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Equilibrium interbank lending networks

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Abstract—We propose a model to study short-term interbank lending from a network formation perspective. Banks, being provided with public and private signals about the solvency of other banks, decide on interbank lending by also considering the decision of other banks to lend. We observe that the dominant equilibrium networks are those where banks follow each others' decisions, making the equilibria very vulnerable to shifts in expectations. The networks range from fully connected (highly liquid markets) to empty networks (frozen markets) and we derive the conditions under which they emerge.

I. INTRODUCTION

As evidenced, for example, during the 2007 global financial turmoil, a characteristic of a financial crisis is contagion where a relatively small event, like the failure of a single financial institution, may trigger a chain reaction. This failure can spread to the whole financial system and eventually reach out to the real economy. In order to capture the connections between financial institutions a network approach has been chosen that focuses on interbank lending.

Financial systems can be seen as networks, with nodes representing individual financial institutions, and links representing their bilateral exposures, such as interbank loans, credit lines or derivatives positions. Starting from the parsimonious financial network of [1] which consists of only four banks, scholars have investigated various financial networks for a wide range of markets that are increasingly detailed and realistic, see [2] for a review of interbank network properties. The network structure is found to be greatly influential for the systemic risk of a banking system, see e.g. [3], [4], [5]. Although some universal features of financial networks have been discovered and studied, many questions await further exploration, such as how these features arise from interactions between individual financial institutions, how they further influence other banks' behaviors, and how these interactions may help to escalate or mitigate systemic risk.

One of the motives behind this work is to offer some insights into the phenomenon of liquidity freezes in interbank markets as was clearly observed and highly noted in the 2007 financial crisis. Money markets that were used to be highly liquid suddenly saw a "freeze" in liquidity, with extremely high borrowing rates. Two possible explanations are offered as to why market players with excess liquidity are not willing to lend to the market in [6]: fear of counter-party risks and

liquidity hoarding (fear of a future liquidity shortage). These two fears may intertwine together to cause a "freeze" in interbank lending. Among the literature referring to financial network formations, few have focused on explaining liquidity freezes, yet it should be interesting to explore this issue from the network formation perspective and how such freezes might emerge.

In this paper, based on the idea in [7] and developed further, we propose a model for the formation of short-term interbank loan networks, under the assumption that banks can observe each others' lending decisions and adjust their evaluations of borrowers accordingly. Under this assumption, a bank's lending decision generates information to other banks and they may sometimes exhibit herding in making lending decisions. Our primary interest is to find equilibrium structures of interbank networks ranging from a complete network (fully liquid market) to an empty network (market freeze), and show how their occurrence depends on individual behaviour. We find that how likely each structure is to occur depends on banks' risk aversion, the expected returns on interbank loans, and uncertainty of private information. For a stressed market, featuring low expectations of debt paying ability, banks eventually cause a frozen market. For a stressed market featuring high risk aversion, banks may have multiple equilibrium structures including a freeze, because banks may either refuse lending or follow each other's decisions.

The main part of this article is organized as follows: in section 2, we review the related literature and section 3 develops the model of how individual banks form and update their beliefs while section 4 assesses their decisions about interbank lending and describes the algorithm used to determine the equilibrium network structures. We analyse the probability distribution of equilibrium networks in section 5 and section 6 discusses some policy implications before we conclude our article in section 7.

II. DETERMINANTS OF INTERBANK LENDING

Firstly, this work is related to a strand of literature investigating systemic risk in interbank markets. The mechanisms of contagion can be generally classified into two types: contagion through direct links, which refers to a situation where one insolvent bank may cause losses to its creditor bank and consequently triggers insolvency among its creditors; and

secondly contagion through indirect links, which includes liquidity hoarding, common assets exposure under fire sales, and bank runs.

