Equilibrium Using Credit or Money with Indivisible Goods*

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Abstract

This note studies the trade of indivisible goods using credit or money in a frictional market. We show how indivisibility matters for monetary equilibrium under different assumptions about price determination. Bargaining generates a price and allocation that are independent of the nominal interest or inflation rate over some range. This is not the case with price posting and directed search. In either case, we provide conditions (the nominal rate cannot be too high) under which stationary monetary equilibrium exists, and we show it is unique or generically unique.

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1 Introduction

The New Monetarist framework has been increasing in popularity, with applications in areas such as finance, payment systems, and monetary policy analysis (see the survey by Lagos et al. (2016)). The earliest search-based models of Kiyotaki and Wright (1989, 1993) had indivisible goods and indivisible money, making every trade a one-for-one swap. Later, Shi (1995) and Trejos and Wright (1995) kept indivisible money but introduced divisible goods to determine prices. Subsequently, Shi (1997) and Lagos and Wright (2005) made both goods and money divisible. This note completes the picture by exploring the remaining case of indivisible goods and divisible money.

Far from being merely a mathematical exercise on the remaining logical possibility, we think this is substantively important. Many goods are, in fact, indivisible, and continuous divisibility is often an abstraction designed to make the analysis easier, not for the sake of realism. We take indivisibility seriously, and show how it matters in pairwise trading. To make the environment comparable with previous models (e.g., Shi (1995) or Trejos and Wright (1995)), our consumers want to consume exactly one unit. We explore equilibrium with credit and with money. We consider bargaining with random search as well as price posting with directed search.[1] Under credit buyers do not incur a cost from bringing a resource to trade. However, they do with money and this matters, especially under bargaining. We also consider lotteries, which can be part of an efficient trading mechanism with indivisibility.

With pure credit, equilibrium exists uniquely, and all potential buyers participate in the market, with bargaining or with price posting. With money, the outcome depends on the mechanism that determines the terms of trade. With bargaining, as long as the nominal interest rate is not too high, (stationary) monetary equilibrium exists uniquely. In this equilibrium, buyers bring the lowest amount of money necessary to make sellers indifferent between trading or not trading, akin to making take-it-or-leave-it offers. For a low range of nominal interest rates, all potential buyers participate in the market, and money is superneutral – inflation does not affect the real allocation. However, for a higher range of nominal interest rates, not all buyers participate, and the allocation is affected by inflation. These results hold with and without lotteries.

With price posting, it is easy to show lotteries are not necessary. We again show existence, and generic uniqueness, as long as the nominal interest rate is not too high. As with bargaining, when the nominal interest rate is low, all buyers participate. By increasing the nominal rate, eventually, some buyers stop participating, and ultimately

[1] Price posting with directed search is often called competitive search (Moen (1997), Mortensen and Wright (2002)). It has been used in monetary economics by Julien et al. (2008) with indivisible money, and by Rocheteau and Wright (2005) and Lagos and Rocheteau (2005) with divisible money.

the monetary equilibrium ceases to exist. Because the equilibrium price depends on the number of participating buyers under competitive search, while it does not under bargaining, the threshold for the nominal interest rate differs between bargaining and price posting. In summary, the indivisibility of goods and the mechanism for determining the terms of trade have important implications on the ability of money to generate real effects.

There is some related work on divisible money with indivisible goods, but it focuses only on random rather than competitive search. In Green and Zhou (1998), indivisibility together with posting and random search leads to indeterminacy of monetary steady state. Those results are extended to an environment closer to our setup by Jean et al. (2010) with fiat money and Rabinovich (2016) with commodity money. Our results show how this indeterminacy problem vanishes with either bargaining or price posting. Galenianos and Kircher (2008) have indivisible goods but the environment is quite different – they assume multilateral meetings with the terms of trade determined by second-price auction. Liu et al. (2011, 2015) consider indivisible goods but use noisy search as in Burdett and Judd (1983), and focus on different issues.

2 Environment

The environment is based on the alternating markets framework of Rocheteau and Wright (2005).[2] Time is discrete and goes on forever. A continuum of buyers and sellers, with measures N and 1, live forever. Agents discount between periods with factor $\beta \in (0, 1)$, but not across markets within a period, and $r = 1/\beta - 1$ is the discount rate. In each period, there are two consecutive markets. The first market to open is a decentralized market (DM), and the second is a frictionless centralized market (CM). Both buyers and sellers consume a divisible good in the CM, while only buyers consume an indivisible good in the DM.

Buyers' preferences within a period are given by $U(x) - h + u\mathbf{1}$, where x is CM consumption, h is CM labor, u is DM utility from consuming the indivisible good, and $\mathbf{1}$ is an indicator function giving 1 if trade occurs and 0 otherwise. Sellers' preferences are $U(x) - h - c\mathbf{1}$ with DM good produced at constant cost c. We assume $u > c$. Let x be the CM numeraire. We assume that x is produced one-to-one from labor h.

