Regime Change and Critical Junctures*

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Abstract

In this paper we study how a society can transition between different economic and political regimes. When the current regime is elitism, the society is modeled as a collection of units of land where at each of these units there is a member of the elite and a peasant. Members of the elite represent the institutions in place and their role is to choose how extractive the current regime is via setting up different tax rates. The role of peasants is to work the land and pay taxes. At every period with some small probability a critical juncture arrives, giving members of the elite a chance to update institutions (tax rates) and peasants an opportunity to revolt in order to instate a populist regime. Under the populist regime, at each of the units of land there is a citizen whose role is to work the land and enjoy the full output he produces. When a critical juncture arrives in this case, citizens have the option to stage a coup in order to revert back to elitism. In our results we characterize the possible outcomes after a critical juncture and study how the society can transition between regimes depending on the different parameters of the model.

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1 Introduction

Consider a society where few of its members, the elite, have ownership of the different economic activities. The rest of the society, peasants, are the ones that work on each of the different economic activities producing a certain output, a share of which is given to the elite. How extractive institutions are, i.e. how the output is shared between the elite and peasants, is decided by the elite and the only chance peasants have at changing the regime is via a revolution. Examples of these societies are, for instance, feudalism where each economic activity is simply a piece of land, or dictatorships where citizens work in the different economic activities and the ministers of the regime and government officials choose how much output to extract from peasants.

Throughout the history of civilizations societies as the ones described above have been abundant, from the medieval feudalism in Europe in the 10th century, to the military dictatorships in certain Asian and African countries in the present century. A frequent feature of these societies is that whenever they have evolved to different regimes, the change was usually triggered by what is know as a critical juncture (see Collier and Collier (1991) among others). A critical juncture can be defined as a "a major event or confluence of factors disrupting the existing economic or political balance in society" (Acemoglu and Robinson (2012), see also Capoccia and Kelemen (2007)). Examples of critical junctures are international events such as the discovery of the Americas, the Black Death or more recently the Arab Spring, as well as national events like the Cuban Revolution.

In this paper we present a model that tries to explain the factors that influence the possible outcomes after a critical juncture, as well as study how a society can transition between different regimes. To this end, we consider a society that is divided into different units of land. If the current economic and political regime is elitism, we assume that in each of these units of land there is a member of the elite and a peasant. The role of the member of the elite is to decide how extractive institutions in place are via setting up a tax rate. The role of the peasant is to work the land in order to produce a certain output which is shared between the member of the elite and the peasant according to the tax rate. If the current regime is populism, at each of the units of land there is a citizen whose role is to work the land and enjoy the full output he produces.

At every period with some very small probability a critical juncture arrives. In the event of a critical juncture, players have a chance to update their actions. Under elitism, members of the elite react to the critical juncture by adapting the institutions to the new circumstances, i.e. updating the tax rate they charge to peasants. After members of the elite have reacted to the critical juncture, peasants may stage a revolution. The revolution can be successful at overruling the current regime depending on how many peasants join the revolt. If the current regime is overruled, the elite is eliminated and a populist regime is installed: there is no distinction between members of the society and each citizen works on a unit of land and enjoys the full output he produces. When a critical juncture arrives under populism, citizens have a chance to stage a coup in order to revert back to elitism and become the new elite.

Within the setting just described, we study what are the possible outcomes after a critical juncture and whether or not a regime change can occur. In our results, we find that under elitism there are four different types of institutions that could emerge after a critical juncture: first, institutions such that the tax rate is the most extractive one and such that no peasant revolts (oppressive regime), second, institutions that are moderately extractive and such that a regime change is possible (unstable regime), third, institutions such that different units of land have different tax rates (segregated regime) and, finally, institutions where all peasants revolt and a regime change occurs with probability one (regime change). When the current regime is populism, we find that there are three possible outcomes after a critical juncture: first, no citizen stages a coup and populism continues after the critical juncture (stable democracy), second, a few citizens stage a coup and try to become the new elite (unstable democracy), and third, all citizens stage a coup and elitism is re-instated (regime change). We complete our analysis by trying to get a better understanding of how different factors such as output shocks and land profitability affect the specific outcomes after a critical juncture.

Our results illustrate situations such as how the fact that the economy may be going through a period of recession affects the resulting institutions after a critical juncture. For instance, we find that a wider output gap can increase the number of different institutions that are possible after a critical juncture. This helps in understanding why there may be heterogeneity in the institutions established in different countries during the Arab Spring, why different institutions arose in North America compared to South America during the colonization period, or why feudalism disappeared in western Europe but it intensified in eastern Europe after the Black Death (Acemoglu and Robinson (2012))

Moreover, when introducing our results later on we also explain how different current and historical episodes relate to each of the different equilibria of our model. We discuss how, for instance, the segregated regime equilibrium found in our model relates with what was observed after the critical juncture caused by the end of World War II, where Korea was divided in two countries with different autocratic regimes during the Korean War.

Our model is based on the work by Acemoglu and Robinson (2001a).[1] However, a key difference in our modeling approach allow us to study the effects of critical junctures from a perspective that to our knowledge has not been used before. The main difference between

[1]See also Acemoglu and Robinson (2000a, 2000b, 2001b) and Acemoglu et al (2001).

our model and the various models in Acemoglu and Robinson (2000a, 2000b, 2001b) is that we explicitly model the cooperation/coordination problem faced by players (elite, peasants and citizens, depending on the current regime). In our paper, each member of the elite is free to set up any tax rate he desires for his own unit of land and, hence, members the elite are playing a *cooperation* game with each other: each member of the elite would like the other members of the elite to choose a low tax rate so that no peasant revolts, a situation which would allow such member of the elite to set up a high tax rate in his unit of land without the risk of having his peasant join the revolution. In Acemoglu and Robinson (2000a, 2000b, 2001b) the elite is aggregated into a single player (all members of the elite choose the same tax rate) and, thus, they do not study the elite's cooperation problem.[2] Moreover, the probability that there is a regime change (successful revolution or coup) in our paper is an increasing function on the number of players that attempt the regime change. This means that peasants or citizens (depending on the current regime) play a *coordination* game with each other whenever a critical juncture a arrives. In the models of Acemoglu and Robinson (2000a, 2000b, 2001b), a regime change happens if and only if a given number of players attempt a regime change. Thus, the unique equilibrium outcome in their work is that either all players attempt a regime change or none do. In our paper we shall see how equilibria where some players attempt a regime change and others do not are possible. A situation where some but not all peasants revolt, or citizens stage a coup, is not only a more realistic scenario but, crucially, equilibria where only a fraction of the players attempt a regime change will prove important in understanding the possible outcomes after a critical juncture.

Another departure from previous literature (Acemoglu and Robinson (2000a, 2000b, 2001b) and relevant references herein) is that in our setting equilibria multiplicity is possible. Equilibria multiplicity should not be surprising given the inherent coordination problem that a revolution or a coup entails. This is true even though several attempts have been made to reconcile the coordination problem faced by peasants and citizens with the simplifying assumption by which either all or no player attempts a regime change (i.e. the collective action problem, see also Lichbach (1995), Moore (1995) or Popkin (1979)).

Also related to our work is the literature on coordination in games of regime change (global games). Carlsson and van Damme (1993), Morris and Shin (1998) and Morris and Shin (2004) among others solve the equilibria multiplicity problem by adding uncertainty about the game that is actually played, by adding noisy about the fundamentals, or by introducing uncertainty in the number of players needed for a regime change respectively. Edmon (2013) considers a game of regime change where there is uncertainty about the strength of the current regime

[2]In our model, there will be equilibria where all members of the elite choose the same tax rate (and equilibria where they do not). Therefore, the fact that the elite may behave as one single player will be an endogenous result in this paper instead of an exogenous assumption.

and shows how such uncertainty results in the model having a unique equilibrium.

We depart from the papers in the previous paragraph in that we are not interested in generating a unique prediction with our model as we do not believe that a model such as ours can be used to predict the future. Instead of this, our target is to explore the factors that make different institutions possible after a critical juncture and to understand the characteristics of these different institutions, without specifying which particular institution will be selected. This is motivated by the fact that, as Acemoglu and Robinsons' (2013) put it, "A critical juncture is a double-edged sword that can cause a sharp turn in the trajectory of a nation. On the one hand it can open the way for breaking the cycle of extractive institutions and enable more inclusive ones to emerge... Or it can intensify the emergence of extractive institutions...". Finally, our worked is also related to that of Bueno de Mesquita (2010), who studies the role of the vanguard as a tool to inform other players about the likelihood of a successful revolution and finds that there can be two possible equilibria, one where a revolution takes place and one where it does not.

The rest of the paper is organized as follows. In Section 2 we introduce the model while we present our main results in Section 3. In Section 4 we discuss our results more in depth and consider some possible extensions. Finally, Section 5 concludes. All mathematical proofs are presented in the Appendix.

2 The Model

2.1 Revolutions: from Elitism to Populism

Assume that time is discrete and given by $n = 0, 1, \ldots$. The society consists of infinitely many units of land indexed by $i \in \{1, 2, \ldots\}$. In each unit of land i there is one member of the elite and a peasant. The role of the member of the elite is to set a tax rate $t_i^r \in [0, 1]$ while the role of the peasant is to work the land and pay the elite a percentage t_i^r of the output.

The output of each unit of land i at any given time period is given by $y_i \in \mathbb{R}$. The values of $\{y_i\}_{i=1}^{\infty}$ are independent and identically distributed where for all i we have that y_i takes the normalized value 1 with probability $(1 - \varepsilon_n) \in (0, 1)$ and the value $y \in [0, 1]$ with probability ε_n. The random variable ε_n is independent and identically distributed for all time n with mean $\mu_\varepsilon \in (0, 1)$ and support $[\underline{\varepsilon}, \overline{\varepsilon}] \subset (0, 1)$. The value of ε_n represents the likelihood of an output shock at time n while y represents the output in case of a shock. Whenever there is no ambiguity, we refer to the value of ε_n at the current period as ε. The peasant works the land every period by paying a cost $c \in [0, Ey]$ where $Ey = (1 - \mu_\varepsilon) + \mu_\varepsilon y$ is the expected output of each unit of land. Thus, in unit of land i with tax rate t_i^r the payoff of the member of the elite is given by $t_i^r y_i$ and the payoff of the peasant is given by $(1 - t_i^r)y_i - c$.

Each unit of land can be considered as a territory but also as, for example, a certain economic activity or market. The member of the elite of a given unit of land represents the few agents (or a single agent) that own the unit of land or that are given exclusive rights to exploit it. The peasant represents all the agents that are responsible for the actual exploitation of the unit of land.[3] They are the ones that have to spend effort and time to produce a certain output. This output is then split between the two sides, elite and peasant, according to the tax rate set by the member of the elite. Moreover, at every period and at every unit of land, an output shock that lowers the output of the unit of land may be present. The parameter μ_ε captures the long term output gap while the value of ε represents the current state of the economy. A high value of ε correspond to a recession as output shocks are frequent whereas a low value of ε represents a period of economic expansion because output shock are rare. Note that the model can be easily reinterpreted into one where the tax rate is applied not to output but to profit: if this is the case then $c = 0$ and y_i represents profit. In Section 4 we explore this case in more detail.

At each time period and with some probability $\delta \in (0, 1)$ a critical juncture may arrive. During a critical juncture, and after the current state of the economy ε is known but before the actual output of each unit of land y_i for all i is observed, each member of the elite has a chance to change the tax rate for his unit of land. All members of the elite choose their tax rate simultaneously and critical junctures are the only opportunity they have at changing the tax rate they charge. This is motivated by the fact that the tax rate represents more than just income tax. The tax rate represents how extractive the current regime in place is, i.e. institutions. After the new tax rates have been set, the realization of y_i for all i is observed and peasants have a chance to revolt against the regime by simultaneously deciding whether to join the revolution or not. The interpretation of this is that revolutions do not happen spontaneously, they are triggered by critical junctures and the way the elite reacts to them.

When the peasant of a given unit of land revolts the output of that unit of land for the current period is 0 and the peasant does not have to pay the cost c of working the land. If a fraction x of the peasants revolt then the revolution is successful with probability $\gamma(x)$ where $\gamma : [0, 1] \to [0, 1]$. We assume γ is continuous and weakly increasing with $\gamma(0) = 0$ and $\gamma(1) = 1$. The assumption about the continuity of γ is made to simplify calculations but it has no significant impact in the results. Throughout the paper γ is referred to as the

[3] Although throughout the paper we refer to one member of the elite, one peasant or one citizen, these terms may encompass more than one person, i.e. a peasant refers to all those who work on a single unit of land (as, for instance, a family or group of families during feudal times).

technology of the revolution. The function γ reveals how effective peasants are at revolting and includes information on, for example, how well armed the rebels are or whether or not the NATO supports and helps the revolution.

