Pension Scheme Redesign and Wealth Redistribution Between the Members and Sponsor: The USS Rule Change in October 2011

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ABSTRACT
The redesign of defined benefit pension schemes usually results in a substantial redistribution of wealth between age cohorts of members, pensioners, and the sponsor. This is the first study to quantify the redistributive effects of a rule change by a real world scheme (the Universities Superannuation Scheme, USS) where the sponsor underwrites the pension promise. In October 2011 USS closed its final salary scheme to new members, opened a career average revalued earnings (CARE) section, and moved to ‘cap and share’ contribution rates. We find that the pre-October 2011 scheme was not viable in the long run, while the post-October 2011 scheme is probably viable in the long run, but faces medium term problems. In October 2011 future members of USS lost 65% of their pension wealth (or roughly £100,000 per head), equivalent to a reduction of roughly 11% in their total compensation, while those aged over 57 years lost almost nothing. The riskiness of the pension wealth of future members increased by a third, while the riskiness of the present value of the sponsor’s future contributions reduced by 10%. Finally, the sponsor’s wealth increased by about £32.5 billion, equivalent to a reduction of 26% in their pension costs.

Key words: Defined benefit, Pension scheme, Redistribution, USS, Scheme design, Risk shifting, Risk management

JEL: G22, G23, J32
On retirement the sponsor of a UK defined benefit (DB) pension scheme promises to pay a pension according to the rules of the scheme, regardless of the scheme’s financial state. This appears to place all the risks (investment, interest rates, inflation, salaries, longevity, regulation, etc.) on the sponsor, who is usually the employer. But the sponsor can share these risks with active and future members of the scheme by altering the rules applying to future accruals. For example, a large deficit may lead to rule changes such as an increase in the members’ contribution rate, the introduction of limited price indexation, a later retirement age, or a reduction in the accrual rate. Because UK law does not allow accrued benefits to be reduced, rule changes only apply to future accruals. This means that the youngest scheme members are the hardest hit by such action as they will be accruing benefits under the new rules for many years, while those near retirement are largely unaffected since their substantial accrued benefits are legally protected.

Before a rule change the various scheme participants have both accrued benefits and expectations of the net present value (NPV) of their future interactions with the scheme, i.e. contributions to be made and pensions to be received\(^1\). After a rule change these expectations are altered, and the difference between NPVs of the cash flows before and after the rule change for each age cohort quantifies the redistributive effect of the rule change. For example, an increase in the member contribution rate redistributes pension wealth from active and future members to the sponsor. Therefore a rule change leads to the redistribution of pension wealth and risk between the main groups of participant - the sponsor, active members, deferred members\(^2\), pensioners and future members.

When rule changes are proposed, attention usually focusses on the details of these changes such as contribution rates, accrual rates and retirement ages, but with no detailed valuation of the size of the wealth transfer. Almost no explicit consideration is given to the effects of a rule change on the wealth of the different age cohorts, or to the riskiness of this wealth, and these can be substantial. Therefore an important objective of this paper is to stimulate a greater awareness of the redistributive effects on wealth and risk of pension scheme redesign, particularly the

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\(^1\) The resulting changes in cash flows between the members and sponsor are zero sum.

\(^2\) Members who are no longer active contributors, but who have not yet retired.
generational effects. While this paper deals with a particular pension scheme and rule change, the methodology can be applied to investigate the redistributive effects of rule changes by other DB schemes where the sponsor remains responsible for meeting the pension promise, as in countries such as the UK and USA. It can also be used to investigate the long run viability of such DB pension schemes.

Previous investigations of the redistribution of pension wealth by rule changes have been of hypothetical schemes. This is the first paper to quantify the redistributive effects of a major package of rule changes by a large real-world DB pension scheme - the UK Universities Superannuation Scheme (USS). Almost all previous studies have been of hypothetical Dutch schemes where the sponsor has no obligation beyond paying a fixed contribution rate. Therefore the sponsor is not involved, and all the redistribution is between different generations of member, i.e. inter-generational redistribution. In 2011 USS was a ‘balance of cost’ scheme where, unlike Dutch schemes, the sponsor bears the default risk, and so any redistribution of wealth and risk is primarily between the sponsor and members.

To quantify redistribution stemming from the October 2011 rule change, a benchmark must be specified. One possible benchmark is to compute the ‘true’ funding position of USS in October 2011, and then to distribute any deficit among the sponsor and the cohorts of members and pensioners. However, there would be a considerable degree of uncertainty and subjectivity attached to such a benchmark. In October 2011 USS had a well-defined set of rules, the main features of which had remained unchanged since 1975, when USS began. Therefore a reasonable expectation for members in October 2011 was that the pension promises enshrined in the USS rules would be honoured, and so the benchmark we use is the pre-October 2011 scheme.

This paper incorporates many aspects of the problem not included in previous studies - lump sum payments on retirement, deferred pensioners, limited price indexation, spouses’ pensions, increases in the retirement date, both final salary and career revalued benefits (CRB) sections, and consumer price indexation (CPI) of the accrued benefits of the CRB section active members and the accrued benefits of deferred pensioners, as well as pensions in payment. In addition, we compute final salaries using the retail price index (RPI), see Appendix A. This is also the first
study of redistribution by a scheme moving to ‘cap and share’ contribution rates. We model the pension scheme for longer than a working lifetime to avoid the problem of back-loading, where contributions made when young represent worse value than those made when old\(^3\). If the effects of a rule change are quantified for a period shorter than a working lifetime, the presence of back-loading is likely to show that the young receive a less favourable outcome than the old. We also employ a dynamic asset allocation strategy by allowing the asset allocation to respond to the current funding ratio (assets/liabilities), rather than use a fix-mix investment strategy as have most previous studies. With 13 factors the vector auto-regression (VAR) model we use to forecast asset returns and inflation includes many more assets than previous studies, and is only the second study to include the three factors of the yield curve (level, slope and curvature) in the VAR model, rather than selected interest rates. Finally, we model the numbers of new active and deferred scheme members each year as stochastic processes.

Section 1 describes USS, and section 2 outlines our methodology. Section 3 has a literature review, followed in section 4 by details of the data and methodology used to forecast the yield curve, asset returns, inflation and academic salaries each period until the horizon date. Section 5 contains the procedure for forecasting the size of each age cohort, and section 6 explains how the liabilities (i.e. the accrued benefits) of each age cohort are estimated at the end of each period. Section 7 then brings together all these forecasts to calculate the triennial values of the USS funding ratio, revisions to the member and sponsor contribution rates, and adjustments to the asset allocation. In section 8 these are used to generate the cash flows to and from the various participants each time period until the horizon date. The NPVs of these cash flows are valued using stochastic discount factors (SDF) to give the redistribution of wealth generated by the October 2011 rule changes. The results appear in section 9, with robustness checks in section 10, where the use of riskless discount rates also permits estimates of the changes in risk\(^4\). Finally, section 11 has the conclusions.

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\(^3\) Back-loading occurs when the scheme uses age-independent contribution and accrual rates (as does USS) and the rate of return on the scheme’s assets exceeds the rate of salary growth.

\(^4\) It is not possible to use SDFs to measure changes in risk.
1. USS
In 2014 USS was the second largest pension scheme in the UK, and the 36th largest in the world with 316,440 active members, deferred pensioners and pensioners. It is a multi-employer scheme with 374 separate sponsors (or institutions), and assets valued at £42 billion in 2014. Until the rule change implemented in October 2011, USS was an open final salary scheme. In October 2011 USS was split into two sections - a final salary section that was closed to new members in October 2011, and a CRB section, which operates on a career average revalued earnings (CARE) basis, and started operation in October 2011. The rule changes in October 2011 were a matter of heated public controversy between the institutional sponsors of USS, represented by the Employers Pension Forum; and the members and pensioners of USS, represented by the University and College Union (UCU), leading to lengthy industrial action by members of the UCU.5

USS is a very large and complicated scheme with a 295 page rule book, and so any model of USS is bound to be a gross simplification. This study captures the financially important features of USS, including all the rules that changed. The other important changes implemented in October 2011, besides new members joining the CRB section, were (a) an increase in the contribution rate for the final salary section, (b) the introduction of a ‘cap and share’ rule for deficits and surpluses, (c) linking the normal retirement age to the state pension age, and (d) limiting the indexation of pensions and deferred pensions. The rules pre and post-October 2011 are set out in Appendix A. This appendix also details some of the other USS rules incorporated in our model, including lump sum payments, spouses’ pensions, deferred pensioners, and the computation of final salary. Unchanged rules tend to be less important because they have a similar effect on pension wealth before and after the rule change, and so tend not to create redistribution.

