Abstract — A finite element model is presented in which bilayer lead zirconate titanate (PZT) structures that are formed from a dense layer and a porous layer are investigated for their hydrostatic sensing properties. The model simulates the poling of the porous ferroelectric material to determine the distribution of poled material throughout the structure. The fraction of PZT successfully poled is found to be closely related to resulting piezoelectric and dielectric properties of the composite. Structures with high layer porosity (>40 vol.%) and porous layer relative thickness (>0.5) were found to have a significantly improved hydrostatic piezoelectric coefficient, $d_h$, hydrostatic voltage coefficient, $g_h$, and hydrostatic figure of merit, $d_h g_h$. The highest $d_h g_h$ of $7.74 \times 10^{-12}$ m/V was observed in the structure with a porous layer relative thickness of 0.6 and porosity of 60 vol.%, which was more than 100 times higher than that for dense PZT ($d_h g_h = 0.067 \times 10^{-12}$ m/V) and over three times that of PZT with 60 vol.% porosity with 3-3 connectivity ($d_h g_h = 2.19 \times 10^{-12}$ m/V). The results demonstrate the potential for layered porous materials for use in hydrophones.

Keywords—piezoelectric, hydrophones, PZT, finite element

I. INTRODUCTION

Ferroelectric ceramics have long been of interest for both sensor and transducer applications due to their ability to convert mechanical or thermal energy into electrical energy, and vice versa, via the piezoelectric and pyroelectric effect [1]. The introduction of porosity as a low permittivity second phase into a ferroelectric ceramic, such as lead zirconate titanate (PZT), has been found to increase sensitivity of both mechanical and thermal sensors [2, 3]. The interconnectivity between the ceramic and porous phases determines the final mechanical, thermal and electrical properties of these composites, allowing tuning of properties depending on the desired application [4]. Despite the potential benefits of porous ferroelectric materials, the effect of porosity on the poling of these materials is only recently being understood in more detail [5].

As formed, ferroelectric materials display no piezoelectric or pyroelectric properties and so must be poled via the application of a large electric field in order to generate a net alignment of domains within the material. In porous ferroelectrics the electric field tends to preferentially concentrate in the low permittivity pores, resulting in regions of the active material becoming difficult to pole. The degree of poling of a ferroelectric material is closely linked to the final piezoelectric and pyroelectric properties [5, 6]. By developing a better understanding of how the porous structure may affect the poling of ferroelectric materials will allow better design of these materials for their desired application.

Porous PZT ceramics have been extensively studied for low frequency (<100 kHz) hydrophone applications, such as sonar [7-10]. Various figures of merit have been derived to rank the performance of piezoelectric materials for these applications. Firstly, materials should have high hydrostatic piezoelectric charge coefficients, $d_h$, that is related to the longitudinal and transverse piezoelectric charge coefficients ($d_{33}$ and $d_{31}$, respectively) by the relationship:

$$d_h = d_{33} + 2d_{31}.$$  \hspace{1cm} (1)

Monolithic piezoceramics tend to have low $d_h$ values as $d_{31} \sim \frac{1}{3} d_{33}$ due to crystallographic symmetry, however, the introduction of porosity decreases $d_{31}$ whilst $d_{33}$ maintains relatively high values up to ~50 vol.% porosity, leading to increases in $d_h$ [2]. This is important for increasing the sensitivity of hydrophones as it leads to an increase in the hydrostatic voltage coefficient, $g_h$, which is related to the charge coefficient by:

$$g_h = d_h / \varepsilon_{33}^f$$  \hspace{1cm} (2)

where $\varepsilon_{33}^f$ is the permittivity at constant stress. Introducing porosity reduces significantly the bulk permittivity of the composite thus further increasing $g_h$. A figure of merit for materials selection for hydrostatic applications is the product of (1) and (2), i.e. $d_h g_h$. A further benefit of introducing porosity into PZT for use in hydrophones is the resulting reduction in the density of material, which leads to improved acoustic matching between water and the active material, allowing more efficient transfer of mechanical energy into the active phase.