If a revolution is unsuccessful then those peasants that revolted are removed from the game and replaced with new ones.[4] The society then continues to function as before with the new tax rates set by the elite. If the revolution is successful several things occur: first, peasants that did not take part in the revolution are removed from the game a replaced with new ones. Second, the elite is completely removed from the game and all peasants become citizens. Third, the political regime changes to one where citizens enjoy all the product (and costs) of the land. After a successful revolution the payoff per period of a citizen working in unit of land i is $y_i - c$.

Critical junctures are rare significant historical events that may trigger a regime change. During a critical juncture, the elite has a chance to "update" or "modernize" in order to retain its power. This takes the form of an opportunity for each member of the elite to change how extractive institutions are with the peasant, i.e. change the tax rate. After the elite has responded to the new situation posed by the critical juncture, peasants decide whether to revolt or not.[5] Once a critical juncture and possibly a revolution are resolved, the society continues to function under the new regime until the next critical juncture. Note that critical junctures are exogenous occurrences that are not necessarily related to the frequency of shocks ε. Nevertheless, it could be assumed that critical junctures arrive at any period in which the frequency of shocks is above or below a certain threshold. This modification of the model does not affect our results in any way.

When a critical juncture takes place, it is assumed that members of the elite know the frequency of shocks ε but not which units of land suffer an output shock. We assume that at a critical juncture, members of the elite know the current state of the economy (expansion, recession, depression, etc.). Such state is represented in the model by the current value of ε. However, the elite ignores which specific units of lands (or sectors, markets, etc.) of the economy will suffer an output shock. Thus, given our assumptions, members of the elite have to react to the critical juncture knowing the general state of the economy but not the particular state of each unit of land.

Note that we assume that after a failed revolution all peasants that revolted are removed

[4]This new peasants could be, for instance, the offspring of the peasants that revolted and failed in their attempt to change the regime. This is in line with models of evolutionary game theory (see, for instance, Weibull (1996)) and models with dynasties in Macroeconomics (see, for instance, Bertola et al (2005)

[5]In our paper we use critical junctures as revision opportunities: a situation that allows players in the model to change their actions (as opposed to how Acemoglu and Robinson (2000b) use output shocks to solve the collective action problem).

from the game and, similarly, if a revolution succeeds then the elite is removed from the game. We believe this is a realistic assumption as whenever a revolution fails revolutionaries receive very severe punishments (such as torture and death during the Tibetan unrest in 2008)).[6] Similar treatment is given to members of the elite after a successful revolution (from the beheading of aristocrats during the French revolution to the hanging of dictator Saddam Hussein). We assume that in case of a successful revolution, peasants that did not join the revolt are removed from the game. This aims to highlight the loss in private benefits of not joining a successful revolution. As Tullock (1971) writes: "...(revolutionaries) generally expect to have a good position in the new state which is to be established by the revolution. Further, ...(leaders) continuously encourage their followers in such views. In other words, they hold out private gains to them.". Alternatively, peasants that do not join the revolution can be thought of as aligning with the elite and the current regime. Hence, if the revolution is successful these peasants face the same fate as members of the elite.

Critical junctures are rare and infrequent events that are hard to predict. Thus, we are interested in situations where the probability of a critical juncture δ is very small. From the modeling point of view, this means that we look for the limit case when δ tends to zero.

Assume that all agents discount future payoffs at a rate $\beta \in (0, 1)$. Hence, the expected discounted stream of present and future payoffs (expected payoff hereafter) of the member of the elite in unit of land i after a critical juncture where the revolution is unsuccessful is given by

$$V^e(t_i^r) = \sum_{n=1}^{\infty}(1-\delta)^n \beta^{n-1} t_i^r Ey + \delta o^e(t_i^r),$$

where o_e is some function that is bounded below by 0 (in case there is a critical juncture at some point in the future and the elite is removed from the game), and above by $\frac{Ey}{1-\beta}$ (in case there is a critical juncture at some point in the future, the elite sets up tax rate $t_i^r = 1$ and the revolution is unsuccessful). The function o^e may include beliefs of what will happen when the next critical juncture arrives, beliefs about beliefs of what will happen in the next critical juncture, etc. However, as the purpose is to study the situation when $\delta \to 0$, we do not need to have any knowledge about o^e except for the fact that it is bounded above and below. We can rewrite the function V^e as

$$\begin{aligned} V^e(t_i^r) &= (1-\delta)\frac{t_i^r Ey}{1-(1-\delta)\beta} + \delta o^e(t_i^r), \\ \lim_{\delta \to 0} V^e(t_i^r) &= \frac{t_i^r Ey}{1-\beta}. \end{aligned}$$

[6]See the US Department of State report on 2010 Human Rights Report: China (http://www.state.gov/j/drl/rls/hrrpt/2010/eap/154382.htm).

Similarly, after a critical juncture where the revolution is unsuccessful the expected payoff of the peasant working in unit of land i if he did not revolt is given by

$$\begin{aligned} V^p(t_i^r) &= (1-\delta)\frac{(1-t_i^r)Ey-c}{1-(1-\delta)\beta} + \delta o^p(t_i^r), \\ \lim_{\delta\to 0} V^p(t_i^r) &= \frac{(1-t_i^r)Ey-c}{1-\beta}, \end{aligned}$$

where the function o^p belongs to the interval $\left[0, \frac{Ey}{1-\beta}\right]$ and has a similar interpretation to that of o^e.[7]

If the revolution is successful, the expected payoff of each citizen is given by

$$\begin{aligned} V^c &= (1-\delta)\frac{Ey-c}{1-(1-\delta)\beta} + \delta o^c, \\ \lim_{\delta\to 0} V^c &= \frac{Ey-c}{1-\beta}, \end{aligned}$$

where again the function o^c belongs to the interval $\left[0, \frac{Ey}{1-\beta}\right]$ and as a similar interpretation to that of o^e.

The timing of the game at hands is summarized in Figure 1. This figure is not an extensive form game representation of the model but an illustration of the order at which events take place in unit of land i. In Figure 1 the status quo refers to a situation where a critical juncture does not take place and, hence, tax rates do not change and no peasant revolts. In this figure, NR stands for not revolt while R stands for revolt.

At a critical juncture, and after the elite of a given unit of land i sets up the new tax rate t_i^r, the expected payoff of the peasant working on unit of land i if a fraction x of the peasants revolt is given by $u_i^p(t_i^r, x)$ with

$$u_i^p(t_i^r, x) = \begin{cases} (1-t_i^r)y_i - c + \beta(1-\gamma(x))V^p(t_i^r) & \text{if he does not revolt,} \\ \beta\gamma(x)V^c & \text{if he revolts.} \end{cases}$$

Hence, the peasant working on unit of land i chooses not to revolt if and only if

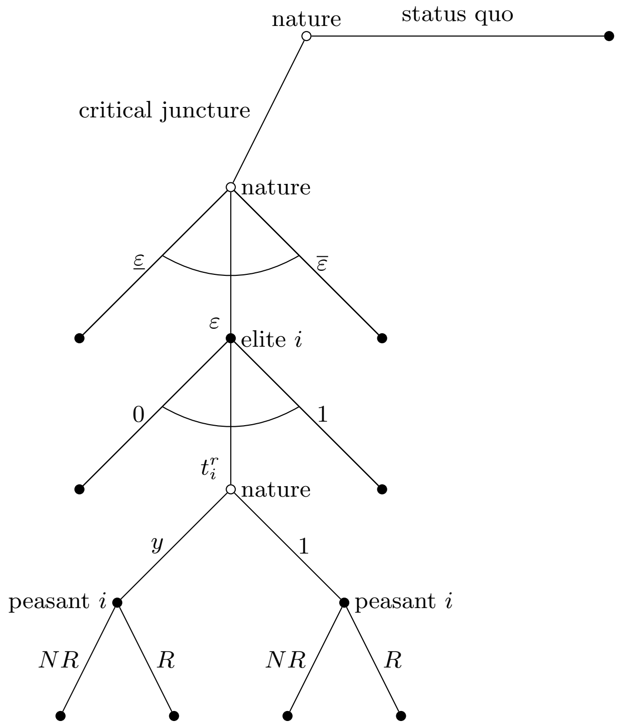
$$(1-t_i^r)y_i - c - \beta[\gamma(x)V^c - (1-\gamma(x))V^p(t_i^r)] \geq 0.$$

Note that in case a peasant is indifferent between joining the revolution or not we assume he does not join the revolution. This assumption does not affect our results in any meaningful way.

Let $t_1^r(x)$ be the maximum tax rate such that the peasant of a given unit of land does not join the revolution if a fraction x of the peasants revolt and if there is an output shock in his

[7]The function o^p could take the value $\frac{Ey}{1-\beta}$ if the peasant becomes a member of the elite in the future. This is explained in more detail in Section 2.2 when we consider coups.

Figure 1: Revolutions - Timing of the game for unit of land i



unit of land. We have that $t_1^r(x)$ is given implicitly by

$$(1-t_1^r(x))y - c - \beta[\gamma(x)V^c - (1-\gamma(x))V^p(t_1^r(x))] \quad = \quad 0,$$

which as $\delta \to 0$ can be rewritten as

$$t_1^r(x) \quad = \quad 1 - \frac{c + \beta\gamma(x)(Ey - 2c)}{(1-\beta)y + \beta(1-\gamma(x))Ey}. \tag{1}$$

Similarly, let $t_2^r(x) \geq t_1^r(x)$ be the maximum tax rate that keeps the peasant of a given unit of land from joining the revolution if a fraction x of the peasants revolt and if there is no output shock in his unit of land. Then, $t_2^r(x)$ when $\delta \to 0$ is given by:

$$t_2^r(x) \quad = \quad 1 - \frac{c + \beta\gamma(x)(Ey - 2c)}{(1-\beta) + \beta(1-\gamma(x))Ey}. \tag{2}$$

Note that it can happen that either $t_1^r(x)$ or both $t_1^r(x)$ and $t_2^r(x)$ are negative for a given value of x. If $t_1^r(x) < 0$ then it is not possible for the elite to keep his peasant from revolting in case of an output shock. If, on top of that, $t_2^r(x) < 0$ then the peasant always revolts regardless of the tax rate set by the member of the elite. Notice that it is never the case that $t_1^r(x)$ nor $t_2^r(x)$ are greater than 1 for any $x \in [0,1]$.

At a critical juncture where a fraction x of the peasants revolt the expected payoff of the member of the elite of a given unit of land i is given by $u_{NR}^e(t_i^r, x)$ if his peasant does not join the revolution and by $u^e_R(t^r_i, x)$ if he does:

$$\begin{aligned} u^e_{NR}(t^r_i, x) &= t^r_i Ey + \beta(1-\gamma(x))V^e(t^r_i), \\ u^e_R(t^r_i, x) &= \beta(1-\gamma(x))V^e(t^r_i). \end{aligned}$$

Note that the superscript i is not needed in the functions u^e_{NR} and u^e_R as members of the elite do not know the realization of output y_i when they choose the tax rate.

Thus, if we denote by $u^e(t^r_i, x)$ the expected payoff of the member of the elite who owns unit of land i and by ε the value of ε_n at the critical juncture we have that

$$u^e(t^r_i, x) = \begin{cases} t^r_i Ey + \beta(1-\gamma(x))V^e(t^r_i) & \text{if } t^r_i \leq t^r_1(x), \\ t^r_i(1-\varepsilon) + \beta(1-\gamma(x))V^e(t^r_i) & \text{if } t^r_i \in (t^r_1(x), t^r_2(x)], \\ \beta(1-\gamma(x))V^e(t^r_i) & \text{otherwise.} \end{cases} \tag{3}$$

If $t^r_i = t^r_1(x) = t^r_2(x)$ we assume that the member of the elite is in effect choosing tax rate $t^r_1(x)$ and, thus, his expected payoff is given by $t^r_i Ey + \beta(1-\gamma(x))V^e(t^r_i)$. This assumption is for consistency with the fact that in case of indifference between revolting or not peasants prefer not to revolt.