2. Methodology
We modelled the effects of the six rule changes as a single package, rather than examining the effects of each rule change separately. This is because we are primarily interested in the effects of the package, the rule changes interact and so the effects of the October 2011 rule changes are only available by treating them as a package, and because repeating the analysis another six

5 No explicit concerns were expressed for the distributional implications of the rule change.
times would be a considerable undertaking. The analysis of the redistributive effects of the USS rules change in October 2011 is divided into two main steps. The first step is to model the evolution of USS over the horizon period, permitting forecasts of the cash flows between each age cohort, the sponsors and USS under two alternative sets of rules - those pre and post-October 2011. This will be done using three year time periods, as this is the frequency of USS actuarial valuations and contribution rate reviews. In the second step, the NPV of the forecast cash flows for each age cohort and the sponsors is computed for both the pre and post-October 2011 rules. This allows the calculation of the NPV of the change in expected pension wealth for each cohort caused by the October 2011 rule changes, which is the standard way of measuring pension redistribution, Bonenkamp (2009).

We concentrate on expected pension wealth, although the October 2011 rule change may well have other effects on members and the sponsor. Pension contributions are an important component of university expenditure. Since the government no longer raises university funding to compensate for increases in the cost of USS, any additional sponsor contributions must be funded by the universities themselves. Apart from raising additional revenue, universities might make cost savings by cutting expenditure on capital projects, reducing salaries or increasing workloads, with an adverse effect on active members. It is also possible that after October 2011 employers used their reduction in pension contributions, relative to the benchmark, to raise salaries to compensate for the drop in expected pension wealth of members. Many empirical studies have tried to quantify the compensating wage differential, i.e. the size of the trade-off between pension benefits and wages, and recent examples of this literature include Disney, Emmerson and Tetlow (2009), Gerakos (2010) and Haynes and Sessions (2013). Attempts to quantify the wage-pension trade-off have encountered substantial econometric and data problems (Allen and Clark, 1987), but subject to these reservations, the empirical evidence suggests the trade-off is well below one-for-one. Consistent with this evidence UK academic salaries have showed no obvious response to the USS rule change of October 20116. Such consequential effects on the membership such as higher or lower salaries, worse conditions of service etc. are outside the scope of this research.

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6 Between October 2011 and October 2015 academic salary scales rose by only about 1% per year.
The members, future members, pensioners and deferred pensioners of the two sections of USS (final salary and CRB) are disaggregated into age-based cohorts, where the age range covered by each cohort is five years. We use five years because this is the period used by USS when they supplied us with some cohort data, and provides computational tractability. For the post-October 2011 final salary section there are eight cohorts each of active members and deferred pensioners aged between 25 and 65 years, and six cohorts of pensioners aged between 65 and 95. In addition, the post-October 2011 CRB section has three cohorts of actives and deferreds aged between 25 and 35 years. This is because new active members (future cohorts) enter directly into the four youngest cohorts each year. The post-October 2011 CRB section also has 11 cohorts of both future actives and deferreds aged from minus 30 to 25 years of age in 2011. This makes a total of 50 cohorts for the post-October 2011 scheme. The continuation of an unchanged pre-October 2011 final salary scheme has a total of 44 cohorts, all of which appear as part of the 50 post-October 2011 scheme cohorts. The other participants in the scheme are the sponsors of USS, i.e. the 374 UK universities and related institutions, who are treated as a single group.

Forecasting asset returns, yield curves, inflation, longevity and salaries for the horizon period (54 years) is a daunting task; as is projecting the membership in each USS age cohort during this period. Therefore the resulting cash flow forecasts are inevitably subject to a considerable degree of estimation risk. Because of the heroic forecasts required, a range of financial and demographic forecasts are employed to generate a distribution of outcomes, and the sensitivity of the conclusions to some of the important assumptions is investigated as a robustness check.

3. Literature Review
In 2001 Chapman, Gordon and Speed suggested taking a much wider view of the effects of changes in pension scheme rules than had previously been the case. They identified six stakeholders who are affected by a change in pension scheme rules - the sponsor’s shareholders, the sponsor’s debt holders, the employees, externals (the sponsor’s suppliers and customers), consultants and advisors, and the government. For a hypothetical UK scheme, they simulated the cash flows between these six stakeholders over a ten year period, and then used SDFs to compute the NPV of the cash flows for each stakeholder. This was done for both a base case and various alternative pension rules, and the average NPV for each set of rules for each group of
stakeholders computed. The changes in these averages gave the redistributive effects of the rule change on the wealth of each stakeholder group.

Ponds (2003) proposed using the approach of Chapman, Gordon and Speed (2001) to quantify the intergenerational redistributive effects of different pension scheme rules. He considered a hypothetical Dutch scheme where the sponsor bears no risk, and analysed redistribution between active members, pensioners and future members; and between age cohorts of these three groups. Using the same methodology, Hoevenaars and Ponds (2007, 2008), Lekniute (2011) and Draper, Van Ewijk, Lever and Mehlkopf (2014) have also illustrated intergenerational redistribution among age cohorts arising from changes in scheme rules for hypothetical Dutch pension schemes; while Hoevenaars, Kocken and Ponds (2009) and Hoevenaars (2011) have investigated redistribution between the sponsor and members (but not between age cohorts of members) for hypothetical Dutch DB schemes. Finally, for a hypothetical US state pension scheme, Lekniute, Beetsma and Ponds (2014) and Beetsma, Lekniute and Ponds (2014) simulated intergenerational redistribution between cohorts of active members and the sponsor (the state’s tax payers) due to rule changes.

4. Forecasting Asset Returns, Inflation and Salaries

In this section we use the Nelson and Siegel (1987) model to estimate the parameters of the yield curve for the data period (1993-2010). We then estimate a VAR(1) model to enable us to forecast asset returns, inflation, and the yield curve for the horizon period. Finally we generate forecasts of the salaries of the various age cohorts of USS active members until the horizon date. In making these forecasts we only use data that would have been available to USS at the time of the rule change.

A. Yield Curves. Diebold and Li (2006) have developed a variation of the Nelson-Siegel model for forecasting yield curves which allows the entire yield curve to be represented by only three parameters.

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7 The data we use to estimate the VAR(1) model starts in 1993. This is because earlier data is not available for some of the maturities involved in the estimation of the three Nelson-Siegel yield curve factors: β₁, β₂ and β₃. The length of the estimation period we use for the VAR(1) model is in line with that used by Ferstl and Weissensteiner (2011) and Gulpinar and Pachamanova (2013).
where \( y_n(t) \) denotes the spot rate (zero coupon) at time \( t \) for a maturity of \( n \) periods, and \( \beta_1, \beta_2 \) and \( \beta_3 \) are the level, slope and curvature respectively of the yield curve. Following Diebold and Li, we set the annual decay rate \( \lambda \) in equation (1) to 0.1827. We use end-of-quarter yields from 1993 to 2010 for zero coupon UK government bonds with maturities of 3 months, 1 year, 2 years, 3 years, 5 years, 10 years and 20 years. Following Diebold and Li (2006), who estimate the Nelson-Siegel yield curve using data for a 15 year period, we apply linear interpolation to compute the nearby maturities and estimate a time series for each of the three parameters in equation (1). These three time series of the parameters of the yield curve are then included in a VAR(1) model to generate forecasts of the yield curve in future years, (Ferstl and Weissensteiner, 2011).

