To appear in the *International Journal of Control*
Vol. 00, No. 00, Month 20XX, 1–27

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Passivity Based Control of Linear Time Invariant Systems Modelled by Bond Graph

R. Galindo[a*] and R. F. Ngwompo[b]

[a] *Autonomous University of Nuevo Leon, Faculty of Electrical and Mechanical Engineering, Av. Universidad, San Nicolas de los Garza, Nuevo Leon, 66450, Mexico;*
[b] *University of Bath, Department of Mechanical Engineering, Faculty of Engineering and Design, Bath, BA2 7AY, UK, Email:r.f.ngwompo@bath.ac.uk*

(*June 2016*)

Closed loop control systems are designed for Linear Time-Invariant (LTI) controllable and observable systems modelled by bond graph. Cascade and feedback interconnections of bond graph models are considered and are realized through active (signal) bonds with no loading effect. The use of active bonds may lead to non-conservation of energy and the overall system is modelled by proposed pseudo-junction structures. These structures are build by adding parasitic elements to the bond graph models which assure that each storage element is connected to a dissipative element and the overall system may become singularly perturbed. The structures for these interconnections can be seen as consisting of inner structures that satisfy energy conservation properties and outer structures including multiport-coupled dissipative fields. These structures are called *pseudo* due to the structural properties of power conservation not being satisfied in the outer structures. The multiport-coupled dissipative fields highlight energy properties like passivity. These properties are useful for control design. In both interconnections, junction structures and multiport-coupled dissipative fields for the controllers are proposed and passivity is guaranteed for the closed loop systems assuring robust stability. The pseudo-junction structure for the cascade interconnection is applied to the structural representation of the closed loop transfer functions, in a one-degree of freedom feedback configuration, when a controller from the parametrisation of all stabilizing controllers is applied to a given nominal plant. Applications are given when the plant and the controller are described by state-space realizations, in this case parasitic elements are not added. Moreover, the feedback interconnection is used and the controller is tuned getting necessary and sufficient stability conditions based on the characteristic polynomial of the closed loop transfer function, solving a pole-placement problem and achieving zero-stationary state error.

Keywords: Bond Graph; Junction structure; Structural properties; Feedback; Physical and Passivity Based Control; Robust stability; Pole-placement; Parametrisation of all stabilizing controllers; Singularly perturbed

1. Introduction

Over the last two decades, the design of control systems in the physical domain has been proposed as a means of integrating controllers within the design of systems from various engineering domains. Advantages of such an approach are to preserve physical insight and to exploit the system's architecture for controller synthesis and analysis. The pioneering work of Sharon, Hogan & Hardt (1991) developed a control system for a robotic manipulator by comparing classical (purely mathematical) and physical-based design approaches. They showed that the latter technique provided guidance for the choice and location of actuators leading to a stable overall system. A modelling technique that naturally lends itself to the above physical approach is the bond graph representation first

*Corresponding author. Email: rgalindo@gama.fime.uanl.mx

proposed by Paynter (1961) and developed by Karnopp & Rosenberg (1975) among others. For a tutorial on bond graph the reader is referred to the works of Tanguy (1995) and Gawthrop (2007). Gawthrop (1995) used bond graphs to propose a generic framework for the design of controllers in the physical domain, where the controller as well as the system to be controlled were all represented by their bond graph models. This *physical model-based control* has more recently found a number of applications like the ones of Gawthrop, Wagg & Nield (2007); Gawthrop, Bhikkaji & Moheimani (2010). In particular in the work of Yeh (2002) the closed-loop system is visualized from the open-loop bond-graph model to derive a control law using a recursive back-stepping procedure for one and two-port systems with a specific cascaded structure. The present article does not focus on an specific physical system as in the work of Sharon, Hogan & Hardt (1991). It focus on an a dynamic feedback rather than on an observer-control design as in the works of Gawthrop (1995) or Gonzalez-A (2016), and it is a closed loop methodology while the work of Yeh (2002) is based on an open-loop design.

In this paper, a closed loop control system is analysed from the junction structures and multiport-coupled dissipative fields point of view with the objective of determining the properties of the control system. Both the system and the dynamic controller are modelled by junction structures and multiport-coupled dissipative fields representations. This description of the system represents a bond graph model of a physical system, and the other is a proposed description for the controller. A common problem in the bond graph representation of control system is that of the interconnection of systems with no loading effect. This usually results in combining *power bonds* and *active (signal) bonds*. An alternative approach, attempting to preserve a common framework of power, was proposed in Li & Ngwompo (2005) through the introduction of power scaling elements in the context of the design of passive control systems.

In the present work, pseudo-junction structures are proposed and analysed for the cascade and feedback interconnections of Linear Time Invariant (LTI) controllable and observable systems, modelled by bond graph. These interconnections are realized through active bonds, which may lead to the overall structure not being conservative. The structures for these interconnections can be seen as inner structures that satisfy energy conservation properties and outer structures including multiport-coupled dissipative fields. These structures are called *pseudo* due to the structural properties of power conservation are not being satisfied in the outer structures. These multiport-coupled dissipative fields highlight energy properties like passivity of the overall system. The aim of the control designer is to keep the physical properties in closed loop. The advantage of the proposed controllers is that they have associated junction structures and multiport-coupled dissipated fields with physical meaning. The proposed structures are based on a certain connection of each storage element to a dissipative element. These connections are achieved adding parasitic elements, so, when small storage elements are added, the overall system may become singularly perturbed (see Kokotovic, Khalil & O'Reilly (1999)). In the work of Gonzalez-A (2016) a state-estimated feedback is designed for singularly perturbed systems modelled by bond graph. A quasi-steady state model of the closed loop system is gotten, based on assigning integral and derivative causalities to the plant and to the observer, respectively.

Passivity Based Control (PBC) is addressed based on the proposed pseudo-junction structures. A lot of works have been realized on PBC, for instance see the survey of Ortega & García (2004), and few works on PBC based on bond graph, one of them is the work of Garcia, Rimaux & Delgado (2006) in which damping is added to a DC/DC power converter such that the closed loop system is passive. A general non-linear control methodology is presented in the work of Ortega & García (2004), that first assigns algebraically a desired interconnection and damping and then the dependency of non-measurable states is removed by the non-parametrised interconnection and damping assignment. This PBC is realized for an specific non-linear state space description in terms of the total stored energy. The present work does not focus on an specific physical system as it focuses on linear systems modelled by bond graph. Passivity implies that a certain transfer function is positive real and that robust stability is achieved by the closed loop system (see Brogliato, Lozano, Maschke & Egeland (2007)), that is, stability is guaranteed under large uncertainties such as unmodelled dynamics or large variations in the parameters. In the present work it is considered that after the subsystem interconnection only power energy external sources are applied to the m-port system. Hence, as stated in the work of Beaman & Rosenberg (1988) whether each element of a model is passive then the system is passive. In the proposed pseudo-junction structures the storage fields of the plant and the controller are assumed passive and remains unchanged, so power conservation is guaranteed if the closed loop multiport-coupled dissipative field is passive, or equivalently if the associated defining matrix is positive semi-definite.

In section 2 the tackled problem is stated. Using the parametrisation of all stabilizing controllers the closed loop transfer functions are affine functions of the free control parameters and can be regarded as the cascade interconnection of certain transfer functions. In this case, applications of the proposed results are given in sections 3 and 5 when the plant and the controller are described by state-space realizations. Parasitic elements are not required and so the system is not singularly perturbed. Also, in section 4, a junction structure and a multiport-dissipative field for the controller are proposed into the pseudo-junction structure for the feedback interconnection. In both interconnections passivity is analysed. Moreover, necessary and sufficient stability conditions are presented based on the characteristic polynomial of the closed loop transfer function and the controller is tuned by solving a pole-placement problem and a constrained pole-placement problem, achieving zero-stationary state error in both cases. In section 4 and 5 an Illustrative example of a two-mass spring damper system is given.

Notation 1: $\mathcal{I}_p$ is the identity matrix of dimension $p \times p$; diag$\{a_1, a_2, \ldots, a_n\}$ is a diagonal matrix of dimension $n \times n$ whose elements are $a_1, a_2, \ldots, a_n$; and a real matrix M is positive semi-definite if and only if the symmetric part $\frac{1}{2}\left(M + M^T\right)$ is positive semi-definite, where M^T is the transpose of M.

2. Background and Problem Statement

A bond graph model of a conservative Linear Time-Invariant (LTI) system in integral causality is represented in Fig. 1 where C and I are storage elements in integral causality, S_e and S_f are sources of effort and flow, R is the dissipative field, D_e and D_f are detectors (sensors) of effort and flow as proposed by Karnopp & Rosenberg (1975). The junction structure $S(0, 1, TF, GY)$, linking these elements, is an assemblage of $0-$ junctions, $1-$ junctions, transformers , TF, and gyrators, GY. Let m, n, p and q be the input, state, output and dissipative space dimensions, respectively. The state vector $x(t) \in \Re^{n\times 1}$ is associated with the energy variables of the storage elements in integral causality; $z(t) = Fx(t) \in \Re^{n\times 1}$ is the co-energy vector, where F is a matrix composed of $1/I$ and $1/C$ elements; $D_o(t) \in \Re^{q\times 1}$ and $D_i(t) \in \Re^{q\times 1}$ are vectors of variables associated with the dissipation field R such that $D_o(t) = LD_i(t)$, where L is a matrix; $u(t) \in \Re^{m\times 1}$ and $v(t) \in \Re^{m\times 1}$ are the system input and $y(t) \in \Re^{p\times 1}$ is the system output.

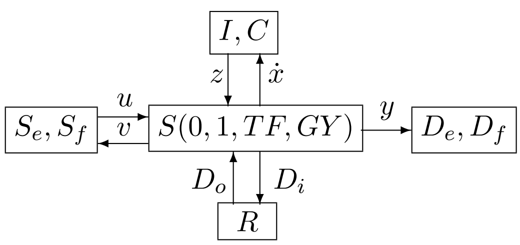

Figure 1. Junction structure associated with a bond graph in integral causality

The relationships for the junction structure are given by:

$$\begin{bmatrix} \dot{x}(t) \\ D_i(t) \\ y(t) \end{bmatrix} = S(0,1,TF,GY) \begin{bmatrix} z(t) \\ D_o(t) \\ u(t) \end{bmatrix} \tag{1}$$

where $S(0,1,TF,GY)$ has a block partition according to the dimensions of $z(t)$, $D_o(t)$ and $u(t)$.

The system of Fig. 1 is power conservative in the sense that the supplied power must be equal to the stored and dissipated powers,

$$\dot{x}^T(t)z(t) + D_i^T(t)D_o(t) = u^T(t)v(t) \tag{2}$$

Moreover, junction structures associated to bond graph models of conservative Linear Time-Invariant (LTI) systems, may be regarded as a special type of dissipative fields that preserves the continuity of power, and their properties (see Karnopp & Rosenberg (1975); Sueur & Dauphin-Tanguy (1989) and Lamb, Woodall & Asher (1997)) are stated as follows:

P1 : S_{11} and S_{22}, are skew-symmetric,
P2 : $S_{12} = -S_{21}^T$,
Moreover, a solvability property is,
P3 : The bond graph model is singular if $\mathcal{I} - S_{22}L$ is a singular matrix. When the elements of R are linearly independent, there are no direct causal paths between these elements and $S_{22} = 0$, meaning that $\mathcal{I} - S_{22}L = \mathcal{I}$ and the model is non-singular.

However, bond graphs that use active bonds do not satisfy Eq. (2). These non-conservative systems may arise in the system interconnection of subsystems or in systems that includes internal modulated sources, that is, the junction structure is also a function of these sources, *i.e.*, $S(0,1,TF,GY,MS_e,MS_f)$ where MS_e and MS_f are the internal modulated sources of effort and flow respectively. This is the case of bond graphs including power-scaling elements of the work of Li & Ngwompo (2005). Also, properties P1 and P2 are not satisfied by non-conservative systems. The proposed inner pseudo-junction structures satisfy these properties and allow focusing on the multiport-coupled dissipative fields.

In the present work it is assumed that,

Assumption 1. *After the subsystem interconnection only power energy external sources or conjugate external input/output signals, i.e., whose product is power, are applied to the m-port system, where m is the input space dimension.*

This assumption characterise the allowable outputs and references for the feedback system. Also, under this assumption, as stated in the work of Beaman & Rosenberg (1988) whether each element of a model is passive then the system is passive. In the proposed pseudo-junction structures the storage fields of the plant and the controller are assumed passive and remains unchanged, so power conservation is guaranteed if the closed loop multiport-coupled dissipative field is passive, or equivalently if the associated defining matrix is positive semi-definite.

Under assumption 1 it is possible to have outputs and references of velocities, and the closed-loop relative degree can be 1, *i.e.*, it can be passivized. However, outputs and references of positions are in general not allowed due to the increase of the closed-loop relative degree, although, as it will be shown in example 2 of section 4, it is possible to design a passivity-based control of velocities and use an approximation of a derivative, to control positions.

