



Citation for published version:

Schaefer, A 2014, 'Technological change, population dynamics, and natural resource depletion', *Mathematical Social Sciences*, vol. 71, pp. 122-136. <https://doi.org/10.1016/j.mathsocsci.2014.06.001>

DOI:

[10.1016/j.mathsocsci.2014.06.001](https://doi.org/10.1016/j.mathsocsci.2014.06.001)

Publication date:

2014

Document Version

Peer reviewed version

[Link to publication](#)

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Technological Change, Population Dynamics, and Natural Resource Depletion

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This Version: April 2014

Abstract

In this paper, we integrate fertility and educational choices into a scale-invariant model of directed technological change with non-renewable natural resources, in order to reveal the interaction between population dynamics, technological change, and natural resource depletion. In line with empirical regularities, skill-biased technological change induces a decline in population growth and a transitory increase in the depletion rate of natural resources. In the long-run, the depletion rate also declines in the skill intensity. A decline in population growth is harmful for long-run productivity growth, if R&D is subject to diminishing technological opportunities. The effectiveness of economic policies aimed at sustained economic growth thus hinges on its impact on long-run population growth given the sign of intertemporal spillovers in R&D with respect to existing technological knowledge. We demonstrate that an increase in relative research productivities or an education subsidy enhances long-run growth, if R&D is subject to diminishing technological opportunities, while an increase in the teacher-student ratio is preferable in terms of positive intertemporal knowledge spillovers.

Keywords: OLG-Model, Endogenous Fertility, Directed Technological Change, Non-renewable Natural Resources

JEL: J13, O13, O41

*Acknowledgments: I would like to thank Lucas Bretschger, David de la Croix, Christian Groth, Klaus Prettnner, Thomas Steger, seminar participants at the University of Leipzig and the Vienna University of Technology for valuable discussions. Moreover, I wish to thank two anonymous Referees and an Associate Editor for thoughtful suggestions and constructive comments. In addition, I thank Kathryn Arsenault and Miriam Kautz for proof-reading. The usual disclaimer applies.

1. INTRODUCTION

In terms of exhaustible natural resources, a sufficiently high growth rate of productivity is a prerequisite for sustained economic growth. Scale invariant growth models suggest that the long-run growth rate of productivity depends positively on the population's growth rate. As economic development is associated with declining fertility rates, the overall effect of productivity gains and declining fertility rates on prospects of long-run growth seems to be all but clear. In other words, is productivity growth digging its own grave through declining population growth rates and what does this imply for economic policies aiming at sustained economic growth?

Recent decades have been characterized by skill-biased technical change, declining fertility rates and an increasing depletion of natural resources. While each of these phenomena have been analyzed by different strands of the literature, an integrated dynamic general equilibrium framework which allows to assess the interaction between the aforementioned events is still missing in the literature. Arguably, the integration of these features into a single framework makes the analysis not only more realistic, but it moreover deepens our understanding with respect to the effectiveness of economic policies aimed at sustained economic growth in view of exhaustible natural resources. The way in which population growth affects productivity growth depends on the existence of positive or negative intertemporal spillover effects in research and development (R&D) with respect to existing technological knowledge. We thus argue that economic policies have to take into account both their effect on population growth and the sign of spillover effects in R&D. In our analysis we focus on the effects of two different shocks: (a) a change in relative research productivities and (b) a change in the teacher-student ratio. While the former captures the relative difficulty in developing machines complementary to skilled labor relative to unskilled labor complementary machines, i.e. the efficiency of the research infrastructure, the latter can be interpreted as a proxy for schooling quality. We find that an increase in relative research productivities raises population growth while an increase in schooling quality reduces fertility. Thus, both shocks exert diametrically opposed effects on long-run growth. An increase in the relative research productivity of inventing skilled labor complementary machines is conducive for long-run growth if R&D is subject to positive spillover effects while an increase in schooling quality is advisable if research labs are subject to diminishing technological opportunities. We demonstrate moreover that an education subsidy enhances long-run growth, if R&D is subject to diminishing technological opportunities.

The economic framework we employ can be summarized as follows: We consider an economy with over-

lapping generations populated by skilled and unskilled households who decide the number of children they wish to raise and their educational attainment. Thus, the composition of the population in terms of skilled and unskilled households is endogenous. The skilled wage premium generates differential fertility between skilled and unskilled households, in the spirit of Galor and Mountford (2006), and de la Croix and Doepke (2003), i.e. fertility is inversely related to wages and education. In consequence, the skilled-unskilled population ratio and the skill bias of technological innovations are jointly determined in equilibrium. In regards to the production side of our model, we look at a scale invariant growth model of directed technological change, in line with Acemoglu (1998;2002), where the production of machines is subject to a non-renewable natural resource as an essential input. We are thus able to analyze the interaction between population dynamics, the skill bias of innovations, and natural resource depletion. In order to guide the reader through the model, we present its structure graphically in Figure 1.

Figure 1 about here

In order to address the transition process, we have performed numerical experiments, because analytical solutions to high dimensional non-linear systems of difference equations cannot be obtained. Moreover, closed-form solutions are, due to the existence of heterogeneities and the endogeneity of labor supply, even for the steady state difficult to obtain.¹ Nevertheless, we are able to demonstrate the main arguments of our theory in a qualitative way.

By integrating the interaction between skill-biased technological change, declining population growth and resource depletion into a theoretical framework,² our paper relates to the following three empirical regularities and theoretical building blocks: *(i)* In the past sixty years, the relative supply of skilled labor has increased sharply in the U.S. as well as in other industrialized countries. Moreover, and contrary to the predictions of a neoclassical framework with concave production technologies, there has been a sharp increase in the skilled wage premium since the 1970s. The standard explanation for this pattern is an acceleration in the skill bias of capital-embodied technological change (Autor et al., 1998 and Hornstein et al., 2005). *(ii)* At the same time, increasing demand for human capital and increasing wages can

¹Note that this is not a particularity of our model, see for example the papers by Galor and Weil (2000), Galor and Mountford (2006), and de la Croix and Doepke (2003). Technically, this problem arises in any general equilibrium model with endogenous labor supply and concave utility and production functions (for example RBC-models).

²Models of directed technical change have been applied in environmental economics by Andre and Smulders (2005), Di Maria and Smulders (2004), Di Maria and van der Werf (2005), and Di Maria and Valente (2008). Di Maria and Smulders (2004) analyze the effects of pollution while Andre and Smulders (2005) study a labor-resource economy. Di Maria and Valente (2008) were likely the first who provide a micro-foundation of purely resource-augmenting technical progress.

be seen as responsible for the decline in average fertility rates - an observation which constitutes one of the major stylized facts that characterized the development process of industrialized countries (Galor and Weil, 1996; 2000).³ The economic channel which links increasing demand for human capital to declining fertility rates works through a trade-off between the number of children that parents wish to raise and the amount of resources they spend on education per child (see for example de la Croix and Doepke 2003).⁴ Our work is also related to He (2012), who quantitatively analyses the question of whether technological change or demographic transition drives the skill wage premium. In He's work, schooling choice is endogenous but investment-specific technological change and demographic change are exogenous forces. It has been reported that technological change drives the skill wage premium while the role of demographic change is limited. Our paper complements these findings in the sense that skill-biased technological change drives the skilled wage premium, in addition, however, there is a general equilibrium effect of the skill-wage premium on the demographic transition through the parental incentives to invest in skills for their offspring.⁵ (iii) In 2007, OECD petroleum consumption amounted to 57% of the world petroleum consumption (U.S. Energy Information Administration, 2008) and per capita energy use differs between the richest and the poorest group of countries by a factor ten (Weil, 2005). In addition, the world depletion rate of crude oil was increasing continuously from roughly 0.03 between 1945 and 1960 to around 0.18 between 1991 and 2002 (Weil, 2005 and own calculations), where OECD petroleum consumption increased by a factor greater than two between 1960 and 2005 (U.S. Energy Information Administration, 2006). This piece of evidence points to the notion that economic development comes with - at least during the transition - declining fertility rates and an increasing depletion rate of natural resources.

The remainder of the paper is organized as follows: Section 2 introduces the optimization problem of households. Section 3 describes the production side of the model. Section 4 presents the equilibrium structure and the dynamic system. Section 5 describes the long-run equilibrium. In Section 6, we calibrate the model and explore its dynamic behavior. Section 7 provides a discussion of policy implications and finally, Section 8 summarizes and concludes.

³For a comprehensive overview of aspects of the demographic transition, see Galor (2005) and Lee (2003). For the emergence of lowest-low fertility rates in Europe, see Kohler et al. (2002).

⁴Given a high intergenerational persistence, fertility decisions and investments in education per child are transferred from one generation to another and interact with macroeconomic aggregates. For empirical and theoretical evidence, see Kremer and Chen (2000); Rosenzweig and Wolpin (1980); de la Croix and Doepke (2003,2004). For a more detailed discussion see Schäfer and Valente (2011).

⁵Similar Guvenen and Kuruscu (2012) find that an increase in skill-biased technical change increases overall wage inequality both in the short and in the long run. Moreover He and Liu (2012) develop a unified framework where the dynamics of skill accumulation and wage inequality arise as an equilibrium outcome in view of investment-specific technical change.

2. HOUSEHOLDS

We consider two groups of households: skilled and unskilled. Households bear children and make decisions regarding the educational attainment of their offspring. In accordance with empirical observations reporting a high intergenerational persistence in education, we assume that the group of skilled households raises skilled offspring only, while unskilled parents raise either unskilled or skilled offspring.⁶ The fraction of unskilled households raising skilled offspring will be determined in equilibrium. Hence, there exist three types of agents, skilled households raising skilled offspring, unskilled households raising unskilled offspring, and unskilled households raising skilled offspring.

2.1 Preferences and Budget Constraints

The economy under consideration is populated by a continuum of overlapping generations composed of skilled and unskilled households, denoted by L_t^i ($i = u, s$), respectively. Time is discrete, indexed by t , and ranges from 0 to ∞ . Households live for three periods: childhood, adulthood, and old age. All economically relevant decisions are made in the second period of life, adulthood. Adult agents supply one unit of work time inelastically to firms, earn a wage income, w_t^i ($i = u, s$), raise children and save. Old agents consume their savings. Moreover, there exist two types of children trained to be either skilled workers or unskilled workers in $t + 1$, denoted by $n_t^{i,j}$, $j = u, s$. Preferences of a member $i = u, s$ of generation t that is born in $t - 1$ are defined over consumption in t and $t + 1$ denoted by c_t and c_{t+1} , respectively, as well as the potential aggregate income of her children, $w_{t+1}^j n_t^{i,j}$. Preferences are specified as

$$u_t^i = \ln c_t^i + \nu \ln(w_{t+1}^j n_t^{i,j}) + \rho \ln c_{t+1}^i, \quad (1)$$

where potential income of an individual's offspring is weighted by the altruism factor ν and ρ represents as usual the individual discount factor of future consumption. We denote the fraction of unskilled households investing in education for their children by $\theta_t \in [0, 1]$. Hence, the two population groups evolve according to

$$L_{t+1}^s = \theta_t n_t^{u,s} L_t^u + n_t^{s,s} L_t^s \quad (2)$$

⁶Downward mobility and discrimination among offspring with respect to educational choices are absent as we focus on developed economies with low fertility rates, which makes the emergence of the former and latter unlikely. We discuss this assumption more in detail at the end of this subsection.

and

$$L_{t+1}^u = (1 - \theta_t)n_t^{u,u}L_t^u. \quad (3)$$

The educational system is privately funded. As teaching requires skilled labor, $L_t^{E,s}$, schooling fees depend on the wage rate for skilled labor, w_t^s , and the exogenously fixed teacher-student ratio, ϕ ,⁷ which reflects in a broader sense the quality of the schooling sector. Moreover, the education sector is subject to a non-deficit condition such that tuition fees must equal the wage sum of teachers

$$w_t^s\phi(n_t^{s,s}L_t^s + n_t^{u,s}\theta_tL_t^u) = w_t^sL_t^{E,s}. \quad (4)$$

Consequently, the cost to educate a child amount to $w_t^s\phi$. Regardless of the type of children parents wish to raise, fertility is subject to forgone wage earnings in terms of opportunity costs and consumption needs of children. To this end, parents must relinquish the fraction z of their wage income per child. Therefore, child-rearing costs for unskilled children amount to $zw_t^i n_t^{i,u}$ with $i = u$ because only unskilled households raise unskilled offspring. On the other hand, total child-rearing costs for skilled offspring amount to $(zw_t^i + w_t^s\phi)n_t^{i,s}$, with $i = u, s$.

In order to finance old age consumption, members of generation t can buy property rights on natural resources (natural capital) and invest in the capital market (man-made capital). We denote the stock of the exhaustible natural resource in period t by M_t^s , and its extraction allocated to production by R_t . The economy is initially endowed with a resource stock $M_0 > 0$. At the beginning of the current period, t , the stock of exhaustible natural resources is determined by the past resource stock minus extraction in the current period, hence, $M_t = M_{t-1} - R_t$. Each member of generation t buys m_t^i units of natural resources from the old age generation at the competitive price p_t^R (in units of the consumption good). In $t + 1$, the level of old age consumption equals revenues from investments in man-made capital on the capital market $((1 + r_{t+1})s_t^i)$, plus the selling of the property rights to natural resources to the adult cohort born in t ($p_{t+1}^R m_t^i$), hence

$$c_{t+1}^i = (1 + r_{t+1})s_t^i + p_{t+1}^R m_t^i, \quad \text{with } i = u, s. \quad (5)$$

Thus, the budget constraints for skilled households raising skilled offspring ($i, j = s$), for unskilled households raising unskilled offspring, ($i, j = u$), and for unskilled households raising skilled offspring,

⁷For similar assumptions regarding the schooling sector see Eicher (1996,1999), and Bhagwati and Srinivasan (1977).

⁸We denote aggregate levels in capital letters and per capita levels in lower case letters.

($i = u$ and $j = s$) read as

$$w_t^s \geq z w_t^s n_t^{s,s} + w_t^s \phi n_t^{s,s} + c_t^s + p_t^R m_t^s + s_t^s, \quad (6)$$

$$w_t^u \geq z w_t^u n_t^{u,u} + c_t^u + p_t^R m_t^u + s_t^u, \quad (7)$$

$$w_t^u \geq (z w_t^u + \phi w_t^s) n_t^{u,s} + c_t^u + p_t^R m_t^u + s_t^u. \quad (8)$$

2.2 Optimization

A member $i = u, s$ of generation t chooses $\{c_t^i, n_t^{i,j}, c_{t+1}^i, s_t^i, m_t^i\}$ in order to maximize the utility function given by Eq. (1), with $j = s$, if $i = s$ and $j = u, s$, if $i = u$. The maximization of lifetime utility with respect to natural resources (m_t^i) and investment in the capital market (s_t^i) implies a non-arbitrage condition between the two assets known as Hotelling's rule

$$1 + r_{t+1} = \frac{p_{t+1}^R}{p_t^R}. \quad (9)$$

Hence, the marginal return of investment in the exhaustible resource stock, $\frac{p_{t+1}^R}{p_t^R}$, must equal the marginal return of investment in the capital market used to finance research and development (R&D). The respective levels of consumption and savings are obtained from the maximization of (1) subject to either (6), (7) or (8) as

$$c_t^i = \frac{1}{1 + \nu + \rho} w_t^i, \quad (10)$$

$$s_t^i = \frac{\rho}{1 + \nu + \rho} w_t^i - p_t^R m_t^i, \quad (11)$$

with $i = u, s$.

When $w_t^u < w_t^s$, unskilled households have fewer resources available for present and future consumption.

Skilled households raise only skilled offspring and maximize (1) subject to (6) implying that

$$n_t^{s,s} = \frac{\nu}{(1 + \nu + \rho)(z + \phi)}. \quad (12)$$

Thus, $n_t^{s,s}$ depends negatively on the child-rearing cost parameter, z , and the student-teacher ratio, ϕ , which steers the educational cost per child. Unskilled households raising unskilled offspring, maximize (1) subject to (7), implying that

$$n_t^{u,u} = \frac{\nu}{(1 + \nu + \rho)z}. \quad (13)$$

A comparison between (12) and (13) shows that skilled parents raise fewer children than unskilled parents.

Unskilled households raising skilled offspring maximize lifetime utility (1) subject to (8), such that

$$n_t^{u,s} = \frac{\nu}{(1 + \nu + \rho)} \frac{w_t^u}{(w_t^u z + w_t^s \phi)} = \frac{\nu}{(1 + \nu + \rho)} \frac{1}{(z + \tilde{w}_t \phi)}, \quad \text{with } \tilde{w}_t = \frac{w_t^s}{w_t^u}. \quad (14)$$

As an increase in \tilde{w}_t reflects higher educational cost per child compared to the wage income of an unskilled household, $n_t^{u,s}$ is adversely affected by the skilled wage premium \tilde{w}_t .

2.3 The Share of Unskilled Households Raising Skilled Offspring

The fraction of unskilled households raising skilled offspring is denoted by $\theta_t \in [0, 1]$. For $\theta_t > 0$, lifetime utility (1) of unskilled parents raising skilled offspring must at least equal lifetime utility of unskilled parents raising unskilled offspring. In light of the solution to the optimization problem of unskilled households (10), (11), (13) and (14), it follows that the group of unskilled households raises both types of children, i.e. $n_t^{u,s} > 0$ and $n_t^{u,u} > 0$, if $u_t^{u,u} = u_t^{u,s}$, such that¹⁰

$$\frac{w_{t+1}^s}{w_{t+1}^u} = \tilde{w}_{t+1} = \frac{z + \tilde{w}_t \phi}{z}. \quad (15)$$

Relation (15) is an indifference condition which will determine θ_t in equilibrium. Intuitively, as parents are altruistic in respect to their offspring's aggregate potential labor income, \tilde{w}_{t+1} must equal the cost ratio of raising skilled or unskilled children. As the right-hand side of (15) is greater than one, it follows that (15) holds only if $w_{t+1}^s > w_{t+1}^u$. Now comparing the respective fertility decisions implies

$$n_t^{u,u} > n_t^{s,s} > n_t^{u,s}, \quad (16)$$

given that (15) holds.¹¹ The number of unskilled children born in unskilled households, $n_t^{u,u}$, is the highest since no resources are allocated to education. On the other hand, the fertility of unskilled households raising skilled offspring, $n_t^{u,s}$, is the lowest because they trade a lower number of children against a higher income for their offspring. At this point it is also worthwhile to note that the absence of downward mobility for skilled households is not as restrictive as it seems. Skilled parents are indifferent between raising skilled or unskilled offspring if the skilled wage premium of the subsequent period is at least equal to $(z + \phi)/z$, which is smaller than the necessary skilled wage premium implied by (15). Thus, skilled parents always invest in human capital for their offspring if (15) holds.¹²

⁹In order to ease the notation of the paper, we indicate an s/u -ratio between variables q^s and q^u by $\tilde{q} = q^s/q^u$.

¹⁰The proof follows directly from (10),(11), and (13),(14) given that: $u_t^{u,u} = u_t^{u,s}$.

¹¹Similar to de la Croix and Doepke (2003), fertility differentials are generated by wage differentials. The difference is that de la Croix and Doepke consider a continuous wage distribution.

¹²Downward mobility would be a problem if human capital investments were subject to capital market discrimination in the sense that the interest rate of borrowers exceeds the equilibrium interest rate as in Galor and Zeira (1993). There, downward mobility is a transitory phenomenon whose extent depends on initial inequality and the size of the gap between interest rates. Moreover, downward mobility applies only to unskilled households that decided to invest in human capital but never for skilled dynasties who never borrow on the capital market. A scenario characterized by a wage differential below $(z + \phi)/z$ could arise in earlier stages of economic development, but this would over-stress our R&D-based theory.

3. PRODUCTION

Final output, Y_t , is composed of two intermediates, Y_t^s and Y_t^u , stemming from two different production processes: one using skilled labor, $L_t^{Y,s}$, and the other one using unskilled labor, $L_t^{Y,u}$. This assumption accounts for different skill-intensities and a low inter-sectoral mobility of skilled and unskilled labor. Moreover, each sector produces with a set of horizontally differentiated machines which are complementary to each type of labor. The production of machines, in turn, requires the existence of technological knowledge (a blueprint or design) and natural resources. Blueprints are the outcome of purposeful investments in research and development (R&D).

3.1 Final Good Production

The elasticity of substitution between Y_t^s and Y_t^u is determined by $\varepsilon \in (0, \infty)$ such that the production of final output is subject to the following nested CES-production function

$$Y_t = \left[\gamma (Y_t^u)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) (Y_t^s)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \quad (17)$$

The parameter $\gamma \in (0, 1)$ is a distribution parameter which determines how important the two goods are for aggregate output. In each period the price of final output is normalized to 1, i.e. $p_t \equiv 1$, where the prices of Y_t^s and Y_t^u are denoted by $p_t^{Y,s}$ and $p_t^{Y,u}$, such that $\left[\gamma^\varepsilon (p_t^{Y,u})^{1-\varepsilon} + (1-\gamma)^\varepsilon (p_t^{Y,s})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} = p_t = 1$.¹³

3.2 Production of Intermediates and Machines

The production of Y_t^s and Y_t^u requires skilled and unskilled labor, $L_t^{Y,i}$, $i = s, u$, as well as a range of labor complementary machines. The quantity of a machine of type l is denoted by $x_t^i(l)$, $i = s, u$. In each period of time, t , there are N_t^i different types of machines available. Production functions of both intermediates read as

$$Y_t^i = \frac{1}{1-\beta} \int_0^{N_t^i} x_t^i(l)^{1-\beta} dl (L_t^{Y,i})^\beta, \quad (18)$$

with $0 < \beta < 1$ and $i = u, s$.

Machines of type l , $x_t^i(l)$, are manufactured with natural resources, $R_t^{x,i}(l)$, $i = u, s$. Moreover, we assume that current technological knowledge in sector $i = u, s$ reflected by N_t^i increases factor productivity in

¹³For details see Appendix B.1.

the machine producing sector.¹⁴ Therefore, production functions for a machine of type l in sector s or u read as

$$x_t^i(l) = G^i N_t^i R_t^{x,i}(l), \quad G^i > 0, \quad (19)$$

where G^i is a scaling parameter which takes the value $G^i = A$, if $i = s$ and $G^i = B$, if $i = u$ with $A, B > 0$.

3.3 Research and Development

R&D constitutes the search for new designs (blueprints) of machines. To this end, research firms rent labor services, capital inputs, and natural resources while taking the current level of technological knowledge as given. It simplifies the analysis considerably, however, if we assume that all the three of these factors combine to produce blueprints in exactly the same way that they combine to produce final output, i.e. we apply the so called lab-equipment approach.¹⁵ As one period encompasses approximately 30 years, we assume that blueprints are depreciated entirely after one period.¹⁶ Both R&D sectors generate new blueprints according to the following scale invariant production functions

$$N_{t+1}^s = \frac{\eta^s}{(N_t^s)^\delta} (D_t^s) \quad \text{and} \quad N_{t+1}^u = \frac{\eta^u}{(N_t^u)^\delta} (D_t^u), \quad (20)$$

where D_t^s and D_t^u are spending on R&D (in units of the final good) for skilled- and unskilled-labor complementary types of machines, respectively. The parameters η^s and η^u are productivity parameters that allow the cost of innovation to differ. Finally, we will distinguish between the cases $\delta > 0$ and $\delta < 0$. In the former, technological advances are partially hedged out by diminishing technological opportunities (Evenson, 1984; Kortum, 1993; Jones and Williams, 2000), when it may become more and more complicated to achieve productivity gains. In the case of $\delta < 0$, there are intertemporal knowledge spillovers - a case which is labeled "standing on the shoulders of giants" in the literature.

¹⁴It is debatable whether this assumption induces an over-optimistic perspective with respect to non-reproducible capital into the model, but it is necessary to generate steady-state growth. If this effect were absent or weaker, marginal production cost of machines would be, contrary to Acemoglu (2002), increasing over time since the price per unit of natural resources, p_t^R , increases with the depletion of natural resources. Hence, operating profits of machine producers would approach zero in the long-run (see also Eq. (22)) which would undermine the incentives to invest in R&D within finite time.

¹⁵By doing so, we also take into account the criticism stressed by resource economists with respect to the so-called "knowledge driven" specification in which (skilled) labor is the only input to R&D and its therefore seemingly overly optimistic assumption that R&D could take place without natural resources.

¹⁶As in our case, complete depreciation is a common assumption in OLG models of the Diamond type with natural resources in order to assure analytical tractability. Without complete depreciation, the old generation would sell its assets on the capital market to the adult cohort. The latter would split up their savings between existing blueprints and investments in R&D. Under full depreciation the amount of savings is entirely allocated to R&D.

