The Interaction between Endogenous Fertility and Inequality in the Political Economy

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Abstract

We simulate a two period OLG-model with heterogeneous agents. Parents receive utility from the quantity and quality of their offspring. An increase in the wage rate leads to higher opportunity costs of child-rearing time, thus implying lower fertility and higher quality per child. This causes intergenerational persistence in fertility decisions and wages. Controlling for the initial distribution of wealth, we show that economic growth increases inequality and fertility differentials. Furthermore, we endogenise redistribution by implementing a median voter-system. Due to fertility differentials, the median-voter moves from upper to lower income percentiles.

JEL: D31,J1, I2, O0

Keywords: OLG-model, endogenous fertility, intergenerational persistence, inequality

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1 Introduction

The purpose of this paper is to examine the link between economic inequality, fertility, and the redistribution of wealth. Recently, Doepke and de la Croix (2003) have argued that it is essential for the understanding of the link between inequality and growth to account for the fertility differentials between rich and poor. Poor agents tend to have more children and invest less resources in their offspring than do rich households. Therefore, the poor population will grow faster than its richer counterpart. We argue further that the emergence of fertility differentials based on inequality also translates into increasing redistributional pressure.

Even before Malthus, economists analysed the relationship between population and economic growth, but it is only recently that fertility has been incorporated into formal micro-founded growth models. The evolution of economies over most of human history was marked by Malthusian stagnation, the so called offsetting effect of population growth on resources per capita. During the course of industrialisation, the developed countries of today escaped the Malthusian trap and experienced an increase in the population and growth rates of income per capita. The increasing role of human capital initiated a decline in population growth, leading to a state of sustained economic growth. Existing literature considers the decline in fertility to be an endogenous phenomenon and introduced the number of children to the parental utility function (Becker (1960)). The first decision-oriented macro models in this area are those from Barro and Becker (1988,1989), and Becker, Murphy, and Tamura (1990). The latter has been criticized because of its dependence on exogenous shocks (luck) to initiate a demographic transition and because of its counterfactual results.[1] First growth theories unifying the different phases of economic development with its interdependencies to population growth were advanced by Galor and Weil (1999,2000). Another channel triggering the demographic transition was the increase in the opportunity costs of time spent in child rearing (Galor and Weil (1996,1999)).[2] If fertility is negatively related to the wage rate and a high wage rate, in turn, is associated with a high quality per child, it makes sense to analyse the latter within a macroeconomic setting that includes income heterogeneity.

[1]The major criticism has been that the Malthusian steady state is lagging the main features of a Malthusian trap and that the model is not able to replicate the characteristics of the demographic transition that is, an increase in population growth, followed by a decline in population growth. For more details, see Galor (2004).

[2]For empirical evidence, see Kögel(2004).

The questions of how inequality relates to economic growth and how growth relates to inequality have a long tradition. In the following, we summarise the research most important to this study. As it seems that recent findings provide little support for the Kuznets curve stating that growth determines inequality (Anhand and Kanbur (1993) and Deininger and Squire (1998)), the question is how reversed causality relates inequality to growth.[3] Forbes (2000) finds a positive short and medium-term relationship between inequality and growth in a country, whereas the coefficient becomes insignificant for long-run considerations. This result, however, cannot be applied to very poor countries as they were excluded from the data set Forbes used. Furthermore, her findings do not contradict the theory that there exists a long-run negative relationship between inequality and growth. In fact, most of the theoretical channels suggesting a negative relationship would have an effect only over longer periods of time (see Galor and Zeira (1993) for capital market imperfection, and Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1996) and Rodrik (1998) for political economy and social conflicts). As we will consider a two-period OLG-framework with endogenous fertility and therefore a period-length between 30 and 40 years, we derive definite implications for the long-run performance of an economy.

Although both Perotti (1996) and Barro (2000) find that demographic variables are important for understanding the relationship between inequality and economic growth, the demographic structure in terms of differential fertility has not been considered by them. Recent research suggests that there is a positive correlation between inequality and fertility differentials, which is measured by the Total Fertility Rate by women's years of education. Kremer and Chen (2000) find that changing from a relatively equal country, such as Indonesia (Gini=0.32), to a country with higher inequality, such as Brazil (Gini=0.545) would lead to an increase in the fertility differential by 0.020. This result can be interpreted as follows. Comparing two women with ten and zero years of schooling means that the relation as to the expected number of children between both women is about 1.22 children higher in Brazil than in Indonesia.

De la Croix and Doepke (2003) close the link between inequality, fertility differentials, and growth. They argue both theoretically and empirically that the above mentioned positive correlation between inequality and fertility differentials

[3] Barro (2000) finds evidence for the Kusnetz curve; however his evidence is most likely based on the presence of transition countries in the data set (Ferreira (1999)).

translates into differentials in education between rich and poor, lowering average education and slowing down growth. In their work, as well as in Dahan and Tsiddon (1998) and Morand (1999), inequality is introduced by the initial distribution of human capital. The steady state, however, is characterised by an equal distribution of human capital.

Building on Schäfer (2003,2002) and similar to Galor and Zeira (1993), Galor and Zang (1997), Persson and Tabellini (1994), and Benabou (1996), we argue in this paper that the distribution of wealth in an economy is essential to overall growth performance. In detail, we analyse a two period OLG-model with endogenous labour supply. Parents receive utility from the quality and quantity of their offspring. Whereas the link between the direction of intergenerational transfers and the demographic transition is analysed in Blackburn and Cipriani (2002), we show that the amount of wealth including human and physical capital bequeasthed to the next generation leads to an intergenerational persistence in wage rates and fertility decisions. We find that growth is conducive to higher inequality and higher fertility differentials. Higher initial inequality slows down growth and increases transitory inequality, leading to redistributional pressure. Lastly, we close the missing link between growth, inequality, fertility, and the political economy by implementing a median voter system. Given the research results of Kremer and Chen (2000) and de la Croix and Doepke (2003), we find that inequality is not only harmful to growth but also leads to increasing redistributional pressure during the transition to the steady state. Hence, due to differential fertility, the median voter moves from upper to lower income percentiles and votes in favour of more redistribution given his/her preferences for economic equality. If the median voter prefers a relatively low tax-rate, then the redistribution of wealth in terms of future opportunities is low and the subsequent growth rate is also low since inequality is high. Contrary to existing literature, redistributional pressure does not cease once the steady state is reached, and it is interacting with demographic variables.