The proposed pseudo-junction structures, combined with the constitutive relationships of the fields, have the following characteristics,

(1) it can represent non-conservative systems and are useful for control design.
(2) it has an inner structure that is power conservative and an outer structure with a multiport-

coupled dissipative field that includes the internal modulated sources (MS_e, MS_f).

(3) the multiport-coupled dissipative field may be decomposed into power scaling elements introduced in Li & Ngwompo (2005) and the original dissipative fields.
(4) it leads to an associated state-space description.
(5) under Assumption 1, power conservation is guaranteed if the multiport-coupled dissipative field is passive (due to the original storage field remaining unchanged).

The aim of the present work is,

Problem 1. *Based on proposed pseudo-junction structures for the cascade and feedback interconnections of LTI systems with no loading effect, the aim is to design a control such that the closed loop system is passive, that is, the control is such that the multiport-coupled dissipative field is passive and thus the overall system becomes conservative.*

The pseudo-junction structure of the work of Gonzalez & Galindo (2009) for systems described by a state-space realizations, can be derived from the presented pseudo-junction structures. However, as stated before, the state-space description matrices (A, B, C, D) might not have physical significance. The proposed pseudo-junction structures require that the number of storage elements be equal to the number of dissipative elements. This condition is consistent with the work of Gonzalez & Galindo (2009) and it can be achieved by,

(1) Connecting "high" resistors in parallel to each C or connecting "small" capacitors in parallel to each R, as required, and
(2) Connecting "small" resistors in serial to each I or connecting "small" inductors in serial to each R, as required.

This building proposition is shown in Fig. 2 where a predefined integral causality assignment is realized. So, the strong bonds impose the causality to all the elements connected to these junctions and assure that,

$$e_R = e_C \text{ and } f_R = f_I \tag{3}$$

Hence, since it is realized for each pair of $R - C$ and $R - I$, then,

$$q = n,\ S_{21} = \mathcal{I}_n,\ S_{22} = 0 \text{ and } S_{23} = 0 \tag{4}$$

and property P3 is achieved. Also, Fig. 2 implies that $S_{12} = -\mathcal{I}_n$ for a conservative system. However, it does not hold for non-conservative systems like the interconnection of subsystems using active bonds.

Connecting "high" resistors in parallel to a C element and "small" resistors in serial to an I element, adds $1/R$ and R elements to the dissipative field, respectively. So, these parasitic elements add almost zero elements into L. Also, when "small" capacitors or inductors are added, the augmented system is singularly perturbed and the added fast dynamics must be stable accordingly to Tikhonov's Theorem (see Kokotovic, Khalil & O'Reilly (1999)). These parasitic elements add $1/\epsilon$ elements into F, where $\epsilon \in \Re$ is a "small" perturbation parameter replacing the added "small" capacitors or inductors. Locate all together the $1/\epsilon$ elements, the state equation of the system becomes of the form,

$$\begin{cases} \dot{x}_1(t) & = A_{11}x_1(t) + A_{12}x_2(t) + B_1u(t) \\ \epsilon\dot{x}_2(t) & = A_{21}x_1(t) + A_{22}x_2(t) + B_2u(t) \end{cases} \tag{5}$$

where the fast dynamics associated to $x_2(t)$ must be stable, and the as it will be shown in section 4 the quasi-steady state can be gotten setting $\epsilon = 0$ or in a bond graph approach using the result of Gonzalez-A (2016). Moreover, the proposed augmented bond graph ensures that all the storage elements accept a predefined integral causality assignment as shown in Fig. 2. Also they accept a predefined derivative causality assignment on the Singularly Perturbed Bond Graph proposed by Gonzalez which implies that A_{22} is a non-singular matrix as required for getting the quasi-steady state model.

In what follows it is assumed that,

Assumption 2. *Each LTI subsystem to be interconnected with no loading effect satisfies Eq. (4).*

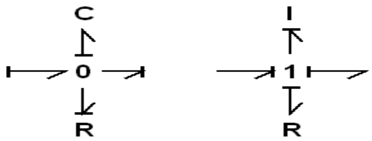

Figure 2. Augmenting the BG model using parasitic elements.

In the following section the cascade interconnection of systems described by state-space realizations is considered, and an application is presented when an ideal plant is described by a state-space description $(A, \mathcal{I}_n, \mathcal{I}_n, 0)$ and the controller belongs to the parametrisation of all stabilizing controllers (see Vidyasagar (1985)). The pseudo-junction structure of the cascade interconnection is used to get closed loop transfer functions. In the last section, the results of this section are extended to plants described by a state-space description $(A, B, \mathcal{I}_n, 0)$.

3. Ideal plant and controller described by state-space realizations

If the plant and the controller are described by state-space realizations, a useful result is the one given by the work of Gonzalez & Galindo (2009). There are several junction structures and possible constitutive relationships for the same state-space description. If the junction structure $S(0, 1, TF, GY, MS_e, MS_f)$ is described in terms of a state-space realization using the work of Gonzalez & Galindo (2009), then, $z(t) = -Fx(t)$ into the pseudo-junction structure. So, the passivity properties are not obtained directly from the dissipative field due to the sign of the storage field. In order to obtain $z(t) = Fx(t)$ a change of sign is proposed into the result of Gonzalez & Galindo (2009), that is, an inner pseudo-junction structure of a BG with predefined integral causality assignment, is derived from a linear time-invariant state-space realization (A, B, C, D) and is given by,

$$\begin{bmatrix} \dot{x}(t) \\ D_i(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 0 & -\mathcal{I}_n & B \\ \mathcal{I}_n & 0 & 0 \\ CF^{-1} & 0 & D \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{D}_o(t) \\ u(t) \end{bmatrix} \tag{6}$$

where $z(t) = Fx(t)$, $\hat{D}_o(t) = \hat{L}D_i(t)$ and $\hat{L} = -AF^{-1}$ with $F = \text{diag}\left\{\frac{1}{I_1}, \ldots, \frac{1}{I_v}, \frac{1}{C_{v+1}}, \ldots, \frac{1}{C_n}\right\}$ and v an arbitrary number of L elements. The storage elements $I_1, \ldots, I_v, C_{v+1}, \ldots, C_n$ are cancelled in the product AF^{-1}, leaving only resistive elements in $\hat{L}$ matrix as expected. Clearly Eq. (6) also reduces to $\dot{x}(t) = Ax(t) + Bu(t)$ and $y(t) = Cx(t) + Du(t)$ as the result of the work of Gonzalez & Galindo (2009).

Since in Eq. (6) each storage element is connected to a dissipative element, then parasitic elements are not added and the system is not singularly perturbed.

Using the parametrisation of all stabilizing controllers the closed loop transfer functions are affine functions of the free control parameters (see Vidyasagar (1985)). In particular in the one degree of freedom feedback configuration shown in Fig. 3, where $P(s)$ is the plant, $\mathcal{K}(s)$ is the controller, $y(t)$ is the plant output and $y_d(t)$ is the output reference, the transfer function from $y_d(t)$ to $y(t)$ is $T_o(s) = N(s)\tilde{N}_{\mathcal{K}}(s)$ (see Vidyasagar (1985)), where $N(s)$ and $\tilde{N}_{\mathcal{K}}(s)$ are the numerators of the

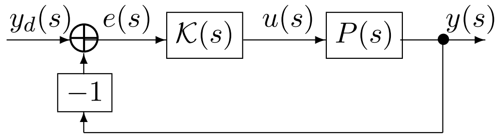

Figure 3. One degree of freedom feedback configuration

coprime factorizations of the plant and the controller, respectively. The objective is to get pseudo-junction structures of these sensitivity functions, so, pseudo-junction structures of the cascade interconnection of $N(s)$ and $\tilde{N}_{\mathcal{K}}(s)$, are required.

The following Theorem states the construction of a pseudo-junction structure for given bond graph models with associated junction structures $S^a(0, 1, TF, GY, MS_e, MS_f)$ and $S^b(0, 1, TF, GY, MS_e, MS_f)$ described by Eq. (6). The subsystems are interconnected in cascade with no loading effect.

Theorem 1: *Suppose that the subsystems described by* $(A_a,\ B_a,\ C_a,\ D_a)$ *and* $(A_b,\ B_b,\ C_b,\ D_b)$ *are interconnected in cascade with no loading effect and under Assumption* 1*. Let* m_a*,* n_a *and* p_a *be the input, state and output space dimensions of the first system, respectively,* m_b*,* n_b *and* p_b *be the input, state and output space dimensions of the second system, and* $u_b(t) = Ky_a(t)$*, where* $K \in \Re^{m_b \times p_a}$ *is a non-singular matrix composed of the gains of* MS_e^b *and* MS_f^b*, and two junction structures* $S^a(0, 1, TF, GY, MS_e, MS_f)$ *and* $S^b(0, 1, TF, GY, MS_e, MS_f)$ *described by Eq. (6),*

$$\begin{bmatrix} \dot{x}_a(t) \\ D_i^a(t) \\ y_a(t) \end{bmatrix} = \begin{bmatrix} 0 & -\mathcal{I}_{n_a} & B_a \\ \mathcal{I}_{n_a} & 0 & 0 \\ C_a F_a^{-1} & 0 & D_a \end{bmatrix} \begin{bmatrix} z_a(t) \\ \hat{D}_o^a(t) \\ u_a(t) \end{bmatrix}, \tag{7}$$

and

$$\begin{bmatrix} \dot{x}_b(t) \\ D_i^b(t) \\ y_b(t) \end{bmatrix} = \begin{bmatrix} 0 & -\mathcal{I}_{n_b} & B_b \\ \mathcal{I}_{n_b} & 0 & 0 \\ C_b F_b^{-1} & 0 & D_b \end{bmatrix} \begin{bmatrix} z_b(t) \\ \hat{D}_o^b(t) \\ u_b(t) \end{bmatrix} \tag{8}$$

that satisfy Eq. (4), where $x_a(t) \in \Re^{n_a \times 1}$*,* $z_a(t) = F_a x_a \in \Re^{n_a \times 1}$*,* $D_i^a(t) \in \Re^{n_a \times 1}$*,* $\hat{D}_o^a(t) = -A_a F_a^{-1} D_i^a(t) \in \Re^{n_a \times 1}$*,* $y_a(t) \in \Re^{p_a \times 1}$*,* $u_a(t) \in \Re^{m_a \times 1}$*,* $x_b(t) \in \Re^{n_b \times 1}$*,* $z_b(t) = F_b x_b(t) \in \Re^{n_b \times 1}$*,* $D_i^b(t) \in \Re^{n_b \times 1}$*,* $\hat{D}_o^b(t) = -A_b F_b^{-1} D_i^b(t) \in \Re^{n_b \times 1}$*,* $y_b(t) \in \Re^{p_b \times 1}$ *and* $u_b(t) \in \Re^{m_b \times 1}$*.*

Then, an inner pseudo-junction structure $S_{ab}^i(0, 1, TF, GY, MS_e, MS_f)$ *for the cascade interconnection, satisfying the energy conservation properties P1 and P2 is,*

$$\begin{bmatrix} \dot{x}(t) \\ D_i(t) \\ y_b(t) \end{bmatrix} = \begin{bmatrix} 0 & -\mathcal{I}_{n_a+n_b} & \hat{S}_{13}^o \\ \mathcal{I}_{n_a+n_b} & 0 & 0 \\ \hat{S}_{31}^o & 0 & \hat{S}_{33}^o \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{D}_o(t) \\ u_a(t) \end{bmatrix} \tag{9}$$

where $\dot{x}(t) := \begin{bmatrix} \dot{x}_a^T(t) & \dot{x}_b^T(t) \end{bmatrix}^T$*,* $D_i(t) := \begin{bmatrix} (D_i^a(t))^T & \left(D_i^b(t)\right)^T \end{bmatrix}^T$*,* $z(t) := \begin{bmatrix} z_a^T(t) & z_b^T(t) \end{bmatrix}^T$*,*

$$\hat{S}_{13}^o := \begin{bmatrix} B_a \\ B_b K D_a \end{bmatrix}, \quad \hat{S}_{31}^o := \begin{bmatrix} D_b K C_a F_a^{-1} & C_b F_b^{-1} \end{bmatrix}, \quad \hat{S}_{33}^o := D_b K D_a, \tag{10}$$

And the multiport-coupled dissipative field is,

$$\hat{D}_o(t) := L_{ab} D_i(t) \tag{11}$$

where,

$$L_{ab} := -\begin{bmatrix} A_a F_a^{-1} & 0 \\ B_b K C_a F_a^{-1} & A_b F_b^{-1} \end{bmatrix} \tag{12}$$

Moreover, the system is passive if L_{ab} is a positive semi-definite matrix.

Proof. See Appendix 1. □

The triangular structure of the dissipative field given in equations (11) and (12), means that the dissipative field of the controller remains decoupled and from the outer pseudo-junction structure for the cascade interconnection, it is clear that the controller subsystem does not change, *i.e.*, as expected due to the cascade interconnection with no loading effect.

The definitions of the elements of equations (9) to (12) clearly shows the dependency of the inner pseudo-junction structure on the gain K of the internal modulated sources.

Control can be designed using the power conservation of $S^i_{ab}(0, 1, TF, GY, MS_e, MS_f)$ and the passivity properties of its dissipative field such that the overall system becomes robustly stable.