4. EQUILIBRIUM

We denote the amount of natural resources allocated to machine production in sector $i = s, u$ by $R_t^{x,i} = \int_0^{N_t^i} R_t^{x,i}(l) dl$ such that in equilibrium an efficient use of extracted natural resources requires $R_t^{x,s} + R_t^{x,u} = \varphi_t^{x,u} R_t + \varphi_t^{x,s} R_t = R_t$ with $\varphi_t^{x,u} + \varphi_t^{x,s} = 1$, and $\varphi_t^{x,i}$, $i = s, u$ representing the share of extracted natural resources allocated to sector s or u . Full employment of labor requires $L_t^s = L_t^{Y,s} + L_t^{E,s}$ and $L_t^u = L_t^{Y,u}$. In addition, an equilibrium consists of a sequence of quantities $\{N_t^i, Y_t, Y_t^i, R_t, x(l)_t^i, L_t^i, L_t^{Y,i}, L_t^{E,s}, s_t^i, m_t^i, c_t^i, n_t^{i,j}\}_{t=0}^\infty$, shares $\{\theta_t, \tau_t, \varphi_t^{R,i}\}_{t=0}^\infty$ with τ_t denoting the depletion rate of natural resources, and prices $\{w_t^i, p_t^R, r_t, p_t^{Y,i}, p(l)_t^{x,i}\}_{t=0}^\infty$, where $p(l)_t^{x,i}$ represents the price of a machine, x , of type l in sector i , with $i, j = s, u$. The usual characteristics of a symmetric equilibrium imply equal prices, $p_t^{x,i}$, and quantities, x_t^i , for each type of machine, l , such that $p(l)_t^{x,i} = p_t^{x,i}$, and $x(l)_t^i = x_t^i$. Machine producers maximize profits $\pi_t^{x,i} = [p_t^{x,i} - c_t^{x,i}]x_t^i$, where marginal production costs amount to $c_t^{x,i} = \frac{p_t^R}{G^i N_t^i}$, with $i = u, s$. In order to ease the notation we denote the ratio of two variables h^a and h^b by \tilde{h} , i.e. $\tilde{h} = h^a/h^b$. Perfect competition on goods and factor markets and monopolistic competition for machine producers imply in equilibrium for periods $t = 0, 1, \dots, \infty$:

Lemma 1

- (i) Demand for machines of type l is obtained from (18) in light of profit maximizing behavior of intermediate producers

$$x_t^i = \left(\frac{p_t^{Y,i}}{p_t^{x,i}} \right)^{\frac{1}{\beta}} L_t^{Y,i}, \quad i = s, u. \quad (21)$$

Machine producers take (21) as given and maximize profits, $\pi_t^{x,i} = [p_t^{x,i} - c_t^{x,i}]x_t^i$, if $p_t^{x,i} = \frac{c_t^{x,i}}{1-\beta} = \frac{p_t^R}{(1-\beta)G^i N_t^i}$, such that

$$\pi_t^{x,i} = \beta(1-\beta)^{\frac{1-\beta}{\beta}} (p_t^{Y,i})^{\frac{1}{\beta}} L_t^{Y,i} (c_t^{x,i})^{\frac{\beta-1}{\beta}}, \quad i = s, u. \quad (22)$$

Moreover, the level of intermediates (18) then reads

$$Y_t^i = (1-\beta)^{\frac{1-2\beta}{\beta}} N_t^i \left(\frac{p_t^{Y,i}}{c_t^{x,i}} \right)^{\frac{1-\beta}{\beta}} L_t^{Y,i}, \quad i = s, u. \quad (23)$$

- (ii) Profit maximizing behavior in final good production, together with (17) and (23), determines the relative price of the two intermediates $\tilde{p}_t^Y = \frac{p_t^{Y,s}}{p_t^{Y,u}}$ as

$$\tilde{p}_t^Y = \frac{1-\gamma}{\gamma} (\tilde{Y}_t)^{-\frac{1}{\varepsilon}} = \left(\frac{1-\gamma}{\gamma} \right)^{\frac{\beta\varepsilon}{\sigma}} (\tilde{N}_t \tilde{L}_t^Y)^{-\frac{\beta}{\sigma}} (\tilde{c}_t^x)^{\frac{1-\beta}{\sigma}}, \quad (24)$$

¹⁷As has been emphasized before, marginal production cost, $c_t^{x,i}$, affects operating profits of machine producers adversely, while the increase in $c_t^{x,i}$ caused by p_t^R is counteracted by the increase in technological knowledge.

with $\sigma \equiv \varepsilon - (\varepsilon - 1)(1 - \beta)$.

In light of Lemma 1, we can specify the skilled wage premium, \tilde{w}_t , the allocation of skilled labor ($L_t^{Y,s}, L_t^{E,s}$), and the allocation of extracted natural resources, $\varphi_t^{x,i}$ ($i = s, u$), which are summarized in the following proposition:

Proposition 1

(i) The skilled wage premium $\tilde{w}_t = \frac{w_t^s}{w_t^u} = \frac{\partial Y^s}{\partial L_t^{Y,s}} / \frac{\partial Y^u}{\partial L_t^{Y,u}}$ reads in light of (24) and (23) as

$$\tilde{w}_t = [(1 - \gamma)/\gamma]^{\frac{\varepsilon}{\sigma}} (A/B)^{\frac{(\sigma-1)(1-\beta)}{\sigma\beta}} (\tilde{N}_t)^{\frac{\sigma-1}{\sigma\beta}} (\tilde{L}_t^Y)^{-\frac{1}{\sigma}}, \quad (25)$$

with $\tilde{N}_t = \frac{N_t^s}{N_t^u}$ and $\tilde{L}_t^Y = \frac{L_t^{Y,s}}{L_t^{Y,u}}$. Furthermore, the equilibrium allocation of labor requires full employment, such that $L_t^{Y,s} = L_t^s - L_t^{E,s}$ and $L_t^{Y,u} = L_t^u$.

(ii) Given the student-teacher ratio, ϕ , demand for labor in the education sector is determined by the total number of children sent to school: $L_t^{E,s} = \phi(n^{s,s}L_t^s + \theta_t n_t^{u,s}L_t^u)$. Given $L_t^{Y,s} = L_t^s - L_t^{E,s}$, the skilled-unskilled employment ratio, \tilde{L}_t^Y , reads therefore as

$$\tilde{L}_t^Y = (1 - \phi n^{s,s})\tilde{L}_t - \phi\theta_t n_t^{u,s}, \quad \text{with } \tilde{L}_t = \frac{L_t^s}{L_t^u}. \quad (26)$$

(iii) The shares of extracted natural resources allocated to the s - or u -sector, $\varphi_t^{x,i}$, $i = s, u$, are driven by the blueprint ratio, \tilde{N}_t , the skilled-unskilled employment ratio, \tilde{L}_t^Y , and the ratio of marginal production costs of machines, \tilde{c}_t^x , such that

$$R_t^{x,u} = \frac{\overbrace{1}^{\varphi_t^{x,u}}}{1 + ((1 - \gamma)/\gamma)^{\frac{\varepsilon}{\sigma}} (\tilde{N}_t)^{\frac{\sigma-1}{\sigma}} (\tilde{L}_t^Y)^{\frac{\sigma-1}{\sigma}} (\tilde{c}_t^x)^{\frac{(\sigma-1)(\beta-1)}{\sigma\beta}}} R_t \quad (27)$$

$$R_t^{x,s} = \frac{((1 - \gamma)/\gamma)^{\frac{\varepsilon}{\sigma}} (\tilde{N}_t)^{\frac{\sigma-1}{\sigma}} (\tilde{L}_t^Y)^{\frac{\sigma-1}{\sigma}} (\tilde{c}_t^x)^{\frac{(\sigma-1)(\beta-1)}{\sigma\beta}}}{1 + \underbrace{((1 - \gamma)/\gamma)^{\frac{\varepsilon}{\sigma}} (\tilde{N}_t)^{\frac{\sigma-1}{\sigma}} (\tilde{L}_t^Y)^{\frac{\sigma-1}{\sigma}} (\tilde{c}_t^x)^{\frac{(\sigma-1)(\beta-1)}{\sigma\beta}}}_{1 - \varphi_t^{x,u} = \varphi_t^{x,s}}} R_t \quad (28)$$

with $R_t^{x,u} + R_t^{x,s} = R_t$.¹⁸

If $\sigma > 1$ then $\varepsilon > 1$ (see Lemma 1, item (ii)), such that the two intermediates Y_t^u and Y_t^s are gross substitutes. In this case, item (i) of Proposition 1 indicates that the skilled wage premium, \tilde{w}_t , is increasing in the blueprint ratio, \tilde{N}_t . In contrast, for a given \tilde{N}_t , the relative factor reward of skilled labor declines in the current skilled-unskilled employment ratio, \tilde{L}_t^Y , since labor is subject to diminishing marginal returns. According to item (ii), the employment ratio of skilled and unskilled labor in production, \tilde{L}_t^Y ,

¹⁸A proof can be found in Appendix A.1.

increases in \tilde{L}_t but declines in the amount of skilled labor allocated to the education sector. Moreover, if $\sigma > 1$, an increase in \tilde{L}_t^Y raises the relative market size of skilled labor complementary machines which increases their marginal productivity in intermediate production and thus relative demand for machines, \tilde{x}_t .¹⁹ Consequently, the share of extracted natural resources allocated to the s -sector, $\varphi_t^{x,s}$ (see item (iii)), increases as well. A symmetric argument holds for an increase in the blueprint ratio, \tilde{N}_t . On the other hand, an increase in the relative production cost ratio, \tilde{c}_t^x , reduces relative demand for skilled labor complementary machines and increases the share of extracted natural resources allocated to the u -sector, $\varphi_t^{x,u}$. The evolution of the blueprint ratio will depend likewise on $\sigma \geq 1$. More specifically, an expected increase in the employment ratio of skilled and unskilled labor \tilde{L}_{t+1}^Y will increase the profitability of skilled-labor complementary innovations by means of the market size effect and will bias technological progress towards the s -sector, whenever $\sigma > 1$, we come back to this point further below. In order to match our model to empirical observations regarding the features of directed technological change, we will assume that $\sigma > 1$ throughout the paper.²⁰

We now turn to the capital market and the depletion rate of natural resources, τ . In both R&D-sectors, free entry drives profits down to zero. The value of each blueprint equals the discounted profit stream (i.e. $\pi_{t+1}^{x,i}/(1+r_{t+1})$) generated by patent owners, that is, machine producers. On the other hand, Eq. (20) implies that the marginal productivity of one unit of final output allocated to R&D equals $\eta^i(N_t^i)^{-\delta}$. As p_t has been normalized to 1, the technology market clearing condition reads as

$$\tilde{\pi}_{t+1}^x = \tilde{\eta}(\tilde{N}_t)^{-\delta}, \quad (29)$$

such that the evolution of the blueprint ratio in the subsequent period is obtained from (22) and (24) as²¹

$$\tilde{N}_{t+1} = \left[\tilde{\eta}^{\sigma\beta} ((1-\gamma)/\gamma)^{\varepsilon\beta} (A/B)^{(\sigma-1)(1-\beta)} \tilde{N}_t^{-\delta\sigma\beta} (\tilde{L}_{t+1}^Y)^{(\sigma-1)\beta} \right]^{\frac{1}{\beta-(1-\beta)(\sigma-1)}}, \quad (30)$$

where $\beta - (\sigma - 1)(1 - \beta) > 0$ within the range of plausible parameter values.

An increase in the future skilled-unskilled employment ratio, \tilde{L}_{t+1}^Y , induces an increase in the next period's blueprint ratio, \tilde{N}_{t+1} , if $\sigma > 1$. Thus, innovations are biased towards skilled labor complementary innovations if it is expected that next period's skilled-unskilled labor ratio increases. An increase in A/B

¹⁹Eq. (21) implies $\tilde{x}_t = \left(\frac{\tilde{p}_t^Y}{\tilde{p}_t^x}\right)^{\frac{1}{\beta}} \tilde{L}_t^Y$, $i = s, u$.

²⁰If $\sigma = 1$, relative market size effects are absent and $\varphi_t^{x,i}$, $i = u, s$ are time invariant, i.e. $\varphi_t^{x,u} = \gamma$ and $\varphi_t^{x,s} = 1 - \gamma$ for all t . Moreover, directed technological change requires $\sigma > 1$.

²¹For details see Appendix B.3.

enhances the efficiency of natural resources in the production of machines in sector s relative to sector u , such that relative marginal production cost, \tilde{c}_t^x , shrink. Consequently, the relative profitability of skilled labor complementary innovations, $\tilde{\pi}_{t+1}$, and the skill bias of future innovations reflected by \tilde{N}_{t+1} increase. If $\delta > 0$, the number of existing blueprints, \tilde{N}_t , dampens the speed of innovations in the future because it is comparatively difficult to innovate skill-complementary machines for the next period. For $\delta < 0$, in contrast, the existing level of blueprints speeds up innovations due to positive intertemporal knowledge spillovers.

Outlays for R&D are financed by aggregate savings, S_t , which are not invested in ownership of natural resources, implying that the amount of aggregate savings can be written as²²

$$S_t = p_t^{Y,u} Y_t^u \underbrace{\left\{ \frac{\rho}{1+\gamma+\rho} \beta \left[1 + \tilde{w}_t \tilde{L}_t \right] - \frac{1-\beta}{\varphi_t^{x,u}} \frac{1-\tau_t}{\tau_t} \right\}}_{Z_t^s}. \quad (31)$$

Free entry in R&D and perfect capital markets imply equality between future aggregate profits and revenues of aggregate savings allocated to R&D-sector u or s such that $N_{t+1}^i \pi_{t+1}^i = (1+r_{t+1})D_t^i$. Thus, we obtain from the technology market clearing condition (29), given that $D_t^s + D_t^u = S_t$:

$$D_t^u = \frac{1}{1+\Omega_t} S_t \quad \text{and} \quad D_t^s = \frac{\Omega_t}{1+\Omega_t} S_t, \quad (32)$$

with $\Omega_t = \tilde{N}_{t+1} \tilde{\eta}^{-1} (\tilde{N}_t)^\delta = \tilde{N}_{t+1} \tilde{\pi}_{t+1}^x$ and \tilde{N}_{t+1} given by (30).

In equilibrium, R&D expenditures, D_t^i (D_t^u), depend in light of (32) positively (inversely) on the future relative profitability, $\tilde{\pi}_{t+1}^x$, of skilled-labor complementary innovations. Clearly, a prerequisite not only for sustained economic growth, but also for a non-trivial interior solution of the model is $S_t > 0$ implied by $Z_t^s > 0$, i.e. $\frac{\rho}{1+\gamma+\rho} \beta \left[1 + \tilde{w}_t \tilde{L}_t \right] > \frac{1-\beta}{\varphi_t^{x,u}} \frac{1-\tau_t}{\tau_t}$. For the emergence of sustained economic growth it is therefore insufficient that the labor income share exceeds the natural resource income share, i.e. $\beta > (1-\beta)$, which would require that: $\beta > 1/2$. Since in reality β should be approximately $2/3$, there arises no threat from a too large resource income share. In this sense, a necessary condition for $S_t > 0$ and sustained economic growth is a sufficiently high expenditure share for old-age consumption $\frac{\rho}{1+\gamma+\rho}$. Aggregate savings fuel productivity growth and enhance the efficiency of natural resource use. Through Hotelling's rule, the evolution of the price per unit of natural resources is tight to the interest rate as expressed by Eq. (9). On the other hand, the technology market clearing condition (29) links the interest rate to the profitability of future innovations such that in equilibrium

$$g_{t+1}^{p,R} = 1 + r_{t+1} = \pi_{t+1}^i \eta^i (N_t^i)^{-\delta}. \quad (33)$$

²²For details see Appendix B.4.

Noting further that in equilibrium the markets for machines clear, i.e. $N_t^i (p_t^{Y,i}/p_t^{x,i})^{\frac{1}{\beta}} L_t^{Y,i} = G^i N_t^i \varphi_t^{x,i} R_t$, the evolution of the depletion rate of natural resources evolves according to

$$\tau_{t+1} = \frac{Z_t^S}{Z_t^N} \frac{\tau_t}{1 - \tau_t} \left(g_{t+1}^{\varphi^{x,u}} \right)^{-1}, \quad (34)$$

with $Z_t^N = \beta(1 - \beta)(1 + \Omega_t)$.²³

As we remarked earlier, a necessary condition for aggregate savings being positive is $Z_t^S > 0$. Otherwise, savings, demand for machines and the depletion rate would jump to zero implying zero output in the machine and the intermediate goods producing sectors. It is precisely the behavior of $\frac{Z_t^S}{Z_t^N}$ that steers natural resource depletion during the transition and in the long-run. During the transition, an increase in Z_t^S relative to Z_t^N - which contains aggregate relative profits of future innovations (see (32)) - increases the depletion rate of natural resources. In summary, the evolution of the economy is governed by a four-dimensional system of difference equations containing the laws of motion for the population ratio, \tilde{L} , the blueprint ratio, \tilde{N} , the depletion rate, τ , and the fraction of unskilled households raising skilled offspring, θ :

$$\tilde{L}_{t+1} = \frac{\theta_t n_t^{u,s} + n^{s,s} \tilde{L}_t}{(1 - \theta_t) n^{u,u}}, \quad (35)$$

$$\tilde{N}_{t+1} = \left[\tilde{\eta}^{\sigma\beta} ((1 - \gamma)/\gamma)^{\varepsilon\beta} (A/B)^{(\sigma-1)(1-\beta)} \tilde{N}_t^{-\delta\sigma\beta} (\tilde{L}_{t+1}^Y)^{(\sigma-1)\beta} \right]^{\frac{1}{\beta - (1-\beta)(\sigma-1)}}, \quad (36)$$

$$\tau_{t+1} = \frac{Z_t^S}{Z_t^N} \frac{\tau_t}{1 - \tau_t} \left(g_{t+1}^{\varphi^{x,u}} \right)^{-1}, \quad (37)$$

and indifference condition (15)

$$\tilde{w}_{t+1} = \frac{z + \tilde{w}_t \phi}{z}, \quad (38)$$

which determines θ_{t+1} implicitly via, since $\tilde{w}_{t+1} = w(\tilde{N}_{t+1}, \tilde{L}_{t+1}^Y) = w(\tilde{L}_{t+1}^Y)$, see also (36), with $\tilde{L}_{t+1}^Y = (1 - \phi n^{s,s}) \tilde{L}_{t+1} - \phi \theta_{t+1} n_{t+1}^{u,s}$ (see (26)).

5. STEADY STATE

In this section we characterize the long-run equilibrium implied by the system of difference equations (35)-(38) as functions of θ_* , where subscript '*' denotes stationary values. Thereafter, we analyze the effects of a change in the student-teacher ratio, ϕ , and in relative research productivities in R&D, $\tilde{\eta}$.

²³For details see Appendix B.5.

These parameters are of special interest, since they capture different policies aiming either at improving schooling quality or a change in relative research infrastructure. It turns out that the parameter shocks under consideration have diametrically different effects on the long-run growth rate of the economy.

Imposing steady state conditions on (35)-(38), the long-run equilibrium can be defined as follows

Definition 1

The long-run equilibrium consists of a set of stationary ratios $\{\tilde{L}_, \tilde{N}_*\}$ and shares $\{\theta_*, \tau_*\}$ which imply constant (gross) growth rates of the population and the productivity, $\{n_*, g_*\}$, a constant employment ratio, \tilde{L}_*^Y , a constant skill wage premium, \tilde{w}_* , and constant relative prices \tilde{p}_*^Y .*

At this point it is important to note that the long-run values of \tilde{L}_*, \tilde{N}_* and θ_* are independent from τ_* while, in contrast, τ_* can be expressed as a function of θ_* . The reason is that the difference equations (35), (36) and (38) are decoupled from the evolution of τ , but τ is not decoupled from the evolution of the other three variables.²⁴ The following proposition presents the relevant long-run values as functions of θ_* (for details see Appendix A.2):

Proposition 2

The unique long-run equilibrium is characterized by $\tilde{L}_ = \tilde{L}(\theta_*)$, $\tilde{N}_* = \tilde{N}(\theta_*)$, $\tau_* = \tau(\theta_*)$ with θ_* implied by (15), such that:*

- (i) *The skilled-unskilled population ratio reads, in light of (35), as*

$$\tilde{L}_* = \tilde{L}(\theta_*) = \frac{\theta_* n_*^{u,s}}{(1 - \theta_*) n_*^{u,u} - n_*^{s,s}}. \quad (39)$$

The employment ratio in production specified in Proposition 2, \tilde{L}_^Y , is thus a function of θ_* :*

$$\tilde{L}_*^Y = \tilde{L}^Y(\theta_*) = (1 - \phi n_*^{s,s}) \tilde{L}_* - \phi \theta_* n_*^{u,s} = \frac{\theta_* n_*^{u,s} [1 - \phi(1 - \theta_*) n_*^{u,u}]}{(1 - \theta_*) n_*^{u,u} - n_*^{s,s}}. \quad (40)$$

- (ii) *The long-run blueprint ratio, \tilde{N}_* , reads*

$$\tilde{N}_* = \tilde{N}(\theta_*) = \tilde{\eta}^{\frac{\sigma\beta}{\psi}} \Gamma^{\frac{1}{\psi}} \left[\tilde{L}_*^Y(\theta_*) \right]^{\frac{(\sigma-1)\beta}{\psi}}, \quad (41)$$

with $\Gamma = ((1 - \gamma)/\gamma)^{\varepsilon\beta} (A/B)^{(\sigma-1)(1-\beta)}$ and $\psi = \beta - (1 - \beta)(\sigma - 1) + \delta(\beta + (\sigma - 1)\beta) > 0$ in the range of plausible parameters.

²⁴This is not a peculiarity of our model but a common feature of endogenous growth models with exhaustible natural resources.

²⁵Note that $n_*^{u,s} = \frac{\nu}{(1+\nu+\rho)(z+\tilde{w}_*\phi)}$, with $\tilde{w}_* = \frac{z}{z-\phi}$, see item (iii) of this proposition.

(iii) The skilled wage premium is obtained as $\tilde{w}(\theta_*) = \tilde{\eta}^{\frac{\sigma-1}{\psi}} \Gamma^{\frac{1+\delta}{\psi}} \left[\tilde{L}^Y(\theta_*) \right]^{\frac{\sigma-1-\beta(1+\delta)}{\psi}}$, such that the share of unskilled households raising skilled offspring, θ_* , is in light of (15) determined by

$$\tilde{w}(\theta)_* = \tilde{\eta}^{\frac{\sigma-1}{\psi}} \Gamma^{\frac{1+\delta}{\psi}} \left[\tilde{L}^Y(\theta_*) \right]^{\frac{\sigma-1-\beta(1+\delta)}{\psi}} = \frac{z}{z-\phi}. \quad (42)$$

(iv) The long-run depletion rate of natural resources is implicitly specified by

$$\tau_* = 1 - \frac{Z_*^S(\theta_*, \tau_*)}{Z_*^N(\theta_*)} \rightarrow \tau(\theta_*), \quad (43)$$

with $Z_*^S < Z_*^N$ for $\tau_* > 0$ and $Z_*^S > 0$ for $\tau_* < 1$.²⁶

(v) Skilled and unskilled-labor complementary innovations evolve along the balanced growth path in compliance with

$$g_* = g_*^{N,i} = [(n_*)^\beta (g_*^R)^{1-\beta}]^{\frac{1}{\delta}}, \quad i = u, s, \quad (44)$$

where $n_* = n(\theta_*) = (1 - \theta_*)n^{u,u}$ (see Eq. (39)) represents the average number of children in steady state and $g_*^R = 1 - \tau_*$ the growth rate of extracted natural resources.

From Proposition 2, item (i), it follows that the skilled-unskilled population ratio, \tilde{L}_* , and thus the employment ratio, \tilde{L}_*^Y , are increasing in the share of unskilled households raising skilled offspring, θ_* . Hence, the long-run blueprint ratio, \tilde{N}_* (see item (ii)), and subsequently the long-run skill premium, \tilde{w}_* , are also rising in θ_* , given that $\sigma > 1 + \beta(1 + \delta)$. Imposing steady state conditions on indifference condition (15), the long-run skill premium, $\tilde{w}(\theta_*)$, is fixed to the constant $z/(z - \phi)$, see (item (iii) and Eq. (42)). Thus \tilde{L}_*^Y is obtained as

$$\tilde{L}_*^Y = \left[\frac{[z/(z - \phi)]^\psi}{\tilde{\eta}^{\sigma-1} \Gamma^{1+\delta}} \right]^{\frac{1}{\sigma-1-\beta(1+\delta)}}, \quad (45)$$

with $\Gamma = ((1 - \gamma)/\gamma)^{\varepsilon\beta} (A/B)^{(\sigma-1)(1-\beta)}$ and \tilde{L}_*^Y given by (40). Hence, an increase in θ_* which induces an increase in \tilde{L}_*^Y , can be caused by an increase in schooling quality ϕ or a reduction in relative research productivities, $\tilde{\eta}$. Thus item (iii) of Proposition 2 implies

$$\frac{\partial \theta_*}{\partial \phi} > 0, \quad \text{and} \quad \frac{\partial \theta_*}{\partial \tilde{\eta}} < 0 \quad (46)$$

for $0 < \phi < z$.²⁷

An increase in ϕ increases the skilled wage premium, \tilde{w}_* , accompanied by an increase in \tilde{L}_*^Y which

²⁶Note that $Z_*^S = \frac{\rho}{1+\gamma+\rho} \beta [1 + \tilde{\omega}_* \tilde{L}_*] - \frac{1-\beta}{\varphi_*^{x,u}} \frac{1-\tau_*}{\tau_*}$ and $Z_*^N = \beta(1 - \beta)(1 + \tilde{N}_*^{1+\delta} \tilde{\eta}^{-1})$.