2.2 Coups: from Populism to Elitism

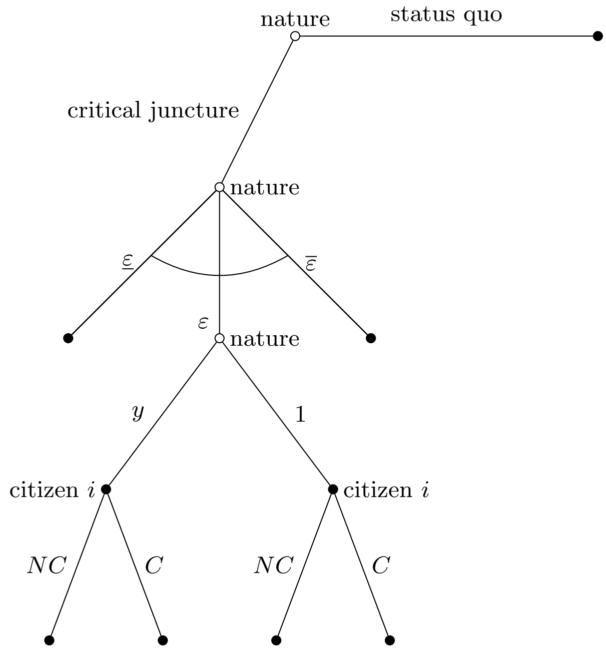
In this section we develop the model for the opposite situation as in the section above: citizens work the land and at each critical juncture they may stage a coup that could reinstate the regime where the society is split between the elite and peasants.

Assume that in each unit of land there is a citizen with an expected payoff of V^c. When a critical juncture arrives, citizens decide whether to stage a coup or not after observing the frequency of shocks ε and the production in each unit of land y_i. The timing of the game for each unit of land i is represented in Figure 2.

If a fraction $z \in [0, 1]$ of the citizens stage a coup, let $\rho(z)$ be the probability that the coup is successful. As with function γ, we assume $\rho : [0, 1] \to [0, 1]$ to be continuous and weakly increasing with $\rho(0) = 0$ and $\rho(1) = 1$. The fact that γ and ρ are different functions is meant to represent that it may be easier to move from one regime to the other than vice versa.

In line with the assumptions made in the previous section, if a citizen joins a successful coup then he becomes a member of the elite and a peasant arrives to his unit of land. Moreover, if a citizen does not join the coup and the coup is successful then he becomes a peasant and a member of the elite arrives to his unit of land. Finally, citizens that join an unsuccessful coup are removed from the game and replaced with new ones.

Figure 2: Coups: Timing of the game for unit of land i



We remind the reader that although throughout the paper we refer to one member of the elite, one peasant or one citizen, these terms may encompass more than one person (i.e. a group of people). Thus, when a coup is successful the regime changes to elitism and a member of the elite is installed in each unit of land, even in those units of land where the citizen did not join the coup. This has the interpretation that members of the newly created elite distribute all units of land amongst themselves. Similarly, when a coup is successful it is assumed that a peasant arrives to each unit of land where the citizen joined the coup. This is because in each unit of land there may be members of the society that do not participate in the economic and political landscape in any way and that they only become relevant from the modeling point of view when they are turned into peasants.

At a critical juncture where a fraction z of the citizens stage a coup, the expected payoff of the citizen in unit of land i is given by $u_i^c(t^c, z)$ with

$$u_i^c(t^c, z) = \begin{cases} y_i - c + \beta\left[(1-\rho(z))V^c + \rho(z)V^p(t^c)\right] & \text{if he does not join the coup,} \\ \beta\rho(z)V^e(t^c) & \text{if he joins the coup,} \end{cases}$$

where t^c is the tax rate implemented after a successful coup. We assume that before the coup is resolved, the citizens that stage a coup commit to setting up a certain tax rate if they become members of the elite, and this tax rate is the same in all units of land. This has the following interpretation: the tax rate represents the institutions in place and, thus, committing to a tax rate is the equivalent for those that stage the coup to commit to a certain manifesto which lays out the institutions to be put in place after the coup. The case where the tax rate can vary between different units of land after a successful coup is discussed as an extension to the main model in Section 4.2.

A citizen does not have incentives to join the coup when $\delta \to 0$ if and only if

$$y_i - c + \frac{\beta}{1-\beta}\left[Ey(1 - 2\rho(z)t^c) - c\right] \quad \geq \quad 0, \tag{4}$$

which for $\rho(z) > 0$ can be rewritten as

$$t^c \quad \leq \quad \frac{(1-\beta)y_i + \beta Ey - c}{2\rho(z)\beta Ey}.$$

Note that in case of indifference a citizen chooses not to join the coup. This assumption is made simply for analytical convenience and does not affect our results in any meaningful way.

Let $t_1^c(z)$ be the maximum tax rate such that the citizen of a given unit of land does not join the coup if a fraction $z > 0$ of the citizens stage a coup and if there is an output shock in his unit of land. We have that $t_1^c(z)$ as $\delta \to 0$ is given by

$$t_1^c(z) \quad = \quad \frac{(1-\beta)y + \beta Ey - c}{2\rho(z)\beta Ey}. \tag{5}$$

Similarly, let $t_2^c(z) \geq t_1^c(z)$ be the maximum tax rate such that a citizen does not join the coup if a fraction $z > 0$ of the citizens stage a coup and if there is no output shock in his unit of land. Then, $t_2^c(z)$ when $\delta \to 0$ is given by

$$t_2^c(z) \quad = \quad \frac{(1-\beta) + \beta Ey - c}{2\rho(z)\beta Ey}. \tag{6}$$

If $z = 0$ then as $c \in [0, Ey]$ no citizen that works on a unit of land that does not suffer an output shock wants to revolt (see equation (4)). However, if $y_i - c + \frac{\beta}{1-\beta}[Ey - c] < 0$ then, even when no other citizen joins the coup, the citizens that work in the units of land that suffer an output shock have incentives to join the coup. Moreover, it is possible that $t_1^c < 0$, in which case there is no tax rate that keeps the citizens that work in the units of land that suffer a production shock from joining the coup. Note that it is always the case that $t_2^c > 0$.

2.3 Equilibrium

In our model, we use Markov Perfect Equilibrium as the equilibrium concept. A Markov Perfect Equilibrium is a (pure strategy) sub-game perfect Nash equilibrium of the game that is played at each critical juncture. At each critical juncture, the state variables are the current frequency of shocks ε and whether the society is elitist (Section 2.1) or populist (Section 2.2).

For each possible frequency of shocks, we study what are the possible sub-game perfect Nash equilibria under each of the two different regimes.

Under the elitist regime, after a critical juncture and after observing ε, each member of the elite simultaneously chooses a tax rate, the realization of y_i for all i is known, and peasants simultaneously choose whether to revolt or not. Therefore, the Markov Perfect Equilibrium in this case prescribes a collection of tax rates for the elite and a fraction of the peasants that revolt such that two conditions are satisfied. First, given the frequency of shocks and the fraction of the peasants that revolt, no member of the elite has incentives to choose a different tax rate. Second, given the frequency of shocks, the tax rates set by the elite, the realization of the value of output in each unit of land and how many peasants revolt, exactly this fraction of the peasants have incentives to revolt.

Under a populist regime, after a critical juncture and after observing ε and the realization of y_i for all i, citizens simultaneously choose whether to stage a coup or not. Thus, the Markov Perfect Equilibrium in this case prescribes a fraction of the citizens that stage a coup and a collection of tax rates such that given the frequency of shocks, the realization of the values of output, how many citizens stage a coup and the taxes set by the elite in case of a successful coup, exactly this fraction of the citizens have incentives to stage a coup.

Definition 1. *A Markov Perfect Equilibrium is a tuple* $\{(\{t_i^r\}_{i=1}^{\infty}, x), (t^c, z)\}_{\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]}$ *which for each possible value of* $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$ *specifies a collection of tax rates* $\{t_i^r\}_{i=1}^{\infty}$ *with* $t_i^r \in [0,1]$ *for each unit of land* i*, a fraction of the peasants that revolt* $x \in [0,1]$*, a tax rate* $t^c \in [0,1]$ *for all units of land, and a fraction of the citizens that stage a coup* $z \in [0,1]$ *such that:*

1. *At a critical juncture, if the society is under elitism and the frequency of shocks is* ε*:*
   - *Given* x*, every member of the elite maximizes* $u^e(t_i^r, x)$ *by choosing tax rate* t_i^r*.*[8]
   - *Given* t_i^r*,* y_i *and* x*, the peasant working in unit of land* i *maximizes* $u_i^p(t_i^r, x)$ *by choosing whether to join the revolution or not, and the fraction of the peasants that choose to revolt equals* x*.*
2. *At a critical juncture, if the society is under populism and the frequency of shocks is* ε*:*
   - *Given* t^c *and* z*, every member of the elite maximizes* $u_i^c(t^c, z)$ *by choosing whether to join the coup or not, and the fraction of the citizens that choose to join the coup equals* z*.*

[8] Alternatively, given that x is calculated as a best response to $\{t_i^r\}_{i=1}^{\infty}$, every member of the elite maximizes $u^e(t_i^r, x)$ given $\{t_j^r\}_{j \neq i}$ and peasants' play the best response to the tax rates set by the elite (i.e. by backwards induction given $\{t_j^r\}_{j \neq i}$). The two statements are equivalent as x does not depend on any individual tax rate t_i^r because there are infinitely many units of land.

For the analysis of Markov Perfect Equilibria, we focus on the limit cases where $\delta \to 0$. Thus, in what follows, although not explicitly stated it is always assumed that the value of δ is taken to zero.

Given the equilibrium definition above, it is useful to separate the analysis of the model in two parts. The first one (Section 3.1) deals with the first component of the Markov perfect Equilibrium: the tuple $(\{t_i^r\}_{i=1}^{\infty}, x)$. This is the sub-game perfect Nash equilibrium of the game that is played after a critical juncture where elitism is the current regime. Such equilibrium does not need to specify the strategies to be played in the critical juncture after the current one as $\delta \to 0$ and $\beta < 1$ and, thus, what occurs in the critical juncture after the current one is payoff irrelevant.

The second part of the analysis (Section 3.2) deals with the second component of the Markov Perfect Equilibrium: the tuple (t^c, z). This is the sub-game perfect Nash equilibrium of the game that is played after a critical juncture where populism is the current regime. Once again and for the same reasons as in the paragraph above, such equilibrium only needs to specify the strategies to be played in the current critical juncture.

Effectively, the fact that $\delta \to 0$ and $\beta < 1$ implies that every critical juncture can be studied independently of each other. The only thing that connects a critical juncture with the next one is what is the regime that results after the critical juncture, which is one of the state variables in our definition of Markov Perfect Equilibrium.

3 Analysis

3.1 Analysis: Revolutions

In this section we analyze the first component of the Markov Perfect Equilibrium. We refer to the tuples $(\{t_i^r\}_{i=1}^{\infty}, x)$ that are part of a Markov Perfect Equilibrium as r-equilibria. Our first result states that only four different types of r-equilibria can exist.

Proposition 1. *For any value of ε there are at most four possible r-equilibria:*

- *r-equilibrium 1:* $(\{t_1^r(0)\}_{i=1}^{\infty}, 0)$.
- *r-equilibrium 2:* $(\{t_2^r(\varepsilon)\}_{i=1}^{\infty}, \varepsilon)$.
- *r-equilibrium 3:* $(\{t_i^r\}_{i=1}^{\infty}, x)$ *with* $x \in [0, \varepsilon]$ *where a fraction* $1 - \frac{x}{\varepsilon}$ *of the elite chooses tax rate* $t_1^r(x)$ *and the rest choose tax rate* $t_2^r(x)$*, and* x *is such that*

$$u^e(t_1^r(x), x) \quad = \quad u^e(t_2^r(x), x).$$

- *r-equilibrium 4:* $(\{t_i^r\}_{i=1}^{\infty}, 1)$ *for any* $\{t_i^r\}_{i=1}^{\infty}$.

In r-equilibrium 1 (oppressive regime) we have that all members of the elite choose a tax rate $t_1^r(0)$ and no peasant revolts. Thus, in this r-equilibrium the current regime whereby the peasant works the land and the elite extracts some of the revenue at no cost continues with probability one after the critical juncture. This regime is observed when all the political and economy power concentrates on the elite after a critical juncture, as it is currently the case in military dictatorships in certain Asian countries, where the strength of the elite increased after the critical juncture caused by the death of Mao Zedong (Acemoglu and Robinson (2011)).

Under r-equilibrium 2 (unstable regime) we have that the tax rate set up by the elite is such that only the peasants that suffer an output shock revolt. The probability that the regime changes is given by $\gamma(\varepsilon)$. Hence, this r-equilibrium leads to a situation where a regime change is possible. We currently observe this regime in certain countries in Western Asia, where the Arab Spring that caused fast transitions to democracy in some countries (as in Tunisia) have left the current regime unstable and in a period of continued civil war (as in Syria).

