Chasing the Gap:
Speed Limits and Optimal Monetary Policy

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Abstract

Speed limit monetary policy rules incorporate a response to the change in the output gap. They have been widely used in the recent DSGE literature, for example in the influential model of Smets and Wouters. However, the optimality of these rules is unclear. Using a simple New-Keynesian DSGE model with habit persistence, we: (a) derive an optimal speed limit policy rule under discretion that has the same functional form as speed limit rules in the recent literature; and (b) show that the type of speed limit policy rule used in the recent literature are not optimal in our model, mainly because they assume a relatively low policy weight on inflation.

Keywords: optimal monetary policy, habit formation, speed limit, DSGE; New Keynesian

JEL Classification: C51, C52, E52, E58

1 Introduction

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Speed limit monetary policy rules, which incorporate a response to the change in the output gap, have been suggested as an alternative to the well-known Taylor rule. Advocates of the use of speed limit rules for the conduct of monetary policy argue that focusing on the change in the output gap rather than on its level may alleviate some of the measurement problems associated with the estimation of the equilibrium level of output (Gramlich, 1999, Orphanides and Williams, 2002, Walsh, 2003, Orphanides, 2003). They also argue that a policy response to the previous value of output, implied by the use of a speed limit rule, might improve the trade-off between inflation and output stability (Walsh, 2003).

This early literature proposed a policy rule that included a speed limit term but not the level of the output gap. This type of policy rule has been shown to be optimal under commitment (McCallum and Nelson, 2004, Blake, 2012). However the value of this result for practical monetary policy is unclear, since it is widely accepted that the framework within which Central Banks operate is closer to discretion than commitment. Walsh (2003) argues that the type of speed limit rule analysed by McCallum and Nelson (2004) can also be shown to be optimal under discretion, if the objective function of the policymaker includes a speed limit instead of an output gap term. While this approach delivers an optimal speed limit rule under discretion, it is unclear how the presence of a speed limit term in the objective function can be justified. As a result, the optimality of speed limit policy rules under discretion remains unclear. This issue has become more pressing in recent years as speed limit policy rules have become more prominent, popularised by their use in influential DSGE models such as Smets and Wouters (2007)[1]. The speed limit policy rules used in the recent literature often differ from those considered in the earlier literature in that they contain both the level and the rate of change of the output gap, in effect augmenting a Taylor rule by adding a speed limit term.

In this paper we argue that this more recent type of speed limit rules

[1] See also An and Schorfeide (2007), Scharnagl et al (2010), Cogley et al (2011), Ilbas (2012), Brendon et at (2013), Theodoridis et al (2013) and Kapinos and Hanson (2013), among others.

can be shown to be optimal under discretion. The key to our derivation is the assumption of habit persistence in household utility (Fuhrer, 2000, Amato and Laubach, 2004). The intuition for our derivation is very simple. An optimal monetary policy rule is obtained by combining the aggregate demand relationship with the optimality condition for monetary policy; if either of these contains speed limit terms, so will the resultant rule. Previous analyses of optimal speed limit rules (McCallum and Nelson, 2004) did not assume habit persistence. In this case, the aggregate demand relationship cannot be written in a form that contains speed limits. Any speed limit terms must therefore come from the optimality condition. This condition contains speed limit terms under commitment, but not under discretion (eg Clarida et al, 1999, Woodford, 2003). Therefore, in such a context, optimal speed limit policy rules can be obtained only under commitment. In this paper, by contrast, we assume habit persistence. In this case, the aggregate demand relationship can be written in a form that contains speed limit terms. As a result, the optimal monetary policy rule contains speed limit effects irrespective of the optimality condition and therefore we are able to obtain optimal speed limit policy rules under discretion. Given that habit persistence is supported by extensive empirical evidence[2], this suggests that speed limits may be a pervasive feature of optimal monetary policy. Moreover, the presence of an output gap level in the policy rule implies that policymakers cannot avoid the issue of measuring the equilibrium level of output, contrary to the hopes of the early literature on speed limits.