Studies of contagion through direct links are perhaps most abundant. Some important early models all follow the standard model of [8] but extend it with a network perspective. However, these models consider only very simple network structures, usually only three structures are studied, a complete network, a ring network and an empty network. [1] and [9] are two such early standard models, which have much similar settings. Banks have incentives to hold bilateral exposures, changing deposits [1] or having credit lines [9], due to uncertainty about deposit withdrawals, uncertainty of when depositors they consume [1] or where they consume [9]. Then under the three different network structures, they examine how the shocks spread out through a liquidity preference shock [1] or risky long-term assets [9]. Similar conclusions are drawn from both models, a complete network is usually more stable than an incomplete network (a ring network) since there are more banks to share the loss of a given shock. Following this strand of literature, [10], [11], and [12] investigate network formations in which banks choose to form a network that maximize their utility through risk diversification, profit maximization or reducing the risk of contagion. Another strand of literature develops models of financial contagion based on more complex networks, mainly random graphs, and most of these studies have noted that some sort of "phase transition" of systemic risk happens in such networks, which is also referred to by [13] as financial networks having "robust-yet-fragile" feature. In models like [4] and [14], this feature means that there exist tipping points for the level of connectivity, and that sharp changes in levels of systemic risk (the extent of contagion) occur around these tipping points. [5] find that tipping points also exist for the size of initial shock. [15] find a necessary condition for contagion to not encompass the entire network and develop a resilience measure which is a function of each bank's connectivity and fraction of contagious links. All tipping points for macroeconomic shocks correspond to positive resilience measure values. [16] develop the work of [4] by adding heterogeneity into the model and show that heterogenous connectivity, the size of banks and degree correlations play a role in determining the stability of a financial system.

Studies of contagion through indirect linkage include liquidity hoarding, common asset exposures under fire sales, and bank runs. [17] present a model showing that banks can be illiquid but solvent. They find that in equilibrium it is possible that solvent banks still fail to obtain liquidity from interbank markets due to the liquidity hoarding of informed investors. For this reason, they argue the importance of a lender of last resort. [18] present a model that highlight the role of asymmetric information in the assessment of counterparty risk leading to liquidity hoarding among liquid potential providers of funds. The result is that the market fails to reach the desirable equilibrium but instead reaches a liquidity freeze. [19] present a model showing precautionary liquidity hoarding

arising from lenders fearing rollover risk that can help to explain market stress, where high rates and low volumes for borrowing are the result of high leverage and the illiquidity of assets. [6] present a similar model of banks' choices between liquid and illiquid assets in a portfolio where banks have incentives to hoard liquidity not only due to rollover risk but also have the opportunity to buy fire sale assets of other banks that face liquidity shortages. Finally, [20] present a model that considers the interplay of market liquidity and funding liquidity and shows that they can be mutually reinforcing as liquidity spirals when margins are destabilising.

Secondly, this work is also related to another growing strand of literature on network formation games as applied to financial systems. One strand sees the network formation as a static game, like [11], [10], [5], where all individuals make decisions simultaneously. They are all based on the standard model of [1] but extend it to N banks and assume banks consider the risk of potential shocks to them or to their neighbors when choosing links. In [11], this is done by a social planner solving for and then proposing an optimal network based on the banks' random initial endowments with banks choosing to accept the proposal or have an empty network. In making decisions, banks try to balance the tradeoff between risk sharing and the risk of contagion. This is because being linked in a network on the one hand offers resources to lend from when one is facing an unexpected liquidity shock, yet on the other hand, one may suffer from losses when others withdraw money. In the model of [10] banks are classified into two types, and banks play a network formation game within each type. The primary concern of a bank is to prevent the risk of contagion, thus a bank chooses the network where no single neighbor's liquidation should lead to its own bankruptcy. In the model of [5] banks propose debt contracts conditional on borrowers' lending behaviour. As a result [11] find the network can be ex ante optimal, but a collapse of the whole system may still occur in some cases. [10] show the network can be resilient with a large number of banks, which means the probability of contagion can be close to zero. [5] find that the equilibrium interbank network formed can be vulnerable to contagion, due to the presence of financial network externalities which cause the emergence of socially inefficient network.

Another strand of this literature see the network formation as a process of evolution, e.g. [21] and [22]. In the model of [21] banks form a network from random rewiring. The possibility a bank links to another bank depends on the latter's profitability, a bank's in-degree is thus a signal for its profitability. How much banks value this signal influences the structure of the network eventually formed. In the model of [22] rollover decisions are made as in a foreclosure game. Whether a bank chooses to rollover depends on its cost of miscoordination and the borrower's asset-to-liability ratio, which are random and time-dependent. Consequently, the network structure evolves overtime, and the average connectivity in the stationary state depends on debt maturity and miscoordination cost. There is also model like [23] that use a combination of

static and dynamic games.

Among the work studying interbank network formation, we especially refer to [7] where rollover decisions of banks depend on the existing architecture of interbank networks. We keep the salient feature of their model, which is a bank's lending decision provides information to other banks, but have notable differences in both the details of the approach taken and aim of the model.

III. A MODEL OF INTERBANK LENDING DECISIONS

We assess a banking system in which N banks are considering lending to each other. For simplicity we assume that all loans are of the same size and mature in a single time period such that we can focus on the rollover decisions by banks. Furthermore, there are no constraints on the ability of banks to lend to each other and banks are always willing to accept any loans they are given. This simplification allows us to concentrate on the lending decision itself rather than having to consider the impact of any such constraints on the outcomes. Banks only differ in their risk of repaying this loan and the signals banks receive about this risk.