For simplicity, first consider an ideal plant with transfer function $(sI_n - A)^{-1}$ obtained by setting $B = C = \mathcal{I}_n$. This plant is controllable and observable, and let $K = \mathcal{I}_n$. From the work of Galindo, Sanchez-Orta & Herrera (2002), a stabilizing controller is $\mathcal{K}(s) = A + \frac{(a+r)s+a^2}{s+a-r} I_n$, so that $\tilde{N}_{\mathcal{K}}(s) = \frac{1}{s+a}\left[((a+r)\mathcal{I}_n + A)s + a^2\mathcal{I}_n + (a-r)A\right]$ and $N(s) = \frac{1}{s+a}\mathcal{I}_n$, where $r \in \Re$ and $0 < a \in \Re$ are control parameters. Hence, the state-space realizations of $\tilde{N}_{\mathcal{K}}(s)$ and $N(s)$ are,

$$\begin{array}{l} (-a\mathcal{I}_n, \mathcal{I}_n, \ -r(a\mathcal{I}_n + A), \ (a+r)\mathcal{I}_n + A) \ \text{and} \\ (-a\mathcal{I}_n, \mathcal{I}_n, \mathcal{I}_n, 0), \end{array} \tag{13}$$

respectively. So, applying the stabilizing controller $\mathcal{K}(s)$ to $(sI_n - A)^{-1}$ in a one degree of freedom feedback configuration, and since $K = \mathcal{I}_n$ then from Theorem 1 the closed loop pseudo-junction structure of the complementary sensitivity function $T_o(s) = N(s)\tilde{N}_{\mathcal{K}}(s)$ is given by Eq. (9), where $\hat{S}^o_{33} = 0$,

$$\hat{S}^o_{13} = \begin{bmatrix} \mathcal{I}_n \\ (a+r)\mathcal{I}_n + A \end{bmatrix}, \quad \hat{S}^o_{31} = \begin{bmatrix} 0 & F_b^{-1} \end{bmatrix}, \tag{14}$$

and the dissipative field is $\hat{D}_o(t) = L_{ab} D_i(t)$, where,

$$L_{ab} = \begin{bmatrix} aF_a^{-1} & 0 \\ r(a\mathcal{I}_n + A)F_a^{-1} & aF_b^{-1} \end{bmatrix} \tag{15}$$

with $F_a = \text{diag}\left\{\frac{1}{I_1}, \ldots, \frac{1}{I_{n_a}}\right\}$ and $F_b = \text{diag}\left\{\frac{1}{I_1}, \ldots, \frac{1}{I_{n_b}}\right\}$. Hence, the system is passive if the real and non-symmetric matrix L_{ab} is a positive semi-definite matrix. Necessary conditions for L_{ab} to be a positive semi-definite matrix, are that aF_a^{-1} and aF_b^{-1} be positive semi-definite matrices. Since F_a and F_b are diagonal matrices of positive elements, so, these matrices are positive definite matrices and the necessary conditions for L_{ab} to be a positive semi-definite matrix are achieved if,

$$a \geq 0 \tag{16}$$

From the work of Galindo, Sanchez-Orta & Herrera (2002), $-a < r < a$ and at low frequencies regulation is achieved when $r \to a$. The large couplings of the dissipative field arise when $r \to a$ or when $r \to -a$, and when $r = 0$ the dissipative field is decoupled.

For a given Eq. (1) and the constitutive relationships of the fields, the state-space description of the model is given by (see Karnopp & Rosenberg (1975)),

$$\begin{array}{ll} A = (S_{11} + S_{12} M S_{21}) F, & B = S_{13} + S_{12} M S_{23} \\ C = (S_{31} + S_{32} M S_{21}) F, & D = S_{33} + S_{32} M S_{23} \end{array} \tag{17}$$

where $M := (\mathcal{I} - L S_{22})^{-1} L$. So, from Eq. (17) a state-space realization of Eq. (9) where $\hat{S}_{13}^{o}$ and $\hat{S}_{31}^{o}$ are given by Eq. (14) is,

$$\begin{array}{ll} A_{cl} = -\begin{bmatrix} a\mathcal{I}_n & 0 \\ r(a\mathcal{I}_n + A) & a\mathcal{I}_n \end{bmatrix}, & B_{cl} = \begin{bmatrix} \mathcal{I}_n \\ (a+r)\mathcal{I}_n + A \end{bmatrix} \\ C_{cl} = \begin{bmatrix} 0 & \mathcal{I}_n \end{bmatrix}, & D_{cl} = 0 \end{array} \tag{18}$$

for which the result can be verified since its transfer function is $T_o(s)$.

Eq. (18) reveals that when the plant and the controller are described by state-space realizations, the closed-loop state-space realization does not depend on F, as apparently depends on Eq. (17).

Example 1: Consider for instance $n = 1$, $F_a = 1$, $F_b = f$, $A = \alpha$, then, from (15), $L_{ab} = \begin{bmatrix} a & 0 \\ r(a+\alpha) & \frac{a}{f} \end{bmatrix}$ and its symmetric part is,

$$\frac{1}{2}\left(L_{ab} + L_{ab}^T\right) = \begin{bmatrix} a & \frac{1}{2} r(a+\alpha) \\ \frac{1}{2} r(a+\alpha) & \frac{a}{f} \end{bmatrix} \tag{19}$$

that from Sylvester's criterion is positive semi-definite if $a \geq 0$ and $\frac{2a}{\pm\sqrt{f}(a+\alpha)} > r$. From the eigenvalues of Eq. (18) the closed loop system is stable if $a > 0$. So, as expected passivity implies stability, however, the converse may not hold. □

In the following sections passivity conditions are presented based on passivity properties of the multiport-coupled dissipative field, and necessary and sufficient stability conditions are given based on the characteristic polynomial of the closed loop transfer function.

4. Control design

The following Theorem presents a pseudo-junction structure for the feedback configuration as shown in Fig. 4, that is Fig. 1 combined with Fig. 3, where the junctions structures associated to the bond graph of $\mathcal{K}(s)$ and $P(s)$ are denoted by S^a and S^b, respectively. It is assumed that the sub-systems are interconnected with no loading effect, that is, these sub-systems are interconnected trough active (signal) bonds that modulates sources of effort or flow. Due to this connection, the overall system may not conserve energy.

Theorem 2: *Suppose that the sub-systems modelled by bond graph are interconnected in feedback with no loading effect as shown in Fig. 4 and under Assumptions* 1 and 2*. Let m_a, n_a and p_a be the input, state and output space dimensions of the first system, respectively, m_b, n_b and p_b be the input, state and output space dimensions of the second system, $u_b(t) = K_b y_a(t)$ and $u_a(t) = K_a(y_d(t) - y_b(t))$, where $K_b \in \Re^{m_b \times p_a}$ and $K_a \in \Re^{m_a \times p_b}$ are non-singular matrices composed of the gains of MS_e^b, MS_f^b, MS_e^a and MS_f^a, and two junction structures $S^a(0, 1, TF, GY, MS_e, MS_f)$*

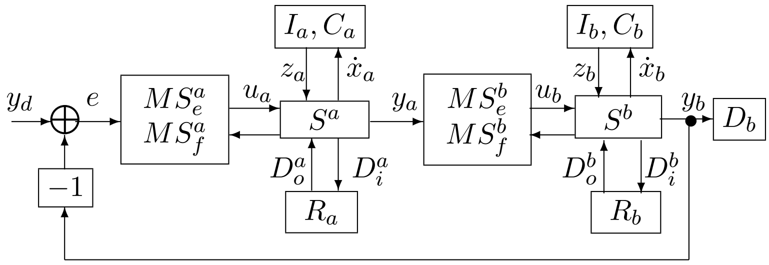

Figure 4. Feedback interconnection of bond graph models

and $S^b(0,1,TF,GY,MS_e,MS_f)$ *of bond graphs modelling conservative or non-conservative LTI systems,*

$$\begin{bmatrix} \dot{x}_a(t) \\ D_i^a(t) \\ y_a(t) \end{bmatrix} = \begin{bmatrix} S_{11}^a & S_{12}^a & S_{13}^a \\ \mathcal{I}_{n_a} & 0 & 0 \\ S_{31}^a & S_{32}^a & S_{33}^a \end{bmatrix} \begin{bmatrix} z_a(t) \\ D_o^a(t) \\ u_a(t) \end{bmatrix}, \tag{20}$$

and

$$\begin{bmatrix} \dot{x}_b(t) \\ D_i^b(t) \\ y_b(t) \end{bmatrix} = \begin{bmatrix} S_{11}^b & S_{12}^b & S_{13}^b \\ \mathcal{I}_{n_b} & 0 & 0 \\ S_{31}^b & S_{32}^b & S_{33}^b \end{bmatrix} \begin{bmatrix} z_b(t) \\ D_o^b(t) \\ u_b(t) \end{bmatrix} \tag{21}$$

that satisfy Eq. (4), where $x_a(t) \in \Re^{n_a \times 1}$, $z_a(t) = F_a x_a(t) \in \Re^{n_a \times 1}$, $D_i^a(t) \in \Re^{n_a \times 1}$, $D_o^a(t) = L_a D_i^a(t) \in \Re^{n_a \times 1}$, $y_a(t) \in \Re^{p_a \times 1}$, $u_a(t) \in \Re^{m_a \times 1}$, $x_b(t) \in \Re^{n_b \times 1}$, $z_b(t) = F_b x_b(t) \in \Re^{n_b \times 1}$, $D_i^b(t) \in \Re^{n_b \times 1}$, $D_o^b(t) = L_a D_i^b(t) \in \Re^{q_b \times 1}$, $y_b(t) \in \Re^{p_b \times 1}$ *and* $u_b(t) \in \Re^{m_b \times 1}$.

Then, an inner pseudo-junction structure $S_{cl}^i(0,1,TF,GY,MS_e,MS_f)$ *for the closed loop system, satisfying the energy conservation properties* $P1$ *and* $P2$ *is,*

$$\begin{bmatrix} \dot{x}(t) \\ D_i(t) \\ y_b(t) \end{bmatrix} = \begin{bmatrix} 0 & -\mathcal{I}_{n_a+n_b} & \hat{S}_{13}^o \Psi K_a \\ \mathcal{I}_{n_a+n_b} & 0 & 0 \\ \left(\mathcal{I} + \hat{S}_{33}^o K_a\right)^{-1} \bar{S}_{31} & 0 & \hat{S}_{33}^o \Psi K_a \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{D}_{ocl}(t) \\ y_d(t) \end{bmatrix} \tag{22}$$

with the following multiport-coupled dissipative field,

$$\hat{D}_{ocl}(t) := \left(L_{ab} + \hat{S}_{13}^o \Psi K_a \bar{S}_{31}\right) D_i(t) \tag{23}$$

where $\dot{x}(t) := \begin{bmatrix} \dot{x}_a^T(t) & \dot{x}_b^T(t) \end{bmatrix}^T$, $D_i(t) := \begin{bmatrix} (D_i^a(t))^T & \left(D_i^b(t)\right)^T \end{bmatrix}^T$, $z(t) := \begin{bmatrix} z_a^T(t) & z_b^T(t) \end{bmatrix}^T$, $L_{ab} := -\begin{bmatrix} S_{11}^a + S_{12}^a L_a & 0 \\ S_{13}^b K_b (S_{31}^a + S_{32}^a L_a) & S_{11}^b + S_{12}^b L_b \end{bmatrix}$, $\bar{S}_{31} := \hat{S}_{31}^o + \hat{S}_{32}^o L$ *and* $\Psi := \left(\mathcal{I}_{n_a} + K_a \hat{S}_{33}^o\right)^{-1}$,

With L*:=diag*$\{L_a, L_b\}$, $\hat{S}_{13}^o := \begin{bmatrix} S_{13}^a \\ S_{13}^b K_b S_{33}^a \end{bmatrix}$, $\hat{S}_{31}^o := \begin{bmatrix} S_{33}^b K_b S_{31}^a & S_{31}^b \end{bmatrix}$, $\hat{S}_{32}^o := \begin{bmatrix} S_{33}^b K_b S_{32}^a & S_{32}^b \end{bmatrix}$ *and* $\hat{S}_{33}^o := S_{33}^b K_b S_{33}^a$.