In the presence of habit formation, the loss function, derived using a quadratic approximation to the representative household's utility (Leith et al, 2012), can be expressed in a form that includes an inflation term, an output gap term and a speed limit term. As a result, this provides a theoretical justification for the presence of a speed limit term in the objective function, as suggested by Walsh (2003)[3]. However, contrary to Walsh (2003),

[2] See Fuhrer and Rudebush (2004), Bouakex et al. (2005), Christiano er al (2005), Ravn et al (2006, 2008)

[3] Although derivation of a speed limit policy rule does not require a speed limit term in the objective function.

the welfare function also contains an output gap term.

The optimal speed limit policy rule we derive under discretion has the same functional form as many of the speed limit policy rules used in the more recent literature. As a consequence the ad hoc policy rules assumed by this literature are potentially optimal; failure to achieve optimality can only arise from the use of non-optimal parameters. We assess this by using simulations of our model, calibrated with parameter values typical of the literature, to compare welfare outcomes from using our optimal speed limit (OSL) policy rule and from using a generic speed limit (GSL) policy rule, representative of the rules used in recent models . We find that social welfare is higher with the optimal rule and conclude that speed limit policy rules used in the recent literature are not optimal in our model, primarily because they assume a relatively low policy weight on inflation.

The remainder of the paper is structured as follows. We outline our model in section 2) and then derive and discuss our optimal speed limit rule in section 3). In section 4), we address the optimality of the speed limit rules used in the recent literature using simulations of a calibrated model. There are a number of caveats to our findings; we discuss these and conclude in section 5).

2 The Model

We use a simple New Keynesian DSGE model with habit persistence in which households supply homogenous labour inputs and purchase differentiated goods, while firms hire labour and produce goods. The goods market is monopolistically competitive but the labour market is competitive. There is a continuum of households; each has an inter-temporal utility function given by

$$(1)\ E_t \sum_{k=0}^{\infty} (e^{\epsilon^D_{t+k}} \beta)^k \left\{ \frac{(C_{t+k}(j) - \mu C_{t+k-1})^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+k}(j)^{1+\eta}}{1+\eta} \right\}$$

where j indexes the household, $C_t(j) = (\int\limits_0^\infty C_t(j,q)^{\frac{\theta-1}{\theta}} dq)^{\frac{\theta}{\theta-1}}$ is a consumption index that aggregates consumption of individual goods $C_t(j,q)$ by household j, $C_t = \int_0^\infty C_t(j)dj$ is aggregate consumption, $N_t(j)$ are hours worked; β is the discount factor, σ is the inverse of the elasticity of inter-temporal substitution, μ measures the strength of habit formation, η is the inverse of the labour supply elasticity and χ denotes the relative weight on hours worked. $e^{\epsilon_t^D}$ represents a preference shock: we assume $\epsilon_t^D = \rho_D \epsilon_{t-1}^D + \varrho_t^D$ where $0 \leqslant \rho_D \leqslant 1$ and ϱ_t^D is distributed as $N(0, \sigma_D^2)$.

We characterise habit persistence in household utility using the external (or "shallow") habit formation approach of Abel (1990) and Campbell and Cochrane (1999) and follow Constantinides (1990) and Smets and Wouters (2003, 2007) in expressing habits in terms of the quasi-difference between current and lagged consumption. Alternatives include expressing habits in terms of the ratios of current and lagged consumption within an external habits framework (eg Amato and Laubach, 2004) and using the internal (or "deep") habit approach of Ravn et al (2006). We use the simple external habits formulation as it allows us to obtain analytic solutions for optimal monetary policy rules[4].

The period t budget constraint of households is

(2) $C_t(j) + \frac{B_t(j)}{P_t} \leqslant \frac{(1+i_{t-1})B_{t-1}(j)}{P_t} + \frac{W_t(j)}{P_t} N_t(j) + \frac{\Pi_t(j)}{P_t} - T_t$

where $B_t(j)$ are holdings of bonds, P_t is the aggregate price level given by $P_t = (\int_0^\infty P_t(q)^{1-\theta} dq)^{\frac{1}{1-\theta}}$ where $P_t(q)$ is the price of good q, i_t is the nominal interest rate, W_t is the nominal wage, $\Pi_t(j)$ are nominal profits distributed to household j and T_t is a lump-sum tax. Optimising with respect to consumption and labour supply implies equality between the real wage and the marginal rate of substitution,

(3) $\frac{W_t(j)}{P_t} = \chi N_t(j)^\eta (C_t(j) - \mu C_{t-1})^\sigma$

[4] As we discuss in the conclusions, we would not expect the use of deep habits to affect our main results. Optimal monetary policy with habit persistence is analysed through simulations in a more complex model of external habit persistence model by Amato and Laubach (2004), Levine et al (2008) and Corrado et al (2012) and in a model with internal habit persistence by Leith et al (2012).

where we note that habit persistence implies that lagged consumption affects the marginal rate of substitution.