Trade in the DM implies a price and quantity bundle $(p, q) \in \mathcal{P} \times Q$ where $\mathcal{P} = \{0 \leq p \leq L\}$ and $Q = \{0, 1\}$. L represents the available liquidity in the economy with credit

[2] The original alternating markets framework by Lagos and Wright (2005) has agents receiving a preference shock in the CM, revealing whether they will be a buyer or a seller in the DM. In Rocheteau and Wright (2005), buyers are always buyers and sellers are always sellers. All our results hold for both frameworks.

or money to be defined below.

In the DM, meetings occur according to a general meeting technology, which is assumed homogeneous of degree one. Given the buyer-seller ratio $n \leq N$, which is also the measure of participating buyers in the DM, the meeting rate for sellers is $\alpha(n)$, and $\alpha(n)/n$ for buyers. Assume $\alpha' > 0$, $\alpha'' < 0$, $\alpha(0) = 0$, $\lim_{n \to \infty} \alpha(n) = 1$, and $\lim_{n \to 0} \alpha'(n) = 1$.

3 Credit

Consider an economy in which commitment is feasible. Agents are not anonymous. Record keeping and punishment devices are available. In this environment, there is no role for money. Rather, sellers in the DM produce for buyers with the buyers promising to deliver x in the subsequent CM. We assume an exogenous credit constraint $p \leq D$, where p is the real price and liquidity constraint $L = D > 0$.

Buyers in the CM obtain

$$W_t^b(d) = \max_{x,h} \left\{ U(x) - h + \beta V_{t+1}^b \right\} \text{ s.t. } x = h - d, \tag{1}$$

where d is the buyer's debt coming out of the DM, i.e. $d = p$ if a purchase, and $d = 0$ otherwise. Buyers participate in the DM if $V_{t+1}^b \geq 0$. Using the budget constraint to eliminate h and solving for optimal x^* yields $W_t^b(d) = \Sigma - d + \beta V_{t+1}^b$ with $\Sigma = U(x^*) - x^*$. Sellers in the CM get

$$W_t^s(d) = \max_{x,h} \left\{ U(x) - h + \beta V_{t+1}^s \right\} \text{ s.t. } x = h + d,$$

where $d = p$ if a sale, and $d = 0$ otherwise. This simplifies to $W_t^s(d) = \Sigma + d + \beta V_{t+1}^s$. Sellers participate in the DM if $V_{t+1}^s \geq 0$. The buyer's payoff in the DM is

$$V_t^b = \frac{\alpha(N)}{N} \left[u + W_t^b(p) \right] + \left[1 - \frac{\alpha(N)}{N} \right] W_t^b(0).$$

A buyer that trades obtains credit p, to be paid in the next CM, and gets utility u from DM consumption. Using linearity of W in d, for buyers and similarly for sellers,

$$V_t^b = W_t^b(0) + \frac{\alpha(N)}{N}(u - p) \text{ and } V_t^s = W_t^s(0) + \alpha(N)(p - c). \tag{2}$$

3.1 Bargaining

Upon meeting, a buyer and a seller solve the generalized Nash bargaining problem

$$\max_p (u-p)^\eta (p-c)^{1-\eta} \text{ s.t. } p \le D.$$

Proposition 1 *In the model with credit and bargaining, $\exists!$ stationary equilibrium (SE) if $D \ge c$, characterized by*

$$p^B = \begin{cases} \bar{p}^B & \text{if } D > \bar{p}^B \\ D & \text{if } D \le \bar{p}^B, \end{cases}$$

where $\bar{p}^B = (1-\eta)u + \eta c$.

All buyers are active in this environment since using credit is costless and $(u - p^B)\alpha(N)/N > 0$, for all p^B. We show below that introducing money in the DM can result in $n < N$ active buyers.

3.2 Competitive Search

We study competitive search equilibrium with price posting. As in Moen (1997) and Rocheteau and Wright (2005), instead of a single DM, there exist a continuum of submarkets, each identified by masses of sellers posting the same terms of trade. Sellers post and commit to DM prices before buyers enter the DM. After observing all posted prices, each buyer chooses a submarket giving the maximum surplus. Each seller can only produce for one buyer in each period. If a seller is visited by multiple buyers, he chooses one with an equal probability. Let n represent the expected queue length for any seller in a submarket offering price p. The meeting rates now depend on the queue length induced by price, instead of the aggregate N.