B. VAR(1) Model. We use a VAR(1) model to generate the future scenarios, as have Hoevenaars and Ponds (2008), Hoevenaars, Kocken and Ponds (2009), Hoevenaars, Molenaar and Ponds (2010), Hoevenaars (2011), Lekniute (2011), Lekniute, Beetsma, and Ponds (2014). The financial data included in our VAR(1) model consists of quarterly excess returns from 1993 to 2010 for UK equities (FTSE All Share Total Return index), European equities (MSCI Europe excluding the UK Total Return index), US equities (S&P500 Composite Total Return index), hedge funds (HFRI Hedge Fund index), commodities (S&P GSCI Total Return index), UK property (UK IPD Index Total Return index), together with quarterly values for UK dividends (FTSE All Share Dividend Yield), US dividends (S&P500 Composite DS Dividend Yield), and UK inflation rates (UK RPI and UK CPI). In addition, we include the three estimated parameters of the Nelson-Siegel yield curve factors, \( \beta_1, \beta_2 \) and \( \beta_3 \), in the VAR(1) model in equation (2).

\[
\begin{align*}
x_{t+1} & = c + Bx_t + \zeta_{t+1} \\
\text{where } \zeta_{t+1} & \sim N(0, \Sigma)
\end{align*}
\]

where \( x_t \) is a column vector of economic factors at time \( t \), \( c \) is a column vector of constants, \( B \) is a square matrix of coefficients, \( \zeta_{t+1} \) is a column vector of disturbances at time \( t+1 \), and \( \Sigma \) is the variance-covariance matrix of the column vector of disturbances. The estimated VAR(1) model with 13 variables appears in Table 1, with the estimated covariance matrix of the disturbances in
Appendix D\textsuperscript{8}. The largest eigenvalue of the estimated coefficient matrix ($B$) is 0.9428, and since this is less than one the system is stable and shocks to the system dampen over time. We also tried including jumps in the VAR(1) model, but the results were inferior.

Following Hoevenaars and Ponds (2008), Hoevenaars, Kocken and Ponds (2009), Hoevenaars, Molenaar and Ponds (2010) and Hoevenaars (2011), we forward iterate this model for the out-of-sample period to produce 5,000 sets of forecasts (i.e. scenarios) of asset returns, inflation rates, and the yield curve until the horizon date\textsuperscript{9}. To produce a scenario we generate an $x$ vector at time $t+1$ ($x_{t+1}^*$) for each out-of-sample period, where the superscript * indicates a value estimated using equation (3).

$$x_{t+1}^* = c + Bx_t^* + \zeta_t^*$$

The value of the $\zeta_{t+1}^*$ vector each period is generated by Monte Carlo simulation using the estimated multivariate normal distribution of the disturbances in equation (2). We computed the maximum and minimum yields for each maturity (3 months to 20 years) for the Nelson-Siegel yield curves estimated in section 4A. The maximum yield for all maturities is almost flat at 8.5%, while the minimum yield rises in a more or less linear manner from 0.4% for 3 months to 3.8% for 20 years. When generating future scenarios we impose these upper and lower bounds on the forecast yield curves.

\textit{C. Salaries.} Salaries rise for two reasons - general increases in the salary scale ($S_s$), and incremental pay rises ($S_i$) as active members age or are promoted. The rise in salaries for an age cohort of active members is the product of these two sources of wage rises, i.e. $(1+S_i)(1+S_s)$. The USS scheme actuary estimates general salary increases as the forecast rate of RPI inflation plus one percent (USS, 2014). Table E in Higher Education Statistics Agency (2014) provides academic salary levels for each age cohort in 2012-13. We use this data to compute the relationship between age and salary for UK academics, with the slope of this curve giving the rate

\textsuperscript{8} VAR(1) is preferable to VAR(2) according to the Schwarz criterion for model selection. $F$-tests for the regressions of the 13 variables show that one lag is preferable to two lags for ten of the variables. For these reasons we prefer the VAR(1) model to the VAR(2) model. All of the 13 variables are stationary. None of the sets of regression residuals for the 13 explanatory variables displayed serial correlation. Ten of the sets of residuals are normally distributed, with plots of the remaining three sets looking approximately normal.

\textsuperscript{9} In a similar context Chen, Pelsser and Ponds (2014) also used 5,000 scenarios.
of incremental pay rises for different age cohorts. Together with the RPI inflation forecasts from the VAR(1) model, this allows \( S_s \) and \( S_i \) to be forecast until the horizon. USS (2014) gives the total pensionable salaries of active members in 2011 as £5.845 billion, and the number of active members in 2011 as 139,931. This implies an average salary of £41,771 for USS active members in 2011, and we use this to calibrate the cohort salary data from HESA (2014) to match the USS active membership average.

5. Forecasting the Size of the Age Cohorts

The forecasts of the numbers of actives, deferreds and pensioners in each age cohort for the pre and post-October 2011 schemes use membership data from the USS annual reports, as well as data for 2014 on the size of each age cohort in the final salary and CRB sections supplied directly to us by USS. Changes in the total number of active members of USS for the years 1997 to 2014 are regressed on a time dummy. By modelling the total number of active members we allow for early leavers and late joiners. When computing the pension wealth effects of the rule changes on each age cohort of active members, we assume they expect to stay in USS until retirement. The estimated slope of this regression is zero, with a highly significant constant term of 5,050, indicating that USS total active membership is increasing by 5,050 per year. We use the residuals from this regression to estimate the standard deviation of annual changes in the total active membership of the pre-October 2011 scheme at 2,366. When forecasting the annual increase in the total active membership we choose a value at random from a normal distribution and add it to the forecast increase to allow for year to year fluctuations. We assume that each year equal proportions of the new active members enter the four youngest age cohorts. This assumption is chosen to ensure that the average distribution of active members by age cohort across all the years until the horizon date approximates the age distribution of active members in 2014 supplied to us by USS.

We follow a similar procedure to forecast the annual changes in the number of deferred pensioners for the pre and post-October 2011 schemes, except that new deferreds enter with a five year lag\(^\text{10}\). In a regression of changes in the total number of deferred pensioners for 1997 to 2014 on a time dummy the estimated slope coefficient is zero, and the highly significant constant

\(^{10}\) So we implicitly assume that, on average, deferreds have previously been actives for five years.
term is 4,292, with a standard deviation of 1,359. The resulting average age distribution of deferred members across all the years until the horizon date approximates the age distribution of deferred members in 2014 supplied to us by USS.

6. Forecasting the Liabilities

We use an actuarial model based on Board and Sutcliffe (2007) and Platanakis and Sutcliffe (forthcoming) to forecast the values of the scheme’s liabilities for each age cohort until the horizon date, and this requires the specification of a number of parameters, see Appendix B. We are modelling the performance and decisions of USS over the horizon period, where the value of the liabilities is a key input to computing the funding ratio and revising the contribution rates and asset allocation. Therefore, to model the decisions of USS, we need to use the same inputs as USS when valuing the liabilities.

We base the life expectancy for each cohort of pensioners on the National Life Tables for the UK, 2011-13 (ONS, 2014), with the number of members of each pensioner cohort reducing each year in accordance with this life table. We generated a blended mortality table using the weights of 55.5% male and 44.5% female, taken from the HESA (2014) data on the gender of academic staff. In 2011 USS members had a blended life expectancy of 25 years at age 65, which incorporates the actuary’s estimate of future improvements in USS longevity, USS (2012). As USS members have a greater life expectancy than the general population, the national life tables are uprated by six years, so that at age 65 USS pensioners are expected to live until they are 90 years of age. Following Carnes and Olshansky (2007), we do not allow for any further increases in longevity over the horizon period. For simplicity, those pensioners who reach the age of 90 years are assumed to die at the age of 95, which matches their expected longevity of five years at the age of 90.

Based on USS (2014) we assume that two thirds of pensioners have an eligible beneficiary (usually a spouse) at the time of their death, and that surviving beneficiaries live for another three years. During this time eligible beneficiaries receive a pension equal to half that of the deceased pensioner. So the extra cost of a spouse’s pension is approximately equal to \(0.667 \times 0.5 \times 3 = 1\) year of the deceased member’s pension. To account for this additional liability, we increase longevity
by one year.

The computation of the liabilities also requires estimates of the number of accrued years for each age cohort. The age of each cohort is taken as the mid-point of the cohort’s age range, and their accrued years are computed from their current age, assuming they joined USS at an average age of 32.5 years. We then adjust these numbers in 2011 for each cohort to match the average accrued years across all the cohorts of active members of 10.4 years given in USS (2014). On a technical provisions basis, the USS liability for deferred pensions in the 2011 actuarial valuation was £2.792 billion (USS, 2012). We used equation B.1 in Appendix B to compute the implied number of accrued years for deferred pensioners at four years, and adjusted the accrued years for deferred members to match the estimated average number of accrued years.