Firms have the production function

(4) $Y_t(q) = N_t(q)$

where $Y_t(q)$ and $N_t(q)$ are output and employment at firm q[5]. Firms are able to reset their price with probability $(1-\xi)$ and maintain the same price with probability ξ. There is no indexation of prices for firms that are not able to reset their price. Real marginal cost is given by

(5) $(1-\nu)(\frac{W_t}{P_t})$

where ν is a subsidy paid to ensure an efficient level of output, financed from lump-sum taxation on households.

Following Benigno and Woodford (2005), we define the output gap as $\hat{x}_t = y_t - y_t^e$ where y represents log output and y^e represents the socially efficient level of log output and assume that the efficient level of output differs from the flexible-price (or "natural") level of log output, y^n, because of a supply shock, so

(6) $y_t^e = y_t^n + \epsilon_t^S$

where $\epsilon_t^S = \rho_S \epsilon_{t-1}^S + \varrho_t^S$ where $0 \leqslant \rho_S \leqslant 1$ and ϱ_t^S is distributed as $N(0, \sigma_S^2)$.

Solving the model and using a first-order linearization around the steady-state yields aggregate demand and supply equations given by

(7) $\hat{x}_t = \frac{\mu}{1+\mu}\hat{x}_{t-1} + \frac{1}{1+\mu}E_t\hat{x}_{t+1} - \frac{1-\mu}{\sigma(1+\mu)}(\hat{\imath}_t - E_t\pi_{t+1}) + \zeta_t$

and

(8) $\pi_t = \beta E_t \pi_{t+1} + \kappa\hat{\varphi}_t$

[5] We do not include productivity shocks as these make it more difficult to obtain analytic solutions for optimal monetary rules using an approximation to social welfare. As discussed in Walsh (2010, section 8.3.5), productivity shocks would be a component of the shock to aggregate demand in a linearised model expressed in terms of the otuput gap.

where $\hat{x}$ is the deviation of the output gap around its' steady-state value, $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ is the inflation rate, $\hat{\imath}$ is the deviation of the nominal interest rate from its' steady-state value, $\hat{\varphi}$ represents log linear deviations of real marginal cost around the steady-state, $\kappa = \frac{(1-\xi)(1-\beta\xi)}{\xi}$ and $\zeta_t = \frac{(1-\mu)(1-\rho_D)}{\sigma(1+\mu)}\epsilon_t^D$. These relationships are familiar from the literature. Equation (7) is an aggregate demand relationship with habit persistence. Equation (8) is a New Keynesian Phillips Curve expressed in terms of marginal cost.

We next combine the defintion of marginal cost in (5), the equality between the real wage and the marginal rate of substitution in (3) and the definition of the efficient level of output in (6), which gives

(9) $\hat{\varphi}_t = (\eta + \frac{\sigma}{1-\mu})\hat{x}_t - \frac{\sigma\mu}{1-\mu}\hat{x}_{t-1} + (\eta + \frac{\sigma}{1-\mu})\epsilon_t^S - \frac{\sigma\mu}{1-\mu}\epsilon_{t-1}^S$

Combining (8) and (9) gives

(10) $\pi_t = \beta E_t \pi_{t+1} + \lambda\hat{x}_t - \delta\hat{x}_{t-1} + \lambda\epsilon_t^S - \delta\epsilon_{t-1}^S$

where $\lambda = \kappa(\eta + \frac{\sigma}{1-\mu})$ and $\delta = \kappa\frac{\sigma\mu}{1-\mu}$. This Phillips curve relationship between inflation and the output gap includes both the current and the lagged output gap. This reflects the impact of habit persistence in the household utility function on the marginal rate substitution between consumption and leisure and hence on marginal cost (Amato and Laubach, 2004). In the absence of habit persistence, (10) simplifies to a standard New Keynesian Phillips Curve.