Buyers' payoff in the CM is

$$W_t^b(d) = \Sigma - d + \beta \max_{\hat{p},\hat{n}} \left\{ \frac{\alpha(\hat{n})}{\hat{n}} (u - \hat{p}) + W_{t+1}^b(0) \right\},$$

where $(\hat{p},\hat{n})$ refers to $t+1$. The seller's payoff in the DM is $V_t^s = W_t^s(0) + \max_{p,n} \{\alpha(n)(p-c)\}$. Let Ω be the equilibrium expected utility of a buyer in the DM. Sellers solve

$$\max_{p,n} \alpha(n)(p-c) \text{ s.t. } \frac{\alpha(n)}{n}(u-p) \ge \Omega,\ p \le D.$$

Solve for p from the buyer's participation constraint, and substitute it into the seller's objective function,

$$\max_{n} \alpha(n) \left[u - c - \frac{n\Omega}{\alpha(n)} \right] \text{ s.t. } u - \frac{n\Omega}{\alpha(n)} \leq D.$$

Proposition 2 *In the model with credit and competitive search, $\exists!$ symmetric SE if $D \geq c$, characterized by*

$$p^c = \begin{cases} \bar{p}^c & \text{if } D > \bar{p}^c \\ D & \text{if } D \leq \bar{p}^c \end{cases}$$

where $\bar{p}^c = [1 - \varepsilon(n)]u + \varepsilon(n)c$, and $n = N$.

This result is identical to the case with bargaining when $\varepsilon(N) = \eta$ (Hosios (1990)). Again, $(u - p^c)\alpha(N)/N > 0$ for all p^c, and all buyers are active in the DM.

4 Money

Now assume agents in the DM cannot commit and there are no enforcement or punishment mechanisms. Therefore, buyers must pay sellers with cash in the DM. Let M_t^s be the money supply per buyer at time t, with $M_t^s = \gamma M_{t-1}^s$ and the growth rate of money, γ, is constant. Changes in M^s occur in the CM via lump-sum transfers (taxes) if $\gamma > 1$ ($\gamma < 1$). Nominal interest rate is given by the Fisher equation $1 + i = \gamma/\beta$ and we assume $\gamma > \beta$. The Friedman rule is the limiting case $i \to 0$. Define ϕ_t as the CM price of money in terms of x_t, and $1/\phi_t$ as the nominal price level. Liquidity available is then $L = \phi_t m_t$. In stationary equilibrium, $\phi_t/\phi_{t+1} = \gamma$. Since there is a cost of carrying money, which may not be covered by the buyer's surplus from DM trade, we allow endogenous participation by buyers and let n denote the ratio of active buyers to sellers in the DM.

Buyers with money holding m in the CM solve

$$W_t^b(m) = \max_{x,h,\hat{m}} \left\{ U(x) - h + \beta V_{t+1}^b(\hat{m}) \right\} \text{ s.t. } x = \phi_t(m + T) + h - \phi_t \hat{m}, \tag{3}$$

where $\hat{m}$ is the money holding carried to the next DM, and T represents net transfers from the government only given to buyers. Eliminating h from the budget equation,

$$W_t^b(m) = \Sigma + \phi_t(m + T) + \max_{\hat{m}} \left\{ \beta V_{t+1}^b(\hat{m}) - \phi_t \hat{m} \right\}. \tag{4}$$

Sellers do not bring money into the DM. Thus, $W_t^s(m) = \Sigma + \phi_t m + \beta V_{t+1}^s$ represents their CM value.

Buyers' payoff in the DM is

$$V_t^b(m) = \frac{\alpha(n)}{n}\left[u + W_t^b\left(m - \frac{p}{\phi_t}\right)\right] + \left[1 - \frac{\alpha(n)}{n}\right] W_t^b(m), \tag{5}$$

where n represents the number of active buyers in the DM. If a buyer gets to trade, he pays p and gets u. Linearity, $\partial W_t^b/\partial m = \phi_t$, allows us to write

$$V_t^b(m) = \frac{\alpha(n)}{n}(u-p) + W_t^b(m) \text{ and } V_t^s = \alpha(n)(p-c) + W_t^s(0).$$

4.1 Bargaining

The generalized Nash bargaining problem is

$$\max_p (u-p)^\eta (p-c)^{1-\eta} \text{ s.t. } p \leq \phi m,\ u-p \geq 0,\ p-c \geq 0. \tag{6}$$

As is standard when $\gamma > \beta$, the feasibility constraint, $p \leq \phi m$, binds and $c \leq \phi m \leq \bar{p}^B$, where $\bar{p}^B = (1-\eta)u + \eta c$ as in Proposition 1. Any negotiated price $p^B \in [c, \bar{p}^B]$ is a potential bargaining solution. Substituting V_{t+1}^b into W_t^b yields the following CM value function

$$W_t^b(m) = \Sigma + \phi_t(m+T) + \beta W_{t+1}^b(0) + \max_{\hat{m}\in[\underline{m},\bar{m}]} \beta\left\{\frac{\alpha(n)}{n}\left(u - \phi_{t+1}\hat{m}\right) - i\phi_{t+1}\hat{m}\right\}, \tag{7}$$

where $\underline{m} = \frac{c}{\phi_{t+1}}$ and $\bar{m} = \frac{\bar{p}^B}{\phi_{t+1}}$. Since buyers' surplus decreases in $\hat{m}$, optimal money holding decision in (7) reduces to $\phi_{t+1}\hat{m} = c$. A buyer effectively commits to not paying more than c by bringing exactly $\phi_{t+1}\hat{m} = c$. The solution is akin to buyers making a take-it-or-leave-it offer to sellers in pairwise meetings.

