Monetary Policy in the Small Open Economy with Market Segmentation *

Jae Hun Shim†

Department of Economics, University of Bath

September 2016 (This Version, July 2017)

Abstract

We extend a New Keynesian small open economy DSGE model with non-tradable goods and intermediate inputs. Firstly, we show that the optimal monetary policy faces a trade-off between composite domestic inflation and output gap stabilization due to net exports externalities. Secondly, we rank alternative monetary policy rules associated with welfare and show that setting graduate interest rates towards their target levels rather than an immediate response is desirable. However, when the economy is highly exposed to foreign goods market and non-tradable productivity shocks, the CPI-based Taylor rule can be the best alternative policy. Lastly, we identify linkages between final and intermediate sectors and explain "sectoral heterogeneity" under the optimal policy and alternative monetary policy regimes.

Keywords: Small open economy, Net exports externalities, Optimal monetary policy, Non-tradable goods, Intermediate inputs

JEL Classification Numbers: E52, F3, F41

1 Introduction

There has been a great debate whether central banks in an open economy should take into account fluctuations in the exchange rate for the monetary policy design or follow a purely inward looking monetary policy by targeting domestic inflation. The open economy macroeconomic literature has explored the optimal monetary policy in the absence of non-tradable sectors and argues that the monetary policy in a

*I am greatly indebted to Christopher Martin for his valuable comments. Also, I would like to thank Charles Engel, Jordi Gali, Alexander Mihailov, Bruce Morley and Harald Uhlig for useful comments and help.

†Address: Department of Economics, University of Bath, Bath, BA2 7AY, UK, e-mail: *jhs30@bath.ac.uk*.

small open economy is isomorphic to the closed economy (see, for example, Clarida et al. (2001), Gali & Monacelli (2005), Faia & Monacelli (2007) and De Paoli (2009)).

This paper analyses the optimal monetary policy in a small open economy characterised by relative prices (the terms of trade and the relative home producer price), and compare different monetary policy rules in a welfare analysis. In our model, the relative prices through the expenditure switching effect between non-tradable goods and home (foreign) tradable goods are key factors for explaining the dynamics of the small open economy and the optimal monetary policy, and capturing a role for non-tradable sectors. Empirically, roughly 40% of GDP comes from consumption of non-tradable goods as shown by Dotsey & Duarte (2008) and Stockman & Tesar (1995)[1], and non-tradable goods play critical roles in explaining the real exchange rate (see, for example, Burstein et al. (2005), Obstfeld & Rogoff (2007), Dotsey & Duarte (2008) and Rabanal & Tuesta (2013)). This implies that the presence of non-tradable sectors alters the dynamics of the economy by stochastic shocks and the optimal policy through risk sharing and market clearing conditions.

Extensive studies have analysed the optimal monetary policy and terms of trade effects in the absence of non-tradable sectors. Corsetti & Pesenti (2001) find that in contrast to the closed economy, positive domestic monetary shocks can reduce domestic welfare due to terms of trade effects. In the open economy, a positive monetary shock induces depreciation of the nominal exchange rate and the terms of trade and CPI inflation worsens purchasing power for a given short term sticky wage or home production price. The negative terms of trade effects working through nominal exchange rate dynamics can be larger than the positive effects of aggregate demand. Clarida et al. (2001) and Gali & Monacelli (2005) develop a small open economy New Keynesian model with Calvo (1983) type staggered price setting and monopolistic competition and argue that strict domestic tradable inflation targeting is the optimal policy. Also, they compare alternative monetary policy regimes using a welfare analysis and argue that while the domestic tradable inflation-based Taylor rule generates the lowest welfare losses, the exchange peg (PEG) policy leads to the highest welfare losses in response to the composite of productivity and foreign output shocks. Faia & Monacelli (2007) analyze the optimal monetary policy using a Rotemberg (1982) type sticky price model which is characterized by home bias in consumption and show that since the optimal monetary policy requires quantatively negligible volatile domestic tradable inflation in response to the productivity shocks, strict domestic tradable inflation targeting is a good approximation to the optimal policy. Also, De Paoli (2009) shows that the domestic

[1]The literature computes the average share of personal consumption of services in GDP for the share of non-tradable consumption. Stockman & Tesar (1995) use data from the seven largest industrial countries (Canada, France, Germany, Italy, Japan, UK and US) during 1970-1984 and Dotsey & Duarte (2008) use the US data during 1973-2004.

tradable inflation-based policy rule is superior to consumer price inflation-based policy rule and PEG except for implausibly high values of the elasticity of substitution between home and foreign goods in response to the composite shocks (productivity, markup, fiscal and external shocks)

So far, there have been many developments embedding non-tradable sectors in the open economy macroeconomic models such as Obstfeld & Rogoff (2000), Obstfeld & Rogoff (2007), Dotsey & Duarte (2008) and Rabanal & Tuesta (2013). The literature mostly focuses on the role of non-tradable sectors in explaining real exchange rate movement rather than investigating the optimal monetary policy or externalities from the movement in the relative prices. While Lipińska (2015) investigates the optimal monetary policy with non-tradable goods using the welfare loss function and gives penalties on the fluctuations away from the target levels, the optimal policy is characterised by fixed loss function coefficients from calibrated parameter values. Also, Devereux et al. (2006) examine the optimal monetary policy with non-tradable goods but they do not embed tradable sectors. Thus, their model does not capture a trade-off between composite domestic inflation and the output gap in the optimal policy.

Our analysis has three main contributions. Firstly, by embedding non-tradable goods, we identify an additional expenditure switching effect between home non-tradable goods and home (foreign) tradable goods. Households optimally choose the combination between home tradable and non-tradable goods, and imports depending on the relative prices. Thus, in addition to the choice between home and foreign tradable goods, they also choose between non-tradable goods and home (foreign) tradable goods. This implies that we observe net exports externalities arising from imperfect substitutability between tradable goods, non-tradable goods and imports. We identify net exports externalities in the optimal allocation and the optimal monetary policy. The optimal allocation requires eliminating net exports externalities along with the monopolistic distortions. Thus, strict domestic tradable inflation targeting in the open economy is no longer optimal in the presence of non-tradable sectors. Rather, the optimal policy requires to stabilize both tradable and non-tradable inflation by eliminating the externalities. However, since stabilizing tradable inflation, non-tradable inflation and the output gap is not attainable without fiscal instruments, the central bank faces a trade-off between stabilizing composite domestic inflation and the output gap due to the externalities.

Secondly, we compare alternative monetary policy rules in a welfare analysis and show that ranking of monetary rules crucially depends on types of shocks and parameter values. In response to tradable productivity shocks, the agumented composite domestic inflation-based Taylor rule (ADT) leads to the lowest welfare losses. The partial adjustment of the interest rates of ADT reduces a volatility of composite domestic inflation and welfare losses. For non-tradable productivity shocks, the CPI-based Taylor rule (CTR) outperforms other alternative policy rules. While CTR and ADT symmetrically stabilize composite domestic inflation, CTR further reduces a volatility of the relative price gap and thereby stabilizing the output gap. Importantly, we find that under the conventional inflation coefficient, the composite domestic inflation-based Taylor rule (DTR) generates the highest welfare losses by tradable productivity, non-tradable productivity and composite shocks. Thus, setting graduate interest rates towards their target levels rather than an immediate response appears to be desirable unless the central bank responds sufficiently aggressive to composite domestic inflation. However, when the economy is highly exposed to foreign goods markets (i.e., high degree of openness, weights on tradable goods and elasticities of substitution between home and foreign tradable goods) and non-tradable productivity shocks, CTR can be the best alternative policy.

Lastly, in contrast to the standard open economy DSGE model, we also introduce intermediate sectors in order to analyse the impact of stochastic shocks on the intermediate sectors which consist of approximately 50% of gross output across countries as shown by Jones (2011). In our model, markets are segmented so that we have three sectors; a final non-tradable sector, a final tradable sector and an intermediate sector. Dotsey & Duarte (2008) introduce intermediate sectors along with the presence of non-tradable sectors using an otherwise standard open-economy macro model and highlight the critical role of nontradable goods in explaining the real exchange rate. They assume that in the non-tradable sectors, output of monopolistically competitive firms is used as both final non-tradable goods and non-tradable intermediate inputs, and final tradable sectors combine intermediate inputs (home and foreign tradable intermediate inputs and non-tradable intermediate inputs) so that they do not explicitly distinguish final and intermediate sectors, thereby excluding an analysis of intermediate sectors. Thus, the main departure in this paper is that in order to shed light on the intermediate sectors, we distinguish final and intermediate sectors by embedding separate final and intermediate non-tradable sectors and assuming the standard production function of final tradable sectors. We compare dynamics of main variables in intermediate sectors under alternative monetary regimes and find that under the optimal policy, intermediate sectors are symmetrically beneficial from the stochastic shocks (tradable and non-tradable sector productivity shocks). However, under CTR, ADT and PEG, while the stochastic shocks have symmetric impact on the final and intermediate tradable sectors, it does not have symmetric impact on the final and intermediate non-tradable sectors due mainly to the sectoral capacity to engage in international trade and the substitutability between home and foreign tradable goods (intermediate inputs).

The paper is organized as follows: Section 2 briefly describes sectoral output and productivity fluctuations in response to sectoral productivity shocks. Section 3 describes the small open economy model with non-tradable goods and intermediate sectors, and derives the optimal monetary policy. Section 4 presents calibrated values and quantitative results. Section 5 concludes.

2 Stylised Facts of Market Segmentation

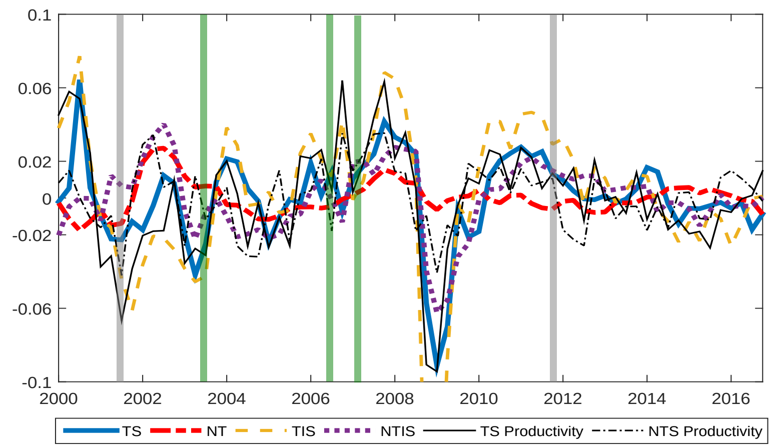

Figure 1: Sectoral Output and Productivity in Korea

NOTE: TS, NTS, TIS and NTIS refer to tradable sectors, non-tradable sectors, tradable intermediate sectors and non-tradable intermediate sectors, respectively. Variables are presented in percentage deviations from an Hodrick-Prescott (HP) filtered trend and estimated from 2000q1 to 2016q4. Following Burstein et al. (2005), Corsetti et al. (2008) and Dotsey & Duarte (2008), we regard non-tradable intermediate output as wholesale and retail services and transportation (distribution services). Source: The Bank of Korea and Federal Reserve Economic Data.

In order to explore the sectoral heterogeneity in a small open economy in response to sectoral productivity shocks, Figure 1 shows sectoral output and productivity fluctuations in South Korea[2] during 2000q1-2016q4. The green (grey) line corresponds to tradable (non-tradable) productivity shocks[3]. Three main stylized facts stand out. Firstly, we observe the heterogeneity of sectoral output fluctuations between tradable and non-tradable sectors (final and intermediate sectors) in response to sectoral productivity shocks. Sec-

[2]Korea has one of the most open goods markets in the world and a small open economy which is unable to influence the foreign interest rate, output and prices, and the central bank in Korea is following the CPI-based Taylor rule. During the period given, the share of consumption of non-tradable goods and that of intermediate goods consist of approximately 40% and 50% of GDP, respectively.

[3]As Figure 1 shows, tradable and non-tradable productivity shocks are highly correlated, $corr(\sigma_{H,T}, \sigma_{H,N}) = 0.53$ and thus, the vertical green (grey) line corresponds to the quarter when tradable (non-tradable) sector productivity changes due to changes in sectoral labour market conditions and distinctively dominates the other.

ondly, while non-tradable sector productivity shocks led to broadly symmetric fluctuations of sectoral output, positive tradable productivity shocks had a negative effect on non-tradable intermediate sector. Lastly, we observe relatively large fluctuations of tradable intermediate sector output in response to tradable productivity shocks[4].

3 A Small Open Economy Model

The model is a two-country New Keynesian DSGE model with non-tradable goods and intermediate sectors. The baseline framework for the open economy follows Gali & Monacelli (2005), Sutherland (2005), Obstfeld & Rogoff (2007) and Meier (2013). We extend the benchmark for small open economy New Keynesian DSGE model with non-tradeable goods and intermediate inputs.

3.1 Households

The world is composed of a small home economy and the large foreign country. The home country is small since it is unable to influence the foreign interest rate, output and prices. Households on the subinterval [0, n) live in the home country (denoted by h) and households on the subinterval (n, 1] live in the foreign country (denoted by f).

The intertemporal utility of a representative household in the home economy is given by

$$E_0 \sum_{t=0}^{\infty} \beta^{t} U(C_t, L_t) \tag{1}$$

where per-period utility specifically is given by

$$U(C_t, L_t) = \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{L_t^{1+\varphi}}{1+\varphi} \right]$$

where ρ is the coefficient of relative risk aversion and φ is the inverse (Frisch) labour supply elasticity. The household gains utility from consumption, C_t and disutility from producing domestic goods, L_t. Aggregate consumption of a representative home (foreign) household is given by

$$C_t = \left[\sigma^{\frac{1}{\omega}} (C_{T,t})^{\frac{\omega-1}{\omega}} + (1-\sigma)^{\frac{1}{\omega}} (C_{H,N,t})^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}; \; C_t^f = \left[\sigma^{f\,\frac{1}{\omega^f}} (C_{T,t}^f)^{\frac{\omega^f-1}{\omega^f}} + (1-\sigma^f)^{\frac{1}{\omega^f}} (C_{F,N,t}^f)^{\frac{\omega^f-1}{\omega^f}} \right]^{\frac{\omega^f}{\omega^f-1}} \tag{2}$$

[4]Since the global financial crisis, started in 2008 influenced the small open economy mainly though negative foreign financial shocks, we do not investigate the sectoral heterogeneity by the shocks during the crisis.

where $C_{T,t}$ ($C^{f}_{T,t}$) is the consumption of tradable goods from both countries and $C_{H,N,t}$ ($C^{f}_{F,N,t}$) is the consumption of non-tradable goods. σ (σ^{f}) $\in [0,1]$ is the weight on tradable goods in the overall consumption and ω (ω^{f}) is the elasticity of substitution between tradable and non-tradable goods.