Interbank lending can be represented by a directed network where each node represents a bank and the edges the existence of a loan between two banks. This network can be represented by an adjacency matrix $A = \{a_{ij}\}_{i,j=1,\dots,N}$, where $a_{ij} = 1$ if bank i lends to bank j and zero otherwise. Obviously we require that $\forall i = 1, \dots, N : a_{ii} = 0$.

Decisions on interbank lending are assumed to be done decentrally such that each loan is assessed individually by a risk averse decision-maker maximizing expected utility. Approximating the expected utility of bank i giving a loan to bank j in the usual way with absolute risk aversion $\lambda_i \geq 0$, we get

$$U_{ij} = a_{ij}\mu_{ij} - \frac{1}{2}\lambda_i a_{ij}\sigma_{ij}^2, \quad (1)$$

where μ_{ij} and σ_{ij}^2 denote the expected value and variance of the return bank i believes to be generated from lending to bank j . This return will not only include the interest charged, but most importantly also include the possible default of bank j and the subsequent losses arising from this.

The following section will now explore how these expected values and variances are determined. We will use information based on public and private signals in a first step to assess each loan and then in addition also use information based on the lending decisions of other banks.

A. Assessment of private signals on interbank lending returns

Banks know that the true return of bank i , r_i , is a random variable that is normally distributed with mean μ_i and common variance σ^2 :

$$r_i \sim N(\mu_i, \sigma^2). \quad (2)$$

This true return, however, cannot be directly observed. Instead each bank receives a private noisy signal that is independent of the true return as well as independent across banks:

$$\begin{aligned} \hat{r}_{ij} &= r_j + \varepsilon_{ij}, \\ \varepsilon_{ij} &\sim N(0, \sigma_\varepsilon^2). \end{aligned} \quad (3)$$

Banks can now use their noisy signal \hat{r}_{ij} to infer the true return of the bank using conditional expectations:

$$\begin{aligned} \mu_{j|i} &= E[r_j|\hat{r}_{ij}] = \mu_j + \frac{\sigma^2}{\sigma_r^2}(\hat{r}_{ij} - \mu_j), \\ \sigma_{j|i}^2 &= Var[r_j|\hat{r}_{ij}] = \frac{(\sigma_r^2 - \sigma^2)\sigma^2}{\sigma_r^2}, \end{aligned} \quad (4)$$

where $\sigma_r^2 = Var[\hat{r}_{ij}] = \sigma^2 + \sigma_\varepsilon^2$. Hence the return bank i receives from lending to bank j , $r_{j|i}$, is believed to be a random variable distributed as follows:

$$r_{j|i} \sim N(\mu_{j|i}, \sigma_{j|i}^2). \quad (5)$$

Similarly we know that private signals for bank j will also affect our assessment of bank k 's signal about bank j due to the correlation between the two signals:

$$\begin{aligned} \mu_{kj|ij} &= E[\hat{r}_{kj}|\hat{r}_{ij}] = \mu_k + \frac{\sigma^2}{\sigma_r^2}(\hat{r}_{ij} - \mu_k), \\ \sigma_{kj|ij}^2 &= Var[\hat{r}_{kj}|\hat{r}_{ij}] = \frac{\sigma_r^4 - \sigma^4}{\sigma_r^2}, \end{aligned} \quad (6)$$

and hence

$$r_{kj|ij} \sim N(\mu_{kj|ij}, \sigma_{kj|ij}^2). \quad (7)$$

Finally we can easily verify following [24] that the correlation between these updated signals is given by

$$\rho_{j|i,kj|ij} = \frac{\sigma^2}{\sqrt{\sigma^2 + \sigma_r^2}}, \quad (8)$$

This correlation can now be used to determine the joint distribution of $r_{kj|ij}$ and $r_{j|i}$.

B. Updating beliefs from lending decisions

In addition to the private signal, banks can also extract information from the behavior of other banks, i. e. whether they lend or not to a specific bank. This decision will reveal partially the private signal the other bank has received and can be taken into account when assessing one's own lending decision. A bank will only lend if $U_{ij} \geq 0$ as for non-lending we have that due to $a_{ij} = 0$ it is $U_{ij} = 0$.