The next r-equilibrium, r-equilibrium 3 (segregated regime), is a combination of r-equilibria 1 and 2, where some members of the elite set up tax rate $t_1^r(x)$ and other set up tax rate $t_2^r(x)$. As $x \in [0, \varepsilon]$ implies $t_1^r(x) \leq t_2^r(x)$, there is segregation between the units of land where the more extractive tax rate $t_2^r(x)$ is in place and where some of the peasants revolt, and the units of land where the less extractive tax rate $t_1^r(x)$ is in place and where no peasant revolts. This type of equilibrium was observed after the critical juncture caused by the end of World War II, where Korea was divided in two countries with different autocratic regimes during the Korean War. One of these countries then evolved after another critical juncture (the June Democratic Uprising in 1987, Adesnik and Kim (2008)) and became a full democracy.[9]

Finally, r-equilibrium 4 (regime change) represents a situation where all peasants revolt. If all peasants revolt then the elite can do little to stop a revolution as when all peasants revolt the optimal strategy for a peasant is, under most parameter values, to revolt (Proposition 2 below specifies the parameter values for which this is true). As discussed above, this equilibrium was observed during the Arab Spring, where Tunisia quickly evolved into a democracy.

An observation worth mentioning is that everything else equal the higher the fraction of the peasants that revolt, the lower the r-equilibrium tax rates t_1^r and t_2^r. This can be seen

[9]In the language of the model, the recent history of South Korea can be described by a critical juncture where r-equilibrium 3 was selected followed by another critical juncture 50 years later where r-equilibrium 4 was selected.

in equations (1) and (2), which are both decreasing in x. Therefore, a situation where few peasants revolt represents a more extractive society than a situation where a higher fraction of the peasants revolt. The explanation for this is that if few peasants revolt, the incentives for each peasant to revolt are low as the chances of a successful revolution are also low. Thus, the elite can take advantage of this by charging a higher tax rate. If a high fraction of the peasants revolt then the revolution is likely to succeed and, hence, if a member of the elite wants to avoid his peasant from revolting the tax rate he sets has to be relatively low. The implications of this are intuitive: members of the elite prefer an r-equilibrium where no peasant revolts and a higher tax rate can be implemented while peasants prefer a r-equilibrium with a lower tax rate with some social unrest where a regime change is possible.

Our next result shows that there always exists an r-equilibrium and, furthermore, it states the conditions under which each of the four different r-equilibria are possible. Conditions for r-equilibria 1-3 to exist are stated implicitly, their explicit forms are presented in the appendix, where we also present the proof of the proposition.

Proposition 2. *For any value of ε there always exists an r-equilibrium. Furthermore, define*

$$\begin{aligned} \Delta u^e(x) &= u^e(t_1^r(x), x) - u^e(t_2^r(x), x) \\ &= t_1^r(x)\varepsilon y - (t_2^r(x) - t_1^r(x))\left[(1-\varepsilon) + (1-\gamma(x))\frac{\beta E y}{1-\beta}\right], \end{aligned}$$

we have the following:

1. *The tuple $(\{t_1^r(0)\}_{i=1}^{\infty}, 0)$ is an r-equilibrium if and only if $\Delta u^e(0) \geq 0$.*

2. *The tuple $(\{t_2^r(\varepsilon)\}_{i=1}^{\infty}, \varepsilon)$ is an r-equilibrium if and only if $t_2^r(\varepsilon) \geq 0$ and $\Delta u^e(\varepsilon) \leq 0$.*

3. *The tuple $(\{t_i^r\}_{i=1}^{\infty}, x)$ with $x \in [0, \varepsilon]$ where a fraction $1 - \frac{x}{\varepsilon}$ of the elite chooses tax rate $t_1^r(x)$ and the rest choose tax rate $t_2^r(x)$ is an r-equilibrium if and only if $\Delta u^e(x) = 0$.*

4. *The tuple $(\{t_i^r\}_{i=1}^{\infty}, 1)$ is an r-equilibrium for any $\{t_i^r\}_{i=1}^{\infty}$ if and only if*

$$\frac{\beta}{1-\beta} V^c \quad > \quad 1 - c.$$

Proposition 2 states the conditions under which each of the different r-equilibria are possible. A crucial function is that of $\Delta u^e(x)$, which specifies what is the increase in the expected payoff of a member of the elite from choosing tax rate $t_1^r(x)$ instead of tax rate $t_2^r(x)$ when a fraction x of the peasants revolt. Thus, for instance, r-equilibrium $(\{t_1^r(0)\}_{i=1}^{\infty}, 0)$ is possible if and only if the increase in expected payoff from choosing tax rate $t_1^r(0)$ instead of tax rate $t_2^r(0)$ when no peasant revolts is positive: i.e. all members of the elite have incentives to choose tax rate $t_1^r(0)$ (when all the other members of the elite choose this tax rate) and, hence, no peasant has incentives to revolt. The second r-equilibrium, $(\{t_2^r(\varepsilon)\}_{i=1}^{\infty}, \varepsilon)$, is possible if and only if the increase in expected payoff from choosing tax rate $t_1^r(\varepsilon)$ instead of tax rate $t_2^r(\varepsilon)$ when a fraction ε of the peasants revolt is negative: i.e. all members of the elite have incentives to choose tax rate $t_2^r(\varepsilon)$ and, hence, only those peasants that work on a unit of land that suffers an output shock revolt. Finally, if for some $x \in [0, \varepsilon]$ we have that $\Delta u^e(x) = 0$ then when exactly a fraction x of the peasants revolt, members of the elite are indifferent between setting tax rate $t_1^r(x)$ and tax rate $t_2^r(x)$. Hence, if a fraction $1 - \frac{x}{\varepsilon}$ of the members of the elite choose tax rate $t_1^r(x)$ and the rest choose tax rate $t_2^r(x)$ then exactly a fraction x of the peasants have incentives to revolt.

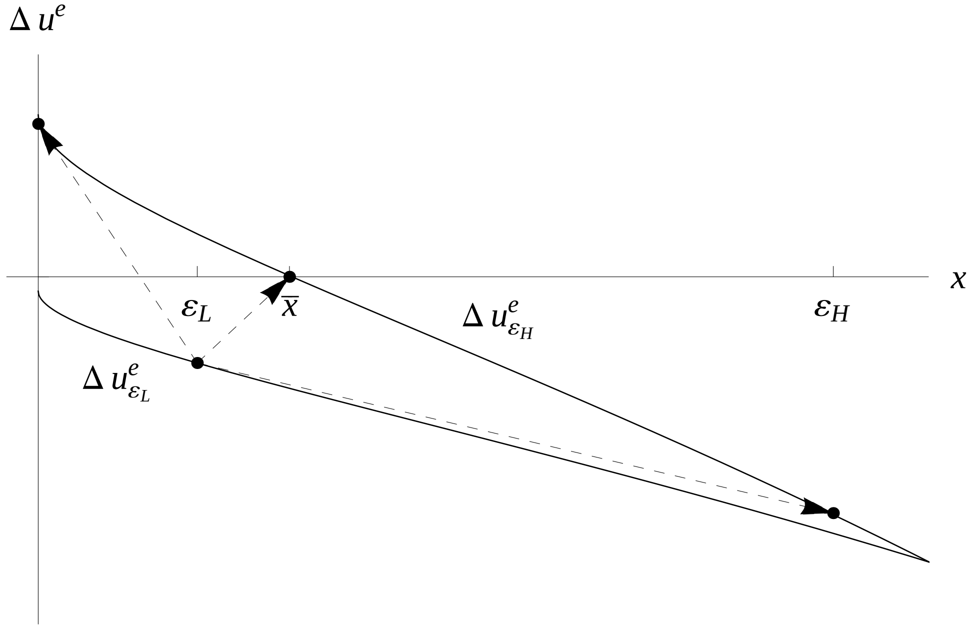
As stated by Proposition 2, a necessary but not sufficient condition for r-equilibrium 2 to exist is that $t_2^r(\varepsilon) \geq 0$. This is a requirement as otherwise the elite cannot set up tax rate $t_2^r(\varepsilon)$ given that tax rates to belong to the interval $[0, 1]$. As sufficient condition for $t_2^r(\varepsilon) \geq 0$ is that $\gamma(\varepsilon) \leq \frac{1}{2\beta}$ (see Lemma 1 in the Appendix).

For most of the rest of this section we shall focus our attention away from r-equilibrium 4 and concentrate on r-equilibria 1-3. Nevertheless, a further discussion on r-equilibrium 4 is presented at the end of this section. In order to obtain a graphical representation on when r-equilibria 1-3 are possible, we present Figures 3-5, where the parameter values are set to $\beta = 0.95$, $y = 0.8$ and $\mu_\varepsilon = 0.2$, and ε takes two possible values: $\varepsilon_L = 0.05$ and $\varepsilon_H = 0.25$. In Figures 3 and 4 the cost of working the land is set to $c = 0.2$ while in Figure 5 this value is set to $c = 0.5$. As it can be checked using Lemma 1 in the Appendix, in all three figures it is true that $t_2^r(\varepsilon) \geq 0$. Each figure plots the function Δu^e from Proposition 2 for each of the two values of ε: $\Delta u^e_{\varepsilon_k}$ equals function Δu^e when $\varepsilon = \varepsilon_k$ with $k \in \{L, H\}$.

In Figure 3 we assume that the probability of a revolution to be successful given the proportion of the peasants that join the revolt is given by $\gamma(x) = \sqrt{x}$. In Figure 3, we can see that $\Delta u^e_{\varepsilon_L}(0) < 0$, $\Delta u^e_{\varepsilon_L}(\varepsilon_L) < 0$ and there is no $x \in [0, \varepsilon_L]$ such that $\Delta u^e_{\varepsilon_L}(x) = 0$. Hence, by Proposition 2 the unique r-equilibrium (apart from r-equilibrium 4) is r-equilibrium 2. If $\varepsilon = \varepsilon_H$ then we have that $\Delta u^e_{\varepsilon_H}(0) > 0$, $\Delta u^e_{\varepsilon_H}(\varepsilon_H) < 0$ and $\Delta u^e_{\varepsilon_H}(\bar{x}) = 0$. Hence, by Proposition 2 we have that r-equilibria 1-3 are possible. In Figure 3 each possible r-equilibrium is represented by a dot and the arrows point from r-equilibrium 2 when $\varepsilon = \varepsilon_L$ to r-equilibria 1-3 when $\varepsilon = \varepsilon_H$.

Suppose that at a given critical juncture where the current regime is elitism it is true that $\varepsilon = \varepsilon_L$. Ignoring r-equilibrium 4, this would lead members of the elite to set a tax rate $t_2^r(\varepsilon_L)$. Consider now that the elitist regime survives the revolution initiated by a fraction ε_L of the peasants. Suppose a new critical juncture arrives and assume that this time the society is going through a period of economic crisis that makes output shocks more likely: $\varepsilon_H > \varepsilon_L$. This new critical juncture can lead to three very distinct situations. Firstly, the

Figure 3: $\beta = 0.95$, $y = 0.8$, $c = 0.2$, $\gamma(x) = \sqrt{x}$, $\mu_\varepsilon = 0.2$, $\varepsilon_L = 0.05$ and $\varepsilon_H = 0.25$.



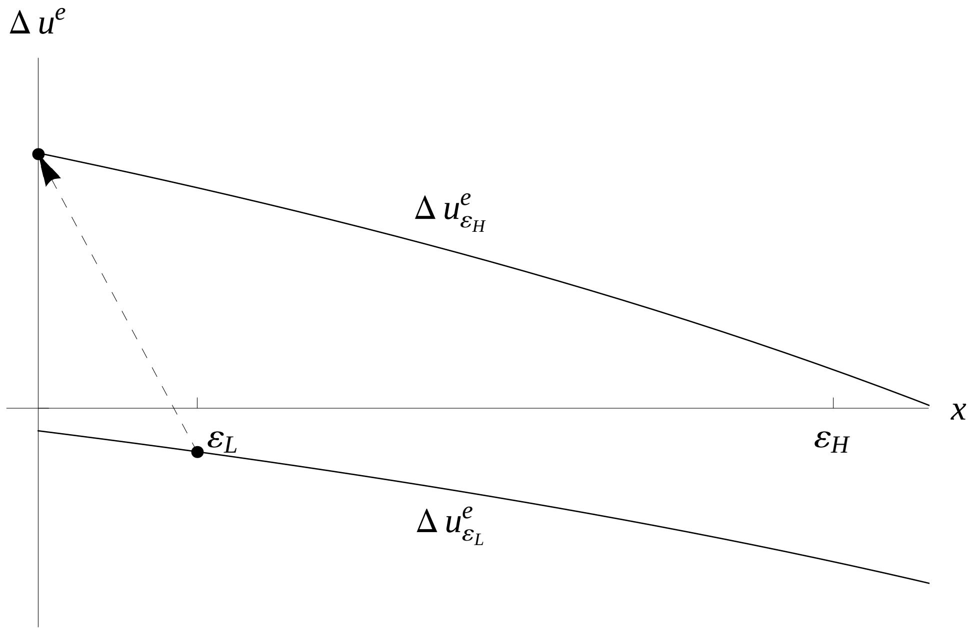
society could move to an r-equilibrium where the tax rate is $t_1^r(0)$ and such that no peasant revolts. Secondly, the society could move to an r-equilibrium where tax rate is $t_2^r(\varepsilon_H)$ and where the chances of the revolution to be successful are $\gamma(\varepsilon_H) = 50\%$ and, therefore, a regime change is likely. Thirdly, the society could move to a situation where each unit of land belongs to either one of two different groups: one where tax rate $t_1^r(\bar{x})$ is in place and as such no peasant revolts, and another one where tax rate $t_2^r(\bar{x})$ is in place and where a fraction ε_H of the peasants revolts.