The consumption of home and foreign tradable goods of the representative home (foreign) household is assumed to be

$$C_{T,t}=\left[\lambda^{\frac{1}{\eta}}(C_{H,T,t})^{\frac{\eta-1}{\eta}}+(1-\lambda)^{\frac{1}{\eta}}(C_{F,T,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}};\ C^{f}_{T,t}=\left[\lambda^{f\,\frac{1}{\eta^{f}}}(C^{f}_{F,T,t})^{\frac{\eta^{f}-1}{\eta^{f}}}+(1-\lambda^{f})^{\frac{1}{\eta^{f}}}(C^{f}_{H,T,t})^{\frac{\eta^{f}-1}{\eta^{f}}}\right]^{\frac{\eta^{f}}{\eta-1}} \quad (3)$$

where $C_{H,T,t}$ ($C^{f}_{F,T,t}$) is the consumption of home (foreign) tradable goods and $C_{F,T,t}$ ($C^{f}_{H,T,t}$) is the consumption of foreign (home) tradable goods. $\lambda \in [0,1]$ is the weight on home tradable goods in the overall tradable goods. Following Sutherland (2005), $(1-\lambda)=\alpha(1-n)$ is the weight on imported goods from foreign country, reflecting the relative size of the home country n and the degree of openness α. As a small open economy is characterised by $n \rightarrow 0$, the degree of openness requires $\alpha < 1$. The weight on imports from home country is defined as $\left(1-\lambda^{f}\right)=n\alpha$. Foreign weights on tradable goods are analogous to the weight on imports from home country. η (η^{f}) is the elasticity of substitution between home tradable goods and foreign tradable goods. For simplicity, we assume the same elasticity of substitution between different varieties across countries.

The consumer price index (CPI) corresponding to aggregate consumption in home and foreign country is given by

$$P_{t}=\left[\sigma(P_{T,t})^{1-\omega}+(1-\sigma)(P_{H,N,t})^{1-\omega}\right]^{\frac{1}{1-\omega}};\ P^{f}_{t}=\left[\sigma^{f}(P^{f}_{T,t})^{1-\omega}+(1-\sigma^{f})(P^{f}_{F,N,t})^{1-\omega}\right]^{\frac{1}{1-\omega}} \quad (4)$$

where $P_{T,t}$ ($P^{f}_{T,t}$) is the price of tradable goods and $P_{H,N,t}$ ($P^{f}_{F,N,t}$) is the price of non-tradable goods, measured both in home (foreign) currency. The corresponding price of tradable goods determined by the price of home (foreign) and imports is given by

$$P_{T,t}=\left[\lambda(P_{H,T,t})^{1-\eta}+(1-\lambda)(P_{F,T,t})^{1-\eta}\right]^{\frac{1}{1-\eta}};\ P^{f}_{T,t}=\left[\lambda^{f}(P^{f}_{F,T,t})^{1-\eta}+(1-\lambda^{f})(P^{f}_{H,T,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \quad (5)$$

Home (foreign) households allocate consumption expenditure on home (foreign) tradable and non-tradable goods, and imported goods in order to minimize the cost of reaching a specified level of utility.

We assume complete international financial markets in which consumers in different economies have symmetric state contingent nominal claims and identical preferences. Since, total consumption of the home household is given by $\mathrm{P}_t\mathrm{C}_t = \mathrm{P}_{H,T,t}\mathrm{C}_{H,T,t} + \mathrm{P_{F,T,t}C_{F,T,t}} + \mathrm{P_{H,N,t}}\mathrm{C}_{H,N,t}$, the per-period budget constraint for the household is given by

$$\mathrm{P}_t\mathrm{C}_t + \sum_{q^{t+1}\in q} Q^a\left(q^{t+1}|q^t\right)B\left(q^{t+1}|q^t\right) \leq B_t + \mathrm{W}_t\mathrm{L}_t + \mathrm{T}_t + \Pi_t \tag{6}$$

where T_t are the lump sum taxes, Π_t are equally shared profits earned by firms producing goods, W_t is the nominal wage and L_t is the labour supply. $B\left(q^{t+1}|q^t\right)$ are units of an international state-contingent security bought in period t at one unit of home currency price $Q^a\left(q^{t+1}|q^t\right)$ in state q^{t+1} for a given history of events by period t, $q^t = (q_0, q_1, q_2, q_3 \cdots\cdot q_t) \in q$ where q are states of nature[5]. The household receives payments of each asset in the portfolio B_{t+1} if only the event q^{t+1} occurs.

Symmetrically, the per-period budget constraint for a representative foreign household is given by

$$\mathrm{P^f_t C^f_t} + \sum_{q^{t+1}\in q} Q^a\left(q^{t+1}|q^t\right)\left(\frac{1}{\mathrm{X_t}}\right)B^f\left(q^{t+1}|q^t\right) \leq B^f_t\left(\frac{1}{\mathrm{X_t}}\right) + \mathrm{W^f_t L^f_t} + \mathrm{T^f_t} + \Pi^f_t \tag{7}$$

where X_t is the nominal exchange rate. Under the assumption of complete international financial markets where households in both countries have access to a complete set of international contingent claims, the stochastic discount factor is symmetric. Thus, the optimality conditions with respect to consumption, international bonds and labour supply in both countries implied by maximizing intertemporal utility subject to a sequence of dynamic budget constraints can be written as

$$Q^a_{t,t+1} = \beta\mathrm{E}_t\left\{\left(\frac{\mathrm{C}_{t+1}}{\mathrm{C}_t}\right)^{-\rho}\left(\frac{\mathrm{P}_t}{\mathrm{P}_{t+1}}\right)\right\} \tag{8}$$

$$Q^a_{t,t+1} = \beta\mathrm{E}_t\left\{\left(\frac{\mathrm{C^f_{t+1}}}{\mathrm{C^f_t}}\right)^{-\rho}\left(\frac{\mathrm{P^f_t}}{\mathrm{P^f_{t+1}}}\right)\left(\frac{\mathrm{X}_t}{\mathrm{X}_{t+1}}\right)\right\} \tag{9}$$

$$L^{\varphi}_t C^{\rho}_t = \frac{W_t}{P_t} \tag{10}$$

where $Q^a_{t,t+1} \equiv \dfrac{Q^a\left(q^{t+1}|q^t\right)}{\kappa(q^{t+1}|q^t)}$ is the one unit of home currency price in $t+1$. Log-linearization around the steady state without inflation of these conditions gives the standard Euler equation for the optimal intertem-

[5]The market value of the state contingent claims obtained in period t can be written in terms of expected value as $\sum_{q^{t+1}\in q} \dfrac{Q^a\left(q^{t+1}|q^t\right)}{\kappa(q^{t+1}|q^t)}\kappa(q^{t+1}|q^t)B\left(q^{t+1}|q^t\right) = E_t(Q^a_{t,t+1}B_{t+1})$ where $\kappa(q^{t+1}|q^t)$ is the probability of particular event of q^{t+1} in period $t+1$ for a given history of events by period t and $Q^a_{t,t+1} \equiv \dfrac{Q^a\left(q^{t+1}|q^t\right)}{\kappa(q^{t+1}|q^t)}$ is so called the stochastic discount factor

poral allocation of consumption

$$\hat{C}_t = \mathrm{E}_t \left\{ \hat{C}_{t+1} \right\} - \frac{1}{\rho} (\hat{i}_t - \mathrm{E}_t \left\{ \pi_{t+1} \right\}) \tag{11}$$

where $\hat{i}_t \equiv -\log Q^a_{t,t+1}$ is the home nominal interest rate and variables with a 'hat' denotes log deviations from the steady state and steady state values are denoted with letters without time scripts. $\pi_{t+1} \equiv \hat{P}_{t+1} - \hat{P}_t$ is the inflation rate (in terms of the CPI price index).

3.2 The terms of trade, the real exchange rate, the real interest rate and risk sharing condition

The terms of trade is the relative price between exports and imports. Hence, the terms of trade between the home economy and the foreign country can be defined as $\mathrm{S}_t \equiv \frac{\mathrm{P}_{F,\ T,t}}{\mathrm{P}_{H,T,t}} \equiv \frac{\mathrm{X}_t \mathrm{P}^f_{F,T,t}}{\mathrm{P}_{H,T,t}}$ where $\mathrm{P}^f_{F,T,t}$ is the price of imports measured in foreign currency. We assume the law of one price, $\mathrm{P}_{F,T,t} = X_t \mathrm{P}^f_{F,T,t}$ and $\mathrm{P}_{H,T,t} = X_t \mathrm{P}^f_{H,T,t}$ so that the price of the same good should be equal across countries without market discrimination and trade barriers such as transaction costs or tariffs. We can define the real exchange rate between the domestic economy and country f as $Q_t \equiv \frac{\mathrm{X}_t P^f_t}{P_t}$. Thus, Q_t is the relative price of goods between the domestic and foreign country, expressed in domestic currency. Since the home economy is small and unable to influence the foreign economy, the foreign economy is analogous to a closed economy. Thus, the foreign CPI, the price of foreign tradable and non-tradable are symmetric $P^f_t = \mathrm{P}^f_{F,T,t} = \mathrm{P}^f_{F,N,t}$. Let $\hat{e}_{\mathrm{t}} \equiv \hat{\mathrm{P}}_{\mathrm{H,N,t}} - \hat{P}_{\mathrm{H,T,t}}$ be the log relative home producer price. Then, the log real exchange rate can be written as

$$\hat{Q}_t = [1 - \sigma(1-\lambda)]\,\hat{S}_t - (1-\sigma)\hat{e}_{\mathrm{t}} \tag{12}$$

Thus, the terms of trade, relative price of home producer goods, the weight on tradable goods and the home bias determines the real exchange rate. If the economy only produces tradable goods the real exchange rate equation become $\hat{Q}_t = \lambda \hat{S}_t$ so that real exchange rate and the terms of trade are proportional. However, in the presence of non-tradable goods, the variation and weights of non-tradable price also influence the real exchange rate while the terms of trade plays a more important role for the fluctuations in the real exchange rate implied by $[1-\sigma(1-\lambda)] > (1-\sigma)$, which is broadly consistent with the data.

Aggregating (8) and (9) and assuming zero initial net foreign assets, we have the equilibrium risk-sharing condition

$$\begin{aligned}\hat{C}_t - \hat{C}_t^f &= (\frac{1}{\rho})\hat{Q}_t \\ &= (\frac{1}{\rho})\left[[1-\sigma(1-\lambda)]\hat{S}_t - (1-\sigma)\hat{e}_\mathrm{t}\right]\end{aligned} \tag{13}$$

Risk sharing implies that since households purchase contingent claims, idiosyncratic shocks can be insured away. Thus, the marginal utility of consumption of both countries, weighted by the real exchange rate should be equalized, as noted by Backus & Smith (1993). While an increase in the price of non-tradable goods reduces consumption through a higher relative price of home producer goods for given prices of foreign and home tradable goods, lower consumption induced by an increase in the price of home tradable goods through an appreciation of the terms of trade partially offset by a fall in the relative price of home producer goods.

We define the Fisher equation as $1+i_t \equiv \left(\frac{\mathrm{P}_{t+1}}{\mathrm{P}_t}\right)R_t$ and $1+i_t^f \equiv \left(\frac{P_{t+1}^f}{P_t^f}\right)R_t^f$ where R_t and R_t^f are the home and foreign gross real interest rate. Combining with the uncovered interest parity condition under the assumption of complete international financial markets , $\hat{i}_t = \hat{i}_t^f + E_t(\triangle\hat{X}_{t+1})$ and log linearizing yields

$$\hat{R}_t = \hat{R}_t^f + E_t(\triangle\hat{Q}_{t+1}) \tag{14}$$

Thus, the real interest rate depends on the foreign real interest rate and expected fluctuations of the real exchange rate. Since the real exchange rate is influenced by prices of final non-tradable goods, final non-tradable sectors have a critical role for the real interest rate.

3.3 Firms

As explained earlier, in our model, markets are segmented so that we have three sectors; final non-tradable sectors, final tradable sectors and intermediate sectors. While final non-tradable sectors use labour only for production, final tradable sectors use both labour inputs and intermediate inputs (home tradable and non-tradable intermediate inputs and foreign tradable intermediate inputs). Final tradable sectors are mainly manufacturing and final non-tradable sectors are mainly services which are consumed by households such as education and housing. We assume that firms across different sectors are monopolistically competitive and follow Calvo type sticky price setting in order to allow the same speed of adjustment in response to stochastic shocks. Final domestic firms are indexed by $z \in [0,\lambda\sigma;\ \lambda^i\sigma^i,\sigma^i;\ \sigma,1]$ where $[0,\lambda\sigma;\ \lambda^i\sigma^i,\sigma^i]$ represents final tradable firms producing goods for home and foreign consumers and $[\sigma,1]$ represent firms in non-tradable sectors.

3.3.1 Final Tradable Sectors

The production function for a representative domestic firm that produces a differentiated final tradable good is given by

$$\mathrm{Y}_{T,t}(z) = a_{H,T,t}\mathrm{L}_{H,T,t}^{1-\alpha^p}(z)\mathrm{YC}_t^{i\,\alpha^p}(z),\ 0 \leq \alpha^p < 1 \tag{15}$$

where $a_{H,T,t}$ is a tradable sector productivity shock that follows the AR(1) process in logs $\hat{a}_{H,T,t} = \rho_{\mathrm{aht}}\hat{a}_{H,T,t-1} + \varepsilon_{H,T,t}$. For $0 < \alpha^p$, the firms use both labour and intermediate inputs, YC_t^i. When $\alpha^p = 0$, then final tradable firms do not use intermediate inputs in production and production function is written as $\mathrm{Y}_{T,t}(z) = a_{H,T,t}\mathrm{L}_{H,T,t}(z)$. Since we assume complete exchange rate pass-through (producer currency pricing), the consumption-based prices of final tradable goods, $P_{H,T,t}$ is equal to the price set by final tradable firms producing goods for home and foreign consumers, denominated in home currency.