The paper is organized as follows. In Section 2, we introduce the set-up of the model. Section 3 introduces heterogeneity to the model. In Section 4, we explore the development of wealth distribution and Section 5 endogenises redistributional policies by means of the median-voter system. There follows a conclusion in Section 6.

2 The Model

We consider an economy populated by a continuum of households, limiting our analysis to two periods of life: childhood and adulthood. During childhood, each child consumes a constant fraction z of the parental time budget, with $n_t^{t-1,i}$ representing the number of children (quantity) and $l_t^{t-1,i}$ the household's labour supply. Normalising the total available time to one, the time budget constraint reads:

$$zn_t^{t-1} + l_t^{t-1} = 1.^4 \tag{1}$$

Consequently, the decision of a household to have a certain number of children runs analogously to the decision to be active in the labour market for a certain amount of time. When adult individuals receive bequest x_t^{t-1} and derive utility from their own number of children n_t^{t-1}, their own consumption c_t^{t-1} and x_t^t, with x_t^t representing the amount of wealth bequeathed to the children (quality). The overall fertility costs consist of opportunity costs in terms of foregone wage income during child-rearing time $w_t z n_t^{t-1}$ and the amount of wealth bequeathed to the offspring $x_t^{t,i} n_t^{t-1,i}$.[5]

For further interpretations, it is important to use a broad definition of capital x that includes physical and human capital and thus education too. Therefore, the amount of bequest x has to be interpreted as the sum of efforts undertaken by parents to equip their offspring with quality in terms of future opportunities.

The production technology is common knowledge, so that each adult has access to the following production technology:

$$y_t^{t-1} = B\left(x_t^{t-1}\right)^{\alpha}\left(l_t^{t-1}\right)^{1-\alpha}. \tag{2}$$

In addition, each household is equipped with some non-labour income h, provided exogenously.[6]

[4]Superscript $t-1$ indicates the period of birth and the indexes of the current period.

[5]The latter is responsible for the convexity of the budget constraint that governs the quality-quantity trade-off.

[6]h can be endogenised easily, for example by child or family allowances (see Schäfer (2004)).

The optimisation problem of household i is given by

$$\max_{\{n_t^{t-1,i};x_t^{t,i};c_t^{t-1,i}\}} u_t^{t-1,i} = \gamma \ln n_t^{t-1,i} + \beta \ln x_t^{t,i} + \phi \ln c_t^{t-1,i}, \tag{3}$$

subject to:

$$\underbrace{y_t^{t-1,i} + h}_{I_t^{t-1,i}} = w_t^{t-1,i} z n_t^{t-1,i} + c_t^{t-1,i} + n_t^{t-1,i} x_t^{t,i}, \tag{4}$$

and
$$1 = z n_t^{t-1,i} + l_t^{t-1,i}, \tag{5}$$

where
$$y_t^{t-1,i} = \frac{\partial y_t^{t-1,i}}{\partial l_t^{t-1,i}} l_t^{t-1,i} + \frac{\partial y_t^{t-1,i}}{\partial x_t^{t-1,i}} x_t^{t-1,i}. \tag{6}$$

In equilibrium, factors are compensated according to their marginal products and profits are zero:

$$\frac{\partial y_t^{t-1,i}}{\partial l_t^{t-1,i}} = w_t^{t-1,i}, \tag{7}$$

$$\frac{\partial y_t^{t-1,i}}{\partial x_t^{t-1,i}} = r_t^{t-1,i}. \tag{8}$$

Solving problem (3)-(5) leads to optimal decisions

$$c_t^{t-1,i} = \frac{\phi}{\gamma + \phi} I_t^{t-1,i}, \tag{9}$$

$$n_t^{t-1,i} = \frac{\gamma - \beta}{\phi + \gamma} \frac{I_t^{t-1,i}}{z w_t^{t-1,i}}, \tag{10}$$

$$x_t^{t,i} = \frac{\beta}{\gamma - \beta} z w_t^{t-1,i}, \quad \text{with } \gamma > \beta. \tag{11}$$

It is apparent from Eq.(10) and (11) that the model is characterised by a quality-quantity trade-off between the number of children and the amount of wealth. Parents of high wealth receive high wage incomes, have high opportunity costs of

child rearing, and hence low fertility, and they leave a high amount of wealth to their offspring. When their children are adult they will receive a high wage rate, and choose low fertility and a high quality per child. This leads to an intergenerational persistence in fertility decisions and wealth.
If the initial value of x is smaller than its steady state value ($x_0 < x^*$), then $\frac{x_t^t}{x_{t-1}^{t-1}} > 1$ and the economy converges as a whole, with declining fertility and increasing wages towards its steady state. The latter is characterised by constant fertility and zero growth in per capita terms:

$$n_t^{t-1,i} = n^* \quad \text{and} \quad x_t^{t,i} = x^* \qquad \forall \quad t, i. \tag{12}$$

If agents differ in their position in the wealth distribution, they will choose different levels of fertility and quality per child.

3 Heterogeneity in Terms of Wealth and Abilities

In this section, we extend the basic framework introduced above by heterogeneity and derive analytically the law of motion of the moments of wealth distribution during the transition to the steady state.