Hence, the arrival of a critical juncture when the frequency of shocks increases creates different alternatives in the possible history that the society could follow. The critical juncture may lead to a more extractive and repressing regime where no peasant revolts, to a more inclusive environment where regime change is likely, or to segregation between the different units of land. In this respect, our model illustrates Acemoglu and Robinsons' (2013) statement that "A critical juncture is a double-edged sword that can cause a sharp turn in the trajectory of a nation. On the one hand it can open the way for breaking the cycle of extractive institutions and enable more inclusive ones to emerge... Or it can intensify the emergence of extractive institutions...".

The reason why the frequency of shocks affects the r-equilibria that are possible in this case is the following. During times of expansion (ε_L), if a member of the elite charges the high tax rate t_2^r then the probability that his peasant revolts is small given that output shocks are infrequent. Hence, members of the elite are better off by charging a high tax rate and having the risk that their peasant revolts than by charging a smaller tax rate that ensures that their peasant does not revolt. On the other hand, during times of economic crisis (ε_H), members of the elite would like to set up a tax rate so that no peasant revolts (r-equilibrium 1) as the number of the peasants that could revolt in this case is more significant and, hence, a regime change would be likely. However, if peasants coordinate and many of them revolt then the elite can only stop from revolting those that do not suffer an output shock (r-equilibrium 2). An intermediate case also exists where in some units of land no peasant revolts and in some others a fraction of the peasants revolt (r-equilibrium 3).

Figure 4 represents the same situation as Figure 3 when instead $\gamma(x) = x$. As $x \leq \sqrt{x}$ for all $x \in [0, 1]$, a successful revolution is harder to achieve compared to the case depicted in Figure 3. As we can see in Figure 4 , if $\varepsilon = \varepsilon_L$ then the unique r-equilibrium (apart from r-equilibrium 4) is again r-equilibrium 2, where a fraction ε_L of the peasants revolt and the tax rate is set to $t_2^r(\varepsilon_L)$. However, if $\varepsilon = \varepsilon_H$ then only r-equilibrium 1 is possible among r-equilibria 1-3. Hence, the fact that a successful revolution is harder to achieve combined with a period of recession suggests that at a critical juncture the society could move to more extractive institutions where no peasant revolts.

Figure 4: $\beta = 0.95$, $y = 0.8$, $c = 0.2$, $\gamma(x) = x$, $\mu_\varepsilon = 0.2$, $\varepsilon_L = 0.05$ and $\varepsilon_H = 0.25$.

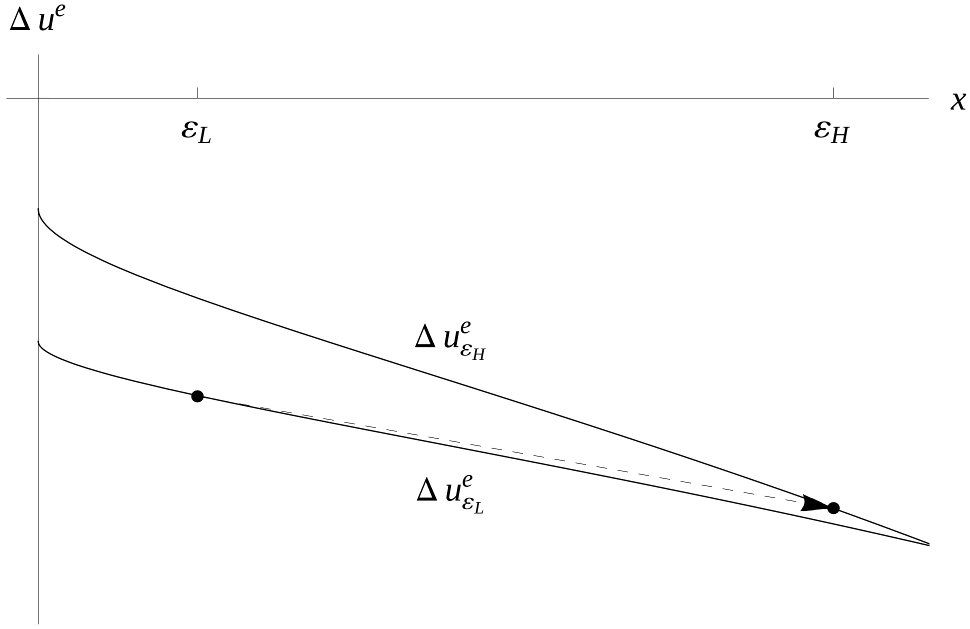


Comparing Figures 3 and 4 we can deduce that if a successful revolution is harder to achieve then peasants are less likely to revolt and over-rule the elite. That is, a less effective technology of the revolution leads to a decrease in the likelihood that the regime changes.

This is true not only because those peasants that revolt are less effective at doing so, but also because less peasants have incentives to join the revolution.

Figure 5 plots the same situation as Figure 3 when instead the cost of working the land is given by $c = 0.5$. As we can observe, there is a unique r-equilibrium (except for r-equilibrium 4) given by $(\{t_2^r(\varepsilon)\}, \varepsilon)$. Hence, with respect to Figure 3, increasing the cost of working the land implies that the r-equilibrium where no peasant revolts is less likely to be present. This is intuitive as if c increases then peasants enjoy less benefit from the land and, hence, are more likely to revolt as the potential loss if the revolution is unsuccessful is lower. This also suggests that societies that enjoy lower land profitability $(Ey - c)$ are less prone to equilibria multiplicity as r-equilibria 1 and 3 are less likely to be present.

Figure 5: $\beta = 0.95$, $y = 0.8$, $c = 0.5$, $\gamma(x) = \sqrt{x}$, $\mu_\varepsilon = 0.2$, $\varepsilon_L = 0.05$ and $\varepsilon_H = 0.25$.



In all three figures above we have ignored r-equilibrium 4, which is the equilibrium where all peasants revolt. Proposition 2 states that r-equilibrium 4 is possible if and only if $\frac{\beta}{1-\beta}V^c > 1-c$, a condition that is satisfied for most sensible parameter values. For example, if $\beta = 0.95$ then r-equilibrium 4 is possible unless land profitability, $Ey-c$, is more than 380 times smaller than the maximum profit of a unit of land, $1-c$. The reasons we have chosen to focus on r-equilibrium 4 is that this equilibrium is present in all the graphs plotted above. This is not surprising, however, if all peasants revolt then the revolution is successful and, hence, every single peasant wants to revolt.[10]

[10]Unless his discount factor is such that he prefers the one-off payoff from working the land today instead

A question that may arise is that of whether or not it is possible for r-equilibrium 4 to be the unique equilibrium. Our next result states that this is a possibility:

Proposition 3. *If* $(1-\beta)y + \beta Ey < c$ *then r-equilibrium 4 is the unique equilibrium.*

If r-equilibrium 4 is the unique equilibrium then the regime change happens with probability one and the society turns to a populist regime where the population consists of citizens.

3.2 Analysis: Coups

In this section we analyze the second component of the Markov Perfect Equilibrium. We refer to the tuples $(\{t_i^c\}_{i=1}^{\infty}, z)$ that are part of a Markov Perfect Equilibrium as c-equilibria. The next result characterizes the set of c-equilibira and states the conditions under which each of the possible c-equilibria can be present.

Proposition 4. *For any value of* ε *there always exists a c-equilibrium and there are at most three possible c-equilibria:*

- *c-equilibrium 1:* $(\{t^c\}_{i=1}^{\infty}, 0)$ *for any* $t^c \in [0,1]$. *This c-equilibrium exists if and only if*

$$(1-\beta)y + \beta Ey - c \quad \geq \quad 0.$$

- *c-equilibrium 2:* $(\{t^c\}_{i=1}^{\infty}, \varepsilon)$ *for any* $t^c \in (t_1^c(\varepsilon), t_2^c(\varepsilon)]$. *This c-equilibrium exists if and only if*

$$(1-\beta)y + \beta(1-2\rho(\varepsilon))Ey - c \quad < \quad 0.$$

- *c-equilibrium 3:* $(\{t^c\}_{i=1}^{\infty}, 1)$ *for any* $t^c > t_2^c(1)$. *This c-equilibrium exists if and only if*

$$(1-\beta) - \beta Ey - c \quad < \quad 0.$$

In c-equilibrium 1 (stable democracy) we have that no citizen attempts a coup and the society continues to function under the populist regime after the critical juncture with probability one.[11] In this situation, citizens do not have incentives to stage a coup for one main reason: no other citizen joins the coup. c-equilibrium 1 is not possible if $y-c+\frac{\beta}{1-\beta}[Ey-c] < 0$ or, in other words, if the expected payoff of a citizen who works on a unit of land that suffers a production shock is negative. In this case, each citizen working on a unit of land that suffers an output shock would be better off staging a coup even if he is the only citizen doing so. This

of the future stream of payoffs from eliminating the elite and becoming a citizen.

[11]Note that although we make use of the term democracy the model has no democratic process built in. We chose this name given that under populism there is no elite and all citizens are the same.

woulds imply that all citizens that suffer an output shock stage a coup, which contradicts the fact that in a c-equilibrium 1 no citizen stages a coup.

The behavior of most democracies in the western world fall into this type of equilibrium. After a critical juncture, like the financial crisis in 2008, the country continues to function as a democracy. The role of a financial crisis in the institutions in a democratic regime is discussed further after we describe c-equilibrium 2 and c-equilibrium 3.

The second c-equilibrium, c-equilibrium 2 (unstable democracy), is such that those citizens that work on a unit of land that suffers an output shock join the coup. In this case, the tax rate to be set by the future elite is such that only a fraction of the citizens want to stage a coup. The coup is successful at changing the current regime back to elitism with a probability of $\rho(\varepsilon)$. This type of equilibrium is observed in democratic countries where a critical juncture has caused a civil war, like the civil war that started in South Sudan in 2013 after a former minister was removed from his duties and staged a coup.[12]

Finally, in c-equilibrium 3 (regime change) citizens from all units of land join the coup and a regime change happens with probability 1. The regime change implies that from next period on the society is back elitism. This equilibrium is present for most sensible parameter values because if all citizens join the coup then the coup will succeed with probability one.[13] Hence, the only factor that may stop a citizen from joining the coup is a very low discount factor β, so that the cost of not having any output today is greater than the future benefits of joining the elite. The fact that c-equilibrium 3 is a possibility for most sensible parameter values should not be surprising and has a very natural interpretation: even in the most robust democracies in the developed world, if all politically active people coordinate to change the regime there will be a regime change.

Proposition 4 also implies the following. Suppose that a severe economic crisis causes a critical juncture. In the language of the model, the fact that the economic crisis is severe implies that ε is high, which in turn implies that Ey is low. This makes c-equilibrium 1 less likely and c-equilibrium 3 more likely. However, whether c-equilibrium 2 is more or less likely depends on how likely the coup is to succeed given the number of citizens that join the coup (function ρ).