For simplicity, we assume that the composition of intermediate inputs is analogous to consumption

$$\mathrm{YC}_t^i = \left[\sigma^{i\,\frac{1}{\omega^i}}(\mathrm{YC}_{T,t}^i)^{\frac{\omega^i-1}{\omega^i}} + (1-\sigma^i)^{\frac{1}{\omega^i}}(\mathrm{YC}_{H,N,t}^i)^{\frac{\omega^i-1}{\omega^i}}\right]^{\frac{\omega^i}{\omega^i-1}} \tag{16}$$

where $\mathrm{YC}_{T,t}^i$ are tradable inputs, $\mathrm{YC}_{H,N,t}^i$ are home non-tradable inputs, $\sigma^i \in [0,1]$ is relative weights on tradable inputs and ω^i is the elasticity of substitution between tradable and non-tradable inputs.

The composition of tradable inputs is given by

$$\mathrm{YC}_{T,t}^i = \left[\lambda^{i\,\frac{1}{\eta^i}}(\mathrm{YC}_{H,T,t}^i)^{\frac{\eta^i-1}{\eta^i}} + (1-\lambda^i)^{\frac{1}{\eta^i}}(\mathrm{YC}_{F,T,t}^i)^{\frac{\eta^i-1}{\eta^i}}\right]^{\frac{\eta^i}{\eta^i-1}} \tag{17}$$

where $\mathrm{YC}_{H,T,t}^i$ is home tradable inputs, $\mathrm{YC}_{F,T,t}^i$ is foreign tradable inputs, $\lambda^i \in [0,1]$ is relative weights on home tradable inputs and η^i is the elasticity of substitution between home tradable and foreign tradable inputs.

The cost minimizing decision of the firm with respect to intermediate inputs and labour equate the relative price of inputs to the relative marginal productivities as

$$\frac{W_t}{P_t^i} = \left(\frac{1-\alpha^p}{\alpha^p}\right)\frac{\mathrm{YC}_t^i(z)}{\mathrm{L}_{\mathrm{H,T,t}}(z)} \tag{18}$$

where P_t^i is price of the aggregate input. The real marginal cost of producing final goods is given by

$$\mathrm{MC}_{H,T,t} = \frac{W_t L_{H,T,t}^{\alpha^p}(z)\mathrm{YC}_t^{i^{-\alpha^p}}(z)}{(1-\alpha^p)a_{H,T,t}P_{H,T,t}} \tag{19}$$

Log linearizing around the steady state yields

$$\begin{aligned}\widehat{MC}_{H,T,t} &= \hat{W}_t - \hat{P}_{\mathrm{H,T,t}} - \hat{a}_{H,T,t} + \alpha^p(\hat{L}_{\mathrm{H,T,t}}(z) - \widehat{YC}_{\mathrm{t}}^i(z)) \\ &= (1-\alpha^p)\hat{W}_{\mathrm{t}} - \hat{P}_{\mathrm{H,T,t}} - \hat{a}_{H,T,t} + \alpha^p\hat{P}_t^i\end{aligned} \tag{20}$$

Thus, real marginal cost in terms of the price of tradable goods is common across the domestic firms that produce final tradable goods. While an increase in wage and input costs increase real marginal costs, an increase in the productivity reduces costs.

Due to analogous cost minimizing decisions, demand for each inputs yields

$$\mathrm{YC}_{H,T,t}^i = \lambda^i\sigma^i\left[\frac{\mathrm{P}_{H,T,t}^i}{\mathrm{P}_{T,t}^i}\right]^{-\eta^i}\left[\frac{\mathrm{P}_{T,t}^i}{\mathrm{P}_t^i}\right]^{-\omega^i}\mathrm{YC}_t^i \tag{21}$$

$$\mathrm{YC}_{F,T,t}^i = (1-\lambda^i)\sigma^i\left[\frac{\mathrm{P}_{F,T,t}^i}{\mathrm{P}_{T,t}^i}\right]^{-\eta^i}\left[\frac{\mathrm{P}_{T,t}^i}{\mathrm{P}_t^i}\right]^{-\omega^i}\mathrm{YC}_t^i \tag{22}$$

$$\mathrm{YC}_{H,N,t}^i = (1-\sigma^i)\left[\frac{\mathrm{P}_{H,N,t}^i}{\mathrm{P}_t^i}\right]^{-\omega^i}\mathrm{YC}_t^i \tag{23}$$

where $\mathrm{P}_{T,t}^i$, $\mathrm{P}_{H,T,t}^i$ and $\mathrm{P}_{H,N,t}^i$ are the prices of tradable inputs, home tradable inputs and home non-tradable inputs. The terms of trade in intermediate sectors is the relative price between intermediate exports and imports and defined as $S_t^i \equiv \dfrac{P_{F,T,t}^i}{P_{H,T,t}^i}$. Analogous to the final goods sectors, we assume the law of one price in intermediate sectors.

A randomly selected proportion $1-\theta$ of home tradable firms sets new prices each period while a fraction θ keep their prices unchanged following the Calvo (1983) framework. The firms who can set new prices each period maximize the expected present discounted profits, given by.

$$max\sum_{\mathrm{k}=0}^{\infty}(\theta\beta)^{\mathrm{k}}\mathrm{E}_t[\overline{\mathrm{Y}}_{T,t+k}\overline{\mathrm{P}}_{H,T,t} - \mathrm{TC}_{H,T,t+k}^n(\overline{\mathrm{Y}}_{T,t+k})] \tag{24}$$

subject to the sequence of demand functions $\overline{\mathrm{Y}}_{T,t+k} \leq \left(\frac{\overline{\mathrm{P}}_{H,T,t}}{\mathrm{P}_{H,T,t+k}}\right)^{-\varepsilon}[\left(\frac{1}{\lambda\sigma}\right)^{\varepsilon}C_{H,T,t+k} + \left(\frac{1}{(1-\lambda^f)\sigma^f}\right)^{\varepsilon}C_{H,T,t+k}^f]$ where $\mathrm{TC}_{H,T,t+k}^n$ is the nominal total cost of producing final tradable goods, $\overline{\mathrm{P}}_{H,T,t}$ is the prices set by firms adjusting their prices in period t and $\overline{\mathrm{Y}}_{T,t+k}$ is the corresponding output in period $t+k$.

The first order condition yields

$$\sum_{\mathrm{k}=0}^{\infty}(\beta\theta)^{\mathrm{k}}\mathrm{E}_t\left\{\overline{\mathrm{Y}}_{T,t+k}\left(\overline{\mathrm{P}}_{H,T,t}-\Psi MC_{H,T,t+k}\mathrm{P}_{H,T,t+k}\right)\right\}=0 \tag{25}$$

where $\Psi\equiv\frac{\varepsilon}{\varepsilon-1}$ is the markup of price over marginal cost in steady-state. The optimal price setting strategy for firms setting a new price,$\tilde{\overline{P}}_{H,T,t}$ in terms of the logs of nominal marginal costs $\widetilde{MC}_t^n$ is given by

$$\tilde{\overline{P}}_{H,T,t}=\mu+(1-\beta\theta)\sum_{\mathrm{k}=0}^{\infty}(\beta\theta)^{\mathrm{k}}\mathrm{E}_t\{\widetilde{MC}^n_{H,T,t+k}\} \tag{26}$$

where letters with tilde denote the logs of the respective variables and $\mu\equiv log\frac{\varepsilon}{\varepsilon-1}$ is the logs of markup of price over marginal cost in steady-state. The domestic tradable goods price index is given by $\mathrm{P}_{\mathrm{H,T,t}}=\left[\theta(\mathrm{P}_{\mathrm{H,T,t-1}})^{1-\varepsilon}+(1-\theta)(\overline{\mathrm{P}}_{\mathrm{H,T,t}})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$, which, when log linearized around the zero inflation steady state yields $\pi_{\mathrm{H,T,t}}=(1-\theta)(\hat{\overline{P}}_{\mathrm{H,T,t}}-\hat{\mathrm{P}}_{\mathrm{H,T,t-1}})$. Combining this with the log linearized optimal price setting strategy around the steady state, we have the marginal cost based domestic tradable inflation

$$\pi_{\mathrm{H,T,t}}=\beta E_t(\pi_{\mathrm{H,T,t+1}})+\nu\widehat{MC}_{H,T,t} \tag{27}$$

where $\nu\equiv\dfrac{(1-\beta\theta)(1-\theta)}{\theta}$

3.3.2 Final Non-Tradable Sectors and Intermediate Sectors

We assume that the final non-tradable sectors and intermediate sectors have symmetric production functions and Calvo-type sticky price setting decisions. Thus, firms use only labour inputs in production.

The production function for a representative domestic firm that produces a differentiated non-tradable good is given by

$$\mathrm{Y}_{H,N,t}(\mathrm{z})=a_{H,N,t}\mathrm{L}_{H,N,t}(\mathrm{z}) \tag{28}$$

where $a_{H,N,t}$ is a non-tradable goods producing sector productivity shock that follows the AR(1) process in logs.

The optimal decision of the firm yield the labour demand

$$\mathrm{MC}_{H,N,t}=\frac{\mathrm{W_t}}{\mathrm{P_{H,N,t}}\mathrm{a}_{H,N,t}} \tag{29}$$

where $\mathrm{MC}_{H,N,t}$ is real marginal cost of producing non-tradable goods in terms of the price of non-tradable goods. Since we assume perfect labour mobility across sectors, workers in different sectors receive identical wages.

Log linearizing around the steady state gives

$$\widehat{MC}_{H,N,t} = \hat{W}_{\mathrm{t}} - \hat{P}_{\mathrm{H,N,t}} - \hat{a}_{H,N,t} \tag{30}$$

We assume intermediate tradable and non-tradable sectors are analogous to the final non-tradable sector so that the production function is

$$\mathrm{Y}^{i}_{T,t}(z^{i}) = \mathrm{a}_{H,T,t}\mathrm{L}^{i}_{H,T,t}(z^{i});\;\; \mathrm{Y}^{i}_{H,N,t}(z^{i}) = \mathrm{a}_{H,N,t}\mathrm{L}^{i}_{H,N,t}(z^{i}) \tag{31}$$

where $\mathrm{L}^{i}_{H,T,t}$ is employment in home tradable input sectors and $\mathrm{L}^{i}_{H,T,t}$ is employment in home non-tradable input sectors. Price setting is analogous to (26) and the intermediate sector market clearing condition is analogous to the final goods market clearing condition.

3.4 The New Keynesian Philips Curve

Log linearizing (4) and (5) around the steady state yields $\hat{P}_{\mathrm{t}} = \sigma\hat{P}_{\mathrm{T,t}} + (1-\sigma)\hat{P}_{\mathrm{H,N,t}}$ and $\hat{P}_{\mathrm{T,t}} = \lambda\hat{P}_{\mathrm{H,T,t}} + (1-\lambda)\hat{P}_{\mathrm{F,T,t}}$. Combining with the terms of trade, domestic tradable and non-tradable inflation yields the marginal cost and the terms of trade based New Keynesian Philips curve

$$\begin{aligned}
\pi_{\mathrm{t}} &= \sigma\pi_{\mathrm{H,T,t}} + (1-\sigma)\,\pi_{\mathrm{H,N,t}} + \sigma(1-\lambda)\widehat{\Delta S}_{\mathrm{t}} \\
&= \sigma[\beta E_t(\pi_{\mathrm{H,T,t+1}}) + \nu\widehat{MC}_{H,T,t}] + (1-\sigma)\,[\beta E_t(\pi_{\mathrm{H,T,t+1}}) + \nu\widehat{MC}_{H,T,t}] + \sigma(1-\lambda)\widehat{\Delta S}_{\mathrm{t}} \\
&= \beta \mathrm{E}_{\mathrm{t}}\{\pi_{\mathrm{t+1}}\} + \nu[\sigma\widehat{MC}_{H,T,t} + (1-\sigma)\widehat{MC}_{H,N,t}] - \sigma(1-\lambda)(\beta\widehat{\Delta S}_{\mathrm{t+1}} - \widehat{\Delta S}_{\mathrm{t}})
\end{aligned} \tag{32}$$

Thus, expected future inflation, an increase in marginal costs of both sectors and expected future appreciation of the terms of trade generates current inflation. While an increase in marginal costs push up production cost and thereby generates inflation, expected future appreciation of the terms of trade implies a current depreciation under the assumption of trade balance in the long run and thus raises current inflation.

3.5 Consumption, Exports, Imports and Net Exports

We assume that households are identical within each country. The aggregate consumption of home households, $\mathrm{C}_{\mathrm{t}} = \mathrm{C}_{\mathrm{H,T,t}} + \mathrm{C}_{\mathrm{F,T,t}} + \mathrm{C}_{\mathrm{H,N,t}}$ consists of demand for home tradable goods, imports from the foreign country and home non-tradable goods. We assume the trade balance in the steady state $\mathrm{C}_{\mathrm{F,T}} = \mathrm{C}^{\mathrm{f}}_{\mathrm{H,T}}$ where steady state variables are denoted without time scripts so that in the steady state $\mathrm{C}_{\mathrm{H,T}} = \mathrm{C}\lambda\sigma$, $\mathrm{C}_{\mathrm{F,T}} =$

$C^f_{H,T} = C(1-\lambda)\sigma$ and $C_{H,N} = C(1-\sigma)$. Log linearizing the aggregate consumption around the steady state yields

$$\hat{C}_t = \sigma\lambda\hat{C}_{H,T,t} + \sigma(1-\lambda)\hat{C}_{F,T,t} + (1-\sigma)\hat{C}_{H,N,t} \tag{33}$$

Foreign consumption of home tradable goods $C^f_{H,T,t}{=}(1-\lambda^f)\sigma^f\left[\frac{P^f_{H,T,t}}{P^f_{T,t}}\right]^{-\eta}\left[\frac{P^f_{T,t}}{P^f_t}\right]^{-\omega}C^f_t$ represents the demand for exports to the foreign country. Log linearizing yields

$$\widehat{EX}_t = \eta\hat{S}_t + \hat{C}^f_t \tag{34}$$

where $\widehat{EX}_t$ is log linearized demand for exports. An increase in the terms of trade, allowing higher relative foreign price, and higher elasticity of substitution between foreign and home tradable goods raises home exports.