For equilibrium, it has to be individually rational for buyers to participate. Define

$$v(\hat{m}) = \frac{\alpha(n)}{n}\left(u - \phi_{t+1}\hat{m}\right) - i\phi_{t+1}\hat{m}. \tag{8}$$

The buyer's free entry condition $v(c/\phi_{t+1}) = 0$ implies

$$i = \frac{\alpha(n)}{n}\frac{(u-c)}{c} = \Psi(n). \tag{9}$$

Given i, (9) uniquely determines the measure of active buyers in the DM, n^*. The matching rate $\alpha(n)/n$ is decreasing in n, and so is $\Psi(n)$, with $\partial n^*(i)/\partial i < 0$. Higher i, leads to fewer active buyers in the DM, reduces congestion and increases the marginal gain of entering the DM. Define $\bar{\imath}^N = \Psi(N)$ and $\bar{\imath}^B = (u - c)/c$.

Proposition 3 *In the model with money and bargaining: (i) For $i \leq \bar{\imath}^N$, $\exists!$ stationary monetary equilibrium (SME) with $n^* = N$; (ii) for $i \in (\bar{\imath}^N, \bar{\imath}^B)$, $\exists!$ SME with $n^* < N$; (iii) for $i \geq \bar{\imath}^B$, $\nexists$ SME.*

Real balances in equilibrium only depend on c, and not on the bargaining power η or the nominal rate i. Buyers move first by choosing money balances and commit to bringing the lowest level of real balances acceptable for trade. The nominal interest rate has no effects on the DM real price, buyer's real balances, or the real value of money. For case (i), all buyers participate in the DM and the total output is not affected by i, either. Therefore, money is superneutral in this model for small nominal interest rates. For case (ii), as i increases, n^* falls and increases $\alpha(n^*)/n^*$. Buyers trade faster, a "hot potato" effect as in Liu et al. (2011).

This result differs from most of the New Monetarist literature, which generally features neutrality of money but real allocations are affected by changes in inflation. The generalized Nash bargaining mechanism determines the buyer's share of surplus according to exogenous bargaining power, which then determines the unique optimal real balance. Monetary variables do not play a role in the determination of real variables, but only affect the price of money ϕ.

When it is costless to carry money to the DM, i.e. $i = 0$, the monetary economy is comparable to the credit economy in Section 3.1, but with different price in the DM. When $i = 0$, buyers still choose to carry just enough real balance to cover the seller's reservation price c, so as to maximize their surplus from trade. As shown in Proposition 1, the equilibrium price with credit is almost always higher than the seller's reservation price. This is because when facing an exogenous credit constraint, buyers do not have the power to effectively commit to paying c ex ante.

4.1.1 Lotteries

We introduce lotteries because lotteries are usually efficient mechanisms in non-convex economies. Let $E = \mathcal{P} \times \{0, 1\}$ denote the space of trading events, and $\mathcal{E}$ the Borel σ-algebra. Define a lottery to be a probability measure ω on the measurable space $(E, \mathcal{E})$. We can write $\omega(p, q) = \omega_q(q)\omega_{p|q}(p)$ where $\omega_q(q)$ is the marginal probability measure of q and $\omega_{p|q}(p)$ is the conditional probability measure of p on q. Without loss of generality, as shown in Berentsen et al. (2002), we restrict attention to $\tau = \Pr\{q = 1\}$ and $1 - \tau = \Pr\{q = 0\}$, and $\omega_{p|0}(p) = \omega_{p|1}(p) = 1$. Randomization is only useful on q because Q is non-convex. Thus, $\tau \in [0,1]$ is the probability that the good is produced and traded.[3]

The bargaining problem with lotteries becomes

$$\max_{p,\tau} (\tau u - p)^{\eta} (p - \tau c)^{1-\eta} \text{ s.t. } p \le \phi m,\ \tau \le 1, \text{ and } \tau u \ge p \ge \tau c.$$

Lemma 1 *The solution to the bargaining problem with lotteries is*

$$\left(p^B, \tau^B\right) = \begin{cases} \left(\bar{p}^B, 1\right) & \text{if } \phi m > \bar{p}^B \\ (\phi m, 1) & \text{if } \underline{p}^B \le \phi m \le \bar{p}^B \\ \left(\phi m, \phi m/\underline{p}^B\right) & \text{if } c \le \phi m < \underline{p}^B \\ (0,0) & \text{if } \phi m < c \end{cases}$$

where $\bar{p}^B = (1-\eta)\, u + \eta c$ *and* $\underline{p}^B = uc/(\eta u + (1-\eta)c)$.