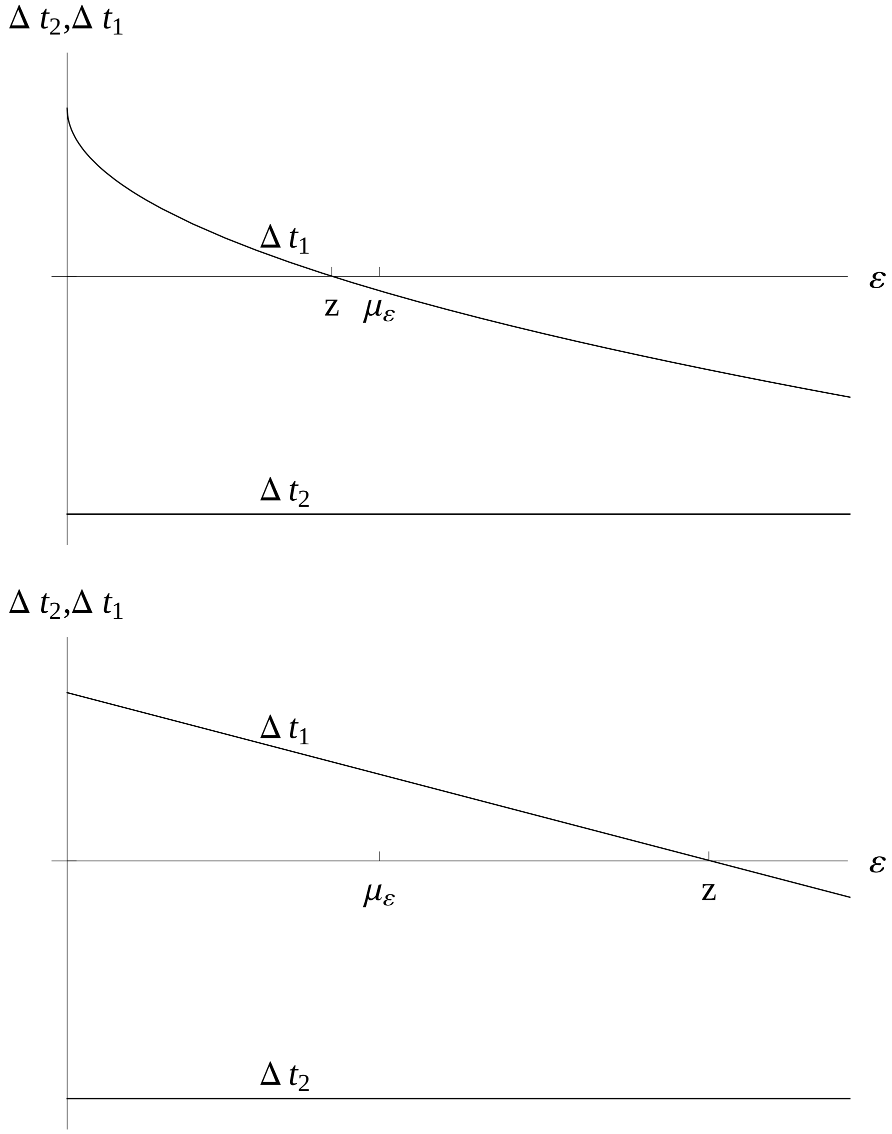
In order to get a better understanding of the result in Proposition 4, define the functions $\Delta t_1(\varepsilon) = (1-\beta)y + \beta(1-2\rho(\varepsilon))Ey - c$ and $\Delta t_2 = (1-\beta) - \beta Ey - c$. Thus, according to

[12] See, for instance, `http://www.aljazeera.com/video/africa/2013/07/20137287019670555.html` and `http://www.trust.org/item/20131223195244-2j16n/?source=hptop#`.

[13] With regard to the interpretation of the model, note that all citizens joining the coup means that in every unit of land all those who are politically active attempt a regime change to turn the rest (politically inactive people) into peasants.

Proposition 4, c-equilibrium 1 is possible if and only if $\Delta t_1(0) \geq 0$, c-equilibrium 2 is possible if and only if $\Delta t_1(\varepsilon) < 0$ and, finally, c-equilibrium 3 is possible if and only if $\Delta t_2 < 0$. Figure 6 below depicts the functions Δt_1 and Δt_2 where in the top graph we set $\rho(z) = \sqrt{z}$ and in the bottom graph we set $\rho(z) = z$. Both figures use the same parameter values as Figures 3 and 4.

Figure 6: $\beta = 0.95$, $y = 0.8$, $c = 0.2$, $\rho(x) = \sqrt{x}$ (top), $\rho(x) = x$ (bottom), $\mu_\varepsilon = 0.25$.



Both plots in Figure 6 depict a similar situation: c-equilibrium 1 and c-equilibrium 3 are both possible while c-equilibrium 2 is possible if and only if ε is high enough at the critical juncture. The difference between the plot at the top of the figure and the plot at the bottom is that the infimum value of ε for which c-equilibrium 2 is possible (referred to as z in the graph) is lower under the function $\rho = \sqrt{z}$ (top) than under the function $\rho(z)$ (bottom). That is, c-equilibrium 2 is more likely to exist under the function $\rho = \sqrt{z}$ than under the function $\rho = z$. The interpretation of this is that if a coup is easier to succeed given how many citizens join it, then ceteris paribus citizens have more incentives to join the coup. In c-equilibrium 2 not all citizens want to join the coup because although the are some chances that the coup is successful, there is still the possibility that the coup fails. Thus, when the citizens that work on a unit of land that does not suffer an output shock decide whether to join the coup or not, the fact that the coup may fail makes them not to want to join the coup. The citizens that work on a unit of land that suffers and output shock have less to lose (lower opportunity cost from joining the coup) and, hence, the fact that the coup has some chances of succeeding means that they have enough incentives to join the coup.

The function ρ is interpreted as how likely a coup is to succeed given how many citizens join it. Therefore, just as it is the case with function γ, the function ρ could have information on how armed those that stage a coup are or whether or not those that stage the coup receive any type of external support from other countries or organizations (funds, food, use of military equipment, etc.). As it was the case with function γ in the previous section, we observe a significant role played by the function that represents how likely a coup is to succeed.

3.3 Example of a Markov Perfect Equilibrium

In sections 3.1 and 3.2 we have studied each of the two components of all possible Markov Perfect Equilibria separately. In this section we present an example of a full description of a Markov Perfect Equilibrium using the parameter values that we have used in Figures 3 and 6.

Example. *Assume that $\beta = 0.95$, $y = 0.8$, $c = 0.2$, $\gamma(x) = \sqrt{x}$, $\rho(x) = \sqrt{x}$, $\mu_\varepsilon = 0.2$, $\underline{\varepsilon} = 0.1$ and $\overline{\varepsilon} = 0.3$. A Markov Perfect Equilibrium is given by:*

$$\{(\{t_i^r\}_{i=1}^{\infty}, x), (t^c, z)\}_\varepsilon = \begin{cases} \{(\{t_2^r(\varepsilon)\}_{i=1}^{\infty}, \varepsilon), (t^c, 0)\}_\varepsilon & \text{if } \varepsilon \in [\underline{\varepsilon}, \mu_\varepsilon], \\ \{(\{t_i^r\}_{i=1}^{\infty}, 1), (t^c, \varepsilon)\}_\varepsilon & \text{if } \varepsilon \in (\mu_\varepsilon, \overline{\varepsilon}], \end{cases}$$

for any $\{t_i^r\}_{i=1}^{\infty}$ with $t_i^r \in [0,1]$ for all i and any $t^c > t_2^c(1)$.

In the example above we have that under the elitist regime, if a critical juncture arrives and the economy is going through a period of expansion ($\varepsilon \leq \mu_\varepsilon$) then only a few peasants revolt. If, however, the economy is going through a period of contraction ($\varepsilon > \mu_\varepsilon$) then all peasants revolt and a regime change happens with probability one. On the other hand, if the current regime is populism and a critical juncture arrives during a time of economic expansion, no citizen will stage a coup and populism will continue with probability one. Finally, under populism, if a critical juncture arrives and the economy is going through a period of contraction then a few citizens will stage a coup and try to reinstate the elitist regime. This example illustrates how the society could transition between regimes depending on the current circumstances at the time the critical juncture arrives.

Note that the outcome of a critical juncture can have long lasting effects, spanning several future critical junctures. In the example above, suppose that the initial regime is elitism. If at the first critical juncture the economy is going through a period of recession, the regime changes to populism. Such populist regime could survive for many future critical junctures, provided that conditions during future critical junctures are appropriate. If, on the other hand, during the first critical juncture the economy is going through a period of expansion and the few that join the revolution fail at changing the regime, the elitist regime could itself survive for many future critical junctures. This illustrates the phenomenon of path dependence; as Skocpol and Pierson (2002) put it, "outcomes at a critical juncture trigger feedback mechanisms that reinforce the recurrence of a particular pattern into the future …once actors have ventured far down a particular path, they are likely to find it very difficult to reverse course".[14] In our model, the feedback mechanism Skocpol and Pierson (2002) refer to is the current regime at the critical juncture.

Note that the Markov Perfect Equilibrium above is simply an example, as for the parameter values used there is not a unique Markov Perfect Equilibrium. The purpose of this paper is not to predict the outcome of a critical juncture but to understand the different possible scenarios that could arise after a critical juncture and to improve our knowledge of the intuition behind each of these different possibilities, as well as to study how the society can transition between different regimes.

Given the nature of the Markov Perfect Equilibrium and the fact that the current value of ε is a state variable, for each value of ε the Markov Perfect Equilibrium specifies a unique r-equilibrium and a unique c-equilibrium. It could be that at two different critical junctures the society evolves differently depending on the equilibrium it selects even if both state variables (ε and current regime) are the same. We do not explicitly allow for this possibility but the model presented here is rich enough to accommodate for this alternative: one simply has to look at what are the possible r-equilibria and c-equilibria for the different values of ε.

[14] For more information on the relationship between critical junctures and path dependence see Stockpol (2000).

4 Discussion

4.1 Tax on Profits: $c = 0$

A particular case of our model is the situation where $c = 0$. If $c = 0$ the model can be reinterpreted as one where y_i is not output but the profit of each unit of land i. A salient feature of the model when $c = 0$ is that the equilibria where no peasant revolts and no citizen joins the coup are always possible. This is our next result:

Proposition 5. *If $c = 0$ then the tuple $(\{t_1^r(0)\}, 0)$ is always a r-equilibrium and the tuple $(\{t^c\}_{i=1}^{\infty}, 0)$ is always an c-equilibrium.*

The intuition for the result above when the current regime is elitism is that if $c = 0$ then the elite can always set up a tax rate such that no peasant wants to revolt. Consider the extreme situation where the member of the elite in unit of land i sets up a tax rate $t_i^r = 1$. In this case, when no peasant revolts then the peasant working in unit of land i does not have incentives to revolt as if he does then he obtains an expected payoff of zero (the revolution fails for sure), while if he does not revolt then he also gets an expected payoff of zero per period (the elite extracts all his surplus). When $c > 0$ if the output in case of a shock is lower than the cost of working the land, $y < c$, and the peasant is impatient enough, low β, the peasant is better off joining the revolution even if no other peasant revolts: the revolution fails for sure but the peasant who revolts guarantees himself an expected payoff of zero, which is better than a negative payoff.

If the current regime is populism, the fact that $c = 0$ implies that the expected payoff of a citizen is always strictly positive and, hence, if no other citizen joins the coup then staging a coup gives the citizen an expected payoff of zero, as the coup fails for sure. Thus, $c = 0$ implies that when no citizen joins the coup then no other citizen has incentives to stage a coup regardless of the other parameters of the model.

Note that, as already discussed, the fact that no peasant revolts does not mean that the profit of each unit of land is distributed equally between the elite and the peasant. As a matter of fact, when $c = 0$ the equilibrium where no peasant revolts is such that the elite sets up a tax rate as extractive as possible ($t_1^r(0) = 1$). This is because the elite no longer has to compensate the peasant for the costs of exploiting the land as the tax is applied to profit, not just to output.

4.2 Different Tax Rates after a Coup

In this section we relax the assumption by which all citizens that stage a coup commit to the same tax rate. In particular, we now consider the possibility of setting up different tax rates for each unit of land after a successful coup.