Consumption of foreign tradable goods, $C_{F,T,t} = (1-\lambda)\sigma\left[\frac{P_{F,T,t}}{P_{T,t}}\right]^{-\eta}\left[\frac{P_{T,t}}{P_t}\right]^{-\omega}C_t$ represents the demand for imports from the foreign country and combining with the risk sharing condition and log linearizing yields

$$\widehat{IM}_t = \Lambda\hat{e}_t - \Upsilon\hat{S}_t + \hat{C}^f_t \tag{35}$$

where $\Lambda \equiv [\omega - (1/\rho)](1-\sigma)$, $\Upsilon \equiv \left\{(1-\lambda)\left[\omega(1-\sigma) + \left(\frac{1}{\rho}\right)\sigma\right] + \lambda\eta - \left(\frac{1}{\rho}\right)\right\} > 0$ and $\widehat{IM}_t$ is log linearized imports. The demand for imports from the foreign country depends on the relative home producer price and the terms of trade for given foreign consumption. Firstly, note that while depreciation of the terms of trade unambiguously reduces imports, the impact of the relative home producer price depends on the relative parameter values. When $\omega > (1/\rho)$, the expenditure switching effect between home non-tradable goods and imports dominate the (state contingent) income effects induced by changes in purchasing power. While an increase in the price of non-tradable goods leads to a switching of consumption from non-tradable goods to imports, appreciation of the real exchange rate due to the higher aggregate price level reduces aggregate consumption including imports and partially offsets the switching effect. The reverse is true if $\omega < (1/\rho)$. When we do not have non-traded sectors, the demand for imports equation represents the substitution between home and foreign tradables $\widehat{EWT}_t = -[\lambda(\eta - \frac{1}{\rho})]\hat{S}_t + c^f_t$ so that coupled with the substitution between home and foreign tradable consumption, we observe additional substitution between consumption of home non-tradable goods and foreign tradable goods that is captured by $\widehat{EWN}_t = \Lambda\hat{e}_t - \left[(1-\sigma)(1-\lambda)(\omega - \left(\frac{1}{\rho}\right))\right]\hat{S}_t$. Thus, imports can be rewritten in terms of expenditure switching between different varieties

$$\widehat{IM}_t = \widehat{EWT}_t + \widehat{EWN}_t \tag{36}$$

Combining the risk sharing condition and log linearizing net exports of final goods, $\mathrm{NX}_t \equiv \mathrm{C}^{\mathrm{f}}_{\mathrm{H,T,t}} - \frac{\mathrm{P_{F,T,t}}}{\mathrm{P_{H,T,t}}}\mathrm{C_{F,T,t}}$ around the steady state yields

$$\widehat{NX}_t = \xi\hat{S}_{\mathrm{t}} - \Theta\hat{e}_{\mathrm{t}} \tag{37}$$

where $\xi \equiv \sigma(1-\lambda)\left\{\eta-1+(1-\lambda)\left[\omega(1-\sigma)+\left(\frac{1}{\rho}\right)\sigma\right]+\lambda\eta-\left(\frac{1}{\rho}\right)\right\} > 0$ and $\Theta \equiv \sigma(1-\lambda)(1-\sigma)[\omega-\left(\frac{1}{\rho}\right)]$. A higher (lower) degree of foreign (home) openness or a depreciation of the terms of trade improve the trade balance while the impact of the relative home producer price depends on the relative parameter values. An economy without a trade imblance is extensively discussed in the literature by Obstfeld & Rogoff (1995), Benigno & Benigno (2003), Gali & Monacelli (2005) and De Paoli (2009) among others; they show that under the assumption of $\eta = \rho = 1$, the economy never experiences a trade imbalance. However, in the presence of non-tradable goods, the trade imbalance condition requires additional assumptions. Thus, if $\eta = \rho = \omega = 1$, the economy never experiences a trade imbalance. An increase in the price of imports (foreign tradable goods) proportionally leads to the substitution towards home tradable and non-tradable goods.

Log linearizing relative net exports, $\frac{\mathrm{C}^{\mathrm{f}}_{\mathrm{H,T,t}}}{\mathrm{C_{F,T,t}}}$ yields

$$\begin{aligned}\widehat{RNX}_t &= (\hat{C}^f_t - \hat{C}_{\mathrm{t}}) - \omega(1-\sigma)\hat{e}_{\mathrm{t}} + [\eta+\lambda\eta+\omega(1-\sigma)-\lambda\omega(1-\sigma)]\hat{S}_{\mathrm{t}} \\ &= (\hat{C}^f_t - \hat{C}_{\mathrm{t}}) - \omega(1-\sigma)\hat{e}_{\mathrm{t}} + \upsilon\hat{S}_{\mathrm{t}} \\ &= \delta\hat{S}_{\mathrm{t}} - \Lambda\hat{e}_{\mathrm{t}}\end{aligned} \tag{38}$$

where $\upsilon \equiv \eta+\lambda\eta+\omega(1-\sigma)-\lambda\omega(1-\sigma)$ and $\delta \equiv [\left(\frac{1}{\rho}\right)[\sigma(1-\lambda)-1]+\eta+\lambda\eta+\omega(1-\sigma)-\lambda\omega(1-\sigma)] > 0$ so that an increase in the terms of trade increases relative net exports.

3.6 Aggregate Demand

Since home tradable goods are consumed by home and foreign households, log linearizing demand for the goods around the steady state with balanced trade and the market clearing condition[6] yields

$$\hat{Y}_t = \sigma\lambda\hat{C}_{\mathrm{H,T,t}} + \sigma(1-\lambda)\hat{C}^{\mathrm{f}}_{\mathrm{H,T,t}} + (1-\sigma)\hat{C}_{\mathrm{H,N,t}} \tag{39}$$

We can find the links between domestic output and consumption by combining (33) and (39)

$$\begin{aligned}\hat{Y}_t &= \hat{C}_t + \sigma(1-\lambda)\widehat{RNX}_t \\ &= \hat{C}_t + \sigma(1-\lambda)\hat{C}^{\mathrm{f}}_{\mathrm{t}} + \sigma(1-\lambda)\eta\hat{S}_t - \sigma(1-\lambda)(\widehat{EWT}_t + \widehat{EWN}_t) \\ &= \hat{C}_t + \varpi\hat{Q}_t + \Xi\hat{S}_t\end{aligned} \tag{40}$$

where $\varpi \equiv [\sigma(1-\lambda)\left(\omega - \frac{1}{\rho}\right)]$ and $\Xi \equiv \{\sigma(1-\lambda)[\eta+\lambda(\eta-\omega)]\} > 0$. Thus, the discrepancy between demand for home goods and consumption of home households arises from exports and imports. Home output increases due to higher consumption and relative net exports.

Combining this with the risk sharing condition and foreign economy market clearing condition $\mathrm{Y}^{\mathrm{f}}_{\mathrm{t}} = \mathrm{C}^{\mathrm{f}}_{\mathrm{t}}$[7] yields aggregate demand

$$\begin{aligned}\hat{Y}_t &= \hat{C}^{\mathrm{f}}_{\mathrm{t}} + \sigma(1-\lambda)\widehat{RNX}_t + (\frac{1}{\rho})\hat{Q}_t \\ &= \hat{C}^{\mathrm{f}}_{\mathrm{t}}[1+\sigma(1-\lambda)] + \sigma(1-\lambda)\eta\hat{S}_t - \sigma(1-\lambda)(\widehat{EWT}_t + \widehat{EWN}_t) + (\frac{1}{\rho})\hat{Q}_t \\ &= \hat{C}^{\mathrm{f}}_{\mathrm{t}} + \Gamma\hat{Q}_t + \Xi\hat{S}_t \\ &= \hat{Y}^{\mathrm{f}}_{\mathrm{t}} + \mu\hat{S}_t - \tau\hat{e}_t\end{aligned} \tag{41}$$

where $\Gamma \equiv \varpi + \frac{1}{\rho} > 0$, $\mu \equiv \{\Gamma[1-\sigma(1-\lambda)] + \Xi\} > 0$ and
$\tau \equiv \Gamma(1-\sigma) > 0$. This shows a spillover effect from the foreign country since foreign consumption raises the demand for the domestic goods and so increases domestic output. Depreciation in the real exchange generates an income effect raising home consumption and in turn, raising home output. Also, relative net exports influence home output so that if exports are larger than imports, home production will increase

[6] We have three goods market clearing condition; market clearing condition for aggregate final goods, $Y_t = \mathrm{C}_{\mathrm{H,T,t}} + \mathrm{C}^{\mathrm{f}}_{\mathrm{H,T,t}} + \mathrm{C}_{\mathrm{H,N,t}}$, for final tradable goods, $Y_{H,T,t} = \mathrm{C}_{\mathrm{H,T,t}} + \mathrm{C}^{\mathrm{f}}_{\mathrm{H,T,t}}$ and for final non-tradable goods,$Y_{H,N,t} = \mathrm{C}_{\mathrm{H,N,t}}$. Since intermediate sectors also require labour, labour market clearing condition can be shown to be $L_t = L_{H,T,t} + L_{H,N,t} + L^i_{H,T,t} + L^i_{H,N,t}$

[7] Since the foreign country is assumed to be large, the home country is not able to influence foreign aggregate consumption, interest rate and price level, and tradable sector becomes negligible. Thus, the foreign country is analogous to the closed economy and equilibrium condition yields $Y^{\mathrm{f}}_{\mathrm{t}} = C^{\mathrm{f}}_{\mathrm{t}}$

and this influence is amplified with higher openness and weights on tradable consumption. Relative net exports depends on the terms of trade and the relative prices between home tradable and non-tradable goods. This implies that we observe relative home producer price effects along with terms of trade effects in the presence of non-tradable goods. In other words, the terms of trade and the relative home producer prices influence relative net exports and the real exchange rate thereby influencing aggregate output and generating an externality. In sum, aggregate output can be characterized by the spillover effects in consumption, income effects, the terms of trade effects and the expenditure switching effects between different varieties where the expenditure switching effects and income effects are influenced by relative home producer price effects.

3.7 The Real Value of Home Production

The real value of home production is given by

$$\mathrm{V}_t = \left(\frac{\mathrm{P_{H,T,t} Y_{T,t} + P_{H,N,t} Y_{H,N,t}}}{P_t} \right)$$

Combining aggregate demand and log linearizing around the steady state yields

$$\begin{aligned} \hat{V}_t &= \hat{C}_t + \widehat{NX}_t \\ &= \hat{C}^{\mathrm{f}}_{\mathrm{t}} + \widehat{NX}_t + (\frac{1}{\rho})\hat{Q}_t \\ &= \hat{C}^{\mathrm{f}}_{\mathrm{t}} + \Gamma \hat{Q}_t + \daleth \hat{S}_t \\ &= \hat{C}^{\mathrm{f}}_{\mathrm{t}} + \{\Gamma[1-\sigma(1-\lambda)] + \daleth\} \hat{S}_t - \Gamma(1-\sigma)\hat{e}_t \end{aligned} \tag{42}$$

where $\daleth \equiv \{\sigma(1-\lambda)[\eta+\lambda(\eta-\omega)-1]\} > 0$. If $\eta = \omega = \rho = 1$, we have $\hat{V}_t = \hat{Q}_{\mathrm{t}} + \hat{C}^{\mathrm{f}}_{\mathrm{t}}$ so that depreciation of the real exchange rate has a proportional effect on the real value of production and thereby generating a proportional effect on consumption without net export effects. Thus if we have PPP then stochastic shocks do not influence the real value of production. This implies that under complete markets, although positive real income effects do not influence on home consumption itself due to consumption insurance, real income effects are indirectly reflected through the risk sharing condition.

Under the assumption of $\eta > \omega$, which is consistent with empirical observations, an increase (decrease) of the terms of trade (relative non tradable prices) always has a positive effect on real value of home production under complete financial markets.

3.8 Optimal Monetary Policy

In this section, we show the dynamics of the small open economy in terms of the output gap, the relative price gap, the net exports gap and the New Keynesian Philips curve, and compare with the small open economy without non-tradable sectors and net exports. Also, we define net exports externalities in the optimal allocation and the optimal monetary policy following Benigno & Benigno (2003) and Gali & Monacelli (2005). We characterize the optimal allocation in the small open economy in terms of social planner. In order to generate tractability of derivation of the optimality monetary policy, we exclude intermediate input sectors, $\alpha^p \to 0$ and assume common value of the risk aversion, $\rho = 1$. Thus, in contrast to Gali & Monacelli (2005), we do not impose balanced trade for all t and maintain open economy features in a tractable way. Firstly, by combining (10), (13), (20), (30) and (40), log linearized marginal cost of tradable and non-tradable goods can be rewritten as[8]

$$\widetilde{MC}_{H,T,t} = \varphi\tilde{Y}_t + \tilde{Y}_t^f + \tilde{S}_t - \varphi\tilde{a}_t - \tilde{a}_{H,T,t} \tag{43}$$

$$\widetilde{MC}_{H,N,t} = \varphi\tilde{Y}_t + \tilde{Y}_t^f + \tilde{S}_t - \tilde{e}_t - \varphi\tilde{a}_t - \tilde{a}_{H,N,t} \tag{44}$$

Due to the analogous production function of tradable and non-tradable sectors, the economy wide real marginal cost, $\widetilde{MC}_t = \sigma\widetilde{MC}_{H,T,t} + (1-\sigma)\widetilde{MC}_{H,N,t}$ can be written as $\widetilde{MC}_t = \varphi\tilde{Y}_t + \tilde{Y}_t^f - (1+\varphi)\tilde{a}_t + \tilde{S}_t - (1-\sigma)\tilde{e}_t$. Analogously, the aggregate productivity shocks can be written as $\widetilde{a}_t = \sigma\widetilde{a}_{H,T,t} + (1-\sigma)\widetilde{a}_{H,N,t}$. Then, rearranging this equation, we obtain $\tilde{Y}_t = (1/\varphi)[\widetilde{MC}_t + (1+\varphi)\tilde{a}_t - \tilde{Y}_t^f - \tilde{S}_t + (1-\sigma)\tilde{e}_t]$. By imposing $\widetilde{MC}_t = -\wp$ which is optimal marginal cost in a flexible price economy, the natural level of output is

$$\begin{aligned}\overline{Y}_t &= (1/\varphi)[-\wp + (1+\varphi)\tilde{a}_t - \tilde{Y}_t^f - \overline{S}_t + (1-\sigma)\overline{e}_t] \\ &= -\hbar\wp + \hbar(1+\varphi)\tilde{a}_t + \hbar(\frac{1-\mu}{\mu})\tilde{Y}_t^f - \hbar[\frac{\tau - \mu(1-\sigma)}{\mu}]\overline{e}_t\end{aligned} \tag{45}$$

where $\hbar \equiv \mu/(1+\varphi\mu)$. By defining the gap of respective variables as $Y_t^g \equiv \tilde{Y}_t - \overline{Y}_t$, the relation between real marginal cost, the output gap, the relative price gap $[S_t^g - (1-\sigma)e_t^g]$ and the net exports gap can be shown to be

$$\begin{aligned}\widehat{MC}_t &= \varphi Y_t^g + [S_t^g - (1-\sigma)e_t^g] \\ &= (1+\varphi)Y_t^g - NX_t^g\end{aligned} \tag{46}$$