²⁷Note that $\phi > z$ is meaningless and $\theta_* = 0$, if $\phi = 0$, since then $n_*^{u,s} = n^{u,u}$.

ultimately directs technological change towards skilled labor complementary innovations reflected by the increase in \tilde{N}_* (see item (ii) of Proposition 2). In light of item (iii), the depletion rate of natural resources is inversely related to the ratio between aggregate savings and aggregate relative profits of future innovations, Z_*^S/Z_*^N . An increase in Z_*^S would raise aggregate expenditures on R&D, enhance productivity growth, and thus reduce τ_* . Before we discuss this point more in detail, we turn to the long-run growth rate of productivity, g_* , which is affected by n_* as well as τ_* , see item (iv) of Proposition 2. Obviously, the sign of δ , which represents the degree of intertemporal knowledge spillovers in R&D, is crucial for the impact of population growth and natural resource depletion on productivity growth in the long-run. If there are diminishing technological opportunities, i.e. $\delta > 0$, an increase in n_* would increase productivity growth but an increase in τ_* would reduce it, and vice versa for $\delta < 0$. According to United Nation's long-run projections, a stationary world population seems to be the most plausible assumption for the long-run, i.e. $n_* = 1$. Under these circumstances, the prospects of long-run productivity growth are rather pessimistic if R&D is subject to diminishing technological opportunities, i.e. $\delta > 1$.²⁸ Thus, in view of stationary or even shrinking populations sufficiently large intertemporal knowledge spillovers in R&D with respect to the existing stock of blueprints ($\delta < 0$) are mandatory for sustained economic growth.

So far we have gained insights into the behavior of the productivity growth rate, the population's growth rate and the depletion rate in the long-run. However, in order to assess the reaction of household income, we need to know the growth rate of wage incomes which steers the level of overall savings, $s_t^i + p_t^R m_t^i$, and the level of consumption, c_t^i , per households. In light of Proposition 2, we obtain for the evolution of wages, $g_*^{w,i}$, with $i = u, s$, in steady state (for details see Appendix A.4):

$$g_*^{w,i} = \left[\frac{g_*}{(g_*^{p,R})^{1-\beta}} \right]^{\frac{1}{\beta}} = (n_*)^{\frac{1-(1-\beta)(1+\delta)}{\delta}} (1-\tau_*)^{\frac{(1-\beta)(1+\delta)}{\delta}}. \quad (47)$$

The economy moves along a sustainable growth path, whenever $g_*^{w,i} \geq 1$, which implies at least non-declining per-capita consumption levels in the long-run. For sustainable development to arise in its weakest form, Eq. (47) suggests that productivity growth, g_* , must at least compensate for the increase in the natural resource price, $g_*^{p,R}$, caused by an increasing shortage of natural resources.

Given that $g_* = [(n_*)^{1/\beta}(1-\tau_*)^{1-\beta}]^{1/\delta}$, the reaction of g_* in response to a change in ϕ or $\tilde{\eta}$ depends on:

²⁸Under these circumstances the maximum productivity growth rate would be 0 for $\tau_* = 0$ and $\delta > 0$. Hence for any interior solution $0 < \tau_* < 1$, the economy would exhibit negative productivity growth and a negative growth rate of the wage rate in both sectors, see also (47) below.

(a) $\delta \geq 0$, and (b) on the response of n_* and τ_* . The following proposition summarizes the effects of a change in ϕ on n_* , τ_* , and g_* :

Proposition 3

(i) The long-run growth rate of the population is inversely related to ϕ , i.e. $\frac{\partial n_*}{\partial \phi} < 0$.

(ii) The long-run depletion rate of natural resources is inversely related to ϕ , i.e. $\frac{\partial \tau_*}{\partial \phi} < 0$, if $0 < \phi < \phi^{crit} < z$ and the degree of intertemporal knowledge spillover with respect to existing technological knowledge is sufficiently large, in the sense that $\delta < \delta^{crit}$.²⁹

(iii) The reaction of long-run productivity growth, g_* , in response to changes schooling quality, ϕ , i.e.

$\frac{\partial g_*}{\partial \phi}$ is determined by

$$\frac{\partial g_*}{\partial \phi} = \frac{1}{\delta} \left[n_*^\beta (1 - \tau_*)^{1-\beta} \right]^{\frac{1-\delta}{\delta}} \underbrace{\left[\beta \frac{\partial n_*}{\partial \phi} \left(\frac{1 - \tau_*}{n_*} \right)^{1-\beta} - (1 - \beta) \left(\frac{n_*}{1 - \tau_*} \right)^\beta \frac{\partial \tau_*}{\partial \phi} \right]}_{\mathcal{G}^\phi}. \quad (48)$$

Thus $\frac{\partial g_*}{\partial \phi} > 0$, if either $1/\delta < 0$ and $\mathcal{G}^\phi < 0$ or $1/\delta > 0$ and $\mathcal{G}^\phi > 0$. Moreover, $\mathcal{G}^\phi \geq 0$, if

$$-\beta \frac{\frac{\partial \theta_*}{\partial \phi}}{1 - \theta_*} \geq (1 - \beta) \frac{\frac{\partial \tau_*}{\partial \phi}}{1 - \tau_*}. \quad (49)$$

Item (i) of Proposition 3 directly follows from (46): an increase in ϕ reduces $n^{s,s}$ and $n_*^{u,s}$ but leaves $n^{u,u}$ unaffected. Since, on the other hand, θ_* increases, the number of households raising unskilled offspring shrinks. Thus population growth must decline as well because $n_* = (1 - \theta_*)n^{u,u}$. The reasoning behind item (ii) is as follows: an increase in ϕ stimulates aggregate savings because the population grows at a slower pace but generates a higher level of wage incomes which implies that Z_*^S increases by more than aggregate relative profits Z_*^N . This results from the combined effect of an increase in \tilde{L}_* and \tilde{w}_* . At the same time, relative aggregate profits increase, due to an increase in \tilde{L}_*^Y as well, but due to a higher amount of skilled labor allocated to education, the increase in \tilde{L}_*^Y is smaller than the increase in \tilde{L}_* . Consequently, aggregate relative profits, Z_*^N , increase by less than Z_*^S . Thus Z_*^S/Z_*^N increases and ultimately reduces the long-run depletion rate of natural resources. Nevertheless, this mechanism requires sufficiently large intertemporal spillovers with respect to existing technological knowledge, i.e. $\delta < \delta^{crit}$. If this condition is not met, the just described effect reverses in the sense that Z_*^N increases by

²⁹A sketch of the proof can be found in Appendix A.3. Furthermore, note that $\phi \geq z$ is meaningless because \tilde{w}_* is infinity for $\phi = z$ and negative for $z < \phi$. In reality this restriction never becomes binding (see Section 6.1). Whether or not ϕ^{crit} is binding is not analytically clear. If it is binding, the sign of $\frac{\partial \tau_*}{\partial \phi}$ turns positive for $\delta > \delta^{crit}$. The other restriction is $\phi < \phi^{crit}$. If ϕ exceeds this threshold the amount of skilled labor available for production may actually shrink, due to the high amount of teachers, see (40). Numerically, it turned out that ϕ^{crit} plays only a role for very large ϕ close to z and δ approximately around 0.25 which is very unrealistic scenario, see Section 6.1. Compare also Figure 2 and Appendix B.11.

³⁰Since $n_* = (1 - \theta_*)n^{u,u}$, it follows that $\frac{\partial n_*}{\partial \phi} = -\frac{\partial \theta_*}{\partial \phi}$.

more than Z_*^S such that τ_* rises in response to increases in ϕ . The intuition behind item (iii) is similar. Note that within the range of plausible parameter values $\beta/(1-\theta_*) > (1-\beta)/(1-\tau_*)$.³¹ Given that we just observed that the reaction of Z_*^S is stronger than the reaction of Z_*^N in response to changes in ϕ , we should also expect that $\frac{\partial \theta_*}{\partial \phi} > -\frac{\partial \tau_*}{\partial \phi}$ and conjecture that $\mathcal{G}^\phi < 0$, such that $\frac{\partial g_*}{\partial \phi} > 0$ if $\delta < 0$.

In Figure 2, we illustrate the results discussed above with respect to changes in ϕ graphically for the cases $\delta < 0$ and $\delta > 0$ under realistic assumptions regarding the set of parameters (see Section 6.1). An increase in schooling quality clearly reduces population growth and the depletion of natural resources in the long-run. In line with the above discussion in light of Proposition 3, the negative effect of ϕ on n_* is greater than the negative effect of ϕ on τ_* . Thus, productivity growth and the growth rate of wages shrink if there are diminishing technological opportunities in R&D, i.e. $\delta > 0$ because $\mathcal{G}^\phi < 0$. Whenever, ϕ exceeds a critical level, the growth rate of productivity and wage income even turns negative. However, if there are intertemporal knowledge spillovers with respect to existing technological knowledge, i.e. $\delta < 0$, schooling quality has to be sufficiently high in order to generate positive growth rates in productivity and wage income.

Figure 2 about here

The effects of a change in the relative research productivity $\tilde{\eta}$ are summarized in the following proposition.

Proposition 4

- (i) The long-run growth rate of the population is positively related to $\tilde{\eta}$, i.e. $\frac{\partial n_*}{\partial \tilde{\eta}} > 0$.
- (ii) The long-run depletion rate of natural resources is positively related to $\tilde{\eta}$, i.e. $\frac{\partial \tau_*}{\partial \tilde{\eta}} > 0$, for $\tilde{\eta} \in (\tilde{\eta}^\#, \tilde{\eta}^{\#\#})$, with $\tilde{\eta}^\# \geq \tilde{\eta}^{crit} > 0$ and $\tilde{\eta}^{\#\#} \leq +\infty$.³²
- (iii) The reaction of long-run productivity growth, g_* , in response to changes in relative research productivities, $\tilde{\eta}$, i.e. $\frac{\partial g_*}{\partial \tilde{\eta}}$ is determined by

$$\frac{\partial g_*}{\partial \tilde{\eta}} = \frac{1}{\delta} \left[n_*^\beta (1-\tau_*)^{1-\beta} \right]^{\frac{1-\delta}{\delta}} \underbrace{\left[\beta \frac{\partial n_*}{\partial \tilde{\eta}} \left(\frac{1-\tau_*}{n_*} \right)^{1-\beta} - (1-\beta) \left(\frac{n_*}{1-\tau_*} \right)^\beta \frac{\partial \tau_*}{\partial \tilde{\eta}} \right]}_{\mathcal{G}^{\tilde{\eta}}}. \quad (50)$$

Thus $\frac{\partial g_*}{\partial \tilde{\eta}} > 0$, if either $1/\delta < 0$ and $\mathcal{G}^{\tilde{\eta}} < 0$ or $1/\delta > 0$ and $\mathcal{G}^{\tilde{\eta}} > 0$. Thus $\mathcal{G}^{\tilde{\eta}} \gtrless 0$, if

$$-\beta \frac{\frac{\partial \theta_*}{\partial \tilde{\eta}}}{1-\theta_*} \gtrless (1-\beta) \frac{\frac{\partial \tau_*}{\partial \tilde{\eta}}}{1-\tau_*} \quad (51)$$

³¹ Given that $\beta = 0.65$ and $\theta_* \in [0.1; 0.15]$, we find that $\beta/(1-\theta_*) > (1-\beta)/(1-\tau_*)$ for $0 < \tau_* < 0.54$, if $\theta_* = 0.15$. A plausible value of τ_* would be around 0.3. For details, see Section 6.1.

Item (i) of Proposition 4 states that an increase in relative research productivity is associated with an increase in population growth, n_* , because the skilled-unskilled employment ratio, \tilde{L}_*^Y , adjusts endogenously through \tilde{L}_* in order to meet indifference condition (42) again. The right-hand side of (42) remains constant while the left-hand side experiences an increase through an increase in $\tilde{\eta}$, equality is again assured through a decline in θ_* (see also (46)). Thus \tilde{L}_* and \tilde{L}_*^Y shrink and the long-run population's growth rate increases. This result is in sharp contrast to item (i) of Proposition 3.

Item (ii) implies that an increase in relative research productivities increases the depletion rate of natural resources. The intuition behind this result is symmetric to item (ii) of Proposition 3 in the sense that the induced reduction in \tilde{L}_* reduces aggregate savings, i.e. Z_*^S , by more than Z_*^N , such that τ_* increases. In light of this results (and contrary to Proposition 3), item (iii) of Proposition 4 suggests that we should expect that $\mathcal{G}^{\tilde{\eta}} > 0$, given that realistically $\beta/(1-\theta_*) > (1-\beta)(1-\tau_*)$ and that, as suggested by item (ii), the induced change in θ_* exceeds the induced change in τ_* , such that $-\frac{\partial\theta_*}{\partial\tilde{\eta}} > \frac{\partial\tau_*}{\partial\tilde{\eta}}$. If $\mathcal{G}^{\tilde{\eta}} > 0$ it follows that $\frac{\partial g_*}{\partial\tilde{\eta}} > 0$ if and only if $\delta > 0$. The effects of an increase in $\tilde{\eta}$ are shown in Figure 3. It turns out that an increase in relative research productivities has diametrically opposed effects on long-run growth when compared to an increase in the teacher student ratio. An increase in the student teacher ratio is conducive for economic growth if R&D is subject to positive intertemporal spillovers with respect to existing technological knowledge ($\delta < 0$), while an increase in relative research productivity enhances long-run growth in the opposite case, i.e. if R&D is subject to diminishing technological opportunities ($\delta > 0$). The reason for the different effects is that the reaction of n_* is of opposite sign in response to changes in ϕ or $\tilde{\eta}$ while the induced change (in absolute terms) in population growth exceeds the change in the depletion rate.

Figure 3 about here

Furthermore it is worth wile to note that g_*^v is like g_*^N a function of long-run population growth, n_* , and the depletion rate, τ_* . Thus, the insights from Proposition 3 and 4 also apply for (47).³³

In the next section, we explore of the transitional dynamics of our model.

³²A sketch of the proof can be found in Appendix A.4. Note that there exists a critical $\tilde{\eta} = \tilde{\eta}^{crit}$ inducing a corresponding level of θ_*^{crit} such that $(1 - \theta_*^{crit})n^{u,u} - n^{s,s} = 0$ and \tilde{L}_* approaches $+\infty$, see (39). Moreover, the second threshold level $\tilde{\eta}^{##}$ seems to be irrelevant. We couldn't exclude the existence analytically but never found a $\tilde{\eta}^{##} < +\infty$, numerically, within the range of plausible parameter values. Note moreover that an increase in δ increases θ_* and thus \tilde{L}_*^Y . At the same time $\tilde{\eta}^{crit}$ increases. The proof follows simply from (45) and is available upon request.

³³Note also that $\frac{1-(1-\beta)(1+\delta)}{\delta} > \frac{(1-\beta)(1+\delta)}{\delta}$, if $2\beta > 1/(1+\delta)$ which is for β around $2/3$ and $\delta > 0$ always full filled. Only for very small δ , i.e. very large and unrealistically high intertemporal knowledge spillover, this restriction may be violated.

6. Computational Experiments

In the previous section we found that ϕ and $\tilde{\eta}$ affect the population's growth rate and the depletion rate of natural resources along the balanced growth path. In order to address the dynamic implications of our theory, we calibrate the model and consider the dynamic implications of an increase in the teacher-student ratio, ϕ , which may mirror quality improvements in the educational system.³⁴ This numerical experiment serves to reveal the interaction between endogenous variables during the transition to a long-run equilibrium in response to a shock. In this sense our exercise has to be understood as a thought experiment.³⁵ In order to match our model to realistic parameter values and observed data points, we have to adjust our model slightly. The reason for this is that the resource income share $1 - \beta$ is far too high if β is set to $2/3$, which implies a far too low investment share and under reasonable assumptions no income growth while the depletion rate is accordingly above 0.9. In order to circumvent this problem, we introduce labor, $L_t^{x,i}$, $x = u, s$ into the production functions of machines.³⁶ Thus, production functions for machines modify to

$$x_t^i(l) = G^i N_t^i (L_t(l)^{x,i})^{1-\alpha} (R_t(l)^{x,i})^\alpha, \quad G^i > 0. \quad (52)$$

Hence, the labor income share reads now $\xi^L = \beta + (1 - \alpha)(1 - \beta)$ and the resource income share ξ^R equals $\alpha(1 - \beta)$, such that $\xi^L + \xi^R = 1$.³⁷

6.1 Calibration

Since one period encompasses approximately thirty years, we chose for the discount factor of future consumption, ρ , a value that is standard in real-business-cycle literature: 0.99 per quarter, i.e. $\rho = 0.99^{120}$ in our context. The parameter β represents the labor share in intermediate goods production and is set to 0.65. In the U.S., energy expenditures as a share of GDP amounted to 8.8% in 2006 with a maximum close to 14% at the beginning of the eighties (see Energy Information Administration, 2009). Hence, 8.8% constitutes an upper limit for the resource income share of non-renewables in our model. We therefore set $\alpha = 0.08$, which implies $\xi_R = \alpha(1 - \beta) = 0.028$ in each intermediate sector. The parameter ϕ reflects

³⁴We omit the discussion of a shock in $\tilde{\eta}$, since the effects do qualitatively not differ to a shock in ϕ . The only difference is that \tilde{w}_* is invariant with respect to changes in $\tilde{\eta}$.

³⁵This experiment is important, since it is compatible with an increase in both the skill wage premium and the relative supply of skills in the long-run.

³⁶We assume sector specific labor in order to keep the model tractable and in line with the analytical results because this assumption assures that the skill ratios in intermediate and machine production are identical, i.e. $\tilde{L}_t^x = \tilde{L}_t^Y$.

³⁷The modified system contains the previous system as a special case for $\alpha = 1$, such that the theoretical results remain valid. The system is presented in Appendix B.8.

the teacher-student ratio and is set to $1/20$. Moreover, child-rearing is subject to forgone consumption possibilities and losses in potential lifetime earnings which amount to 13% for highly educated women and higher if women drop out of the labor market completely (Dankmeyer, 1996). The direct time cost for parents raising a child to adulthood amounts to 50% of parents' time endowment (see de la Croix and Doepke, 2003), which would imply $z = 0.075$ as a lower limit. Taking losses in lifetime earnings into account, we set $z = 0.15$, which matches the U.S. skilled wage premium $\tilde{w}_* = 1.5$ (Acemoglu, 2002). The weight of children, ν , in the utility function drives the growth rate of the population. We choose a value of $\nu = 0.26$, which generates approximately zero population growth in the long-run ($-0.0007 \approx 0$ per year). Now there are five parameters left: the elasticity of substitution between intermediate goods in final good production, ε , the weight of Y_t^u in final good production, γ , the ratio of the productivity parameters in R&D, $\tilde{\eta}$, the ratio of productivity parameters in machine production, $\frac{A}{B}$, and the externality parameter in R&D, δ . We fixed these parameters by matching our model to a long-run productivity growth rate of 2.4%, an investment share, $\frac{I}{Y_t}$, in the vicinity of 14.43% - fitting the 10 year average of US private fixed capital formation as a share of GDP³⁸ (OECD Economic Outlook Database), and an employment share in education of around 2%. In addition, the long-run decline in the natural resource stock is matched to 2.4% per year which causes via Hotelling's rule a long-run interest rate close to 4%. Moreover, we fit the skilled population ratio, \tilde{L}_* , to approximately 0.6 which is in line with the relative supply of college skills in the US (Acemoglu, 1998). The remaining parameters are therefore fixed as follows: $\varepsilon = 2.4, \gamma = 0.655, \tilde{\eta} = 2.2, \delta = -0.045$, and $\frac{A}{B} = 8$.³⁹

6.2 Equilibrium Dynamics

In our numerical exercise (see Figure 4), we increase the teacher-student ratio ϕ by 1% which may reflect an increase in the quality of schools. We start with the discussion of the long-run effects. Indifference condition (15) implies that the long-run wage differential adjusts to $\tilde{w}_* = \frac{z}{z-\phi}$. After an increase in ϕ , it is more beneficial for the unskilled population group to raise skilled offspring. Consequently, θ_* increases.

Figure 4 about here

With a higher fraction of unskilled households raising skilled offspring, the long-run skilled-unskilled

³⁸For the derivation of the investment share, see Appendix B.7.

³⁹The numeric method is described in Appendix B.9. Given the parameter constellation the dynamic system exhibits two positive eigenvalues within the unit circle and two outside the unit circle, such that the economy is subject to saddle-point stability. This constellation is robust for a very large range of parameters with the exception of $\varepsilon < 2$. Note also that the transitory behavior is independent from $\delta \geq 0$.

population ratio, $\tilde{L}_* = \left(\frac{L^s}{L^u}\right)_*$, must increase as well. Consequently, relative demand for skilled-labor complementary machines, $\left(\frac{X^s}{X^u}\right)_*$, increases which generates an increase in the ratio of R&D expenditures, $\left(\frac{D^s}{D^u}\right)_*$, and the blueprint ratio, $\tilde{N}_* = \left(\frac{N^s}{N^u}\right)_*$. Therefore, the share of extracted natural resources allocated to the s -sector, $\varphi_*^{x,s}$, must increase as well. With a higher fraction of unskilled households wishing to educate their offspring to become skilled workers, the long-run growth rate of the population, n_* , must decline. In light of Proposition 3, item (ii), a higher share of unskilled households raising skilled offspring in response to an increase in ϕ is associated with a lower depletion rate of natural resources. At the same time the investment share of GDP, $\left(\frac{I}{Y}\right)_*$, increases in the long-run (Z_*^S increases), where long-run productivity growth amounts to 2.62% in comparison to 2.4% in the baseline scenario given that we assumed $\delta < 0$, here.

We now turn to the transitory effects of an increase in ϕ . Since the current wage differential is below its long-run value as well as the skilled-unskilled population ratio, a smooth convergence of the latter to its new long-run value requires that the share of unskilled households raising skilled offspring, θ_t , adjusts from above to the new long-run value. Hence, the growth rate of the skilled population group must be above its long-run value too. The opposite is true for the unskilled population group. As $n^{u,u} > n^{s,s} > n_t^{u,s}$, the population's growth rate converges from below to its actual long-run value. The increase in θ_t raises demand for skilled labor in the education sector such that $\frac{L_t^{E,s}}{L_t}$ increases at the expense of employment ratios of skilled labor in production, \tilde{L}_t^Y . Consequently, the wage differential between skilled and unskilled labor increases given the state of technology, \tilde{N}_t . Since the (relative) market size for skilled-labor complementary innovations is currently reduced, the share of natural resources allocated to the u -sector must increase, i.e. $\varphi_t^{x,s}$ declines. At the same time, the depletion rate increases in order to satisfy aggregate demand for machines. During the transition, θ_t converges from above to its higher long-run value, such that relative demand for skilled labor declines in the education sector as well. As this process comes along with an increasing skilled-unskilled population ratio, the employment ratios of skilled labor in production must rise. Therefore, skill-biased technological change meets an increase in the skilled population group. The (transitory) decline in θ_t and the increase in relative wages that is responsible for a decline in $n_t^{u,s}$ cause a decline in the growth rate of the skilled population group and an increase in the growth rate of the unskilled population group. With a lower demand for labor in the education sector and increasing employment ratios of skilled labor in production the depletion rate of natural resources declines during the transition while the growth rate of the population increases.

7. POLICY IMPLICATIONS

At the core of our theory is the negative relationship between population growth and human capital investments. As we have seen in the previous sections, a change in ϕ and a change in $\tilde{\eta}$ have opposite effects on long-run growth. An increase in ϕ enhances long-run growth prospects if there are intertemporal knowledge spillovers in R&D ($\delta < 0$), but reduces growth in the case of diminishing technological opportunities ($\delta > 0$). An increase in relative research productivity, $\tilde{\eta}$, is in turn conducive for long-run growth if $\delta > 0$ but it is harmful for productivity growth if $\delta < 0$.