If we denote by t_i^c the tax rate set up in unit of land i in case of a successful coup we have that a citizen does not have incentives to join the coup when $\delta \to 0$ if and only if

$$y_i - c + \frac{\beta}{1-\beta}\left[Ey(1 - 2\rho(z)t_i^c) - c\right] \quad \geq \quad 0,$$

which for $\rho(z) > 0$ can be rewritten as

$$t_i^c \quad \leq \quad \frac{(1-\beta)y_i + \beta Ey - c}{2\rho(z)\beta Ey}.$$

In order to accommodate for the fact that different tax rates are allowed after a successful coup, we modify the definition of a Markov Perfect Equilibrium as follows:

Definition 2. *A Markov Perfect Equilibrium with different tax rates after a coup is a tuple* $\{(\{t_i^r\}_{i=1}^{\infty}, x), (\{t_i^c\}_{i=1}^{\infty}, z)\}_{\varepsilon}$ *which for each possible value of* $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$ *specifies a collection of tax rates* $\{t_i^r\}_{i=1}^{\infty}$ *with* $t_i^r \in [0,1]$ *for each unit of land* i*, a fraction of the peasants that revolt* $x \in [0,1]$*, a collection of tax rates* $\{t_i^c\}_{i=1}^{\infty}$ *with* $t_i^c \in [0,1]$ *for each unit of land* i*, and a fraction of the citizens that stage a coup* $z \in [0,1]$ *such that:*

1. *At a critical juncture, if the society is under elitism and the frequency of shocks is* ε*:*
   - *Given* x*, every member of the elite maximizes* $u^e(t_i^r, x)$ *by choosing tax rate* t_i^r*.*
   - *Given* t_i^r*,* y_i *and* x*, the peasant working in unit of land* i *maximizes* $u_i^p(t_i^r, x)$ *by choosing whether to join the revolution or not, and the fraction of the peasants that choose to revolt equals* x*.*
2. *At a critical juncture, if the society is under populism and the frequency of shocks is* ε*:*
   - *Given* $\{t_i^c\}_{i=1}^{\infty}$ *and* z*, every member of the elite maximizes* $u_i^c(t_i^c, z)$ *by choosing whether to join the coup or not, and the fraction of the citizens that choose to join the coup equals* z*.*

As in the previous section, a c-equilibrium refers to the second component of a Markov Perfect Equilibrium: the tuple $(\{t_i^c\}_{i=1}^{\infty}, z)$. We have the following result:

Proposition 6. *For all* $z \in [0,1]$ *there exists a collection of tax rates* $\{t_i^c\}_{i=1}^{\infty}$ *such that* $(\{t_i^c\}_{i=1}^{\infty}, z)$ *is a c-equilibrium.*

The result in Proposition 6 states that for any possible fraction of the citizens that join the coup, we can construct a c-equilibrium where exactly this fraction of the citizens have incentives to join the coup. The intuition is that for every unit of land and fraction of the citizens that join the coup z, there is a tax rate above which a citizen has incentives to join the coup and below which a citizen does not have incentives to join the coup. Thus, by choosing the right collection of tax rates $\{t_i^c\}_{i=1}^{\infty}$ a c-equilibrium can be constructed such that the fraction of the citizens that join the coup is as desired. The result in Proposition 6 is as expected: by choosing different tax rates we are effectively manipulating the expected payoff of each citizen when he chooses to join the coup. Hence, we can choose tax rates such that each citizen behaves a we require.

We believe that a situation where each citizen that joins the coup is allowed to choose a different tax rate if the coup is successful is less realistic than a situation where all citizens that stage a coup commit to a unique tax rate. As discussed before, the tax rate represents the institutions in place and, hence, it can be viewed as the manifesto of the coup. As a result, those that join the coup explicitly adhere to such manifesto by committing to setting up a common tax rate in case the coup is successful.

5 Conclusions

In this paper we studied regime change in the presence of critical junctures and established what are the different types of institutions that could emerge after a critical juncture. We completed our analysis by looking at various examples that illustrated how different factors, such as frequency of output shocks, affect the possible outcomes after a critical juncture.

In our results, we used the different equilibria that may arise in the model to explain different institutional settings that have appeared at various points in time in the history of civilizations. On top of that, we have used several current and historical examples to illustrate how each of the different equilibria of the model matches the different institutions observed in different countries. As opposed to previous literature, we have explicitly considered the cooperation/coordination problem faced by peasants in the presence of an elite. This lead to a richer set of possible equilibria and allowed us to characterize a wide variety of institutions. For instance, we found that an equilibrium where some but not all peasants revolt is possible, and an equilibrium where there is segregation, are both possible. We have deliberately left open the question of which of the specific equilibria is selected because we do believe such equilibrium selection depends of minor institutional differences not modeled here (also know as institutional drift, see Acemoglu and Robinson (2011)).

In this paper we have focused on what happens after a critical juncture arrives. Future work could look at the causes of a critical juncture and how these came to exists, or to study empirically the various variables that contribute to the appearance of a critical juncture.

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Appendix

Proof of Proposition 1. Any r-equilibrium $(\{t_i^r\}_{i=1}^{\infty}, x)$ where x is such that $t_2^r(x) < 0$ must have $x = 1$, as if x peasants revolt and $t_2^r(x) < 0$ then all peasants have incentives to revolt regardless of the tax rate set by members of the elite (see equation (2)). Thus, in r-equilibrium either $(\{t_i^r\}_{i=1}^{\infty}, 1)$ with $t_2^r(1) < 0$ or $(\{t_i^r\}_{i=1}^{\infty}, x)$ with $t_2^r(x) \geq 0$.

If $t_2^r(1) < 0$ and all peasants revolt then the tax rates set by the elite $\{t_i^r\}_{i=1}^{\infty}$ are irrelevant as there is no tax rate that will keep a peasant from revolting. Furthermore, if $x = 1$ and $t_2^r(1) < 1$ then all peasants have incentives to revolt regardless of the tax rate. This is r-equilibrium 4, $(\{t_i^r\}_{i=1}^{\infty}, 1)$, in the proposition.

Consider r-equilibria $(\{t_i^r\}_{i=1}^{\infty}, x)$ with $t_2^r(x) \geq 0$. If $t_1^r(x) < 0$ then it must be that all members of the elite set a tax rate equal to $t_2^r(x)$ (as it can be deduced from equation (3),

$u^e(t_i^r, x)$ has a maximum in $t_2^r(x)$ whenever $t_2^r(x) \geq 0$ and $t_1^r(x) < 0$). If all members of the elite choose tax rate $t_2^r(x)$ and $x > \varepsilon$ then there are peasants that revolt even though their land did not suffer an output shock. If this is true then it must be that all peasants revolt; this leads to the only possibility of $(\{t_2^r(1)\}_{i=1}^{\infty}, 1)$, which is a particular case of r-equilibrium 4. If, on the other hand, $x < \varepsilon$, then there are peasants that do not revolt even though their land suffered an output shock, a contradiction to the definition of $t_2^r(x)$. Thus, if all members of the elite set up tax rate $t_2^r(x)$ then the only possibility is $x = \varepsilon$ so that the chosen tax rate is given by $t_2^r(\varepsilon)$. This is r-equilibrium 2 in the proposition.

Consider now r-equilibria $(\{t_i^r\}_{i=1}^{\infty}, x)$ with $t_1^r(x) \geq 0$. We have that members of the elite either set a tax rate equal to $t_1^r(x)$ or to $t_2^r(x)$. This can be seen easily by looking at equation (3) and noting that $u^e(t_i^r, x)$ has a maximum in either $t_1^r(x)$ or $t_2^r(x)$ whenever $t_1^r(x) \geq 0$. We have now three possibilities: all members of the elite choose tax rate $t_1^r(x)$, all choose $t_2^r(x)$, or some choose $t_1^r(x)$ and others choose $t_2^r(x)$. We have already dealt with the case where all members of the elite set tax rate $t_2^r(x)$ in the paragraph above. Thus, we focus now on the situations where either all members of the elite choose tax rate $t_1^r(x)$, or some choose $t_1^r(x)$ and others choose $t_2^r(x)$.

In any r-equilibrium $(\{t_1^r(x)\}_{i=1}^{\infty}, x)$ it must be that $x = 0$ as if $x > 0$ and all members of the elite set up tax rate $t_1^r(x)$ then when a fraction x of the peasants revolt, by the definition of $t_1^r(x)$ in equation (1) no peasant has incentives to revolt, which represents a contradiction to the fact that $x > 0$. Thus, the only possibility is that $(\{t_1^r(0)\}_{i=1}^{\infty}, 0)$, which is r-equilibrium 1 in the proposition.

Finally, consider any r-equilibrium $(\{t_i^r\}_{i=1}^{\infty}, x)$ where some members of the elite choose $t_1^r(x)$ and others choose $t_2^r(x)$. Then it must be that exactly a fraction $1 - \frac{x}{\varepsilon}$ of the members of the elite choose tax rate $t_1^r(x)$ and a fraction $\frac{x}{\varepsilon}$ choose tax rate $t_2^r(x)$. Otherwise, if fraction $\delta \neq \frac{x}{\varepsilon}$ of the members of the elite choose tax rate $t_2^r(x)$ then a proportion $\delta\varepsilon \neq x$ of the peasants choose to revolt (those peasants working on a unit of land subject to tax rate $t_2^r(x)$ and where there is an output shock). This represents a contradiction to the fact that $(\{t_i^r\}_{i=1}^{\infty}, x)$ is an r-equilibrium. Thus, the only possibility is that $(\{t_i^r\}_{i=1}^{\infty}, x)$ where a fraction $1 - \frac{x}{\varepsilon}$ of the members of the elite choose tax rate $t_1^r(x)$ and the rest choose tax rate $t_2^r(x)$. This is r-equilibrium 3 in the proposition, which is only possible if $x \leq \varepsilon$ and no member of the elite that chooses tax rate $t_1^r(x)$ wants to choose tax rate $t_2^r(x)$ and vice-versa, i.e. $u^e(t_1^r(x), x) = u^e(t_2^r(x), x)$. □

Proof of Proposition 2. We prove the result separately for each of the different r-equilibria and then prove that at least one of them is always present.

1. $(\{t_1^r(0)\}_{i=1}^{\infty}, 0)$:

From equation (2), we have that

$$(1-t_1^r(0))y-c \quad = \quad \frac{(1-t_1^r(0))Ey-c}{1-\beta}$$

Hence, if $t_1^r(0)=0$ then the right hand side of the equation above is greater than the left hand side, while if $t_1^r(0)=1$ then the opposite occurs. Thus, since the expression above is continuous in $t_1^r(0)$, by Bolzano's Theorem we have that $t_1^r(0)\in[0,1]$.

The tuple $(\{t_1^r(0)\}_{i=1}^{\infty},0)$ is an r-equilibrium if both no peasant wants to revolt and no member of the elite wants to set up tax rate $t_2^r(0)$. By definition, if a fraction 0 of the peasants revolt then $t_1^r(0)$ is such that no peasant wants to revolt.

Moreover, no member of the elite wants to set up tax rate $t_2^r(0)$ if and only if $u^e(t_1^r(0),0)\geq u^e(t_2^r(0),0)$. This can be rewritten as $\Delta u^e(0)\geq 0$. The explicit functional form for this condition $\Delta u^e(0)\geq 0$ is

$$1-\frac{c+\beta\gamma(0)(Ey-2c)}{(1-\beta)y+\beta(1-\gamma(0))Ey}\varepsilon y-\left(\frac{c+\beta\gamma(0)(Ey-2c)}{(1-\beta)y+\beta(1-\gamma(0))Ey}-\frac{c+\beta\gamma(0)(Ey-2c)}{(1-\beta)+\beta(1-\gamma(0))Ey}\right)$$
$$\left[(1-\varepsilon)+(1-\gamma(0))\frac{\beta Ey}{1-\beta}\right]\geq 0.$$

2. $(\{t_2^r(\varepsilon)\}_{i=1}^{\infty},\varepsilon)$:

   Firstly, it must be that $t_2^r(\varepsilon)\geq 0$ as otherwise the elite is not able to set up tax rate $t_2^r(\varepsilon)$. The tuple $(\{t_2^r(\varepsilon)\}_{i=1}^{\infty},\varepsilon)$ is an r-equilibrium if both no peasant wants to revolt and no member of the elite wants to set up tax rate $t_1^r(\varepsilon)$. By definition, if a fraction ε of the peasants revolt then $t_2^r(\varepsilon)$ is such that only those peasant who suffer an output shock revolt, i.e. a fraction ε of the peasants want to revolt.

   Moreover, no member of the elite wants to set up tax rate $t_1^r(\varepsilon)$ if and only if $u^e(t_2^r(\varepsilon),\varepsilon)\geq u^e(t_1^r(\varepsilon),\varepsilon)$. This can be rewritten as $\Delta u^e(\varepsilon)\leq 0$. The explicit functional form for this condition is

$$1-\frac{c+\beta\gamma(\varepsilon)(Ey-2c)}{(1-\beta)y+\beta(1-\gamma(\varepsilon))Ey}\varepsilon y-\left(\frac{c+\beta\gamma(\varepsilon)(Ey-2c)}{(1-\beta)y+\beta(1-\gamma(\varepsilon))Ey}-\frac{c+\beta\gamma(\varepsilon)(Ey-2c)}{(1-\beta)+\beta(1-\gamma(\varepsilon))Ey}\right)$$
$$\left[(1-\varepsilon)+(1-\gamma(\varepsilon))\frac{\beta Ey}{1-\beta}\right]\leq 0.$$

3. $(\{t_i^r\}_{i=1}^{\infty},x)$ where a fraction $1-\frac{x}{\varepsilon}$ of the elite chooses tax rate $t_1^r(x)$ and the rest choose tax rate $t_2^r(x)$:

   By definition, if a fraction x of the peasants revolt then $t_1^r(x)$ and $t_2^r(x)$ are such that no peasant on a unit of land where the tax rate is $t_1^r(x)$ wants to revolt and only those peasants that work on a unit of land where the tax rate is $t_2^r(x)$ and that suffer an output shock revolt. Thus, since a fraction $\frac{x}{\varepsilon}$ of the members of the elite set up tax rate $t_2^r(x)$ and a fraction ε of those have their peasants revolt, we have that a fraction x of the peasants revolt, as required.