An identity of the relative producer price gap by relating the changes in domestic tradable inflation,

[8] We show the dynamics of main variables in terms of the logs of variables rather than the log deviation around the steady state in this section in order to represent the economy in terms of the output gap.

non-tradable inflation and the natural relative producer price is

$$e_t^g \equiv e_{t-1}^g - \pi_{\mathrm{H,T,t}} + \pi_{\mathrm{H,N,t}} - \Delta\bar{e}_t \tag{47}$$

We can rewrite the New Keynesian Philips curve in terms of CPI inflation, the output gap, the net exports gap and the terms of trade by combining (32) and (46)

$$\pi_{\mathrm{t}} = \beta \mathrm{E_t}\{\pi_{\mathrm{t+1}}\} + \nu\{(1+\varphi)Y_t^g - NX_t^g]\} - \sigma(1-\lambda)(\beta\widetilde{\Delta S}_{\mathrm{t+1}} - \widetilde{\Delta S}_{\mathrm{t}}) \tag{48}$$

Also, the New Keynesian Philips curve for domestic tradable and non-tradable sectors can be written as

$$\pi_{\mathrm{H,T,t}} = \beta E_t(\pi_{\mathrm{H,T,t+1}}) + \nu[\varphi Y_t^g + S_t^g];\;\; \pi_{\mathrm{H,N,t}} = \beta E_t(\pi_{\mathrm{H,N,t+1}}) + \nu[\varphi Y_t^g + S_t^g - e_t^g] \tag{49}$$

Combining (11), (32), (40) and (45) yields the so-called dynamic IS equation for the small open economy with tradable and non-tradable goods in terms of the output gap and net exports[9].

$$Y_t^g = E_t\{Y_{t+1}^g\} - \widetilde{\Delta NX}_{\mathrm{t+1}} - [\tilde{i}_t - \sigma E_t\{\pi_{H,T,t+1}\} - (1-\sigma)E_t\{\pi_{H,N,t+1}\} - \overline{R}_t] \tag{50}$$

where $\overline{R}_t \equiv \hbar[\frac{\tau-\mu(1+\sigma\varphi)}{\mu}]\tilde{a}_{H,T,t}(1-\rho_{aht}) - \hbar[\frac{\tau+\mu\varphi(1-\sigma)}{\mu}]\tilde{a}_{H,N,t}(1-\rho_{ahn}) + \hbar(\frac{1-\mu}{\mu})\widetilde{\Delta Y}^f$ is the natural real interest rate in terms of domestic goods (tradable and non-tradable goods).

Social planner maximizes utility of households subject to the risk sharing condition, production function and the goods market clearing condition. The optimal allocation implies constant employment

$$L = \Omega^{1/1+\varphi} \tag{51}$$

where $\Omega \equiv [1-\sigma(1-\lambda)]/\{(1+\varpi)[1-\sigma(1-\lambda)]+\Xi\}$. In order to identify net exports externalities coupled with the monopolistic distortions, we add an employment subsidy or tax for the tradable, $\Im_{T,t}$ and non-tradable sectors, $\Im_{N,t}$. Then, the optimal decision of tradable and non-tradable firms in a flexible price equilibrium can be shown to be

$$(\frac{\varepsilon-1}{\varepsilon}) = (1-\Im_{T,t})\Omega S_t^{1-\mu} e_t^{\tau}(a_t/a_{H,T,t});\;\; (\frac{\varepsilon-1}{\varepsilon}) = (1-\Im_{N,t})\Omega S_t^{1-\mu} e_t^{\tau-1}(a_t/a_{H,N,t}) \tag{52}$$

[9]In order to derive the IS equation, we use definitions of the natural level of relative prices and net exports, $\overline{e}_t = \tilde{a}_{H,T,t} - \tilde{a}_{H,N,t}$, $\overline{S}_t = (\frac{1}{\mu})(\overline{Y}_t - \tilde{Y}_t^f + \tau\overline{e}_t)$ and $\widetilde{NX}_{\mathrm{t}} = \sigma(1-\lambda)(\widetilde{RNX}_{\mathrm{t}} - \tilde{S}_{\mathrm{t}})$.

We obtain an employment subsidy for tradable and non-tradable sectors by rearranging these equations

$$\Im_{T,t} = 1-(\frac{\varepsilon-1}{\varepsilon})\frac{S_t^{\mu-1}e_t^{-\tau}}{\Omega}(a_{H,T,t}/a_t);\ \ \Im_{N,t} = 1-(\frac{\varepsilon-1}{\varepsilon})\frac{S_t^{\mu-1}e_t^{1-\tau}}{\Omega}(a_{H,N,t}/a_t) \tag{53}$$

Thus, the optimal flexible price allocation can be obtained by imposing a subsidy, which ensures optimal marginal cost in a flexible price economy, in a way to eliminate both terms of trade and relative home producer price effects. Since optimal subsidies lead to constant relative producer price for all t and $NX_t = S_t^{\mu-1}$, the subsidies effectively eliminate net exports externalities along with the monopolistic distortions.

In the open economy with only tradable goods where $\sigma \to 1$, $a_{H,T,t} = a_t$ and balanced trade for all t (i.e., $\rho = \eta = \omega = 1$), the optimal allocation leads to constant employment $L = \lambda^{1/1+\varphi}$ and employment subsidy $(\frac{\varepsilon-1}{\varepsilon}) = (1-\Im)\lambda$ so that for a given subsidy in place, strict domestic tradable inflation targeting is optimal as implied by (27), (41), (46) and $\pi_{\mathrm{H,T,t}} = \beta E_t(\pi_{\mathrm{H,T,t+1}}) + \nu(1+\varphi)Y_t^g$. and the small open economy needs to stabilize domestic tradable inflation as Clarida et al. (2001), Gali & Monacelli (2005), Faia & Monacelli (2007) and De Paoli (2009) among others point out.

However, when we allow non-tradable sectors and net exports, we also need to stabilize non-tradable inflation by eliminating net exports externalities and this in turn, leads to $Y_t^g = S_t^g = e_t^g = 0$ as implied by (27), (41) and (49). Thus, there is no trade-off between output gap, terms of trade gap and relative producer price gap stabilization, and we achieve "divine coincidence" as emphasized by Gali & Monacelli (2005) and Corsetti et al. (2010). The optimal policy implies

$$Y_t^g = S_t^g = e_t^g = \pi_{\mathrm{H,T,t}} = \pi_{\mathrm{H,N,t}} = 0 \tag{54}$$

and

$$\pi_{\mathrm{t}} = (\frac{1}{\mu})(\Delta\overline{Y}_t - \Delta Y_t^f + \tau\Delta\overline{e}_t) \tag{55}$$

CPI inflation fluctuates according to the growth rate of the natural relative producer price and the difference between the growth rate of the natural output and foreign output[10].

Following Rotemberg & Woodford (1999) and Galí (2009), we can derive a second order approximation to the welfare losses of households. The welfare loss function as a fraction of steady state consumption is

[10]Under the specific values of $\rho = \eta = \omega = 1$, we have trade balance for all t and CPI inflation can be written as $[\frac{\sigma(1-\lambda)}{1-\sigma(1-\lambda)}]\Delta\overline{Q}_t$ so that the relative price distortions which distort purchasing power of domestic households through changes in the growth rate of the natural real exchange rate without improvement of trade balance need to be removed

shown to be

$$WLF_t = -\sum_{t=0}^{\infty} \beta^t (\frac{\Omega}{2}) \{ \frac{\varepsilon}{\nu} [\sigma\pi_{H,T,t}^2 + (1-\sigma)\pi_{H,N,t}^2] + (1+\varphi) Y_t^{g\,2} \} \tag{56}$$

Additional source of welfare losses arises from non-tradable sectors in terms of composite domestic inflation. Notice that the presence of non-tradable sectors generates a trade-off between stabilizing tradable inflation, non-tradable inflation and the output gap without the fiscal instruments. Thus, the optimal policy inevitably needs to deviate from stabilizing the three variables.

Without optimal subsidies in place, the central bank which can resort to commitment will minimizes (56) subject to (41), (47), (49), and (50). Then, optimality conditions[11] imply

$$\sigma\pi_{H,T,t} + (1-\sigma)\,\pi_{H,N,t} = -\frac{\mu(1+\varphi)}{\varepsilon(\varphi\mu+1)} \Delta Y_t^g \tag{57}$$

Combining (49) and (57), domestic price yields

$$\hat{P}_{H,t} = \kappa \hat{P}_{H,t-1} + \kappa\beta \hat{P}_{H,t-1} + \kappa\nu [S_t^g - (1-\sigma) e_t^g] \tag{58}$$

for all t where $\hat{P}_{H,t} = [\sigma\hat{P}_{H,T,t} + (1-\sigma)\hat{P}_{H,N,t}]$ and $\kappa \equiv [\frac{\mu(1+\varphi)}{\mu(1+\varphi)(1+\beta)+\varepsilon\nu\varphi(1+\mu\varphi)}]$. Thus, the stationary solution yields

$$\begin{aligned} \hat{P}_{H,t} - \varsigma \hat{P}_{H,t-1} &= \frac{\varsigma\nu}{1-\varsigma\beta} [S_t^g - (1-\sigma) e_t^g] \\ &= \frac{\varsigma\nu}{1-\varsigma\beta} [Y_t^g - NX_t^g] \end{aligned} \tag{59}$$

for all t where $\varsigma \equiv \frac{1-\sqrt{1-4\beta\kappa^2}}{2\beta\kappa}$.

Rather than stabilizing tradable and non-tradable inflation, the central bank now faces a trade-off between stabilizing composite domestic inflation and the output gap due to net exports externalities.

In the absence of net exports externalities (i.e., $\rho = \eta = \omega = 1$), the output gap is equal to the relative price gap, $Y_t^g = [S_t^g - (1-\sigma)e_t^g]$ and composite domestic inflation can be written as $\pi_{H,t} = \beta E_t(\pi_{H,t+1}) + \nu(1+\varphi)Y_t^g$. Thus, stabilizing composite domestic inflation for all t is the optimal policy by closing the output gap and satisfying the stationary solution.

$$Y_t^g = [S_t^g - (1-\sigma)e_t^g] = \pi_{H,t} = 0 \tag{60}$$

[11]The optimality conditions are $\nu(\frac{\varphi\mu+1}{\mu})(\lambda_t^1 + \lambda_t^2) = -\Omega(1+\varphi)Y_t^g$ and $\frac{\Omega\varepsilon}{\nu}(\sigma\pi_{H,T,t} + (1-\sigma)\pi_{H,N,t}) = \Delta\lambda_t^1 + \Delta\lambda_t^2$ where λ_t^1 and λ_t^2 are the sequence of Lagrange multipliers.

While the optimal policy implies $\tilde{i}_t = \overline{R}_t$, it leads to an indeterminacy problem as shown by Blanchard & Kahn (1980) and Bullard & Mitra (2002). Thus, the central bank can implement the optimal policy by committing to a rule

$$\tilde{i}_t = \overline{R}_t + \rho_\pi \pi_{\mathrm{H,t}} + \rho_y Y_t^g \tag{61}$$

where $\nu(1+\varphi)(\rho_\pi - 1) + (1-\beta)\rho_y > 0$. Since $\pi_{\mathrm{H,t}} = Y_t^g = 0$ in equilibrium, $\rho_\pi \pi_{\mathrm{H,t}} + \rho_y Y_t^g$ will disappear and this in turn leads to $\tilde{i}_t = \overline{R}_t$ for all t.

Thus, in contrast to the close economy[12] and the open economy without net exports, the central bank additionally needs to eliminate net exports externalities arising from imperfect substitutability between tradable goods, non-tradable goods and imports. However, since composite domestic inflation is a major determinant of net exports as implied by (46), (49) and (50), strict composite domestic inflation targeting is a good approximation to the optimal policy.

4 A Numerical Analysis

4.1 Calibration

Since the literature has assumed two large economies and correspondingly, calibrated for large economies, we use small open economy (Korean) data in order to calibrate our model. We use the share of consumption of non-tradable goods in GDP, 0.42, the share of consumption of imports in GDP, 0.1, the share of intermediate inputs in GDP, 0.5, the share of distribution servies in GDP, 0.13 and the share of intermediate imports in GDP, 0.2 in South Korea during 1990-2016 in order to set the weights on consumption of tradable goods $\sigma = 0.58$, on consumption of imports $1-\lambda = 0.17$, on demand for tradable intermediate inputs $\sigma^i = 0.74$ and on demand for intermediate imports $1-\lambda^i = 0.54$[13]. We follow the standard assumption that final tradable firms demand fixed proportions of tradable and non-tradable intermediate inputs, $\omega^i = 0.001$ (see, for example, Burstein et al. (2003), Corsetti et al. (2008) and Dotsey & Duarte (2008)). The elasticity of substitution between home and foreign intermediate input is set $\eta^i = 3.14$ which is es-

[12] In the close economy with two sectors, market clearing condition, the output gap and the New Keynesian Philips curve can be shown as $\tilde{Y}_t = \tilde{C}_t$, $Y_t^g = -(1-\sigma)e_t^g$ and $\pi_\mathrm{t} = \beta \mathrm{E_t}\{\pi_{\mathrm{t+1}}\} + \nu(1+\varphi)Y_t^g$. There is no trade-off between stablizing composite inflation, the output gap and the relative price gap. Thus, strict composite inflation targeting is the optimal policy as emphasized by Aoki (2001).