Buyer's payoff in the CM is

$$W_t^b(m) = \Sigma + \phi_t (m + T) + \beta W_{t+1}^b (0) + \beta \max_{\hat{m}} v(\hat{m}),$$

where $v(\hat{m}) = \alpha(n)(\tau^B u - p^B)/n - i\phi_{t+1}\hat{m}$.

Proposition 4 *In the monetary model with bargaining and lotteries: (i) For* $i \le i^N$, $\exists!$ *SME with* $\phi_{t+1}\hat{m} = \underline{p}^B$, $\tau^B = 1$ *and* $n^* = N$*; (ii) for* $i \in (i^N, \bar{\imath}^B)$, $\exists!$ *SME with* $\phi_{t+1}\hat{m} = \underline{p}^B$, $\tau^B = 1$ *and* $n^* < N$*; (iii) for* $i \ge \bar{\imath}^B$, $\nexists$ *SME.*

First, given that $\phi_{t+1}\hat{m} = \underline{p}^B$, n^* does not decrease with inflation when $i \le i^N$. Money is still superneutral. For $i \in (i^N, \bar{\imath}^B)$, real balances stay constant but n^* changes with i. Second, lotteries benefit sellers. With lotteries, the seller's surplus from a DM trade is $\underline{p}^B - c$, compared to zero surplus from a trade without bargaining over lotteries. Because of lotteries, buyers now bring exactly enough money to achieve the maximum expected surplus from trade at $\tau^B = 1$. Third, introducing lotteries makes it harder for a monetary equilibrium to exist, which is easy to see from $\bar{\imath}^B = \eta(u-c)/c$ in the lottery case being smaller than $\bar{\imath}^B = (u-c)/c$ without lotteries. Fourth, with lotteries, the nominal interest rate cut off values increase with the buyer's bargaining power η. Finally, compared to Berentsen et al. (2002), the probability τ^B does not change with respect to the buyer's bargaining power or the inflation rate. Introducing lotteries with *indivisible goods* and *divisible money*, the total surplus from trade is affected but not price. However, introducing lotteries with *indivisible money* and *divisible goods*, the total

[3] We only study lotteries in money and bargaining because lotteries are trivial for other cases. In competitive search $\tau = 1$ always holds, and in credit and bargaining, $\tau < 1$ only if D is severe enough.

surplus from trade stays the same but price changes according to the value of lotteries in equilibrium.

4.2 Competitive Search

Under competitive search and price posting, the buyer's DM value function becomes

$$V_t^b(p, m) = \frac{\alpha(n)}{n}(u - p) + W_t^b(m), \tag{10}$$

where p is the price posted by the buyer's chosen seller. From (4) and (10) buyers' value is

$$W_t^b(m) = \Sigma + \phi_t(m + T) + \beta W_{t+1}^b(0) + \max_{\hat{m},p,n} \beta \left\{ \frac{\alpha(n)}{n}(u - p) - i\phi_{t+1}\hat{m} \right\}. \tag{11}$$

Since sellers post p before buyers choose their money holdings, $\phi_{t+1}\hat{m} = p$ as long as $i > 0$. Let Ω again be the equilibrium expected utility of a buyer in the DM. Sellers maximize

$$\max_{p,n} \pi(n) = \alpha(n)(p - c) \text{ s.t. } \frac{\alpha(n)}{n}(u - p) - ip \geq \Omega, \tag{12}$$

or

$$\max_{n} \pi(n) = \alpha(n)\left[\frac{\alpha(n)u - n\Omega}{\alpha(n) + ni} - c\right], \tag{13}$$

and the seller's optimal price is

$$p^c(i) = \frac{\alpha(n^*)\{[1 - \varepsilon(n^*)]u + \varepsilon(n^*)c\} + \varepsilon(n^*)n^*ic}{\alpha(n^*) + \varepsilon(n^*)n^*i}. \tag{14}$$

In equilibrium, n^* is consistent with the free entry condition

$$\frac{\alpha(n^*)}{n^*}(u - p^c(i)) - ip^c(i) \geq 0. \tag{15}$$

Thus, (14) and (15) determine $(p^c(i), n^*(i))$. Unlike bargaining, p^c depends on i, and jointly determined with n^*, the measure of active buyers in the market.

We follow Gu and Wright (2016) in establishing the existence and uniqueness of monetary equilibrium. Define the aggregate demand of liquidity, $L^d = n^*p^c$, with n^* and p^c both depending on i. Monetary equilibrium is then characterized by the intersection of L^d and the relevant supply curve, which is horizontal at the exogenous policy variable, i. The nominal interest rate is the price of holding liquidity. It determines the equilibrium quantity via L^d.