We assume two sources of heterogeneity. Firstly, the initial distribution of wealth in a society, for example, by way of norms concerning equal living standards, access to education or any other asset that can be accumulated and is important to production. For reasons of tractability and in accordance with empirical findings, we follow the common assumption that wealth is log-normal distributed.[7] The advantage of this specification is that the moments of wealth distribution are described by μ and σ. Furthermore, for any log-normal distributed random variable it must be true that the logarithm of this variable is normal distributed. The initial distribution of wealth is therefore given by

[7]See, for example, Diaz-Gimenez et al. (1997).

$$x^i_{t=0} \sim LN\left(\mu^x_{t=0}, (\sigma^x_{t=0})^2\right), \tag{13}$$

or equivalently

$$\ln x^i_{t=0} \sim N\left(\mu^x_{t=0}, (\sigma^x_{t=0})^2\right). \tag{14}$$

Consequently, the distribution and its moments are described in full by μ^x and σ^x, so that

$$E[x^i_{t=0}] = \exp\left(\mu^x_{t=0} + \frac{\sigma^{x^2}_{t=0}}{2}\right), \tag{15}$$

$$Var[x^i_{t=0}] = \exp\left(2\mu^x_{t=0} + \sigma^{x^2}_{t=0}\right)\left(\exp\left(\sigma^{x^2}_{t=0}\right) - 1\right). \tag{16}$$

Second, we allow for some social mobility (even in the steady state) by assuming that intellectual ability ϵ^i is log-normal distributed as well (Loury (1981)). Contrary to the distribution of wealth, the distribution of abilities does not change over time

$$\epsilon^{t-1,i}_t \sim LN\left(\mu^\epsilon, (\sigma^\epsilon)^2\right), \qquad \text{with } \mu^\epsilon = 0 \text{ and } \sigma^\epsilon = \text{const.} \ \forall\ t. \tag{17}$$

Since abilities govern individual abilities in production, individual output is subject to

$$y^{t-1,i}_t = \epsilon^{t-1,i}_t B\left(x^{t-1,i}_t\right)^\alpha \left(l^{t-1,i}_t\right)^{1-\alpha}. \tag{18}$$

Individual wealth develops according to (11) as

$$x^{t,i}_t = \frac{\beta}{\gamma - \beta} z w^{t-1,i}_t, \tag{19}$$

with $w^{t-1,i}_t = (1-\alpha)\epsilon^{t-1,i}_t B(x^{t-1,i}_t)^\alpha (l^{t-1,i}_t)^{-\alpha}$.

The amount of wealth received in period t is a function of the parental wage rate, which, in turn, is a positive function of the wealth the parents received and the parent's intellectual ability. Substituting the wage rate in (19) and taking logarithms leads to

$$\ln x_t^{t,i} = C + \ln \epsilon_t^{t-1,i} + \alpha \ln x_t^{t-1,i} - \alpha \ln l_t^{t-1,i}, \tag{20}$$

with $C \equiv \ln\left[\frac{\beta}{\gamma-\beta}\right] + \ln z + \ln B + \ln(1-\alpha)$.

Obviously, quality per child is governed by the parental ability term and their endowment with wealth. As the ability shock translates into endogenous variables of the model, the assumption of uncorrelated abilities over time is not as limiting as it might appear. Parents of low ability will leave lower bequests. Therefore, their children will perform worse, independent of their own ability.
Developing the expected value and the variance over Eq.(20) leads to two difference equations governing the development of the moments of wealth distribution

$$E[\ln x_t^{t,i}] = \mu_{t+1}^x = \ln C + \alpha\mu_t^x - \alpha\mu_t^l, \tag{21}$$

$$Var[\ln x_t^{t,i}] = (\sigma_{t+1}^x)^2 = (\sigma^\epsilon)^2 + \alpha^2\left((\sigma_t^x)^2 + (\sigma_t^l)^2\right). \tag{22}$$

As labour supply and fertility are a function of individual wealth, μ_t^l and σ_t^l can be expressed as functions of μ_t^x and σ_t^x. Defining $\ln l_t = \ln(1 - zn_t(x_t^{t-1})) \equiv g(x_t^{t-1})$ and developing a Taylor-series approximation yields:

$$E\left[\ln l_t^{t-1,i}\right] = E\left[g(\overline{x}) + (x-\overline{x})g'(\overline{x}) + \frac{1}{2}(x-\overline{x})^2 g''(\overline{x})\right]. \tag{23}$$

Using (23) and setting $\overline{x} = \mu_t^x$ leads to

$$E[\ln l_t] = \mu_t^l = g(\mu_t^x) + \frac{1}{2}(\sigma_t^x)^2 g''(\mu_t^x), \tag{24}$$

$$Var[\ln l_t] = (\sigma_t^l)^2 = (g'(\mu_t^x))^2(\sigma_t^x)^2. \tag{25}$$

The transition towards the steady state is entirely driven by the moments of wealth distribution and the structural parameters of the model. An increase in $(\sigma_t^x)^2$ leads to an increase in wealth inequality, accelerated by an increase in labour supply and fertility differentials. Hence, economic and demographic variables are interacting with each other.

As the steady state is characterised by zero growth in per-capita terms, such that $E[x_t^{t-1,i*}] = E[x_t^{t,i*}] = E[x^*]$, which implies that $E[w_{t-1}^{t-2,i*}] = E[w_t^{t-1,i*}] = E[w^*]$ and that $E[n_{t-1}^{t-2,i*}] = E[n_t^{t-1,i*}] = E[n^*]$, any steady state is characterised by a stationary distribution of wealth, so that $\mu_t^x = \mu_{t-1}^x = \mu^{x^*}$ and $(\sigma_t^x)^2 = (\sigma_{t-1}^x)^2 = (\sigma^{x^*})^2$. Consequently,

$$E[\ln x^*] = \mu^{x^*} = \ln C + \alpha\mu^{x^*} - \alpha\mu^{l^*}, \tag{26}$$

$$Var[\ln x^*] = (\sigma^{x^*})^2 = (\sigma^\epsilon)^2 + \alpha^2\left((\sigma^{x^*})^2 + (\sigma^{l^*})^2\right), \tag{27}$$

with $E[\ln l^*] = \mu^{l^*} = g(\mu^{x^*}) + \frac{1}{2}(\sigma^{x^*})^2 g''(\mu^{x^*})$,
and $Var[\ln l^*] = (\sigma^{l^*})^2 = (g'(\mu^{x^*}))^2(\sigma^{x^*})^2$.