[13] Firstly, steady state values of the share of each consumption in GDP pin down the values of α and σ . Then, steady state values of the share of intermediate inputs, the share of distribution servies and the share of intermediate imports in GDP subsequently pin down the values of α^p, λ^i and σ^i

Table 1: Parameters

Households		
Discount rate	β	0.99
Risk aversion	ρ	1
Relative weight on tradable goods in the final consumption (unless specified otherwise)	σ	0.58
Inverse elasticity of labour supply (unless specified otherwise)	φ	3
Degree of trade openness (unless specified otherwise)	α	0.17
Elast. of substitution $C_{H,T}$ and $C_{F,T}$ (unless specified otherwise)	η	1.5
Elast. of substitution C_T and $C_{H,N}$ (unless specified otherwise)	ω	0.74
Elast. of substitution individual varieties (unless specified otherwise)	ε	6
Firm		
Elast. of output with respect to intermediate inputs (unless specified otherwise)	α^p	0.86
Degree of price stickiness (unless specified otherwise)	θ	0.75
Elast. of substitution $YC^i_{H,T}$ and $YC^i_{F,T}$	η^i	3.14
Elast. of substitution YC^i_T and $YC^i_{H,N}$	ω^i	0.001
Relative weight on intermediate non-tradable inputs	$1-\sigma^i$	0.26
Relative weight on home intermediate tradable input	λ^i	0.46
Share of intermediate input in GDP	Y^i/Y	0.5
Share of intermediate imports in GDP	$\mathrm{YC}^i_{F,T}/Y$	0.2
Share of distribution services in GDP	$\mathrm{YC}^i_{H,N}/Y$	0.13
Monetary policy		
Inflation coefficient of the Taylor rule	ρ_π	1.5
Degree of interest rate smoothing	ρ_i	0.8

timated by Kasahara & Rodrigue (2008). β is set equal to 0.99 and thus the steady state Euler equation $\beta = 1/R$ implies a riskless steady state annual return of approximately 4%. Following Mendoza (1995) and Benigno & Thoenissen (2008), the elasticity of substitution between tradable and non-tradable goods is set as $\omega = 0.74$. The elasticities of substitution between home tradable and foreign tradable goods and between same category are set $\eta = 1.5$ and $\varepsilon = 6$ respectively. The calibration assumes common values of the risk aversion, $\rho = 1$ and the inverse (Frisch) labour supply elasticity $\varphi = 3$. The Calvo probability of not being able to adjust price is set equal to 0.75 implying an average of four periods between price adjustment. The elasticity of output with respect to intermediate inputs is set to be $\alpha^p = 0.86$[14] in order to pin down the share of intermediate input in GDP.

We estimate the stochastic properties of the exogenous driving processes, using US real GDP (the proxy for foreign output), Korean tradable and non-tradable sector labour productivity (the proxies for tradable and

[14] The economy without intermediate input sectors, $\alpha^p \to 0$ and correspondingly, $\alpha \to 0.5$ in order to ensure that the share of imports in GDP is 0.3

non-tradable productivity) log de-trended data during 2001q1-2016q4 in order to calibrate the properties. The estimates are given by

$$\tilde{a}_{H,T,t} = \underset{(0.07)}{0.61}\tilde{a}_{H,T,t-1} + \varepsilon_{H,T,t}, \sigma_{H,T} = 0.0242$$

$$\tilde{a}_{H,N,t} = \underset{(0.08)}{0.52}\tilde{a}_{H,N,t-1} + \varepsilon_{H,N,t}, \sigma_{H,N} = 0.0169$$

$$\tilde{Y}^f_t = \underset{(0.04)}{0.89}\tilde{Y}^f_{t-1} + \varepsilon^f_t, \sigma^f = 0.0054$$

with $corr(\sigma_{H,T}, \sigma_{H,N}) = 0.53$, $corr(\sigma_{H,T}, \sigma^f) = 0.48$ and $corr(\sigma_{H,N}, \sigma^f) = 0.23$.

4.2 Evaluation of Monetary Rules

This section evaluates monetary policy rules by excluding intermediate input sectors, $\alpha^p \to 0$. While the central bank needs to set an interest rate which achieves the stationary solution, it is nearly implausible to conduct the optimal policy in practice since it is difficult to monitor the natural levels. Thus, we evaluate alternative monetary policy rules associated with welfare. We have four Taylor-type rules responding to composite domestic inflation and CPI inflation, and a rule for the fixed exchange rate regime. Specifically, the composite domestic inflation-based Taylor rule (DTR) and the CPI-based Taylor rule (CTR) follow

$$\tilde{i}_{\mathrm{t}} = \rho_\pi \pi_{\mathrm{H,t}}; \;\; \tilde{i}_{\mathrm{t}} = \rho_\pi \pi_{\mathrm{t}} \tag{62}$$

CPI inflation can be rewritten as $\pi_{\mathrm{t}} = \pi_{\mathrm{H,t}} + \sigma(1-\lambda)\Delta\tilde{S}_{\mathrm{t}}$ so that under CTR, the central bank additionally responds to changes in the terms of trade. As values of $(1-\lambda)$ and σ raise, it responses more to the changes.

The augmented composite domestic inflation-based Taylor rule (ADT) and the augmented CPI-based Taylor rule (ACT) allow partial adjustment

$$\tilde{i}_{\mathrm{t}} = \rho_i \tilde{i}_{\mathrm{t-1}} + (1-\rho_i)\rho_\pi \pi_{\mathrm{H,t}}; \;\; \tilde{i}_{\mathrm{t}} = \rho_i \tilde{i}_{\mathrm{t-1}} + (1-\rho_i)\rho_\pi \pi_{\mathrm{t}} \tag{63}$$

The exchange rate peg (PEG) follows

$$\tilde{X}_t = 0 \tag{64}$$

In this welfare analysis, the source of fluctuations are tradable productivity, non-tradable productivity, foreign output and composite shocks[15]. In addition to monetary rules, we also report the welfare losses of

[15]While our interests are mainly focused on tradable and non-tradable shocks, we include foreign output and composite shocks in order to compare with a welfare analysis presented by Gali & Monacelli (2005) and consider correlated shocks

Table 2: Evalaution of Alternative Monetary Policy Rules

	TS Productivity Shocks						NTS Productivity Shocks					
	OP	DTR	CTR	ADT	ACT	PEG	OP	DTR	CTR	ADT	ACT	PEG
σ ($\pi_{H,T}$)	0.13	0.57	0.51	0.5	0.52	0.54	0.08	0.17	0.12	0.11	0.11	0.13
σ ($\pi_{H,N}$)	0.2	0.32	0.22	0.19	0.21	0.24	0.12	0.33	0.29	0.3	0.3	0.32
σ (Y^g)	0.04	0.63	0.83	0.94	1.02	1.12	0.03	0.28	0.32	0.43	0.44	0.51
σ (π_H)	0.01	0.46	0.38	0.37	0.39	0.41	0.01	0.23	0.19	0.19	0.18	0.21
σ (π)	0.45	0.4	0.23	0.19	0.21	0.25	0.18	0.21	0.15	0.14	0.13	0.17
σ (RP^g)	0.21	0.29	0.45	0.54	0.61	0.69	0.15	0.39	0.43	0.52	0.53	0.59
WL	0.0053	0.0524	0.0432	0.0429	0.0475	0.054	0.0019	0.0132	0.0101	0.0111	0.0111	0.0136
	Foreign Output Shocks						Composite Shocks					
	OP	DTR	CTR	ADT	ACT	PEG	OP	DTR	CTR	ADT	ACT	PEG
σ ($\pi_{H,T}$)	0	0.03	0.1	0.02	0.05	0.22	0.11	0.68	0.59	0.57	0.58	0.56
σ ($\pi_{H,N}$)	0	0.03	0.1	0.02	0.05	0.22	0.17	0.56	0.46	0.44	0.44	0.47
σ (Y^g)	0	0.01	0.14	0.03	0.04	0.39	0.04	0.82	0.98	1.23	1.29	1.33
σ (π_H)	0	0.03	0.1	0.02	0.05	0.22	0.01	0.62	0.52	0.5	0.51	0.5
σ (π)	0.14	0.15	0.12	0.17	0.13	0.15	0.52	0.57	0.4	0.38	0.37	0.34
σ (RP^g)	0	0.01	0.12	0.03	0.03	0.32	0.18	0.6	0.73	0.94	0.98	1.04
WL	0	0.0001	0.0022	0.0001	0.0005	0.0116	0.004	0.089	0.069	0.072	0.075	0.076

the optimal policy (OP) in order to provide a useful benchmark.

Table 2 reports the standard deviations of main variables and the average period welfare losses in response to tradable productivity, non-tradable productivity, foreign output and composite shocks for the optimal policy and alternative monetary policy rules. Surprising result is that ironically, since DTR moderately stabilize composite domestic inflation under the conventional inflation coefficient, $\rho_\pi = 1.5$, it allows more volatile tradable, non-tradable and composite domestic inflation[16], thereby generating the highest welfare losses except for PEG by tradable productivity, non-tradable productivity and composite shocks. On the other hand, ADT leads to the lowest welfare losses in response to tradable productivity shocks. The partial adjustment of the interest rates of ADT reduces volatilities of tradable, non-tradable and composite domestic inflation. For non-tradable productivity shocks, CTR outperforms other policy rules. As implied by (48) and (50), CTR stabilizes the relative price gap which in turn reduces a volatility of the output gap, and composite domestic inflation. While CTR and ADT symmetrically stabilize composite domestic inflation, CTR further reduces a volatility of the relative price gap and obtains lower welfare losses.

Turning to foreign output shocks. DTR and ADT stablize the output gap and composite domestic infla-

[16]This feature is also shown in comparing between CTR and ACT in terms of CPI inflation volatility. Thus, the moderate stabilization of target inflation leads to a higher volatility than partial adjustment of the interest rates towards the target inflation.

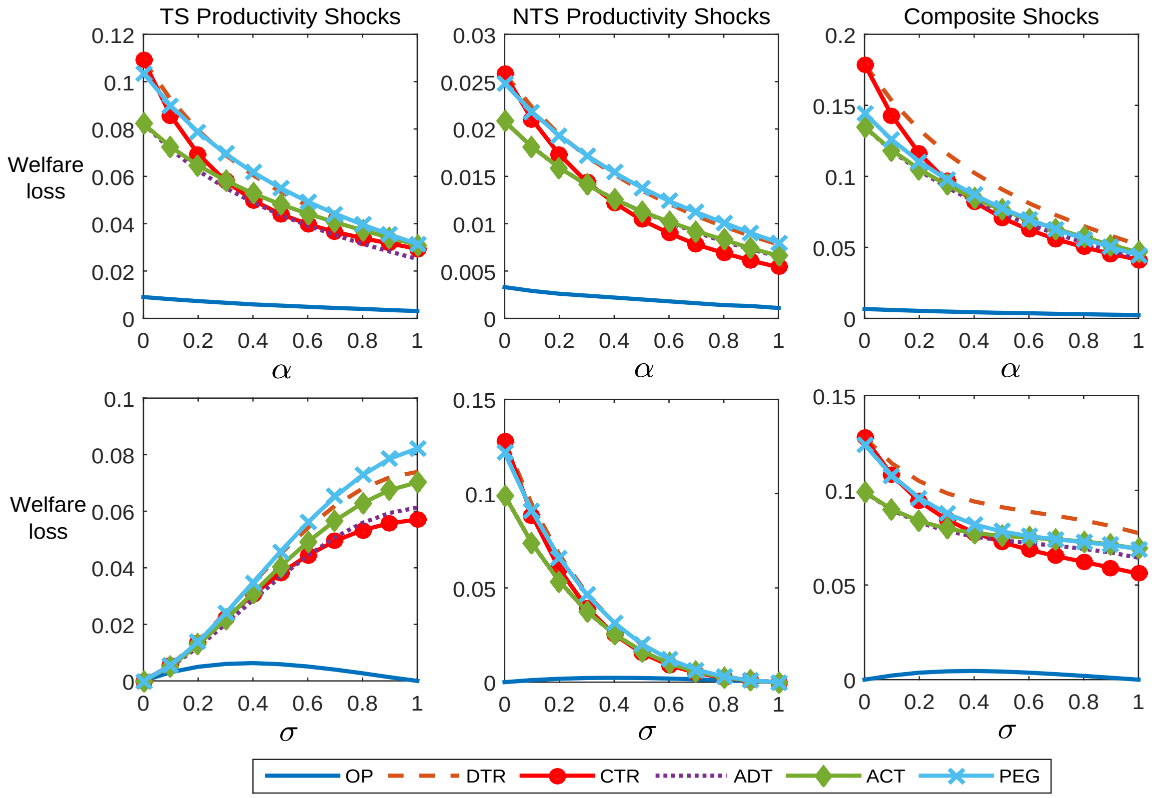

Figure 2: Welfare Losses according to α and σ

NOTE: TS and NTS refer to tradable sectors and non-tradable sectors, respectively.

tion, and thereby generate the lowest welfare losses. This implies that when the economy is influenced by composite shocks which are dominantly determined by foreign output shocks and there is no non-tradable sectors and trade-off between stablizing composite domestic inflation and the realtive price gap, DTR and ADT can generate lower welfare losses than CTR and PEG as emphasized by Gali & Monacelli (2005). Notice that PEG partially stabilizes tradable and non-tradable inflation. Constant exchange rate and exogenous foreign inflation imply that the terms of trade conversely fluctuates with tradable inflation and thus helps to stabilize composite domestic inflation, outperforming DTR in response to composite shocks. Since tradable productivity shocks lead to nearly identical welfare losses for CTR and ADT, and foreign output shocks do not play a significant role for welfare in our model calibration, CTR generates the lowest welfare losses by composite shocks.