Moreover, the closed loop system is passive if $L_{ab} + \hat{S}_{13}^o \Psi K_a \bar{S}_{31}$ *is a positive semi-definite matrix.*

Proof. See Appendix 2. □

Remark 1: Theorems 1 and 2 allows the interconnection of conservative and non-conservative subsystems, so that each subsystem can be another interconnection of subsystems. Also, due to the storage fields of Theorems 1 and Theorem 2 being identical to the original fields, then passivity depends only on the multiport-coupled dissipative field. In order to solve Problem 1 it is required to design the control such that the real and non-symmetric matrices L_{ab} of Theorem 1 and $L_{ab} + \hat{S}^o_{13}\Psi K_a \bar{S}_{31}$ of Theorem 2 be positive semi-definite matrices. These requirements are simplified when the plant and the controller are power conservative systems, that is, $S^a_{12} = -\mathcal{I}_{n_a}$ and $S^b_{12} = -\mathcal{I}_{n_b}$, and S^a_{11} and S^b_{11} are skew symmetric matrices. Hence, $D_i^T(t) S^a_{11} D_i(t) = 0$ and $D_i^T(t) S^b_{11} D_i(t) = 0$, $\forall \mathrm{D}_i(t) \neq 0$. Then, the cascade and feedback interconnections are passive if,

$$\hat{L} := \begin{bmatrix} L_a & 0 \\ -S^b_{13} K (S^a_{31} + S^a_{32} L_a) & L_b \end{bmatrix} \tag{24}$$

and $\hat{L} + \hat{S}^o_{13}\Psi K_a \bar{S}_{31}$, respectively, are positive semi-definite matrices. □

Remark 2: Collecting all together the added "small" capacitors and inductors, such that, $F = \begin{bmatrix} F_1 & 0 \\ 0 & \frac{1}{\epsilon}\mathcal{I} \end{bmatrix}$, where ϵ is a perturbation parameter replacing the "small" inductors and capacitors. Applying Eq. (17) to the closed loop system described by equations (22) and (23), then the state-space equation is,

$$\dot{x}(t) = -\left(L_{ab} + \hat{S}^o_{13}\Psi K_a \bar{S}_{31}\right) F x(t) + \hat{S}^o_{13}\Psi K_a u(t) \tag{25}$$

and let $L_{ab} + \hat{S}^o_{13}\Psi K_a \bar{S}_{31} := \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$ and $\hat{S}^o_{13}\Psi K_a := \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$ be block partitioned accordingly to the dimensions of the block partition of F, so, the singularly perturbed model is,

$$\begin{cases} \dot{x}_1(t) & = -L_{11} F_1 x_1(t) - L_{12}\frac{1}{\epsilon} x_2(t) + B_1 u(t) \\ \epsilon \dot{x}_2(t) & = -\epsilon L_{21} F_1 x_1(t) - L_{22} x_2(t) + \epsilon B_2 u(t) \end{cases} \tag{26}$$

Assuming that the fast dynamics associated to $x_2(t)$ are stable and L_{22} be a non-singular matrix, hence the quasi-steady state model is,

$$\bar{x}_2(t) = \epsilon L_{22}^{-1}\left(-L_{21} F_1 \bar{x}_1(t) + B_2 u(t)\right) \tag{27}$$

and the reduced model is,

$$\dot{\bar{x}}_1(t) = \left(L_{12} L_{22}^{-1} L_{21} - L_{11}\right) F_1 \bar{x}_1(t) + \left(B_1 - L_{12} L_{22}^{-1} B_2\right) u(t) \tag{28}$$

In the following Corollaries that are useful for control design, a junction structure $S^a(0, 1, TF, GY)$ associated to a bond graph model for the controller is proposed as,

$$\begin{bmatrix} \dot{x}_a(t) \\ D^a_i(t) \\ y_a(t) \end{bmatrix} = \begin{bmatrix} S^a_{11} & -\mathcal{I}_{n_a} & S^a_{13} \\ \mathcal{I}_{n_a} & 0 & 0 \\ S^a_{31} & 0 & S^a_{33} \end{bmatrix} \begin{bmatrix} z_a(t) \\ D^a_o(t) \\ u_a(t) \end{bmatrix} \tag{29}$$

where S^a_{11} is a skew symmetric matrix, $x_a(t) \in \Re^{n_a \times 1}$, $z_a(t) = F_a x_a(t) \in \Re^{n_a \times 1}$, $D^a_i(t) \in \Re^{n_a \times 1}$, $D^a_o(t) = L_a D^a_i(t) \in \Re^{n_a \times 1}$, $y_a(t) \in \Re^{p_a \times 1}$ and $u_a(t) \in \Re^{m_a \times 1}$, and it is assumed that,

(1) the controllable and observable plant is a conservative system, that is, $S_{12}^b = -\mathcal{I}_n$ and S_{11}^b is a skew symmetric matrix, and
(2) the output of the plant $y_b(t)$, is not related to the dissipative field $D_o^b(t)$, that is, $S_{32}^b = 0$.

Consider a junction structure $S^b(0, 1, TF, GY)$ associated to a bond graph model of the plant, under these assumptions,

$$\begin{bmatrix} \dot{x}_b(t) \\ D_i^b(t) \\ y_b(t) \end{bmatrix} = \begin{bmatrix} S_{11}^b & -\mathcal{I}_{n_b} & S_{13}^b \\ \mathcal{I}_{n_b} & 0 & 0 \\ S_{31}^b & 0 & S_{33}^b \end{bmatrix} \begin{bmatrix} z_b(t) \\ D_o^b(t) \\ u_b(t) \end{bmatrix} \tag{30}$$

where S_{11}^b is a skew symmetric matrix, $x_b(t) \in \Re^{n_b \times 1}$, $z_b(t) = F_b x_b(t) \in \Re^{n_b \times 1}$, $D_i^b(t) \in \Re^{n_b \times 1}$, $D_o^b(t) = L_b D_i^b(t) \in \Re^{n_b \times 1}$, $y_b(t) \in \Re^{p_b \times 1}$ and $u_b(t) \in \Re^{m_b \times 1}$.

Suppose that the proposed controller and the plant are connected in closed loop as shown in Fig. 4, where,

$$u_b(t) = K_b y_a(t) \text{ and } u_a(t) = K_a (y_d(t) - y_b(t)), \tag{31}$$

with $K_a \in \Re^{m_a \times p_b}$ and $K_b \in \Re^{m_b \times p_a}$ being non-singular matrices composed of the gains of MS_e^a, MS_f^a, MS_e^b and MS_f^b.

The following Corollaries present stability analysis based on the passivity properties of the dissipative field and based on the characteristic polynomial of the closed loop transfer function. The aim is to select the gains K_a, K_b, the elements of the proposed junction structure and the multiport-coupled dissipative field for the controller given by Eq (29).

Corollary 1: *Suppose that two bond graph models are connected in closed loop as shown in Fig. 4, satisfying Assumptions* 1 *and* 2*. Consider two junction structures associated to these bond graph models, $S^a(0, 1, TF, GY)$ and $S^b(0, 1, TF, GY)$ given by equations (29) and (30). Then, an equivalent inner pseudo-junction structure $S_{cl}^i(0, 1, TF, GY, MS_e, MS_f)$ for the closed loop system, satisfying the energy conservation properties P1 and P2 is,*

$$\begin{bmatrix} \dot{x}(t) \\ D_i(t) \\ y_b(t) \end{bmatrix} = \begin{bmatrix} 0 & -\mathcal{I}_{n_a+n_b} & \hat{S}_{13}^o \Psi K_a \\ \mathcal{I}_{n_a+n_b} & 0 & 0 \\ \left(\mathcal{I}+\hat{S}_{33}^o K_a\right)^{-1} \hat{S}_{31}^o & 0 & \hat{S}_{33}^o \Psi K_a \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{D}_{ocl}(t) \\ y_d(t) \end{bmatrix} \tag{32}$$

with the following multiport-coupled dissipative field,

$$\hat{D}_{ocl}(t) := \left(L_{ab} + \hat{S}_{13}^o \Psi K_a \hat{S}_{31}^o\right) D_i(t) \tag{33}$$

where $D_i(t) := \begin{bmatrix} (D_i^a(t))^T & \left(D_i^b(t)\right)^T \end{bmatrix}^T$, $x(t) := \begin{bmatrix} x_a^T(t) & x_b^T(t) \end{bmatrix}^T$, $z(t) := \begin{bmatrix} z_a^T(t) & z_b^T(t) \end{bmatrix}^T$, $L_{ab} := \begin{bmatrix} L_a - S_{11}^a & 0 \\ -S_{13}^b K_b S_{31}^a & L_b - S_{11}^b \end{bmatrix}$, $\Psi := \left(\mathcal{I}_{n_a}+K_a \hat{S}_{33}^o\right)^{-1}$,

$$\hat{S}_{13}^o := \begin{bmatrix} S_{13}^a \\ S_{13}^b K_b S_{33}^a \end{bmatrix} \text{ and } \hat{S}_{31}^o := \begin{bmatrix} S_{33}^b K_b S_{31}^a & S_{31}^b \end{bmatrix},\ \hat{S}_{33}^o := S_{33}^b K_b S_{33}^a. \tag{34}$$

The feedback system is passive if,

$$\begin{bmatrix} L_a & 0 \\ -S_{13}^b K_b S_{31}^a & L_b \end{bmatrix} + \hat{S}_{13}^o \Psi K_a \hat{S}_{31}^o \tag{35}$$

is a positive semi-definite matrix.

Moreover, for strictly proper systems, i.e., for $S^b_{33} = 0$, Eq. (35) reduces to,

$$\begin{bmatrix} L_a & S^a_{13}K_aS^b_{31} \\ -S^b_{13}K_bS^a_{31} & L_b - S^b_{13}K_bS^a_{33}K_aS^b_{31} \end{bmatrix} \tag{36}$$

Furthermore, let $S^b_{13} = -(S^b_{31})^T$, $S^a_{13} = -(S^a_{31})^T$, $S^a_{33} = 0$ and $K_b = -K_a^T$, then the feedback system is passive.

Proof. From equations (29) and (30), and Theorem 2, $\hat{S}^o_{32} = 0$, so, $\bar{S}_{31} = \hat{S}^o_{31}$. Thus, from the same Theorem the results of equations (32) and (33) follows. Moreover, from Theorem 2, the closed loop system is passive if,

$$L_{ab} + \hat{S}^o_{13}\Psi K_a\hat{S}^o_{31} \tag{37}$$

is a positive semi-definite matrix. Since by assumption S^a_{11} and S^b_{11} are skew-symmetric matrices, hence, $D_i^TS^a_{11}D_i = 0$ and $D_i^TS^b_{11}D_i = 0$, $\forall D_i \neq 0$. Then, $L_{ab}+\hat{S}^o_{13}\Psi K_a\hat{S}^o_{31}$ is a positive semi-definite matrix if $\hat{L} + \hat{S}^o_{13}\Psi K_a\hat{S}^o_{31}$ is a positive semi-definite matrix, where $\hat{L}$ with $K = K_b$ is given by Eq. (24). Thus, the result of (35) follows. Also, if $S^b_{33} = 0$, then $\hat{S}^o_{33} = 0$ and $\hat{S}^o_{31} = \begin{bmatrix} 0 & S^b_{31} \end{bmatrix}$, which implies $\Psi = \mathcal{I}_{n_a}$ and matrix (35) reduces to matrix (36). Furthermore, if $S^b_{13} = -(S^b_{31})^T$, $S^a_{13} = -(S^a_{31})^T$, $S^a_{33} = 0$ and $K_b = -K_a^T$, then matrix (36) becomes decomposed as a positive definite matrix $\begin{bmatrix} L_a & 0 \\ 0 & L_b \end{bmatrix}$ plus a skew symmetric matrix $\begin{bmatrix} 0 & S^a_{13}K_aS^b_{31} \\ -S^b_{13}K_bS^a_{31} & 0 \end{bmatrix}$. Thus, matrix (36) is a positive definite matrix. □

In the following Corollary, the pole placement problem is solved for a particular class of systems, assigning a desired characteristic polynomial to the closed loop transfer function. It is assumed that the plant inputs and outputs are linearly independent.

Corollary 2: *Consider two junction structures $S^a(0, 1, TF, GY)$ and $S^b(0, 1, TF, GY)$ given by equations (29) and (30) associated to bond graph models for the controller and the plant, respectively. Suppose that the controller and the plant are connected in a feedback configuration as shown in Fig. 4 with no loading effect, the plant is a strictly proper system, i.e., $S^b_{33} = 0$, rank$\left(S^b_{13}\right) = m$ and rank$\left(S^b_{31}\right) = p$. Let $\det\left(s^2\mathcal{I}_m + \Lambda_1 s + \Lambda_2\right)$ be the desired closed loop characteristic polynomial of the transfer function from $y_d(t)$ to $y_b(t)$ where $\Lambda_1 \in \Re^{m\times m}$, $\Lambda_2 \in \Re^{m\times m}$ and let,*

$$S^a_{11} = 0,\ S^a_{13} = S^b_{13} \text{ and } S^a_{31} = S^b_{31} \tag{38}$$

Then, the transfer function from $y_d(t)$ to $y_b(t)$ is,

$$y_b(s) = \left[\Omega_{11} + K_b\Gamma K_a - \Omega_{12}\Omega_{22}^{-1}\Omega_{21}\right]^{-1} K_b\Gamma K_a y_d(s) \tag{39}$$

where $\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} := U\left(sF_b^{-1} + L_b - S^b_{11}\right)V$ *and* $\Gamma := S^a_{33} + S^b_{31}F_a\left(s\mathcal{I}_{n_a} + L_aF_a\right)^{-1}S^b_{13}$*, with, $U \in \Re^{n_b\times n_b}$ and $V \in \Re^{n_b\times n_b}$ being non-singular matrices such that,*

$$US^b_{13} = \begin{bmatrix} \mathcal{I}_m \\ 0 \end{bmatrix} \text{ and } S^b_{31}V = \begin{bmatrix} \mathcal{I}_p & 0 \end{bmatrix}. \tag{40}$$

Moreover, let $z_a(t) := diag\{F^a_1,\ F^a_2\}\,x_a(t)$, $z_b(t) := diag\{F^b_1,\ F^b_2\}\,x_b(t)$,

$D_o^a(t)$:=diag$\{L_1^a, L_2^a\}$ $D_i^a(t)$ and $D_o^b(t)$:=diag$\{L_1^b, L_2^b\}$ $D_i^b(t)$, with $F_1^a \in \Re^{m\times m}$, $F_1^b \in \Re^{m\times m}$, $L_1^b \in \Re^{m\times m}$, $F_2^a \in \Re^{(n-m)\times(n-m)}$, $F_2^b \in \Re^{(n-m)\times(n-m)}$ and $L_2^b \in \Re^{(n-m)\times(n-m)}$.