Economic theory indicates that the cash flows in each future year should be discounted using the rate of return on a portfolio that replicates the risk and return of this cash flow, leading to the use of a different discount rate for each year. However, to model USS decisions on contribution rates and asset allocation we need to use the same discount rate as USS, which is the average of the current yield curve for the next 20 years plus 1.7%, and so we do likewise, (USS, 2014). However, when discounting the cash flows to compute the wealth changes for each cohort and the sponsor in section 8, we will use SDFs. Finally the total liabilities computed using our model for 2011 were calibrated to equal their value on a USS technical provisions basis of £35.3437 billion in 2011, with the liabilities for each of the constituent age cohorts correspondingly adjusted, (USS, 2012).

The number of people in each cohort was estimated in section 5, and the current salary for the members of each cohort is their initial salary increased by the forecasts of salary growth from section 4. Our estimates of salary increases follow the USS methodology. Section 4 supplies the forecasts of CPI and RPI, and the estimates of the number of members and their salaries in each cohort are then calibrated to match the 2011 aggregate numbers published by USS (2014).

In contrast to the Dutch research, because the sponsor remains liable for the pension promise, the liabilities for the various cohorts of USS take no account of the overall scheme surplus or deficit.
However, in section 8 when computing the future cash flows for pensioners and sponsor, a share of the scheme surplus or deficit at the horizon date is allocated to the sponsor.

7. Generating the Cohort Cash Flows

The cash flows for each cohort per time period are the pensions and lump sums pensioners receive, less the contributions active members make to the scheme, while the sponsor just pays contributions per time period to the scheme. Contributions to the scheme for each cohort are the number of people in the cohort, times the average cohort salary, times the sum of the current contribution rates for active members and the sponsor. The size and average salary of each cohort were computed in section 4. The contribution rate for members of the final salary section increased from 6.35% to 7.5% in October 2011, while the sponsor’s contribution rate remained at 16%. The members’ contribution rate for the CRB section is 6.5%, and that for the CRB sponsor is 16%. The total pension payment to each cohort is the number of people in the cohort times their initial pension, adjusted for subsequent limited price indexation, computed using the rules in Appendix A. The lump sum calculation for each cohort also follows Appendix A.

The cash flow calculations use the contribution rates and asset allocation for that period, both of which can change over time in response to the scheme’s funding ratio. The liabilities were estimated in section 6. The total value of the scheme’s assets at the end of each time period is the value of the investments at the start of the period, plus asset returns, the contributions received from the active members and sponsor during the period, less the lump sums and pensions paid out. Asset returns are computed using the forecasts of asset returns in section 4. The USS contribution rates and asset allocation are adjusted each period in response to the current value of the funding ratio, whose initial value in 2011 was 92% on a technical provisions basis (USS, 2012).

7A. Adjusting the Contribution Rates. Given the volatility of the funding ratio and the costs of change, we only adjust the contribution rates for the final salary and CRB sections when the funding ratio is below 90% or above 120%. They are adjusted so as to extinguish any surplus or deficit over a 15 year spread period, leading to a funding ratio of unity. For the post-October 2011 scheme, the difference between the final salary and CRB contribution rates for active
members remains fixed at 1%. At the request of USS, Ernst and Young assessed the sponsor’s covenant and concluded that the maximum contribution rate the majority of universities can pay is 25% (USS, 2014). Given the ‘cap and share’ rule, this implies a member contribution rate of 10.27% for members of the final salary section, making a total contribution rate of 35.27%. To prevent the contribution rate reaching unrealistically high levels, we impose an upper bound of 35% on the total final salary contribution rate (34% for the CRB section) for both the pre and post-October 2011 schemes. We also investigate a maximum contribution rate of 29%, or 20.275% for the sponsor and 8.725% for members of the final salary section, (28% for the CRB section) as a robustness check in section 10.

7B. Adjusting the Asset Allocation. As well as changing the contribution rates in response to the funding ratio, the asset allocation may also be altered. There are two rival theories of how a scheme’s funding ratio and the probability of default affect its asset allocation. The risk management view is that as the probability of default rises, e.g. the funding ratio falls, schemes shift out of high risk assets into low risk assets; while the risk shifting view is the opposite\(^{11}\). The empirical evidence on these rival views is mixed, and so alternative sets of results are generated for each of these views. Based on their past volatility, we divide the assets into three groups: high risk (UK, European and US equities and commodities), medium risk (real estate and hedge funds) and low risk (10 year bonds and cash). Adopting a very simple rule, the difference between the current funding ratio and the initial funding ratio in 2011 is used to adjust the money allocated to high and low risk assets, subject to the constraints of not allocating more than the available funds, or negative funds, to the high and low risk assets.

For example, suppose the initial asset allocation in 2011 is 65% to high risk assets, 15% to medium risk assets and 20% to low risk assets; the initial funding ratio in 2011 is 0.80 and that by 2014 this has risen to 0.95. Using the risk management approach the asset allocation to high risk assets rises by \(0.95 - 0.80\) = 0.15 to 80%, the allocation to low risk assets drops by a

corresponding amount to 5%, and the allocation to medium risk assets is unchanged at 15%. The allocations to the individual asset classes within each risk category are rebalanced in proportion to the change in the total funds available for that risk category. A similar rule applies for risk shifting, except the directions of change are reversed. Therefore we investigate the effects of three alternative asset allocation strategies - the actual USS allocation in 2011 in conjunction with a fix-mix strategy of rebalancing the asset weights back to the initial allocation every three years, risk management and risk shifting. The initial asset allocation in 2011 is UK equities 23.06%, EU equities 18.32%, US equities 18.32%, cash 5%, 10-year UK government bonds 12.3%, UK property 7%, hedge funds 8%, and commodities 8%.

8. Redistribution

The objective is to estimate the magnitude of the pension wealth transfer from each cohort of scheme members to the sponsor. The NPV of each series of cash flows is computed using risk-neutral valuation, which has the advantage that it does not require any knowledge of the preferences of active members, deferred pensioners, pensioners or the sponsor. For this reason SDFs have previously been used to compute the NPVs of pension scheme cash flows by Chapman, Gordon and Speed (2001), Hoevenaars and Ponds (2008), Hoevenaars, Kocken and Ponds (2009), Hoevenaars, Molenaar and Ponds (2010), Hoevenaars (2011), and Draper, Van Ewijk, Lever and Mehlkopf (2014). Lekniute (2011) and Lekniute, Beetsma and Ponds (2014) used risk neutral probabilities to value pension cash flows, which is logically equivalent to using SDFs. We follow these previous authors in using risk neutral valuation, and use SDFs (or pricing kernels) to compute the NPVs of the cash flows for the member and pensioner cohorts and the sponsor.

In using SDFs to value the pension liabilities we are treating all the liabilities as potentially risky. Although the Pension Protection Fund (PPF) insures roughly 90% of UK DB pension liabilities in the event of default, this does not make USS liabilities riskless. Default by USS requires all UK universities to be in default, and in such circumstances it is likely that the PPF, which has no explicit government guarantee, will also be in default (Blake, Cotter and Dowd, 2007). While pensions in payment have priority in the event of default, we have used the same risky discount rate for all liabilities. Splitting the liabilities into two tranches does not alter the total default risk,
although it would lead to slightly higher discount rates for actives and deferreds, and slightly lower discount rates for pensioners. However the effects of such an adjustment on the conclusions would be minimal.

The use of a unique set of positive SDFs to discount stochastic cash flows relies on the assumptions of complete and arbitrage-free markets in which the law of one price applies. Where markets exist, competition tends to ensure an absence of arbitrage opportunities and the validity of the law of one price. But if markets are incomplete, many alternative sets of positive SDFs exist. The valuation of DB pension liabilities faces the problem of a missing market for trading or hedging future salaries, and an imperfect market for hedging longevity. This leads to the contradiction that the use of risk neutral valuation to value pension scheme liabilities relies on complete markets, but if markets were complete pension schemes would lose their reasons for existence, e.g. risk sharing, economies of scale, low transactions costs, etc. (McCarthy, 2005).

Our model of USS assumes no cohort longevity risk, leaving only diversifiable longevity risk. Since USS has a very large number of members, diversifiable longevity risk is roughly zero, Aro (2014), Donnelly (2014). Future salary increases for USS members are split into two components: a general uplift in the salary scale which is assumed by USS to be RPI inflation plus 1%, and a promotional salary increase which is specific to each age cohort. The general RPI-linked uplift in salaries can be hedged using RPI-linked bonds or RPI-linked swaps, leaving just the promotional increases. Our model assumes that, apart from the annual RPI + 1% uplift to all scale points, the salary scale remains constant over time, making the average promotional increase for each age cohort highly predictable. So, while no instrument exists for trading or hedging promotional increases, they are probably low risk, and can more or less be replicated using the riskless asset, making the market approximately complete.