4 Development of Wealth Distribution

In this section, the development of wealth distribution is analysed. In order to run numerical experiments matching empirical features in industrialised countries, the set of parameters $\{\alpha, \beta, \gamma, \phi, z, B, h, \sigma^\epsilon\}$ has to be specified.[8] We choose a capital share α equal to 0.3. Empirical evidence suggests that the opportunity cost of a child amounts to 15% of parents time endowment. Assuming that a child lives with the parents 15 years (at least) and that the adult period lasts for 30 years implies a value of z equal to 0.0775. ϕ is set to 2.1, B equals 12, and we choose $\beta = 0.1925$ which implies an expenditure share close to 0.88 and a log-run interest rate between three and four percent. The weight of fertility in the parental utility function γ is set 0.2975, in order to guarantee long-run fertility around the reproduction level.[9] Non-labour market income h equals 3.5 and amounts to one third of the average long-run wage rate. Finally, we choose $\sigma^\epsilon = 0.5$ assuring that the long-run (average) fertility differential remains in the intervall between 0.4 and 1.5.[10]

We first conduct an 'empirical' experiment by generating an artificial sample of households, whether the log-normality of wealth distribution is maintained during the transition process or not. In a second step, we simulate the development

[8] Furthermore, the choice of parameters is closely related to De la Croix and Doepke (2003,2004).

[9] Fertility rates in most industrialised countries are around the reproduction level and long-run projections, for example by the United Nations predict a stable population in the long-run.

[10] The results presented below are for reasonable parameter values robust.

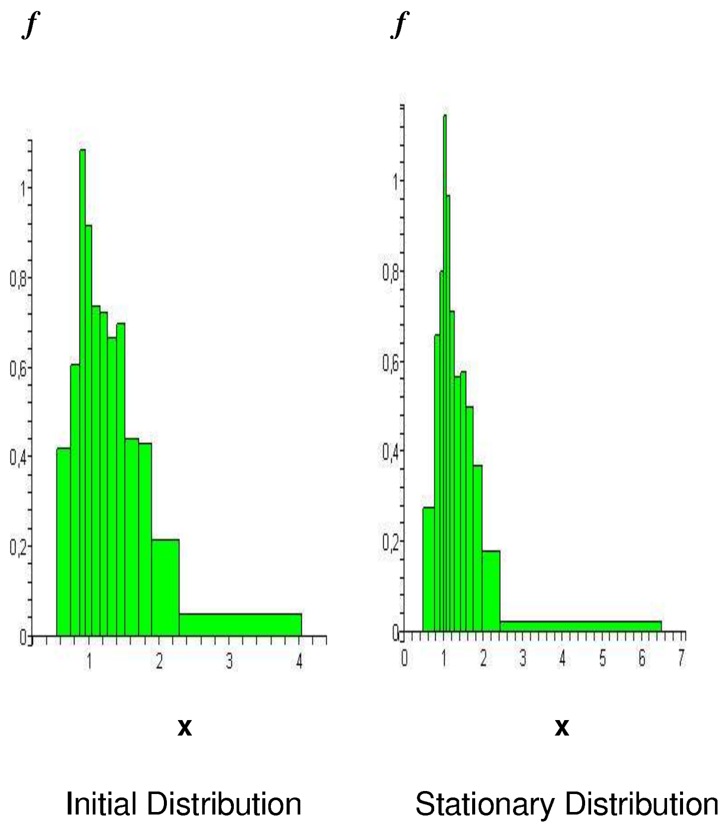

Figure 1: Dynamics of wealth distribution within an artificial sample for $\mu^x_{t=0} = 0.2, \sigma^x_{t=0} = 0.4$ and N=1000.

of theoretical wealth distribution as it can be expected by the considerations above. Since we consider agents who differ in their innate abilities and in their factor endowments, the first period of the economy ($t = 0$) is characterised by the initial parameters of wealth distribution $\mu^x_{t=0}$ and $\sigma^x_{t=0}$ determining the amount of inequality in the economy, and the set of parameters.

Given the set of parameters, thousand artificial households are generated by drawing a stochastic vector $\vec{\epsilon}_{t=0} = [\epsilon^{i=1}_{t=0}, ..., \epsilon^{i=1000}_{t=0}]'$ from the log-normal distribution, satisfying Eq.(17). The endowment vector of the initial period is generated similarly, such that $\vec{x}_{t=0} = [x^{i=1}_{t=0}, ..., x^{i=1000}_{t=0}]'$, given $\mu_{t=0} = 0.2$ and $\sigma_{t=0} = 0.4$. Each of the thousand agents takes the decisions according to the underlying optimisation problem, which is known and the same for all of them. Solving this

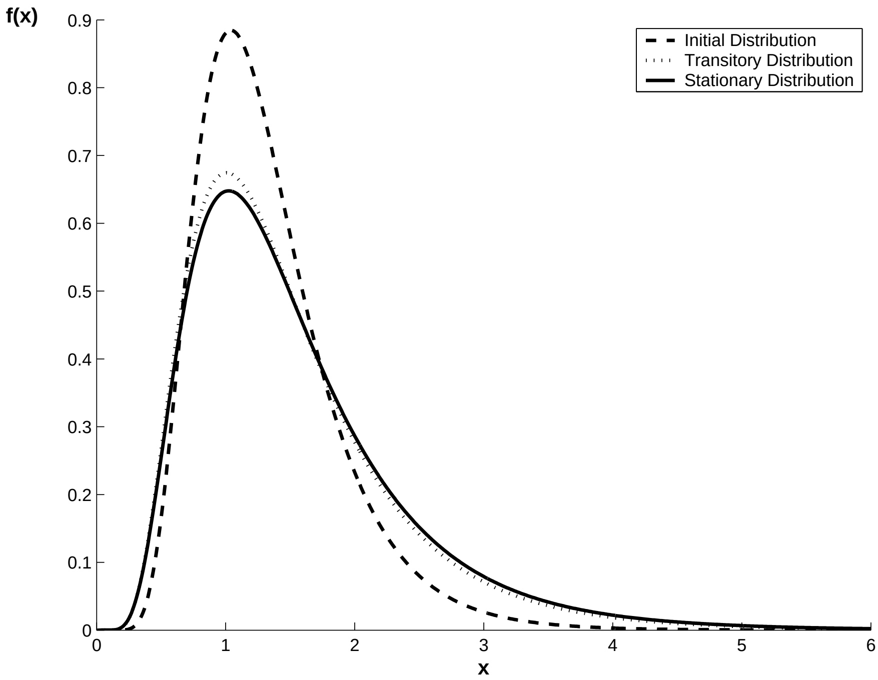

Figure 2: Dynamics of wealth distribution $f(x)$ for $\mu^x_{t=0} = 0.2, \sigma^x_{t=0} = 0.4$.

