \]
\[ = -b(1 + \rho \Delta L_t) \text{Cov}(p_{t-1}, \eta_{t-1}) = -b \Delta L_{t+1} \delta^2, \]

\[ \text{Cov}(q^1_t, \eta_{t-2}) = \text{Cov}(-b \rho \Delta L_t (p_{t-1} - p_{t-2}) + d_{t-1}, \eta_{t-2}) \]
\[ = -b \rho \Delta L_t \text{Cov}(p_{t-1}, \eta_{t-2}) + \text{Cov}(a - b \rho \Delta L_t + \epsilon_{t-1}, \eta_{t-2}) \]
\[ = b \rho (1 - \rho) \Delta L_t \delta^2 - b \rho \delta^2 = b \rho ((1 - \rho) \Delta L_t - 1) \delta^2, \]

\[ \text{Cov}(q^1_{t+1}, \epsilon_{t-1}) = 0, \text{Cov}(q^1_{t+1}, \epsilon_{t-2}) = \sigma^2, \text{Cov}(q^1_{t+1}, \eta_{t-1}) = 0, \text{Cov}(q^1_{t+1}, \eta_{t-2}) = -b \Delta L_{t+1} \delta^2. \]

Substituting the above equations into Equation (E.1) and dividing \( \text{Var}(d_t) \) on both sides of Equation (E.1), we can prove Theorem 3 using Equation (23). This completes the proof. \( \square \)

Appendix F

Proof: The variance of the order quantity under end-demand information can be derived from Equation (22) as follows:

\[ \text{Var}(q^1_t) = b^2 \rho^2 (\Delta L_t + \Delta L_{t+1})^2 \text{Var}(p_{t-1} - p_{t-2}) + \text{Var}(d_{t-1}) \]
\[ - 2b \rho (\Delta L_t + \Delta L_{t+1}) \text{Cov}(d_{t-1}, p_{t-1} - p_{t-2}), \]

(F.1)

where

\[ \text{Var}(p_{t-1} - p_{t-2}) = 2 \delta^2 / (1 + \rho), \]

see Equation (C.2), and

\[ \text{Cov}(d_{t-1}, p_{t-1} - p_{t-2}) = -b \delta^2 / (1 + \rho), \]

see Equation (C.3).

Substituting the above two equations into Equation (F.1) and dividing both sides of Equation (F.1) by \( \text{Var}(d_t) \), and because \( \text{Var}(d_{t+1}) = \text{Var}(d_t) = \sigma^2 + b^2 \delta^2 / (1 - \rho^2) \), we can prove Theorem 4. This completes the proof. \( \square \)
Appendix G

Proof of Proposition 1, Relation (1). \( \frac{\partial(V_{NS-IS1})}{\partial \theta} \leq 0 \) for \( L_1 = 0 \).

\( V_{NS-IS1} \) can be derived from Equation (27) as follows:

\[
V_{NS-IS1} = \frac{f_1(p, L_2) \cdot f_2(b, \rho, \sigma^2, \delta^2)}{1 + f_1(p, L_2) \cdot f_2(b, \rho, \sigma^2, \delta^2) + \frac{\partial f_1(p, L_2)}{\partial L_2} \cdot f_2(b, \rho, \sigma^2, \delta^2)}.
\]

(G.1)

where \( f_1(p, L_2) = 2p(1 - \rho^2)A_{L_2}A_{L_2+1} \), \( f_2(b, \rho, \sigma^2, \delta^2) = \frac{\sigma^2}{(1 - \rho^2)\sigma^2 + \delta^2} \), and \( f_2(b, \rho, \sigma^2, \delta^2) = \frac{\delta^2}{(1 - \rho^2)\sigma^2 + \delta^2} \).

Thus, \( \frac{\partial(V_{NS-IS1})}{\partial \theta} = \frac{-2b(1 + p)(1 + p + f_1(p, L_2))f_1(p, L_2)\delta^2}{[1 + p(1 - \rho^2)A_{L_2}A_{L_2+1}]^2 + b^2(1 + p + f_1(p, L_2))\delta^2} \leq 0 \). This completes the proof for relation (1).

Proof of Proposition 1, Relation (2). \( \frac{\partial(V_{NS-IS1})}{\partial \sigma^2} \geq 0 \) for \( L_1 = 0 \).

\( V_{NS-IS1} \), which is shown by Equation (G.1), can be rewritten as:

\[
V_{NS-IS1} = \frac{f_2(b, \rho, \sigma^2, \delta^2)}{2p(1 - \rho^2)A_{L_2}A_{L_2+1} + f_2(b, \rho, \sigma^2, \delta^2) + \frac{\partial f_1(p, L_2)}{\partial \sigma^2} \cdot f_2(b, \rho, \sigma^2, \delta^2)}.
\]

It is easy to see that \( \frac{\partial(V_{NS-IS1})}{\partial \sigma^2} \geq 0 \). This completes the proof for Relation (2).