Furthermore, let $S_{11}^b := \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ -\Theta_{12}^T & 0 \end{bmatrix}$, $S_{13}^b = \begin{bmatrix} \mathcal{I}_m \\ 0 \end{bmatrix}$, $S_{31}^b = \begin{bmatrix} \mathcal{I}_m & 0 \end{bmatrix}$, $L_1^a = 0$, $K_a = \mathcal{I}_m$ and $L_2^b = 0$ then,

$$\begin{array}{l} K_b := \left[\Lambda_1 \left(F_1^b\right)^{-1} - L_1^b + \Theta_{11}\right] \left(S_{33}^a\right)^{-1} \text{ and} \\ F_1^a := K_b^{-1} \left[\Lambda_2 \left(F_1^b\right)^{-1} - \Theta_{12} F_2^b \Theta_{12}^T\right] \end{array} \tag{41}$$

if S_{33}^a is a non-singular matrix, and,

$$\begin{array}{l} K_b := \Lambda_2 \left(F_1^b\right)^{-1} - \Theta_{12} F_2^b \Theta_{12}^T \\ F_1^a := \mathcal{I}_m \end{array} \tag{42}$$

if $S_{33}^a = 0$. The reference gain,

$$y_d(t) = \left(F_1^a\right)^{-1} K_b^{-1} \Lambda_2 \left(F_1^b\right)^{-1} \bar{y}_d(t) \tag{43}$$

assigns the following desired closed loop transfer function from $\bar{y}_d(t)$ to $y_b(t)$,

$$F_1^b \left(s^2 \mathcal{I}_m + \Lambda_1 s + \Lambda_2\right)^{-1} \left[s K_b S_{33}^a \left(F_1^a\right)^{-1} K_b^{-1} + \mathcal{I}_m\right] \Lambda_2 \left(F_1^b\right)^{-1} \tag{44}$$

if S_{33}^a is a non-singular matrix, and,

$$F_1^b \left[s^2 \mathcal{I}_m + \left(L_1^b - \Theta_{11}\right) F_1^b s + \Lambda_2\right]^{-1} \Lambda_2 \left(F_1^b\right)^{-1} \tag{45}$$

if $S_{33}^a = 0$.

Proof. For strictly proper systems $S_{33}^b = 0$ and using equations (38), then $\hat{S}_{31}^o := \begin{bmatrix} 0 & S_{31}^b \end{bmatrix}$, $\hat{S}_{33}^o := 0$ and $\Psi := \mathcal{I}_{n_a}$. Hence, from Corollary 1, an equivalent inner pseudo-junction structure $\hat{S}^i$ for the closed loop system, satisfying the energy conservation properties P1 and P2 is,

$$\begin{bmatrix} \dot{x}(t) \\ D_i(t) \\ y_b(t) \end{bmatrix} = \begin{bmatrix} 0 & -\mathcal{I}_{n_a+n_b} & \hat{S}_{13}^o K_a \\ \mathcal{I}_{n_a+n_b} & 0 & 0 \\ \begin{bmatrix} 0 & S_{31}^b \end{bmatrix} & 0 & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{D}_{ocl}(t) \\ y_d(t) \end{bmatrix} \tag{46}$$

with the following multiport-coupled dissipative field,

$$\hat{D}_{ocl}(t) := \begin{bmatrix} L_a & S_{13}^b K_a S_{31}^b \\ -S_{13}^b K_b S_{31}^b & L_b - S_{11}^b + S_{13}^b K_b S_{33}^a K_a S_{31}^b \end{bmatrix} D_i(t) \tag{47}$$

So, from Eq. (17) the closed loop state-space realization is given and hence the closed loop transfer function from $y_d(t)$ to $y_b(t)$ is,

$$\begin{bmatrix} 0 & S_{31}^b F_b \end{bmatrix} \left(s\mathcal{I}_{n_a+n_b} - A_{cl}\right)^{-1} \hat{S}_{13}^o K_a \tag{48}$$

where

$$A_{cl}=\left[\begin{array}{cc} -L_aF_a & -S^b_{13}K_aS^b_{31}F_b \\ S^b_{13}K_bS^b_{31}F_a & \left(S^b_{11}-L_b-S^b_{13}K_bS^a_{33}K_aS^b_{31}\right)F_b \end{array}\right] \tag{49}$$

Using Eq. (34) and (see Zhou, Doyle & Glover (1985), p. 23) $\left[\begin{array}{cc} A_{11} & A_{12} \\ A_{21} & A_{22} \end{array}\right]^{-1} = \left[\begin{array}{cc} * & * \\ -\Delta^{-1}A_{21}A_{11}^{-1} & \Delta^{-1} \end{array}\right]$ where $\Delta := A_{22}-A_{21}A_{11}^{-1}A_{12}$ and $*$ are some finite values not important in this context, then,

$$y_b\left(s\right)=S^b_{31}F_b\Delta^{-1}S^b_{13}K_b\Gamma K_a y_d\left(s\right) \tag{50}$$

where $\Delta = s\mathcal{I}_{n_b} + \left(L_b - S^b_{11}\right)F_b + S^b_{13}K_b\Gamma K_a S^b_{31}F_b$. Since by assumption $\text{rank}\left(S^b_{13}\right) = m$ and $\text{rank}\left(S^b_{31}\right) = p$, then there are $U \in \Re^{n_b\times n_b}$ and $V \in \Re^{n_b\times n_b}$ non-singular matrices such that Eq. (40) is satisfied. Hence $y_b\left(s\right) = \left[\begin{array}{cc} \mathcal{I}_p & 0 \end{array}\right]\left[U\left(sF_b^{-1}+L_b-S^b_{11}\right)V + US^b_{13}K_b\Gamma K_aS^b_{31}V\right]^{-1}\left[\begin{array}{c} \mathcal{I}_m \\ 0 \end{array}\right]K_b\Gamma K_a y_d\left(s\right)$, that is,

$$y_b\left(s\right)=\left[\begin{array}{cc} \mathcal{I}_p & 0 \end{array}\right]\left[\begin{array}{cc} \Omega_{11}+K_b\Gamma K_a & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{array}\right]^{-1}\left[\begin{array}{c} K_b\Gamma K_a \\ 0 \end{array}\right]y_d\left(s\right) \tag{51}$$

Using (see Zhou, Doyle & Glover (1985), p. 23),

$$\left[\begin{array}{cc} A_{11} & A_{12} \\ A_{21} & A_{22} \end{array}\right]^{-1}=\left[\begin{array}{cc} \bar{\Delta}^{-1} & * \\ * & * \end{array}\right] \tag{52}$$

where $\bar{\Delta} := A_{11}-A_{12}A_{22}^{-1}A_{21}$ and $*$ are some finite values, then the result of Eq. (39) follows. Moreover, if $S^b_{13} = \left[\begin{array}{c} \mathcal{I}_m \\ 0 \end{array}\right]$, $S^b_{31} = \left[\begin{array}{cc} \mathcal{I}_m & 0 \end{array}\right]$, *i.e.*, $V = U = \mathcal{I}_{n_b}$ and $L^a_1 = 0$, $K_a = \mathcal{I}_m$ and $L^b_2 = 0$ then $\Gamma := S^a_{33} + \frac{1}{s}F^a_1$, and

$$\left[\begin{array}{cc} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{array}\right]=\left[\begin{array}{cc} (F^b_1)^{-1}s+L^b_1-\Theta_{11} & -\Theta_{12} \\ \Theta^T_{12} & (F^b_2)^{-1}s \end{array}\right] \tag{53}$$

So from (39), the transfer function from $y_d\left(t\right)$ to $y_b\left(t\right)$ is,

$$F^b_1\left[s^2\mathcal{I}_m+\left(L^b_1-\Theta_{11}+K_bS^a_{33}\right)F^b_1 s+\left(K_bF^a_1+\Theta_{12}F^b_2\Theta^T_{12}\right)F^b_1\right]^{-1}K_b\left(S^a_{33}s+F^a_1\right) \tag{54}$$

Hence, using the definitions given by equations (41) and (42),

$$y_b\left(s\right)=F^b_1\left(s^2\mathcal{I}_m+\Lambda_1 s+\Lambda_2\right)^{-1}K_b\left(S^a_{33}s+F^a_1\right)y_d\left(s\right) \tag{55}$$

if S^a_{33} is a non-singular matrix, and,

$$y_b\left(s\right)=F^b_1\left[s^2\mathcal{I}_m+\left(L^b_1-\Theta_{11}\right)F^b_1 s+\Lambda_2\right]^{-1}K_bF^a_1 y_d\left(s\right) \tag{56}$$

if $S_{33}^a = 0$, that in stationary state is,

$$y_{bss} = F_1^b \Lambda_2^{-1} K_b F_1^a y_{dss} \tag{57}$$

Thus, the reference gain given by Eq. (43) assigns the desired closed loop transfer functions from $\bar{y}_d(t)$ to $y_b(t)$ given by equations (44) and (45). □

The proposed junction structure associated to a bond graph model for the controller $S^a(0, 1, TF, GY)$ given by Eq. (29) used in Corollaries 1 and 2, has a similar structure as the plant, simplifying the controller implementation. Also, the sufficient stability conditions given by (35) and (36) are easy to check, as illustrated in the following example.

Corollary 2 shows that the characteristic polynomial of the transfer function from $y_d(t)$ to $y_b(t)$ can be freely assigned when $S_{33}^a = \mathcal{I}_{na}$ and a constrained pole-placement can be achieved when $S_{33}^a = 0$. In the last case the implementation of the controller is easier since $F_1^a = \mathcal{I}_m$. If $S_{33}^a = 0$, the fixed coefficient of the characteristic polynomial $L_1^b - \Theta_{11}$ is a positive definite matrix, because L_1^b is a symmetric matrix and Θ_{11} is a skew symmetric matrix. This condition is consistent with the sufficient stability condition of (36) and in this case leads to a single condition compared to those given in Galindo (2015), which require that $L_1^b - \Theta_{11}$ be a positive definite and symmetric matrix with an additional commutability condition that must be satisfied.

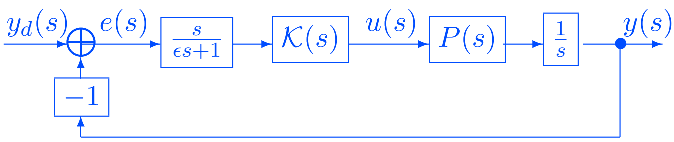

Figure 5. One degree of freedom feedback configuration including an approximation of a derivative

Accordingly to Assumption 1, outputs and references of velocities are selected in the next example, and the passivity-based control is designed using Corollaries 1 and 2. This control is applied to control output positions using an approximation of a derivative, as shown in Fig. 5, where $0 < \epsilon \in \Re$ is a “small” parameter.