An increasing number of skilled households reduces long-run population growth and reduces the depletion rate of natural resources. In this way, an increase in schooling quality increases the wage premium for skilled labor and increases the share of unskilled households that wish to raise skilled offspring. If R&D is subject to diminishing technological opportunities, i.e. $\delta > 0$, our framework suggests a negative association between an increase in ϕ and long-run productivity growth. The described channel gains in importance if one accepts long-run projections of world population growth and the argument that, in the long-run, positive population growth on a finite planet is infeasible. Given that $n_* = 1$, the long-run growth prospects are rather pessimistic, if $\delta > 0$. In this scenario our theory suggests that an increase in relative research productivities, $\tilde{\eta}$ is preferable. But as we will see below, it might still be reasonable to subsidize education. In order to keep the discussion brief, we assume as a limiting case of a progressive tax scheme that only skilled-labor is taxed while the tax rate on unskilled labor is zero. Tax revenues are used to subsidize education for unskilled households. Denote by $0 \leq T^s \leq 1$, the constant tax rate on skilled labor and by s_t^E the subsidy share allocated to unskilled households that wish to raise skilled offspring. Fertility decisions of agents $i = s, u$ modify as follows⁴⁰

$$n^{s,s} = \frac{\nu}{(1 + \nu + \rho)(z(1 - T^s) + \phi)}, \quad (53)$$

$$n_t^{u,s} = \frac{\nu}{(1 + \nu + \rho)(z + \tilde{w}_t \phi(1 - s_t^E))}, \quad (54)$$

$$n^{u,u} = \frac{\nu}{(1 + \nu + \rho)z}. \quad (55)$$

In light of fertility decisions (53)-(55), $n_t^{u,u}$ remains constant because unskilled labor income is not taxed while the number of skilled children born in unskilled as well as in skilled households increases. The latter effect is caused by the reduction in opportunity cost of child rearing time, $(1 - T^s)zw_t^s$ (see Eq.

⁴⁰The budget constraints read as: $(1 - T^S)w_t^s = ((1 - T^S)z + \phi)w_t^s n_t^{s,s} + c_t^s + p_t^R m_t^s + s_t^s$; $w_t^u = (zw_t^u + \phi(1 - s_t^E)w_t^s)n_t^{u,s} + c_t^u + p_t^R m_t^u + s_t^u$; $w_t^u = zw_t^u n_t^{u,u} + c_t^u + p_t^R m_t^u + s_t^u$. A balanced budget in the education sector requires $w_t^s \phi(n_t^{s,s} L_t^s + (1 - s_t^E)n_t^{u,s} \theta_t L_t^u) + SUB_t = w_t^s L_t^{E,s}$, with $SUB_t = T^s w_t^s L_t^s = s_t^E n_t^{u,s} \theta_t L_t^u w_t^s \phi$.

(53)), and the former by the education subsidy (see (54)).⁴¹ Assuming moreover that parents internalize the taxation of future skilled labor income in their utility function, skilled offspring is weighted by the expected post-tax income in the subsequent period, i.e. $(1 - T^s)w_{t+1}^s n_t^{i,s}$, $i = s, u$. Thus, indifference condition (15) modifies in the long-run to⁴²

$$\tilde{w}_* = \frac{z}{(1 - T^s)z - \phi(1 - s_*^E)}. \quad (56)$$

$$\text{with } s_*^E = \frac{T^s \tilde{L}_*}{\phi \theta_* n_*^{u,s}} = \frac{T^s}{\frac{\phi}{1+\nu+\rho} \left[\frac{(1-\theta_*)\nu}{z} - \frac{\nu}{(1-T^s)z+\phi} \right]}.$$

At this point it is worthwhile to note that we can expect that an increase in taxes on skilled labor will reduce the long-run skill wage premium compatible with indifference condition (56), because $\phi \ll z$. This reduces the share of unskilled households raising skilled offspring in the long-run such that θ_* shrinks. At the same time the number of skilled offspring raised in skilled and unskilled households increases. Both effects in combination induce an increase in the long-run growth rate of the population, n_* . Given that $\delta > 0$, i.e. R&D is subject to diminishing technological opportunities, an increasing subsidy share s_*^E would increase the growth rate of productivity and wage incomes.⁴³

8. SUMMARY AND CONCLUSIONS

In this paper, we integrate the features of (skill-biased) directed technological change, fertility decline and natural resource use into a comprehensive framework. More specifically, we consider an overlapping generations economy populated by skilled and unskilled households who decide on the number of children they wish to raise and their educational attainment. Thus, the composition of the population in terms of skilled and unskilled households is endogenous, while the skilled wage premium generates differential fertility between skilled and unskilled households, such that fertility is inversely related to wages and education. Consequently, the skilled-unskilled population ratio and the skill bias of technological innovations are jointly determined in equilibrium. Regarding the production side of our model, we consider a scale invariant growth model of directed technological change, in line with Acemoglu (1998;2002), where the

⁴¹Note that taxation of unskilled households would also reduce their opportunity cost of child rearing time. If skilled households would also receive a subsidy, their level of fertility would also increase due to a reduction of education cost while s^E shrinks given T^s and ϕ .

⁴²During the transition, the indifference condition reads as $(1 - T^s)\tilde{w}_{t+1} = \frac{z + \tilde{w}_t \phi(1 - s_t^E)}{z}$ which equals (15) for $T^s = 0$ implying $s_t^E = 0$.

⁴³Given the set of parameters discussed in Section 6.1, an economy with $\phi = 1/20$ and $\delta = 0.045$ would be unable to generate positive productivity growth ($g_* \approx -0.03$), in the long-run. A numerical experiment revealed that such an economy needed a subsidy share, s_*^E , above 0.4 in order to generate positive long-run growth.

production of machines is subject to a non-renewable natural resource as an essential input. We are thus able to analyze the interaction between population dynamics, the skill bias of innovations, and natural resource depletion. We believe that our results have strong implications for economic policies aimed at sustainable development in view of nonrenewable natural resources and zero population growth in the long-run, especially as the long-run productivity growth rate is not policy invariant in our framework. In regards to the relationship between the depletion rate of natural resources and population growth, we find a negative association during the transition, but a positive association in the long-run. Long-run productivity growth is determined by the population's growth rate, the depletion rate of natural resources, and the sign of intertemporal spillovers in R&D with respect to existing technological knowledge. If R&D is subject to negative spillovers, i.e. diminishing technological opportunities, a decline in the population's growth rate will have an adverse impact on productivity growth while the associated decline in the depletion rate will have a positive impact on productivity growth, and vice versa for positive intertemporal spillovers. We show that in general, the impact of a change in population growth on productivity growth will be larger than the impact of a change in the depletion rate on productivity growth. In our analysis we focus on changes in relative research productivities and the teacher-student ratio because we consider these parameters as particularly policy relevant. While the former captures the efficiency of the research infrastructure, a change in the teacher-student ratio can be seen as a metaphor for schooling quality. We find that an increase in relative research productivities has diametrically opposed effects on long-run growth in comparison to an increase in the teacher student ratio. The latter is conducive for economic growth if R&D is subject to positive intertemporal spillovers, while an increase in relative research productivities enhances long-run growth if R&D is subject to diminishing technological opportunities. The reason for this is that an increase in the teacher-student ratio reduces population growth while an increase in relative research productivity increases population growth. Moreover, our results suggests that a progressive taxation of skilled-labor income used to finance an education subsidy for unskilled households increases population growth. Hence, this policy is beneficial for long-run growth if R&D is subject to diminishing technological opportunities.

REFERENCES

- Acemoglu, D., 2002. Directed Technological Change, *Review of Economic Studies* 69, 781-809.
- Acemoglu, D., 1998. Why Do New Technologies Complement Skills? *Directed Technological Change*

- and Wage Inequality, *Quarterly Journal of Economics* 113, 1055-1089.
- Andre, F., Smulders, S., 2005. Energy Use, Endogenous Technical Change and Economic Growth, Mimeo, CentER-Tilburg University.
- Autor, D., Krueger, A., Katz, L., 1998. Computing Inequality: Have Computers Changed the Labor Market?, *Quarterly Journal of Economics* 113, 1169-1214.
- Baghwati, J.N., Srinivasan, T.N., 1977. Education in a 'Job Ladder' Model and the Fairness- in-Hiring Rule, *Journal of Public Economics* 7, 1-22.
- Dankmeyer, B., 1996. Long Run Opportunity-Costs of Children According to Education of the Mother in the Netherlands, *Journal of Population Economics* 9, 349-361.
- Dasgupta, P., Heal, G.M., 1974. The Optimal Depletion of Exhaustible Resources, *Review of Economic Studies Symposium* 41, 3-28.
- de la Croix, D., Doepke, M., 2003. Inequality and Growth: Why Differential Fertility Matters, *American Economic Review* 93, 1091-1113.
- de la Croix, D., Doepke, M., 2004. Public versus Private Education when Fertility Differentials Matter, *Journal of Development Economics* 73, 607-629.
- di Maria, C., Smulders, S., 2004. Trade Pessimists vs. Technology Optimists: Induced Technical Change and Pollution Havens, *Advances in Economic Analysis & Policy* 4, Article 7.
- di Maria, C., Valente, S., 2008. Hicks Meets Hotelling: The Direction of Technical Change in Capital-Resource Economies, *Environment and Development Economics* 13, 691-717.
- di Maria, C., van der Werf, E., 2005. Carbon Leakage Revisited: Unilateral Climate Policy under Directed Technical Change, CentER Discussion Paper Series, No. 2005-68, Tilburg University.
- Eicher, T.S., 1996. Interaction between Endogenous Human Capital and Technological Change, *Review of Economic Studies* 63, 127-144.
- Eicher, T.S., 1999. Trade, Development and Converging Growth Rates: Dynamic Gains from Trade Reconsidered, *Journal of International Economics* 48, 179-198.
- Evenson, R.E., 1984. International Invention: Implications for Technology Market Analysis. R&D: Patents and Productivity, ed. by Zvi Griliches, University of Chicago Press, Chicago.

- Galor, O., J. Zeira, 1993. Income Distribution and Macroeconomics, *Review of Economic Studies* 60, 35-52.
- Galor, O., 2005. From Stagnation to Growth: Unified Growth Theory. In Aghion, P., Durlauf, S., (eds), *Handbook of Economic Growth*. Elsevier, North-Holland: Amsterdam.
- Galor, O., Mountford, A., 2006. Trade and the Great Divergence: The Family Connection. *American Economic Review* 96, 229-233.
- Galor, O., Weil, D.N., 1996. The Gender Gap, Fertility, and Growth, *American Economic Review* 86, 374-387.
- Galor, O., Weil, D.N., 2000. Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond, *American Economic Review* 90, 806-824.
- Groth, C., 2007. A New-Growth Perspective on Non-renewable Resources. In L. Bretschger, Smulders, S., (eds.) *Sustainable Resource Use and Economic Dynamics*, Springer, Dordrecht.
- Guvenen, F., B. Kuruscu, 2012. *Journal of the European Economic Association* 10, 482-517.
- Hornstein, A., Krusell, P., Violante, G.L., 2005. The Effects on Technical Change on Labor Market Inequalities. In Aghion, P., Durlauf, S., (eds), *Handbook of Economic Growth*. Elsevier, North-Holland: Amsterdam.
- He, H., 2012. What drives the skill premium: Technological Change or Demographic Variation?, *European Economic Review* 56, 1546-1572.
- He, H, Z. Liu, 2008. Investment-specific Technological Change, Skill Accumulation, and Wage Inequality, *Review of Economic Dynamics* 11, 314-334.
- Jones, C.I., Williams, J.C., 2000. Too Much of a Good Thing? The Economics of Investment in R&D, *Journal of Economic Growth* 5, 65-85.
- Kohler, H.P., Billari, F.C., Ortega, J.A., 2002. The Emergence of Lowest-Low-Fertility in Europe during the 1990s. *Population and Development Review* 28, 641-688.
- Kortum, S., 1993. Equilibrium R&D and the Patent-R&D Ratio: U.S. Evidence, *American Economic Association Papers and Proceedings* 83, 450-457.

- Kremer, M., Chen, D., 2000. Income Distribution Dynamics with Endogenous Fertility, NBER, Working Paper 7530.
- Lee, R., 2003. The Demographic Transition: Three Centuries of Fundamental Change, Journal of Economic Perspectives 17, 167-190.
- OECD, 2009. Economic Outlook No. 86.
- Pittel, K., 2002. Sustainability and Endogenous Growth, Edward Elgar, Cheltenham.
- Rosenzweig, M.R., Wolpin, K.I., 1994. Are there Increasing Returns to the Intergenerational Production of Human Capital, Journal of Human Resources 29, 670-639.
- Schäfer, A., Valente, S., 2011. Habit Formation, Dynastic Altruism, and Population Dynamics, Macroeconomic Dynamics, Vol. 15, 365-397.
- Solow, R.M., 1974. Intergenerational Equity and Exhaustible Resources, Review of Economic Studies Symposium 41, 29-45.
- Stiglitz, J.E., 1974. Growth with Exhaustible Resources: The Competitive Economy, Review of Economic Studies Symposium 41, 139-152.
- U.S. Energy Information Administration, 2006. Annual Energy Review.
- U.S. Energy Information Administration, 2008. Annual Energy Review.
- U.S. Energy Information Administration, 2009. Annual Energy Review.
- Weil, D.N., 2005. Economic Growth. Pearson, Boston.

A. MATHEMATICAL APPENDIX

A.1 Proof of Proposition 1, item(iii)

Given profit maximizing behavior and perfect competition on factor markets, the value marginal products of natural resources in machine production are equalized between the s - and the u -sector and are both equal to p_t^R . Note further that the ratio of marginal production costs reads

$$(\tilde{c}_t^x)^{-1} = \frac{A}{B} \tilde{N}_t. \quad (\text{A.1})$$

Taking account for profit maximizing demand for machines (21) and relative prices of intermediates (24), we obtain

$$R_t^{x,s} = ((1-\gamma)/\gamma)^{\frac{\varepsilon}{\sigma}} (\tilde{N}_t)^{\frac{\sigma-1}{\sigma}} (\tilde{L}_t^Y)^{\frac{\sigma-1}{\sigma}} (\tilde{c}_t^x)^{\frac{(\sigma-1)(\beta-1)}{\sigma\beta}} R_t^{x,u}. \quad (\text{A.2})$$

As $R_t = R_t^{x,u} + R_t^{x,s}$, we obtain (27) and (28).

A.2 Proof of Proposition 2

Along the balanced growth path, the blueprint ratio, employment ratios of labor and the wage differential of skilled labor are constant, we therefore know from (30) that

$$\tilde{N}_* = (\tilde{\eta})^{\frac{\sigma}{1+\delta\sigma}} ((1-\gamma)/\gamma)^{\frac{\varepsilon}{1+\delta\sigma}} (\tilde{L}_*^Y)^{\frac{\sigma-1}{1+\delta\sigma}} (\tilde{c}_*^x)^{\frac{(\sigma-1)(\beta-1)}{\beta(1+\delta\sigma)}}. \quad (\text{A.3})$$

Substituting for \tilde{c}_*^x yields (41) which implies together with (25), \tilde{w}_* as specified in (42).

Uniqueness θ_* :

Since $\tilde{L}_* = \frac{\theta_* n_*^{u,s}}{(1-\theta_*)n_*^{u,u} - n_*^{s,s}}$, $\tilde{L}_*^Y = (1 - \phi n_*^{s,s})\tilde{L}_* - \phi\theta_* n_*^{u,s}$ and $\omega_* = \frac{z}{z-\phi}$, we obtain

$$(1 - \phi n_*^{s,s})\tilde{L}_* - \phi\theta_* n_*^{u,s} = \bar{\Phi},$$

with $\bar{\Phi} = \left[\left(\frac{z}{z-\phi} \right)^\psi \left(\frac{\eta^s}{\eta^u} \right)^{1-\sigma} \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon\beta(1+\delta)} \left(\frac{A}{B} \right)^{(1-\sigma)(1-\beta)(1+\delta)} \right]^{\frac{1}{\sigma-1-\beta(1+\delta)}}$, and $\psi = \beta - (1-\beta)(\sigma-1) + \delta(\beta + (\sigma-1)\beta)$, such that

$$\theta_*^2 + \theta_* \frac{\Phi_2}{\Phi_1} - \frac{\Phi_3}{\Phi_1} = 0, \quad (\text{A.4})$$

with $\Phi_1 = \phi n_*^{u,u} n_*^{u,s}$; $\Phi_2 = (1 - \phi n_*^{s,s})n_*^{u,s} + \bar{\Phi} n_*^{u,s} - (n_*^{u,u} - n_*^{s,s})\phi n_*^{u,s}$; $\Phi_3 = \bar{\Phi}(n_*^{u,u} - n_*^{s,s})$. With one positive and one negative root, the solution to (A.4) is unique. Hence, \tilde{L}_* , \tilde{L}_*^Y and \tilde{N}_* are unique and constant as well which implies that $p_*^{Y,s}$ and $p_*^{Y,u}$ are constant and unique as well.

Uniqueness τ_* :

The long-run rate of depletion is implicitly defined by

$$F(\theta_*, \tau_*) = 1 - \frac{Z_*^S(\theta_*, \tau_*)}{Z_*^N(\theta_*)} - \tau_* = 0, \quad (\text{A.5})$$

with

$$Z_*^S(\theta_*, \tau_*) = \frac{\rho}{1+\nu+\rho} \beta \overbrace{\left(1 + \frac{z}{z-\phi} \tilde{L}_* \right)}^{Z_*^{S,A}} - \overbrace{(1-\beta)(1+\Psi_*)}^{Z_*^{S,B}} \frac{1-\tau_*}{\tau_*}, \quad (\text{A.6})$$

$$= Z_*^{S,A} - Z_*^{S,B} \frac{1-\tau_*}{\tau_*}, \quad (\text{A.7})$$

$$\Psi_* = ((1-\gamma)/\gamma)^{\frac{\varepsilon}{\sigma}} (A/B)^{\frac{(\sigma-1)(1-\beta)}{\sigma\beta}} (\tilde{N}_*)^{\frac{\sigma-1}{\sigma}} (\tilde{L}_*^Y)^{\frac{\sigma-1}{\sigma}} \quad (\text{A.8})$$

and

$$Z_*^N = \beta(1 - \beta) \left(1 + \tilde{\eta}^{-1} \tilde{N}_*^{1+\delta}\right). \quad (\text{A.9})$$

Eq. (B.106) exhibits two real solutions $\tau_*^{1,2}$ (one negative and one positive), such that the only economically meaningful solution is

$$\tau_* = \frac{1}{2} \left[1 - \overbrace{\left(\frac{Z_*^{S,A}}{Z_*^N} + \frac{Z_*^{S,B}}{Z_*^N} \right)}^{Z_1} \right] + \left\{ \frac{1}{4} \left[1 - \overbrace{\left(\frac{Z_*^{S,A}}{Z_*^N} + \frac{Z_*^{S,B}}{Z_*^N} \right)}^{Z_1} \right]^2 + \overbrace{\frac{Z_*^{S,B}}{Z_*^N}}^{Z_2} \right\}^{1/2} \quad (\text{A.10})$$

$$\tau_* = \frac{1 - Z_1}{2} + \left\{ \frac{(1 - Z_1)^2}{4} + Z_2 \right\}^{1/2} \quad (\text{A.11})$$

Note that the discriminant is always positive since $Z_*^B, Z_*^N > 0 \rightarrow Z_2 > 0$.

Moreover, as τ_* and θ_* are unique, there exists a unique balanced growth path.

Item (iv):

Given a symmetric equilibrium, the production function of intermediate $Y^i, i = u, s$ writes as

$$Y_t^i = N_t^i (R_t^{x,i})^{(1-\beta)} (L^{Y,i})^\beta, \quad (\text{A.12})$$

such that the steady state gross-growth rate of $Y_t^i, g_*^{Y,i}, i = u, s$ is given by

$$g_*^{Y,i} = g_*^{N,i} (g_*^R)^{1-\beta} (g_*^L)^\beta. \quad (\text{A.13})$$

Since innovations evolve according to (20) and are fueled by aggregate savings, we obtain in steady state $g_*^{D,i} = g_*^{Y,i}$. Moreover, in steady state, we yield from (20)

$$1 = \frac{g_*^{D,i}}{(g_*^{N,i})^{1+\delta}} = \frac{g_*^{Y,i}}{(g_*^{N,i})^{1+\delta}}. \quad (\text{A.14})$$

Hence, $(g_*^{N,i})^{1+\delta} = g_*^{Y,i} = g_*^{N,i} (g_*^L)^\beta (g_*^R)^{1-\beta}$ and

$$g_*^{N,i} = \left[n_*^\beta (1 - \tau_*)^{(1-\beta)} \right]^{\frac{1}{\delta}}, \quad (\text{A.15})$$

since $g_{t+1}^R = (1 - \tau_t) \frac{\tau_{t+1}}{\tau_t}$ and $\tau_* = \tau_t = \tau_{t+1}$.

Since the wages in units of final output read $w_t^i = (1 - \beta) \frac{1-2\beta}{\beta} (p_t^{Y,i})^{\frac{1}{\beta}} (N_t^i)^{\frac{1}{\beta}} (c_t^{x,i})^{\frac{\beta-1}{\beta}}$, the corresponding growth rates can be expressed by⁴⁴

$$g_*^w = g_*^{w^s} = g_*^{w^u} = \left[\frac{g_*^N}{(g_*^R)^{1-\beta}} \right]^{\frac{1}{\beta}} = \left[n_*^\beta (1 - \tau_*)^{1-\beta} \right]^{\frac{1}{\delta}} \left[\frac{1 - \tau_*}{n_*} \right]^{1-\beta}. \quad (\text{A.16})$$

⁴⁴In Appendix B.6, we exploit the general equilibrium structure more in detail.

A.3 Proof of Proposition 3, item (ii)

In light of (A.11), $\frac{\partial \tau_*}{\partial \phi} < 0$, if⁴⁵

$$\frac{\partial \tau_*}{\partial \phi} = -\frac{\partial Z_1}{\partial \phi} + \left[\frac{(1 - Z_1)^2}{4} + Z_2 \right]^{-1/2} \left[-\frac{\frac{\partial Z_1}{\partial \phi} (1 - Z_1)}{2} + \frac{\partial Z_2}{\partial \phi} \right] < 0 \quad (\text{A.17})$$

which requires that

$$\frac{\frac{\partial Z_1}{\partial \phi}}{\frac{\partial Z_2}{\partial \phi}} Z_2 > -(1 - Z_1) + \frac{\frac{\partial Z_2}{\partial \phi}}{\frac{\partial Z_1}{\partial \phi}} \quad (\text{A.18})$$

Note that:

(i) $Z_*^N, Z_*^{S,A}, Z_*^{S,B} > 0$ with $Z_*^N < Z_*^{S,B}$ and $Z_*^N > Z_*^{S,A}$ in the range of plausible parameter values, such that $1 - (Z_*^{S,A}/Z_*^N + Z_*^{S,B}/Z_*^N) < 0$.

(ii) Thus $(1 - Z_1) < 0$.

(iii) It follows from (i) and (ii) that $Z_2 > -(1 - Z_1)$, since $\frac{Z_*^{S,B}}{Z_*^N} > -[1 - \frac{Z_*^{S,A} + Z_*^{S,B}}{Z_*^N}] \Rightarrow Z_*^N > Z_*^{S,A}$.

(iv) $\frac{\partial Z_2}{\partial \phi} = \frac{\partial \frac{Z_*^{S,B}}{Z_*^N}}{\partial \phi} < 0$

(v) $\frac{\partial Z_1}{\partial \phi} > \frac{\partial Z_2}{\partial \phi}$, since

$$\frac{\left[\frac{\partial Z_*^{S,A}}{\partial \phi} + \frac{\partial Z_*^{S,B}}{\partial \phi} \right] Z_*^N - [Z_*^{S,A} + Z_*^{S,B}] \frac{\partial Z_*^N}{\partial \phi}}{(Z_*^N)^2} > \frac{\frac{\partial Z_*^{S,B}}{\partial \phi} Z_*^N - Z_*^{S,B} \frac{\partial Z_*^N}{\partial \phi}}{(Z_*^N)^2} \quad (\text{A.19})$$

$$\frac{\frac{\partial Z_*^{S,A}}{\partial \phi} Z_*^N - Z_*^{S,A} \frac{\partial Z_*^N}{\partial \phi}}{(Z_*^N)^2} > 0, \quad (\text{A.20})$$

which is in light of the Implicit function theorem a sufficient condition for $\frac{\partial \tau_*}{\partial \phi} < 0$.