3 Optimal Monetary Policy

In this section, we use the model outlined above to derive an optimal speed limit policy rule under discretion[6] using a model-based approximation to social welfare to represent the objectives of the policymaker. Leith et al

[6] We also derived the optimal speed limit policy rule under commitment. This rule differs from that derived in McCallum and Nelson (2004) in containing two speed limit terms. Unlike discretion, the speed limit term in the policy rule is not dependent on habit persistence. Details are availabe upon request

(2012) derive a second-order approximation to household utility in a model comparable to that in section 2) above. The per-period loss function is given by

(11) $L(\pi_t, \hat{x}_t, \hat{x}_{t-1}) = \frac{\theta}{2\kappa}\pi_t^2 + \frac{\eta}{2}\hat{x}_t^2 + \frac{\omega}{2}(\hat{x}_t - \mu\hat{x}_{t-1})^2$

where $\omega = \frac{\sigma}{(1-\mu)(1-\beta\mu)}$. This can be expressed as

(12) $L(\pi_t, \Delta\hat{x}_t) = \frac{\theta}{2\kappa}\pi_t^2 + \frac{\eta+\omega(1-\mu)}{2}\hat{x}_t^2 + \frac{\mu\omega}{2}(\Delta\hat{x}_t)^2$

This loss function contains a speed limit term, providing a theoretical rationale for the inclusion of a speed limit in the objective function of the policymaker, as proposed by Walsh (2003). This term is a consequence of habit persistence in household utility. However, in contrast to Walsh, the loss function also contains the level of the output gap. In the absence of habit persistence ($\mu = 0$), the speed limit term disappears and (12) simplifies to a standard New Keynesian loss function (Woodford, 2003).

To derive the optimal policy rule, we note that, in the reduced form policy function of the model, inflation is a function of the state variables[7]:

(13) $\pi_t = \rho_\pi^x\hat{x}_{t-1} + \rho_\pi^S\epsilon_t^S + \rho_\pi^D\epsilon_t^D$

We note that optimal policy fully offsets the impact of demand shocks on inflation and the output gap, so $\rho_\pi^D = 0$. We can then express expected inflation as $E_t\pi_{t+1} = \rho_\pi^x\hat{x}_t + \rho_\pi^S\rho_S\epsilon_t^S$, so the Phillips Curve can be written as

(14) $\pi_t = \lambda_1\hat{x}_t - \delta\hat{x}_{t-1} + \lambda_2\epsilon_t^S - \delta\epsilon_{t-1}^S$

where $\lambda_1 = \lambda + \beta\rho_\pi^x$ and $\lambda_2 = \lambda + \beta\rho_S\rho_\pi^S$. The policymaker's intertemporal loss function is

(15) $\sum_{k=0}^{\infty}\beta^k L_{t+k}$

[7] For standard parameter values, this has roots within the unit circle.

where the per-period loss function is

(16) $L(\pi_t, \hat{x}_t, \hat{x}_{t-1}) = L(\lambda_1 \hat{x}_t - \delta \hat{x}_{t-1} + \lambda_2 \epsilon_t^S - \delta \epsilon_{t-1}^S, \hat{x}_t, \hat{x}_{t-1})$

where the functional form of (16) is given in (11) above. The policymakers' problem can be expressed as the Bellman equation

(17) $V(\hat{x}_{t-1}) = \min\{L(\lambda_1 \hat{x}_t - \delta \hat{x}_{t-1} + \lambda_2 \epsilon_t^S - \delta \epsilon_{t-1}^S, \hat{x}_t) + \beta E_t V(\hat{x}_t)\}$

The optimality condition is

(18) $L_{\hat{x}_t} + \beta E_t V'(\hat{x}_t) = 0$

Using (11), $L_{\hat{x}_t} = \frac{\theta \lambda_1}{\kappa} \pi_t + \eta \hat{x}_t + \omega(\hat{x}_t - \mu \hat{x}_{t-1})$ and the Envelope Theorem implies $V'(\hat{x}_{t-1}) = -\frac{\theta\delta}{\kappa}\pi_t - \omega\mu(\hat{x}_t - \mu\hat{x}_{t-1})$. Combining these, the optimality condition is

(19) $\eta \hat{x}_t + \omega(\hat{x}_t - \mu \hat{x}_{t-1}) - \beta\mu\omega(E_t \hat{x}_{t+1} - \mu \hat{x}_t) = -\frac{\lambda_1 \theta}{\kappa}\pi_t + \frac{\beta\delta\theta}{\kappa} E_t \pi_{t+1}$