Lemma 2 *There exist i^N and $\bar{\imath}^C$ with $i^N < \bar{\imath}^C$, defined in the proof, such that: (i) for $i < i^N$, $\exists!$ L^d with $n^* = N$ and $dL^d/di < 0$; (ii) for generic $i \in [i^N, \bar{\imath}^C]$, $\exists!$ L^d with $n^* \leq N$ and $dL^d/di < 0$; (iii) for $i > \bar{\imath}^C$, $\nexists$ $n^* > 0$ and L^d is not well-defined.*

Then,

Proposition 5 *In the model with money and competitive search: (i) for $i < i^N$, $\exists!$ symmetric SME with $n^* = N$; (ii) for generic $i \in [i^N, \bar{\imath}^C]$, $\exists!$ symmetric SME with $n^* \leq N$ ($<$ if $i > i^N$); (iii) for $i > \bar{\imath}^C$, $\nexists$ SME.*

From Proposition 5, buyers' real balance, i.e. p^c, is always affected by i, and money is not superneutral, but still neutral. For case (i) p^c is strictly decreasing in i, and $n^* = N$. Hence, buyers' expected surplus is increasing in i, and indirectly buyers' bargaining power afforded by the market is increasing. For case (ii) as shown in the proof, $\partial n^*(i)/\partial i < 0$. High inflation then increases buyers' probability of trade, and expected surplus seen from $ip^c(i)$ in (15). It is easy to show $\partial ip^c/\partial i > 0$. This is in line with the "hot potato" effect from Liu et al. (2011). They show that more inflation leads to higher probability of trade (and surplus) for buyers with divisible goods, money, and bargaining, through the extensive margin. They do not study price posting.

Lagos and Rocheteau (2005) consider bargaining and competitive search with divisible goods and money, and derive how buyers' surplus is affected by search intensity. With bargaining, search intensity decreases with inflation, and buyers' surplus decreases with i. Both buyers' share of surplus and probability of trade decrease with i. Thus, they find no "hot potato" effect, while we find this effect for high inflation. With competitive search, for small i, they generate an increasing share of expected surplus for buyers, while the expected surplus still decreases with i. When i is close to zero, the fall of expected surplus is second-order effect, while the increase of buyers' share is first-order. Hence, buyers' surplus increases for a range of i, and so does the probability of trade. Therefore, they find a "hot potato" effect for a low inflation range. After a threshold of i, they find that both buyers' expected surplus and probability of trade decreases with i. Thus, no "hot potato" effect for high range of inflation. We find the reverse, no "hot potato" effect for low range of inflation, but the effect comes in for high range of inflation. Thus, indivisibility matters.

Competitive search provides a natural environment to get (generically) unique equilibrium. Buyers direct their search to sellers that give the highest expected payoff. Competition among sellers guarantees that a buyer gets Ω from the DM trade. The expected queue length adjusts continuously with the posted price, and the market-clearing price in the DM is uniquely determined. The fact that this adjustment mechanism does not exist under price posting and random search leads to the existence of a continuum of monetary equilibria, as in Green and Zhou (1998) and Jean et al. (2010).[4]

5 Conclusion

In this note, we study the trade of indivisible goods in frictional markets. Overall, the consequences of indivisibility on the goods side matter and differ from indivisibility on the money side. In particular, when terms of trade are determined by bargaining with money, the bargained price gives sellers no surplus and is independent of the nominal interest rate. Money is superneutral as long as all buyers participate in the market. Introducing lotteries cannot reestablish the link between real balances and anticipated inflation under bargaining. However, price posting with competitive search does reestablish this link because the equilibrium price of the indivisible good depends on the nominal interest rate and the number of buyers in the market.

In the pure credit economy, we show uniqueness under bargaining and competitive search. We also show uniqueness under bargaining in the monetary economy. Under competitive search, we get uniqueness for generic parameters. While we have focused on stationary equilibrium, the model can easily be used to study price dynamics. We leave that for future research.

Appendix

Proof of Proposition 1. The stationary equilibrium with credit is characterized by the solution to the bargaining problem. Using λ as the multiplier on the credit constraint yields the following Kuhn-Tucker conditions:

$$0 = (1-\eta)(u-p)^{\eta}(p-c)^{-\eta} - \eta(u-p)^{\eta-1}(p-c)^{1-\eta} - \lambda \text{ and } 0 = \lambda\,(D-p)\,.$$

If $\lambda = 0$, then $p = (1-\eta)u + \eta c \equiv \bar{p}^B$. However, if $\lambda > 0$, then $p = D$. Finally, we need $D \geq c$ to guarantee non-negative surplus for sellers. ■

Proof of Proposition 2. The proof is similar to Proposition 1. ■

Proof of Proposition 3. First, $i \leq \bar{\imath}^N = \Psi(N)$ implies $v_N(c) \geq v_N(0)$ and hence (i). For (ii), we need $\lim_{n\to 0}\Psi(n) = (u-c)/c = \bar{\imath}^B$, which is assured by the assumptions of

[4] Apart from existence, our results differ quite substantially from those of Jean et al (2010). They consider price posting and random search to show a continuum of equilibria indexed by different real balances. Their result is driven by coordination failure from simultaneous moves by buyers and sellers. To obtain a unique equilibrium, they impose the assumptions of finite agents and sequential move.