Hoevenaars and Ponds (2008), Hoevenaars, Molenaar and Ponds (2010), Hoevenaars (2011) and Draper, Van Ewijk, Lever and Mehlkopf (2014) simply assumed zero real wage growth to circumvent the incomplete markets problem. This is the same as our model, if promotional salary increases are excluded. De Jong (2008) discusses four methods to value salary-indexed stochastic future cash flows in the presence of incomplete markets, and advocates utility-based valuation
assuming that individuals have a specified utility function. We prefer not to assume utility
functions for actives, deferreds, pensioners and the sponsor, but to rely on the observed market
prices used in the SDF computation, recognising that the assumptions required for the use of
SDFs are not fully met. Pukthuanthong and Roll (2015) have recently found that “the SDF
theory’s main prediction, that the same SDF prices all assets during the same time period, cannot
be rejected with our tests, data, or time periods. ... These results are consistent with complete
markets and an absence of arbitrage.” This empirical finding supports our view that, while the
assumptions may not be fully met, SDFs are still a useful way of valuing a sequence of risky cash
flows. As a robustness check, in section 10 we also compute the NPVs using the forecast riskless
rates, relying on the assumption that USS is backed by the UK university system, and so has
minimal default risk. Using the riskless interest rate as the discount rate for valuing the liabilities
of DB schemes is advocated by Broeders, Chen and Rijsbergen (2013), and has the advantage of
allowing us to estimate the risks attached to expected changes in pension wealth.

If markets are complete, the law of one price applies, and current asset prices are arbitrage free;
then a unique set of positive state prices exists such that each asset’s current price is the sum of
the cash flows from the asset in each future state multiplied by the corresponding state price. This
is the fundamental theorem of asset pricing (see, for instance, Cochrane, 2001), and no
knowledge of individual preferences is required to compute the state prices. Following Ang,
Bekaert and Wei (2008) and Cochrane and Piazzesi (2005); as well as Nijman and Koijen (2006),
Hoevenaars and Ponds (2008), Hoevenaars, Kocken and Ponds (2009), Hoevenaars, Molenaar
and Ponds (2010) and Hoevenaars (2011) from the pensions literature, we define SDFs \((m_{t+1})\) as:

\[-\log(m_{t+1}) = y_{t,3\text{ month}} + \frac{1}{2} \phi_t^T \Sigma \phi_t + \phi_t^T \zeta_{t+1} \]

(4)

where \(\zeta_{t+1} \sim N(0, \Sigma)\) denotes a column vector of disturbances from the VAR(1) model, \(y_{t,3\text{ month}}\) is
the 3 month UK interest rate at time \(t\) estimated using the Nelson-Siegel yield curve (see section
4), and \(\phi_t\) is a column vector of the time-varying prices of risk which is defined as in Cochrane
and Piazzesi (2005), see appendix C. SDFs are scenario dependent, and for a given scenario the
SDF for a cash flow in year \(k\) (denoted \(m_{t+k}\)) is the product of the SDFs for each of the first \(k\)
years, i.e. \(m_{t+k} = m_{t+1} \times m_{t+2} \times \ldots \times m_{t+k}\).
For every scenario, i.e. each sequence of future returns, salaries, inflation rates, cohort size, and contribution rates until the horizon date, we generate annual cash flows. We then discount these back to the present using the set of SDFs specific to that cash flow sequence to get an NPV for each cohort. Finally, for each cohort we compute the average NPV across all cash flow sequences to place a value on this risky asset.

At the horizon date some age cohorts will still have future cash payments to make or receive, i.e. they are pensioners, actives or deferreds. These terminal obligations, which have not yet become cash flows, must be valued. Rather than forecast the cash flows until all the new joiners in the horizon year have died, i.e. until (2065+70) = 2135, we forecast the cash flows for another 25 years until 2090. This allows us to compute the subsequent cash flows for all those who are pensioners in the horizon year, but not for those who are actives or deferreds. To avoid the problem of back-loading we only compute NPVs for cohorts whose members are pensioners or deceased at the horizon date, and not for cohorts with active or deferred members.

Given the ‘cap and share’ rule, the terminal surplus or deficit at the horizon date for the post-October 2011 scheme is shared between the active members and the sponsor. The scheme liabilities in 2065 are estimated as the present value in 2065 of the cash flows between 2065 and 2090 for actives, deferreds and pensioners in 2065, plus the USS valuation using Appendix B of the scheme’s liabilities to these actives and deferreds in 2090. Using these liabilities, together with the total value of USS assets in 2065, the sponsor is allocated 65% of this horizon year surplus or deficit. There is no need to allocate the remaining 35% between the active members at the horizon date as the NPVs of their cash flows are not being computed.

For a given set of rules, assumptions and forecasts, the average NPV represents the expected increase or decrease in pension wealth in October 2011 for each age cohort from the continued operation of USS according to a specified set of rules. We compute the wealth effects of the October 2011 rule change by examining differences in the NPVs for the pre and post-October 2011 schemes. This assumes that each of the alternative scheme designs remains unchanged for

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12 The use of the USS valuation tends to understate the liabilities as it uses a high discount rate. It also ignores subsequent investment, salary, inflation, and contribution rate risk. However, since these cash flows occur at least 79 years in the future, their discounted value will be relatively small.
the horizon period. In the present case, if the pre-October 2011 scheme is not reformed there is an increased risk of financial distress for the sponsor, which may then impact on active and future members in the form of lower salaries, fewer jobs and subsequent scheme redesign. In common with previous studies of the redistributive effects of pension rule changes, we have not valued this risk, and assumed that members expect the chosen scheme to be unchanged.

9. Results

The time series of the forecast mean funding ratios, contribution rates and asset allocations are plotted in figures 1 to 6. They are interdependent, as the asset allocation and contribution rates are adjusted in response to the current funding ratio. The contribution rates then affect the funding ratio in subsequent periods. Cohorts with a mean age of 42 years and older in 2011 contain only members of the final salary section, while cohorts aged 22 years and younger in 2011 are all members of the CRB scheme. The cohorts initially aged 27, 32 and 37 years contain members of both the final salary and the CRB sections.

For the post-October 2011 scheme figure 1 shows that, while the mean funding ratio at first declines to below 80%, it steadily recovers. The improvement in the funding ratio from the mid-2020s onwards is due to the steady shift in the active membership from the final salary section to the cheaper CRB section. By 2053 all the active members are in the CRB section and the funding ratio has stabilized. At this time the long run funding ratio for fix-mix and risk shifting is over 115%, and for risk management it is over 100%. Figure 2 indicates that in the long run the pre-October 2011 scheme has an inadequate funding ratio for risk shifting and fix-mix of about 80%, and for risk management it is a disastrous 65%.

The results in figures 3 and 4 for the mean final salary contribution rates lead to broadly similar conclusions to those from figures 1 and 2. For the post-October 2011 scheme, after a rise to 29%, the total contribution rates for fix-mix and risk shifting steadily decline to a long run rate of below 23%, which is less than the 23.5% rate in 2011. The risk management contribution rate

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13 We modelled the problem using Visual Basic for Applications (VBA). On average it takes 50 seconds to run each scenario on a desktop computer with a 3.2 GHz processor, 12 GB of RAM and running in Windows 7 Professional. Therefore it took 17 days of CPU time to run 5,000 scenarios with three different asset-allocation strategies and two robustness checks.
does not drop below 29% until roughly 2030, and then declines to just below 25%. For the pre-October 2011 scheme, risk shifting and fix-mix generate a long run contribution rate of around 27%, and for risk management it is about 30%. These results suggest that the pre-October 2011 scheme was not viable in the long run, irrespective of the asset allocation strategy adopted. Table 2 has the means and standard deviations of the funding ratios and contribution rates from 2011 to 2065.