Functionally graded structures have also received interest for hydrophone and medical transducer applications [11], whereby materials are manufactured with a porosity gradient.
that acts to match the acoustic impedance between the dense piezoceramic and the media in which it is working, whilst also reducing the bulk permittivity and increasing piezoelectric anisotropy resulting in an improved piezoelectric hydrostatic response [12-14]. Another variation on this is the use of sandwich layer structures, whereby a porous interlayer is surrounded by dense outer-layers. Recent work on barium titanate sandwich structures found it was possible to increase the longitudinal piezoelectric response with increasing bulk porosity, which is a result of the complex poling behavior of ferroelectric materials with high fractions of porosity concentrated into a porous layer in the material, as well as achieving large reductions in permittivity, thus yielding improved energy harvesting figures of merit [15]. Experimental results showed that similar reductions in $d_{31}$ could be achieved for layered porous structures as in uniform porous materials, indicating these structures may be of interest for hydrophone applications, see (1) and (2), particularly when coupled with the reductions in permittivity that were reported. Common manufacturing techniques of layered porous structures include tape casting [13, 14] and sequential uniaxial pressing of powders with varying weight fractions of pore forming agents [12, 15] prior to sintering.

In this work, finite element modelling has been used to determine the effect of bilayer structures, i.e. a dense backing layer and a porous matching layer, on the hydrostatic properties of PZT-5H, a commonly used soft PZT ferroelectric ceramic manufactured by Morgan Advanced Materials. The model simulates the poling process of layered dense-porous structures so as to obtain the hydrostatic sensing properties of interest, i.e. $d_{33}, d_{31}, e_{33}, d_{33}, g_{33}$ and $d_{33}, g_{33}$.

Previous work has demonstrated that models that do not take into account poling distributions within porous materials, but rather assume all the material to be fully poled regardless of the volume and structure of porosity present, tend to overestimate the longitudinal and transverse piezoelectric response as well as the permittivity, which in turn leads to errors when calculating relevant figures of merit [5].

II. MODELLING PROCEDURE

Finite element analysis (Ansys APDL) was used to investigate dense-porous layered PZT structures. A cubic mesh was divided into $30^3$ elements with a dense and a porous layer. The relative thickness of the porous layer was varied from $0 \leq t_{rel} \leq 1$, so that when $t_{rel} = 0$ the structure was fully dense and when $t_{rel} = 1$ the structure was fully porous (3-0 or 3-3 connectivity depending on porosity). Elements in the porous layer were randomly assigned the properties of either air or unpoled PZT, depending on the desired final porosity of the layer. The porosity of the porous layer was varied from 10 to 60 vol.%. Higher porosities were not investigated as this leads to high fractions of ‘floating’ PZT elements, i.e. those surrounded by air, that lead to frequent errors when running the model. All elements in the dense layer were assigned the properties of unpoled PZT. Electrodes were simulated by coupling the voltages of both the top surface and the bottom surface. The properties of unpoled PZT are summarized in Table 1 (from [16]).

After the network of elements had been generated an electric field (0.5 kV/mm) was applied across the geometry to simulate the poling field; this field was selected as it gave a good fit to experimental data for uniform porous structures previously [5]. After the application of the poling field the
the origin of this, as can be seen in Fig. 3. The capacitance was calculated. Data was averaged from 25 runs of each bilayer using measured from the charge that developed at the electrodes, each direction used to calculate the effective was applied between the electrodes and the resulting strain in poled PZT elements was established, a potential difference was generated due to a hydrostatic pressure is possible if evaluation of (1) indicates that an increase in the charge gained interest as materials for hydrostatic pressure an increase in the overall amount of material poled. However, as the overall porosity is increased there is a decoupling of stress transferred from the transverse to the longitudinal direction as high stiffness material is removed, resulting in relatively constant, low \( d_{31} \) values for \( \rho_{\text{pol}} > 0.1 \) at high layer porosities (>40 vol.%). This enhanced decoupling between longitudinal and transverse piezoelectric coefficients, due to the presence of porosity, is one of the reasons why porous piezoceramics gained interest as materials for hydrostatic sensors, as evaluation of (1) indicates that an increase in the charge generated due to a hydrostatic pressure is possible if \( d_{31} \) can be

\[ \text{TABLE I. MATERIAL PROPERTIES OF UNPOLED PZT-5H (FROM [16])} \]

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg m(^{-3}))</th>
<th>Relative permittivity</th>
<th>Young’s Modulus (GPa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT-5H</td>
<td>7500</td>
<td>1585</td>
<td>65</td>
<td>0.37</td>
</tr>
</tbody>
</table>

\[ \text{TABLE II. MATERIAL PROPERTIES OF POLED PZT-5H (FROM [17])} \]

<table>
<thead>
<tr>
<th>( C_{11} ) (GPa)</th>
<th>( C_{12} ) (GPa)</th>
<th>( C_{13} ) (GPa)</th>
<th>( C_{33} ) (GPa)</th>
<th>( C_{44} ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>126</td>
<td>79.5</td>
<td>84.1</td>
<td>117</td>
<td>23.0</td>
</tr>
</tbody>
</table>

\[ \text{\( \varepsilon_{33} \) (C m\(^{-2}\))} \]