$\alpha(n)$. Finally, for $i \geq \bar{\imath}^B$, $v_n(c) < v_n(0)$ for all $n > 0$, and the DM is inactive. ■

Proof of Lemma 1. We have the following Kuhn-Tucker conditions

$$\begin{aligned} 0 &= -\eta\left(\tau u-p\right)^{\eta-1}\left(p-\tau c\right)^{1-\eta}+\left(1-\eta\right)\left(\tau u-p\right)^{\eta}\left(p-\tau c\right)^{-\eta}-\lambda_1 && (16)\\ 0 &= \eta u\left(\tau u-p\right)^{\eta-1}\left(p-\tau c\right)^{1-\eta}-c\left(1-\eta\right)\left(\tau u-p\right)^{\eta}\left(p-\tau c\right)^{-\eta}-\lambda_2 && (17)\\ 0 &= \lambda_1\left(\phi_t m-p\right),\, 0=\lambda_2\left(1-\tau\right), \end{aligned}$$

with λ_1 and λ_2 being the multipliers on the monetary and lotteries constraint. If $\lambda_1 = 0$, $p^B = \tau\bar{p}^B$. Substituting into (17) implies $\lambda_2 > 0$, and hence $\tau^B = 1$, which requires $\phi m > \bar{p}^B$. If $\tau^B < 1$, then $\lambda_2 = 0$ and $p^B = \tau\underline{p}^B$. Substituting into (16) implies $\lambda_1 > 0$ and $p^B = \phi m$, hence $\phi m < \underline{p}^B$. If $\lambda_1 > 0$ and $\lambda_2 > 0$, $p^B = \phi m$ and $\tau^B = 1$. $\lambda_1 > 0$ implies $\phi m < \bar{p}^B$, and $\lambda_2 > 0$ implies $\phi m > \underline{p}^B$. Finally, there is no trade if $\phi m < c$. ■

Proof of Proposition 4. First, the buyer does not want to bring $\phi_{t+1}\hat{m} > \bar{p}^B$ if $i > 0$, or $\phi_{t+1}\hat{m} < c$ for no trade. Then, for $\phi_{t+1}\hat{m} \in (\underline{p}^B, \bar{p}^B)$, $v'(\hat{m}) = -\phi_{t+1}\alpha(n)/n - i\phi_{t+1} < 0$, and the optimal money holding is $\phi_{t+1}\hat{m} = \underline{p}^B$. For $\phi_{t+1}\hat{m} \in (c, \underline{p}^B)$, $v'(\hat{m}) = \phi_{t+1}\left[\alpha(n)\eta(u-c)/nc - i\right]$. Since $\underline{p}^B[\alpha(n)\eta(u-c)/nc - i] = \alpha\left(n\right)\left(u-\underline{p}^B\right)/n - i\underline{p}^B$, $v'(\hat{m})$ shares the same sign as $\alpha\left(n\right)\left(u-\underline{p}^B\right)/n - i\underline{p}^B$. Suppose $v'(\hat{m}) < 0$, then buyers choose $\phi_{t+1}\hat{m} = c$ and $\tau^B = 0$. Suppose $v'(\hat{m}) > 0$, then buyers with measure n^* choose $\phi_{t+1}\hat{m} = \underline{p}^B$. The cutoff i satisfying $v'(\hat{m}) = 0$ is $\bar{\imath}^B = \lim_{n\to 0}\alpha(n)(u-\underline{p}^B)/n\underline{p}^B = \eta(u-c)/c$. Therefore, if $i < \bar{\imath}^B$, $\exists!$ SME with $\phi_{t+1}\hat{m} = \underline{p}^B$ and $\tau^B = 1$; otherwise, there is no monetary equilibrium. Define $i^N = \alpha(N)(u-\underline{p}^B)/N\underline{p}^B = \alpha(N)\eta(u-c)/Nc < \bar{\imath}^B$. If $i \leq i^N$, $n^* = N$; otherwise, $n^* < N$. ■