problem leads to the set of optimal decisions of the sample, especially fertility decisions $n^i_{t=0}$ and optimal choices for bequests $x^i_{t=0}$. The two latter ones constitute the endowment vector $\vec{x}_{t=1}$.
In order to take account of the different fertility decisions, each element i of $\vec{x}_{t=1}$ is weighted by the relative fertility of household i with respect to the mean of the economy. At the end of period one, the working generation dies and their children enter the labour market, equipped with some $x^i_{t=1} \in \vec{x}_{t=1}$. Given the endogenously determined vector $\vec{x}_{t=1}$, the amount of inequality for period one is given as well, so that the loop starts again by generating a new stochastic ability vector $\vec{\epsilon}_{t=1}$. The procedure of this loop continues until the distribution of wealth in this artificial sample becomes stationary.
The histograms are shown in Figure 1. Apparently, the characteristics of a log-normal distribution are preserved during the transition to the steady state, whereas the steady state itself is characterised by a stationary distribution.
In order to examine the interaction between inequality, fertility, and growth, we now explore the theoretically derived dynamics given by the set of difference

Parameter: $\mu^x_{t=0} = 0.2$ and $\sigma^x_{t=0} = 0.4$

t	$E[x_t^{t-1}]$	$Var[x_t^{t-1}]$	$E[l_t^{t-1}]$	$Var[l_t^{t-1}]$	$\frac{\mu^x_{t+1}}{\mu^x_t}$	μ^x_t	σ^x_t	μ^l_t	$(\sigma^l_t)^2$
1	1.3231	0.3037	0.8887	$0.2668\ 10^{-3}$	-	0.2	0.4	-0.1180	$0.3377\ 10^{-3}$
2	1.4985	0.6797	0.8950	$0.2002\ 10^{-3}$	1.3615	0.2723	0.5142	-0.1110	$0.2499\ 10^{-3}$
3	1.5354	0.7425	0.8978	$0.1753\ 10^{-3}$	1.0719	0.2918	0.5232	-0.1088	$0.2175\ 10^{-3}$
4	1.5436	0.7531	0.8987	$0.1655\ 10^{-3}$	1.0168	0.2968	0.5240	-0.1068	$0.2048\ 10^{-3}$
..	.....	......	.....	......	......	.....	.....	.....	.....
19	1.5463	0.7561	0.8988	$0.1648\ 10^{-3}$	1	0.2985	0.5241	-0.1067	$0.2040\ 10^{-3}$
20	1.5463	0.7561	0.8988	$0.1648\ 10^{-3}$	1	0.2985	0.5241	-0.1067	$0.2040\ 10^{-3}$

Table 1: Dynamics of average wealth $E[x_t^{t-1}]$ and labour supply $E[l_t^{t-1}]$, respective variances, and moments of respective distributions for $\mu^x_{t=0} = 0.2$ and $\sigma^x_{t=0} = 0.4$

Parameter: $\mu^x_{t=0} = 0.2$ and $\sigma^x_{t=0} = 0.55$

t	$E[x_t^{t-1}]$	$Var[x_t^{t-1}]$	$E[l_t^{t-1}]$	$Var[l_t^{t-1}]$	$\frac{\mu^x_{t+1}}{\mu^x_t}$	μ^x_t	σ^x_t	μ^l_t	$(\sigma^l_t)^2$
1	1.4208	0.7131	0.8699	$0.4834\ 10^{-3}$	-	0.2	0.55	-0.1395	$0.6384\ 10^{-3}$
2	1.5180	0.7363	0.8654	$0.2091\ 10^{-3}$	1.3938	0.2787	0.5265	-0.1105	$0.2483\ 10^{-3}$
3	1.5390	0.7496	0.8978	$0.1815\ 10^{-3}$	1.0534	0.2936	0.5243	-0.1078	$0.2127\ 10^{-3}$
4	1.5445	0.7544	0.8985	$0.1660\ 10^{-3}$	1.0125	0.2973	0.5242	-0.1071	$0.2056\ 10^{-3}$
..	.....	......	.....	......	......	......	.....	.....	.....
19	1.5463	0.7561	0.8986	$0.1648\ 10^{-3}$	1	0.2985	0.5241	-0.1067	$0.2040\ 10^{-3}$
20	1.5463	0.7561	0.8986	$0.1648\ 10^{-3}$	1	0.2985	0.5241	-0.1067	$0.2040\ 10^{-3}$

Table 2: Dynamics of average wealth $E[x_t^{t-1}]$ and labour supply $E[l_t^{t-1}]$, respective variances, and moments of respective distributions for $\mu^x_{t=0} = 0.2$ and $\sigma^x_{t=0} = 0.55$

equations (21) and (22). Furthermore, we distinguish two scenarios that have different degrees of inequality:

1. $x^i_{t=0} \sim LN(\mu^x_{t=0} = 0.2;\ \sigma^x_{t=0} = 0.4)$,
2. $x^i_{t=0} \sim LN(\mu^x_{t=0} = 0.2;\ \sigma^x_{t=0} = 0.55)$.

The simulation results are shown in Tables 1 and 2 and Figure 2.

Economies with different degrees of initial inequality but identical parameters converge to the same steady state. Obviously, the initial distribution of wealth has, everything else being equal, 'only' transitory effects.

The results of the numerical simulation presented in Tables 1 and 2 and plotted in Figure 2 reveal that growth leads to a higher expected value of wealth $E[x_t^{t-1,i}]$ and also to higher inequality ($Var[x_t^{t-1,i}]$ is increasing). This is due to the fact that, given the degree of inequality, households are endowed with different abilities $\epsilon_t^{t-1,i}$ and wealth $x_t^{t-1,i}$ determining their respective wage rates

$w_t^{t-1,i}$. As $x_t^{t-1,i}$ is increasing $\forall$ i during the transition to the steady state (see Eq.(19)), labour supply increases and fertility declines. Consequently, the expected value of wealth must increase. Independently of the initial amount of inequality, households are getting wealthier (on average) during the transition to the steady state. However, as the model exhibits an intergenerational persistence in wage rates, fertility decisions, and quality (wealth) per child, inequality is also increasing. Poor households will invest less in their children and exhibit higher fertility. Their children, in turn, will receive a relatively low wage income, due to a relatively low amount of x and so on. Consequently, poor households will do worse than richer ones with the same ability. However, as the expected value of x is increasing and agents are getting richer, the distance between rich and poor is increasing, too. Furthermore, it becomes apparent from comparing Table 1 with Table 2 that the described effects are increasing in strength with rising initial inequality.