This generalises the familiar optimality condition under discretion, reflecting the lagged output gap term in the Phillips Curve; if there are no habit effects, (19) simplifies to the "leaning against the wind" condition, $\hat{x}_t = -\frac{\lambda_1 \theta}{\alpha(\kappa+\eta)}\pi_t$. Re-writing the aggregate demand relationship as

(20) $\hat{x}_t = \frac{1}{1+\mu} E_t(\hat{x}_{t+1} - \hat{x}_{t-1}) + \hat{x}_{t-1} - \frac{1-\mu}{\sigma(1+\mu)}(\hat{\imath}_t - E_t \pi_{t+1}) + \zeta_t$

and combining with (19), we obtain

(21) $\hat{\imath}_t = \omega_1 \lambda_1 \pi_t + (1 - \omega_1 \beta\delta) E_t \pi_{t+1} + \omega_2 (E_t \hat{x}_{t+1} - \hat{x}_{t-1}) + \omega_3 \hat{x}_{t-1} + \omega_4 \zeta_t$

where $\omega_1 = \frac{\sigma(1+\mu)\theta}{\kappa\Theta(1-\mu)}$, $\omega_2 = \frac{\sigma}{(1-\mu)}(1 - \frac{(1+\mu)\beta\mu\omega}{\Theta})$, $\omega_3 = \frac{\sigma(1+\mu)}{(1-\mu)}(1 - \frac{(1+\beta)\mu\omega}{\Theta})$, $\omega_4 = \frac{\sigma(1+\mu)}{(1-\mu)}$ and $\Theta = (\eta + \omega(1 + \beta\mu^2))$.

We can write the policy function for the output gap as

(22) $\hat{x}_t = \rho_x^x \hat{x}_{t-1} + \rho_x^S \epsilon_t^S$

Combining (13) and (22) gives

(23) $E_t \pi_{t+1} = \rho_x^x \pi_t + \rho \epsilon_t^S$

where $\rho = \rho_\pi^x \rho_x^S - \rho_x^x \rho_\pi^S + \rho_\pi^S \rho_S$. Using (22) and (23), we can express (21) as {note new notation for error term}

(24) $\hat{\imath}_t = \gamma_\pi \pi_t + \gamma_{SL} \Delta \hat{x}_t + \gamma_{\hat{x}} \hat{x}_t + \upsilon_t^{OSL}$

where $\upsilon_t^{OSL} = \rho_x^x(1-\omega_1\beta\delta) + \omega_1(\lambda + \beta\rho_\pi^x)$, $\gamma_{SL} = \omega_2$, $\gamma_{\hat{x}} = \frac{w_3}{\rho_x^x} - \omega_2(1-\rho_x^x)$ and $\upsilon_{1t} = \omega_4 \zeta_t + \{(1-\omega_1\beta\delta)\rho + \rho_x^S(\omega_2\rho_S - \frac{w_3}{\rho_x^x})\}\epsilon_t^S$.

Equation (24) is the optimal policy rule for this model. It contains a speed limit term, demonstrating that an optimal speed limit policy rule can be obtained under discretion. The speed limit term is a consequence of habit persistence; the policy rule in (24) simplifies to a Taylor Rule if $\mu = 0$[8]. There is no comparable parameter restriction that will eliminate the output gap term[9].

4 Optimality of Speed Limit Rules in the Literature

In this section we address the optimality of speed limit policy rules used in the recent literature. Several papers, including Smets and Wouters (2003, 2007), An and Schorfheide (2007), Scharnagl et al (2010), Cogley et al (2011), Ilbas (2012), Brendon et al (2013), Theodoridis et al (2013) and

[8] This is most easily seen in (21), since $\omega_2 = \omega_3$ if $\mu = 0$

[9] A policy rule with the same functional form is obtained if one uses loss functions that contain quadratic terms in inflation and the output gap or quadratic terms in inflation and a speed limit.