<table>
<thead>
<tr>
<th>( \varepsilon_{33} ) (C m(^{-2}))</th>
<th>( \varepsilon_{44} ) (C m(^{-2}))</th>
<th>( \varepsilon_{11} ) (C m(^{-2}))</th>
<th>( \varepsilon_{11}/\varepsilon_{33} )</th>
<th>( \varepsilon_{11}/\varepsilon_{33} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6.55</td>
<td>23.3</td>
<td>17.0</td>
<td>1700</td>
<td>1470</td>
</tr>
</tbody>
</table>

\[ \text{III. RESULTS AND DISCUSSION} \]

\[ \text{A. Poling and piezoelectric coefficients} \]

The effect of porous layer porosity and relative thickness on \( d_{33} \) and \( d_{31} \) is shown in Fig. 2. At low layer porosities (≤ 30 vol.%) there is a small reduction in \( d_{33} \) and \( d_{31} \) tends towards zero with increasing porous layer relative thickness. When the layer porosity is ≥ 40 vol.% a large drop in \( d_{33} \) is observed at low porous layer thicknesses, before a gradual increase. At very high layer porosities (≥ 50 vol.%) a local maxima in \( d_{33} \) is observed before it begins to reduce as the structure approaches one with uniformly distributed porosity, i.e. no dense layer. The origin of this behavior is discussed in detail for barium titanate sandwich layer structures elsewhere and a good fit with experimental data was achieved [15], in which the presence of a dense and porous layers was found to concentrate the poling field in the low permittivity porous layer. This leads to incomplete poling in the dense layer as the layer porosity increases and a high proportion of the material in the porous layer becoming poled, as can be seen in Fig. 3 where the poled distribution can be seen for three different porous layer thickness and layer porosity is 60 vol.%. As this porous layer increases in thickness, the overall fraction of poled material also increases as there is a higher volume of PZT in the porous layer, which is subject to local electric fields that exceed the coercive field and therefore becomes poled. A drop off in \( d_{33} \) is observed as \( \rho_{\text{pol}} \) approaches unity, i.e. uniform porous structure with no dense layer, as the field is no longer sufficiently concentrated by the presence of the high permittivity dense layer and so the fraction of poled material reduces, hence the fall in \( d_{33} \), see Fig. 2 and 3.

As the inner layer porosity increases the overall reduction in \( d_{31} \) becomes more pronounced, as in 3-3 connected porous material [2, 5]. Generally, when there is a point of inflection in the \( d_{33} \) curve as a function of porous layer thickness the same applies to \( d_{31} \) indicating that once more there is a dependence on the overall amount of material poled. However, as the overall porosity is increased there is a decoupling of stress transferred from the transverse to the longitudinal direction as high stiffness material is removed, resulting in relatively constant, low \( d_{31} \) values for \( \rho_{\text{pol}} > 0.1 \) at high layer porosities (>40 vol.%). This enhanced decoupling between longitudinal and transverse piezoelectric coefficients, due to the presence of porosity, is one of the reasons why porous piezoceramics gained interest as materials for hydrostatic sensors, as evaluation of (1) indicates that an increase in the charge generated due to a hydrostatic pressure is possible if \( d_{31} \) can be
reduced at a faster rate than $d_{33}$.

The hydrostatic piezoelectric coefficient, $d_h$, is shown in Fig. 4 as a function of porous layer thickness and porosity. In all cases introducing a porous layer is found to increase $d_h$ compared to fully dense PZT, however when the layer porosity is $\geq 40$ vol.% an optimum porous layer thickness is observed. This is a result of the high $d_{33}$ values for these structures that arises from the complex poling behavior and the decoupling of the longitudinal and transverse piezoelectric responses. The maximum $d_h$ was found to be 237 pC/N for a structure with a 60 vol.% porosity layer with $t_{pol} = 0.7$, compared to 45 pC/N for dense PZT and 109 pC/N for uniformly porous PZT with 50 vol.% porosity, which has the highest $d_h$ of the uniformly porous material ($t_{pol} = 1$).

**Fig. 4: Hydrostatic piezoelectric coefficient plotted as a function of porous layer relative thickness for varying layer porosity.**

B. Permittivity

The effect of porous layer thickness and porosity on the relative permittivity ($\varepsilon_r = \varepsilon_{33}/\varepsilon_0$, where $\varepsilon_0$ is the permittivity of free space) is shown in Fig. 5. Permittivity reduces in all cases with the introduction of a porous layer as the amount of low permittivity phase increases. The degree of poling also has an effect on the permittivity as poled PZT has a higher permittivity than when it is unpoled, which explains the steep decline in permittivity observed for structures with layer porosities $\geq 40$ vol.% at low $t_{pol}$ values. The layer thicknesses where these declines begin coincide with significant reductions in $d_{33}$ that can be seen in Fig. 2. The lowest permittivity for a given layer porosity occurs when $t_{pol} = 0.1$ in all, i.e. uniform porous structures where the overall volume of porosity is at a maximum.