Moreover, no member of the elite wants to set up a tax rate different from the one he is currently setting if and only if $u^e(t_1^r(x), x) = u^e(t_2^r(x), x)$, which can be rewritten as $\Delta u^e(x) = 0$. The explicit functional form for this condition is

$$1 - \frac{c + \beta\gamma(x)(Ey - 2c)}{(1-\beta)y + \beta(1-\gamma(x))Ey}\varepsilon y - \left(\frac{c + \beta\gamma(x)(Ey - 2c)}{(1-\beta)y + \beta(1-\gamma(x))Ey} - \frac{c + \beta\gamma(x)(Ey - 2c)}{(1-\beta) + \beta(1-\gamma(x))Ey}\right)$$
$$\left[(1-\varepsilon) + (1-\gamma(x))\frac{\beta Ey}{1-\beta}\right] = 0.$$

Finally, we must show that both $t_1^r(x)$ and $t_2^r(x)$ are positive as otherwise this r-equilibrium is not possible. If x is such that $u^e(t_1^r(x), x) = u^e(t_2^r(x), x)$ then it must be that

$$\varepsilon t_1^r(x)y \quad = \quad (1-\varepsilon)(t_2^r(x) - t_1^r(x)) + \beta(1-\gamma(x))[V^e(t_2^r(x) - V^e(t_1^r(x))].$$

Since $t_1^r(x) \leq t_2^r(x)$ for all $x \in [0, 1]$ implies $V^e(t_2^r(x)) - V^e(t_1^r(x)) \geq 0$ then it is true that $(1-\varepsilon)(t_2^r(x) - t_1^r(x)) + \beta(1-\gamma(x))[V^e(t_2^r(x) - V^e(t_1^r(x))] \geq 0$ for all x and, hence, $\varepsilon t_1^r(x)y \geq 0$. This implies that $t_1^r(x) \geq 0$ and, as $t_1^r(x) \leq t_2^r(x)$, it is also true that $t_2^r(x) \geq 0$ as required.

4. $(\{t_i^r\}_{i=1}^{\infty}, 1)$ is an r-equilibrium for any $\{t_i^r\}_{i=1}^{\infty}$:

   From the proof of Proposition 1, the tuple $(\{t_i^r\}_{i=1}^{\infty}, 1)$ is an r-equilibrium for any $\{t_i^r\}_{i=1}^{\infty}$ if and only if $t_2^r(1) < 0$. Hence, the definition of V^c and equation (2) when $x = 1$ together with the fact that $\gamma(1) = 1$ leads to the desired result.

For the proof that at least one the four possible r-equilibria is always present, consider first the case where $t_2^r(\varepsilon) < 0$. In this situation, we have that if ε peasants revolt then all peasants revolt as no tax rate can keep a peasant from revolting. Hence, since for any given tax rate the incentives to revolt are increasing in the number of peasants that revolt, $t_2^r(\varepsilon) < 0$ implies that if all peasants revolt then all peasants have incentives to revolt and the elite can do nothing to stop peasants from revolting. This means that if $t_2^r(\varepsilon) < 0$ then r-equilibrium 4 exists.

Consider now the case with $t_2^r(\varepsilon) \geq 0$. If $t_1^r(\varepsilon) < 0$ then $(\{t_2^r(\varepsilon), \varepsilon\})$ is an r-equilibrium as if ε peasants revolt members of the elite maximize their expected payoff by choosing $t_2^r(\varepsilon)$. If $t_2^r(\varepsilon) \geq 0$ and $t_1^r(\varepsilon) \geq 0$ and neither $(\{t_1^r(0), 0\})$ nor $(\{t_2^r(\varepsilon), \varepsilon\})$ are r-equilibria it must be because $\Delta u^e(0) < 0$ and $\Delta u^e(\varepsilon) > 0$ (see the first part of the proof). However, since the function $\Delta u^e(x)$ is continuous, by Bolzano's Theorem there exists a $\bar{x} \in (0, \varepsilon)$ such that

$\Delta u^e(\bar{x}) = 0$. Moreover, since $t_1^r(\varepsilon) \geq 0$ and $t_2^r(\varepsilon) \geq 0$ and both $t_1^r(x)$ and $t_2^r(x)$ are decreasing functions in their argument (see equations (1) and (2) and recall that γ is an increasing function), then for all $x \leq \varepsilon$ it must be that $t_1^r(x) \geq 0$ and $t_2^r(x) \geq 0$. In particular, $t_1^r(\bar{x}) \geq 0$ and $t_2^r(\bar{x}) \geq 0$. Thus, the tuple $(\{t_i^r\}_{i=1}^{\infty}, \bar{x})$ where a fraction $1 - \frac{\bar{x}}{\varepsilon}$ of the elite chooses tax rate $t_1^r(\bar{x})$ and the rest choose tax rate $t_2^r(\bar{x})$ is an r-equilibrium. □

Lemma 1. *A sufficient condition for $t_2^r(\varepsilon) \geq 0$ is that $\gamma(\varepsilon) \leq \frac{1}{2\beta}$.*

Proof of Lemma 1. Using equation (2) we have that $t_2^r(\varepsilon) \geq 0$ if and only if

$$(1 - 2\beta\gamma(\varepsilon))(Ey - c) \quad \geq \quad -(1 - \beta)(1 - Ey).$$

Thus, as $Ey - c \geq 0$ and $1 - Ey \geq 0$ a sufficient condition for the inequality above to be satisfied is that $1 - 2\beta\gamma(\varepsilon) \geq 0$, which leads to the condition in the lemma. □

Proof of Proposition 3. r-equilibria 1-3 do not exists if $t_2^r(0) < 0$ as in this case $t_1^r(x) \leq t_2^r(x) < 0$ for all $x \in [0, 1]$ and the elite cannot set up tax rates $t_1^r(x)$ or $t_2^r(x)$ for any $x \in [0, 1]$. Using equation (2) we have that $t_2^r(0) < 0$ if and only if

$$y + \beta(Ey - y) < c.$$

Combining the fact that an r-equilibrium always exists (Proposition 2) with the inequality above gives the desired result. □

Proof of Proposition 4. First note that if a citizen working on a unit of land that does not suffer an output shock has incentives to join the coup then all citizens have incentives to join the coup. On the one hand, the citizens that work on a unit of land that does not suffer an output shock have incentives to join the coup as they are all the same and, hence, if one of them has incentives to join the coup then all of them have incentives to join the coup. On the other hand, the citizens that work on a unit of land that suffers an output shock have always more incentives to join the coup than a citizen working on a unit of land that does not suffer an output shock (see equation (4)).

Similarly, if a citizen working on a unit of land that suffers an output shock does not have incentives to join the coup then no citizen has incentives to join the coup. On the one hand, the citizens that work on a unit of land that suffers an output shock do not have incentives to join the coup as they are all the same and, hence, if one of them does not have incentives to join the coup then none of them has incentives to join the coup. On the other hand, the citizens that work on a unit of land that does not suffer an output shock have always less incentives to join the coup than a citizen working on a unit of land that suffers an output shock (see equation (4)).

Any c-equilibrium $(t^c, 0)$ must be such that $t^c \leq t_1^c(0)$ as otherwise when no citizen joins the coup then all citizens that suffer an output shock have incentives to join the coup and, hence, at least a fraction ε of the citizens join the coup, a contradiction. Since $t^c \leq t_1^c(0)$ if and only if $t_1^c(0) \geq 0$, this c-equilibrium is possible if and only if $(1-\beta)y + \beta Ey - c \geq 0$ (see equation (4)).

In any c-equilibrium (t^c, z) where $z \in (0,1)$ it must be that $z = \varepsilon$ as if $z > \varepsilon$ then there are citizens working on a unit of land that does not suffer an output shock that decide to join the coup. This means that all citizens have incentives to join the coup as argued above and, therefore, we must have that $z = 1$, a contradiction. Similarly, if $z < \varepsilon$ then there are citizens working on a unit of land that suffers an output shock that decide not to join the coup. As discussed above, this implies that no citizen has incentives to join the coup and, therefore, we must have that $z = 0$, a contradiction.

Moreover, any c-equilibrium (t^c, ε) must have $t^c \in (t_1^c(\varepsilon), t_2^c(\varepsilon)]$. Otherwise, if $t^c > t_2^c(\varepsilon)$ then when a fraction ε of the citizens join the coup we have that all citizens have incentives to join the coup and, hence, $z = 1$, a contradiction. By the same token, if $t^c \leq t_1^c(\varepsilon)$ then when a fraction ε of the citizens join the coup we have that no citizen has incentives to join the coup and, hence, $z = 0$, a contradiction. Thus, any c-equilibrium where the fraction of the citizens that join the coup belongs to the interval $(0,1)$ must be such that (t^c, ε) with $t^c \in (t_1^c(\varepsilon), t_2^c(\varepsilon)]$. This c-equilibrium is possible as long as $t_1^c(\varepsilon) < 1$ and $t_2^c(\varepsilon) \geq 0$ (otherwise t^c would be outside the interval $[0,1]$). From equation (5) we have that $t_1^c(\varepsilon) < 1$ if and only if $(1-\beta)y + \beta(1-2\rho(\varepsilon))Ey - c < 0$. Moreover, from equation (6) as $c \in [0, Ey]$ it is always the case that $t_2^c(\varepsilon) \geq 0$.

Finally, any c-equilibrium with $(t^c, 1)$ must be such that $t^c > t_2^c(1)$ as otherwise when all citizens join the coup then all citizens that do not suffer an output shock do not have incentives to join the coup and, hence, at most a fraction ε of the citizens join the coup, a contradiction. This c-equilibrium is possible if and only if $t_2^c(1) \in [0,1)$. From equation (6) we have that $t_2^c(1) < 1$ if and only if $(1-\beta) - \beta Ey - c < 0$. Moreover, as $c \in [0, Ey]$ it is always the case that $t_2^c(1) \geq 0$.

In order to prove that an equilibrium always exists it suffices to observe that if c-equilibrium 1 does not exists then it follows that $(1-\beta)y + \beta Ey - c < 0$, which implies $(1-\beta)y + \beta(1-2\rho(\varepsilon))Ey - c < 0$ and, hence, c-equilibrium 2 exists. $\square$

Proof of Proposition 5. Under elitism, if $c = 0$ and no peasant revolts $(x = 0)$ then equations (1) and (2) imply that $t_1^r(0) = t_2^r(0) = 1$. Thus, $\Delta u^e(0) \geq 0$ and by Proposition 2 no member of the elite prefers tax rate $t_2^r(0)$ over tax rate $t_1^r(0)$.[15] Moreover, since the elite is choosing

[15]We remind the reader that for all $x \in [0,1]$ if $t_1^r(x) = t_2^r(x)$ then the elite is in effect choosing tax rate

tax rate $t_1^r(0) \in [0,1]$, by definition of $t_1^r(0)$ no peasant wants to revolt when a fraction of 0 peasants revolt.

To prove the result under populism simply use the result in Proposition 4 when $c = 0$. $\square$

Proof of Proposition 6. Fix any fraction of the citizens that join the coup $z \in [0,1]$. We claim that a situation where a fraction $\lambda \in \left[\max\left\{1 - \frac{z}{\varepsilon}, \varepsilon - z\right\}, 1 - z\right] \cap [0,1]$ of the citizens set up any tax rate $t_\lambda^c \leq t_1^c(z)$ if they join the coup and the coup is successful, a fraction $z_1 = \frac{(1-\lambda)-z}{1-\varepsilon}$ of the citizens set up any tax rate $t_{z_1}^c \in (t_1^c(z), t_2^c(z)]$ if they join the coup and the coup is successful, and a fraction $z_2 = \frac{z-\varepsilon(1-\lambda)}{1-\varepsilon}$ of the citizens set up any tax rate $t_{z_2}^c \geq t_2^c(z)$ if they join the coup and the coup is successful, is a c-equilibrium.

First, note that given the restrictions on λ we have that $\lambda \in [0,1]$, $z_1 \in [0,1]$ and $z_2 \in [0,1]$. Second, we have that $\lambda + z_1 + z_2 = 1$, i.e. the three fractions account for the whole population of citizens. Third, it is true that the total fraction of the citizens that want to join the coup equals $\varepsilon z_1 + z_2 = z$.

Therefore, given the three tax rates t_λ^c, $t_{z_1}^c$ and $t_{z_2}^c$, and the fact that a fraction z of the citizens join the coup, we have that every citizen maximizes his utility by choosing whether to join the coup and the fraction of the citizens that choose to join the coup equals z, as required. $\square$

$t_1^r(x)$ so that his peasant does not revolt.