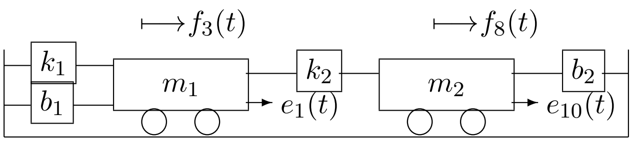

Figure 6. Two-cart system

Example 2: Consider the two-mass-spring-damper system shown in Fig. 6, where m_i, k_i and b_i, $i = 1, 2$, are the mass, the elasticity and friction coefficients, respectively, $e_1(t)$ and $e_{10}(t)$ are forces applied to masses m_1 and m_2, respectively, and $f_3(t)$ and $f_8(t)$ are the velocities of the masses m_1 and m_2, respectively. These velocity outputs satisfies Assumption 1. Fig. 7 shows a BG of this two mass spring damper system. In order to apply Theorem 2 or Corollary 1, assuring a non-singular matrix S_{21}^b, first high gain resistors R_3 and R_4 are added as show in Fig. 8. The junction structure of this augmented BG is,

$$\begin{bmatrix} \dot{x}(t) \\ D_i^b(t) \\ y_b(t) \end{bmatrix} = \begin{bmatrix} S_{11}^b & -\mathcal{I}_4 & S_{13}^b \\ \mathcal{I}_4 & 0 & 0 \\ S_{31}^b & 0 & 0 \end{bmatrix} \begin{bmatrix} z_b(t) \\ D_o^b(t) \\ u_b(t) \end{bmatrix} \tag{58}$$

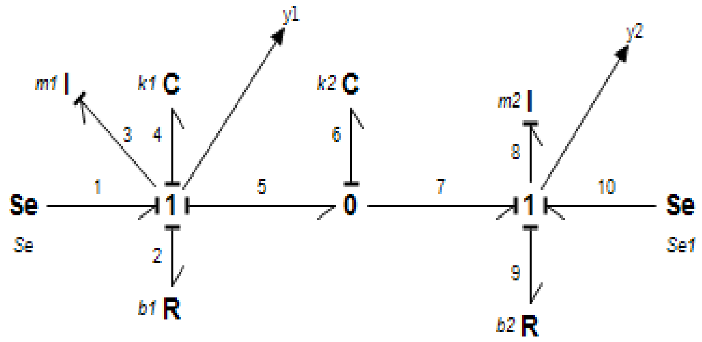

Figure 7. BG of two mass system spring damper system

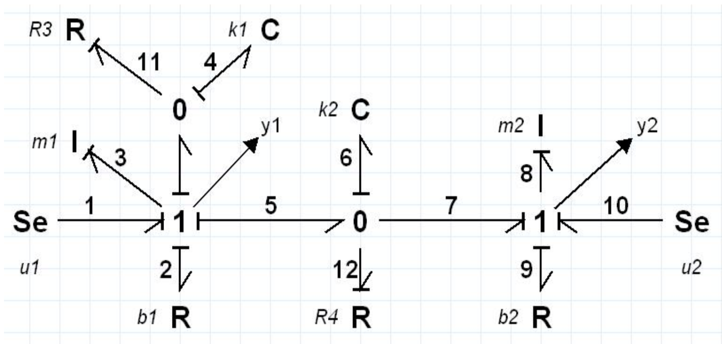

Figure 8. Augmented bond graph of a two mass spring damper system

where $S_{11}^b = \begin{bmatrix} 0_2 & \Theta_{12} \\ -\Theta_{12}^T & 0_2 \end{bmatrix}$ with $\Theta_{12} = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$, $S_{13}^b = \begin{bmatrix} \mathcal{I}_2 \\ 0_2 \end{bmatrix}$, $S_{31}^b = \begin{bmatrix} \mathcal{I}_2 & 0_2 \end{bmatrix}$, $\dot{x}(t) = \begin{bmatrix} e_3 & e_8 & f_4 & f_6 \end{bmatrix}^T$, $D_i^b(t) = \begin{bmatrix} f_2 & f_9 & e_{11} & e_{12} \end{bmatrix}^T$, $y(t) = \begin{bmatrix} f_3 & f_8 \end{bmatrix}^T$, $z(t) = \begin{bmatrix} f_3 & f_8 & e_4 & e_6 \end{bmatrix}^T$, $D_o^b(t) = \begin{bmatrix} e_2 & e_9 & f_{11} & f_{12} \end{bmatrix}^T$ and $u_b(t) = \begin{bmatrix} e_1 & e_{10} \end{bmatrix}^T$. Since the junction structure associated to the bond graph model of the two mass system given by Eq. (58) is strictly proper and has the structure of Eq. (30), in order to apply Corollaries 1 and 2, the controller with the proposed junction structure given by Eq. (29) is applied to the nominal plant with the junction structure given by Eq. (58) in the feedback interconnection of Fig. 4.

From Corollary 1, an equivalent inner pseudo-junction structure for the closed loop system, satisfying the energy conservation properties P1 and P2 is given by Eq. (32) with the multiport-coupled dissipative field given by Eq. (33). The added high resistors R_3 and R_4 introduce some zero terms into $L_{ab} + \hat{S}_{13}^o \Psi K_a \hat{S}_{31}^o$ of Eq. (33) as shown in the next example. Since no storage elements were added then the system is not singularly perturbed. Also, let $S_{33}^a = 0$, $S_{13}^a = S_{13}^b$ and $S_{31}^a = S_{31}^b$ into the junction structure associated to the bond graph for the controller, so, the passivity condition given by (36) is satisfied if,

$$K_a = K_b^T. \tag{59}$$

Moreover, from Corollary 2, $U = \mathcal{I}$, $V = \mathcal{I}$ and as R_3 and R_4 tends to infinity then L_2^b approaches zero, in order to obtain the closed loop transfer function given by (45), let $K_a = \mathcal{I}_2$, $L_1^a = 0$, $L_2^a = \mathcal{I}_2$,

$F_2^a = F_2^b$, $\Lambda_1 = 2\mathcal{I}_2$ and $\Lambda_2 = \mathcal{I}_2$, so, from Eq. (42),

$$\begin{array}{l} K_b = \begin{bmatrix} m_1 - (k_1 + k_2) & k_2 \\ k_2 & m_2 - k_2 \end{bmatrix}, \text{ if } S_{33}^a = 0 \\ K_b = \begin{bmatrix} 2m_1 - b_1 & 0 \\ 0 & 2m_2 - b_2 \end{bmatrix}, \text{ if } S_{33}^a = \mathcal{I}_2 \\ F_1^a = \mathcal{I}, \text{ if } S_{33}^a = 0 \\ F_1^a = \begin{bmatrix} \frac{m_1 - k_1 - k_2}{2m_1 - b_1} & \frac{k_2}{2m_1 - b_1} \\ \frac{k_2}{2m_2 - b_2} & \frac{m_2 - k_2}{2m_2 - b_2} \end{bmatrix}, \text{ if } S_{33}^a = \mathcal{I}_2 \end{array} \tag{60}$$

and from Eq. (43) the reference gain is,

$$y_{dss} = \frac{1}{\phi} \begin{bmatrix} (m_2 - k_2)\, m_1 & -k_2 m_2 \\ -k_2 m_1 & (m_1 - k_1 - k_2)\, m_2 \end{bmatrix} \bar{y}_d \tag{61}$$

where $\phi := (m_1 - k_1)(m_2 - k_2) - k_2 m_2$.

In order to control positions using the above control design of velocities, the designed controller is implemented in MATLAB/Simulink in the feedback configuration of Fig. 5 where the plant parameters are $m_1 = m_2 = 1$ Kg, $b_1 = b_2 = 1$ Ns/m and $k_1 = k_2 = 100$ N/m. The positions of the mass and the applied forces are shown in figures 9 and 10, if $S_{33}^a = 0$ and in figures 11 and 12, if $S_{33}^a = \mathcal{I}_2$, when a step reference of $\bar{y}_d = \begin{bmatrix} 1 & 0.5 \end{bmatrix}^T$ is applied. This is accomplished with a stable response having zero stationary state error as shown in figures 9 and 11, and due to the passivity properties the required forces are smooth and into an admissible range as shown figures 10 and 12. The initial oscillations of the mass velocities of Fig. 9 are due to the assigned characteristic polynomial $\det((s + s + 1)\mathcal{I}_2)$, and can be removed choosing instead $\Lambda_2 = 0.25\mathcal{I}_2$ with a bigger time response. On the other hand a smooth response is shown in Fig. 11 where the desired characteristic polynomial of the transfer function from $y_d(t)$ to $y_b(t)$ is $\det\left((s+1)^2 \mathcal{I}_2\right)$ and in this case it is freely assigned.

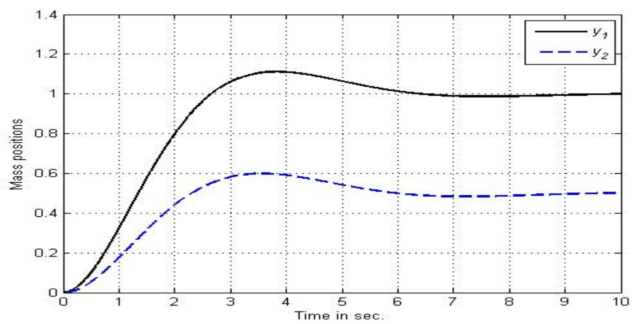

Figure 9. Mass positions when $S_{33}^a = 0$

In the next section an application of Corollary 1 is presented when the plant is described by a state-space description $(A, B, \mathcal{I}_n, 0)$. The controller belongs to the parametrisation of all stabilizing controllers, is designed for an ideal plant $(A, \mathcal{I}_n, \mathcal{I}_n, 0)$ and is implemented using a left inverse of B.

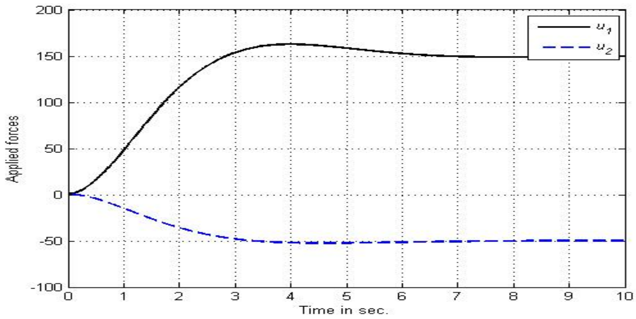

Figure 10. Applied forces when $S^a_{33} = 0$

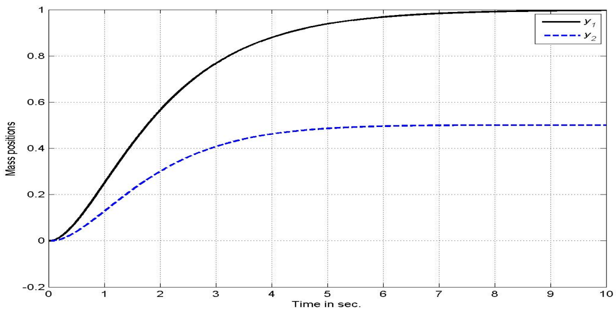

Figure 11. Mass positions when $S^a_{33} = \mathcal{I}$

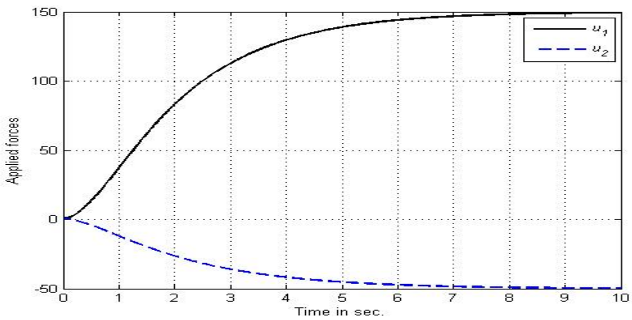

Figure 12. Applied forces when $S^a_{33} = \mathcal{I}$

5. Plant and controller described by state-space realizations

Let the plant and the controller be described by state-space realizations and be given by Eq. (6), then, an equivalent inner pseudo-junction structure for the closed loop system is given by Eq. (32) and Eq. (33) of Corollary 1 where,

$$\hat{S}^o_{13} = \begin{bmatrix} B_a \\ B_b K_b D_a \end{bmatrix}, \quad \hat{S}^o_{31} = \begin{bmatrix} D_b K_b C_a F_a^{-1} & C_b F_b^{-1} \end{bmatrix}, \tag{62}$$

$$\hat{S}^o_{33} = D_b K_b D_a, \quad L_{ab} = -\begin{bmatrix} A_a F_a^{-1} & 0 \\ B_b K_b C_a F_a^{-1} & A_b F_b^{-1} \end{bmatrix} \tag{63}$$

Moreover, let the controller (see Galindo (2006)) $\mathcal{K}(s) = B^L \left(A + \frac{(a+r)s+a^2}{s+a-r} \mathcal{I}_n \right)$ be applied to the plant $P(s) := (sI_n - A)^{-1} B$, in the feedback configuration of Fig. 4, where $0 < a \in \Re$ and $r \in \Re$ are control parameters. These controller and plant have state-space realizations,

$$\mathcal{K}(s) : \left((r-a)\,\mathcal{I}_n,\, \mathcal{I}_n,\, r^2 B^L,\, D_a \right) \tag{64}$$

and

$$P(s) : (A,\, B,\, \mathcal{I}_n,\, 0) \tag{65}$$

respectively, where $D_a := B^L [A + (a+r)\,\mathcal{I}_n]$. Let $K_a = \mathcal{I}_n$ and $K_b = \mathcal{I}_m$. Hence, an inner pseudo-junction structure for the closed loop system is given by equations (32) and (33) of Corollary 1 where,

$$\hat{S}^o_{13} = \begin{bmatrix} \mathcal{I}_{n_a} \\ BD_a \end{bmatrix}, \quad \hat{S}^o_{31} = \begin{bmatrix} 0 & F_b^{-1} \end{bmatrix}, \tag{66}$$

$$\hat{S}^o_{33} = 0, \quad L_{ab} = -\begin{bmatrix} (r-a)\,F_a^{-1} & 0 \\ r^2 BB^L F_a^{-1} & AF_b^{-1} \end{bmatrix} \tag{67}$$