Once the information from the private signal has been assessed as in the previous section, banks will assess the information available from other banks' lending decisions separately. The following lemma provides the the results of these considerations:

Lemma 3.1: Observing the decision of another bank k to lend to bank j , the assessment of the expected return of bank j and its variance by bank i are given by

$$\begin{aligned}\widehat{\mu}_{ij|k} &= E[r_{j|i}|U_{kj} > 0] \\ &= \mu_{j|i} + \rho_{j|i,kj} \sigma_{j|i} \frac{\phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{1 - \Phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}, \\ \widehat{\sigma}_{ij|k} &= \text{Var}[r_{j|i}|U_{kj} > 0] \\ &= \sigma_{j|i}^2 \left(1 - \rho_{j|i,kj} \frac{\phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{1 - \Phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)} \times \right. \\ &\quad \left. \left(\frac{\phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{1 - \Phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)} - \frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}} \right) \right),\end{aligned}\quad (9)$$

where $\gamma = \left(\frac{\sigma_r^2 - \sigma^2}{\sigma^2}\right) \left(\frac{1}{2} \lambda_i \sigma^2 - \mu_i\right)$. Similarly we can get those moments for the case that bank k does not lend to bank j :

$$\begin{aligned}\widehat{\mu}_{ij|-k} &= E[r_{j|i}|U_{kj} < 0] \\ &= \mu_{j|i} - \rho_{j|i,kj} \sigma_{j|i} \frac{\phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{\Phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}, \\ \widehat{\sigma}_{ij|-k} &= \text{Var}[r_{j|i}|U_{kj} < 0] \\ &= \sigma_{j|i}^2 \left(1 - \rho_{j|i,kj} \frac{\phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{\Phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)} \times \right. \\ &\quad \left. \left(\frac{\phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{\Phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)} - \frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}} \right) \right),\end{aligned}\quad (10)$$

Proof The proof is a straightforward application of the moments of truncated normal distributions, where we note from (1) that $U_{kj} > 0$ is equivalent to $\widehat{r}_{kj} > \gamma$ if we replace μ_{ij} with $\mu_{j|i}$ and σ_{ij} with $\sigma_{j|i}$. ■

Based on this lemma we can thus determine the expected returns and variance of bank j as assessed by bank i :

$$\begin{aligned}\mu_{ij} &= \begin{cases} \widehat{\mu}_{ij|k} & \text{if } a_{jk} = 1 \\ \widehat{\mu}_{ij|-k} & \text{if } a_{jk} = 0 \end{cases}, \\ \sigma_{ij} &= \begin{cases} \widehat{\sigma}_{ij|k} & \text{if } a_{jk} = 1 \\ \widehat{\sigma}_{ij|-k} & \text{if } a_{jk} = 0 \end{cases}.\end{aligned}\quad (11)$$

These expressions can now be inserted into equation (1) to assess the utility of bank i from lending to bank j . As this utility will depend on the behavior of another bank, k , we will rewrite this utility as $U_{ij}(a_{jk})$. The coming section will now discuss how the equilibrium in this model can be determined for the special case of $N = 3$.

IV. DETERMINATION OF EQUILIBRIA IN A THREE-BANK SYSTEM

In the coming sections we will restrict our analysis to a banking system with three banks. While such a restriction

	Conditions	Probability	Strategy
1	$U_{ij}(0) < 0, U_{ij}(1) < 0$	$\Phi\left(\frac{\widehat{r}_{ij k} - \mu}{\sigma^2 + \sigma_\varepsilon^2}\right)$	not lending
2	$U_{ij}(0) > 0, U_{ij}(1) > 0$	$1 - \Phi\left(\frac{\widehat{r}_{ij -k} - \mu}{\sigma^2 + \sigma_\varepsilon^2}\right)$	lending
3	$U_{ij}(0) < 0, U_{ij}(1) > 0$	$\Phi\left(\frac{\widehat{r}_{ij -k} - \mu}{\sigma^2 + \sigma_\varepsilon^2}\right) - \Phi\left(\frac{\widehat{r}_{ij k} - \mu}{\sigma^2 + \sigma_\varepsilon^2}\right)$	following
4	$U_{ij}(0) > 0$ and $U_{ij}(1) < 0$	0	anti-following

TABLE I
STRATEGIES FOR BANK i LENDING TO BANK j

seems unrealistic for most actual banking systems, it allows us to provide a complete characterisation of the possible equilibrium lending structures and gain some generalizable insights into interbank markets. The number of possible network structures is $2^{N(N-1)}$ and thus for $N = 3$ consists of 64 potential equilibria to consider while for $N = 4$ this becomes an untractable 4096 potential equilibria.