C. Hydrostatic figures of merit

The variation of the hydrostatic voltage coefficient, $g_h$, with porous layer thickness is shown in Fig. 6a. Increasing the porosity of the porous layer was found to increase $g_h$ in all cases due to the increase in $d_h$ and reduction in $\varepsilon_r$. When the layer porosity is 30 vol.% or less the uniform porous material is predicted to have higher $g_h$ values than a bilayer structure. However, as the porosity is increased a maximum $g_h$ value is observed when $0.5 < t_{pol} < 1$. The maximum $g_h$ (34.6 x 10$^{-3}$ V m$^{-1}$ Pa$^{-1}$) was found in the bilayer structure with 60 vol.% layer porosity and $t_{pol} = 0.7$, which is over 20 times that of dense PZT ($g_h = 1.50 \times 10^{-3}$ V m$^{-1}$ Pa$^{-1}$) and 1.5 times higher than uniformly porous PZT with 60 vol.% porosity ($g_h = 22.5 \times 10^{-3}$ V m$^{-1}$ Pa$^{-1}$).

The hydrostatic figure of merit, $d_h g_h$, is plotted in Fig. 6b as a function of porous layer thickness. For high layer porosities ($\geq 40$ vol.%) a broad peak is observed for $0.5 < t_{pol} < 0.9$, with the highest value predicted for 60 vol.% porous layer PZT with $t_{pol} = 0.7$, where $d_h g_h = 7.74 \times 10^{-12}$ m$^2$/N. This is a significant increase compared to dense PZT ($d_h g_h = 0.067 \times 10^{-12}$ m$^2$/N) and uniform porous PZT with 60 vol.% porosity, where $d_h g_h = 2.19 \times 10^{-12}$ m$^2$/N.

These are significant predicted increases in hydrostatic sensitivity that have not yet been demonstrated experimentally, although similar increases in energy harvesting figure of merit have been reported for barium titanate with porous layers sandwiched between dense outer-layers [15]. Previous experimental work on layered porous PZT has focused on layer porosities $< 30$ vol.% [12-14], however these results indicate that higher layer porosities ($>40$ vol.%) may be necessary to achieve further increases in hydrostatic piezoelectric properties.

**IV. CONCLUSIONS**

A finite element modelling study has been presented that investigated the poling characteristics of PZT-5H with bilayer dense-porous structures. Once the distribution of poled and unpoled PZT and porosity was established the structures were characterized in terms of their piezoelectric and dielectric properties, before being evaluated in terms of relevant figures of merit for hydrophone applications. Complex poling behavior was observed that resulted in $d_{33}$ increasing with increasing overall fraction of porosity under certain conditions, a similar phenomenon to that observed previously in barium titanate ceramics with a porous layer sandwiched between dense outer-layers [15]. The transverse coefficient, $d_{31}$, appeared less dependent on the fraction of material poled at high layer porosities $\geq 50$ vol.% and was consistently low at high layer porosities and thicknesses, which lead to significant...
increases in the hydrostatic piezoelectric response, $d_{33}$, at high porous layer relative thicknesses, i.e. $t_p^{rel} > 0.5$. The highest $d_{33}$ (237 pC/N) was found for PZT with a porous layer thickness of $t_p^{rel} = 0.7$ and 60 vol.% porosity, which was 20 times that of dense PZT and 1.5 times that of PZT with 60 vol.% uniformly distributed porosity.

The permittivity of the structures was found to decrease in all cases, which is beneficial for increasing the hydrostatic voltage response, $g_h$, and the hydrostatic figure of merit, $d_{33}g_h$, which are both inverse functions of the permittivity. Both $g_h$ and $d_{33}g_h$ were significantly increased compared to dense PZT in structures where the porous layer is over half the total thickness of the material and when porosity in this layer exceeds 50vol.%. The highest predicted hydrostatic figure of merit ($d_{33}g_h = 7.74 \times 10^{-12} \text{ m}^2$/N) was found when $t_p^{rel} = 0.7$ and the layer porosity was 60 vol.%, which was over 100 times that of dense PZT and almost 3.5 times that predicted for uniformly porous PZT. The use of a porous matching layer is also likely to facilitate better transfer of energy into the material if these structures were implemented in hydrophones.

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