<table>
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<th></th>
<th>Fix-Mix</th>
<th>Risk Mgt.</th>
<th>Risk Shifting</th>
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<td>Pre</td>
<td>Post</td>
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Table 2: Summary Statistics for the Three Asset Allocation Strategies

Figures 5 and 6 show the very different mean asset allocations for the risk shifting and risk management asset allocation strategies. For risk shifting in figure 5, both of the pre and post-October 2011 schemes retain their high allocation to risky assets and low allocation to low risk assets, with the difference being more extreme for the pre-October 2011 scheme.

For risk management in figure 6 the pre-October 2011 scheme moves to a long run allocation of over 50% in low risk assets, and about 35% in risky assets; while the post-October 2011 scheme allocation moves to over 50% in risky assets, and about 35% in low risk assets.

Figures 1 to 6 present information on the generation of the cash flows until the horizon date, and we can now compute the NPVs of these cash flows for the age cohorts and sponsor. Figure 7 shows the mean percentage drop in the NPV for each of the age cohorts due to the rule change.

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Table 2 shows that for risk shifting the mean post-October 2011 allocation to risky assets (62.19%) is lower than for the fix-mix strategy. This is because, when the funding ratio is below its initial value of 92%, the risk shifting model seeks to increase the allocation to risky assets. However the risky asset proportion cannot rise above 85%, i.e. not more than 17.3% above its opening level of 92%. So if the funding ratio drops below 74.7% there can be no further increase in the allocation to risky assets, and this lowers the average risky asset proportion so that it is lower than for the fix-mix strategy.
There are separate lines for the CRB and final salary sections, which overlap for the 27 to 37 years cohorts. This shows that young cohorts lose a much higher percentage of their pension wealth than do older cohorts, and that CRB members lose much more than do final salary members when they are the same age. Future members (cohorts 17 and 22) lose about 65% of their pension wealth in 2011, which is the present value of their future cash flows with USS (i.e. member contributions, pension and lump sum), while the youngest final salary members lose about a quarter of their NPV, and pensioners lose virtually nothing. This difference is because in 2011 older members had accrued substantial benefits based on contributions made in previous years. These benefits are legally protected from the rule change, and so their NPVs do not drop, while future members are subject to a much larger NPV drop. In addition, because the NPV computation excludes contributions made before 2011 while the accrued benefits these contributions created are carried forwards, the NPVs for older members are larger than those for younger members. So a given absolute NPV reduction represents a smaller proportionate drop for older members.

We converted the proportionate losses per age cohort in figure 7 into approximate losses per head in monetary terms for both actives \( AL = LY \times AY \) and deferreds \( DL = LY \times DY \), using equation (5) to estimate the mean loss of NPV per accrued year \( LY \).

\[
LY = TL/(TN[AP(AY)+DP(DY)])
\]

where \( TL \) is the mean total loss of NPV for the age cohort, \( TN \) is the total number of members of the age cohort, \( AP \) in the proportion of the age cohort who are actives, \( DP \) is the proportion of the age cohort who are deferreds, \( AY \) is the mean number of accrued years at retirement for each active member, and \( DY \) is the mean number of accrued years at retirement for each deferred pensioner. Figure 8 shows the results for the actives and these have a broadly similar shape to figure 7, although figure 7 has percentages, and figure 8 has losses in £ per head. Figure 8 shows that future active members of the CRB scheme lose about £100,000 per head, while the youngest members of the final salary scheme lose £40,000 per head. The corresponding diagram for the deferreds has an identical shape to figure 8, but with losses that are \( (27.5/32.5) = 84.6\% \) lower.

Figure 9 shows the present values of the total loss in £bn. to each cohort of the two sections. The total loss in October 2011 to all the cohorts of the final salary section is £3.87 billion, or 12% of
the total loss to all cohorts of both sections; and so 88% of the total loss fell on the CRB section. The total loss for the age cohorts we have analysed (17 to 92) is £18.0 billion, and the total loss for age cohorts 28 to 12 until 2065 is approximately £14.5 billion.

Figure 10 plots the mean percentage drop in the pension received at age 65 for each age cohort. Future scheme members experience a drop of about 38% in the pension they will receive at age 65. Those in the age 42 cohort and older are all in the final salary scheme and will make higher contributions post-October 2011, make contributions for an extra one or two years, and draw their pension for one or two years less due to an increase in their retirement age. They will also benefit from an extra one or two years of additional accrual and salary increases, which will increase their pension at retirement. These factors lead to the total loss of £3.87 bn. for active members of the final salary scheme shown Figure 9, but none of these factors alter their pension at age 65. As figure 10 shows, the pension at age 65 for members of the final salary scheme is unaffected by the rule changes.

In addition to estimating the monetary loss to the members of each age cohort, we compute the corresponding total monetary gain to the sponsor resulting from the rule change. Since our horizon date is 2065, we quantify the gain for the 2011 to 2065 period. This is the present value of the reduction in the sponsor’s contributions to USS until 2065 plus the present value of the sponsor’s 65% share of the surplus or deficit in that year. The resulting change in NPV for the sponsor is a gain of £32.5 billion, equivalent to 26% of their pension cost for the pre-October 2011 scheme.

10. Robustness Checks
We have previously studied three alternative asset allocation strategies, and we now investigate two additional changes to the base model. So far the sponsor contribution rate for the final salary section has been capped at 25%, but some universities would be unhappy with contributing this much, and so we investigated capping the sponsor’s contribution rate at just over 20%, to give a cap on the total final salary contribution rate of 29% (or 28% for the CRB section). For the post-October 2011 scheme and the fix-mix strategy, the funding ratio drops to about 75% in the medium term, and the final salary contribution rate rises to roughly 26%. In the long run the
funding ratio rises to almost 110%, and the final salary contribution rate falls to just above 22%. These results suggest that, even if the total contribution rate is capped at 29%, the post-October 2011 scheme is viable, albeit with an uncomfortable period of low funding before the long run equilibrium is reached.

Another robustness check involves replacing the SDFs with the riskless discount rates from the VAR(1) model when computing the NPVs. We use riskless rates because USS is backed by the UK university system on a last-man-standing basis, making default very unlikely. Figure 11 shows the expected percentage drop in the NPVs; with broad agreement between the SDF and riskless rate estimates of the percentage drop in the NPVs of pension wealth. For example, as for SDFs, the drop is 65% for future members. Figure 11 also shows the 10% and 90% percentiles of the estimated percentage drop in the NPVs, and these range from about 55% to almost 90% for future cohorts, with progressively less variation for older cohorts. Figure 12 shows the mean loss in £ per head using the riskless rate. For future members this is about £90,000, compared with £100,000 for SDFs, and ranges from £40,000 to £155,000. As before, the loss per head declines rapidly with age, and is markedly lower for members of the final salary section.

Using the riskless rates, the sponsor’s total gain for the 2011-2065 period is £30.0 billion, compared to £32.5 billion using SDFs, with a 10% percentile of £10.5 billion, and a 90% percentile of £55.7 billion. As for SDFs, the sponsor’s total gain using riskless rates is 26% of their costs for the pre-October 2011 scheme. The total loss for the cohorts aged 17 to 92 is £16.1 billion for riskless rates (compared with £18.0 billion when using SDFs), with 10% and 90% percentiles of £6.6 billion and £27.5 billion respectively; while the total loss for the -28 to 12 cohorts is £13.8 billion (compared with £14.5 billion when using SDFs). Therefore the use of riskless rates to discount the cash flows rather than SDFs, supports the general conclusions reached using SDFs.

Finally Figure 13 compares the riskiness (coefficient of variation) of the NPVs computed using riskless rates before and after the rule change. For the younger members of the final salary scheme in October 2011 there is a modest increase in the coefficient of variation of their NPVs with the move to the new scheme, but no change for the older members and pensioners. For
future members in 2011 the riskiness of their pension wealth increases by about one third, relative to the risks they would have faced if they had joined the old final salary scheme. The October 2011 rule changes reduce the coefficient of variation for the sponsor by about 10%.

11. Conclusions
For members close to retirement the value of their pension wealth may be their largest single asset. So changes in a scheme’s rules can have an important effect on a member’s total wealth. When redesigning DB pension schemes, modelling the long run effects on the sponsor and members is generally neglected. What is needed is a dynamic long-term model that incorporates the interactions between the funding ratio, contribution rate, asset allocation and asset returns, as well as the differential effects on the various age cohorts. This research has built and estimated such a model for the USS rule change in October 2011. Although we modelled all the rules that changed, as well as other important rules, the complexity of the problem necessitates ignoring inconsequential rules. It also requires making heroic forecasts of asset returns, salaries, numbers of members and inflation far into the future. The actual situation of USS in 2065 will inevitably be substantially different from the mean forecast of our model, but since pension schemes have very distant horizons, such long term forecasts are necessary when analysing the effects of a rule change. Therefore the model can only give broad indications, rather than precise estimates. However, when comparing two alternative sets of rules using exactly the same model and forecasts, we have a level playing field.