A. Individual lending strategies

As we can see from equation (11), the expected returns and risks of bank i lending to bank j depend on the behavior of the remaining bank, k . Banks will lend if the expected utility from doing so exceeds the expected utility from not lending, with the former given by equation (1) and the latter easily being verified to be zero by inserting $a_{ij} = 0$. We can now investigate the different potential outcomes in the lending decision of bank i towards bank j . If $U_{ij}(0) < 0$ and $U_{ij}(1) < 0$, then the bank will not lend as regardless of the behavior of the other bank as the expected utility from doing so is negative. Similarly, if $U_{ij}(0) > 0$ and $U_{ij}(1) > 0$, then the bank will lend as regardless of the behavior of the other bank as the expected utility from doing so is positive. If $U_{ij}(0) < 0$ and $U_{ij}(1) > 0$, then the bank will only lend if the other bank also lends as only with the information that the other bank lends, the expected utility becomes positive. We will refer to this situation as "bank i following bank k ". Finally, if $U_{ij}(0) > 0$ and $U_{ij}(1) < 0$, then the bank will only lend if the other bank does not do so as the expected utility from doing so is negative. We will refer to this situation as "bank i anti-following bank k ". Table I summarizes these situations. We can now determine the probabilities for each of these situations by firstly defining

$$\begin{aligned}\widehat{r}_{ij|k} &\in \{\widehat{r}_{ij}|U_{ij}(1) = 0\}, \\ \widehat{r}_{ij|-k} &\in \{\widehat{r}_{ij}|U_{ij}(0) = 0\}.\end{aligned}\quad (12)$$

We can solve for $\widehat{r}_{ij|k}$ and $\widehat{r}_{ij|-k}$ numerically. As we can show that $U_{ij}(0)$ is monotonic in \widehat{r}_{ij} , the solution for $\widehat{r}_{ij|-k}$ will be unique. While $U_{ij}(1)$ is not monotonic in \widehat{r}_{ij} in general, it is so in the range of realistic parameters, such that the solution will be unique in the relevant range. We furthermore can easily show that $\widehat{r}_{ij|k} \geq \widehat{r}_{ij|-k}$.

Given the definition of $\widehat{r}_{ij|k}$ and $\widehat{r}_{ij|-k}$ in equation (12) it is obvious that $U_{ij}(0) < 0$ is equivalent to $\widehat{r}_{ij} < \widehat{r}_{ij|-k}$ and $U_{ij}(1) < 0$ corresponds to $\widehat{r}_{ij} < \widehat{r}_{ij|k}$. The probabilities for each of the four lending strategies can now easily be determined where the "not lending" corresponds to

Strategy (i, j)	Observed outcome (i, j)
(not lending, not lending)	(not lending, not lending)
(following, not lending)	(not lending, not lending)
(lending, not lending)	(lending, not lending)
(not lending, following)	(not lending, not lending)
(following, following)	(lending, lending), (not lending, not lending)
(lending, following)	(lending, lending)
(not lending, lending)	(not lending, lending)
(following, lending)	(lending, lending)
(lending, lending)	(lending, lending)

TABLE II

STRATEGY COMBINATIONS FOR BANKS i AND k LENDING TO BANK j

$Prob(\hat{r}_{ij} < \hat{r}_{ij|-k} \leq \hat{r}_{ij|k})$ and "lending" has a probability of $Prob(\hat{r}_{ij} > \hat{r}_{ij|k} \geq \hat{r}_{ij|-k})$. The strategy "following" has a probability of $Prob(\hat{r}_{ij|-k} < \hat{r}_{ij} \leq \hat{r}_{ij|k})$ and the strategy "anti-following" corresponds to $Prob(\hat{r}_{ij|-k} > \hat{r}_{ij} \geq \hat{r}_{ij|k})$, which is impossible as $\hat{r}_{ij|k} \geq \hat{r}_{ij|-k}$ and hence we can neglect this strategy. The probabilities are shown in table I and the derivation is straightforward when using the distribution of the returns from equations (2) and (3).

In order to obtain the equilibrium lending structures we will also need to consider the distribution of lending decisions to bank j by banks i and k . We can easily derive that

$$\begin{bmatrix} \hat{r}_{ij} \\ \hat{r}_{kj} \end{bmatrix} \sim N \left(\begin{bmatrix} \mu_j \\ \mu_j \end{bmatrix}, \begin{bmatrix} \sigma^2 + \sigma_\varepsilon^2 & \sigma^2 \\ \sigma^2 & \sigma^2 + \sigma_\varepsilon^2 \end{bmatrix} \right). \quad (13)$$

Table II shows the possible strategy combinations and the outcome of the lending decision which we would observe. The corresponding probabilities could easily be derived from the joint distribution in equation (13). With these results we can now continue to apply an algorithm to find all possible equilibria in the coming section.

