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Dirty history versus clean expectations: Can energy policies provide momentum for growth?*

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Abstract

We study the impact of economic policy on the importance of history and expectations for the macroeconomic performance of an economy. In our model the energy mix is based on the conversion of heterogeneous energy sources. Markups over marginal costs are endogenous so that the marginal revenue product of capital becomes non-monotonic in capital. We derive multiple steady states and identify regions in which initial conditions are insufficient as a selection criterion for development. In these situations, pure expectations determine the equilibrium selection process which is crucial for long-run performance. Energy policy affects the interplay between history and expectations by shifting the region where expectations matter and by affecting the location of the equilibria in the dirty and the clean economy. We find that taxes and subsidies should be used simultaneously to guide an energy transition. We argue that expectations and momentum effects are important for the energy transition because they decrease policy costs and thus raise political acceptance.

Keywords:
Clean production, multiple equilibria, history vs. expectations, energy transformation, endogenous markups.

JEL Classification: Q43; O44; Q50; O11.

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1 Introduction

1.1 Transforming the energy system

World energy supply relies heavily on the use of fossil fuels. The associated side effects of global warming as well as regional and local air and water pollution strongly suggest transforming current energy systems. This can be done by reducing energy use, increasing energy efficiency, and promoting renewable and cleaner energies. Based on the decisions of the UN climate conferences, all countries will have to contribute to solving the global climate problem by lowering their carbon emissions. The European Union has decided the target of cutting per capita carbon emissions by 40 percent in 2030 against 1990 levels. The presidents of China and the US recently announced their CO2 abatement targets. For China it was stated that CO2 emissions will peak in 2030 and that the share of non-fossil fuels in primary energy should be 20% by then. For the US it was announced that it intends to reduce its CO emissions by 26-28% below its 2005 level in 2025.

Air quality has increasingly become a cause of major concern in many emerging economies. Prominently, in cities like China's capital Beijing, the effects of the extreme levels of air pollution on daily life can increasingly be seen in the form of health problems, deserted bike lanes, and people often staying at home or retreating to conditioned environments of hermetically-sealed malls. Accordingly, the transition to a less polluting energy sector and a cleaner and more sustainable economy is high on the political agenda.

But, what will be the impact of lower and cleaner energy use on the macroeconomic performance of an economy? Can appropriate policy help avoiding unfavorable income effects and how? There is widespread public concern that energy and climate policies cause major costs. Then, efficient environmental policies in the form of Pigovian taxes or pollution permits are difficult to implement politically. More optimistic analyses have highlighted the positive impact of new energy technologies on general productivity and economic dynamics. Therefore, subsidies for renewable energies and active technology policies can be suitable alternatives or at least complements to taxes and permits. But subsidies have to be financed by public funds which compete with other public needs and duties. As a consequence, situations in which environmental policies cause limited costs but have strong impacts on emission reduction and income growth appear to be especially desirable. Are such cases realistic? The literature distinguishes between history and expectations as determinants of an equilibrium selection process. If past development ("history") determines the transition to a long-run equilibrium, a shift to a new steady

state requires significant and potentially expensive policy interventions which might be hard to get approved by the political process. But if the equilibrium is determined by "expectations" of a cleaner future production, policy has only to be active in an initial phase. After that, induced investments may create speeding moments and policy-enforcing momentum might materialize. Hence, expectation-driven equilibria can support initial policy and lower the costs of the transformation of energy systems.

The paper at hand studies the macroeconomic effects of a policy-induced transition from dirty to clean production. Notably, we identify macroeconomic conditions under which expectations affect the trajectory chosen by market participants. We show how policy instruments are able to trigger development with sufficient momentum, fostering at the same time environmental quality, and increasing incomes. The cases in which long-run equilibria are driven by the history of production or by expectations are formally derived. Moreover, we study how economic policy affects the importance of expectations compared to history.

We present a generic macroeconomic model with capital accumulation and a detailed energy sector to study the interplay between policies and the multiplicity of equilibria. In the model, energy is not simply a homogeneous input but, closer to real conditions, an aggregate of heterogenous services. We derive how a policy promoting energy efficiency can, under certain conditions, generate broad momentum, moving an economy to a permanently higher activity and welfare level. This constitutes an especially attractive option for policy making.

1.2 Approach and findings

To analyze the transformation of the energy sector we assume that final output is produced by two types of intermediate input: dirty or clean. Dirty intermediates rely on capital and fossil energy services while clean intermediates employ capital and renewable energy services. Both intermediates are prefect substitutes and, initially, only the dirty sector is active.[3]

We incorporate a number of stylized facts into our model. First, the energy sectors feature characteristic elements of the industry. We assume that energy services are based on the conversion of heterogeneous energy sources like oil, coal, wind, solar, etc. which is done by specialized firms. Heterogeneity may result from specific attributes of each

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[3]It is a limiting case of a model with directed technical change where both technologies are active, which we consider to keep the model analytically and numerically tractable; a model version where both sectors are active simultaneously is available from the authors upon request.

energy source, such as fixed costs, supply intermittency, back-up capacity, and pollution intensity or by the specific supply conditions of the different firms like tariff structure and reliability. As a consequence of these heterogeneities, both the quantity and the variety of energy services have a productive value. Second, because energy services are incomplete substitutes for each other, they are supplied under the market form of incomplete competition. Energy producers can charge a markup over marginal cost. In equilibrium with free market entry, monopoly profits are used to cover the fixed costs. Third, capital productivity is determined by the variety of energy services and economic policy. Moreover, capital is accumulated endogenously by investment decisions of the firms. Fourth, following the standard Ramsey-Cass-Koopmans model, returns to capital are decreasing in the capital stock. However, this useful and broadly used approach is not consistent with the empirical observations of lacking convergence in per capita income (Barro, 1991), the absence of large cross-country differences in interest rates, and the failure of capital to flow from rich to poor countries (Lucas, 1990). Moreover, it cannot explain the deficiency of real wages to develop in a countercyclical fashion and the decrease of the capital share in the course of economic development. This is why we add a second main contribution to our framework, which is to incorporate results on endogenous markups from the IO literature. Specifically, we rely on Rotemberg and Woodford (1991, 1995), who find that mark-ups interact with business formation, and Jaimovich (2007, 2008) who analyses the interaction of markups with business cycles. Also, we build on the macroeconomic literature studying transitory behavior and the long-run performance of an economy using endogenous markups, see Gali (1994, 1995). Assuming that markups are endogenous and inversely related to the capital stock provides an attractive explanation for the different empirical observations mentioned above. Accordingly, we incorporate endogenous markups and thoroughly explore the impact of the assumption on the transformation of an economy from dirty to clean production. Fifth, to add another realistic element of capital accumulation, we posit that capital cannot be increased without any frictions i.e. we include capital adjustment costs in the model.

As a consequence of endogenous markups, the marginal revenue product of capital may become non-monotonic in the capital stock. Hence, the possibility of multiple steady states and multiple equilibrium paths for given initial conditions arises. Policies reducing (end-user) energy prices or diminishing production costs of energy services lead to decreasing markups and thus business creation. The main mechanism is that the competition on energy markets becomes more intensive with economic development. Energy policy then triggers the interplay between (dirty) history and (clean) expectations. Energy prices,

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energy use, pollution, and policy all affect capital productivity and, by this, the long-run performance of the economy as well as the equilibrium selection process. To the best of our knowledge, the present paper is the first to show how economic policy determines the relevance of history versus expectations within a dynamic general equilibrium framework comprising an accumulable stock.

Our framework distinguishes two regimes: A dirty regime in which the dirty technology is active i.e. the dirty energy inputs are used (regime D) and a clean regime (labeled by C) in which the clean technology is active i.e. the clean energy inputs are used. Each regime is characterized by multiple steady states. An exemplary phase diagram in the q/K space of the economy applying qualitatively to both regimes (D) and (C) is presented in Figure 1, with q representing the net present value of an additional unit of capital, K. We show below that the interior steady state is unstable while the two exterior steady states are saddle point stable. Now, in the neighborhood of the unstable steady state, the evolution of the economy may be subject to global indeterminacy in the sense that the trajectories leading to either the superior or the inferior steady state overlap such that expectations determine the equilibrium selection process. Outside the region of the overlap initial conditions (history) determine whether the inferior or the superior equilibrium is reached. Economic policy and the state of the economy, expressed by installed capital equipment, determine whether history or expectations shape the transition to the inferior or superior equilibrium. The reason is that energy prices and production costs of energy services affect factor productivity and thus the shape as well as the position of the $\dot{q}=0$-isocline. In our framework q relates to the net present value of one additional unit of capital. This implies that the selection of the superior (inferior) transition path must be associated to favorable (unfavorable) fundamentals in an admissible fashion. An entrepreneur has optimistic expectations if she expects that everybody else has optimistic expectations regarding the net-present value of additional capital equipment. In this sense our model shares the common feature of self-fulfilling prophecy equilibria which is coordination failure. If expectations matter, the state can align expectations and provide a momentum effect. In an extreme case, energy policy determines even the number of equilibria: if factor productivity is reduced (increased) drastically, only the inferior (superior) equilibrium may survive. Moreover, we show that energy policy affects the relevance of history versus expectations for the equilibrium selection process. Policy is thus not only determining transitory behavior of an economy but also its long-run performance.
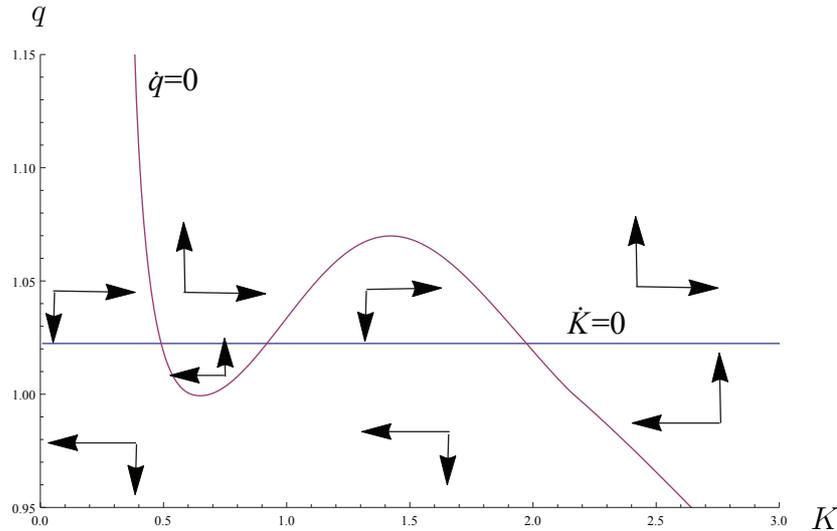
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Figure 1: Multiple steady states

1.3 Relation to the literature

The paper refers to the seminal contribution of Krugman (1991) who shows in a (partial-analytic) model, featuring external economies and adjustment costs, that history and expectations may both matter for economic development. He demonstrates that pure expectations may determine a market equilibrium under certain conditions, substantiating the notion of "self-fulfilling prophecy". This model result is contrary to most of the preceding literature, especially in environmental policy, where mainly past development ("history") sets initial conditions determining long-run steady state. The Krugman model derives in detail how the parameters of the economy separate the relative impact of history versus expectations. Schäfer and Steger (2014) extend this analysis for the equilibrium selection process in a setting with capital and labor mobility. We take up the fundamental concept and apply it to the energy transition. To introduce a potential role for expectations in our framework we rely on a strand of literature stressing that the intensity of market competition is not constant but depends on several determinants. Specifically, we follow Gali (1994, 1995) and the empirical literature cited therein by introducing markups for the intermediate goods producers which depend on the level of economic activity, given by the capital stock. We then show that, under certain conditions, an energy transition is able to trigger expectation-driven growth. For such a development one might use the term of Lucas (1993) to call it an 'economic miracle' or, even in a more moderate way, conclude that such a policy can have favorable effects for the economy turning it easier

to get it through the political process.