(vi) Since $\frac{\partial Z_2}{\partial \phi} < 0$, $\frac{\partial Z_1}{\partial \phi} < 0$ is in light of (iii) and (iv) a necessary condition for $\frac{\partial \tau_*}{\partial \phi} < 0$.

Next steps: $\text{sign} \left\{ \frac{\partial \frac{Z_*^{S,A}}{Z_*^N}}{\partial \phi} \right\}$ and $\text{sign} \left\{ \frac{\partial \frac{Z_*^{S,B}}{Z_*^N}}{\partial \phi} \right\}$.

Proposition 5

The ratio $Z_*^{S,B}/Z_*^N$ is monotonically declining in the interval $\phi \in (0, z)$, i.e. $\frac{\partial Z_*^{S,B}/Z_*^N}{\partial \phi} < 0$.

The proof makes use of the curvatures of $Z_*^{S,B}$ and Z_*^N in ϕ and demonstrates $Z_*^{S,B}$ exhibits a lower curvature than Z_*^N . Since both function are increasing and convex in ϕ , it follow that $Z_*^{S,B}/Z_*^N$ is a declining function in ϕ .

⁴⁵A detailed version of this proof can be found in Appendix B.11.

Proposition 6

The ratio $Z_*^{S,A}/Z_*^N$ is increasing in ϕ , i.e. $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi} > 0$, if the degree of intertemporal knowledge spillovers in R&D with respect to existing technological knowledge is sufficiently high, i.e. $\delta < \delta^{crit}$.

Proof:

Since $\tilde{N}_*^{1+\delta} = \tilde{\eta}^{\frac{\sigma\beta(1+\delta)}{\psi}} \Gamma^{\frac{1+\delta}{\psi}} (\tilde{L}_*^Y)^{\frac{(\sigma-1)\beta(1+\delta)}{\psi}}$, with $\psi = \beta - (1-\beta)(\sigma-1) + \delta(\beta + (\sigma-1)\beta)$, we obtain further

$$\frac{Z_*^{S,A}}{Z_*^N} = \frac{\frac{\rho}{1+\nu+\rho} (1 + \hat{L}_*)}{(1-\beta) (1 + \tilde{\Gamma}[\tilde{L}_*^Y]^{\tilde{\psi}})}. \quad (\text{A.21})$$

Thus, $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi} > 0$, if

$$\frac{\partial \hat{L}_*}{\partial \phi} + \tilde{\Gamma}(\tilde{L}_*^Y)^{\tilde{\psi}} \left[\frac{\partial \hat{L}_*}{\partial \phi} - \tilde{\psi} \frac{(1 + \hat{L}_*)}{\tilde{L}_*^Y} \frac{\partial \tilde{L}_*^Y}{\partial \phi} \right] > 0, \quad (\text{A.22})$$

with $\tilde{\psi} = \frac{(\sigma-1)\beta(1+\delta)}{\psi}$ and $\hat{L}_* = \frac{z}{z-\phi} \tilde{L}_*$. Note moreover that

- $\frac{\partial \hat{L}_*}{\partial \phi \partial \delta}; \frac{\partial \tilde{L}_*^Y}{\partial \phi}; \frac{\partial \tilde{L}_*^Y}{\partial \phi \partial \delta} > 0$
- $\frac{\partial (1+\hat{L}_*)}{\partial \delta} \geq 0$, if $\frac{\partial \hat{L}_*}{\partial \delta} / \frac{\partial \tilde{L}_*^Y}{\partial \delta} > (1 + \hat{L}_*) / \tilde{L}_*^Y$ for $\phi > \tilde{\phi} < z$.⁴⁶

Thus it follows that There exists a critical δ^{crit} for a $\phi = \phi^{crit} < z$ given , such that $\delta \geq \delta^{crit}$ implies

$$\left[\frac{\partial \hat{L}_*}{\partial \phi} - \tilde{\psi} \frac{(1 + \hat{L}_*)}{\tilde{L}_*^Y} \frac{\partial \tilde{L}_*^Y}{\partial \phi} \right]_{\phi=\phi^{crit}} \leq 0, \quad (\text{A.23})$$

which is a necessary condition for $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi} < 0$.

A.4 Proof of Proposition 4, item (ii)

Note that:⁴⁷

- (i) An increase in $\tilde{\eta}$ induces a decline in θ_* , i.e. $\frac{\partial \theta_*}{\partial \tilde{\eta}} < 0$, and thus a decline in \tilde{L}_* as well as \tilde{L}_*^Y , i.e.

$$\frac{\partial \tilde{L}_*}{\partial \tilde{\eta}} < 0 \text{ and } \frac{\partial \tilde{L}_*^Y}{\partial \tilde{\eta}} < 0.$$

- (ii) Since $\tilde{L}_* = \frac{\theta_* n_*^{u,s}}{(1-\theta_*)n_*^{u,u} - n_*^{s,s}}$, it follows that there exists a critical $\tilde{\eta} = \tilde{\eta}^{crit}$, such that $(1-\theta_*)n_*^{u,u} - n_*^{s,s} = 0$. Hence, \tilde{L}_* exhibits a vertical asymptote at $\tilde{\eta} = \tilde{\eta}^{crit}$, with $\tilde{L}_* > 0$ for $\tilde{\eta} > \tilde{\eta}^{crit}$ and $\tilde{L}_* < 0$ for $\tilde{\eta} < \tilde{\eta}^{crit}$. Moreover, $\lim_{\tilde{\eta} \rightarrow \tilde{\eta}^{crit}} = +\infty$, if $\tilde{\eta} > \tilde{\eta}^{crit}$ and $\lim_{\tilde{\eta} \rightarrow \tilde{\eta}^{crit}} = -\infty$, if $\tilde{\eta} < \tilde{\eta}^{crit}$.

⁴⁶If $\phi > \tilde{\phi}$ we know that \tilde{L}_*^Y increases in δ . If this is the case, \tilde{L}_* must increase as well. Since this increases the demand for teachers, the increase in \tilde{L}_* must be stronger than the increase in \tilde{L}_*^Y .

⁴⁷A detailed version of the proof can be found in Appendix B.12.

Consequently, economic meaningful solutions are obtained, if $\tilde{\eta} > \tilde{\eta}^{crit}$. Since furthermore

$$(a) \quad \frac{\partial Z_*^{S,B}/Z_*^N}{\partial \tilde{\eta}} = 0$$

(b) thus $\frac{\partial \tau_*}{\partial \tilde{\eta}} \geq 0$ depends in light of the Implicit function theorem on the sign of $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \tilde{\eta}} \leq 0$ because

$$\frac{\partial \tau_*}{\partial \tilde{\eta}} = -\frac{F_{\tilde{\eta}}}{F_{\tau_*}}, \text{ with } F_{\tau_*} < 0 \text{ and } F_{\tilde{\eta}} = -\frac{\partial Z_*^{S,A}}{\partial \tilde{\eta}}.$$

(c) $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \tilde{\eta}} < 0$ for $\eta \in (\tilde{\eta}^\#, \tilde{\eta}^{\#\#})$, with $\tilde{\eta}^{\#\#} \leq +\infty$ and $\tilde{\eta}^\# \geq \tilde{\eta}^{crit}$ and thus

$$\frac{\partial \tau_*}{\partial \tilde{\eta}} > 0. \tag{A.24}$$

B. Further Derivations - Not for Publication

B.1 Derivation of the price index

For readers' convenience we omit the time index. Aggregate output Y is composed out of two intermediates Y^u and Y^s and is subject to the following CES-production function:

$$Y = \left[\gamma(Y^u)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(Y^s)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \quad (\text{B.1})$$

Competitive behavior implies the following optimality conditions for profit maximizing factor demand, given that $p^Y \equiv 1$

$$\frac{\partial Y}{\partial Y^u} = \left[\gamma(Y^u)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(Y^s)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}-1} \gamma(Y^u)^{\frac{\varepsilon-1}{\varepsilon}-1} - p^{Y,u} = 0, \quad (\text{B.2})$$

$$\frac{\partial Y}{\partial Y^s} = \left[\gamma(Y^u)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(Y^s)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}-1} (1-\gamma)(Y^s)^{\frac{\varepsilon-1}{\varepsilon}-1} - p^{Y,s} = 0. \quad (\text{B.3})$$

Hence, relative factor prices for intermediates read

$$\frac{p^{Y,s}}{p^{Y,u}} = \frac{1-\gamma}{\gamma} \left(\frac{Y^s}{Y^u} \right)^{-\frac{1}{\varepsilon}}, \quad (\text{B.4})$$

implying that

$$Y^s = \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} \left(\frac{p^{Y,u}}{p^{Y,s}} \right)^{\varepsilon} Y^u. \quad (\text{B.5})$$

Moreover, production costs write as

$$C = p^{Y,u}Y^u + p^{Y,s}Y^s, \quad (\text{B.6})$$

$$C = p^{Y,u}Y^u + p^{Y,s} \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} \left(\frac{p^{Y,u}}{p^{Y,s}} \right)^{\varepsilon} Y^u, \quad (\text{B.7})$$

$$C = Y^u \left[p^{Y,u} + p^{Y,s} \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} \left(\frac{p^{Y,u}}{p^{Y,s}} \right)^{\varepsilon} \right]. \quad (\text{B.8})$$

In light of the last expression we are able to express factor demand for intermediates in final good production in terms of production costs, C , factor prices and parameters of the production function

$$Y^u = \frac{C}{p^{Y,u} + p^{Y,s} \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} \left(\frac{p^{Y,u}}{p^{Y,s}} \right)^{\varepsilon}}, \quad (\text{B.9})$$

$$Y^u = \frac{C}{(p^{Y,u})^{\varepsilon} \gamma^{-\varepsilon} [(p^{Y,u})^{1-\varepsilon} \gamma^{\varepsilon} + (1-\gamma)^{\varepsilon} (p^{Y,s})^{1-\varepsilon}]}, \quad (\text{B.10})$$

$$Y^u = \frac{C(p^{Y,u})^{-\varepsilon} \gamma^{\varepsilon}}{(p^{Y,u})^{1-\varepsilon} \gamma^{\varepsilon} + (1-\gamma)^{\varepsilon} (p^{Y,s})^{1-\varepsilon}}. \quad (\text{B.11})$$

Manipulating terms, yields

$$(Y^u)^{\frac{\varepsilon-1}{\varepsilon}} = \frac{C^{\frac{\varepsilon-1}{\varepsilon}} (p^{Y,u})^{1-\varepsilon} \gamma^{\varepsilon-1}}{[(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{\frac{\varepsilon-1}{\varepsilon}}}, \quad (\text{B.12})$$

$$\gamma(Y^u)^{\frac{\varepsilon-1}{\varepsilon}} = \frac{C^{\frac{\varepsilon-1}{\varepsilon}} (p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon}{[(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{\frac{\varepsilon-1}{\varepsilon}}}, \quad (\text{B.13})$$

and similarly

$$(1-\gamma)(Y^s)^{\frac{\varepsilon-1}{\varepsilon}} = \frac{C^{\frac{\varepsilon-1}{\varepsilon}} (p^{Y,s})^{1-\varepsilon} (1-\gamma)^\varepsilon}{[(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{\frac{\varepsilon-1}{\varepsilon}}}. \quad (\text{B.14})$$

Taking this two results together yields

$$\gamma(Y^u)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(Y^s)^{\frac{\varepsilon-1}{\varepsilon}} = \quad (\text{B.15})$$

$$= \frac{C^{\frac{\varepsilon-1}{\varepsilon}} (p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon}{[(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{\frac{\varepsilon-1}{\varepsilon}}} + \frac{C^{\frac{\varepsilon-1}{\varepsilon}} (p^{Y,s})^{1-\varepsilon} (1-\gamma)^\varepsilon}{[(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{\frac{\varepsilon-1}{\varepsilon}}}, \quad (\text{B.16})$$

$$= \frac{C^{\frac{\varepsilon-1}{\varepsilon}}}{[(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{\frac{\varepsilon-1}{\varepsilon}}} [\gamma^\varepsilon (p^{Y,u})^{1-\varepsilon} + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}], \quad (\text{B.17})$$

$$= \frac{C^{\frac{\varepsilon-1}{\varepsilon}}}{[(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{-\frac{1}{\varepsilon}}}. \quad (\text{B.18})$$

Since, $Y = \left[\gamma(Y^u)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(Y^s)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[\frac{C^{\frac{\varepsilon-1}{\varepsilon}}}{[(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{-\frac{1}{\varepsilon}}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$,

it follows immediately that

$$Y = C[(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{\frac{1}{\varepsilon-1}}, \quad (\text{B.19})$$

and finally

$$Y[(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = C, \quad (\text{B.20})$$

with $p \equiv 1 = [(p^{Y,u})^{1-\varepsilon} \gamma^\varepsilon + (1-\gamma)^\varepsilon (p^{Y,s})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$ representing the price index of Y .

B.2 Allocation of Extracted Natural Resources

Perfect competition and profit maximizing behavior imply

$$p_t^R = p_t^{x,u} \frac{X_t^s}{R_t^{x,s}} = p_t^{x,s} \frac{X_t^u}{R_t^{x,u}}. \quad (\text{B.21})$$

Since $p_t^{x,i} = \frac{c_t^{x,i}}{1-\beta}$, we obtain

$$\tilde{c}_t^x \tilde{X}_t = \tilde{c}_t^x \tilde{N}_t \tilde{x}_t = (\tilde{c}_t^x)^{\frac{1-\beta}{\beta}} (\tilde{p}_t^Y)^{\frac{1}{\beta}} \tilde{N}_t \tilde{L}_t^{Y,s} = \frac{R_t^{x,s}}{R_t^{x,u}}. \quad (\text{B.22})$$

Observing that $\hat{p}_t^Y = \frac{p_t^{Y,s}}{p_t^{Y,u}}$ yields

$$\frac{R_t^{x,s}}{R_t^{x,u}} = ((1-\gamma)/\gamma)^{\frac{\varepsilon}{\sigma}} (\tilde{N}_t)^{\frac{\sigma-1}{\sigma}} (\tilde{L}_t^{Y,s})^{\frac{\sigma-1}{\sigma}} (\tilde{c}_t^x)^{\frac{(\sigma-1)(1-\beta)}{\sigma\beta}}. \quad (\text{B.23})$$

As an efficient use of extracted natural resources requires $R_t^{x,s} + R_t^{x,u} = R_t$,

we finally arrive to

$$R_t^{x,u} = \frac{1}{1 + ((1-\gamma)/\gamma)^{\frac{\varepsilon}{\sigma}} (\tilde{N}_t)^{\frac{\sigma-1}{\sigma}} (\tilde{L}_t^{Y,s})^{\frac{\sigma-1}{\sigma}} (\tilde{c}_t^x)^{\frac{(\sigma-1)(1-\beta)}{\sigma\beta}}} = \varphi_t^{x,u} R_t, \quad (\text{B.24})$$

$$R_t^{x,s} = \frac{((1-\gamma)/\gamma)^{\frac{\varepsilon}{\sigma}} (\tilde{N}_t)^{\frac{\sigma-1}{\sigma}} (\tilde{L}_t^{Y,s})^{\frac{\sigma-1}{\sigma}} (\tilde{c}_t^x)^{\frac{(\sigma-1)(1-\beta)}{\sigma\beta}}}{1 + ((1-\gamma)/\gamma)^{\frac{\varepsilon}{\sigma}} (\tilde{N}_t)^{\frac{\sigma-1}{\sigma}} (\tilde{L}_t^{Y,s})^{\frac{\sigma-1}{\sigma}} (\tilde{c}_t^x)^{\frac{(\sigma-1)(1-\beta)}{\sigma\beta}}} = \varphi_t^{x,s} R_t. \quad (\text{B.25})$$

B.3 Dynamics of the Blueprint Ratio

In light of (29) and (22), we know that

$$\tilde{N}_{t+1} = \tilde{\eta}^\sigma [(1-\gamma)/\gamma]^\varepsilon (\tilde{L}_{t+1}^Y)^{\sigma-1} (\tilde{N}_t)^{-\delta\sigma} (\tilde{c}_{t+1}^x)^{\frac{(\sigma-1)(1-\beta)}{\beta}}, \quad (\text{B.26})$$

such that substitution for ω_{t+1} in the marginal cost ratio yields

$$\tilde{N}_{t+1} = \tilde{\eta}^\sigma [(1-\gamma)/\gamma]^\varepsilon (\tilde{L}_{t+1}^Y)^{\sigma-1} (\tilde{N}_t)^{-\delta\sigma} \left(A/B \tilde{N}_{t+1} \right)^{\frac{(\sigma-1)(1-\beta)}{\beta}}, \quad (\text{B.27})$$

$$\tilde{N}_{t+1} = \left[\tilde{\eta}^{\sigma\beta} [(1-\gamma)/\gamma]^{\varepsilon\beta} (A/B)^{(\sigma-1)(1-\beta)} (\tilde{N}_t)^{-\delta\sigma\beta} (\tilde{L}_{t+1}^Y)^{(\sigma-1)\beta} \right]^{\frac{1}{\beta - (1-\beta)(\sigma-1)}}. \quad (\text{B.28})$$

B.4 Aggregate Savings

Aggregate savings are given by the sum of savings in both population groups

$$S_t = (s_t^u - p_t^R m_t^u) L_t^u + (s_t^s - p_t^R m_t^s) L_t^s. \quad (\text{B.29})$$

Hence,

$$S_t = \frac{\rho}{1 + \gamma + \rho} (w_t^u L_t^u + w_t^s L_t^s) - p_t^R M_t, \quad (\text{B.30})$$

$$= \frac{\rho}{1 + \gamma + \rho} w_t^u L_t^u \left[1 + \tilde{w}_t \tilde{L}_t \right] - p_t^R M_t, \quad (\text{B.31})$$

where M_t represents the aggregate natural resource stock in period t . Since

$$w_t^u = p_t^{Y,u} \beta \frac{Y_t^u}{L_t^{Y,u}}, \quad (\text{B.32})$$

$$p_t^R = p_t^{Y,u} (1-\beta) \frac{Y_t^u}{R_t^{x,u}}, \quad (\text{B.33})$$

and

$$L_t^{Y,u} = L_t^u, \quad (\text{B.34})$$

$$R_t = \tau_t M_{t-1}, \quad (\text{B.35})$$

$$R_t^{x,u} = \varphi_t^{x,u} R_t, \quad (\text{B.36})$$

we obtain

$$S_t = \frac{\rho}{1 + \gamma + \rho} \beta p_t^{Y,u} Y_t^u \left[1 + \tilde{w}_t \tilde{L}_t \right] - p_t^{Y,u} (1 - \beta) \frac{Y_t^u}{R_t^{x,u}} M_t, \quad (\text{B.37})$$

$$= \frac{\rho}{1 + \gamma + \rho} \beta p_t^{Y,u} Y_t^u \left[1 + \tilde{w}_t \tilde{L}_t \right] - \frac{p_t^{Y,u} (1 - \beta) Y_t^u M_t}{\varphi_t^{x,u} \tau_t M_{t-1}}. \quad (\text{B.38})$$

Since $\frac{M_t}{M_{t-1}} = 1 - \tau_t$, it follow outright that

$$S_t = p_t^{Y,u} Y_t^u \left\{ \frac{\rho}{1 + \gamma + \rho} \beta \left[1 + \tilde{w}_t \tilde{L}_t \right] - \frac{1 - \beta}{\varphi_t^{x,u}} \frac{1 - \tau_t}{\tau_t} \right\}. \quad (\text{B.39})$$

B.5 Depletion Rate of Natural Resources

The derivation proceeds in the following steps

- (1) The evolution of the resource price $g_{t+1}^{p^R}$
- (2) The evolution of the technology stock $g_{t+1}^{N^u}$
- (3) The evolution of price-cost ratio $\frac{g_{t+1}^{p^{Y,u}}}{g_{t+1}^{x,u}}$ using the technology market clearing condition a) and the market clearing condition for machines b)
- (4) (3) a) and b) determine the evolution of τ

In equilibrium, the evolution of the depletion rate of natural resource takes account for: (i) the evolution of the price per unit of natural resources is tight to the interest factor by Hotelling's rule ($1 + r_{t+1} = g_{t+1}^{p^R}$) while (ii) the free-entry condition in R&D ($1 + r_{t+1} = \pi_{t+1}^u \eta^u (N_t^u)^{-\delta}$) links the interest rate to the profitability of future innovations. To begin with we start with the free-entry condition in R&D

$$1 + r_{t+1} = \pi_{t+1}^u \eta^u (N_t^u)^{-\delta} \quad (\text{B.40})$$

which implies together with Hotelling's rule

$$\pi_{t+1}^u \eta^u (N_t^u)^{-\delta} = g_{t+1}^{p^R}. \quad (\text{B.41})$$

- (1) **Evolution of $g_{t+1}^{p^R}$:**

Since $M_t = M_{t+1} + R_{t+1}$, it follows $R_{t+1} = M_t \left[1 - \frac{M_{t+1}}{M_t} \right]$ and

$$\frac{R_{t+1}}{R_t} = \frac{M_t}{M_{t-1}} \left[\frac{1 - \frac{M_{t+1}}{M_t}}{1 - \frac{M_t}{M_{t-1}}} \right]. \quad (\text{B.42})$$

As additionally $R_{t+1} = \tau_{t+1} M_t$, we yield

$$M_{t+1} = M_t - R_{t+1} = (1 - \tau_{t+1}) M_t, \quad (\text{B.43})$$

$$\frac{M_{t+1}}{M_t} = (1 - \tau_{t+1}). \quad (\text{B.44})$$

Therefore,

$$g_{t+1}^R = \frac{R_{t+1}}{R_t} = (1 - \tau_t) \left[\frac{1 - (1 - \tau_{t+1})}{1 - (1 - \tau_t)} \right] = (1 - \tau_t) \frac{\tau_{t+1}}{\tau_t}. \quad (\text{B.45})$$

As the inverse demand function for machines in the u -sector reads as

$$p_t^{x^u} = p_t^{Y^u} (x_t^u)^{-\beta} (L_t^u)^\beta = \frac{c_t^u}{1 - \beta} = \frac{p_t^R}{(1 - \beta)BN_t^u}, \quad (\text{B.46})$$

$$\Rightarrow p_t^R = (1 - \beta)BN_t^u p_t^{Y^u} (x_t^u)^{-\beta} (L_t^u)^\beta \quad (\text{B.47})$$

and noting further that $x_t^u = X_t^u/N_t^u = B\varphi_t^{x^u} R_t$, we obtain

$$p_t^R = (1 - \beta)BN_t^u p_t^{Y^u} (B\varphi_t^{x^u} R_t)^{-\beta} (L_t^u)^\beta, \quad (\text{B.48})$$

such that the evolution of the resource price is given by

$$g_{t+1}^{p^R} = g_{t+1}^{N^u} g_{t+1}^{p^{Y^u}} (g_{t+1}^{\varphi^{x^u}} g_{t+1}^R)^{-\beta} (g_{t+1}^{L^u})^\beta = 1 + r_{t+1} = \pi_{t+1}^u \eta^u (N_t^u)^{-\delta}. \quad (\text{B.49})$$

(2) Evolution of $g_{t+1}^{N^u}$:

Since $p_t^{Y^u} Y_t^u = w_t^u L_t^{Y,u} + p_t^{x^u} X_t^u$, it follows

$$p_t^{Y^u} Y_t^u = \beta p_t^{Y^u} \frac{Y_t^u}{L_t^{Y,u}} L_t^{Y,u} + p_t^{x^u} X_t^u, \quad (\text{B.50})$$

and $p_t^{Y^u} Y_t^u = \frac{p_t^{x^u} X_t^u}{1 - \beta}$.