B. Algorithm to determine equilibria

Each bank has three possible strategies, "lending", "not lending", and "following". In a network consisting of three banks there are six possible lending decisions, hence a total of $3^6 = 729$ scenarios have to be considered. For each scenario we can now determine the probability of its occurrence. Using the possible strategy combinations from table II we can determine the probability for each pair of banks by applying the joint distribution from equation (13). As r_i is by assumption independently distributed across banks, the probability of a scenario is given by the product of the probabilities for each of the three pairs.

In a scenario those strategies that are "lending" or "not lending" are taken as examined because they do not depend on the action of other banks. Strategies that are "following" are classified as unexamined.

For each scenario we now need to determine the equilibrium outcome by applying the following steps:

- 1) All examined strategies are fixed at that value and if there are no unexamined strategies we have established a unique equilibrium for that scenario. If there are unexamined strategies, we continue with the next step.

- 2) If there is an examined strategy from bank k to bank j and an unexamined relationship from bank i to bank j , then this unexamined relationship is set to the same value as the examined relationship and marked itself as examined and we have established a unique equilibrium for this scenario. If there are any more unexamined strategies, we continue with the final step.
- 3) If banks i and k both follow a strategy of "following" for lending to bank j , then multiple equilibria will occur, namely both "following" or both "not following". Depending on how many pairs of banks have unexamined strategies, the type and number of equilibria are different and shown in table III. We assume that each of the possible equilibria has an equal probability of occurrence.

Having now determined the equilibria for each scenario and calculated their probabilities, we now have to aggregate those probabilities for networks that are observationally identical. We thus have established a probability distribution for the equilibria and we can instantly see that each of the 64 possible networks can be an equilibrium network, the probability of its occurrence will vary though with the parameters employed, namely the expected returns μ_i , variance σ^2 , signal precision σ_ε and the risk aversion of the banks λ_i .

The following section will now evaluate the properties of the resulting equilibria, firstly for a homogenous banking system and then for a banking system with banks of different expected returns.

V. EQUILIBRIUM STRUCTURES

Using the procedure outlined above we can now continue to assess the interbank lending that would emerge in equilibrium. We will assess how key variables affect the equilibrium interbank lending behavior of banks firstly in the case of all banks having the same expected returns and differing only in the private signals they receive about the expected returns for lending to other banks. We will then extend this restrictive case to incorporate banks with different returns as a more realistic alternative.

A. Homogeneous banks

Let us firstly assume that the expected return of providing interbank loans to banks is identical for all banks, i. e. $\forall i : \mu_i = \mu$. As the private signal banks receive cannot be observed by other banks or an outside spectator, all banks are ex-ante identical and the number of potential networks reduces from 64 to 16 as we can ignore the identity of banks and aggregate networks that are therefore looking alike, i. e. topologically equivalent.

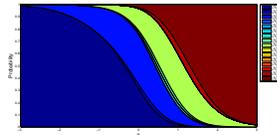
With the aforementioned, each of these 16 interbank lending networks will be an equilibrium for any parameter constellation, the probability of observing a specific network will, however, vary. It are these probabilities of observing a specific network structure that we focus our subsequent analysis on. We set $\sigma = 1$ as a normalization of the amount of risk in the banking system, having verified that the results presented

Unexamined strategies	Equilibrium networks
1 unexamined strategy (i and k towards j)	
2 unexamined strategies (i and k for j , i and j for k)	
3 unexamined strategies (i and k for j , i and j for k , j and k for i)	

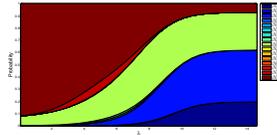
TABLE III
MULTIPLE EQUILIBRIA WITH UNEXAMINED STRATEGIES

here are not substantially affected by this normalization. The parameters we are varying in the following analysis are the risk aversion of banks, λ_i , which for simplicity we assume to be identical across banks, i. e. $\forall i : \lambda_i = \lambda$, the expected returns μ , and the precision of the private signal σ_ε .