Four major differences distinguish our contribution from previous literature. First, we consider a dynamic general equilibrium setup with an accumulable stock which enables us to derive stable interior steady states; hence we do not have to restrict ourselves to corner solutions. Second, we do not rely on Marshallian externalities to generate the necessary scale effects. Third, we explicitly derive how economic policy affects both the emergence and the relevance of expectations for long-run development. Fourth, to the best of our knowledge, the paper is the first to introduce and isolate the effects of market expectations to the energy and climate literature. In fact, to derive the importance of green policies for a transition to a cleaner economy, Acemoglu et al. (2012) focus on the role of history. They use a two-sector model with directed technical change. Because learning effects are sector specific, history favors the larger sector which is usually the dirty sector of the economy. To change the pattern of development, policy is needed to give the green sector the decisive initial push. Closest to our contribution is Van der Meijden and Smulders (2016) which extends the approach of Acemoglu et al. (2012) by introducing expectations in a directed technical change framework. The main difference between their paper and ours is that they assume that changes in expectations stem from new outside information about technical opportunities, while - in our case - agents are fully informed about all the different technology options from the beginning.

While the literature acknowledges the relevance of both history and expectations for the prospects of economic development, the two topics are usually not analyzed within a common framework. To capture both within a single setup, two key ingredients are necessary: (1) multiple steady states for history and (2) indeterminacy for expectations. Regarding the latter, the literature distinguishes between local and global indeterminacy. Local indeterminacy refers to the existence of a multiplicity of equilibrium trajectories leading to a certain steady state. We argue that global indeterminacy is more relevant for the given topic, which can be explained by using again Figure 1. Assume the economy is initially located in a region around the interior (unstable) equilibrium, around which the trajectories to either (stable) steady state may overlap. Notably, the existence of such an overlap region requires that the superior (inferior) steady state is reached from a region to the left (right) of the unstable interior steady state. Expectations about the net present value of one unit of installed capital then trigger the selection of the trajectory to the superior or the inferior equilibrium. It is precisely the existence of this overlap region which gives rise to self-fulfilling prophecies and coordination failures.

In light of our research question, endogenous markups turn out to be a solid foundation for

our theoretical model. Following this approach, multiple steady states arise through non-monotonicities in the marginal product of capital while (global) indeterminacy emerges when the interior unstable equilibrium exhibits complex eigenvalues. We will develop the specific conditions for the emergence of the overlap region in the following. A special focus will be how economic policy can affect both the emergence and the size of the overlap region and hence the relevance of expectations. In this respect we depart from the earlier literature (Benhabib and Farmer 1994, 1996) which discusses the role of expectations within a framework of (local) indeterminacies by implementing externalities into the production function and abstracts from economic policies.[4] We do not want to challenge the models using externalities but have chosen to incorporate endogenous markups because of the microeconomic foundation, the empirical relevance, and the feasibility of interior steady states.[5]

With respect to the emergence of self-fulfilling equilibria our work is also related to the big-push literature of Murphy, Shleifer and Vishny (1989) and Rosenstein-Rodan (1943).[6] The self-determination of the equilibrium path and the implications of coordination failure in our approach are similar to this literature. But we have to stress that we do not model investment decisions which are affected by demand externalities. We rather follow Krugman (1991) by assuming that the selection of the self-fulfilling equilibrium path depends on expectations about the evolution of the co-state variable, i.e. the expected net return of an additional unit of capital. As regards the emergence of coordination failure, our work has certain parallels to Cooper and John (1988) who analyze interactions between the agents at the level of payoffs (spillovers) and "strategic complementarities", i.e. the interaction of strategies. Here, it is important to emphasize that we consider a continuum of agents such that each agent is of mass zero. Thus, only the aggregate of

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[4]Benhabib and Farmer (1994) showed that a necessary and sufficient condition for indeterminacy in a one-sector growth model is that externalities are large enough: an alternative path characterized by faster capital accumulation can materialize as a self-fulfilling equilibrium, if this path generates higher returns to capital supported by a reallocation of labor from leisure to production. The critical parameters are thus the magnitude of increasing returns and the elasticity of labor supply. Subsequent estimates have called the required magnitude of the parameters into question, see for example Basu and Fernald (1994). Benahbib and Framer (1996) provide a version of a standard real business cycle model with sector-specific rather than aggregate externalities that leads to indeterminacy for much smaller magnitudes of external effects.

[5]Using the assumption of externalities, Krugman (1991) and Schäfer and Steger (2014) abstract from accumulable stocks and do not derive interior steady states.

[6]The reason for the emergence of multiple equilibria in their model is the existence of a superior technology and aggregate demand externalities. The superior technology becomes active if aggregate demand is sufficiently high. Aggregate demand surpasses the critical threshold, however, only if everybody invests in this superior technology which is the case if each entrepreneur expects that everybody invests. In the opposite case, when expectations are pessimistic, the available technology is not implemented since it is not able to break-even as aggregate demand falls short of a critical threshold. In light of this theory, coordination failure leads to the selection of an inferior equilibrium while government action may align expectations.

individual actions will have repercussions on an individual agent but each agent alone is not able to affect the payoffs of other agents. In our framework there is, hence, scope for an interdependency of actions (Krugman, 1991) reflected by the size of the overlap, but the term strategic interaction would not be appropriate in our context.

To characterize capital accumulation we take up several important insights from different strands of literature. With new growth theory the importance of markups to provide incentives for innovation and growth has been stressed; in the seminal contribution of Romer (1990) markups determine profits and thus guide investors when inventing new intermediate goods varieties. Peretto and Smulders (2002) exploit the interaction between endogenous growth and the market structure to remove the scale effect of the Romer model. Similar to their approach we focus on the impact of market structure and derive long-run policy effects which do not rely on scale effects. Le Van et al. (2010) combine capital accumulation with non-renewable resource extraction and assume a convex–concave technology so that multiple equilibria arise; like them we study under which conditions a country will escape from an inferior equilibrium with low welfare but add pollution and a scope for expectations to the theory.

Confronting business cycle theory with actual data it was found in the industrial organization literature, see Rotemberg and Woodford (1991, 1995), that endogenous markups provide an attractive explanation of failure of real wages to be countercyclical while raw material prices are more procyclical than final goods prices. Our approach of medium-term development in the energy sector is in line with the finding of Jaimovich (2007, 2008) who found that mark-ups interact with business formation. It also builds on the DSGE-models making use of endogenous markups to analyze oil price shocks, e.g. Sanchez (2011). Empirical evidence further suggests that markups are countercyclical (Banerjee and Russell, 2004; Wilson and Reynolds, 2005; Jaimovich, 2006).

Looking at the longer run, it has been stressed that endogenous mark-ups drive both transitory behavior and the long-run performance of an economy (Gali, 1994;1995). In particular, using data of an international cross-section of countries, Gali (1994) finds empirical evidence for a significantly negative correlation between markups and per capita income. Gali (1994) also develops a theoretical model to show that markup variations caused by changing demand may have a significant impact on the growth path of imperfectly competitive economies: they can generate multiple steady states and multiple equilibrium paths for given initial conditions.

Following Gali (1995) we use the implications of endogenous markups for the dynamics

of capital accumulation, where the degree of competition increases with economic development. Specifically, in our model markups are inversely related to the level of economic activity, given by aggregate capital stock. Then, the marginal revenue product of capital may be non-monotonic in capital giving rise to the possibility of multiple steady states and different equilibrium paths for given initial conditions. Since this is a crucial building block of our theory, it is worth to elaborate a little bit more in detail on the reasoning of endogenous markups. The inverse relationship between markups and the aggregate capital stock implies that the elasticity of demand for differentiated intermediates is positively related to the capital stock (Gali, 1994;1995). An increase in the capital stock raises aggregate output and thus demand for differentiated intermediates as well as profits of each incumbent firm. The latter induces an entry of new firms, hence, more competition and lower markups. Moreover, the entry of new firms raises the range of available types of intermediates and increases the marginal product of physical capital if this effect overcompensates the decline owed to the initial increase in physical capital. This establishes the economic mechanism behind a non-monotonic revenue product of capital.

2 The framework

2.1 Households

We consider a Ramsey-type economy in which the utility function of households takes a standard CRRA form

$$U(t) = \int_0^\infty e^{-\rho t} \Big[\frac{c(t)^{1-\sigma} - 1}{1 - \sigma} - \frac{P(t)^{1-\varepsilon} - 1}{1 - \varepsilon} \Big] dt, \tag{1}$$

where c is consumption, P denotes pollution, ρ the discount rate, t the time index, ε represents the elasticity of marginal disutility from pollution and $1/\sigma$ is the intertemporal substitution elasticity. Each household supplies inelastically one unit of labor services, receives a wage income w and dividends from firms, π. Moreover, households can borrow and lend freely abroad at the world interest rate, r. Thus, the flow budget constraint of the representative household describing the change in economic wealth, b, reads

$$\dot{b} = w + \pi + rb - c. \tag{2}$$

2.2 Final output and pollution

For the production of final output, Y, we distinguish between two energy regimes: Regime (D) which is characterized by the use of a polluting technology, and regime (C) where only the clean technology is used. Pollution, P, is thus generated by the dirty technology and assumed to harm final output. Specifically, pollution adversely affects total factor productivity, provided that the output level of the dirty technology, Y_d, exceeds a critical threshold, Y_d^{crit}. If Y_d is below Y_d^{crit}, the level of flow pollution is set to its minimum level $\psi > 0$. Thus, pollution P is determined by

$$P = \begin{cases} \psi Y_d, & \text{if} & Y_d > Y_d^{crit} \\ \psi, & \text{if} & 0 \leq Y_d \leq Y_d^{crit}. \end{cases} \quad {}^{7} \tag{3}$$

In regime (D) final output production takes place according to

$$Y = P^{-\gamma} Y_d \tag{4}$$

with $\gamma \in (0,1)$. In (C), the dirty technology is inactive and

$$Y = \psi^{-\gamma} Y_c. \tag{5}$$

Y_j, $j = c, d$, is produced by a $[0,1]$-continuum of identical and fully competitive firms employing capital, K_j, and a range differentiated energy services, $\omega_j \in [0, N_j]$, with $x_j(\omega_j)$ denoting the quantity of intermediate ω_j and N_j representing the number of available differentiated energy services, in the clean or the dirty sector. The production function of a representative firm reads

$$Y_j = (K_j)^\alpha \left(\int_0^{N_j} x_j(\omega_j)^{\frac{1}{m_j}} d\omega_j \right)^{m_j(1-\alpha)}, \tag{6}$$

where $\alpha \in (0,1)$. In our framework, m_j is endogenous and determines the elasticity of substitution, $s_j = \frac{m_j}{m_j - 1}$, for each pair of intermediates and thus the markup over marginal production costs. Following Gali (1994, 1995) and the underlying empirical literature we assume that the intensity of competition between (intermediate) firms increases in a growing economy so that $s_j = s_j(\overset{+}{K_j})$. In particular we specify

$$s_j = \overline{\mu} + \underline{\mu} \cdot K_j^\kappa \tag{7}$$

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[7]Note also that we assume without loss of generality $Y_d^{crit} = 1$. We abstract from stock pollution and neglect the accumulative force of pollution, since we aim to reduce the number of state variables to a minimum. This enables us to provide graphical illustrations of our results in the two-dimensional space. In any case, we do not expect that our results would change qualitatively with stock pollution. We also refrain from introducing a separate abatement technology, which would e.g. represent the possibility of using carbon capture and sequestration. Depending on the efficiency of the abatement technology, a switch to the green regime could be delayed, but there would not be a substantial difference between regime (D) with significant abatement and our setting with a clean technology. Thus one could consider our case as the limiting case of complete abatement.

where $\overline{\mu}, \underline{\mu}, \kappa > 0$, implying that

$$m_j = m_j(K_j) > 0$$

$$m'_j(K_j) < 0.^8$$

Capital investment, I_j, is subject to a cost, C_{I_j}, given by the convex function

$$C_{I_j} = I_j\Big[1 + \theta\Big(\frac{I_j}{K_j}\Big)^\eta\Big], \qquad \eta,\ \theta > 0. \tag{8}$$

Denote the price of intermediate ω_j by $p_{x_j}(\omega_j)$, the instantaneous cash-flow of the representative firm at date t, $V(t)_j$, reads as

$$V(t)_j = p_j Y_j - \int_0^{N_j} p_{x_j}(\omega_j) x_j(\omega_j) d\omega_j - I_j\Big[1 + \theta\Big(\frac{I_j}{K_j}\Big)^\eta\Big].^9 \tag{9}$$

Recalling that the interest rate is given by r, firms maximize

$$\max_{\{x_j(\omega_j), I_j, K_j\}} \Big\{ \int_0^\infty e^{-rt}\, V(t)_j\, dt \Big\},$$

subject to

$$\dot{K}_j(t) = I_j(t) - \delta K_j(t), \tag{10}$$

such that optimal demand for energy services reads as[10]

$$x_j(\omega'_j) = \frac{(1-\alpha) p_j Y_j p_{x_j}(\omega'_j)^{\frac{m_j}{1-m_j}}}{\int_0^{N_j} p_{x_j}^{\frac{1}{1-m_j}} d(\omega_j)}. \tag{11}$$

Moreover, the optimal level of investments implies that

$$q_j = 1 + (1+\eta)\theta\Big(\frac{I_j}{K_j}\Big)^\eta, \tag{12}$$

where q_j denotes the shadow value of one additional unit of capital installed under technology j. The evolution of q_j is governed by

$$\dot{q}_j = (r+\delta)q_j - \Big[MPK_j - \frac{\partial C_{I_j}}{\partial K_j}\Big] = (r+\delta)q_j - \Big[\alpha\frac{Y}{K_j} + \theta\eta\Big(\frac{I_j}{K_j}\Big)^{1+\eta}\Big]. \tag{13}$$

Obviously, in the absence of capital adjustment costs ($\theta = 0$), the shadow value of capital is constant and equal to one, see (12), such that $\dot{q} = 0$ and the marginal product of capital takes the usual value $MPK_j = r + \delta$. In the presence of capital adjustment costs ($\theta > 0$), the marginal product of capital deviates from $r + \delta$ since investment costs have to be taken into account. Thus $\dot{q} > 0 (< 0)$ is owed to the fact that $(r+\delta)q$ is larger (smaller) than the net marginal product of one additional unit of capital, such that further increases (reductions) in the capital stock of the representative firm are indicated.