So, from Eq. (17) a state-space description of the closed loop system is,

$$\begin{array}{ll} A_{cl} = \begin{bmatrix} (r-a)\,\mathcal{I}_{n_a} & -K_a \\ r^2 BB^L & A - BD_a K_a \end{bmatrix}, & B_{cl} = \begin{bmatrix} \mathcal{I}_{n_a} \\ BD_a \end{bmatrix} K_a, \\ C_{cl} = \begin{bmatrix} 0 & \mathcal{I} \end{bmatrix}, & D_{cl} = 0, \end{array} \tag{68}$$

that has previously been given in the work of Galindo (2006). Stability and performance were analysed by Galindo (2006) based on the characteristic polynomial. Applying Corollary 1, the feedback system is passive if the non-symmetric matrix,

$$L_{ab} + \hat{S}^o_{13} \Psi K_a \hat{S}^o_{31} = \begin{bmatrix} (a-r)\,F_a^{-1} & F_b^{-1} \\ -r^2 BB^L F_a^{-1} & M_{22} \end{bmatrix} \tag{69}$$

is a positive semi-definite matrix, where $M_{22} := (BD_a - A)\,F_b^{-1}$, that is, the feedback system is passive if the symmetric part,

$$\begin{bmatrix} (a-r)\,F_a^{-1} & Q_{21}^T \\ Q_{21} & \frac{1}{2}\left(M_{22} + M_{22}^T \right) \end{bmatrix} \tag{70}$$

is a positive semi-definite matrix, where $Q_{21} := \frac{1}{2}\left(F_b^{-1} - r^2 BB^L F_a^{-1} \right)$. Necessary conditions are that $(a-r)\,F_a^{-1}$ and $\frac{1}{2}\left(M_{22} + M_{22}^T \right)$ be positive semi-definite matrices. So, since F_a is a positive definite matrix, then from Sylvester's criterion a necessary condition is,

$$r \le a \tag{71}$$

that also assures a stable controller and regulation is achieved when $r \to a$ (see Galindo (2006)). Let $F_a = \mathcal{I}_n$, $F_b = diag\left\{F_1^b, F_2^b\right\}$, $B = \mathrm{S}_{13}^b = \left[\begin{array}{cc} \mathcal{I}_m & 0 \end{array}\right]^T$ and $B^L = \left[\begin{array}{cc} \mathcal{I}_m & G \end{array}\right]$ be a left-inverse matrix of B, *i.e.*, $B^L B = \mathcal{I}_m$, where $G \in \Re^{m\times(n-m)}$. From the work of Galindo (2006) whether $G = 0$ and invertible the characteristic polynomial depends only on $A_{11} \in \Re^{m\times m}$, where A_{11} is a sub-matrix of A. So, $G \neq 0$ is needed to assign a desired characteristic polynomial. Hence, $BB^L = \left[\begin{array}{cc} \mathcal{I}_m & G \\ 0 & 0 \end{array}\right]$ and the feedback system is passive if,

$$\left[\begin{array}{cc|cc} (a-r)\,\mathcal{I}_m & 0 & \frac{1}{2}\left((F_1^b)^{-1} - r^2\mathcal{I}_m\right) & 0 \\ 0 & (a-r)\,\mathcal{I}_{n-m} & -r^2G^T & \frac{1}{2}(F_2^b)^{-1} \\ \hline \frac{1}{2}\left((F_1^b)^{-1} - r^2\mathcal{I}_m\right) & -r^2G & (a+r)\,(F_1^b)^{-1} + W_{11} & -\frac{1}{2}(F_1^b)^{-1}A_{21}^T + W_{21}^T \\ 0 & \frac{1}{2}(F_2^b)^{-1} & -\frac{1}{2}A_{21}(F_1^b)^{-1} + W_{21} & W_{22} \end{array}\right] \tag{72}$$

is a positive semi-definite matrix, where $W_{11} := \frac{1}{2}\left(GA_{21}(F_1^b)^{-1} + (F_1^b)^{-1}A_{21}^TG^T\right)$, $W_{21} := \frac{1}{2}(F_2^b)^{-1}\left(A_{22}^T + (a+r)\mathcal{I}_m\right)G^T$ and $W_{22} := -\frac{1}{2}\left(A_{22}(F_2^b)^{-1} + (F_2^b)^{-1}A_{22}^T\right)$. A necessary condition for $\frac{1}{2}\left(M_{22} + M_{22}^T\right)$ be a positive semi-definite matrix is that,

$$(a+r)\,(F_1^b)^{-1} + W_{11} \tag{73}$$

be a positive semi-definite matrix.

Using the result that (see Boyd, Ghaoui, Feron & Balakrishnan (1994)), $\left[\begin{array}{cc} A_{11} & A_{12} \\ A_{21} & A_{22} \end{array}\right]$ is a positive semi-definite matrix if and only if A_{11} and the Schur complement $A_{22} - A_{21}A_{11}^{-1}A_{12}$ are positive semi-definite matrices, then under (71), $(a-r)\,\mathcal{I}_n$ is a positive semi-definite matrix and (72) is a positive semi-definite matrix if and only if,

$$\left[\begin{array}{cc} Z & -\frac{1}{2}(F_1^b)^{-1}A_{21}^T + W_{21}^T - \frac{1}{2}r^2G(F_2^b)^{-1} \\ -\frac{1}{2}A_{21}(F_1^b)^{-1} + W_{21} - \frac{1}{2}r^2(F_2^b)^{-1}G^T & W_{22} - \frac{1}{4}(F_2^b)^{-2} \end{array}\right] \tag{74}$$

be a positive semi-definite matrix, where $Z := (a+r)\,(F_1^b)^{-1} + W_{11} - \frac{1}{a-r}\left(\frac{1}{4}\left((F_1^b)^{-1} - r^2\mathcal{I}_m\right)^2 - r^4GG^T\right)$.

Corollary 2 shows that whether, $S_{13}^b = \left[\begin{array}{c} \mathcal{I}_m \\ 0 \end{array}\right]$ and $S_{31}^b = \left[\begin{array}{cc} \mathcal{I}_m & 0 \end{array}\right]$ then the transfer function from $y_d(t)$ to $y(t)$ does not depend on F_2^b, and so passivity does not depend on F_2^b and hence does not depend on $W_{22} - \frac{1}{4}(F_2^b)^{-2}$. Let $S_{31}^b = \left[\begin{array}{cc} \mathcal{I}_m & 0 \end{array}\right]$ then (72) is a positive semi-definite matrix if and only if Z be a positive semi-definite matrix.

As r and G approach a and 0, respectively, since F_1^b is a positive definite matrix, then from (72) and Sylvester's criterion a necessary condition is,

$$-a \leq r \tag{75}$$

Let F_1^b =diag$\{f_1, \ldots, f_m\}$, the leading principal minors of Z are,

$$\begin{array}{l} \det\left((a+r)\frac{1}{f_1} - \frac{1}{4(a-r)}\left(\frac{1}{f_1} - r^2\right)^2 \right), \\ \det\begin{bmatrix} (a+r)\frac{1}{f_1} - \frac{1}{4(a-r)}\left(\frac{1}{f_1} - r^2\right)^2 & 0 \\ 0 & (a+r)\frac{1}{f_2} - \frac{1}{4(a-r)}\left(\frac{1}{f_2} - r^2\right)^2 \end{bmatrix}, \\ \vdots \end{array} \tag{76}$$

So, considering inequality (71), the feedback system is passive if,

$$\left(a^2 - r^2\right)\frac{1}{f_i} - \frac{1}{4}\left(\frac{1}{f_i} - r^2\right)^2 \geq 0, \tag{77}$$

for $i = 1, \ldots, m$, that are equivalent to $\frac{a^2}{f_i} \geq \frac{1}{4}\left(\frac{1}{f_i} + r^2\right)^2$ for $i = 1, \ldots, m$, that is,

$$-\frac{1}{f_i} - \frac{2a}{\sqrt{f_i}} \leq r^2 \leq -\frac{1}{f_i} + \frac{2a}{\sqrt{f_i}} \tag{78}$$

for $i = 1, \ldots, m$. Since $f_i > 0$ for $i = 1, \ldots, m$ and $a > 0$ to have a stable feedback system (see Galindo (2006)), so, the left hand side of inequality (78) gives imaginary solutions to r. Hence, the closed loop system is passive if,

$$r^2 \leq \frac{-1}{f_i} + \frac{2a}{\sqrt{f_i}} \text{ and } a \geq \frac{1}{2\sqrt{f_i}} \tag{79}$$

for $i = 1, \ldots, m$, leading to $r \in \Re$.

Example 3: The results given by inequalities (71), (75) and (79) are applied to the previous example of a two mass spring damper system that is shown in Fig. 7. As in example 2, in order to apply Theorem 2 and Corollary 1, assuring a non-singular matrix S_{21}^b, first high gain resistors R_3 and R_4 are added as show in Fig. 8. The junction structure of this augmented BG is given by Eq. (58), and from Eq. (17), a state-space description for this system is $\left(\left(S_{11}^b - L_b\right)F_b, S_{13}^b, \mathcal{I}_4, 0\right)$, that is,

$$\begin{array}{ll} A = \begin{bmatrix} \frac{b_1}{m_1} & 0 & k_1 & k_2 \\ 0 & \frac{b_2}{m_2} & 0 & -k_2 \\ \frac{-1}{m_1} & 0 & \frac{k_1}{R_3} & 0 \\ \frac{-1}{m_1} & \frac{1}{m_2} & 0 & \frac{k_2}{R_4} \end{bmatrix}, & B = \begin{bmatrix} \mathcal{I}_2 \\ 0_2 \end{bmatrix}, \\ C = \mathcal{I}_4, & D = 0 \end{array} \tag{80}$$

Let $K_a = \mathcal{I}_4$, $K_b = \mathcal{I}_2$, $k_1 = k_2 = 100$ N/m, $b_1 = b_2 = 1$ Ns/m and $m_1 = m_2 = 1$ Kg, $B^L = \begin{bmatrix} \mathcal{I}_2 & 0 \end{bmatrix}$ and $F_a = \mathcal{I}_n$. Then, F_b =diag$\{F_1^b, F_2^b\}$ where F_1^b =diag$\{1, 1\}$ and F_2^b =diag$\{100, 100\}$. Hence, applying the controller with the state-space realization given by Eq. (64), an equivalent inner pseudo-junction structure for the closed loop system is given by equations (32) and (33) of Corollary 1. Hence, for $i = 1, 2$ from inequalities (71), (75) and (79), as r and G approach a and 0, respectively, the passivity conditions for the closed loop system are,

$$\begin{array}{l} -a \leq r \leq a, \\ r^2 \leq -1 + 2a \quad \text{and } 0.5 \leq a \end{array} \tag{81}$$

considering that A approaches $\begin{bmatrix} \frac{b_1}{m_1} & 0 & k_1 & k_2 \\ 0 & \frac{b_2}{m_2} & 0 & -k_2 \\ \frac{-1}{m_1} & 0 & 0 & 0 \\ \frac{-1}{m_1} & \frac{1}{m_2} & 0 & 0 \end{bmatrix}$ as R_3 and R_4 tends to infinity. As a is increased, the time response is decreased and the control energy is increased. Also, as a is increased, the feedback system becomes more robust in the sense that additive disturbance at the output are well attenuated. However, decreasing a, robust stability is achieved, that is, stability is preserved under large uncertainties due to unmodelled dynamics or parameter variations. Thus, a criterion is to select the smallest value of a that achieves the desired performance. Also, from inequalities (81), the difference between r and a is increased as the value of a is different from 1. Selecting $a = 1$, then from inequalities (81), $-1 \leq r \leq 1$. Passivity is achieved for values of a and r close to 1.

Tracking to the reference can be gotten if $G = 0$, however $G \neq 0$ is needed to assure stability, so instead the regulation problem is solved. Tracking to the reference and robustness under external disturbances, *i.e.*, robust performance are out of the scope of this work.

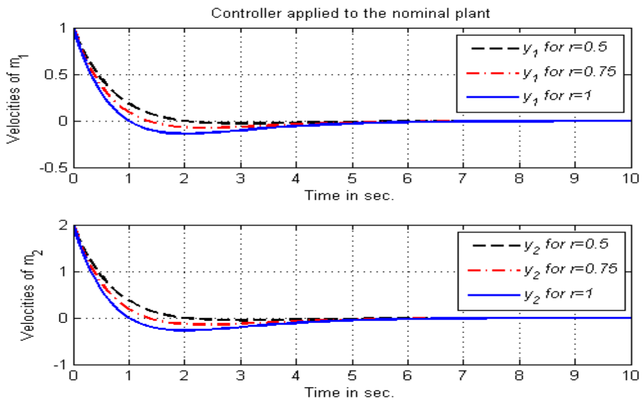

Figure 13. Velocities y_1 and y_2 of m_1 (top) and m_2 (bottom), respectively, for $r = 0.5$, $r = 0.75$ and $r = 1$ when the control is applied to the nominal plant.