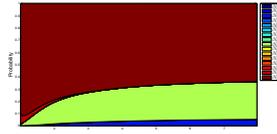
Holding λ and σ_ε constant, we firstly analyse the impact the expected return μ has on the main equilibrium networks that emerge. Figure 1 shows the probabilities of all 16 networks for a range of expected returns. it is obvious from the figure that the networks that can actually be observed will be dominated by only four of the 16 possible networks, which are shown separately in figure 2. Firstly we observe that for low expected returns the empty network dominates all other equilibrium networks. The low expected returns make the provision of interbank loans unprofitable unless a very large positive private signal is received. Such a large private signal is very unlikely to be received and thus anything but a non lending decision has a small probability. Once the expected return increases, the private signal compensating a large negative expected return becomes more likely. Furthermore the likelihood of a following strategy increases as the private signal itself might not be sufficient to induce lending, but in combination with the lending decision of the other bank this might be sufficient. Hence we will slowly observe the emergence of lending to at least one bank. As the expected return increases even more, it becomes more and more likely that the same will become true for the private signals regarding two banks and we observe the emergence of a third equilibrium in which two banks are lent money. A further increase of the expected return will then make the lending to all three banks more and more likely, first arising from following strategies and then once the expected return is sufficiently positive also based directly on the expected return. We note that the full network emerges only once the expected return is significantly above zero due to the risk aversion of the banks. Once the expected returns are sufficiently positive the entire reasoning reverses.



(a) Varying μ



(b) Varying λ



(c) Varying σ_r

Fig. 1. Probability of equilibrium network structures for varying parameters (base case $\mu = 2$, $\lambda = 2.5$, $\sigma_r = 1.6$)

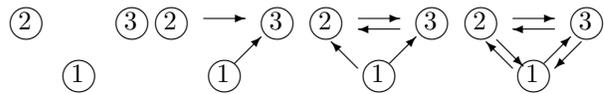


Fig. 2. Dominant equilibrium networks

From this reasoning of the observed dominant equilibrium networks we see that they are those networks that allow a following strategy as shown in table III. The origin of this result arises from the fact that a following strategy is the most likely observation for expected returns that are neither too large or too small.

For positive expected returns, the likelihood of observing a

full network reduces as the risk aversion λ increases as we can see from figure 1. The reasoning is obvious: as the risks are becoming more and more important, it does allow the private signal to be less and less negative in order to generate positive expected utility. For the same reason the empty network becomes more and more likely. Once again we observe an intermediate range with other network structures that allow for following strategies with the same arguments as above. The main difference, however, is that as the risk aversion increases, the empty network does not become dominant but rather the four main network stabilise in fixed proportions. The origin of this observation is that while a higher risk aversion reduces the expected utility of lending, the very same consideration will also be true of the other banks, hence observing another bank lending implies a very high private signal, making the adjustment to the expected return and variance in lemma 3.1 more pronounced, offsetting each other in the variance and expected return and thereby causing the stability of the probabilities of equilibrium network structures as the risk aversion increases.

Looking at the impact of the total risk, σ_r , on the probability of observing specific equilibrium networks, we observe a similar pattern as in the case of increasing risk aversion. The reason here is firstly along the same lines as with risk aversion, but in addition we can also see that the update of expected returns and variance reduces as the variance of the private signal increases due to its much more limited informational content. Hence banks rely less on their private information and observing other banks will also have limited informational value.

In summary, we find that the equilibrium network structures are dominated by four networks that are all consistent with banks adopting a following strategy.

B. Banks with different returns

We now relax the assumption that the expected returns of all banks are identical and instead focus on a situation where $0 = \mu_1 \leq \mu_2 \leq \mu_3 = 2$, with other parameter constellations showing comparable results. As the middle-ranking bank increases its expected returns, μ_2 , we see from figure 3 that the empty network becomes less likely and the full network more likely. An increase in μ_2 will, ceteris paribus, make the lending to the middle ranking bank more attractive and thus the absence of any lending will become less likely. Similarly the likelihood of a full network will increase. The other two dominant networks, which are identical to the ones identified in the homogeneous case previously, and the argument on their emergence are unchanged.

Assessing the impact of changing the risk aversion of banks is shown in figure 4. Here we observe the same properties as in the homogenous case and the results are only varying to the extent that the incomplete non-empty networks are found. If the mid-ranking bank has a high expected return, we observe that it receiving interbank loans is higher and thus the probability of these network structures is increased accordingly. The same observation we also make when analysing the effect a

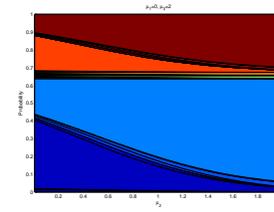


Fig. 3. Probability of equilibrium network structures for varying μ_2 ($\lambda = 2$, $\sigma_r = 1.6$)