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[8]Note that $m_j(K_j) = \frac{s_j(K_j)}{s_j(K_j)-1}$. Thus, $m'_j(K_j) = -\frac{s'_j(K_j)}{(s_j(K_j)-1)^2}$ and $m'_j(K_j) < 0$ since $s'_j(K_j) > 0$. We need a specific functional form of s_j in order to generate closed form solutions but as we will clarify further below the qualitative results of our paper do not hinge on this specific form.

[10]See also A.1 in the Mathematical Appendix.

2.3 Energy services

Energy services are heterogeneous and produced under monopolistic competition. The production of $x_j(\omega_j)$ units of intermediate $\omega_j \in [0, N_j]$ is subject to fixed costs, ϕ_j, which capture the overhead cost in units of the intermediate for a firm to enter the market. The producer of intermediate variety ω_j uses energy $e_j(\omega_j)$ and labor $l_j(\omega_j)$ as inputs to produce a level of gross output $x_j(\omega_j) + \phi_j$. The level of net output of intermediate $x_j(\omega_j)$ is determined by

$$x_j(\omega_j) = B_j l_j(\omega_j)^{1-\beta} e_j(\omega_j)^{\beta} - \phi_j \tag{14}$$

with $\beta \in (0, 1)$ and $B_j > 0$ representing the total factor productivity in producing energy services. Denote marginal production costs by $c_{x_j}(\omega_j)$, operating profits write

$$\pi_{x_j}(\omega_j) = [p_{x_j}(\omega_j) - c_{x_j}(\omega_j)] x_j(\omega_j). \tag{15}$$

The first-order condition, $\frac{\partial \pi_{x_j}(\omega_j)}{\partial p_{x_j}(\omega_j)} = 0$, implies together with the profit maximizing demand for energy services (11) the usual relationship between prices and mark-up over marginal production costs

$$p_{x_j}(\omega_j) = m_j(K_j) \cdot c_{x_j}(\omega_j), \tag{16}$$

where the profit maximizing mark-up over marginal production cost of intermediates depends here inversely on aggregate capital, K_j, reflecting the level of economic activity under technology j, i.e. $m_j' < 0$. Dirty energy is imported while clean energy is produced domestically, according to the following linear production function

$$e_c(\omega_c) = B_{E,c} l_{E,c}(\omega_c), \tag{17}$$

with $B_{E,c} > 0$ representing the factor productivity in the production of green energy.[11]

2.4 Trade and prices

Regarding energy supply, the model incorporates the global dimension of the markets for (polluting) fossil fuels. As a matter of fact, the oil market is one of the most important and most globalized markets, gas and coal have also become very international in recent decades. Hence, (net) prices for fossil fuels are formed on the world market. Moreover, it is specific to the energy sector that most forms of energy services are either taxed or subsidized, with specific rates. In many countries, revenues of oil taxation are used

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[11]We abstract from physical capital in the production of clean energy for notational convenience. Of course, this assumption affects the opportunity costs of green technologies, but apart from that, the main arguments of our research remain unaffected.

to build transportation infrastructure or to finance general public expenditures. On the contrary, several types of (green) renewables are heavily subsidized by the government. In cross-country comparisons it is usually found that the variation of energy consumer prices are almost entirely explained by country-specific taxes and subsidies.

Accordingly, we assume energy prices (net of taxes) of fossils to be predetermined for a single country and consider the case of a small open economy where the energy price in the dirty regime, p_{E_d}, is exogenously given by world market conditions and domestic policy, i.e. we have

$$p_{E_d} = \bar{p}_{E_d} + \tau \tag{18}$$

where $\bar{p}_{E_d}$ is the world market price and τ the tax or subsidy of fossils set by policy.[12] Assuming the analogous small country assumption for the capital market leads to a world interest rate which is given for a single country, i.e. we have $r = \bar{r}$.[13]

3   Equilibrium

Without loss of generality we write expressions of aggregate intermediate goods in terms of average values. Notably, average cost, prices, and quantities of intermediate goods are written as $c_{x_j} = c_{x_j}(\omega_j)$, $p_{x_j} = p_{x_j}(\omega_j)$, and $x_j = x_j(\omega_j) \, \forall \, \omega_j$, while average energy productivity is B_j and average fixed costs ϕ_j, i.e. we abstract from heterogeneities in terms of fixed costs which allows to make use of the elegant properties of a symmetric equilibrium.[14] Free entry drives profits down to zero, such that

$$p_{x_j} x_j = c_{x_j}(x_j + \phi_j). \tag{19}$$

As $p_{x_j} = m_j \, c_{x_j}$, we obtain

$$x_j = \frac{\phi_j}{m_j - 1}. \tag{20}$$

Denoting further aggregate quantities of a variable z_j by $N_j z_j = Z_j$, the level of production of differentiated energy services is given by

$$N_j x_j = B_j E_j^\beta L_j^{1-\beta} - N_j \phi_j. \tag{21}$$

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[12]Regarding the clean regime, we will implement a subsidy to marginal production costs of clean energy services. This modeling strategy seems to be the most realistic fit.

[13]We exclude the existence of natural resources in the domestic economy for simplicity. In a small open economy the resource price is tight to global conditions. If resources were present domestically it would simply increase the present value of consumption due to an income effect. This would not change any of our results but possibly obscure the main mechanisms of our theory since then we would have to include sustainability aspects as well. Moreover, we would again increase the number of state variables which complicates both the presentation and the discussion of the results.

[14]Note that our framework still allows for differing fixed costs between clean and dirty energy services.

Substituting now for x_j by making use of (20), the available number of services compatible to technology j is obtained as

$$N_j = \frac{(m_j - 1) B_j E_j^{\beta} L_j^{1-\beta}}{\phi_j \, m_j}, \tag{22}$$

such that the level of Y_j reads as[15]

$$Y_j = (K_j)^{\alpha} \left(\frac{B_j E_j^{\beta} L_j^{1-\beta}}{m_j} \right)^{1-\alpha}. \tag{23}$$

According to (23), the output level grows with total factor productivity in the production of energy services and the inputs capital, energy, and labor, while a higher markup decreases final output. We normalize aggregate labor supply to unity, such that in regime (D)

$$L_d = 1$$

and in regime (C)

$$L_c + L_{E,c} = 1.$$

The energy price of fossils is determined according to (18) by the world market price and taxes. Consequently, in regime (D) profit maximizing behavior implies $p_{E_d} = \frac{\partial Y_d}{\partial E_d}$, such that aggregate demand for fossils is obtained as

$$E_d = \left[\frac{P^{-\gamma}\tilde{\beta}}{p_{E_d}} (K_d)^{\alpha} \left(\frac{B_d}{m_d} \right)^{1-\alpha} \right]^{\frac{1}{1-\tilde{\beta}}},$$

with $\tilde{\beta} = (1-\alpha)\beta$. Observing (3), the pollution level reads then

$$P = \psi^{\frac{1-\tilde{\beta}}{1-\tilde{\beta}(1-\gamma)}} \left(\frac{\tilde{\beta}}{p_{E,d}} \right)^{\frac{\tilde{\beta}}{1-\tilde{\beta}(1-\gamma)}} \left(\frac{B_d}{m_d} \right)^{\frac{1-\alpha}{1-\tilde{\beta}(1-\gamma)}} K_d^{\frac{\alpha}{1-\tilde{\beta}(1-\gamma)}} \tag{24}$$

and the level of final output in regime (D) including pollution is obtained as[16]

$$Y = \tilde{A}_d \left(\frac{B_d}{m_d} \right)^{\frac{(1-\alpha)(1-\gamma)}{1-\tilde{\beta}(1-\gamma)}} K_d^{\frac{\alpha(1-\gamma)}{1-\tilde{\beta}(1-\gamma)}}, \tag{25}$$

where $\tilde{A}_d = \psi^{\frac{-\gamma}{1-\tilde{\beta}(1-\gamma)}} \left(\frac{\tilde{\beta}}{p_{E,d}} \right)^{\frac{\tilde{\beta}(1-\gamma)}{1-\tilde{\beta}(1-\gamma)}}$. In regime (C), aggregate energy supply is produced domestically according to (17). The labor market equilibrium implies[17]

$$L_{E,c} = \beta \tag{26}$$

$$L_c = 1 - \beta. \tag{27}$$

---

[15]In order to ease the notation, we modified the production function of final output slightly, in the sense that now $Y_j = (K_j)^{\alpha} \left[N_j^{1-m_j} \left(\int_0^{N_j} x_j(\omega_j)^{\frac{1}{m_j}} d\omega_j \right)^{m_j} \right]^{1-\alpha} = (K_j)^{\alpha} \left[N_j^{1-m_j} N_j^{m_j} x_j \right]^{1-\alpha} = (K_j)^{\alpha}(N_j x_j)^{1-\alpha}$, which is a standard procedure in literature, see Jaimovich (2007).

[16]Note that $Y_d = \psi^{\frac{-\gamma\tilde{\beta}}{1-\tilde{\beta}(1-\gamma)}} \left(\frac{\tilde{\beta}}{p_{E,d}} \right)^{\frac{\tilde{\beta}}{1-\tilde{\beta}(1-\gamma)}} \left(\frac{B_d}{m_d} \right)^{\frac{(1-\alpha)}{1-\tilde{\beta}(1-\gamma)}} K_d^{\frac{\alpha}{1-\tilde{\beta}(1-\gamma)}}$.

[17]See A.2 in the Appendix.

As the pollution level is fixed to its minimum possible value, ψ, we obtain[18]

$$Y = \tilde{A}_c \left(\frac{B_c}{m_c} \right)^{1-\alpha} K_c^\alpha, \tag{28}$$

with $\tilde{A}_c = \psi^{-\gamma} \left[(B_{E,c}\beta)^\beta (1-\beta)^{1-\beta} \right]^{1-\alpha}$. Like in the other cases, output is increasing in capital and total factor productivity, $\tilde{A}_j$. The latter will become central because policy affects through variations in τ, $\tilde{A}_d$ and by this final output.[19] Households maximize (1) subject to (2) and take $\bar{r} = \rho$ as given, such that the level of consumption is obtained by

$$\int_0^\infty c(t) e^{-\rho t} dt = \int_0^\infty [w + \pi] e^{-\rho t} dt + b_0 \tag{29}$$

$$= \int_0^\infty e^{-\rho t} \left[\mathcal{I}_c(\psi^{-\gamma} Y_c - C_{I_c}) + (1 - \mathcal{I}_c)[(P^{-\gamma} Y_d - C_{I_d}) - \bar{p}_{E,d} E_d] \right] dt$$
$$+ b_0 \tag{30}$$

$$= \nu_0, \tag{31}$$

with $b_0 \lessgtr 0$ representing the initial value of wealth or debts, $Y - C_{I_j}$ represents net-output of technology j, and $\bar{p}_{E,d} E_d$ the import value of fossils. Moreover, $\mathcal{I}_c$ is an indicator variable, with $\mathcal{I}_c = 1$ if the clean technology is applied and zero otherwise. Apparently, the present value of consumption equals the present value of net output.[20] Integrating the left-hand side and noting that $\rho = \bar{r}$, we obtain

$$c(t) = \bar{r} \, \nu_0. \tag{32}$$

Eq. (32) says that consumption is constant with given ν_0, which - according to Eq. (29) - materializes when the regime is determined and no policy change affecting the output is implemented. Energy policy e.g. in the form of taxes on dirty energy or subsidies on clean energy alters total factor productivity inducing a permanent change in net output, consumption, and welfare. As welfare is negatively affected by pollution, a switch from the dirty to the clean regime ceteris paribus increases welfare in the economy.