Figure 2 illustrates welfare losses of monetary rules according to the degree of openness and weights on tradable goods. For each rule, total welfare losses are presented in percent terms. Firstly, notice that

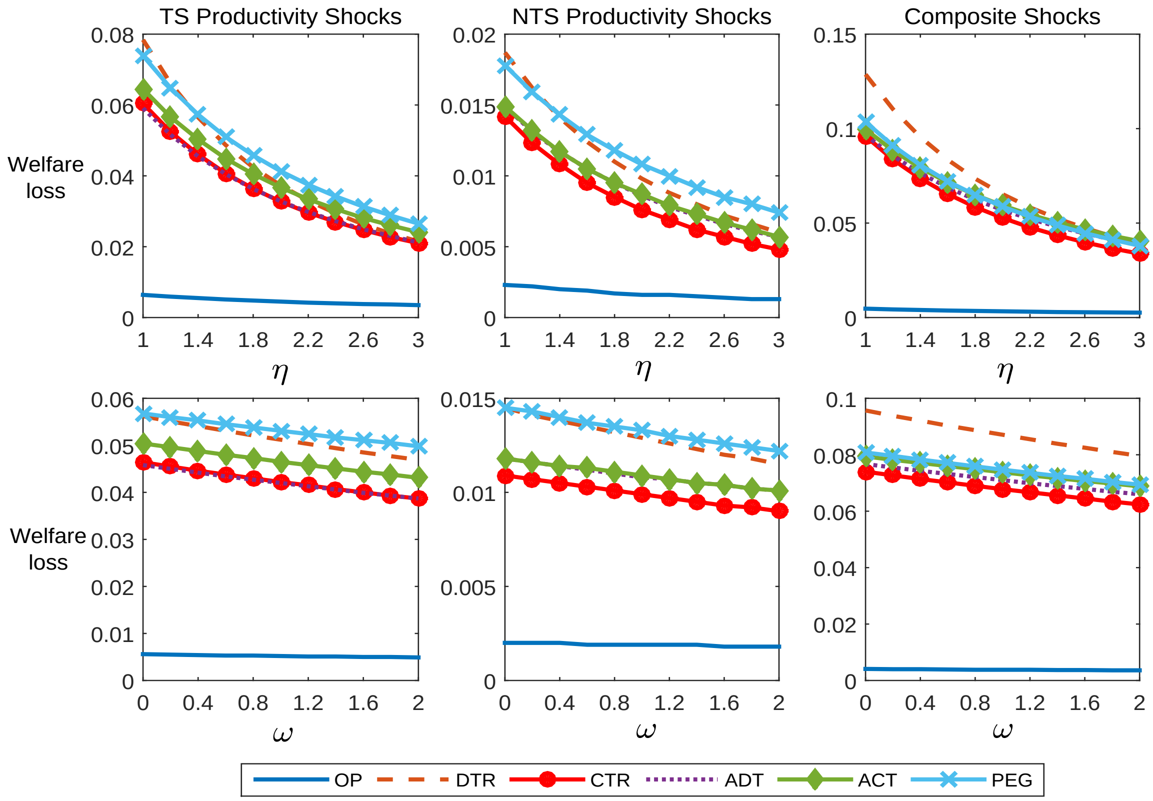

Figure 3: Welfare Losses according to η and ω

NOTE: TS and NTS refer to tradable sectors and non-tradable sectors, respectively.

regardless of the driving forces and the monetary policy regimes, an increase in the degree of openness monotonically reduces welfare losses. The higher openness implies the smaller domestic tradable sectors for given non-tradable sectors and this in turn, reduces a volatility of tradable inflation. In parcitualr, for the values of higher α, the central bank increases responses to fluctuations of the terms of trade and this in turn, reduces a volatility of composite domestic inflation and thus coupled with an increase in net exports externalities for higher levels of openness, welfare losses of CTR exponentially decrease. For tradable shocks, ADT generates the lowest welfare losses across all parameter values, while welfare losses of CTA and ADT show similar magnitude at around $\alpha = 0.5$ which is our benchmark case. In response to non-tradable shocks, CTA leads to lower welfare losses than ADT for sufficiently high values of α (i.e.,$\alpha > 0.4$) . Notice that as $\alpha \rightarrow 0$, the economy converges to the close economy with two sectors so that DTR (ADT) equals to CTR (ACT). Thus, in the close economy with two sectors, the agumented composite inflation-based Taylor rule leads to the lowest welfare losses. As for the degree of weights on tradable goods, an increase in the weights on tradable (non-tradable) goods monotonically raises the welfare gap between alternative policy rules due to a limited capability to control tradable (non-tradable) inflation of PEG and an increase (decrease) in responses towards fluctuations of the terms of trade, along with an increase in net exports externalities in response to tradable (non-tradable) shocks.

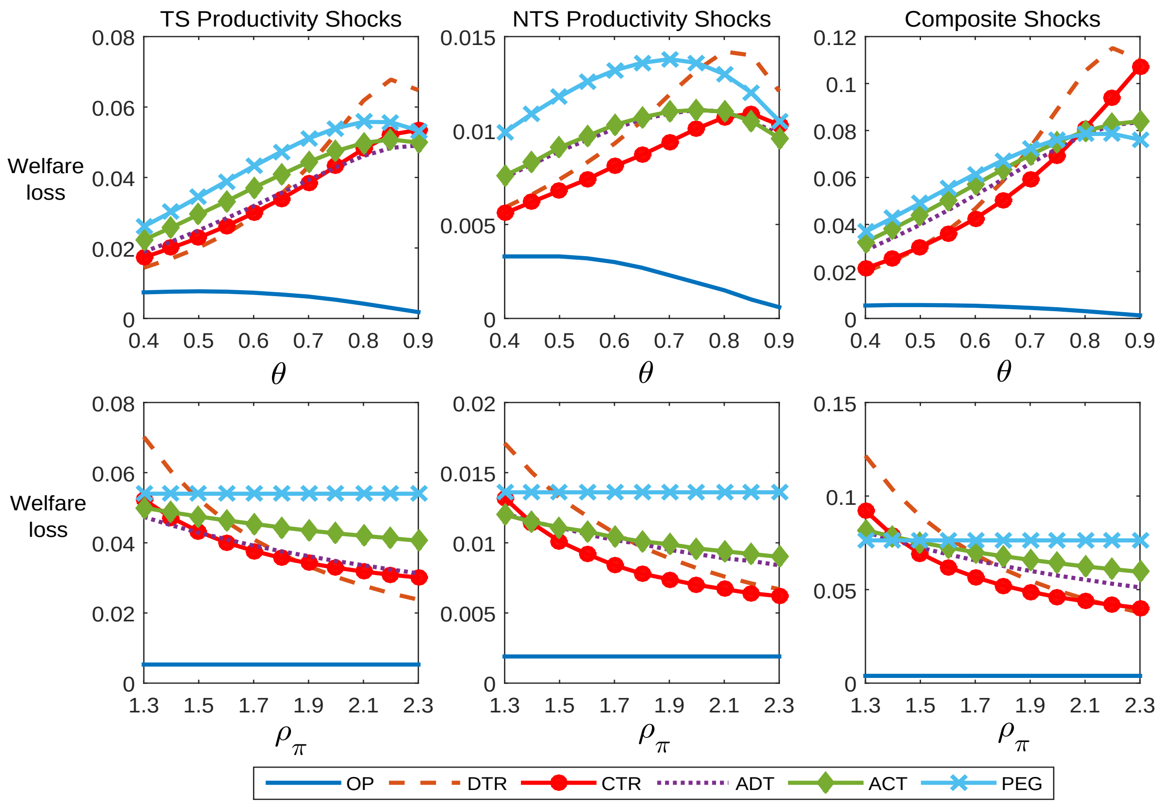

Figure 4: Welfare Losses according to θ and ρ_π

NOTE: TS and NTS refer to tradable sectors and non-tradable sectors, respectively.

Figure 3 represents welfare losses as a function of the elasticities of substitution between home tradable and foreign tradable goods and between same category. Higher values of η and ω monotonically increase the welfare gap between alternative policy rules except for DTR. The higher values imply a higher volatility of the relative price gap and greater net exports externalities so that CTA increasingly dominates other policy rules as the elasticities increase.

Figure 4 displays welfare losses in the plausible degree of price stickiness and the inflation coefficient of the Taylor rule. For low price stickiness, DTR obtains the lowest welfare losses in response to tradable productivity and composite shocks. However, when $\theta > 0.75$, DTR leads to the highest welfare losses by

Table 3: Welfare Losses according to ε and φ

	TS Productivity Shocks						NTS Productivity Shocks					
	OP	DTR	CTR	ADT	ACT	PEG	OP	DTR	CTR	ADT	ACT	PEG
WL ($\varepsilon = 6, \varphi = 2$)	0.005	0.046	0.037	0.036	0.040	0.044	0.002	0.011	0.009	0.009	0.009	0.011
WL ($\varepsilon = 6, \varphi = 3$)	0.005	0.052	0.043	0.043	0.048	0.054	0.002	0.013	0.010	0.011	0.011	0.014
WL ($\varepsilon = 6, \varphi = 4$)	0.005	0.057	0.048	0.049	0.055	0.063	0.002	0.015	0.011	0.013	0.013	0.016
WL ($\varepsilon = 11, \varphi = 2$)	0.010	0.080	0.062	0.058	0.063	0.070	0.004	0.020	0.015	0.015	0.015	0.018
WL ($\varepsilon = 11, \varphi = 3$)	0.010	0.092	0.073	0.070	0.077	0.087	0.004	0.024	0.018	0.019	0.019	0.022
WL ($\varepsilon = 11, \varphi = 4$)	0.010	0.101	0.082	0.081	0.090	0.103	0.004	0.026	0.020	0.022	0.022	0.027

tradable and non-tradable productivity shocks, reaching a pick at around $\theta = 0.85$ and showing a substantial welfare difference. Notice that for the most plausible degree of price stickiness (i.e., $0.7 \leq \theta \leq 0.8$), CTA and ADT still outperform other policy rules, independently of the exogenous driving forces. Turning to ρ_π, since strict composite domestic inflation targeting is a good approximation to the optimal policy, DTR outperforms other alternative policy rules when the central bank aggressively sets the coefficient regardless of types of shocks given. Thus, unless it responds sufficiently aggressive to composite domestic inflation, our results are invariant with respect to the parameter values given.

Table 3 evaluates the robustness of our results with alternative values of the elasticity of substitution between goods in the same category, ε and the inverse (Frisch) labour supply elasticity, φ. We find that higher values of ε and φ monotonically increase welfare losses across all policy regimes.

4.3 Impulse Responses

In this section, we show the dynamics of main variables under CTR, ADT and PEG, and compare with the optimal policy as a benchmark case. Since calibrated parameter $\alpha_p = 0.86$ leads to almost indistinguishable dynamics of main macroeconomic variables and volatility from the economy without intermediate sectors, we include the intermediate sectors for impulse responses.

Firstly, we analyze the effects of positive tradable sector productivity shocks. The impulse responses to the shock are depicted in Figure 5. Since the shocks reduce tradable inflation, an increase in non-tradable inflation is required in order to stabilize composite domestic inflation under the optimal policy. For a given constant foreign consumption, an increase in consumption leads to depreciation of the terms of trade and the real exchange rate. This in turn, raise net exports and output in the second round. A fall in the nominal interest rate is required to support a rise in output and consumption. While employment falls in tradable

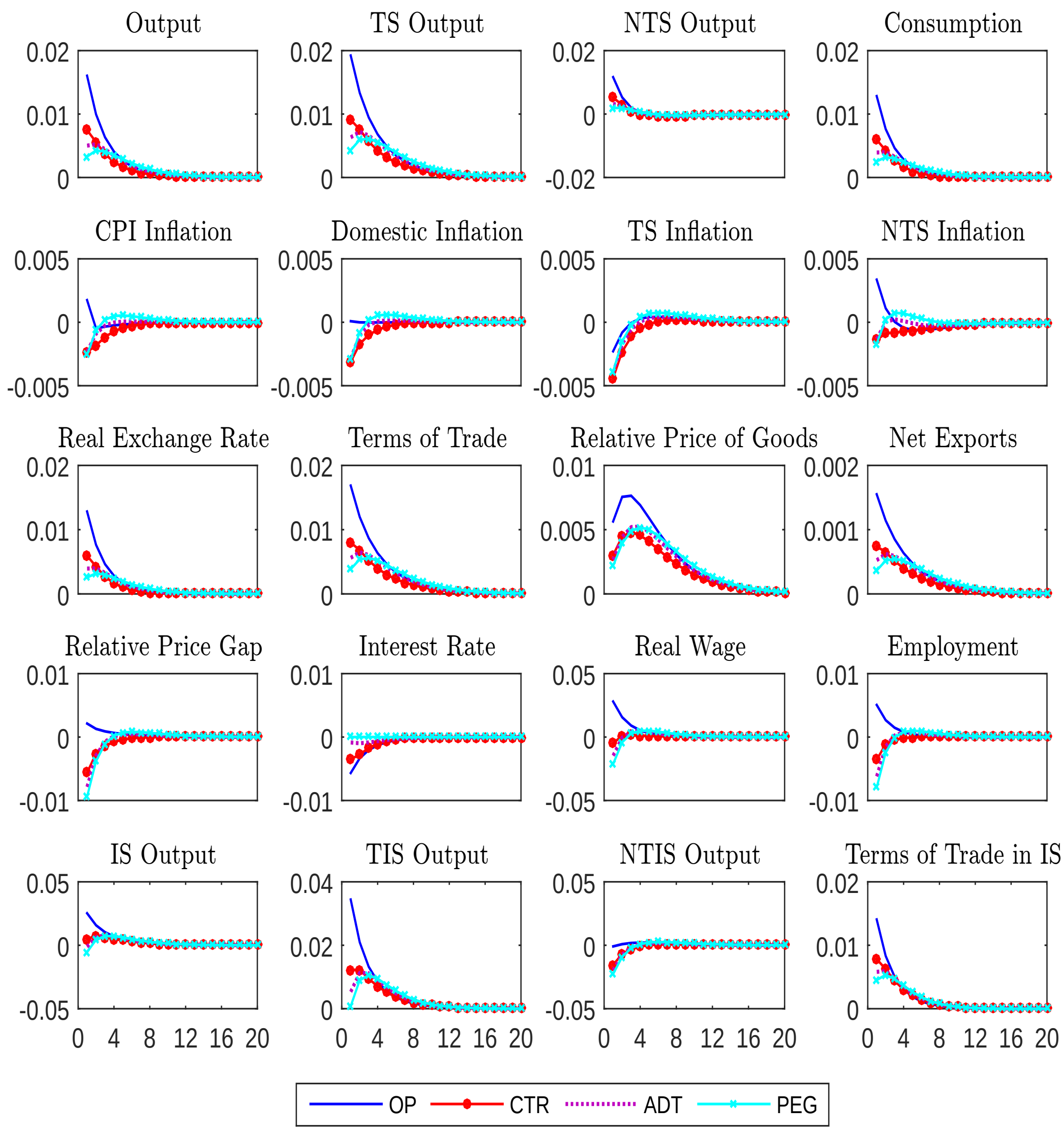

Figure 5: Impulse Responses to Tradable Productivity Shocks

NOTE: TS, NTS, IS, TIS and NTIS refer to tradable sectors, non-tradable sectors, intermediate sectors, tradable intermediate sectors and non-tradable intermediate sectors, respectively. Variables are presented in percentage deviations from the steady state

sectors and increases in non-tradable sectors, an increase in aggregate employment across final and intermediate sectors leads to a rise in real wage. An output expansion of tradable sectors requires more tradable and non-tradable intermediate inputs and coupled with a depreciation of terms of trade and higher productivity in tradable intermediate sectors, intermediate sectors are symmetrically beneficial from the shocks. Due to the higher elasticity of substitution between home and foreign tradable inputs than final tradable sectors, output of tradable intermediate inputs further increases.