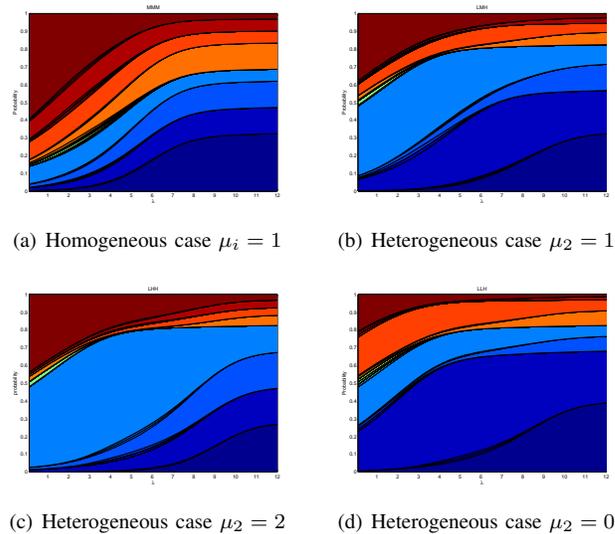


Fig. 4. Probability of equilibrium network structures for varying λ ($\sigma_r = 1.6$)

change of the risk has on the observed network structures as in figure 5.

We can thus conclude that the introduction of heterogeneity in the banks' expected returns does not affect the outcome substantially and the results obtained for the homogeneous case can be shown to be robust. We will thus focus on the homogeneous case in the following discussion of the policy implications.

VI. POLICY IMPLICATIONS

The credit crisis 2007-08 was characterized, among other things, by the withdrawal of interbank lending facilities of banks. Analysis showed that the uncertainty surrounding the solvency of other banks made banks very cautious in advancing new interbank loans or extending the maturity of existing arrangements. We can use our model to explain these observations. If the risk of banks increases our model suggests that the likelihood of networks that show less interbank lending or even its absence become more likely, explaining the reduction in interbank lending that was observed. This effect might have been well exacerbated by an increase in the risk aversion of banks in times their own solvency was questioned as confirmed by our model. Hence even without a reduction in the expected returns, due to the questionable solvency of many banks, we should observe a reduction in interbank lending.

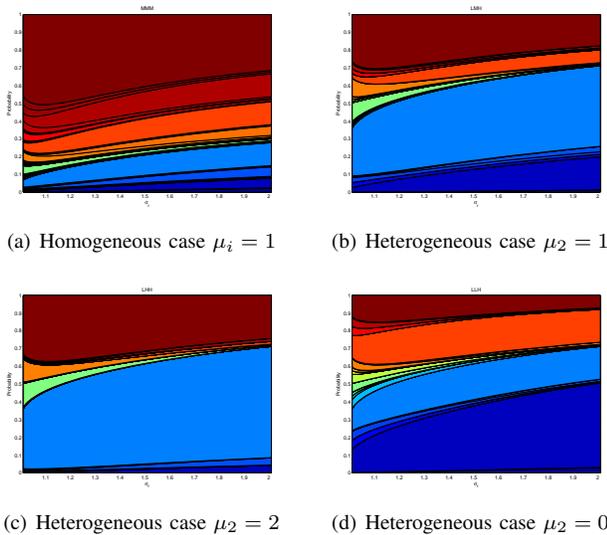


Fig. 5. Probability of equilibrium network structures for varying σ_r ($\lambda = 2$)

From our model we can also deduct the importance of the following strategy in the emergence or absence of interbank lending. Hence the existence of interbank lending will to a large extent depend on expectation formation, and thus it is important to maintain trust in the solvency of banks. A reduction in the quality of the private signals banks have about each other will also be detrimental to the existence of a flourishing interbank market. Any regulator might want for this reason seek to ensure that information on a banks' solvency is easily available as to reduce the risks banks expose themselves to.

VII. CONCLUSIONS

We provided a model of interbank lending where banks seek to maximize expected utility in the presence of uncertainty regarding the risks of a counterparty bank. Banks assess the risk of other banks by relying on a public signal, their individual private signal as well as extracting information from the lending behavior of other banks. We showed that in equilibrium four network structures of interbank lending dominate, the empty network (no lending), the full network (all banks lend to each other), a network in which one bank receives interbank loans from the other banks, and a network where two banks receive interbank loans from the other banks. These network structures were arising mainly from a "following" strategy in which a bank would only lend if the other bank would also do so, providing equilibria that are vulnerable to small shocks that can change the equilibrium structure easily.

The analysis of the model presented here was limited to banking systems with only three banks. An extension to include more banks is in principle straightforward but comes at the cost of significantly increased computational complexity such that additional constraints on the network structure would have to be imposed to make it tractable. A further extension

might be that the expected returns are made endogenous to the network structure and the exposure of the bank to interbank loans, thus depending on the network structure itself. Using such extensions would allow us to provide a more general equilibrium of the interbank lending network.

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