4 Steady states

In this section, we present conditions for the emergence of multiple steady states and the corresponding stability properties.

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[18] Where $Y_c = \left[\frac{B_c}{m_c} (B_{E,c}\beta)^\beta (1-\beta)^{1-\beta} \right]^{1-\alpha} K_c^\alpha$.

[19] A subsidy of green energy services will change $N_c x_c$ and thus affect $\frac{B_c}{m_c}$ directly through the subsidy and indirectly through the change in the marginal productivity of capital, and thus m_c, but this is qualitatively equivalent to variations in $\tilde{A}_c$ as will become clear further below.

[20] After taking account for the import value of fossils in the dirty regime, of course.

Since the level of consumption is time invariant, see Eqs. (29) and (32), the dynamics of the economy is, due to the existence of convex capital adjustment cost, solely driven by the evolution of the capital stock K_j and its shadow price q_j

$$\dot{q}_j = (\bar{r} + \delta)q_j - \left[MPK_j + \theta\eta\left(\frac{I_j}{K_j}\right)^{1+\eta} \right] \tag{33}$$

$$\dot{K}_j = I_j - \delta K_j, \tag{34}$$

with $j = c, d$ and the marginal product of capital MPK reading

$$MPK_d = \alpha\tilde{A}_d\left(\frac{B_d}{m_d}\right)^{\frac{(1-\alpha)(1-\gamma)}{1-\tilde{\beta}(1-\gamma)}} K_d^{\frac{\alpha(1-\gamma)}{1-\tilde{\beta}(1-\gamma)}-1}, \tag{35}$$

$$MPK_c = \alpha\tilde{A}_c\left(\frac{B_c}{m_c}\right)^{1-\alpha} K_c^{\alpha-1}. \tag{36}$$

In steady state, labeled by *, we have $\dot{K}_j = \dot{q}_j = 0$, such that investments equal the amount of depreciated capital

$$I_{j,*} = \delta K_{j,*}. \tag{37}$$

Eq. (37), together with optimal investment decisions obtained from (12), implies that

$$q_{j,*} = 1 + (1+\eta)\theta\delta^\eta \tag{38}$$

which is obviously independent from K_j.[21] On the other hand, $\dot{q}_j = 0$ determines, in light of (33), $q_{j,*}$ implicitly as a function of K_j

$$q_{j,*} = \frac{MPK_{j,*} + \theta\eta\left(\frac{q_{j,*}-1}{(1+\eta)\theta}\right)^{\frac{1+\eta}{\eta}}}{\bar{r} + \delta}. \tag{39}$$

Moreover, as $m_j = m_j(K_j)$, with $m_j'(K_j) < 0$, the marginal product of capital is not necessarily declining in K_j. Thus, $q_{j,*}$ implicitly defined by (39) may intersect (38) more than once in the $\{K_j; q_j\}$-plane, which gives rise to multiple steady states. The following proposition characterizes the conditions for the emergence of multiple steady states.[22]

Proposition 1

(i) *Exogenous mark-ups: if m_j is constant and independent from K_j, there exists a unique saddle-point stable steady state.*

(ii) *Endogenous mark-ups: if $m_j = m_j(K_j)$, and $\kappa > 1$*

(a) *$\lim_{K\to 0} MPK_j = \infty$ and $\lim_{K\to\infty} MPK_j = 0$.*

---

[21]The derivative of the Hamiltonian of a representative firm with respect to I_j implies $q_j = 1 + (1 + \eta)\theta\left(\frac{I_j}{K_j}\right)^\eta$. For further details we refer to the Appendix.

[22]For a proof see Appendix A.4.

(b) *The necessary condition for multiple steady states is*

$$(\alpha\tilde{\gamma}_j - 1)\frac{m_j}{K_j} = \tilde{\gamma}_j(1-\alpha)\frac{\partial m_j}{\partial K_j}, \tag{40}$$

with $\tilde{\gamma}_d = \frac{(1-\gamma)}{1-\tilde{\beta}(1-\gamma)}$ *and* $\tilde{\gamma}_c = 1$.

The parameter κ, reflecting the intensity of the impact of capital on the substitution elasticity, see Eq. (7), governs the shape of the $\dot{q}_j = 0$ locus in the $\{K_j; q_j\}$-plane. Given item (ii),(b) of the above proposition, the necessary condition for the emergence of three steady states, together with (a), is $\kappa = 2$.[23] The emergence of multiple steady states stems from the inverse relationship between the number of intermediate firms and markups and the requirement that this relationship overcompensates the decline in the marginal productivity of physical capital in response to an increase in K_j. Therefore it is important to note that our qualitative results do not hinge on the specific functional form of s_j as specified by Eq. (7).[24]

From Eq. (12) and Eq. (38), we observe that $q_j \gtreqless q_{j,*}$ implies $(I_j/K_j) \gtreqless \delta$, such that in light of Eq. (34), $\dot{K}_j \gtreqless 0$. This situation is reflected by the horizontal vector arrows in Figure 1. Above (below) the $\dot{q}_j = 0$-isocline determined by Eq. (33), the sum of the marginal product of capital and marginal installation cost, $MPK_j + \theta\eta\left(\frac{I_j}{K_j}\right)^{1+\eta}$, is for a given q_j below (above) the level that would assure $\dot{q}_j = 0$. Thus for the region above the $\dot{q}_j = 0$-isocline it follows that $\dot{q}_j > 0$ while below it we have $\dot{q}_j < 0$. In Figure 1, this situation is characterized by the vertical arrows. We therefore conclude that the case of three steady states in the figure is characterized by two exterior saddle-point stable steady states and one interior unstable steady state. In the dirty regime, the steady state with the highest capital stock implies the highest value of net output and consumption given any level of initial wealth. However, as utility is negatively affected by pollution, highest output does not imply highest welfare; it depends on the shape of the utility function,

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[23]With $\kappa = 2$ our model generates, depending on the scenario, a markup of around 1.5 at the superior steady state which is above average markups estimated for example by Norrbin (1993) implying a range between 1.05 and 1.4. If we acknowledge that these values represent average estimates and if we look at services and the energy sector alone we find that markups are usually higher than the above interval. Christopoulou and Vermeulen (2012) estimate for services a range of 1.26 for France to 1.87 for Italy and an average of 1.56 for the Euro area. Molnar and Bottini (2008) estimate for the energy sector a range of 1.16 for Denmark to 1.93 for Finland and for a small group of countries like France even a value of 2.8. Thus $\kappa = 2$ lies in the empirically relevant range. Note that we choose the specific parametrization only to illustrate the results in an intuitive way. The aim of this paper is not a quantitative analysis of policy shocks.

[24]Obviously, a functional form different from eq.(7) would just change parameter restrictions and/or impede analytical closed form solutions but not change the reasoning of our paper. Thus, the results in our paper as well as in Gali (1994,1995), depend on the presence of a negative relationship between the number of firms and equilibrium markups, but neither on the particular functional form nor a particular mechanism chosen to generate this relationship, see Gali 1995, p. 41 and Gali (1994) for a version with markups depending on the savings rate.

specifically on the parameter ε. This is the main difference to the clean regime, where highest net output and consumption directly translate into highest welfare.

5   Energy transition

We now show how expectations can provide momentum for a policy induced transition from a polluting to a clean technology. In the next subsection, we discuss the importance of history versus expectations for the equilibrium selection process and how economic policy affects the relevance of expectations. Then, we discuss two conventional policy instruments, a tax on fossils and a subsidy on clean energy services, that may induce a speedy transition from dirty to clean technologies.

5.1   History, expectations, and policy

We assume that the economy starts in regime (D). The clean technology is known but not active, which may be due to a lower productivity in the production of energy services. For the sake of brevity we will focus the discussion on regime (D); the analysis of regime (C) is qualitatively identical.[25] The switch from the dirty to the clean regime will be the subject of the following subsection. The level of final output is given by (25) and the dynamics of the economy is governed by (33) and (34) with $j = d$. Whether expectations play a role for the model solution depends on the existence of a range of initial states expressed by the capital stock, for which the saddle point trajectories to the exterior steady states overlap. If such a range of states exists, knowledge about the initial state of the economy is insufficient to select the relevant trajectory for economic development. Then, the equilibrium selection process is driven by expectations about the value of the co-state variable, q_d. Outside the overlap region the equilibrium selection is purely driven by initial conditions, the usual situation in most economic models. In our case, the overlap consists of a range of capital stocks $K_d \in [\underline{K}_d; \overline{K}_d]$ in the neighborhood of the interior steady state. With favorable expectations, the superior steady state can be reached from a region lying to the left of the unstable steady state. Conversely, there is the risk that the inferior steady state may be reached, with pessimistic expectations, from a region to the right of the interior steady state. Technically, the emergence of the overlap depends on whether or not the vector field allows for a transitions from the left (right) of the interior steady state to the superior (inferior) steady state. Therefore, the existence of the overlap

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[25]Note again, that economic policy affects $\tilde{A}_d$ in regime (D) and $\frac{B_c}{m_c}$ in regime (C). The latter is qualitatively similar to a change in total factor productivity.
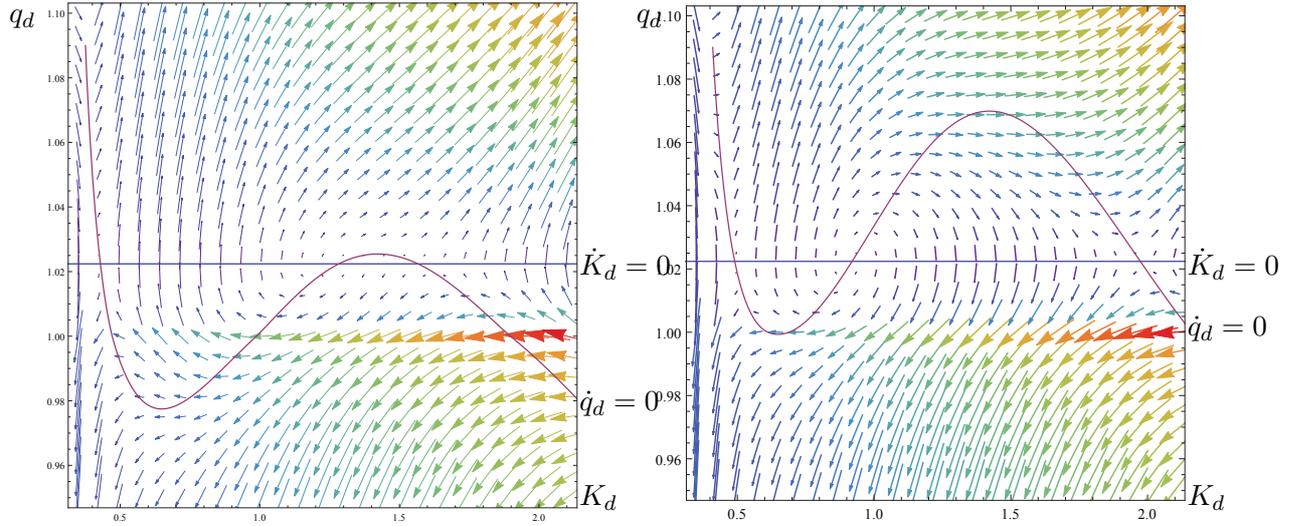
Figure 2: Vector fields for real eigenvalues at the interior steady state (left-hand side) and complex conjugate eigenvalues (right-hand side).

is tied to the emergence of complex conjugate eigenvalues at the interior steady state. We present an illustration of this argument in Figure 2. The left-hand panel is characterized by real eigenvalues at the interior equilibrium such that the existence of an overlap region becomes infeasible. If, in contrast, the interior steady state exhibits complex conjugate eigenvalues, the vector field illustrates that a transition to either equilibrium becomes feasible in a region around the unstable steady state. We provide more details on this issue below.