Profits of a machine producing firm in sector u read as

$$\pi_t^u = p_t^{x^u} x_t^u - c_t^u x_t^u. \quad (\text{B.51})$$

Hence, $\pi_t^u + c_t^u x_t^u = p_t^{x^u} x_t^u$, where substitution for π_t^u and x_t^u yields

$$p_t^{x^u} x_t^u = \beta(1 - \beta)^{\frac{1-\beta}{\beta}} (p_t^u)^{\frac{1}{\beta}} (c_t^u)^{\frac{\beta-1}{\beta}} L_t^{Y,u} + c_t^u \left(\frac{(1 - \beta)p_t^{Y^u}}{c_t^u} \right)^{\frac{1}{\beta}} L_t^{Y,u}, \quad (\text{B.52})$$

$$= (1 - \beta)^{\frac{1}{\beta}} (p_t^u)^{\frac{1}{\beta}} (c_t^u)^{\frac{\beta-1}{\beta}} L_t^{Y,u} \left[\frac{\beta}{1 - \beta} + 1 \right], \quad (\text{B.53})$$

$$= (1 - \beta)^{\frac{1-\beta}{\beta}} (p_t^u)^{\frac{1}{\beta}} (c_t^u)^{\frac{\beta-1}{\beta}} L_t^{Y,u} = \frac{\pi_t^u}{\beta}. \quad (\text{B.54})$$

Therefore, we are allowed to specify

$$p_t^u Y_t^u = \frac{p_t^u X_t^u}{1 - \beta} = \frac{N_t^u \pi_t^u}{\beta(1 - \beta)}. \quad (\text{B.55})$$

Combining the last expression with aggregate savings (31), we yield

$$S_t = \frac{N_t^u \pi_t^u}{\beta(1 - \beta)} Z_t^S. \quad (\text{B.56})$$

As $D_t^u = \frac{1}{1+\Omega_t} S_t$ we yield together with (20)

$$N_{t+1}^u = \frac{\eta^u (N_t^u)^{1-\delta} \pi_t^u}{\beta(1-\beta)} \frac{Z_t^S}{1+\Omega_t}. \quad (\text{B.57})$$

Defining $Z_t^N = \beta(1-\beta)(1+\Omega_t)$ yields

$$N_{t+1}^u = \eta^u (N_t^u)^{1-\delta} \pi_t^u \frac{Z_t^S}{Z_t^N}, \quad (\text{B.58})$$

$$g_{t+1}^{N^u} = \eta^u (N_t^u)^{-\delta} \pi_t^u \frac{Z_t^S}{Z_t^N}. \quad (\text{B.59})$$

(3) **Evolution of $\frac{g_{t+1}^{p^{Y,u}}}{g_{t+1}^{c^u}}$:**

(a)

Plugging the last expression into (B.49) yields

$$\eta^u (N_t^u)^{-\delta} \pi_t^u \frac{Z_t^S}{Z_t^N} g_{t+1}^{p^{Y,u}} (g_{t+1}^{\varphi^{x,u}} g_{t+1}^R)^{-\beta} (g_{t+1}^{L^u})^\beta = \pi_{t+1}^u \eta^u (N_t^u)^{-\delta} \quad (\text{B.60})$$

such that

$$g_{t+1}^{\pi^u} = \frac{Z_t^S}{Z_t^N} g_{t+1}^{p^{Y,u}} (g_{t+1}^{\varphi^{x,u}} g_{t+1}^R)^{-\beta} (g_{t+1}^{L^u})^\beta = (g_{t+1}^{p^{Y,u}})^{\frac{1}{\beta}} (g_{t+1}^{c^u})^{\frac{\beta-1}{\beta}} g_{t+1}^{L^u}, \quad (\text{B.61})$$

while the last part of the equation stems from the definition of operating profits of machine producers. Hence

$$\left(\frac{g_{t+1}^{p^{Y,u}}}{g_{t+1}^{c^u}} \right)^{\frac{1-\beta}{\beta}} = \frac{Z_t^S}{Z_t^N} (g_{t+1}^{\varphi^{x,u}} g_{t+1}^R)^{-\beta} (g_{t+1}^{L^u})^{\beta-1}. \quad (\text{B.62})$$

(b)

Market clearing requires

$$N_t^u \left(\frac{p_t^{Y,u}}{p_t^{x,u}} \right)^{\frac{1}{\beta}} = B N_t^u \varphi_t^{x,u} R_t. \quad (\text{B.63})$$

Therefore,

$$\left(\frac{g_{t+1}^{p^{Y,u}}}{g_{t+1}^{c^u}} \right)^{\frac{1}{\beta}} = g_{t+1}^{\varphi^{x,u}} g_{t+1}^R (g_{t+1}^{L^u})^{-1}. \quad (\text{B.64})$$

(4) **Evolution of τ :**

Combining the last expression with (B.62) yields

$$g_{t+1}^{\varphi^{x,u}} g_{t+1}^R = \frac{Z_t^S}{Z_t^N} (g_{t+1}^{\varphi^{x,u}} g_{t+1}^R)^{-\frac{1}{1-\beta}} \quad (\text{B.65})$$

$$\Rightarrow g_{t+1}^R = \frac{\tau_{t+1}(1-\tau_t)}{\tau_t} = \frac{Z_t^S}{Z_t^N} (g_{t+1}^{\varphi^{x,u}})^{-1} \quad (\text{B.66})$$

$$\tau_{t+1} = \frac{\tau_t}{1-\tau_t} \frac{Z_t^S}{Z_t^N} (g_{t+1}^{\varphi^{x,u}})^{-1}. \quad (\text{B.67})$$

B.6 Productivity Growth

This proof exploits the general equilibrium structure more in detail in order to verify the growth rate of innovations $g_*^{N^i}$.

From the previous proof we know that

$$g_{t+1}^{N,u} = \eta^u (N_t^u)^{-\delta} \pi_t^u \frac{Z_t^S}{Z_t^N}. \quad (\text{B.68})$$

Substituting for π_t^u gives

$$g_{t+1}^{N,u} = \eta^u (N_t^u)^{-\delta} \beta(1-\beta)^{\frac{1-\beta}{\beta}} (p_t^{Y,u})^{\frac{1}{\beta}} L_t^{Y,u} (c_t^u)^{\frac{\beta-1}{\beta}} \frac{Z_t^S}{Z_t^N}. \quad (\text{B.69})$$

Since along the bgp: $g_{t+1}^{N,u} = g_t^{N,u} = g_*^{N,u}$, the previous equation implies

$$\frac{g_{t+1}^{N,u}}{g_t^{N,u}} = 1 = (g_*^{N,u})^{-\delta} n_* (g_*^{c,u})^{\frac{\beta-1}{\beta}}. \quad (\text{B.70})$$

Marginal production cost evolve according to

$$g_*^{c,u} = \frac{g_*^{p,R}}{g_*^{N,u}} \quad (\text{B.71})$$

As $p_t^R = p_t^{Y,u} (1-\beta) \frac{Y_t^u}{R_t^u}$, we yield: $g_{t+1}^{p,R} = g_{t+1}^{Y,u} g_{t+1}^{N,u} \left(\frac{L_{t+1}^u}{L_t^u} \right)^\beta \left(g_{t+1}^{\varphi^x,u} g_{t+1}^R \right)^{-\beta}$. Since $p_t^{Y,u}$ and φ_t^x,u are constant in steady state, we obtain

$$g_*^{p,R} = g_*^{N,u} n_*^\beta (1-\tau_*)^{-\beta}. \quad (\text{B.72})$$

Combining the last expression with $g_*^{c,u}$ gives

$$g_*^{c,u} = n_*^\beta (1-\tau_*)^{-\beta}. \quad (\text{B.73})$$

Combining the last equation with $1 = \left(g_*^{N,u} \right)^{-\delta} \left(g_*^{c,u} \right)^{\frac{\beta-1}{\beta}} n_*$, yields

$$g_*^{N,u} = g_*^{N,s} = \left[n_*^\beta (1-\tau_*)^{1-\beta} \right]^{\frac{1}{\delta}}. \quad (\text{B.74})$$

B.7 Investment Share of GDP

Since we consider a closed economy: $I_t = S_t$. Investment as a share of GDP, $\frac{I_t}{Y_t}$, writes as

$$\frac{I_t}{Y_t} = \frac{p_t^u Y_t^u Z_t^S}{Y_t}. \quad (\text{B.75})$$

Since $p_t^Y \equiv 1 = \left[\gamma^\varepsilon (p_t^{Y,u})^{1-\varepsilon} + (1-\gamma)^\varepsilon (p_t^{Y,s})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$, we obtain

$$1 = p_t^{Y,u} \left[\gamma^\varepsilon + (1-\gamma)^\varepsilon \left(\frac{p_t^{Y,s}}{p_t^{Y,u}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \quad (\text{B.76})$$

$$p_t^{Y,u} = \left[\gamma^\varepsilon + (1-\gamma)^\varepsilon (\tilde{p}_t^Y)^{1-\varepsilon} \right]^{\frac{1}{\varepsilon-1}}. \quad (\text{B.77})$$

For $\frac{Y_t^u}{Y_t^s}$, we yield

$$\frac{Y_t^u}{Y_t^s} = \frac{Y_t^u}{\left[\gamma \left(Y_t^u \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) \left(Y_t^s \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}}, \quad (\text{B.78})$$

$$\frac{Y_t^u}{Y_t^s} = \frac{1}{\left[\gamma + (1-\gamma) \left(\frac{Y_t^s}{Y_t^u} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}}, \quad (\text{B.79})$$

with $\frac{Y_t^s}{Y_t^u} = \frac{N_t^s}{N_t^u} p_t^{\frac{1-\beta}{\beta}} \frac{c_t^u}{c_t^s} \frac{1-\beta}{\beta} \frac{L_t^{Y,s}}{L_t^{Y,u}}$.

Hence,

$$\frac{I_t}{Y_t} = \frac{p_t^u Y_t^u Z_t^S}{Y_t}, \quad (\text{B.80})$$

$$\frac{I_t}{Y_t} = \frac{\left[\gamma^\varepsilon + (1-\gamma)^\varepsilon (\tilde{p}_t^Y)^{1-\varepsilon} \right]^{\frac{1}{\varepsilon-1}}}{\left[\gamma + (1-\gamma) \left(\frac{Y_t^s}{Y_t^u} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}} Z_t^S, \quad (\text{B.81})$$

with $\left(\frac{I}{Y} \right)_* = \text{const.}$

B.8 The model with labor and natural resources in machine production

(1) Marginal production cost of machines

$$c_t^{x,i}(w_t^i, p_t^R) = \frac{(w_t^i)^{1-\alpha} (p_t^R)^\alpha}{AN_t^i (1-\alpha)^{1-\alpha} \alpha^\alpha}, \quad (\text{B.82})$$

with $i = s, u$ (for details see Appendix B.2).

(2) Labor market

$$L_t^{Y,s} = \frac{\beta}{\xi^L} (L_t^s - L_t^{E,s}), \quad L_t^{x,s} = \frac{\xi^{Lx}}{\xi^L} (L_t^s - L_t^{E,s}), \quad (\text{B.83})$$

$$L_t^{Y,u} = \frac{\beta}{\xi^L} L_t^u, \quad L_t^{x,u} = \frac{\xi^{Lx}}{\xi^L} L_t^u, \quad (\text{B.84})$$

with $\xi^{Lx} = (1-\alpha)(1-\beta)$, $\xi^L = \beta + (1-\alpha)(1-\beta)$, $L_t^{Y,s} + L_t^{x,s} + L_t^{E,s} = L_t^s$, $L_t^{Y,u} + L_t^{x,u} = L_t^u$ (for details see Appendix B.3). Moreover, $\frac{L_t^{Y,s}}{L_t^{Y,u}} = \frac{L_t^{x,s}}{L_t^{x,u}} = (1 - \phi n^{s,s}) \frac{L_t^s}{L_t^u} - \phi \theta_t n_t^{u,s}$.

⁴⁸Note that this verifies also that $p_*^{Y,u}$ is constant as \tilde{p}_*^Y is constant. As $p_*^{Y,s} = \tilde{p}_*^Y p_*^{Y,u}$, it follows outright that $p_*^{Y,s} = \text{const.}$

(3) The dynamic system

$$\tilde{L}_{t+1} = \frac{\theta_t n_t^{u,s} + n^{s,s} \tilde{L}_t}{(1 - \theta_t) n^{u,u}}, \quad (\text{B.85})$$

$$\tilde{N}_{t+1} = \left[\left(\frac{\eta^s}{\eta^u} \right) \tilde{N}_t^{-\delta} \right]^{\frac{(\beta + (\sigma-1)\xi^L)}{\beta - \xi^R(\sigma-1)}} \quad (\text{B.86})$$

$$\left[\left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon\beta} (\tilde{L}_{t+1}^Y)^{(\sigma-1)\xi^L} \left(\frac{A}{B} \right)^{(\sigma-1)(1-\beta)} \right]^{\frac{1}{\beta - \xi^R(\sigma-1)}}, \quad (\text{B.87})$$

$$\tau_{t+1} = \frac{Z_t^S}{Z_t^N} \frac{\tau_t}{1 - \tau_t} (g_{t+1}^{R,xu})^{-1}, \quad (\text{B.88})$$

with $\tilde{L}_{t+1}^Y = (1 - \phi n^{s,s}) \tilde{L}_{t+1} - \phi \theta_{t+1} n_{t+1}^{u,s}$ and indifference condition (15)

$$\omega_{t+1} = \frac{z + \omega_t \phi}{z}.$$

which determines θ_{t+1} implicitly via (25).

(4) The steady state

$$\tilde{L}_* = \frac{\theta_* n_*^{u,s}}{(1 - \theta_*) n^{u,u} - n^{s,s}}. \quad (\text{B.89})$$

$$\tilde{w}_* = \frac{z}{z - \phi}, \quad (\text{B.90})$$

where $\tilde{w}_* = \left[(\tilde{\eta})^{\sigma-1} \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon\beta(1+\delta)} (\tilde{L}_*^Y)_*^{\sigma-1-\beta(1+\delta)} \left(\frac{A}{B} \right)^{(\sigma-1)(1-\beta)(1+\delta)} \right]^{\frac{1}{\psi}}$.

$$\tilde{N}_* = \left[(\tilde{\eta})^{\beta+(\sigma-1)\xi^L} \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon\beta} (\tilde{L}_*^Y)_*^{(\sigma-1)\xi^L} \left(\frac{A}{B} \right)^{(\sigma-1)(1-\beta)} \right]^{\frac{1}{\psi}}, \quad (\text{B.91})$$

$$\tau_* = 1 - \frac{Z_*^S}{Z_*^N}, \quad (\text{B.92})$$

where $\psi = \beta - \xi^R(\sigma - 1) + \delta(\beta + (\sigma - 1)\xi^L) > 0$ in the range of plausible parameters. Skilled and unskilled-labor complementary innovations evolve along the balanced growth path in compliance with

$$g_* = g_*^{N^i} = \left[n_*^{\xi^L} (1 - \tau_*)^{\xi^R} \right]^{\frac{1}{\delta}}, \quad i = u, s. \quad (\text{B.93})$$

B.9 Numeric Method

As described above, the dynamic behavior is fully determined by a four-dimensional system of difference equations - as given by (B.85)-(B.88) and (15) - involving two state variables: \tilde{L} and \tilde{N} , as well as two

jump variables: θ and τ . The Jacobian of the dynamic system evaluated at the steady state, J_* , is equal to

$$J_* = \begin{pmatrix} \left. \frac{\partial \tilde{L}_{t+1}}{\partial \theta_t} \right|_* & \left. \frac{\partial \tilde{L}_{t+1}}{\partial L_t} \right|_* & \left. \frac{\partial \tilde{L}_{t+1}}{\partial \tilde{N}_t} \right|_* & \left. \frac{\partial \tilde{L}_{t+1}}{\partial \tau_t} \right|_* \\ \left. \frac{\partial \theta_{t+1}}{\partial \theta_t} \right|_* & \left. \frac{\partial \theta_{t+1}}{\partial L_t} \right|_* & \left. \frac{\partial \theta_{t+1}}{\partial \tilde{N}_t} \right|_* & \left. \frac{\partial \theta_{t+1}}{\partial \tau_t} \right|_* \\ \left. \frac{\partial \tilde{n}_{t+1}}{\partial \theta_t} \right|_* & \left. \frac{\partial \tilde{N}_{t+1}}{\partial L_t} \right|_* & \left. \frac{\partial \tilde{N}_{t+1}}{\partial \tilde{N}_t} \right|_* & \left. \frac{\partial \tilde{N}_{t+1}}{\partial \tau_t} \right|_* \\ \left. \frac{\partial \tau_{t+1}}{\partial \theta_t} \right|_* & \left. \frac{\partial \tau_{t+1}}{\partial L_t} \right|_* & \left. \frac{\partial \tau_{t+1}}{\partial \tilde{N}_t} \right|_* & \left. \frac{\partial \tau_{t+1}}{\partial \tau_t} \right|_* \end{pmatrix}. \quad (\text{B.94})$$

The dynamic system exhibits numerically two unstable eigenvalues ($\lambda_1, \lambda_4 > 1$) and two stable eigenvalues ($\lambda_2, \lambda_3 < 1$), such that the dynamics of the economy is subject to saddle-point stability along a two-dimensional manifold.⁴⁹ The solution of the linearized system reads as

$$\begin{pmatrix} \tilde{L}_t \\ \theta_t \\ \tilde{N}_t \\ \tau_t \end{pmatrix} = P \begin{pmatrix} A_1 & \lambda_1^t \\ A_2 & \lambda_2^t \\ A_3 & \lambda_3^t \\ A_4 & \lambda_4^t \end{pmatrix} + \begin{pmatrix} \tilde{L}_* \\ \theta_* \\ \tilde{N}_* \\ \tau_* \end{pmatrix}, \quad (\text{B.95})$$

where P contains the eigenvectors p_1, p_2, p_3, p_4 and A_1, A_2, A_3, A_4 represent arbitrary constants. With $\lambda_1, \lambda_4 > 1$, it follows immediately that $A_1 = A_4 = 0$, such that

$$\tilde{L}_t = p_{2,1} A_2 \lambda_2^t + p_{3,1} A_3 \lambda_3^t + \tilde{L}_* \quad (\text{B.96})$$

$$\theta_t = p_{2,2} A_2 \lambda_2^t + p_{3,2} A_3 \lambda_3^t + \theta_* \quad (\text{B.97})$$

$$\tilde{N}_t = p_{2,3} A_2 \lambda_2^t + p_{3,3} A_3 \lambda_3^t + \tilde{N}_* \quad (\text{B.98})$$

$$\tau_t = p_{2,4} A_2 \lambda_2^t + p_{3,4} A_3 \lambda_3^t + \tau_*. \quad (\text{B.99})$$

With $\frac{L_{t=0}^s}{L_{t=0}^u} = \tilde{L}_0 > 0$ and $\frac{N_{t=0}^s}{N_{t=0}^u} = \tilde{N}_0 > 0$ given, the unknown constants A_2, A_3 are determined by the solution of

$$\tilde{L}_0 = p_{2,1} A_2 + p_{3,1} A_3 + \tilde{L}_*, \quad (\text{B.100})$$

$$\tilde{N}_0 = p_{2,3} A_2 + p_{3,3} A_3 + \tilde{N}_*, \quad (\text{B.101})$$

such that the initial values $0 < \tau_0 < 1$ and $0 \leq \theta_0 \leq 1$ are known.

B.10 Growth rate of wages

Since $Y_t^u = (1 - \beta)^{\frac{1-2\beta}{\beta}} N_t^u (p_t^{Y,u} / c_t^u)^{\frac{1-\beta}{\beta}} L_t^{Y,u}$ and $w_t^u = p_t^{Y,u} \frac{\partial Y_t^u}{\partial L_t^{Y,u}}$, it follows that

$$w_t^u = (1 - \beta)^{\frac{1-2\beta}{\beta}} N_t^u (p_t^{Y,u})^{\frac{1}{\beta}} (c_t^u)^{\frac{\beta-1}{\beta}}, \quad (\text{B.102})$$

⁴⁹In the second row of J_* we applied the Implicit function theorem numerically to (15). Moreover we plotted the characteristic polynomial with respect to changes in one parameter. It turned out that the constellation of eigenvalues is robust. For $\varepsilon < 2$, the system becomes unstable.

with $c_t^u = \frac{p_t^R}{BN_t^u}$, such that

$$w_t^u = (1 - \beta)^{\frac{1-2\beta}{\beta}} (N_t^u)^{\frac{1}{\beta}} (p_t^{Y,u})^{\frac{1}{\beta}} (p_t^R)^{\frac{\beta-1}{\beta}}. \quad (\text{B.103})$$

Thus w_t^u evolves along the bgp as follows

$$g_*^{w,u} = \left[\frac{g_*^{N,u}}{(g_*^{p,R})^{1-\beta}} \right]^{\frac{1}{\beta}}. \quad (\text{B.104})$$

Substituting for $g_*^{p,R}$ and $g_*^{N,u}$ yields finally

$$g_*^{w,u} = g_*^{w,s} = (n_*)^{\frac{1-(1-\beta)(1+\delta)}{\delta}} (1 - \tau_*)^{\frac{(1-\beta)(1+\delta)}{\delta}}. \quad (\text{B.105})$$

B.11 Proof of Proposition 3, item (ii), Detailed Version

The long-run rate of depletion is implicitly defined by

$$F(\theta_*, \tau_*) = 1 - \frac{Z_*^S(\theta_*, \tau_*)}{Z_*^N(\theta_*)} - \tau_* = 0, \quad (\text{B.106})$$

with

$$Z_*^S(\theta_*, \tau_*) = \overbrace{\frac{\rho}{1 + \nu + \rho} \beta \left(1 + \frac{z}{z - \phi} \tilde{L}_* \right)}^{Z_*^{S,A}} - \overbrace{(1 - \beta)(1 + \Psi_*)}^{Z_*^{S,B}} \frac{1 - \tau_*}{\tau_*}, \quad (\text{B.107})$$

$$= Z_*^{S,A} - Z_*^{S,B} \frac{1 - \tau_*}{\tau_*}, \quad (\text{B.108})$$

$$\Psi_* = ((1 - \gamma)/\gamma)^{\frac{\varepsilon}{\sigma}} (A/B)^{\frac{(\sigma-1)(1-\beta)}{\sigma\beta}} (\tilde{N}_*)^{\frac{\sigma-1}{\sigma\beta}} (\tilde{L}_*)^{\frac{\sigma-1}{\sigma}} \quad (\text{B.109})$$

and

$$Z_*^N = \beta(1 - \beta) \left(1 + \tilde{\eta}^{-1} \tilde{N}_*^{1+\delta} \right). \quad (\text{B.110})$$

Eq. (B.106) exhibits two real solutions $\tau_*^{1,2}$ (one negative and one positive), such that the only economically meaningful solution is

$$\tau_* = \frac{1}{2} \left[1 - \overbrace{\left(\frac{Z_*^{S,A}}{Z_*^N} + \frac{Z_*^{S,B}}{Z_*^N} \right)}^{Z_1} \right] + \left\{ \frac{1}{4} \left[1 - \overbrace{\left(\frac{Z_*^{S,A}}{Z_*^N} + \frac{Z_*^{S,B}}{Z_*^N} \right)}^{Z_1} \right]^2 + \overbrace{\frac{Z_*^{S,B}}{Z_*^N}}^{Z_2} \right\}^{1/2} \quad (\text{B.111})$$

$$\tau_* = \frac{1 - Z_1}{2} + \left\{ \frac{(1 - Z_1)^2}{4} + Z_2 \right\}^{1/2} \quad (\text{B.112})$$

Thus $\frac{\partial \tau_*}{\partial \phi}$ reads as follows

$$\frac{\partial \tau_*}{\partial \phi} = -\frac{\partial Z_1}{\partial \phi} + \frac{1}{2} \left[\frac{(1 - Z_1)^2}{4} + Z_2 \right]^{-1/2} \left[\frac{-\partial Z_1}{\partial \phi} (1 - Z_1) + \frac{\partial Z_2}{\partial \phi} \right]. \quad (\text{B.113})$$

Apparently, $\frac{\partial \tau_*}{\partial \phi} < 0$, if

$$\frac{\partial \tau_*}{\partial \phi} = -\frac{\partial Z_1}{\partial \phi} + \left[\frac{(1 - Z_1)^2}{4} + Z_2 \right]^{-1/2} \left[\frac{-\partial Z_1}{\partial \phi} (1 - Z_1) + \frac{\partial Z_2}{\partial \phi} \right] < 0 \quad (\text{B.114})$$

which requires that

$$\frac{\frac{\partial Z_1}{\partial \phi}}{\frac{\partial Z_2}{\partial \phi}} Z_2 > -(1 - Z_1) + \frac{\frac{\partial Z_2}{\partial \phi}}{\frac{\partial Z_1}{\partial \phi}} \quad (\text{B.115})$$

Note that:

(i) $Z_*^N, Z_*^{S,A}, Z_*^{S,B} > 0$ with $Z_*^N < Z_*^{S,B}$ and $Z_*^N > Z_*^{S,A}$ in the range of plausible parameter values, such that $1 - (Z_*^{S,A}/Z_*^N + Z_*^{S,B}/Z_*^N) < 0$.

(ii) Thus $(1 - Z_1) < 0$.