Turning to CTR, ADT and PEG. the productivity shocks also increase output of non-tradable sectors due to an increase in aggregate consumption and the low elasticity of substitution between final tradable and non-tradable sectors. In other words, while aggregate consumption and thus non-tradable consumption increase due to the depreciation of the real exchange rate reflected in the risk sharing condition, the expenditure switching effect between final tradable and non-tradable consumption partially offsets an increase in non-tradable consumption. Since the shocks increase the relative price of home producer goods and the elasticity of substitution is low, households moderately substitute from non-tradable goods to tradable goods. The terms of trade and the real exchange rate depreciate and thus demand for exports increase and imports decline which is reflected in higher net exports. Higher consumption coupled with higher net export leads to a rise in the real value of home production. Final tradable firms demand fewer workers with higher productivity so that a reduction of labour demand, reflected in lower wage and higher productivity reduces marginal cost and this in turn, reduces domestic and CPI inflation. In contrast to PEG, ADT gradually reduces the nominal interest rate and CTR immediately reduces the nominal interest rate in response to a fall in composite domestic and CPI inflation, respectively. This in turn, increases consumption and output further.

In intermediate input sectors, due to the higher productivity in tradable sectors (both final tradable and tradable intermediate sectors), final tradable firms demand fewer intermediate inputs. However, the higher productivity in tradable intermediate sectors implies lower prices of tradable intermediate inputs so that analogous to final goods, the terms of trade in tradable intermediate sector depreciates and final tradable firms substitute foreign intermediate inputs for home tradable intermediate inputs. This leads to a fall (rise) in intermediate input imports (exports) and raises intermediate sector output. Thus, while final tradable firms reduce demand for inputs which have fixed proportions of tradable and non-tradable intermediate inputs, tradable intermediate sectors are positively influenced by the shock. Since non-tradable intermediate firms are unable to engage in international trade, lower demand for intermediate inputs has a negative impact on intermediate non-tradable sectors.

Figure 6 shows the impacts of positive non-tradable sector productivity shocks. Under the optimal policy, the shocks lead to a fall in non-tradable inflation and correspondingly, a rise in tradable inflation as expected. Since the shocks lead to depreciation of the terms of trade and the real exchange rate, net exports increase and output increases symmetrically across different sectors.

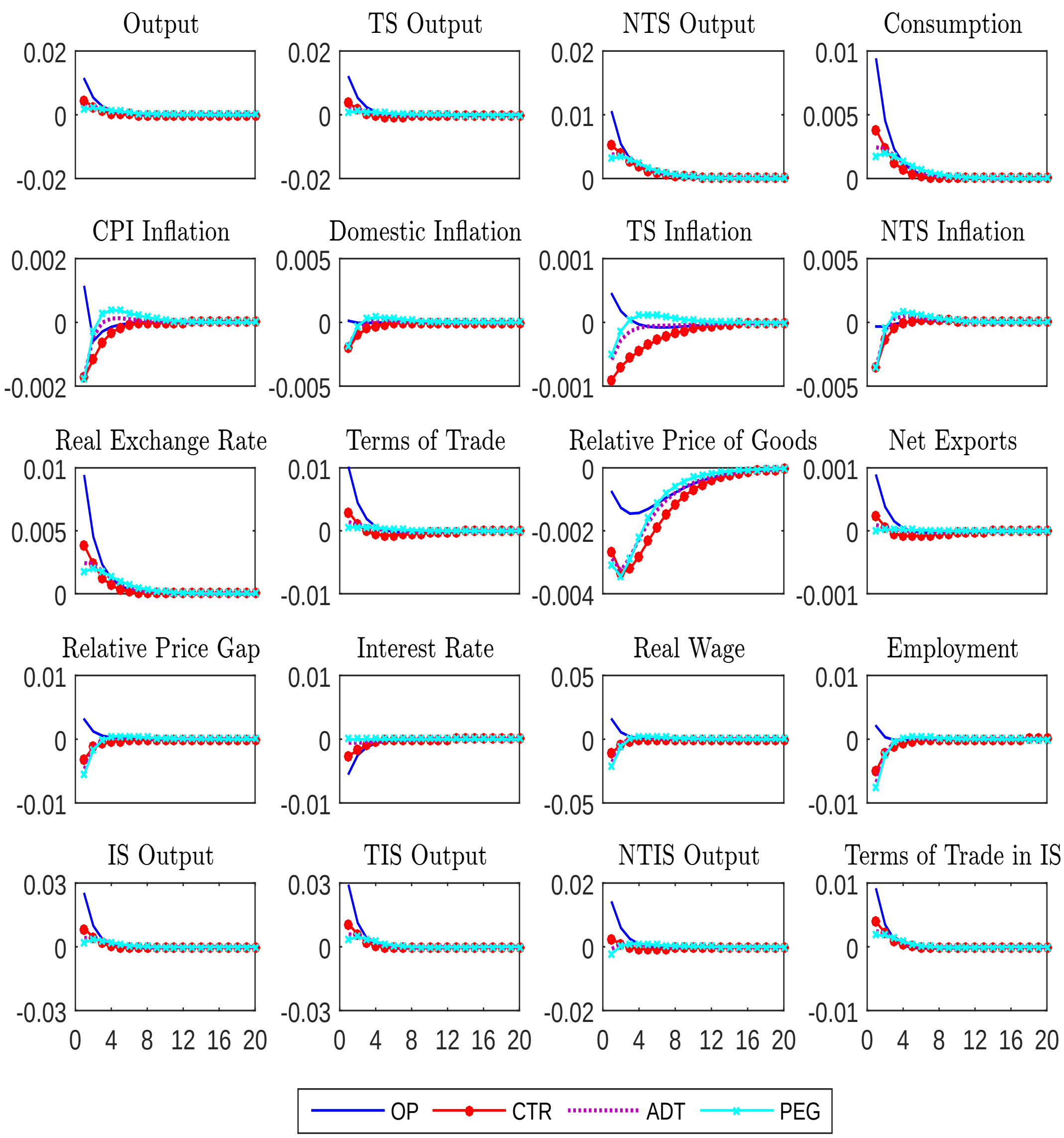

Figure 6: Impulse Responses to Home Non-Tradable Productivity Shocks

NOTE: TS, NTS, IS, TIS and NTIS refer to tradable sectors, non-tradable sectors, intermediate sectors, tradable intermediate sectors and non-tradable intermediate sectors, respectively. Variables are presented in percentage deviations from the steady state

With regard to alternative policy rules, while CTR, ADT and PEG show similar dynamics of the economy, CTR outperforms the others by immediately reducing the nominal interest rate in response to a fall in CPI inflation and generating a lower response of the relative price gap, and PEG amplifies responses of composite domestic inflation and the relative price gap due mainly to a very limited capability to generate depreciation of the terms of trade and the real exchange rate in response to the shocks. Under CTR, ADT and PEG, the shocks reduce the marginal cost of producing final non-tradable goods and this leads to depreciation (appreciation) of the terms of trade (relative home producer goods). Depreciation of the real exchange rate due to changes of relative prices increases aggregate consumption as implied by the risk sharing condition and output. With a lower price of non-tradable goods, households change their consumption (domestic tradable and imports) towards non-tradable goods. While depreciation on the terms of trade increases the real value of exports, an increase in the real value of imports due to the income effect which dominates the expenditure switching effects (consumption of imports towards home non-tradable goods), has a negligible impact on net exports. This implies that in response to the non-tradable productivity shocks, output symmetrically fluctuates with consumption and the real exchange rate under complete financial markets. Turning to intermediate input sectors, since the shocks reduce the costs of non-tradable intermediate inputs and thereby the marginal cost of final tradable firms and output, final tradable firms demand more intermediate inputs for both tradable and non-tradable intermediate inputs due to fixed proportion of inputs' demand, while tradable intermediate input sectors raise output further due to a depreciation of the terms of trade in intermediate sectors. Thus, the shocks are symmetrically beneficial for final and intermediate sectors.

5 Concluding Remarks

In this paper, we have extended a New Keynesian small open economy DSGE model with non-tradable goods and intermediate inputs. We have investigated the optimal policy and evaluated alternative monetary policy rules associated with welfare. Three main findings stand out.

Firstly, while the optimal policy requires stabilizing tradable inflation, non-tradable inflation and the output gap by eliminating net exports externalities arising from imperfect substitutability between tradable goods, non-tradable goods and imports, the optimal policy without fiscal instruments faces a trade-off between stabilizing composite domestic inflation and the output gap due to the externalities.

Secondly, ranking of alternative monetary policy rules crucially depends on types of shocks and parameter values. For tradable productivity shocks, the agumented composite domestic inflation-based Taylor rule (ADT) outperforms other alternative policy rules by reducing a volatility of composite domestic inflation. In response to non-tradable shocks, the CPI-based Taylor rule (CTR) leads to the lowest welfare losses by additionally stabilizing the relative price gap and thereby reducing a volatility of the output gap. We find that the composite domestic inflation-based Taylor rule (DTR) generates sizeable welfare losses under the conventional inflation coefficient. Thus, setting graduate interest rates towards their target levels rather than an immediate response appears to be desirable unless the central bank responds sufficiently aggressive to composite domestic inflation. However, when the economy is highly exposed to foreign goods markets and non-tradable productivity shocks, CTR can be the best alternative policy.

Thirdly, while final and intermediate sectors symmetrically beneficial from the stochastic shocks under the optimal policy, asymmetric impact on different sectors arises under CTR, ADT and the exchange rate peg (PEG) due mainly to the sectoral capacity to engage in international trade and the substitutability between home and foreign tradable goods (intermediate inputs).

References

Aoki, K. (2001). Optimal monetary policy responses to relative-price changes. *Journal of monetary economics*, *48*, 55–80.

Backus, D. K., & Smith, G. W. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, *35*, 297–316.

Benigno, G., & Benigno, P. (2003). Price stability in open economies. *The Review of Economic Studies*, *70*, 743–764.

Benigno, G., & Thoenissen, C. (2008). Consumption and real exchange rates with incomplete markets and non-traded goods. *Journal of International Money and Finance*, *27*, 926–948.

Blanchard, O. J., & Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica*, *48*, 1305–1312.

Bullard, J., & Mitra, K. (2002). Learning about monetary policy rules. *Journal of monetary economics*, *49*, 1105–1129.

Burstein, A., Eichenbaum, M., & Rebelo, S. (2005). Large devaluations and the real exchange rate. *Journal of Political Economy*, *113*, 742–784.

Burstein, A. T., Neves, J. C., & Rebelo, S. (2003). Distribution costs and real exchange rate dynamics during exchange-rate-based stabilizations. *Journal of monetary Economics*, *50*, 1189–1214.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, *12*, 383–398.

Clarida, R., Gali, J., & Gertler, M. (2001). Optimal monetary policy in open versus closed economies: An integrated approach. *The American Economic Review*, *91*, 248.

Corsetti, G., Dedola, L., & Leduc, S. (2008). International risk sharing and the transmission of productivity shocks. *The Review of Economic Studies*, *75*, 443–473.

Corsetti, G., Dedola, L., & Leduc, S. (2010). Optimal monetary policy in open economies. In B. M. Friedman, & M. Woodford (Eds.), *Handbook of Monetary Economics* chapter 16. (pp. 861–933). Elsevier volume 3.

Corsetti, G., & Pesenti, P. (2001). Welfare and macroeconomic interdependence. *Quarterly Journal of Economics*, (pp. 421–445).

De Paoli, B. (2009). Monetary policy in a small open economy: the role of the asset market structure. *Journal of Money, Credit and Banking*, *41*.

Devereux, M. B., Lane, P. R., & Xu, J. (2006). Exchange rates and monetary policy in emerging market economies. *The Economic Journal*, *116*, 478–506.

Dotsey, M., & Duarte, M. (2008). Nontraded goods, market segmentation, and exchange rates. *Journal of Monetary Economics*, *55*, 1129–1142.

Faia, E., & Monacelli, T. (2007). Optimal interest rate rules, asset prices, and credit frictions. *Journal of Economic Dynamics and Control*, *31*, 3228–3254.

Galí, J. (2009). *Monetary Policy, inflation, and the Business Cycle: An introduction to the new Keynesian Framework*. Princeton University Press.

Gali, J., & Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies*, *72*, 707–734.

Jones, C. I. (2011). Intermediate goods and weak links in the theory of economic development. *American Economic Journal: Macroeconomics*, (pp. 1–28).

Kasahara, H., & Rodrigue, J. (2008). Does the use of imported intermediates increase productivity? plant-level evidence. *Journal of development economics*, *87*, 106–118.

Lipińska, A. (2015). Optimal monetary policy for the emu accession countries. a new keynesian approach. *Macroeconomic Dynamics*, *19*, 1427–1475.

Meier, S. (2013). *Financial globalization and monetary transmission*. Working Paper Swiss National Bank.

Mendoza, E. G. (1995). The terms of trade, the real exchange rate, and economic fluctuations. *International Economic Review*, (pp. 101–137).

Obstfeld, M., & Rogoff, K. (1995). Exchange rate dynamics redux. *Journal of Political Economy*, (pp. 624–660).

Obstfeld, M., & Rogoff, K. (2000). Perspectives on oecd economic integration: implications for us current account adjustment. *Global Economic Integration: Opportunities and Challenges*, (pp. 168–209).

Obstfeld, M., & Rogoff, K. (2007). The unsustainable us current account position revisited. In *G7 current account imbalances: Sustainability and adjustment* (pp. 339–376). University of Chicago Press.

Rabanal, P., & Tuesta, V. (2013). Nontradable goods and the real exchange rate. *Open Economies Review*, *24*, 495–535.

Rotemberg, J. J. (1982). Sticky prices in the united states. *The Journal of Political Economy*, (pp. 1187–1211).

Rotemberg, J. J., & Woodford, M. (1999). Interest rate rules in an estimated sticky price model. In *Monetary policy rules* (pp. 57–126). University of Chicago Press.

Stockman, A., & Tesar, L. (1995). Tastes and technology in a two-country model of the business cycle: Explaining international comovements. *American Economic Review*, *85*, 168–85.

Sutherland, A. (2005). Incomplete pass-through and the welfare effects of exchange rate variability. *Journal of international economics*, *65*, 375–399.