At this point it is interesting to note that the emergence of both central model elements, multiple steady states and an overlap region seems to depend on the position of the $\dot{q} = 0$-isocline and even more important: The position of this isocline and the role of expectations can be influenced by economic policies. In the following proposition, we summarize how both, the location of the $\dot{q} = 0$-isocline and the role of expectations depend on the total factor productivity of technology j $(\tilde{A}_j)$.

Proposition 2

(i) *An increase (decline) in total factor productivity $\tilde{A}_j$ moves the $\dot{q}_j = 0$-isocline in the $\{K_j; q_j\}$-plane upwards (downwards).*

(ii) *If $\kappa = 2$ there are three steady states, if $\tilde{A}_j \in [\tilde{A}_{j,min}, \tilde{A}_{j,max}]$. If $\tilde{A}_j < \tilde{A}_{j,min}$ only the inferior steady state survives. If $\tilde{A}_j > \tilde{A}_{j,max}$ only the superior steady state survives. In both cases the dynamics of the economy are driven by initial conditions.*
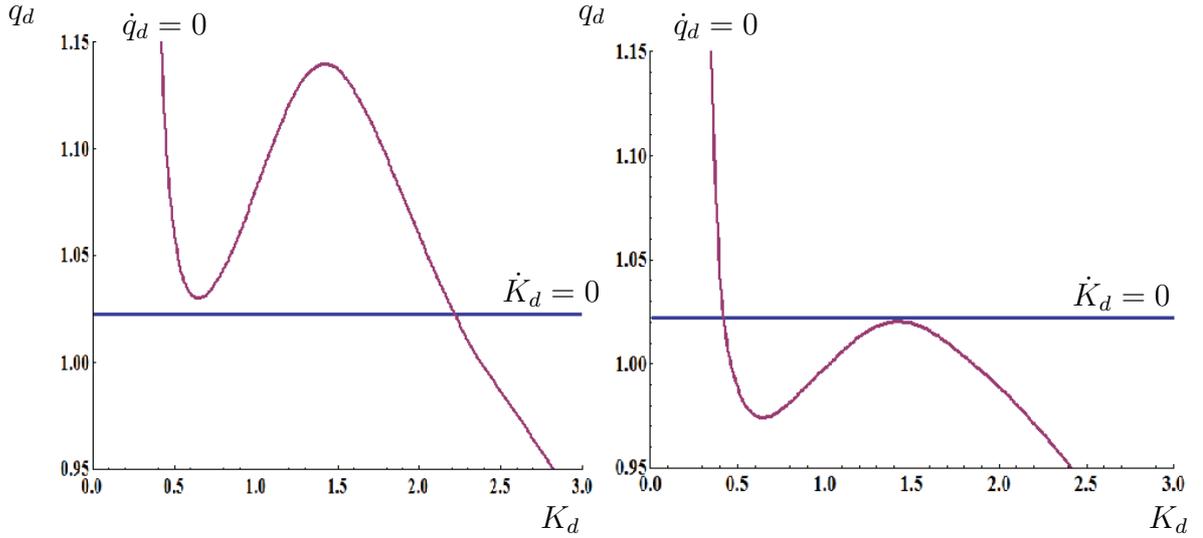
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Figure 3: Variation in fossil energy prices, $p_{E,d}$, through a decline in taxes , τ: left-hand side versus an increase: right-hand side.

(iii) *If $\tilde{A}_j \in [\tilde{A}_{j,min}, \tilde{A}_{j,max}]$ and $\tilde{A}_j \leq \tilde{A}_{j,crit}$ the interior steady state exhibits real eigenvalues and complex conjugate eigenvalues if $\tilde{A}_j > \tilde{A}_{j,crit}$.*

In Figure 3, we illustrate items (i) and (ii) of the above proposition, i.e. the effect of an increase or a decline in energy prices through a variation in taxes per unit of fossil energy. Noting Eq. (35), it becomes apparent that a change in energy prices alters the marginal product of capital. Then, given (39), an increase in energy prices shifts the $\dot{q}_d = 0$-locus downwards while a decline shifts it upwards. For sufficiently strong variations in energy prices, the multiplicity of steady states vanishes such that the economy transits to a steady state characterized by either a low or a high level of installed capital and thus aggregate output.

Item (iii) is illustrated by Figure 2. If $\tilde{A}_d$ is such that the existence of multiple steady states is assured, i.e. $\tilde{A}_j \in [\tilde{A}_{j,min}, \tilde{A}_{j,max}]$, it is not guaranteed that expectations matter. This is only the case, if in addition $\tilde{A}_j$ exceeds within this range a critical threshold $\tilde{A}_{j,crit}$ meaning that $\tilde{A}_{j,min} < \tilde{A}_{j,crit} < \tilde{A}_j \leq \tilde{A}_{j,max}$. This result is important since economic policy triggers in both energy regimes (i) the number and the position of steady states, and (ii) the existence of an overlap region. Moreover, we will show below that not only the existence of an overlap region but also its size will be determined by economic policy

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in a systematic fashion, affecting the risks and chances to converge to the exterior steady states.

Before analyzing the impact of policy we discuss the overlap region and the role of expectations in more detail. The existence of an overlap region is illustrated in Figure 4, where the interior steady state exhibits two complex conjugate eigenvalues with positive real parts. Obviously, there exists a range of capital stocks $K_d \in [\underline{K}_d; \overline{K}_d]$, in the neighborhood around the interior steady state, where the saddle-point trajectories to either steady state overlap. Thus, the equilibrium selection process within this region is entirely driven by expectations and outside the overlap by history. The detection of this region requires the solution of a non-linear system of differential equations and thus the application of numerical methods. Here, we employ the Relaxation algorithm and search the lowest feasible $\underline{K}_d$ as an initial point for the transition to the superior steady state and the highest feasible $\overline{K}_d$ as an initial value for the saddle-point trajectory to the inferior steady state.[26]

As regards the determination of expectations several remarks are at order. Expectations are reflected by the selection of the costate variable, q_j, which is by no means arbitrary but related to fundamental characteristics of the economy. This can be seen very clearly by integrating (33) for $j = d$

$$q_d = \int_0^\infty \left[MPK_d - \frac{\partial C_{I_d}}{\partial K_d} \right] e^{-\int_0^\tau (\bar{r}+\delta)ds} d\tau. \tag{41}$$

Obviously, q_d relates to the present value of the marginal product of capital minus the change in installation costs, i.e. the net present value of one additional unit of capital. This implies that the selection of the superior (inferior) transition path must be associated to favorable (unfavorable) fundamentals in an admissible fashion. An entrepreneur has optimistic expectations if she expects that everybody else has optimistic expectations regarding the net-present value of additional capital equipment. In this sense our model shares the common feature of self-fulfilling prophecy equilibria which is coordination failure. If expectations matter, the state can align expectations and provide a momentum effect. The important aspect here is, again, that expectations relate to fundamentals of

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[26]We employ the Relaxation algorithm since this method is numerically the most efficient one in order to detect a transition path from a certain initial point, i.e. $K_{j,0}$ given, to a steady state characterized by $(q_{j,*}; K_{j,*})$. In theory we could also use for example backward integration, see Brunner and Strulik (2002). This procedure exhibits drawbacks if the dynamic system is stiff. Moreover, to find a certain trajectory that fulfills all initial problems, an iteration process is required which typically gives rise to problems of convergence reenforced by the fact that we increase the distance to the steady state. Applying the Relaxation algorithm avoids this shortcomings and leads to a numerical error, if the initial capital stock is not part of the trajectory to the steady state under consideration. Clearly, linearization methods do not deliver such a precise criterion of exclusion. For more details on the Relaxation algorithm, see Trimborn et al. (2008).
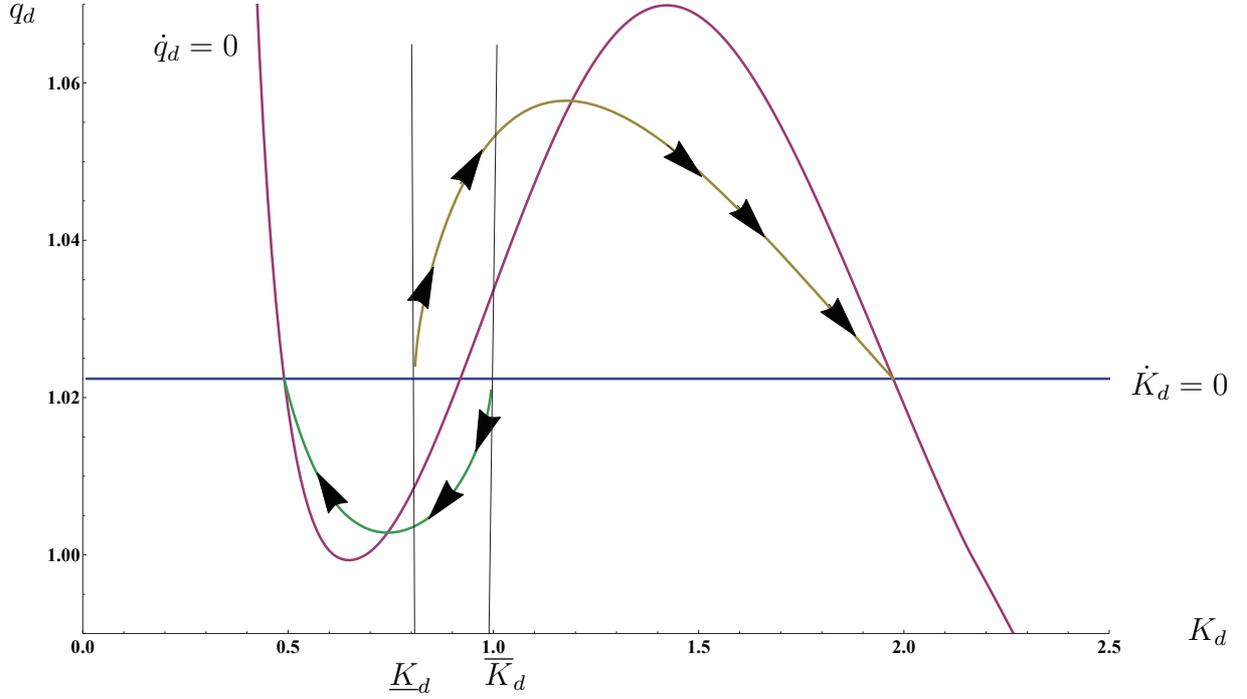
Figure 4: History versus expectations

the economy!