A relatively equal initial distribution of wealth leads to relatively high wage rates for all households and therefore to relatively high labour supply, with a fast decline in fertility and an expected high growth rate.

Eq.(10) and (11) show that the amount of wealth translates into fertility decisions. Therefore, inequality in wealth must also lead to inequality in labour supply and fertility, and hence in fertility differentials. As the scenario presented in Table 2 is characterized by higher initial inequality, the variance of labour supply $Var[l_t^{t-1,i}]$ is higher, too during the transition. This implies also higher fertility differentials due to the intergenerational persistence mentioned above. As wealth (inequality) is an endogenous variable, variations in the initial amount of inequality can only have transitory effects. Economies with different initial conditions are converging, with everything else being equal, to the same steady state.

We can summarise these results as follows:

1. Growth leads to a higher expected value of wealth, as well as higher variance; thus inequality increases.

2. During the transition, higher initial inequality raises both the expected value and variance of x.

3. During the transition, higher initial inequality lowers the expected value of labour supply and increases its variance, hence fertility differentials rise.

4. The more equal the initial distribution, the higher is the expected growth rate.

It follows that the inequality rising (transitory) effect of growth is weakened by an initial distribution of wealth that is more equal. Growth leads to higher labour supply, declining fertility, and decreasing fertility differentials. This means that in the earlier stages of economic development, lower income percentiles have a higher fertility than the upper ones when controlling for initial inequality.
Faster growth caused by a more equal initial distribution leads to a higher growth rate of labour market participation and a faster decline in fertility.
These results correspond to the increasing arm of the Kuznets curve, as higher growth leads to rising inequality and higher fertility differentials when controlling for the initial wealth distribution. If this is the case, higher inequality and higher fertility for lower income percentiles should translate into redistributional pressure. Therefore, it seems to be very unlikely that an economy converges as described to its steady state. One should expect that growth and increasing social pressure leads to the redistribution of widely defined capital and opportunities. A norm of justice in wealth redistribution could be that people of the same innate abilities can engage in the same venture, independent of his/her social background.[11] The effects of wealth redistribution are examined in the following section.

5 Endogenous Redistribution

In this section, we seek to analyse the effects of wealth redistribution when the amount of redistribution is endogenised by a majority-voting system. Note here that redistribution takes place before entering the labour force and not after production. An after-production tax would reverse the outcomes. Such a redistribution scheme would increase the non-labor market income for transfer-receiving households and lower the opportunity costs of child-rearing time for the contributing households. Fertility would increase economy-wide and growth would slow down because of lower wealth or quality per adult.[12]
We assume instead that wealth is redistributed before entering the labour force, according to the following tax scheme (Benabou (1996)):

$$\hat{x}_t^{t-1,i} = (x_t^{t-1,i})^{1-\tau}(\tilde{x}_t)^{\tau}, \tag{28}$$

[11] For a similar argumentation, see Galor and Zeira (1993). We prescind from the existence of a capital market and consider the non-existence of a capital market as a limiting case of capital market discrimination.

[12] For the consequences of labour taxation, see Schäfer (2004).

where $\hat{x}_t^{t-1,i}$ represents after-tax wealth, $\tilde{x}_t^{t-1,i}$ break-even wealth and $0 < \tau < 1$ the tax rate. Obviously, the tax scheme is progressive. Households with wealth lower than $\tilde{x}$ have a higher after-tax wealth, and the opposite is true for the richer ones:

$$\hat{x}_t^{t-1,i} > x_t^{t-1,i} \quad \forall \quad \{x_t^{t-1,i} | x_t^{t-1,i} < \tilde{x}_t\}, \tag{29}$$

$$\hat{x}_t^{t-1,i} < x_t^{t-1,i} \quad \forall \quad \{x_t^{t-1,i} | x_t^{t-1,i} > \tilde{x}_t\}. \tag{30}$$

Given, that $d = x_t^{t-1,i} - \hat{x}_t^{t-1,i}$ represents the net-transfer received or contributed to according to Eq.(28), it has to be taken into account that net-transfers sum up to zero in each period:

$$\int_0^{N_t} x_t^{t-1,i} - \hat{x}_t^{t-1,i} \quad dx^i = 0. \tag{31}$$

Equation (28) and (31) determine the budget constraint for wealth redistribution which is binding as we do not consider intertemporal debt policies. It follows that the expected value of after-tax wealth equals the expected value of pre-tax wealth,

$$\int_0^{N_t} x_t^{t-1,i} f(x_t^{t-1,i}) \quad dx^i = \tilde{x}_t^{\tau} \int_0^{N_t} (x_t^{t-1,i})^{1-\tau} f((x_t^{t-1,i})^{1-\tau}) \quad dx^i, \tag{32}$$

$$\text{and} \quad (x_t^{t-1,i})^{1-\tau} \sim LN\left((1-\tau)\mu_t^x, (1-\tau)^2(\sigma_t^x)^2\right). \tag{33}$$

In light of (28) and (33) it follows that

$$\begin{aligned} &\ln\left[\exp\left(\mu_t^x + \frac{(\sigma_t^x)^2}{2}\right)\right] \\ &= \tau \ln \tilde{x}_t + \ln\left[\exp\left((1-\tau)\mu_t^x + \frac{(1-\tau)^2(\sigma_t^x)^2}{2}\right)\right]. \end{aligned} \tag{34}$$

Hence, the break-even wealth that satisfies the budget constraint is given by the following relation:

$$\ln \tilde{x}_t = \mu_t^x + \frac{1}{\tau}\left[1 - (1-\tau)^2\right]\frac{(\sigma_t^x)^2}{2}. \tag{35}$$