This is the first such study for a real scheme (USS), and also the first where the sponsor bears all or part of the risk, e.g. ‘balance of cost’ or ‘cap and share’. It is also the first to incorporate a range of real world pension scheme features - lump sums, deferred pensioners, limited price indexation, spouses’ pensions, an increase in the retirement age, two sections (final salary and CARE), ‘cap and share’ contribution rates, and an uncertain number of new members each year. It also examines three different asset allocation strategies - fix-mix, risk shifting and risk management - over a 54 year out-of-sample period, which is long enough to avoid the back-loading problem. This has the advantage that none of the actives in 2011 are still active in 2065, allowing the scheme to reach a new equilibrium by the horizon date.
For both schemes the performance of the risk shifting and fix-mix asset allocation strategies is similar, mainly because fix-mix involves a substantial allocation to risky assets, and both strategies are clearly superior to risk management. The results indicate that in the long run the pre-October 2011 scheme was not viable. Using the two best asset allocation strategies (risk shifting and fix-mix) the long run funding ratio would be about 80%, and the contribution rate for the final salary scheme around 27%. For the risk management strategy the long run outcomes are markedly worse - a long run funding ratio of 65% and a contribution rate of 30%. The post-October 2011 scheme appears reasonably viable in the long run for the two best asset allocation strategies, with a funding ratio above 115% and a final salary contribution rate of about 23%, which is slightly below the 2011 rate of 23.5%. However, before this long run state is reached, the post-October 2011 scheme experiences funding ratios of 80% and contribution rates of about 29%, which would be problematic.

So the decision to redesign USS in October 2011 was justified, creating a post-October 2011 scheme that appears to be sustainable in the long run, although with medium term difficulties that are gradually solved as the active membership switches from the final salary section to the cheaper CRB section. These results indicate that a further redesign of USS is needed in the medium term to cope with progressively higher contribution rates and lower funding ratios. However, in the long run, when all the active members are in the cheaper CRB section, USS will become a well-funded scheme with a total contribution rate just above the pre-October 2011 value of 22.35%. Subsequent rule changes to deal with the medium term problems will only increase the long run strength of USS. The robustness checks broadly support these conclusions.

The rule change in October 2011 resulted in the transfer of about £32.5 billion of wealth from the members to the sponsor during the 2011 to 2065 period. This is equivalent to about £600 million per year, or over 60% of the sponsor’s contribution in 2011 of £938.4 million. The reduction in

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15 Error! Main Document Only. In 2014 USS decided to reduce the riskiness of its investments, so that the expected return drops by three basis points each year for the next 20 years, i.e. a total reduction of 60 basis points (USS, 2014). In April 2016 there was another major rule change - the final salary section was closed to future accruals and the CRB section offered to these members. Contribution rates to the CRB scheme increased to 8% for members and 18% for the sponsor, i.e. 26% in total. Pensionable salary for the CRB section was capped (initially £55,000), with earnings above this cap eligible for a new defined contribution section.
the present value in 2011 of the sponsor’s pension contributions over this period is 26% using either SDFs or riskless rates. The cost of this wealth transfer is very unevenly distributed across the various age cohorts and sections, with the burden rising from near zero for pensioners and those close to retirement in 2011, to about 65% of their pension wealth for future members. Since pensions are deferred pay, this represents a substantial pay cut. Before the October 2011 rule change the total annual contributions to the scheme were 22.35% of salaries, but the above analysis suggests that the long run annual cost of providing this scheme was closer to 27% of salaries, of which 6.35% was paid by the members from their salaries, leaving 20.65% to be paid by the sponsor of this ‘balance of cost’ scheme. We have estimated that future members have experienced a drop in their pension wealth of 65%, which is equivalent to a drop of approximately 0.65×0.2065/1.2065 ≈ 11% in their total compensation, or 0.65×0.2065 = 13% in their salaries.

References
Appendix A - USS Rules Pre and Post-October 2011

The final salary section was closed to new members in October 2011, but remains open for accruals by existing members, while the CRB section has been open to new members and accruals since October 2011.

Rules that Changed in October 2011

29
1 Contribution Rate - Final Salary. The contribution rate for active members of the final salary section increased from 6.35% to 7.5%.

2 Contribution Rate – ‘Cap and Share’. Before October 2011 USS was a ‘balance of cost’ scheme, where the sponsor is ultimately responsible for meeting the pensions promise. Post-October 2011 for both the final salary and CRB sections, any increase in the contribution rate is shared between the sponsor and active members in the proportions 65% to the sponsor and 35% to the members. If there is a large surplus, contribution rates are reduced in the same proportions.

3 Inflation Indexation for Pensions in Payment. Until April 2011 RPI was used to fully uprate USS pensions in payment and deferred pensions, but the government changed this to CPI in April 2011. So before October 2011 there was full indexation of pensions in payment using CPI. Post-October 2011 for both the final salary and CRB sections if inflation, as measured by the CPI, is less than 5% there is full indexation. For inflation between 5% and 15% indexation is 5% plus half of the excess over 5%. If inflation is more than 15% indexation is capped at 10%. In periods of negative inflation pensions are not reduced, but no increase is applied. Benefits accrued before October 2011 in the final salary section increase fully in line with official pensions, i.e. uncapped CPI.

4 Up-rating of the Accrued Benefits of Deferred Pensioners. Before October 2011 there was no cap on the up-rating of the accrued benefits of deferred pensioners. After the rule change in October 2011 the accrued benefits of deferred pensioners are uprated by CPI, capped at 2.5%.

5 Normal Retirement Age (NRA). Before October 2011 the NRA was 65 years. In October 2011 this was changed so that the USS NRA for the final salary and CRB sections increases with the state retirement age. This will rise to 66 years in about 2020, 67 years in about 2028, and 68 in about 2046.

Some Other Rules Which Did Not Change in October 2011

6 Accrual Factor. For both the final salary and CRB sections the accrual rate is 1/80th plus a lump sum of three times the pension. Using a commutation factor of 1:16 to convert the lump sum into a pension, the accrual factor including the lump sum is 1/67.37 for both sections.

7 Lump Sum. On retirement, pensioners can choose to take up to 25% of the value of their pension as a tax free lump sum. Pensioners are assumed to follow the USS default and take three times the annual value of their pension as a lump sum. Their subsequent pension payments are then based on an accrual factor of 1/80ths, rather than 1/67.37ths.
8 **Revaluation Rate for the CRB Section.** The revaluation rate used to uprate the average salary for active members of the CRB section each year to allow for inflation is the same as the inflation rate used to up-rate pensions in payment.

9 **Pensionable Salary for the Final Salary Section.** The pensionable final salary is the greater of:

(a) the member’s highest salary for any period of 12 complete months ending on the last day of a month during the last three years before retirement, and

(b) the highest yearly average of the total salary of the member for any three consecutive years ending at the end of any month within the last ten years before retirement. Both amounts are increased, except for the last year before retirement, in proportion to any increase in the RPI between that published at the last day of the relevant year and that published at retirement.

10 **Spouses Pensions.** When a pensioner dies their spouse, civil partner or dependant partner (regardless of sex) receives a pension for life. The spouse’s or civil partner’s pension is $\frac{1}{160}$th times pensionable salary at retirement times pensionable service at retirement, plus pension increases from retirement to death. Note that this calculation ignores the actual lump sum chosen by the pensioner, and assumes they took the standard amount of three times their initial pension.