For the economic intuition behind eq.(41) it is important to see that this equation is conceptually interlinked with the well known no-arbitrage condition implied by an asset market equilibrium: suppose an owner of one unit of capital equipment earns a stream of dividends $\pi(t)^{net}$. In addition to this claim, the unit of capital is subject to gains and losses, expressed by $\dot{q}$. Investors are willing to hold the claim, if the total return equals the return of a save consumption loan of size q, such that the following arbitrage condition has to be satisfied $\pi(t)^{net} + \dot{q}(t) = \hat{r}q(t)$, which immediately implies that

$$\dot{q}(t) - \hat{r}q(t) = -\pi(t)^{net}. \tag{42}$$

Integrating the last equation and noting that $\lim_{t\to\infty} q(t)e^{-\int_0^\tau \hat{r}\, ds} = 0$ yields[27]

$$q(0) = \int_0^\infty e^{-\int_0^\tau \hat{r}\, ds}\pi(t)^{net}dt, \tag{43}$$

where in our case: $\hat{r} = \bar{r} + \delta$ and $\pi(t)^{net} = \left[MPK_j - \frac{\partial C_{I_d}}{\partial K_d}\right]$, such that we obtain again to (41). The time paths of installed capital, K_d, and its shadow price, q_d, are depicted in Figure 5. There, we present the transition to the superior (inferior) steady state in the upper (lower) part of the figure. In light of (41), q_d reflects the present value of the net return of one additional unit of installed capital. Thus, the transition to the superior steady state requires that q_d is above its long-run value $q_{d,*}$ which was just sufficient to

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[27]$\lim_{t\to\infty} q(t)e^{-\hat{r}t} = 0$ follows from the existence of a steady state.
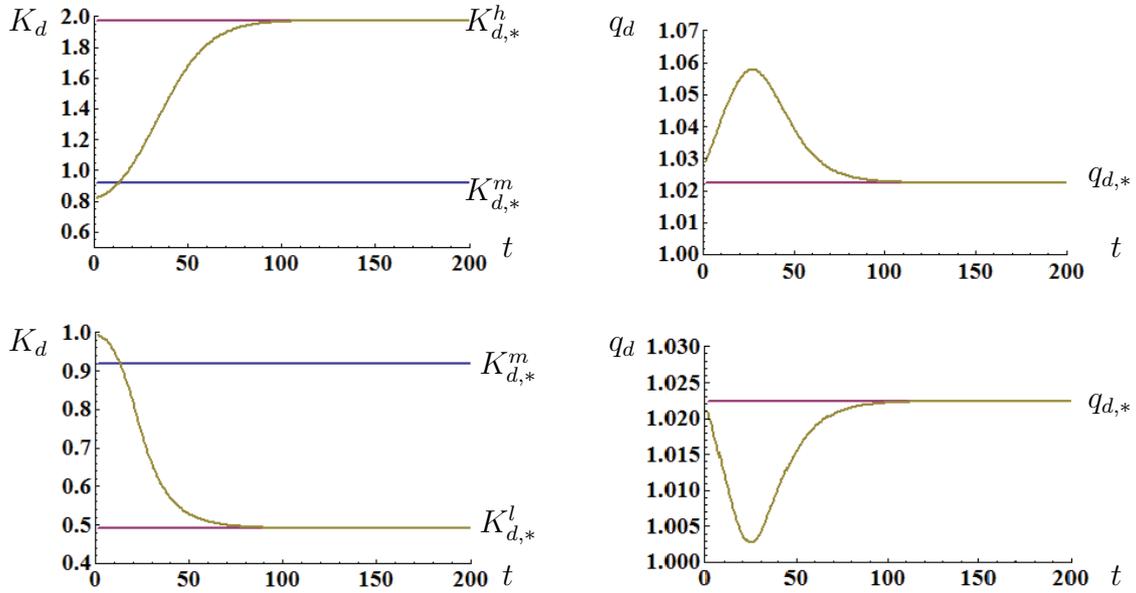
Figure 5: Time paths of K_d and q_d to the superior steady state (upper panel) and the inferior steady state (lower panel)

sustain the long-run level of installed capital, $K_{d,*}$ by guaranteeing that $I_{d,*} = \delta K_{d*}$. Hence, $q_d > q_{d,*}$ induces $I_d > I_{d,*}$ and thus $\dot{K}_d > 0$. Symmetrically, $q_d < q_{d,*}$ implies $I_d < I_{d,*}$ and thus $\dot{K}_d < 0$ (see the lower panel of Figure 5).

From Eq. (24) we know that the pollution level is increasing in K_d. Thus the steady state with the highest level of installed capital generates the highest level of pollution and is associated with the highest level of (net-) output, see (25). Because of our welfare function Eq. (1) where pollution enters negatively, maximum output does not imply maximum welfare, however. The switch to the green technology is beneficial if the adverse effects of pollution on TFP and welfare are strong and the import value of fossils is high.

Based on Propositions 1 and 2, we summarize the effects of policy on the energy transition in the following corollary.

Corollary 1

(i) *Economic policy is decisive for the emergence of multiple steady states and the importance of expectations in the equilibrium selection process.*

(ii) *In regime (D), economic policy affects total factor productivity through pollution taxes, τ. In regime (C), total factor productivity is improved by production subsidies on energy services which reduce markups.*

(iii) *The simultaneous implementation of taxes and subsidies provides the best conditions for reaching highest welfare.*
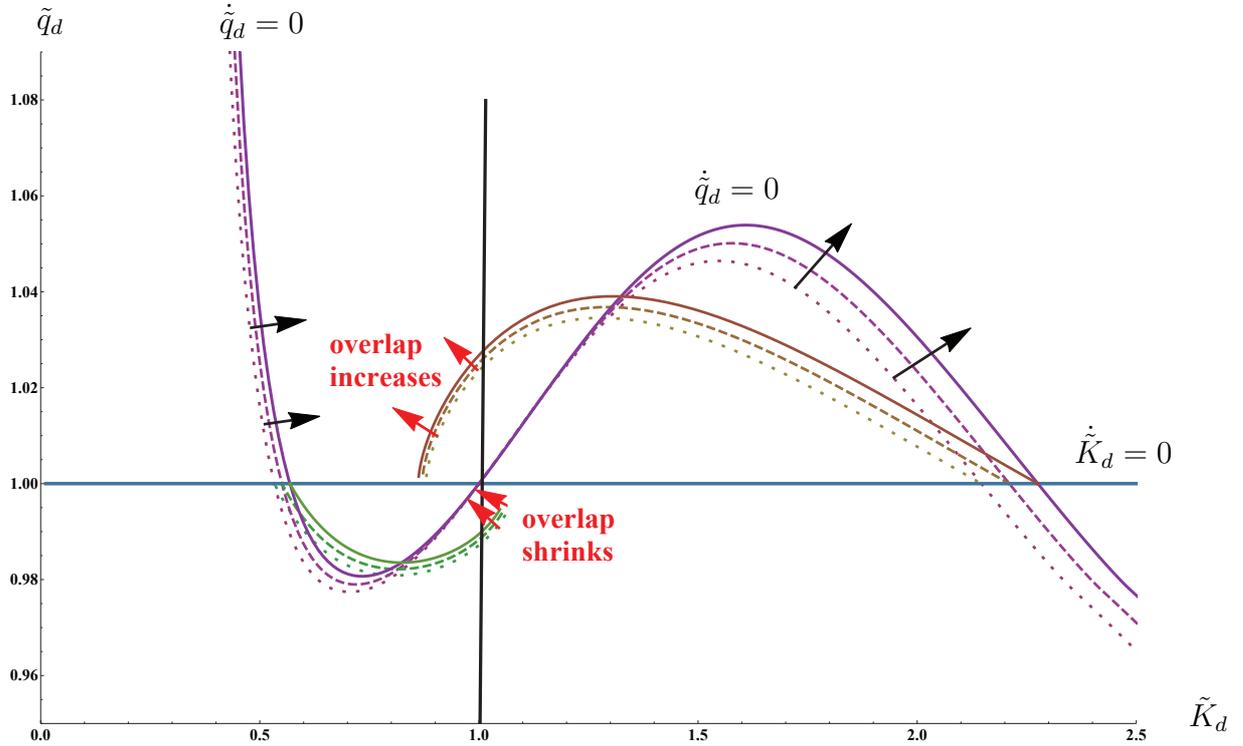
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Figure 6: Increase in TFP for example caused by a change in $p_{E,d}$. Variables $\tilde{q}_d$ and $\tilde{K}_d$ are expressed in ratios to the interior steady state. Dotted line: low TFP; dashed line: increase in TFP of 0.5%; solid line: increase in TFP of 1%.

In both energy regimes, economic policies affecting total factor productivity $\tilde{A}_j$ trigger (i) the number and the position of steady states as well as (ii) the existence and the size of an overlap region. By steering the size of the overlap region, economic policies influence thus the relevance of expectations. In order to elaborate on this argument more in detail, we consider non-drastic changes in $\tilde{A}_j$ meaning that $\tilde{A}_j$ remains above $\tilde{A}_{j,crit}$ and below $\tilde{A}_{j,max}$ which assures both, the existence of multiple steady states and an overlap region. Non-drastic improvements in TFP move both exterior steady states to the right, since the $\dot{q}_d = 0$-isocline moves upwards. This upward shift is owed to the increase in the marginal productivity of capital. In addition, the TFP change affects the size of the overlap in a systematic fashion. We illustrate this effect in Figure 6. There, we increased the TFP by 0.5% (dashed line) and 1% (solid line) relative to the baseline scenario (dotted line). For the sake of visual clarity we normalized q_d and K_d relative to the interior steady state. The new adjusted variables, $\tilde{q}_d$ and $\tilde{K}_d$, reflect thus the distance to the interior steady state which has been normalized to 1. Obviously an increase in TFP moves the transition

path to the inferior and the superior steady state upwards. As can be seen clearly, the relative importance of expectations for the transition path to the superior steady state has increased while the relative importance of expectations for the transition to the inferior equilibrium is reduced. This mechanism follows a clear economic reasoning: From (41) we know that $q_d(t)$ is associated to the net present value of one additional unit of installed capital. Obviously, an increase in TFP must increase $q_d(t)$. This mirrors the upward shift of the transition paths. Thus, an increase in TFP reduces the risk that firms located to the right of the interior steady state mover under comparatively favorable initial conditions but pessimistic expectations to the inferior steady state. At the same time the chance to reach the superior steady state has increased since firms located further to the left of the interior steady state (relatively unfavorable initial conditions) have now a chance to transit towards the superior steady state if they are sufficiently optimistic.

The above reasoning is also valid for regime (C) characterized by an inactive dirty sector. Everything else equal, total factor productivity in the green regime exceeds factor productivity in the dirty regime as aggregate pollution is at its minimum possible value ψ. Net output shrinks in regime (C) compared to regime (D) since labor has to be allocated to energy production. On the other hand, firms switching to the green technology save on energy imports.

5.2 Regime switch

We finally discuss the centerpiece of the energy transition which is the switch from dirty to clean production. We shall assume that the clean technology is known but comparatively less productive. With respect to policy instruments we consider taxes on fossils and a subsidy on marginal production costs of clean energy services.

In Figure 7, we depict again the $\dot{q}_j$- and the $\dot{K}_j$-isocline, i.e. equations (33)-(36), and illustrate the introduction of a tax on fossils, $\tau > 0$, on the location of the steady states induced by variations in $\tilde{A}_d$. Since the dirty technology is more productive, its superior steady state is the highest in the economy. Nevertheless, the inferior steady state of the clean technology is higher compared to the inferior steady state of the dirty technology. A sufficiently high tax on fossils reduces $\tilde{A}_d$ and moves all the steady state levels of the dirty technology in a position below the corresponding level of the clean technology. As the clean technology is less productive, the relative importance of expectations will shrink to the left and increase to the right of the unstable equilibrium. Thus, if firms located in the overlap region are pessimistic in terms of the net present value of one additional unit of installed capital the regime switch may increase the risk that the inferior equilibrium will
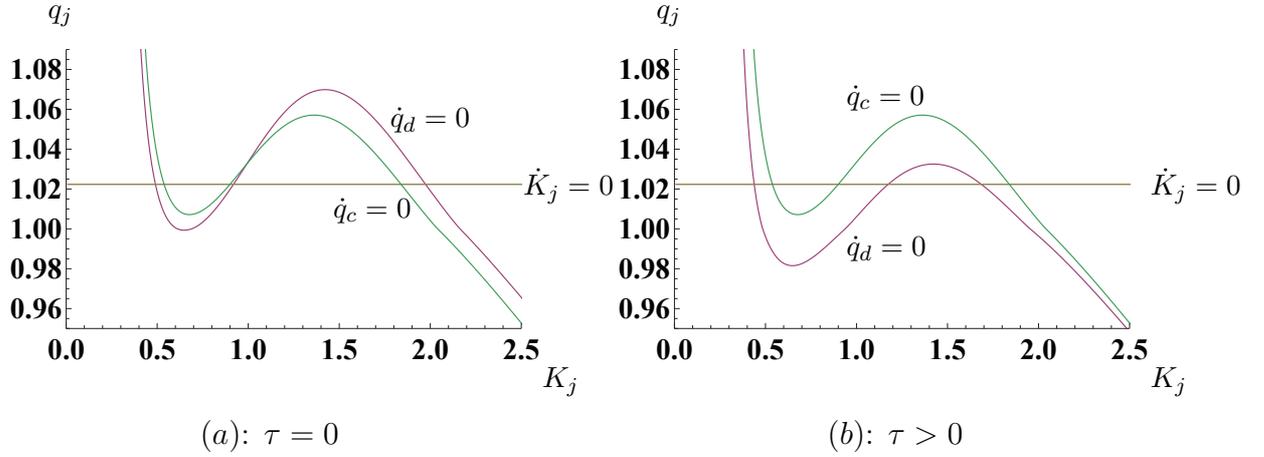
Figure 7: Clean technology is less productive. Panel (a): fossils are not taxed, panel (b): fossils are taxed.

be selected, see also Figure 6. Hence, enforcing a regime switch under these circumstances may improve the situation compared to the dirty regime only if the economy transits to the inferior equilibrium, which is higher in the clean case. This effect depends, however, on the parametrization of the model and will vanish for larger productivity gaps between the clean and the dirty technology. To conclude, taken alone, taxation of fossils also seems to be an inappropriate instrument to generate a momentum effect for the transition from dirty to clean production since the introduction of a tax reduces the overlap region.