Proof of Lemma 2. To prove the existence and uniqueness of L^d, it is sufficient to show the existence and uniqueness of $n^* > 0$. Substitute p^c into (15) and we get $\alpha\varepsilon\left(u-c\right)i + \alpha^2\varepsilon\left(u-c\right)/n^* \geq \alpha\left[\left(1-\varepsilon\right)u+\varepsilon c\right]i + \varepsilon n^* c i^2$. Define $h(n^*, i) = \alpha\varepsilon(u-c)i + \alpha^2\varepsilon(u-c)/n^* - \alpha[(1-\varepsilon)u+\varepsilon c]i - \varepsilon n^* c i^2$. Given any $n \in (0, N]$, $h(n, i) = 0$ is a quadratic function in i, with two real solutions of opposite signs. The positive solution $i_+(n)$, satisfying $h(n, i_+) = 0$, is an implicit function of n. Let $i_+(0) = \lim_{n\to 0} i_+(n) < \infty$. It is easy to show $i_+(n)$ is continuous on $[0, N]$. Define i^N by $h(N, i^N) = 0$ and $\bar{\imath}^C = \max_{n\in[0,N]} i_+(n)$. For $i < i^N$, $h(N, i) > 0$ then $n^* = N$. Thus, $L^d = Np^c(N, i)$ is unique, and $\partial L^d/\partial i = N\partial p^c(N, i)/\partial i < 0$, hence (i). For $i > \bar{\imath}^C$, $h(n^*, i) < 0$ $\forall n^*$, and the free-entry condition does not hold since $\alpha(n^*)(u-p^c)/n^* - ip^c < 0$, hence (iii).

Regarding (ii), for $i \leq \bar{\imath}^C$, $h(n^*, i) = 0$ always holds for some $n^* > 0$, and L^d exists. To show that L^d is generically unique and monotone, consider $L^d = n^*p^c$ and $dL^d/di = \partial L^d/\partial i + (\partial L^d/\partial n^*)(\partial n^*/\partial i)$. Given $h(n^*, i) = 0$, $L^d = \alpha(n^*)n^*u/[\alpha(n^*) + in^*]$, hence $\partial L^d/\partial i < 0$ and $\partial L^d/\partial n^* > 0$. Then, it is sufficient to show n^* is generically unique and $\partial n^*/\partial i < 0$. Next, we want to show that n^* is unique and $\partial n^*/\partial i < 0$ for generic

i. Suppose $\pi(n_1^*, i) = \pi(n_2^*, i) = \max_n \pi(n, i)$ and $n_2^* > n_1^*$. Then, n_1^* is the minimum n maximizing $\pi(n, i)$, and $\pi(n_1^*, i) > \pi(n, i)$, $\forall n < n_1^*$. For small $\varepsilon > 0$, $\pi(n_1^*, i+\varepsilon) > \pi(n, i+\varepsilon)$ also holds for $n < n_1^*$ due to continuity. If $\partial^2\pi/\partial i \partial n^* < 0$, then $\pi(n_1^*, i+\varepsilon) > \pi(n_2^*, i+\varepsilon)$, and there is a unique global maximizer in the neighborhood of n_1^*. Finally, we need to show $\partial^2\pi/\partial i \partial n^* < 0$. Derive $\partial\pi/\partial n$ from (13),

$$\frac{\partial \pi}{\partial n} = \frac{(\alpha + in)\left[(u-c)\,\alpha' - ic\right] - i\,(1-\varepsilon)\left[(u-c)\,\alpha - inc\right]}{(\alpha + in)^2/\alpha}.$$

Define $T(i) = (\alpha + in)[(u-c)\alpha' - ic] - i(1-\varepsilon)[(u-c)\alpha - inc]$, and $T'(i) = n[(u-c)\alpha' - ic] - (\alpha + in)c - (1-\varepsilon)[(u-c)\alpha - inc] + inc(1-\varepsilon)$. Since $T_{n=n^*} = 0$, $\partial^2\pi/\partial i \partial n^* = T'(i)/[(\alpha + in^*)^2/\alpha]$. With $\alpha(u-c) - in^*c > 0$, we have

$$T'_{n=n^*}(i) = \frac{-\left[\alpha\,(u-c) - in^*c\right](1-\varepsilon)\,\alpha - c\,(\alpha + in^*)\,(\alpha + in^*\varepsilon)}{\alpha + in^*} < 0,$$

and $\partial^2\pi/\partial i \partial n^* < 0$ holds. Although $\arg\max_n \pi(n, i)$ may have more than one solution for some $i \geq \bar{\imath}^C$, the set of such i has measure zero, hence (ii). ■

Proof of Proposition 5. First, for $i > \bar{\imath}^C$, $n^* < 0$ as shown in Lemma 2, and there is no monetary equilibrium, hence (iii). For $i < i^N$, L^d is unique and monotonically decreasing in i. Hence, given i, there exists a unique real money holding $\phi_{t+1}\hat{m} = p^c$ and a unique SME. Since $h(N, i) > 0$ and $\alpha(N)(u - p^c)/N - ip^c > 0$, we have $n^* = N$, thus (i). Finally, as shown in the proof of Lemma 2, L^d is generically unique and $\partial L^d/\partial i < 0$ for $i \in [i^N, \bar{\imath}^C]$. Therefore, there exists a generically unique real balance $\phi_{t+1}\hat{m}$ and symmetric SME with $n^* \leq N$. The inequality is strict if $i > i^N$. ■

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