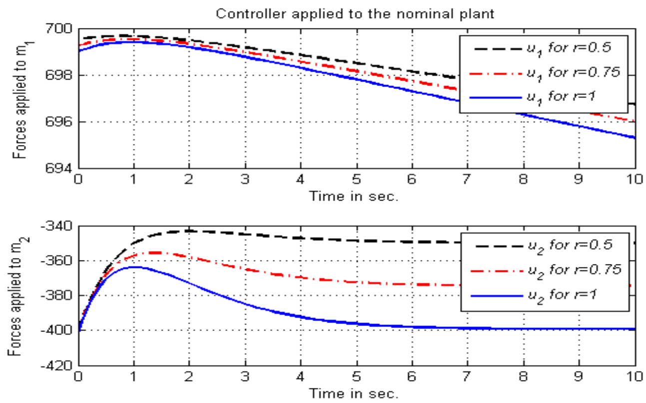

Figure 14. Applied forces u_1 and u_2 to m_1 (top) and m_2 (bottom), respectively, for $r = 0.5$, $r = 0.75$ and $r = 1$ when the control is applied to the nominal plant.

The controller is implemented in MatLab-Simulink using the feedback configuration of Fig. 4, and is applied to the two mass spring damper system that is shown in Fig. 7. The outputs are shown in figures 13 and 14 when the control is applied to the nominal plant and are compared in figures 15 and 16 when a large change of the parameters of $1.2A$ in the state matrix is realized. A state reference $x_d = 0$ and a state initial condition $x(0) = [\ 1 \quad 2 \quad 3 \quad 4\ ]^T$ are considered. In all the cases the values of $r = 0.5$ and $r = 0.75$ for a passive closed loop system, and the value of $r = 1$ for closed loop system at the limit of passivity, are compared. Values of $r > 1$ for active closed loop system are unstable and are not shown. Also, $K_a = \mathcal{I}$, $K_b = \mathcal{I}$ and $B_L = \left[\begin{array}{cc} \mathcal{I}_2 & G \end{array} \right]$ are considered, where $G = \left[\begin{array}{cc} 0 & 0.001 \\ 0.001 & 0 \end{array} \right]$ is close to zero and assigns a stable closed-loop characteristic polynomial. Smooth output responses are achieved in all the cases. Figures 13 and 14 show that when the control is applied to the nominal plant, the stationary state error of the velocities is decreased as r increase, until zero stationary state error is obtained for $r = 1$. The slope of the magnitude of the applied forces is increased as r increase.

Due to the small values of a and r, selected to achieve regulation and passivity, the magnitude of the plant input is 'small" in all the cases. However, robust performance requires bigger values of a. Figures 15 and 16 show that stability is preserved for $r = 0.5$, $r = 0.75$ and $r = 1$, despite the large change of parameters. This robust stability property is expected due to the passivity of the closed loop system. However, the outputs become more oscillatory and in the limit of passivity small numerical errors can lead to instability under large change of parameters.

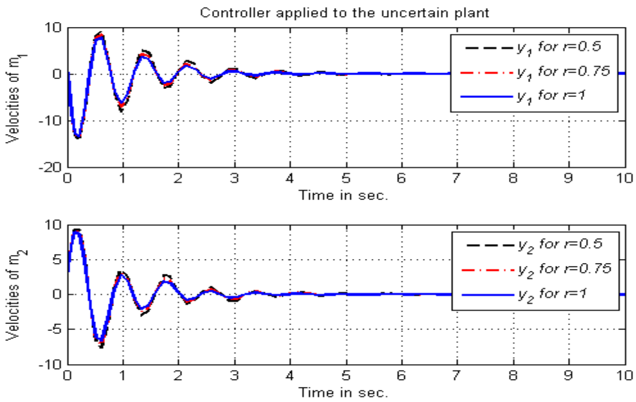

Figure 15. Velocities y_1 and y_2 of m_1 (top) and m_2 (bottom), respectively, for $r = 0.5$, $r = 0.75$ and $r = 1$ when the control is applied to the uncertain plant.

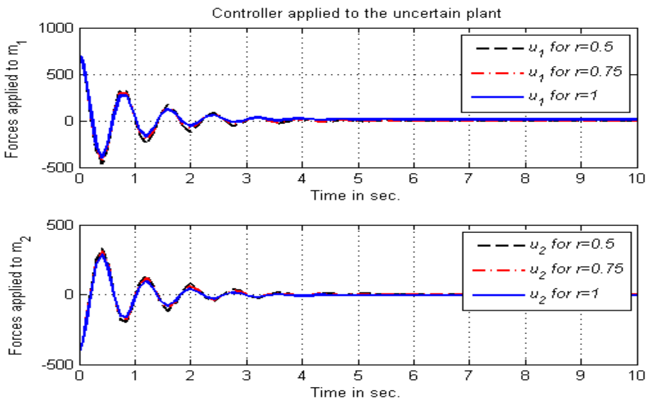

Figure 16. Applied forces u_1 and u_2 to m_1 (top) and m_2 (bottom), respectively, for $r = 0.5$, $r = 0.75$ and $r = 1$ when the control is applied to the uncertain plant.

6. Conclusions

Passivity-Based Control (PBC) design is proposed based on proposed pseudo-junction structures for the cascade and feedback interconnections and the multiport-coupled dissipative fields. The high or small resistances added to the bond graph model implies that certain terms of the matrix defining the relationship of the dissipative field, approach zero. Also, the added small capacitors and inductors to this model lead to a singularly perturbed model. From these unified representations of the closed loop system, conditions for passivity are determined from the passivity of the dissipative fields. It is shown that the method provides guidance in the choice of the structure of the controller and the assignment of relevant parameters. The result shows that the proposed PBC achieves robust stability. Applications of the results are given, when the plant and the controller are described by state-space realizations. In this case the overall system is not singularly perturbed. The results show that the passivity condition of the closed loop system allows to tune the control parameters when only power external sources after the interconnection are considered. An approximation of a derivative is proposed for control of output positions when the controller is designed for control of velocities. The results shows that the tracking control problem is solved when the controller is designed in the physical domain and the regulation control problem is solved when the plant and the controller are described by state space realizations. Moreover, the pole placement problem is considered for a particular class of systems using the proposed representation. The pseudo-junction structures in the representation of closed loop control system provide a framework for consideration of advanced control design in the physical domain. Optimal control or energy-based control can be tackled using this approach. Further investigations can be realized for tracking the output reference when the plant and the controller are described by state space realizations, for robust performance and for . extensions to non-linear systems.

Acknowledgements

This work was completed when the first author was an academic visitor at the University of Bath and was supported by the CONACyT Sabbatical project 000000000261575.

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Appendix A. Proof of Theorem 1

Proof. Energy may not be conserved when the subsystems are interconnected using active bonds. Since $u_b(t) = Ky_a(t)$ and by assumption (A_b, B_b, C_b, D_b) does not have load effect on (A_a, B_a, C_a, D_a), then from the outputs of S^a and S^b in equations (7) and (8), we have $\hat{S}^o_{ab}$ for the cascade interconnection,

$$\begin{aligned} \dot{x}(t) &= \hat{S}^o_{11} z(t) - \bar{D}_o(t) + \hat{S}^o_{13} u_a(t) \\ D_i(t) &= z(t) \\ y_b(t) &= \hat{S}^o_{31} z(t) + \hat{S}^o_{33} u_a(t) \end{aligned} \tag{1}$$

where

$$\begin{aligned} \bar{D}_o &:= \begin{bmatrix} \hat{D}^a_o(t) \\ \hat{D}^b_o(t) \end{bmatrix} = \begin{bmatrix} -A_a F_a^{-1} & 0 \\ 0 & -A_b F_b^{-1} \end{bmatrix} \textit{and} \\ \hat{S}^o_{11} &:= \begin{bmatrix} 0 & 0 \\ B_b K C_a F_a^{-1} & 0 \end{bmatrix}, \end{aligned} \tag{2}$$

that does not satisfy the structural properties of energy conservation, *i.e.*, $\hat{S}^o_{11}$ is not a skew-symmetric matrix. From the outputs of $\hat{S}^o_{ab}$ in Eq. (1), $z(t) = D_i(t)$. So, substituting $\bar{D}_o(t)$ given by Eq. (2) into Eq. (1), and using the definition of the coupled dissipative field given in Eq. (11), the result of Eq. (9) follows. Clearly, Eq. (9) satisfy properties P1 and P2 and thus is power conserving. Moreover, the system is passive whether the elements of the bond graph model must be such that the system be a power conserving physical system, that is, under Assumption 1 the stored and dissipated powers must satisfy,

$$\dot{x}^T(t)z(t) + D_i^T(t)\hat{D}_o(t) = v^T(t)u(t) \tag{3}$$

Since the inner structure $\hat{S}_{ab}^i$ is power conserving, the only element that may not be power conserving is the multiport-coupled dissipative field. Under Assumption 1 whether all the elements are passive then the system is passive according to the work of Beaman & Rosenberg (1988). Hence, the overall system is passive if $\int_0^t D_i^T(\tau)\hat{D}_o(\tau)d\tau > 0$, that is, from Eq. (11) ,$\int_0^t D_i^T(\tau)L_{ab}D_i(\tau)d\tau > 0$, and the condition of passivity follows. □

Appendix B. Proof of Theorem 2

Proof. Energy may not be conserved when the subsystems are interconnected using active bonds. Using $u_b(t) = K_b y_a(t)$, from the outputs of S^a and S^b in equations (20) and (21),

$$\begin{array}{l} \dot{x}(t) = \hat{S}_{11}^o z(t) + \hat{S}_{12}^o D_o(t) + \hat{S}_{13}^o u_a(t) \\ D_i(t) = z(t) \\ y_b(t) = \hat{S}_{31}^o z(t) + \hat{S}_{32}^o D_o(t) + \hat{S}_{33}^o u_a(t) \end{array} \tag{4}$$

where $D_o := \left[\begin{array}{cc} (D_o^a(t))^T & (D_o^b(t))^T \end{array} \right]^T$,

$$\begin{array}{l} \hat{S}_{11}^o := \left[\begin{array}{cc} S_{11}^a & 0 \\ S_{13}^b K_b S_{31}^a & S_{11}^b \end{array} \right] \textit{and} \\ \hat{S}_{12}^o := \left[\begin{array}{cc} S_{12}^a & 0 \\ S_{13}^b K_b S_{32}^a & S_{12}^b \end{array} \right], \end{array} \tag{5}$$

So, $u_a(t) = K_a(y_d(t) - y_b(t))$ is,

$$u_a(t) = \Psi K_a \left(y_d(t) - \hat{S}_{31}^o z(t) - \hat{S}_{32}^o D_o(t) \right) \tag{6}$$

Hence, an outer pseudo-junction structure $\hat{S}_{cl}^o$ for the feedback interconnection is,

$$\left[\begin{array}{c} \dot{x}(t) \\ D_i(t) \\ y_b(t) \end{array} \right] = \left[\begin{array}{ccc} \hat{S}_{11}^o - \hat{S}_{13}^o \Psi K_a \hat{S}_{31}^o & \hat{S}_{12}^o - \hat{S}_{13}^o \Psi K_a \hat{S}_{32}^o & \hat{S}_{13}^o \Psi K_a \\ \mathcal{I}_{n_a+n_b} & 0 & 0 \\ (\mathcal{I}_{p_b} - \hat{S}_{33}^o \Psi K_a)\hat{S}_{31}^o & (\mathcal{I}_{p_b} - \hat{S}_{33}^o \Psi K_a)\hat{S}_{32}^o & \hat{S}_{33}^o \Psi K_a \end{array} \right] \left[\begin{array}{c} z(t) \\ D_o(t) \\ y_d(t) \end{array} \right] \tag{7}$$

Since, $\mathcal{I}_{p_b} - \hat{S}_{33}^o \Psi K_a = \mathcal{I} - \left(\mathcal{I} + \hat{S}_{33}^o K_a\right)^{-1} \hat{S}_{33}^o K_a = \left(\mathcal{I} + \hat{S}_{33}^o K_a\right)^{-1}$, then, from the outputs of $\hat{S}_{cl}^o$ in Eq. (7), $z(t) = D_i(t)$. So, substituting $D_o(t) = LD_i(t)$ into Eq. (7 ,

$$\begin{array}{l} \dot{x}(t) = \left(\hat{S}_{11}^o - \hat{S}_{13}^o \Psi K_a \bar{S}_{31} + \hat{S}_{12}^o L \right) D_i(t) + \hat{S}_{13}^o \Psi K_a y_d(t) \\ D_i(t) = z(t) \\ y_b(t) = \left(\mathcal{I}_{p_b} + \hat{S}_{33}^o K_a \right)^{-1} \bar{S}_{31} D_i(t) + \hat{S}_{33}^o \Psi K_a y_d(t) \end{array} \tag{8}$$

Since $\hat{S}_{11}^o + \hat{S}_{12}^o L = -L_{ab}$ and using the definition of the coupled dissipative field given in Eq. (23), the result of Eq. (22) follows. Clearly, Eq. (22) satisfy properties P1 and P2 and thus is power conserving. Moreover, the system is passive whether the elements of the bond graph model must be such that the system be a power conserving physical system, that is, under Assumption 1, the only element that may not be power conserving is the multiport-coupled dissipative field. So, whether all the elements are passive then the system is passive according to the work of Beaman & Rosenberg (1988). Hence, the overall system is passive if $\int_0^t D_i^T(\tau)\hat{D}_o(\tau)d\tau > 0$, that is, from Eq. (23) the condition of passivity follows. □