Kapinos and Hanson (2013), use similar policy rules, containing inflation, the output gap and a speed limit. We consider a generic speed limit (GSL) representation of these rules, given by[10]

(25) $\hat{\imath}_t = \nu_\pi \pi_t + \nu_{SL} E_t \Delta \hat{x}_t + \nu_{\hat{x}} \hat{x}_t + \upsilon_t^{GSL}$

The optimal speed limit (OSL) policy rule in (24) and the generic rule in (25) have the same functional form. Therefore, sub-optimality in (25) can only arise from the use of sub-optimal parameters. We can assess sub-optimality by comparing welfare outcomes from simulations of a calibration of our model using both (24), which is the optimal policy rule for this model, and (25), which is not the optimal rule if the parameters of (24) and (25) differ[11].

We calibrate the structural parameters of the model and the generic policy rule using the median value of the priors for the relevant parameters in Smets and Wouters (2007), as summarised in Table 1)[12]. We set $\mu = 0.7$, $\sigma = 1.5$, $\eta = 2$, $\xi = 0.5$, $\theta = 4$ and $\beta = 0.99$. The persistence of the supply shocks is assumed to be $\rho_S = 0.9$.

Table 1)

Calibrated Structural Parameters

[10] Not all the cited papers use precisely this policy rule; for example Smets and Wouters (2003) also include the change in the inflation rate in their policy rule, while An and Schorfheide (2007) and Chen et al (2013) do not include an output gap term and use growth in output for the speed limit. Many of the speed limit rules used in the literature also include an interest rate smoothing term. Simulations using a rule that included an interest rate smoothing term gave similar results to those obtained using (24); details are available upon request. Sims (2013) finds that the optimal weights on the lagged interest rate and the output gap are both zero when optimising an assumed speed limt rule.

[11] We should stress that we are investigating the sub-optimality of the generic policy rules in (25) in the context of our simple New Keynesian DSGE model. In particular, we aim to identify the sources of sub-optimality that would arise in the context of our structural model, if one adopted the calibration of the speed limit rules proposed by the recent literature, instead of adopting the calibration implied by our optimal speed limit rule. We are not investigating the optimality of speed limit rules used in the recent literature; these are based on different structural models.

[12] We use the priors as they are representative of values used in the literature. We do not use the posteriors of the parameters as these are estimated using a different, more general, model than that used in Section 2).

μ	σ	η	ξ	θ	β	ρ_S	σ_S
0.7	1.5	2	0.5	4	0.99	0.9	0.09

These calibrated parameters imply weights on inflation, the speed limit and the output gap in the optimal policy rule of 5.79, 0.20 and 0.48 respectively (cf Table 2). For the calibrated policy rule in (25), we again use the median values of the priors in Smets and Wouters (2007), $\nu_\pi = 1.5$, $\nu_{SL} = 0.48$ and $\nu_{\hat{x}} = 0.12$.

Table 2)
Parameters of Optimal and Calibrated Policy Rules

policy rule	parameter values		
OSL	γ_π	γ_{SL}	$\gamma_{\hat{x}}$
weights	5.79	0.20	0.48
GSL	ν_π	ν_{SL}	$\nu_{\hat{x}}$
weights	1.50	0.48	0.12

Using the values in Table 2)[13], we can re-write the systematic components of the policy rules as $\hat{\imath}^{OSL} = 5.79\pi_t + 0.68\hat{x}_t - 0.20\hat{x}_{t-1}$ and $\hat{\imath}^{GSL} = 1.50\pi_t + 0.60\hat{x}_t - 0.48\hat{x}_{t-1}$ respectively. Therefore these alternative calibrations imply a much larger policy response to inflation with the optimal speed limit rule, similar responses to the output gap and a larger response to the lagged output gap with the generic speed limit rule.

We analyse the response to an adverse supply shock using these alternative policy rules and use the simulated values of inflation and the output gap to calculate the implied values of social welfare, from (11). The results are presented in column (i) of Table 3).

[13] These parameter values imply $\rho_\pi^x = 0.048$

Table 3)
Values of Welfare Loss Implied by Simulations

	(i) $\mu = 0.7$	(ii) $\mu = 0$	(iii) $\mu = 0.9$
GSL	0.190	0.092	0.881
OSL	0.119	0.079	0.495

The optimal speed limit policy rule delivers a superior welfare outcome, delivering a level of loss 37% lower than with the GSL rule. The reason for this is apparent in the impulse response functions, presented in Figure 1). Although Figure 1a) shows that output is less volatile with the generic speed limit rule, this only makes a marginal contribution to welfare. Rather the superiority of the optimal speed limit rule reflects the lower volatility of the inflation rate apparent in Figure 1b). The differences in welfare outcomes in Table 3) therefore reflect the large coefficient assigned to inflation in the optimal speed limit policy rule[14]. The remainder of Table 3) explores the sensitivity of this finding to variations in the strength of habit peristence, considering the cases were habit persistence is absent (column (ii)) and very high (column (iii)). The welfare improvement from using optimal policy falls to 14% if $\mu = 0$ and increases to 49% if $\mu = 0.9$. The benefits from using optimal policy are larger when habit persistence is stronger.