(iii) It follows from (i) and (ii) that $Z_2 > -(1 - Z_1)$, since

$$\frac{Z_*^{S,B}}{Z_*^N} > -\left[1 - \frac{Z_*^{S,A} + Z_*^{S,B}}{Z_*^N}\right] \quad (\text{B.116})$$

$$\Rightarrow Z_*^N > Z_*^{S,A} \quad (\text{B.117})$$

(iv) $\frac{\partial Z_2}{\partial \phi} = \frac{\partial \frac{Z_*^{S,B}}{Z_*^N}}{\partial \phi} < 0$

(v) $\frac{\partial Z_1}{\partial \phi} > \frac{\partial Z_2}{\partial \phi}$, since

$$\frac{\left[\frac{\partial Z_*^{S,A}}{\partial \phi} + \frac{\partial Z_*^{S,B}}{\partial \phi}\right] Z_*^N - [Z_*^{S,A} + Z_*^{S,B}] \frac{\partial Z_*^N}{\partial \phi}}{(Z_*^N)^2} > \frac{\frac{\partial Z_*^{S,B}}{\partial \phi} Z_*^N - Z_*^{S,B} \frac{\partial Z_*^N}{\partial \phi}}{(Z_*^N)^2} \quad (\text{B.118})$$

$$\frac{\frac{\partial Z_*^{S,A}}{\partial \phi} Z_*^N - Z_*^{S,A} \frac{\partial Z_*^N}{\partial \phi}}{(Z_*^N)^2} > 0, \quad (\text{B.119})$$

which is a sufficient condition for $\frac{\partial \tau_*}{\partial \phi} < 0$.

Proof:

As τ_* is implicitly defined by

$$F = 1 - \frac{Z_*^{S,A}}{Z_*^N} + \frac{Z_*^{S,B}}{Z_*^N} \frac{1 - \tau_*}{\tau_*} \Rightarrow \frac{d\tau_*}{d\phi} = -\frac{F_\phi}{F_{\tau_*}}. \quad (\text{B.120})$$

Since $F_{\tau_*} = -\frac{Z_*^{S,B}}{Z_*^N \tau_*^2} - 1 < 0$ it follows that $\frac{d\tau_*}{d\phi} \leq 0$ if $F_\phi \leq 0$. As $F_\phi = -\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi} + \frac{\partial Z_*^{S,B}/Z_*^N}{\partial \phi} \frac{1 - \tau_*}{\tau_*}$, with $\frac{\partial Z_*^{S,B}/Z_*^N}{\partial \phi} < 0$ it follows that

$$\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi} > 0 \quad (\text{B.121})$$

is sufficient for $F_\phi < 0$ and thus sufficient for $\frac{d\tau_*}{d\phi} < 0$. Further below we demonstrate that this condition depends on $\delta < \delta^{crit}$.

(vi) Since $\frac{\partial Z_2}{\partial \phi} < 0$, $\frac{\partial Z_1}{\partial \phi} < 0$ is in light of (iii) and (iv) a necessary condition for $\frac{\partial \tau_*}{\partial \phi} < 0$.

Next steps

- $sign\left\{\frac{\partial \frac{Z_*^{S,A}}{Z_*^N}}{\partial \phi}\right\}$
- $sign\left\{\frac{\partial \frac{Z_*^{S,B}}{Z_*^N}}{\partial \phi}\right\}$

Proposition 7

The ratio $Z_*^{S,B}/Z_*^N$ is monotonically declining in the interval $\phi \in (0, z)$, i.e. $\frac{\partial Z_*^{S,B}/Z_*^N}{\partial \phi} < 0$.

Proof:

Using (B.107)-(B.110) implies

$$\frac{Z_*^{S,2}}{Z_*^N} = \frac{1 + \Psi_*}{\beta(1 + \eta^{-1}\tilde{N}_*^{1+\delta})}, \quad (\text{B.122})$$

with Ψ_* and $\tilde{N}_*^{1+\delta}$ being increasing functions in \tilde{L}_*^Y , if $\sigma > 1$ and $\delta > -1$.

Note that:

- \tilde{L}_*^Y is increasing in ϕ .
- For $\phi = 0$, the schooling sector is shut down and skilled labor is not supplied ($n^{u,s} = n^{s,s} = n^{u,u}$), such that $\theta_* = 0$ and $\tilde{L}_* = \tilde{L}_*^Y = 0$, such that $\frac{Z_*^{S,B}}{Z_*^N} = \frac{1}{\beta} > 1$ for $0 < \beta < 1$.
- For $\phi \rightarrow z$, the skilled wage premium, $w_* = z/(z - \phi)$, approaches infinity. Thus $\theta_* = 1$ and the supply of unskilled labor is zero, such that $\lim_{\phi \rightarrow z} \tilde{L}_* = \lim_{\phi \rightarrow z} \tilde{L}_*^Y = +\infty$.
- The curvature of Z_*^N and $Z_*^{S,B}$ with respect to changes in ϕ can be expressed by the corresponding Arrow-Pratt measures as follows:

$$AP^{S,B} = -\frac{(Z_*^{S,B})''}{(Z_*^{S,B})'}\phi = -\frac{[\partial^2 Z_*^{S,B}/\partial L_Y^{*2}][\partial^2 L_Y^*/\partial \phi^2]}{[\partial Z_*^{S,B}/\partial L_Y^*][\partial L_Y^*/\partial \phi]}\phi \quad (\text{B.123})$$

$$AP^{S,N} = -\frac{(Z_*^{S,N})''}{(Z_*^{S,N})'}\phi = -\frac{[\partial^2 Z_*^{S,N}/\partial L_Y^{*2}][\partial^2 L_Y^*/\partial \phi^2]}{[\partial Z_*^{S,N}/\partial L_Y^*][\partial L_Y^*/\partial \phi]}\phi \quad (\text{B.124})$$

It follows that $\frac{Z_*^{S,B}}{Z_*^N}$ is declining (increasing) in ϕ , if $AP^{S,B} \leq AP^N$ implying that

$$\frac{\partial^2 Z_*^{S,B}/\partial L_Y^{*2}}{\partial Z_*^{S,B}/\partial L_Y^*} \leq \frac{\partial^2 Z_*^{S,N}/\partial L_Y^{*2}}{\partial Z_*^{S,N}/\partial L_Y^*} \quad (\text{B.125})$$

$$\rightarrow \frac{\delta\sigma\beta + \beta\sigma^2 - 3\sigma\beta + \sigma - 1 + \beta}{\sigma(\beta\sigma - \sigma + 1 + \delta\sigma\beta)} \leq \frac{(\sigma - 1) - \delta\beta - \beta}{\beta\sigma - \sigma + 1 + \delta\sigma\beta} \quad (\text{B.126})$$

and therefore $AP^{S,B} < AP^{S,N}$ because

$$\delta\sigma\beta + \beta\sigma^2 - 3\sigma\beta + \sigma - 1 + \beta < \sigma(\sigma - 1) - \delta\sigma\beta - \sigma\beta, \quad (\text{B.127})$$

$$\beta(\sigma - 1)^2 + \sigma - 1 < \sigma(\sigma - 1), \quad (\text{B.128})$$

$$\beta < 1, \quad (\text{B.129})$$

with $0 < \beta < 1$.

Proposition 8

The ratio $Z_*^{S,A}/Z_*^N$ is increasing in ϕ , i.e. $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi} > 0$, if the degree of intertemporal knowledge spillovers in R&D with respect to existing technological knowledge is sufficiently high, i.e. $\delta < \delta^{crit}$.

Proof:

In light of (B.107) and (B.110) we obtain

$$\frac{Z_*^{S,A}}{Z_*^N} = \frac{\frac{\rho}{1+\nu+\rho} \left(1 + \overbrace{\frac{z}{z-\phi} \hat{L}_*}^{\hat{L}_*}\right)}{(1-\beta) \left(1 + \tilde{\eta}^{-1} \tilde{N}_*^{1+\delta}\right)}. \quad (\text{B.130})$$

Since $\tilde{N}_*^{1+\delta} = \tilde{\eta}^{\frac{\sigma\beta(1+\delta)}{\psi}} \Gamma^{\frac{1+\delta}{\psi}} (\tilde{L}_*^Y)^{\frac{(\sigma-1)\beta(1+\delta)}{\psi}}$, with $\psi = \beta - (1-\beta)(\sigma-1) + \delta(\beta + (\sigma-1)\beta)$, we obtain further

$$\frac{Z_*^{S,A}}{Z_*^N} = \frac{\frac{\rho}{1+\nu+\rho} \left(1 + \hat{L}_*\right)}{(1-\beta) \left(1 + \underbrace{\tilde{\eta}^{\frac{\sigma\beta(1+\delta)-\psi}{\psi}} \Gamma^{\frac{1+\delta}{\psi}} (\tilde{L}_*^Y)^{\frac{(\sigma-1)\beta(1+\delta)}{\psi}}}_{\tilde{\Gamma}}\right)} \quad (\text{B.131})$$

$$= \frac{\frac{\rho}{1+\nu+\rho} \left(1 + \hat{L}_*\right)}{(1-\beta) \left(1 + \tilde{\Gamma} [\tilde{L}_*^Y]^{\tilde{\psi}}\right)}. \quad (\text{B.132})$$

Thus, $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi} > 0$, if

$$\frac{\partial \hat{L}_*}{\partial \phi} + \tilde{\Gamma} (\tilde{L}_*^Y)^{\tilde{\psi}} \left[\frac{\partial \hat{L}_*}{\partial \phi} - \tilde{\psi} \frac{(1 + \hat{L}_*)}{\tilde{L}_*^Y} \frac{\partial \tilde{L}_*^Y}{\partial \phi} \right] > 0, \quad (\text{B.133})$$

with $\tilde{\psi} = \frac{(\sigma-1)\beta(1+\delta)}{\psi}$.

Denote the function \tilde{L}_*^Y as a function of ϕ for a given δ by $\tilde{L}_*^Y(\phi; \delta)$. We now show that

(i) $\tilde{L}_*^Y(\phi = 0; \delta)$ is declining in δ

(ii) given that $\delta_1 > \delta_0$, $\tilde{L}_*^Y(\phi; \delta_1)$ intercepts the function $\tilde{L}_*^Y(\phi; \delta_0)$ at $\phi = \tilde{\phi}$ from below

(iii) since $\lim_{\phi \rightarrow z} \tilde{L}_Y^*(\phi; \delta_1) = \lim_{\phi \rightarrow z} \tilde{L}_Y^*(\phi; \delta_0) = \infty$, it follows that

$$\frac{\partial \tilde{L}_*^Y}{\partial \phi \partial \delta} > 0 \quad (\text{B.134})$$

and

$$\frac{\partial \tilde{L}_*^Y}{\partial \delta} \geq 0, \quad \text{if} \quad \phi \geq \tilde{\phi} \quad (\text{B.135})$$

ad (i) First note that

$$\tilde{L}_*^Y(\phi = 0, \delta) = \left[\tilde{\eta}^{\sigma-1} ((1-\gamma)/\gamma)^{-\varepsilon\beta(1+\delta)} (A/B)^{-(\sigma-1)(1-\beta)(1+\delta)} \right]^{\frac{1}{\sigma-1+\beta(1+\delta)}} \quad (\text{B.136})$$

and thus

$$\frac{\partial \tilde{L}_*^Y(\phi = 0, \delta)}{\partial \delta} = \frac{\tilde{L}_*^Y(\phi = 0, \delta)}{[1 - \sigma + \delta(1 + \beta)]^2} \left\{ \beta\varepsilon[(1 + \delta)\beta - (\sigma - 1)] \ln((1 - \gamma)/\gamma) \quad (\text{B.137}) \right.$$

$$+ (\sigma - 1)[\beta(\delta + \sigma - \beta(1 + \delta)) - (\sigma - 1)] \ln(A/B) \quad (\text{B.138})$$

$$\left. + \ln \tilde{L}_*^Y(\phi = 0, \delta) \right\} \quad (\text{B.139})$$

Since

– empirically $\tilde{L}_*^Y < 1$. Thus $\tilde{L}_*^Y(\phi = 0; \delta) \ll 1$ which implies that $\ln \tilde{L}_*^Y(\phi = 0, \delta) < 0$

– $\beta\varepsilon[(1 + \delta)\beta - (\sigma - 1)] < 0$, if $1 + \delta < \frac{\sigma-1}{\beta}$

– $(\sigma - 1)[\beta(\delta + \sigma - \beta(1 + \delta)) - (\sigma - 1)] < 0$, if

$$\beta(\delta + \sigma - \beta(1 + \delta)) < (\sigma - 1) \quad (\text{B.140})$$

$$\beta(\sigma - \beta + \delta(1 - \beta)) < (\sigma - 1) \quad (\text{B.141})$$

$$\beta(\sigma + 1 - 1 - \beta + \delta(1 - \beta)) < (\sigma - 1) \quad (\text{B.142})$$

$$\beta(\sigma - 1 + (1 - \beta)(1 + \delta)) < (\sigma - 1) \quad (\text{B.143})$$

$$\beta(\sigma - 1) + \beta(1 - \beta)(1 + \delta) < (\sigma - 1) \quad (\text{B.144})$$

$$\rightarrow 1 + \delta < \frac{\sigma - 1}{\beta} \quad (\text{B.145})$$

Thus it follow that $\tilde{L}_*^Y(\phi = 0, \delta)$ is declining in δ , i.e.

$$\frac{\partial \tilde{L}_*^Y(\phi = 0, \delta)}{\partial \delta} < 0. \quad (\text{B.146})$$

ad (ii) Therefore, $\tilde{L}_*^Y(\phi = 0; \delta_1) < \tilde{L}_*^Y(\phi = 0; \delta_0)$ given that $\delta_1 > \delta_0$, and

$$\tilde{L}_*^Y(\phi; \delta_1) = \tilde{L}_*^Y(\phi; \delta_0) \quad (\text{B.147})$$

at

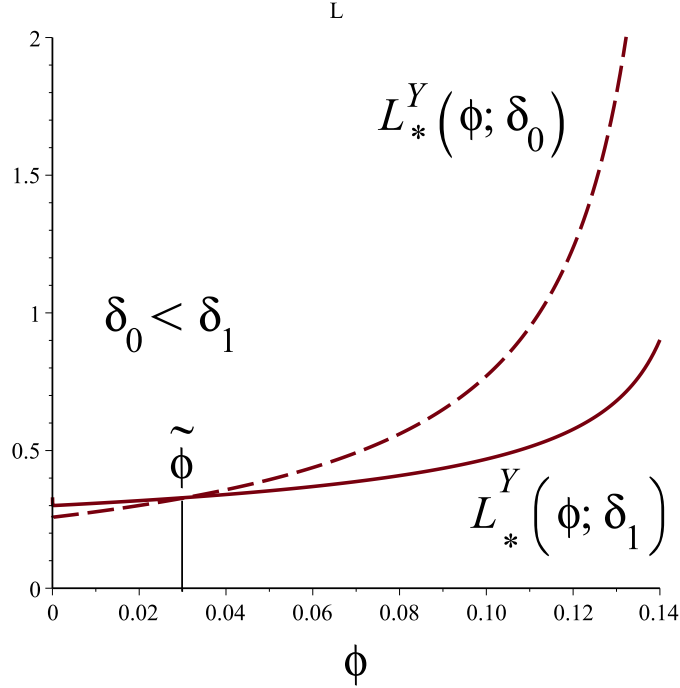
$$\phi = \tilde{\phi} = z \left[1 - \frac{\left(\tilde{\eta}^{\sigma-1} ((1-\gamma)/\gamma)^{\varepsilon\beta(1+\delta_0)} (A/B)^{(\sigma-1)(1-\beta)(1+\delta_0)} \right)^{\frac{\sigma-1-\beta(1+\delta_1)}{\beta(\sigma-1)^2(\delta_1-\delta_0)}}}{\left(\tilde{\eta}^{\sigma-1} ((1-\gamma)/\gamma)^{\varepsilon\beta(1+\delta_1)} (A/B)^{(\sigma-1)(1-\beta)(1+\delta_1)} \right)^{\frac{\sigma-1-\beta(1+\delta_0)}{\beta(\sigma-1)^2(\delta_1-\delta_0)}}} \right] \quad (\text{B.148})$$

As the second term in squared brackets is smaller than one, it follows that $0 < \tilde{\phi} < z$. Thus, $\tilde{L}_*^Y(\phi; \delta_1)$ cuts $\tilde{L}_*^Y(\phi; \delta_0)$ from below. Together with $\lim_{\phi \rightarrow z} \tilde{L}_*^Y(\phi; \delta_1) = \lim_{\phi \rightarrow z} \tilde{L}_*^Y(\phi; \delta_0) = \infty$ we can thus conclude that

$$\frac{\partial \tilde{L}_*^Y(\phi, \delta)}{\partial \delta} \geq 0, \quad \text{if} \quad \phi \geq \tilde{\phi} \quad (\text{B.149})$$

ad (iii) and

$$\frac{\partial \tilde{L}_*^Y(\phi, \delta)}{\partial \phi \partial \delta} > 0. \quad (\text{B.150})$$



The results are visualized in the figure above. As can be verified, a necessary condition for $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi} < 0$ is $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi \partial \delta} < 0$, which requires

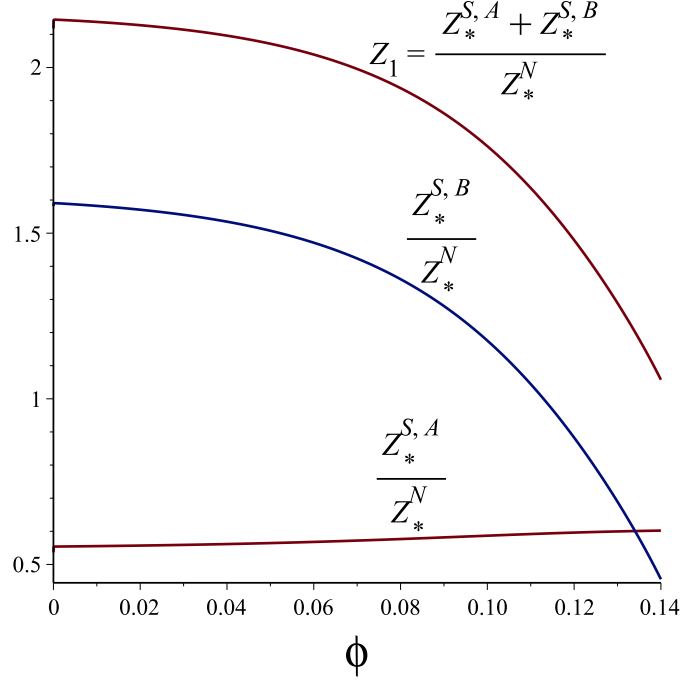
$$\begin{aligned} \frac{\partial \left[\frac{\partial \hat{L}_*}{\partial \phi} - \tilde{\psi} \frac{(1+\hat{L}_*)}{\tilde{L}_*^Y} \frac{\partial \tilde{L}_*^Y}{\partial \phi} \right]}{\partial \delta} &= \frac{\partial \hat{L}_*}{\partial \phi \partial \delta} - \frac{\partial \tilde{\psi}}{\partial \delta} \frac{(1+\hat{L}_*)}{\tilde{L}_*^Y} \frac{\partial \tilde{L}_*^Y}{\partial \phi} - \tilde{\psi} \frac{\partial \left(\frac{1+\hat{L}_*}{\tilde{L}_*^Y} \right)}{\partial \delta} \frac{\partial \tilde{L}_*^Y}{\partial \phi} \\ &\quad - \tilde{\psi} \frac{(1+\hat{L}_*)}{\tilde{L}_*^Y} \frac{\partial \tilde{L}_*^Y}{\partial \phi \partial \delta} < 0 \end{aligned} \quad (\text{B.151})$$

Note that

- $\frac{\partial \hat{L}_*}{\partial \phi \partial \delta}; \frac{\partial \tilde{L}_*^Y}{\partial \phi}; \frac{\partial \tilde{L}_*^Y}{\partial \phi \partial \delta} > 0$
- $\frac{\partial \frac{(1+\hat{L}_*)}{\tilde{L}_*^Y}}{\partial \delta} \geq 0$, if $\frac{\partial \hat{L}_*}{\partial \delta} / \frac{\partial \tilde{L}_*^Y}{\partial \delta} > (1 + \hat{L}_*) / \tilde{L}_*^Y$ for $\phi > \tilde{\phi}$.⁵⁰

The figure below demonstrate the behavior of $Z_1 = \frac{Z_*^{S,A} + Z_*^{S,B}}{Z_*^N}$, $Z_2 = \frac{Z_*^{S,B}}{Z_*^N}$ and $\frac{Z_*^{S,A}}{Z_*^N}$ as functions of ϕ

Thus it follows



- (i) There exists a critical δ^{crit} for a $\phi = \phi^{crit} < z$ given , such that $\delta \geq \delta^{crit}$ implies

$$\left[\frac{\partial \hat{L}_*}{\partial \phi} - \tilde{\psi} \frac{(1 + \hat{L}_*)}{\tilde{L}_*^Y} \frac{\partial \tilde{L}_*^Y}{\partial \phi} \right]_{\phi=\phi^{crit}} \leq 0$$

- (ii) Given the set of parameters assumed in this paper revealed that $\frac{\partial Z_*^{S,A} / Z_*^N}{\partial \phi} < 0$ is a rather unlikely case.

- (iii) Nevertheless it is worthwhile to note that $\frac{\partial Z_*^{S,A} / Z_*^N}{\partial \phi} < 0$ is a sufficient condition for $\frac{\partial \tau_*}{\partial \phi} > 0$.

⁵⁰If $\phi > \tilde{\phi}$ we know that \tilde{L}_*^Y increases in δ . If this is the case, \tilde{L}_* must increase as well. Since this increases the demand for teachers, the increase in \tilde{L}_* must be stronger than the increase in \tilde{L}_*^Y .

The following figures illustrate for the case of $\delta = 0.25$ that for very large ϕ the sign of $\frac{\partial \tau_*}{\partial \phi}$ turns positive because $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi \partial \delta} < 0$.

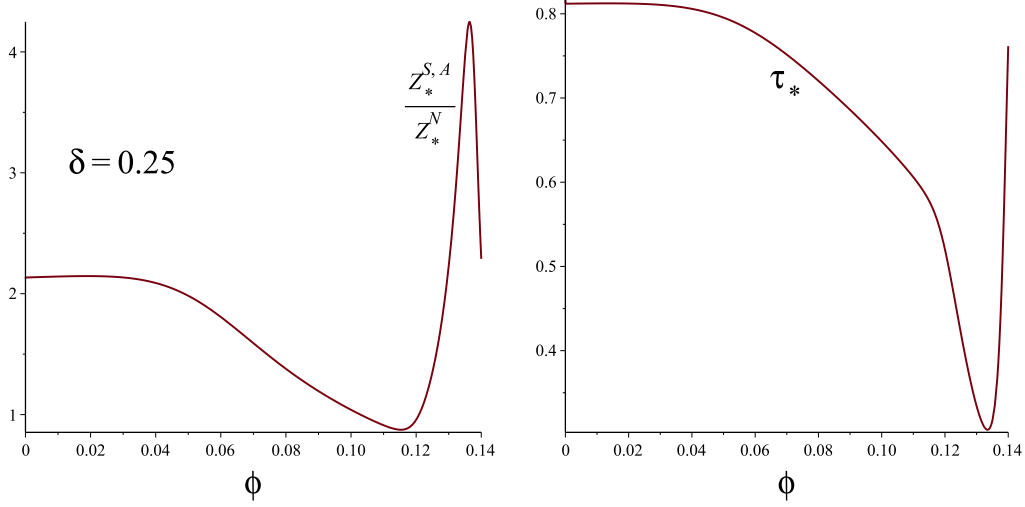


Figure: Example for $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \phi \partial \delta} < 0$ with $\delta = 0.25$ and very large ϕ

B.12 Proof of Proposition 4, item (ii), Detailed Version

Note that:

- (i) An increase in $\tilde{\eta}$ induces a decline in θ_* , i.e. $\frac{\partial \theta_*}{\partial \tilde{\eta}} < 0$, and thus a decline in \tilde{L}_* as well as \tilde{L}_*^Y , i.e. $\frac{\partial \tilde{L}_*}{\partial \tilde{\eta}} < 0$ and $\frac{\partial \tilde{L}_*^Y}{\partial \tilde{\eta}} < 0$.
- (ii) Since $\tilde{L}_* = \frac{\theta_* n_*^{u,s}}{(1-\theta_*)n^{u,u} - n^{s,s}}$, it follows that there exists a critical $\tilde{\eta} = \tilde{\eta}^{crit}$, such that $(1-\theta_*)n^{u,u} - n^{s,s} = 0$. Hence, \tilde{L}_* exhibits a vertical asymptote at $\tilde{\eta} = \tilde{\eta}^{crit}$, with $\tilde{L}_* > 0$ for $\tilde{\eta} > \tilde{\eta}^{crit}$ and $\tilde{L}_* < 0$ for $\tilde{\eta} < \tilde{\eta}^{crit}$. Moreover, $\lim_{\tilde{\eta} \rightarrow \tilde{\eta}^{crit}} = +\infty$, if $\tilde{\eta} > \tilde{\eta}^{crit}$ and $\lim_{\tilde{\eta} \rightarrow \tilde{\eta}^{crit}} = -\infty$, if $\tilde{\eta} < \tilde{\eta}^{crit}$.