We now consider a lump-sum financed subsidy of marginal production cost, c_{x_c}, for clean energy services. The profit maximizing monopoly price modifies to

$$p^s_{x_c} = m_c(K_c)(1 - s_c)c_{x_c},$$

with $0 \leq s_c < 1$ denoting the subsidy rate. Consequently, the zero-profit condition writes as

$$(p^s_{x_c} - (1 - s_c)c_{x_c})x_c = c_{x_c}\phi_c,$$

such that equilibrium demand for each energy service increases to

$$x_c = \frac{\phi_c}{(1 - s_c)(m_c - 1)} \tag{44}$$
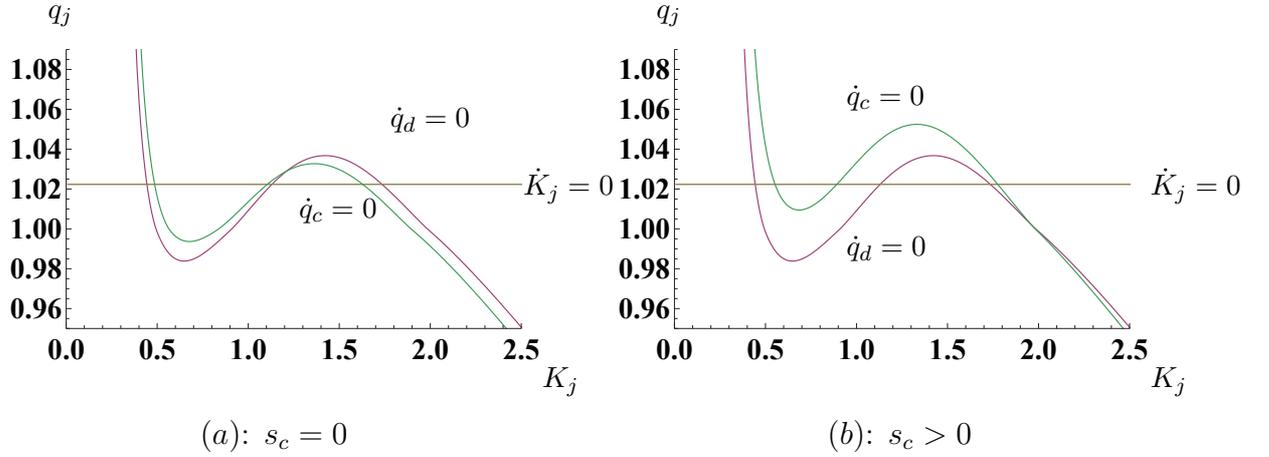
and the level of final output is obtained as

Figure 8: Clean technology is less productive. Panel (a): clean energy services are not subsidized, panel (b): clean energy services are subsidized.

$$Y = \tilde{A}_c \left(\frac{B_c \phi_c}{\phi_c s_c + \phi_c (1 - s_c) m_c} \right)^{1-\alpha} K_c^{\alpha}, \tag{45}$$

which is increasing in s_c. From the last two equations it becomes apparent that s_c increases the aggregate level of clean energy services $N_c x_x$ and thus Y via two channels: (i) clearly, s_c increases demand for intermediates directly, and (ii), indirectly, by an increase in the marginal productivity of physical capital and thus capital accumulation which reduces m_c. The implementation of a subsidy on clean energy services is illustrated in Figure 8. Intuitively, a subsidy on clean energy services increases the demand for intermediates and increases thus final output. Hence the marginal productivity of capital increases, see (45), which shifts the $\dot{q}_c = 0$-isocline upwards. Moreover, in regime (C) the interior steady state is below the interior steady state of regime (D). A higher position of the $\dot{q}_c = 0$-isocline implies in addition that the relative importance of expectations has increased to the left of the interior steady state and has been reduced to the right of it. Thus the implementation of subsidies increases the likelihood that the superior steady state will be reached. This is obviously the best outcome when the clean technology is used. The result on the simultaneous implementation of a tax and a subsidy thus contrasts the effects of an implementation of a pollution tax alone. There are three elements explaining the difference of the policy in our model compared to the more traditional models. First,

the policy in the present model does not need to be permanent but may be temporary up to the point where sufficient momentum for the switch to the new clean trajectory has been built. This contrasts to the more traditional Pigouvian pollution tax or to conventional subsidies, which have to be permanent. In this respect our approach is similar to the findings of Acemoglu et al. (2012) who use an approach with directed technical change and sector-specific learning, generating the momentum effect. Because policy is only temporary and the momentum effects come without costly use of inputs overall costs of the policy are reduced. Second, contrary to Acemoglu et al. (2012), we do not rely on the quite restrictive assumption of pure sector-specific learning but on free market entry in the energy sector which is based on regular profit maximization of firms. Third, our framework is characterized by multiple equilibria, where history and expectations trigger the equilibrium selection process. Moreover, economic policy alters the importance of expectations, i.e. the overlap region from which both steady states can be reached. If economic policy improves the TFP of the clean technology it increases the probability that agents coordinate to reach a superior clean equilibrium. In an extreme case they may already do so without policy or with minimal taxes and subsidies so that the energy transition may become quite inexpensive. Of course, to promote the coordination with given indeterminacy the government has to send out credible signals so that the private sector has an incentive to move. At the same time it is important to stress that we cannot quantitatively compare the costs of these policies to the conventional policy approach because they are based on a different model setup. A direct comparison would require a deeper microeconomic analysis of the process coordinating the expectations, which is certainly an interesting and relevant endeavor, but given the current state of the literature a demanding novel project which is left for future research.

6 Summary and conclusions

The paper at hand studies the macroeconomic effects of a policy-induced switch from dirty to clean production. Notably, we identify macroeconomic conditions under which policy instruments are able to trigger development with sufficient momentum, fostering at the same time economic activity and environmental quality. We present a generic macroeconomic model with capital accumulation and a detailed energy sector to study the interplay between policies and the multiplicity of equilibria. In the model, energy is an aggregate of heterogenous services, which differ in terms of fixed production costs, efficiency, and pollution impact. We derive how economic policy can, under certain con-

ditions, generate broad momentum, moving an economy to a permanently higher activity level. This constitutes an especially attractive option for policy making.

To analyze the transformation of the energy sector we assume that final output is produced by two types of intermediate input: dirty or clean. Dirty intermediates rely on capital and fossil energy services while clean intermediates employ capital and renewable energy services. Initially, only the dirty technology is active. We incorporate a number of crucial stylized facts into our model. First, in our model, energy sectors feature important elements of the industry. Specifically, we assume that energy services are based on the conversion of heterogeneous energy sources like oil, coal, wind, solar, etc. Heterogeneity is given by specific attributes of each energy source, such as fixed costs, supply intermittency, back-up capacity, and pollution intensity. As a consequence of these heterogeneities, both quantity and variety of energy services have a productive value. Put differently, the productivity of the overall energy mix depends both on quantitative and qualitative aspects of the different energy sources.

Our framework distinguishes two regimes: (1) only a dirty technology is active, and (2) only a clean technology is used. Each regime is characterized by multiple steady states. For a range of initial state variables, the evolution of the economy is subject to global indeterminacy in the sense that the trajectories leading to either the superior or the inferior steady state overlap such that expectations determine the equilibrium selection process. Outside the region of the overlap initial conditions, i.e. history, determines whether the inferior or the superior equilibrium is reached. Economic policy and the state of the economy expressed by installed capital equipment determine whether history or expectations shape the transition to the inferior or superior equilibrium. In an extreme case, energy policy determines even the number of equilibria: if factor productivity is reduced (increased) drastically, only the inferior (superior) equilibrium may survive. Energy policy is thus not only determining transitory behavior of an economy but also its long-run performance.

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Mathematical Appendix

A.1 Maximization problem of a typical firm in sector j

The Hamiltonian in current values reads

$$\mathcal{H}_j = p_j Y_j - \int_0^{N_j} p_j(\nu_j) x_j(\nu_j) d\nu_j - I_j \Big[1 + \theta \Big(\frac{I_j}{K_j} \Big)^\eta \Big] + q_j \Big[I_j - \delta K_j \Big]. \tag{A.1}$$

From $\frac{\partial \mathcal{H}_j}{\partial x_j(\nu_j)} = \frac{\partial \mathcal{H}_j}{\partial x_j(\nu'_j)} = 0$ it follows that

$$x_j(\nu_j) = x_j(\nu'_j) \Big(\frac{p_j(\nu_j)}{p_j(\nu'_j)} \Big)^{\frac{m_j}{1-m_j}}. \tag{A.2}$$

Noting that

$$(1-\alpha)Y_j = \int_0^{N_j} p_j(\nu_j) x_j(\nu_j) d\nu_j \tag{A.3}$$

and combining (A.3) with (A.2) yields

$$x_j(\nu'_j) = \frac{(1-\alpha)Y_j p_j(\nu'_j)^{\frac{m_j}{1-m_j}}}{\int_0^{N_j} p_j(\nu_j)^{\frac{1}{1-m_j}} d\nu_j}. \tag{A.4}$$

$\frac{\partial \mathcal{H}}{\partial I} = 0$ implies

$$q_j = 1 + (1+\eta)\theta \Big(\frac{I_j}{K_j} \Big)^\eta. \tag{A.5}$$

From $\frac{\partial \mathcal{H}_j}{\partial K_j} = \bar{r} q_j - \dot{q}_j$ we obtain

$$\dot{q}_j = (\bar{r}+\delta)q_j - \Big[p_j \alpha \frac{Y_j}{K_j} + \theta\eta \Big(\frac{I_j}{K_j} \Big)^{1+\eta} \Big]. \tag{A.6}$$

A.2   Labor market equilibrium in regime (C)

Profit maximizing demand for labor and clean energy implies

$$(1-\alpha)(1-\beta)\frac{Y_c}{L_c} = w \qquad (A.7)$$

$$(1-\alpha)\beta\frac{Y_c}{E_c} = p_{E,c}. \qquad (A.8)$$

Profit maximizing demand for labor in the clean energy sector implies

$$p_{E,c}B_{E,c} = w. \qquad (A.9)$$

Combining the last equation with (A.7), (A.8) and noting that $E_c = B_{E,c}L_{E,c}$ yields

$$L_{E,c} = \frac{\beta}{1-\beta}L_c. \qquad (A.10)$$

Oserving further the resource constraint for the labor market, $L_c + L_{E,c} = 1$, yields

$$L_c = 1-\beta \qquad (A.11)$$

$$L_{E,c} = \beta. \qquad (A.12)$$

A.3   The representative household

The associated Hamiltonian in current values writes as

$$\mathcal{H} = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda(w+\pi+\bar{r}b-c). \qquad (A.13)$$

The necessary conditions are given by

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \rightarrow c^{-\sigma} = \lambda, \qquad (A.14)$$

$$\frac{\partial \mathcal{H}}{\partial b} = \rho\lambda - \dot{\lambda} = \bar{r}\lambda, \qquad (A.15)$$

and $\lim_{t\to\infty} \lambda b e^{-\rho t} = 0$.

The small open economy assumption requires for interior solutions that $\rho = \bar{r}$. Hence, it follows from (A.15) that $\dot{\lambda} = 0$ and $\lambda = const. \; \forall \; t$. Therefore marginal utility of consumption (A.14) is constant and the level of consumption is fixed for all t as well.