With everything else being constant, a higher tax rate implies a lower break-even wealth (differentiating (35) with respect to τ leads to $\frac{\partial \ln \tilde{x}_t}{\partial \tau} = -\tau^2 < 0$), and to a higher redistribution concentrating on the lower income percentiles.
Each household formulates a voting function, given optimal choices for fertility $n_t^{t-1,i}$, quality $x_t^{t-1,i}$, consumption $c_t^{t-1,i}$, and the break-even wealth $\tilde{x}_t$:

$$V_t^i = \gamma \ln n_t^{t-1,i} + \beta \ln x_t^{t,i} + \phi \ln c_t^{t-1,i} + \Upsilon \ln \tilde{x}_t. \tag{36}$$

The last term in the voting function therefore captures the individual preferences for redistribution and hence the tolerated amount of inequality. For break-even wealth, the following relationships must hold (see Eqs.(35) and (28)):

$$\ln \tilde{x}_t = \mu_t^x + \frac{1}{\tau_t}\left[1 - (1-\tau_t)^2\right]\frac{(\sigma_t^x)^2}{2} \tag{37}$$

and

$$\ln \tilde{x}_t = \frac{1}{\tau_t}\left[\ln \hat{x}_t^{t-1,i} - (1-\tau_t)\ln x_t^{t-1,i}\right]. \tag{38}$$

As stated above, break-even wealth is declining in the tax rate and increasing in the degree of inequality. The second equation (see Eq.(38)) captures a rather egoistic behaviour, in the sense that each individual tries to maximize the distance between pre and after-tax wealth, so that each individual seeks to set a tax rate that increases his or her after-tax wealth. Therefore, a low weight Υ of $\tilde{x}_t$ reflects a preference for low distance between $\hat{x}_t^{t-1,i}$ and $x_t^{t-1,i}$, hence less selfishness, and a relatively high preference for a lower σ_t^x associated with a higher tax rate. Alternatively, Υ can be interpreted as voting-power. A high Υ would represent a system which is more biased towards the rich.
In a majority-voting system, we assume that the median-voter is decisive for the size of the tax-rate. Although this is the common assumption in the literature of income distribution, one should take into consideration that this is only true if political power is independent of economic power. Despite the fact that rich as well as poor people have only one vote, the rich have greater resources available to organise and articulate their preferences. In addition the electoral participation of poor people, by contrast, might be lower, due to a lower level of education. Therefore, any majority-voting system might be more biased towards the rich. However, the interval around the median definitively serves as a point of refer-

ence, as it represents more than fifty percent of the population.[13]
The moments of wealth distribution are known, so that the pre-tax wealth of the median is known, too:

$$x_t^{t-1,med} = \exp(\mu_t^x). \tag{39}$$

Consequently, the after-tax wealth of the median is given by

$$\hat{x}_t^{t-1,med} = (\exp(\mu_t^x))^{1-\tau_t}(\tilde{x}_t)^{\tau_t}, \tag{40}$$

implying a median after-tax income of

$$\hat{y}_t^{t-1,med} = \hat{w}_t^{t-1,med} l_t^{t-1,med} + \hat{r}_t^{t-1,med} \hat{x}_t^{t-1,med}. \tag{41}$$

Plugging (41) together with the set of optimal decisions (Eqs. (9)-(11)) into Eq.(36), we obtain the voting function of the median voter $V_t^{med}(\tau_t^{med})$. Forming the first-order condition $\frac{\partial V_t^{med}(\tau_t^{med})}{\partial \tau_t^{med}} = 0$ (and $\frac{\partial^2 V_t^{med}(\tau_t^{med})}{\partial \tau_t^{med 2}} < 0$) leads, together with Eq.(10) to a system of equations with τ_t^{med} and $n_t^{t-1,med}$ as unknowns. Solving this system of equations leads to the preferred tax rate in period t:

$$\tau_t = \tau_t^{med}. \tag{42}$$

With the tax rate known, the break even wealth $\tilde{x}_t$ is also known given the moments of wealth distribution. Consequently, the economy develops according to

$$\begin{aligned} E[\ln x_t^{t-1,i}] \quad = \quad \mu_{t+1}^x \quad = \quad & \ln C + \alpha\mu_t^x + \alpha\left[1 - (1-\tau_t^{med})^2\right]\frac{(\sigma_t^x)^2}{2} \\ & -\alpha\mu_t^l, \end{aligned} \tag{43}$$

and

$$Var[\ln x_t^{t-1,i}] \quad = \quad (\sigma_{t+1}^x)^2 \; = (\sigma^\epsilon)^2 + \alpha^2\left((1-\tau_t^{med})^2(\sigma_t^x)^2 + (\sigma_t^l)^2\right). \tag{44}$$

[13]For an exogenous approach of biased democracies, see Benabou (1996); for an endogenous approach, see Bourguignon and Verdier (2000).

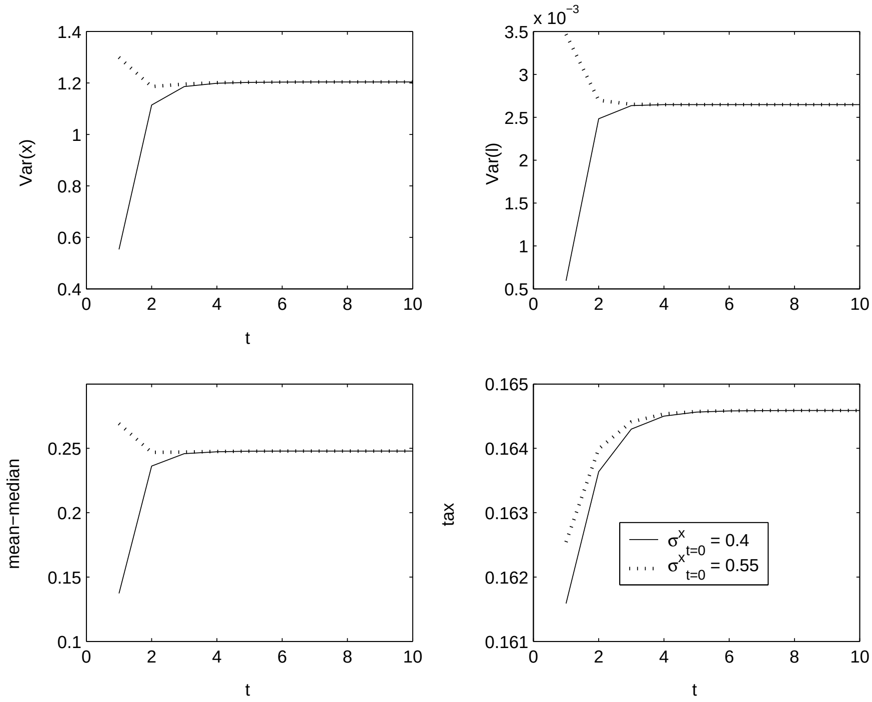

Figure 3: Secenario I: Different Initial Distributions; Identical Preferences for Redistribution