Proof of Proposition 1, Relation (3). \( \frac{\partial(V_{NS-IS1})}{\partial \delta^2} \geq 0 \) for \( L_1 = 0 \).

Using Equation (G.1), it can be shown that:

\[
\frac{\partial(V_{NS-IS1})}{\partial \delta^2} = \frac{-b^2(1 + p)(1 + p + f_1(p, L_2))f_1(p, L_2)\sigma^2}{[1 + p(1 - \rho^2)A_{L_2}A_{L_2+1}]^2 + b^2(1 + p + f_1(p, L_2))\sigma^2} \geq 0.
\]

This completes the proof for Relation (3).

Proof of Proposition 1, Relation (4). \( \frac{\partial(V_{NS-IS1})}{\partial \theta} \leq 0 \) for \( L_1 = 0 \).

Using Equation (G.1), it can be shown that:

\[
\frac{\partial(V_{NS-IS1})}{\partial \theta} = \frac{-b^2(1 + p)(1 + p + f_1(p, L_2))f_1(p, L_2)\sigma^2}{[1 + p(1 - \rho^2)A_{L_2}A_{L_2+1}]^2 + b^2(1 + p + f_1(p, L_2))\sigma^2} \leq 0.
\]

This completes the proof for Relation (4).

Appendix H

Proof of Proposition 3, Relation (1). \( \Delta BWE \leq 0 \).

\( \Delta BWE \) can be derived as follows:

\[
\Delta BWE = g_1(p) \cdot g_2(p, L_1, L_2) \cdot g_3(b, \rho, \sigma^2, \delta^2),
\]

(H.1)

where \( g_1(p) = 2p(1 - \rho) \), \( g_2(p, L_1, L_2) = ((1 + p)\rho^{L_2+L_2+1} - 2p)A_{L_1}A_{L_2} - (1 - \rho)^2A_{L_1}A_{L_1+1}A_{L_2}A_{L_2+1} \), and \( g_3(b, \rho, \sigma^2, \delta^2) = \frac{b^2 \sigma^2}{(1 - \rho^2)\sigma^2 + b^2 \delta^2} \).

Thus,

\[
\Delta BWE \leq g_1(p) \cdot g_2(b, \rho, \sigma^2, \delta^2) \cdot [(1 + p)\rho - 2p]A_{L_1}A_{L_2} - (1 - \rho)^2A_{L_1}A_{L_1+1}A_{L_2}A_{L_2+1} \]

\[
= -g_1(p) \cdot g_2(b, \rho, \sigma^2, \delta^2) \cdot (1 - p)A_{L_1}A_{L_2}(\rho + (1 - \rho)A_{L_1+1}A_{L_2+1}) \leq 0.
\]
This completes the proof for Relation (1).

Proof of Proposition 3, Relation (2).

Note that $g_2(p, L_1, L_2) \leq 0$ in Equation (H.1). Thus,

$$\frac{\partial (\Delta \text{BWE})}{\partial b} = g_1(p) \cdot g_2(p, L_1, L_2) \cdot \frac{\partial (g_3(b, p, \sigma^2, \delta^2))}{\partial b} \leq 0.$$

$$\frac{\partial (\Delta \text{BWE})}{\partial L_1} = g_1(p) \cdot g_3(b, p, \sigma^2, \delta^2) \cdot \frac{\partial (g_2(p, L_1, L_2))}{\partial L_1}$$

$$= g_1(p) \cdot g_3(b, p, \sigma^2, \delta^2) \cdot \frac{\ln(p)}{(1 - p^2)} \cdot (1 - p^2) \cdot (1 - p^2) \cdot (1 - \rho^{L_1 + 1}) \leq 0.$$

$$\frac{\partial (\Delta \text{BWE})}{\partial L_2} \leq g_1(p) \cdot g_3(b, p, \sigma^2, \delta^2) \cdot \rho^{L_1} \cdot (1 - \rho^{L_1}) \cdot \frac{\ln(p)}{1 - \rho} \leq 0.$$

$$\frac{\partial (\Delta \text{BWE})}{\partial \sigma^2} = g_1(p) \cdot \frac{\partial (g_2(p, L_1, L_2))}{\partial \sigma^2} \geq 0.$$

$$\frac{\partial (\Delta \text{BWE})}{\partial \delta^2} = g_1(p) \cdot \frac{\partial (g_3(b, p, \sigma^2, \delta^2))}{\partial \delta^2} \leq 0.$$

This completes the proof for Relation (2).