5 Conclusions

In this paper we have derived an optimal speed limit policy rule under discretion using a simple New Keynesian DSGE model with habit persistence. This optimal rule contains both output gap and speed limit terms and so is similar to the speed limit policy rules used in the recent literature. The presence of an output gap level in the policy rule implies that policymakers cannot avoid the issue of measuring the equilibrium level of output. Simulations suggest that the optimal speed limit policy rule is superior to a generic

[14] Sims (2013) also finds a large corefficient on the inflation rate, based on optimisation of an assumed speed limit policy rule.

speed limit rule similar to those used in the recent literature, mainly because of a larger weight on inflation in the optimal rule. The differences in welfare outcomes identified in this paper suggest that there are potential benefits from using optimal policy. Given all this, we would argue that optimal speed limit rules under discretion merit further investigation.

In particular, we suggest two possible lines of enquiry. Our model can be extended by using the deep habit model of Ravn et al (2006) and by allowing for inflation persistence in the Phillips Curve. We speculate that these extensions would not alter our argument that optimal monetary policy rules include speed limit terms when there is habit persistence. However, the optimal policy weights assigned to inflation, output and the speed limit terms are likely to be different. In the light of the potential welfare gains from using optimal policy identified above, such extensions are a useful future research topic.

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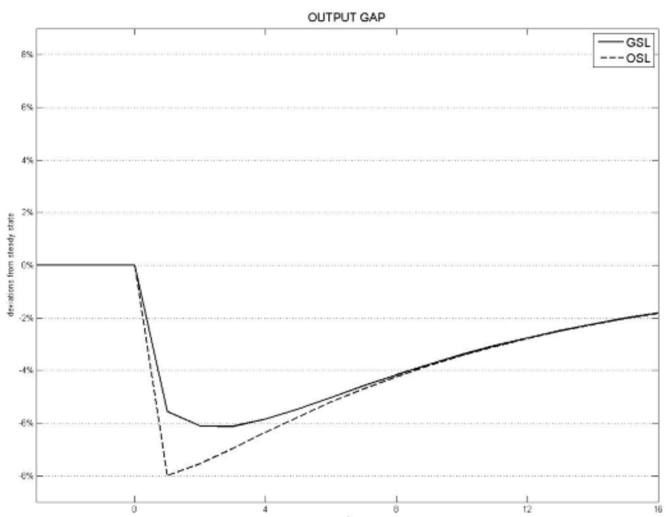
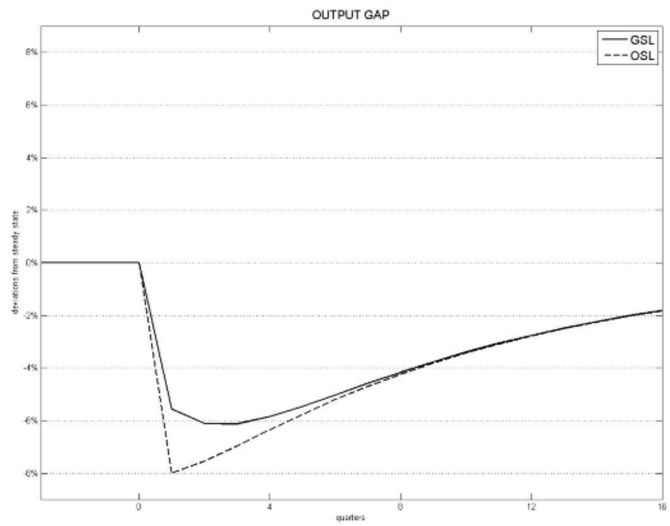
Table 1a)
Impulse Response for Output Gap Following a Supply Shock
x



1.*jpg*

Table 1b)
Impulse Response for Output Gap Following a Supply Shock
x



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