The flow budget constraint of the representative household reads $\dot{b} = w+\pi+\bar{r}b-c$. Thus,

$$\dot{b} = (Y - C_{I_j}) - \bar{p}_{E_d}E_d - c + \bar{r}b. \qquad (A.16)$$

Integrating the last expression yields

$$\int_0^\infty c(t)e^{-\rho t}dt = \int_0^\infty e^{-\rho t}\Big[(Y - C_{I_j}) - \bar{p}_{E,d}E_d\Big]dt + b_0 \qquad (A.17)$$

$$= \nu_0. \qquad (A.18)$$

A.4 Steady state(s)

A.4.1 Model with exogenous mark ups

Here, we discuss the stability properties and the number of steady states in our framework with exogenous, i.e. constant mark-ups, which serves as a tractable benchmark case for the subsequent analysis. We refer to the dirty regime only since the characteristics of the clean regime are qualitatively the same. The dynamic system reads as

$$\dot{K}_d = I_d - \delta K_d, \tag{A.19}$$

$$\dot{q}_d = (\bar{r} + \delta)q_d - \left(\alpha \frac{Y_d}{K_d} - \frac{\partial C_{I_d}}{\partial K_d}\right) \tag{A.20}$$

From the first order conditions of the associated control problem, we obtain

$$I_d = \left(\frac{q_d - 1}{(1 + \eta)\theta}\right)^{\frac{1}{\eta}} K_d, \tag{A.21}$$

such that

$$\dot{K}_d = \left[\left(\frac{q_d - 1}{(1 + \eta)\theta}\right)^{\frac{1}{\eta}} - \delta\right] K_d. \tag{A.22}$$

As moreover

$$\frac{\partial C_{I_d}}{\partial K_d} = \theta\eta\left(\frac{I_d}{K_d}\right)^{1+\eta}, \tag{A.23}$$

it follows that

$$\dot{q}_d = (\bar{r} + \delta)q_d - \left[\alpha\frac{Y}{K_d} - \theta\eta\left(\frac{q_d - 1}{(1 + \eta)\theta}\right)^{\frac{1+\eta}{\eta}}\right]. \tag{A.24}$$

Imposing steady state conditions, yields

$$\dot{K}_d = 0 \rightarrow q_{d*} = 1 + (1 + \eta)\theta\delta^\eta \tag{A.25}$$

$$\dot{q}_d = 0 \rightarrow (\bar{r} + \delta)[1 + (1 + \eta)\theta\delta^\eta] - \eta\theta\delta^{1+\eta} = \alpha\frac{Y}{K_d}. \tag{A.26}$$

In the dirty regime, MPK_d reads as

$$\alpha\tilde{A}\left(\frac{B_d}{m_d}\right)^{\frac{(1-\alpha)(1-\gamma)}{1-\tilde{\beta}(1-\gamma)}} K_d^{\frac{(\alpha+\beta)(1-\gamma)-1}{1-\tilde{\beta}(1-\gamma)}}, \tag{A.27}$$

where m_d is the exogenous markup over marginal production costs.
Thus,

$$K_{d*} = \left(\frac{\alpha\tilde{A}}{(\bar{r} + \delta)[1 + (1 + \eta)\theta\delta^\eta] - \eta\theta\delta^{1+\eta}}\right)^{\frac{1-\tilde{\beta}(1-\gamma)}{1-(\alpha+\beta)(1-\gamma)}} \left(\frac{B_d}{m_d}\right)^{\frac{(1-\alpha)(1-\gamma)}{1-(\alpha+\beta)(1-\gamma)}}. \tag{A.28}$$

The Jacobian of the dynamic system reads

$$
\begin{bmatrix}
\frac{\partial \dot{q}_d}{\partial q_d} & \frac{\partial \dot{q}_d}{\partial K_d} \\
\frac{\partial \dot{K}_d}{\partial q_d} & \frac{\partial \dot{K}_d}{\partial K_d}
\end{bmatrix}.
\tag{A.29}
$$

The eigenvalues are obtained from

$$
\left(\frac{\partial \dot{q}_d}{\partial q_d} - \lambda\right)\left(\frac{\partial \dot{K}_d}{\partial K_d} - \lambda\right) - \frac{\partial \dot{K}_d}{\partial q_d}\frac{\partial \dot{q}_d}{\partial K_d}.
\tag{A.30}
$$

As $\frac{\partial \dot{K}_d}{\partial K_d} = 0$, the characteristic polynomial boils down to

$$
\lambda^2 - \frac{\partial \dot{q}_d}{\partial q_d}\lambda - \frac{\partial \dot{K}_d}{\partial q_d}\frac{\partial \dot{q}_d}{\partial K_d}.
\tag{A.31}
$$

Noting further that in steady state

$$
\frac{\partial \dot{q}_d}{\partial q_d} = r,
\tag{A.32}
$$

$$
\frac{\partial \dot{q}_d}{\partial K_d} = \frac{\alpha[1 - (\alpha+\beta)(1-\gamma)]\tilde{A}[K_d^\alpha(B_d/m_d)^{1-\alpha}]^{\frac{1-\gamma}{1-\tilde{\beta}(1-\gamma)}}}{(1-\tilde{\beta}(1-\gamma))K_d^2},
\tag{A.33}
$$

$$
\frac{\partial \dot{K}_d}{\partial q_d} = \frac{\delta^{1-\eta}K_d}{\eta(1+\eta)\theta},
\tag{A.34}
$$

$$
\frac{\partial \dot{K}_d}{\partial K_d} = 0
\tag{A.35}
$$

it follows that $\frac{\partial \dot{q}_d}{\partial q_d}, \frac{\partial \dot{K}_d}{\partial q_d}\frac{\partial \dot{q}_d}{\partial K_d} > 0$ and

$$
\lambda_{1,2} = \frac{r}{2} \pm \sqrt{\frac{r^2}{4} + \frac{\partial \dot{K}_d}{\partial q_d}\frac{\partial \dot{q}_d}{\partial K_d}},
\tag{A.36}
$$

$$
= \frac{r}{2} \pm \sqrt{\frac{r^2}{4} + \frac{\alpha[1 - (\alpha+\beta)(1-\gamma)]\tilde{A}[K_{d*}^\alpha(B_d/m_d)^{1-\alpha}]^{\frac{1-\gamma}{1-\tilde{\beta}(1-\gamma)}}\delta^{1-\eta}}{(1-\tilde{\beta}(1-\gamma))\eta(1+\eta)\theta K_{d*}\eta(1+\eta)\theta}}.
\tag{A.37}
$$

Thus, $K_{d*} \in \Re^+$ specified by (A.28) is unique and saddle-point stable with $\lambda_1 > 0$ and $\lambda_2 < 0$.

A.4.2 Model with endogenous mark ups

(1) The Jacobian

We set $\kappa = 2$, such that

$$
m_d = \frac{s_j}{s_j - 1},
\tag{A.38}
$$

$$
s_j = M + \mu(K_j)^2.
\tag{A.39}
$$

The entries of the Jacobian are the same, except

$$\frac{\partial \dot{q}_d}{\partial K_d} = \frac{\alpha \tilde{A}[K_d^\alpha (B_d/m_d)^{1-\alpha}]^{\frac{1-\gamma}{1-\tilde{\beta}(1-\gamma)}}}{(1-\tilde{\beta}(1-\gamma))K_d^2} \frac{F}{s_d(s_d-1)}, \tag{A.40}$$

with $F = [1-(1-\gamma)(\alpha+\tilde{\beta})][s_d^2 - M(1+(K_d)^2)] - (1-\alpha)(1-\gamma)2(K_d)^2\mu - (K_d)^2(\mu+M)$.

(2) Necessary condition for multiple steady states

The $\dot{q} = 0$-isocline is implicitly defined by (39). Applying the Implicit function theorem, the slope of the $\dot{q} = 0$-isocline is obtained as

$$\frac{\partial q_j}{\partial K_j} \approx \frac{dq_j}{dK_j} = \frac{\frac{\partial MPK_j}{\partial K_j}}{(\bar{r}+\delta) - \left(\frac{q_j-1}{(1+\eta)\theta}\right)^{\frac{1}{\eta}}}. \tag{A.41}$$

For economically meaningful constellations, the denominator of the above expression is always positive, such that

$$sign\left\{\frac{\partial q_j}{\partial K_j}\right\} = sign\left\{\frac{\partial MPK_j}{\partial K_j}\right\}, \tag{A.42}$$

where

$$MPK_j = \frac{\alpha \tilde{A}_j K_j^{\alpha\tilde{\gamma}_j-1} B_j^{(1-\alpha)\tilde{\gamma}_j}}{m_j^{(1-\alpha)\tilde{\gamma}_j}} \tag{A.43}$$

with $\tilde{\gamma}_d = \frac{(1-\gamma)}{1-\tilde{\beta}(1-\gamma)}$ and $\tilde{\gamma}_c = 1$.

Thus

$$\lim_{K\to 0} MPK_j = \infty \tag{A.44}$$

$$\lim_{K\to\infty} MPK_j = 0, \tag{A.45}$$

as long as $\kappa > 1$.

Moreover, in light of (A.42) the necessary condition for the emergence of multiple steady states is that at least once $\frac{\partial MPK_j}{\partial K_j} = 0$, such that

$$(\alpha\tilde{\gamma}_j - 1)K_j^{-1} + \tilde{\gamma}_j(\alpha-1)m_j^{-1}\frac{\partial m_j}{\partial K_j} = 0, \tag{A.46}$$

$$\Rightarrow (\alpha\tilde{\gamma}_j - 1)\frac{m_j}{K_j} = \tilde{\gamma}_j(1-\alpha)\frac{\partial m_j}{\partial K_j}. \tag{A.47}$$

If $\kappa = 2$, the last expression exhibits two roots $\{K_{j,1}; K_{j,2}\}$. In light of (A.44) and (A.45), $K_{j,1}$ is a local minimum and $K_{j,2}$ is a local maximum of the $\dot{q} = 0$-isocline. Thus, there are three steady states, if in addition

$$q_{j,1}(K_{j,1}) < q_{j,*} \tag{A.48}$$

$$q_{j,1}(K_{j,2}) > q_{j,*}. \tag{A.49}$$

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(3) Emergence of complex conjugate eigenvalues. Since

$$\lambda_{1,2} = \frac{r}{2} \pm \sqrt{\frac{r^2}{4} + \frac{\partial \dot{K}_d}{\partial q_d}\frac{\partial \dot{q}_d}{\partial K_d}}, \tag{A.50}$$

inspection of (A.40) reveals that the emergence of complex conjugate eigenvalues requires $F < 0$. Note moreover that

(i) $F = F(K_d)$ is a polynomial of the fourth degree and exhibits thus four roots.

(ii) Hence $\frac{\partial F}{\partial K_d}$ exhibits three roots, where

$$\frac{\partial F}{\partial K_d} = [1 - (\alpha + \tilde{\beta})(1 - \gamma)]2K_d\mu(2K_d^2\mu + 2M - 1) - 4K_d\mu(1 - \gamma) \tag{A.51}$$

and $\frac{\partial F}{\partial K_d} = 0$ at

$$K_{d1} = 0, \tag{A.52}$$

$$K_{d2} = \frac{1/2\sqrt{\Phi}}{[1 - (\alpha + \tilde{\beta})(1 - \gamma)]\mu}, \tag{A.53}$$

$$K_{d3} = \frac{-1/2\sqrt{\Phi}}{[1 - (\alpha + \tilde{\beta})(1 - \gamma)]\mu}, \tag{A.54}$$

with $\Phi \equiv -2\mu[1-(\alpha+\tilde{\beta})(1-\gamma)][2\mu(\alpha+\tilde{\beta})(\gamma-1)-3(1+\alpha(1-\gamma))+\beta(1-\gamma)+2\gamma]$.

At $K_{d1} = 0$, we obtain

$$F(0) = [(1 - (\alpha + \tilde{\beta})(1 - \gamma)][M^2 + M] > 0. \tag{A.55}$$

As $F(K_d)$ exhibits four roots and three extrema, it follows that $F(0)$ is a local maximum and $F(K_{d2})$; $F(K_{d3})$ are local minima, where $K_{d3} < 0$ and $K_{d2} > 0$.

Since (i) and (ii) imply that $F(K_{d2}) < 0$, there are two complex conjugate eigenvalues, if the interior steady state falls in between the two positive roots of F and $\tilde{A}_j$ is sufficiently large, i.e. $\tilde{A}_j > \tilde{A}_{j,crit}$, see (A.40) and (A.50).

B   Parameters

We set as usual $\alpha = 0.3$. Capital depreciation per year is set to 4%. $\beta = 0.18$ implies realistically an income share of energy around 10%. Total factor productivity in the production of dirty energy is $B_d = 0.95$ and for clean services 95% of B_d. This is in reality not necessarily the case but we need marginal differences in order to observe numerically comparable cases. The remaining parameters are fixed in an iterative way in order to achieve a transition period of around 200 years, a reasonable distance between the steady states and comparability between the energy regimes. The interest rate is set to 6%, $\gamma = -0.1$, $B_{E_c} = 0.56$, $\theta = 20$, $\eta = 2.5$, $\overline{\mu} = 1.095$ and $\underline{\mu} = 0.35$.