In order to explore the effects of inequality in a median-voter system, we simulate two scenarios. Scenario I (see Figure 3) is characterised by different initial degrees of inequality ($\sigma_{t=0} = 0.4$ and $\sigma_{t=0} = 0.55$). The preference for redistribution, however, is identical. The second scenario (see Figure 4) is characterised by the same initial degree of inequality, and this time the preferences for redistribution ($\Upsilon = 0.95$ and $\Upsilon = 0.85$) are different.
As can be clearly seen from the simulations performed in Scenario I, an increase in inequality leads to higher variance in wealth. The mean is richer, therefore the labour supply of the mean is higher, too, and its fertility lower, but the distance between mean and median has also increased. Despite an increase in average wealth and lower fertility of persons with average wealth, the variances for both wealth and labour supply are higher. Higher labour supply differentials imply higher fertility differentials and therefore faster population growth in the lower income percentiles. As the household of average wealth does not coincide with the population's average, a higher distance between the median and the mean

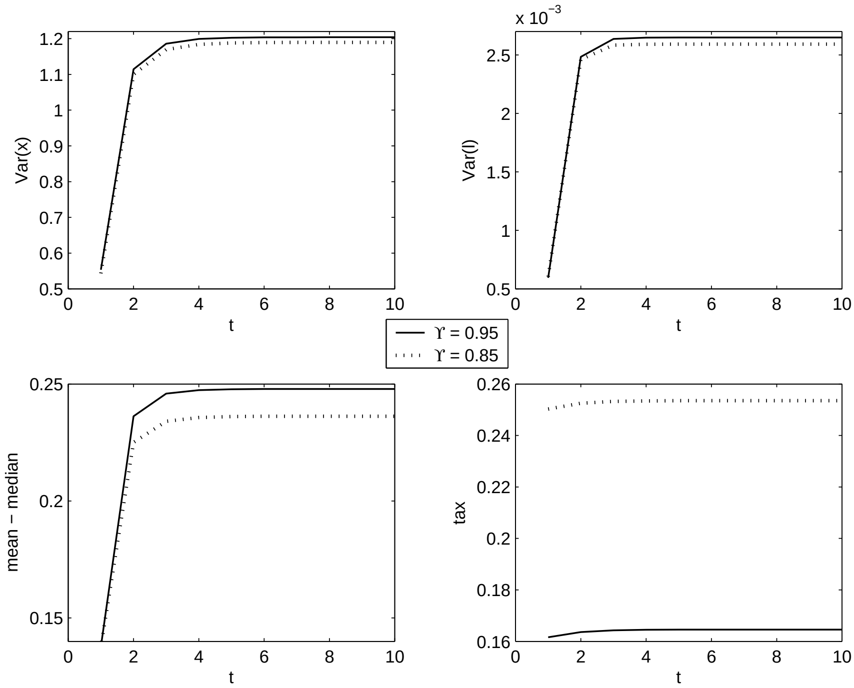

Figure 4: Scenario II: Identical Initial Distributions of Wealth; Different Preferences for Redistribution

leads to a higher tax rate and a lower break-even wealth. With the same parameter constellation, the long-run characteristics of Scenario I are entirely the same. In Scenario II, we seek to explore the effects of different preferences for redistribution. These not only affect the transitory but also the long-run characteristics of the economy. With a lower tax rate, the mean is poorer and the variance of wealth is increasing. A higher amount of inequality translates into higher labour supply and fertility differentials. Due to diminishing returns to wealth in the production function, a higher amount of inequality lowers the growth rate of the economy during the transition to the steady state. By contrast, a higher amount of redistribution lowers fertility differentials and redistributes wealth to households with higher returns. Consequently, the growth rate is higher. Here it is once again important to remember that this scenario considers a pre-production tax. The implementation of an after-production tax would be harmful to growth.

We can summarise the results of this subsection as follows

1. As economic growth increases inequality, the difference between the mean and median of wealth distribution is increasing. Consequently, the median-voter opts for an increasing tax rate during the transition to the steady state.

2. The size of the tax rate and hence the amount of wealth redistribution depends on preferences for equality in the economy, and specifically the preferences of the median-voter.

3. A lower preference for equality implies a lower tax rate during all periods. The preference for redistribution, therefore, translates not only into the transitory but also into the long-run characteristics of the economy.

4. A lower preference for equality leads to higher transitory and long-run inequality and fertility differentials.

During the transition, the median-voter becomes increasingly poorer due to rising fertility differentials. As a consequence, the tax rate is increasing during the transition; the more it does so, the more unequal is the wealth distribution and the higher is the preference for equality.
Contrary to existing literature, redistributional pressure does not cease once the steady state is reached, and it is interacting with demographic variables.

6 Conclusion

Using empirical findings, we have shown that fertility declines as economic development progresses and that fertility interacts with wealth distribution.
Growth by itself leads to higher inequality and rising fertility differentials. This gives rise to redistributional pressure during the process of economic development. In terms of wealth redistribution, once a redistributional policy is applied, the average economic growth rate increases.
In a median-voter system, the preferred tax-rate and the transitory as well as the limiting distribution depend on the preference for redistribution, and hence for societal equality. During the transition, the median-voter becomes increasingly poorer due to rising fertility differentials. The more unequal the wealth distribution is and the higher the preference for equality is, the more is the tax rate

increasing during the transition. Contrary to existing literature, redistributional pressure is interacting with demographic variables and does not cease once the steady state is reached.

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