**Appendix B.1. Final Salary (FS) Scheme**

The actuarial liability for active and deferred members of the cohort $x$ at time $t$ is given by:

$$L_{x,FS}^{A/D,t} = N_{x,FS}^{A/D,t} \times \left( \frac{p_{x,FS}^{A/D,t} \times S_{x,FS}^{A/D,t}}{A} \right) \times \left( \frac{1 + e_{x,FS}^{A/D,t}}{1 + h_{x,FS}^{A/D,t}} \right)^{R_{x,FS}^{A/D,t} - G_{x,FS}^{A/D,t}} \times \left( 1 - \frac{1 + h_{x,FS}^{A/D,t}}{1 + P_{x,FS}^{A/D,t}} \right)^{W_{x,FS}^{A/D,t} - 1} \right) \right) \right) \right) (B.1)$$

where $A$ is the accrual rate (constant),

$h_{x,FS}^{A/D,t}$ is the annual nominal discount rate at time $t$,

$R_{x,FS}^{A/D,t}$ is the forecast retirement age of the active/deferred members in cohort $x$ at time $t$,

$G_{x,FS}^{A/D,t}$ is the average age of the active/deferred members in cohort $x$ at time $t$,

$W_{x,FS}^{A/D,t}$ is the life expectancy at retirement of the active/deferred members in cohort $x$ at time $t$,

$p_{x,FS}^{A/D,t}$ is the annual rate of growth of the price level at time $t$,

$p_{x,FS}^{A/D,t}$ is the past years of service of active/deferred members in cohort $x$ at time $t$,

$s_{x,FS}^{A/D,t}$ is the annual salary of the active/deferred members in cohort $x$ at time $t$,

$e_{x,FS}^{A/D,t}$ is the expected nominal rate of salary growth per annum between time $t$ and retirement of the active/deferred members in cohort $x$,

$N_{x,FS}^{A/D,t}$ is the number of the active/deferred members in cohort $x$ at time $t$.

The actuarial liability for pensioners in cohort $x$ at time $t$ is given by:
\[ L_{x,FS} = N_{x,FS}^{P,t} \times PEN_{x,FS}^{P,t} \times \left[ \frac{1 - \left( 1 + h_{x,FS}^{P,t} \right)^{q_{x,FS}^{P,t}}}{1 + p_{x,FS}^{P,t}} \right] \] (B.2)

where
- \( N_{x,FS}^{P,t} \) is the number of the pensioners in cohort \( x \) at time \( t \),
- \( PEN_{x,FS}^{P,t} \) is the annual pension of the pensioners in cohort \( x \) at time \( t \),
- \( p_{x,FS}^{P,t} \) is the annual rate of growth of the price level at time \( t \),
- \( h_{x,FS}^{P,t} \) is the annual nominal discount rate at time \( t \),
- \( q_{x,FS}^{P,t} \) is the life expectancy of the pensioners in cohort \( x \) at time \( t \).

The total actuarial liability of the FS scheme is given by:
\[ L_{FS, \text{Total}} = \sum_{x \in Z} \left( L_{x,FS}^{A,t} + L_{x,FS}^{D,t} + L_{x,FS}^{P,t} \right) \] (B.3)

**Appendix B.2. Career Revalued Benefit (CRB) Scheme**

The actuarial liability for active and deferred members in cohort \( x \) at time \( t \) is given by:
\[ L_{x,CRB}^{A/D,t} = N_{x,CRB}^{A/D,t} \times \left( \frac{P_{x,CRB}^{A/D,t} \times S_{x,CRB}^{A/D,t}}{A_{x,CRB}^{A/D,t}} \right) \times \left[ \frac{1 + h_{x,CRB}^{A/D,t}}{1 + p_{x,CRB}^{A/D,t}} \right] \times \left[ 1 - \frac{1 + h_{x,CRB}^{A/D,t}}{1 + h_{A/D,t}^{x,CRB}} \right] \] (B.4)

where
- \( A \) is the accrual rate (constant),
- \( h_{x,CRB}^{A/D,t} \) is the annual nominal discount rate at time \( t \),
- \( R_{x,CRB}^{A/D,t} \) is the forecast retirement age of the active/deferred members in cohort \( x \) at time \( t \),
- \( G_{x,CRB}^{A/D,t} \) is the average age of the active/deferred members in cohort \( x \) at time \( t \),
- \( W_{x,CRB}^{A/D,t} \) is the life expectancy at retirement of the active/deferred members in cohort \( x \) at time \( t \),
- \( p_{x,CRB}^{A/D,t} \) is the annual rate of growth of the price level at time \( t \),
- \( P_{x,CRB}^{A/D,t} \) is the past years of service of active/deferred members in cohort \( x \) at time \( t \),
- \( S_{x,CRB}^{A/D,t} \) is the average annual revalued earnings of the active/deferred members in cohort \( x \) at time \( t \),
- \( e_{x,CRB}^{A/D,t} \) is the expected nominal rate of salary growth per annum between time \( t \) and retirement of the active/deferred members in cohort \( x \),
- \( N_{x,CRB}^{A/D,t} \) is the number of the active/deferred members in cohort \( x \) at time \( t \).

The actuarial liability for the pensioners in cohort \( x \) at time \( t \) is given by:
\[ L_{x,CRB}^{P,t} = N_{x,CRB}^{P,t} \times PEN_{x,CRB}^{P,t} \times \left[ 1 - \frac{1 + h_{x,CRB}^{P,t}}{1 + p_{x,CRB}^{P,t}} \right] \] (B.5)

where
- \( N_{x,CRB}^{P,t} \) is the number of the pensioners in cohort \( x \) at time \( t \),
- \( PEN_{x,CRB}^{P,t} \) is the annual pension of the pensioners in cohort \( x \) at time \( t \),
- \( p_{x,CRB}^{P,t} \) is the annual rate of growth of the price level at time \( t \),
- \( h_{x,CRB}^{P,t} \) is the annual nominal discount rate at time \( t \),
- \( q_{x,CRB}^{P,t} \) is the life expectancy of the pensioners in cohort \( x \) at time \( t \).
Finally, the total actuarial liability of the CRB scheme is given by:

\[
L_{t_{CRB, \text{Total}}} = \sum_{x=Z} \left( L_{t_{A,x_{CRB}}} + L_{t_{D,x_{CRB}}} + L_{t_{P,x_{CRB}}} \right)
\]

(B.6)

Appendix C. Time Varying Price of Risk

Following Cochrane and Piazzesi (2005, equation 8) and other studies, we compute the column vector \( \phi_t \) for equation (4) using the following expression:

\[
\phi_t = \Sigma^{-1} \left[ c + \frac{1}{2} \text{diag}(\Sigma) \right] + \Sigma^{-1} B x_t
\]

(C.1)

If investors are risk neutral, an absence of arbitrage opportunities requires the spot rate expected next period to equal the implied forward rate for next period. Following Hoevenaars (2011), Hoevenaars, Molenaar and Ponds (2010), and Hoevenaars and Ponds (2008), we set the implied forward interest rate next period \((t+1)\) (i.e. the first two right hand terms in equation 4) equal to the spot interest rate for next period from the VAR(1) model. The parameters \( \Sigma \) and \( B \) come from the VAR(1) model in equation (2), while the column vector \( x_t \) contains the state variables of the VAR(1) model at time \( t \).
Fig. 1: Post-October 2011 Scheme Mean Funding Ratios for the Three Asset Allocation Strategies

Fig. 2: Pre-October 2011 Scheme Mean Funding Ratios for the Three Asset Allocation Strategies
Fig. 3: Post-October 2011 Mean Contribution Rates for the Three Asset Allocation Strategies

Fig. 4: Pre-October 2011 Mean Contribution Rates for the Three Asset Allocation Strategies
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Fig. 6: Mean Risk Management Asset Allocation for the Pre and Post-October 2011 Schemes
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Fig. 8: £s Loss Per Head for Actives in Each Age Cohort Using SDF
Fig. 9: Losses Per Age Cohort in £bn. Using SDF

Fig. 10: Mean Percentage Drop Per Head for Actives in Pension Received at Age 65.
Fig. 11: Mean Percentage Drop in the NPV for Each Age Cohort Due to the Rule Change Using Riskless Rates

Fig. 12: Mean £s Expected Loss Per Head for Actives in Each Age Cohort Using Riskless Rates
Fig. 13: Coefficient of Variation of the NPVs of the Pre and Post-October 2011 Schemes Using Riskless Rates
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Table 1: VAR(1) Model Used to Generate the Forecasts

***, ** and * represent significance at the 1%, 5% and 10% levels respectively
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<td>0.000029</td>
<td>-0.000032</td>
<td>0.000038</td>
<td>0.000364</td>
<td>0.001159</td>
<td></td>
</tr>
</tbody>
</table>

Appendix D: Covariance Matrix of the VAR(1) Model Residuals