Consequently, economic meaningful solutions are obtained, if $\tilde{\eta} > \tilde{\eta}^{crit}$. Since furthermore

(a) $\frac{\partial Z_*^{S,B}/Z_*^N}{\partial \tilde{\eta}} = 0$

(b) thus $\frac{\partial \tau_*}{\partial \tilde{\eta}} \gtrless 0$ depends in light of the Implicit function theorem on the sign of $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \tilde{\eta}} \lesseqgtr 0$ because

$$\frac{\partial \tau_*}{\partial \tilde{\eta}} = -\frac{F_{\tilde{\eta}}}{F_{\tau_*}}, \text{ with } F_{\tau_*} < 0 \text{ and } F_{\tilde{\eta}} = -\frac{\partial Z_*^{S,A}}{\partial \tilde{\eta}}.$$

(c) Within the range of reasonable parameter values $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \tilde{\eta}} < 0$ and thus

$$\frac{\partial \tau_*}{\partial \tilde{\eta}} > 0 \tag{B.152}$$

ad (a) **Proof:** $\frac{\partial Z_*^{S,B}/Z_*^N}{\partial \tilde{\eta}} = 0$

$$\frac{\partial Z_*^{S,B}/Z_*^N}{\partial \tilde{\eta}} = \frac{\frac{\partial Z_*^{S,B}}{\partial \tilde{\eta}} Z_*^N - \frac{\partial Z_*^N}{\partial \tilde{\eta}} Z_*^{S,B}}{(Z_*^N)^2} = 0, \quad (\text{B.153})$$

$$\rightarrow \frac{\frac{\partial Z_*^{S,B}}{\partial \tilde{\eta}}}{\frac{\partial Z_*^N}{\partial \tilde{\eta}}} = \frac{Z_*^{S,B}}{Z_*^N}, \quad (\text{B.154})$$

Note now that

$$\frac{\partial Z_*^{S,B}}{\partial \tilde{\eta}} = \frac{1}{\psi \tilde{\eta}} \left\{ (1-\beta)(1-\gamma)/\gamma \right\}^{\frac{\varepsilon}{\sigma}} (A/B)^{\frac{(\sigma-1)(1-\beta)}{\sigma\beta}} \left[\tilde{\eta}^{\sigma\beta} (1-\gamma)/\gamma \right]^{\varepsilon\beta} (L_*^Y)^{(\sigma-1)\beta} \left. \right]^{\frac{(\sigma-1)}{\psi\sigma\beta}} (\sigma-1)(L_*^Y)^{\frac{\sigma-1}{\sigma}} \} \quad (\text{B.155})$$

and

$$\frac{\partial Z_*^N}{\partial \tilde{\eta}} = \frac{1}{\psi \tilde{\eta}^2} \left\{ \beta(1-\beta) \left[\tilde{\eta}^{\sigma\beta} (1-\gamma)/\gamma \right]^{\varepsilon\beta} (L_*^Y)^{(\sigma-1)\beta} \right\}^{\frac{1}{\psi}} \quad (\text{B.156})$$

$$(\sigma-1) \left[\tilde{\eta}^{\sigma\beta} (1-\gamma)/\gamma \right]^{\varepsilon\beta} (L_*^Y)^{(\sigma-1)\beta} \right]^{\frac{\delta}{\psi}}, \quad (\text{B.157})$$

with $\Psi = \beta - (1-\beta)(\sigma-1) + \delta(\beta + (\sigma-1)\beta)$.

Thus

$$\frac{\frac{\partial Z_*^{S,B}}{\partial \tilde{\eta}}}{\frac{\partial Z_*^N}{\partial \tilde{\eta}}} = \frac{1}{\beta} \left\{ \tilde{\eta}^0 (1-\gamma)/\gamma \right\}^0 (A/B)^0 (L_*^Y)^0 \} = \frac{1}{\beta}. \quad (\text{B.158})$$

Furthermore

$$\frac{Z_*^{S,B}}{Z_*^N} = \frac{1 + \Psi_*}{\beta(1 + \tilde{\eta}^{-1} \tilde{N}_*^{1+\delta})} \quad (\text{B.159})$$

$$= \frac{\tilde{\eta}(1 + \Psi_*)}{\beta(\tilde{\eta} + \tilde{N}_*^{1+\delta})} = \frac{\frac{1}{\Psi_*} + 1}{\beta(\frac{1}{\Psi_*} + \frac{\tilde{N}_*^{1+\delta}}{\tilde{\eta}\Psi_*})}, \quad (\text{B.160})$$

with

$$\frac{\tilde{N}_*^{1+\delta}}{\tilde{\eta}\Psi_*} = \frac{\left[\tilde{\eta}^{\sigma\beta} ((1-\gamma)/\gamma)^{\varepsilon\beta} (L_*^Y)^{(\sigma-1)\beta} \right]^{\frac{1+\delta}{\psi}}}{\tilde{\eta} \left((1-\gamma)/\gamma \right)^{\frac{\varepsilon}{\sigma}} (A/B)^{\frac{(\sigma-1)(1-\beta)}{\sigma\beta}} (L_*^Y)^{\frac{\sigma-1}{\sigma}} \left[\tilde{\eta}^{\sigma\beta} (1-\gamma)/\gamma \right]^{\varepsilon\beta} (\tilde{L}_*^Y)^{(\sigma-1)\beta} \right]^{-\frac{\sigma-1}{\sigma\beta\psi}}} \quad (\text{B.161})$$

$$= \tilde{\eta}^0 ((1-\gamma)/\gamma)^0 (A/B)^0 (\tilde{L}_*^Y)^0 = 1. \quad (\text{B.162})$$

Thus

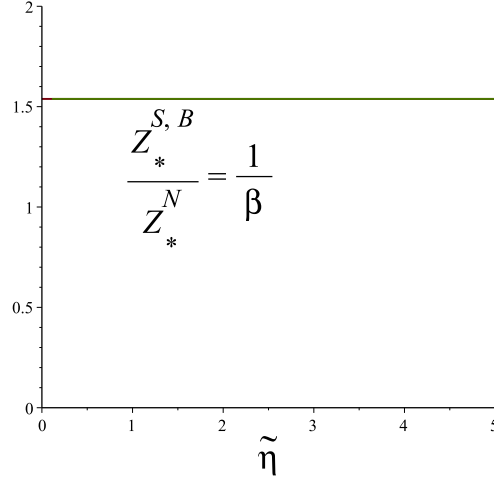
$$\frac{Z_*^{S,B}}{Z_*^N} = \frac{\frac{1}{\Psi_*} + 1}{\beta(\frac{1}{\Psi_*} + \frac{\tilde{N}_*^{1+\delta}}{\tilde{\eta}\Psi_*})} = \frac{\frac{1}{\Psi_*} + 1}{\beta(\frac{1}{\Psi_*} + 1)} = \frac{1}{\beta}. \quad (\text{B.163})$$

Hence

$$\frac{\frac{\partial Z_*^{S,B}}{\partial \tilde{\eta}}}{\frac{\partial Z_*^N}{\partial \tilde{\eta}}} = \frac{Z_*^{S,B}}{Z_*^N} = \frac{1}{\beta} \quad (\text{B.164})$$

and

$$\frac{\partial Z_*^{S,B}/Z_*^N}{\partial \tilde{\eta}} = 0. \quad (\text{B.165})$$



ad (c) **Proof:** $\frac{\partial Z_*^{S,A}}{\partial \tilde{\eta}} < 0$ for some $\eta \in (\tilde{\eta}^\#, \tilde{\eta}^{\#\#})$, with $\tilde{\eta}^{\#\#} \leq +\infty$ and $\tilde{\eta}^\# \geq \tilde{\eta}^{crit}$.

Note that

$$\frac{Z_*^{S,A}}{Z_*^N} = \frac{\frac{\rho}{1+\nu+\rho}(1 + \hat{L}_*)}{(1 - \beta) \left[1 + \tilde{\eta}^{\psi_0} \Gamma^{\frac{1+\delta}{\psi}} (\tilde{L}_*^Y)^{\psi_1} \right]}. \quad (\text{B.166})$$

Thus

$$\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \tilde{\eta}} \geq 0, \quad (\text{B.167})$$

if

$$\frac{\partial \hat{L}_*}{\partial \tilde{\eta}} + \tilde{\eta}^{\psi_0} \Gamma^{\frac{1+\delta}{\psi}} (\tilde{L}_*^Y)^{\psi_1} \left[\frac{\partial \hat{L}_*}{\partial \tilde{\eta}} - \left(\frac{\tilde{\psi}_0}{\tilde{\eta}} + \tilde{\psi}_1 \frac{\partial \tilde{L}_*^Y}{\partial \tilde{\eta}} \right) (1 + \hat{L}_*) \right] \geq 0. \quad (\text{B.168})$$

Remember that: $\frac{\partial \hat{L}_*}{\partial \tilde{\eta}}; \frac{\partial \tilde{L}_*^Y}{\partial \tilde{\eta}} < 0$ and thus $\left(\frac{\tilde{\psi}_0}{\tilde{\eta}} + \tilde{\psi}_1 \frac{\partial \tilde{L}_*^Y}{\partial \tilde{\eta}} \right) \geq 0$ and observe that

$$\frac{\partial \tilde{L}_*^Y}{\partial \tilde{\eta}} = \frac{\sigma - 1}{[1 - \sigma + \beta(1 + \delta)]\tilde{\eta}} \quad (\text{B.169})$$

$$\psi_1 = \frac{(\sigma - 1)\beta(1 + \delta)}{\psi} \quad (\text{B.170})$$

$$\psi_0 = \frac{\sigma\beta(1 + \delta)}{\psi}, \quad (\text{B.171})$$

such that

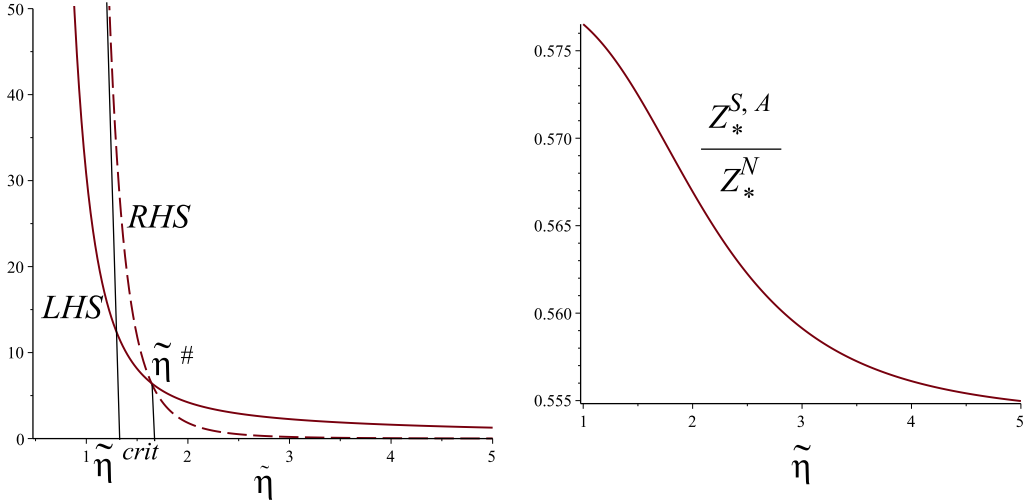
$$\frac{\tilde{\psi}_0}{\tilde{\eta}} + \tilde{\psi}_1 \frac{\frac{\partial \tilde{L}_*^Y}{\partial \tilde{\eta}}}{\tilde{L}_*^Y} = \frac{1}{\tilde{\eta}[1 - \sigma + \beta(1 + \delta)]\psi} \beta(1 + \delta)[1 - \sigma + \sigma\beta(1 + \delta)] \quad (\text{B.172})$$

$$= \frac{\beta(1 + \delta)}{\tilde{\eta}[1 - \sigma + \beta(1 + \delta)]}. \quad (\text{B.173})$$

Thus $\frac{\tilde{\psi}_0}{\tilde{\eta}} + \tilde{\psi}_1 \frac{\frac{\partial \tilde{L}_*^Y}{\partial \tilde{\eta}}}{\tilde{L}_*^Y} < 0$, if $\frac{\sigma-1}{\beta} > 1 + \delta$ which requires in our case that $\delta < 0.4$ such that we can conjecture that $\frac{\tilde{\psi}_0}{\tilde{\eta}} + \tilde{\psi}_1 \frac{\frac{\partial \tilde{L}_*^Y}{\partial \tilde{\eta}}}{\tilde{L}_*^Y} < 0$ is a very likely scenario. A necessary condition for $\frac{\partial Z_*^{S,A}/Z_*^N}{\partial \tilde{\eta}} > 0$ however is that

$$-\frac{\beta(1 + \delta)(1 + \hat{L}_*)}{[1 - \sigma + \beta(1 + \delta)]\tilde{\eta}} > -\frac{\partial \hat{L}_*}{\partial \tilde{\eta}}. \quad (\text{B.174})$$

Since $\lim_{\tilde{\eta} \rightarrow \tilde{\eta}^{crit}} \tilde{L}_*^Y = +\infty$ and $\lim_{\tilde{\eta} \rightarrow \tilde{\eta}^{crit}} \frac{\partial \tilde{L}_*^Y}{\partial \tilde{\eta}} = -\infty$, The right-hand side of (B.174) has a vertical asymptote at $\tilde{\eta} = \tilde{\eta}^{crit}$. Moreover, the right-hand side approaches zero as $\tilde{\eta} \rightarrow \infty$. The left-hand side of (B.174) exhibits a vertical asymptote to the left of $\tilde{\eta}^{crit}$. As both the left-hand side and the right-hand side are declining functions in $\tilde{\eta}$, they must intercept at some $\tilde{\eta}^\# \geq \tilde{\eta}^{crit}$. Nevertheless it cannot be excluded though that there exists a second intercept at $\tilde{\eta} = \tilde{\eta}^{\#\#} < \infty$. Thus it follow that $\frac{\partial Z_*^{S,A}}{\partial \tilde{\eta}} < 0$ for some $\tilde{\eta} \in (\tilde{\eta}^\#, \tilde{\eta}^{\#\#})$, with $\tilde{\eta}^{\#\#} \leq +\infty$ and $\tilde{\eta}^\# \geq \tilde{\eta}^{crit}$, while could not verify the existence of $\eta^{\#\#}$ numerically.



C. FIGURES

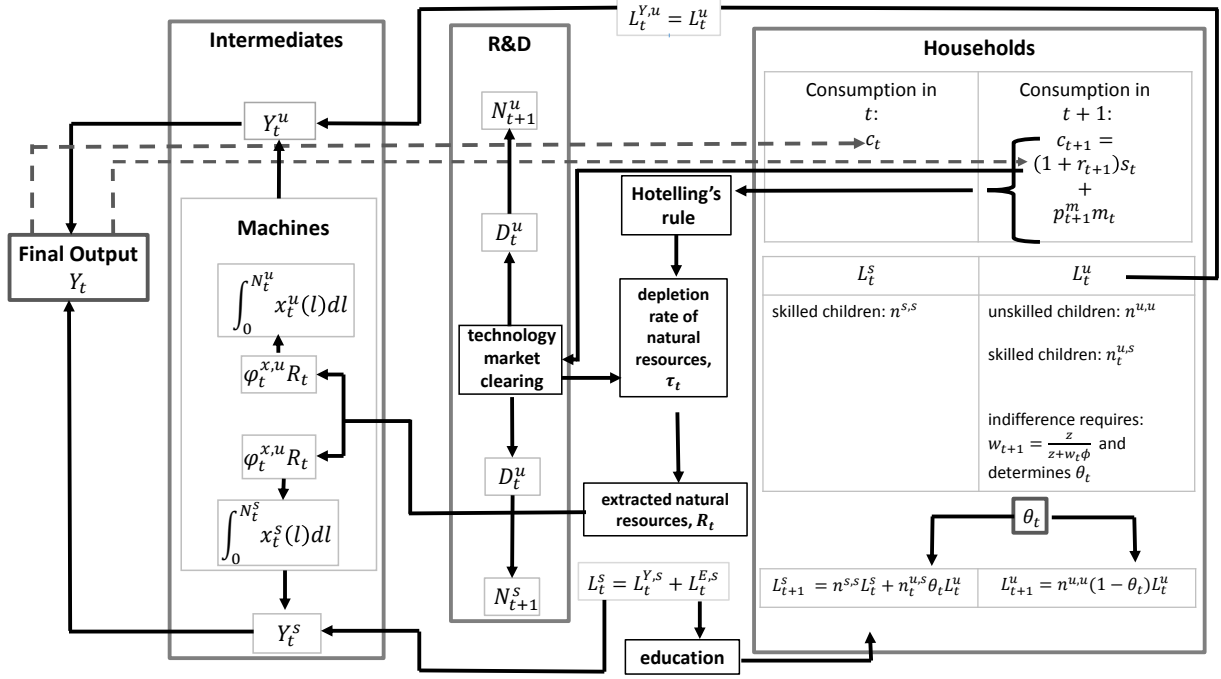


Figure 1: The structure of the model

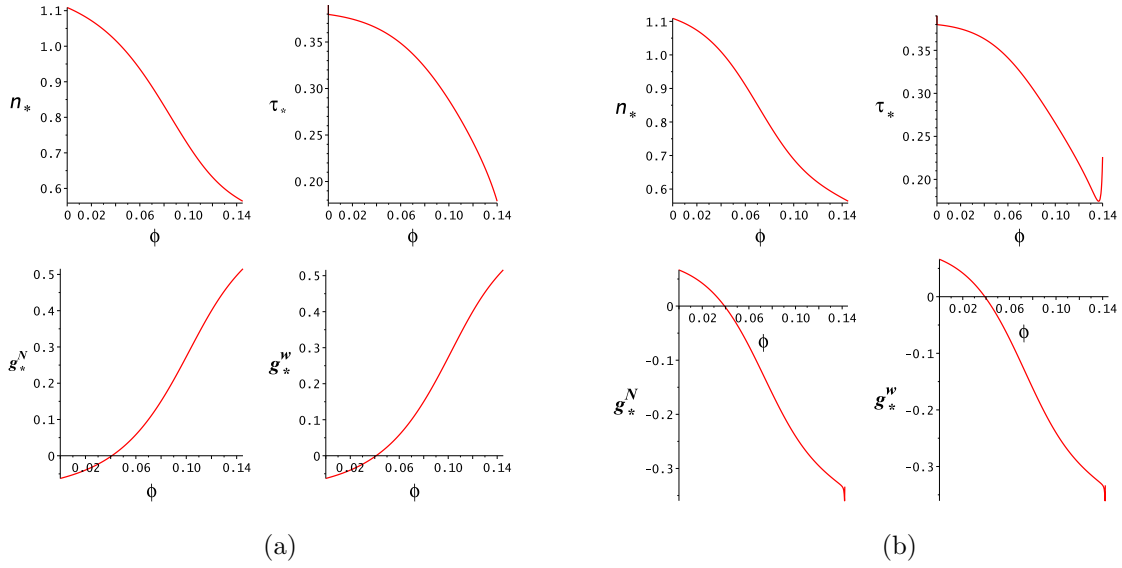


Figure 2: (a) Long-run effects of a change in the teacher-schooling ratio, ϕ , for $\delta < 0$; (b) Long-run effects of a change in the teacher-schooling ratio, ϕ , for $\delta > 0$

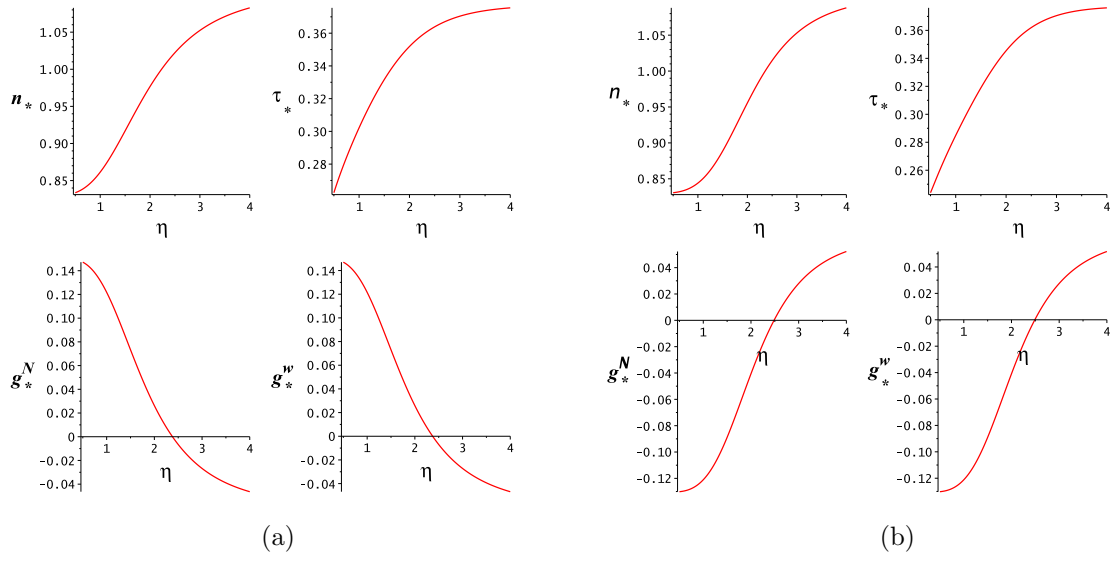


Figure 3: (a) Long-run effects of a change in relative research productivity, $\tilde{\eta}$, for $\delta < 0$; (b) Long-run effects of a change in relative research productivity, $\tilde{\eta}$, for $\delta > 0$

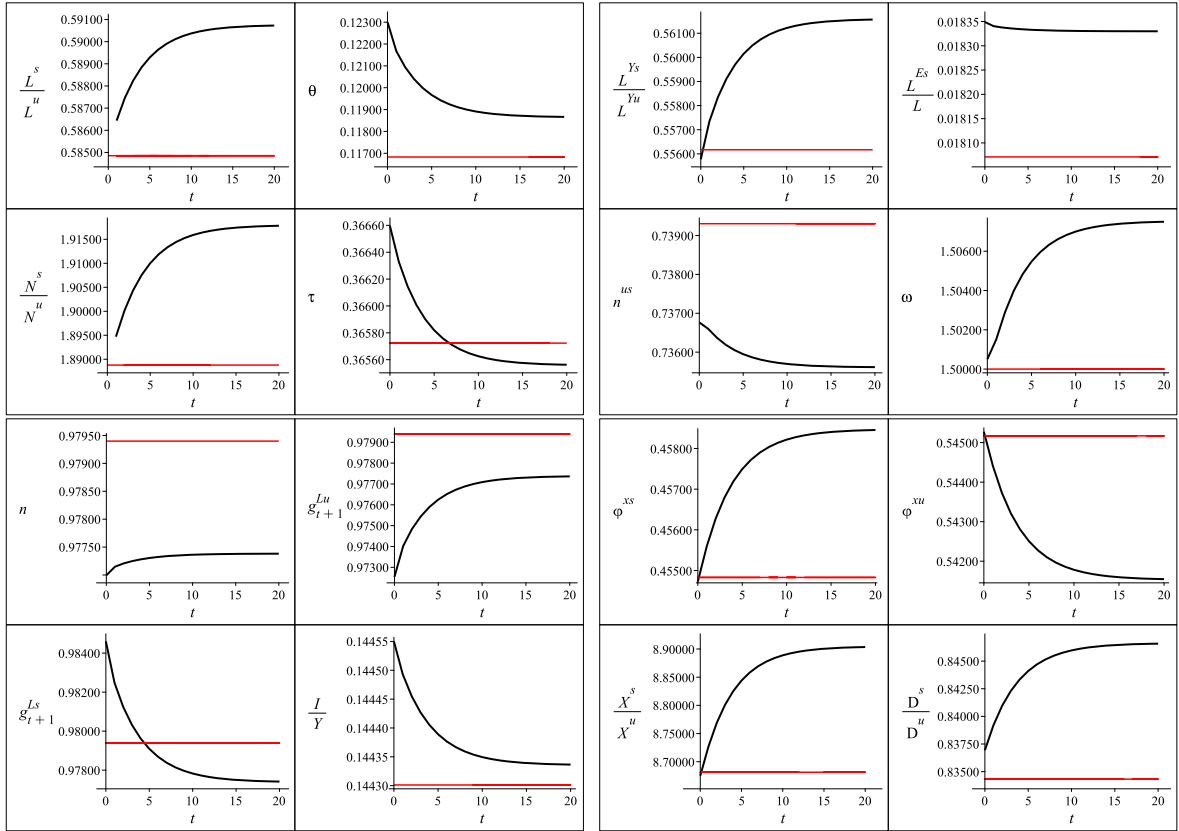


Figure 4: Impulse response functions with respect to an increase in the teacher-student ratio, ϕ .