Distributed Actuation and Control for Morphing Structures

Guanyu Lai

A thesis submitted for the degree of Doctor of Philosophy

University of Bath
Department of Mechanical Engineering

June 2017

COPYRIGHT

Attention is drawn to the fact that copyright of this thesis rests with the author. A copy of this thesis has been supplied on condition that anyone who consults it is understood to recognise that its copyright rests with the author and that they must not copy it or use material from it except as permitted by law or with the consent of the author.

This thesis may be made available for consultation within the University Library and may be photocopied or lent to other libraries for the purposes of consultation.
Abstract

It is believed that structures and actuation systems should be tightly integrated together in the future to create fast moving, efficient, lightweight dynamic machines. Such actuated structures could be used for morphing aircraft wings, lightweight actuated space structures, or in robotics. This requires actuators to be distributed through the structure. A tensegrity structure is a very promising candidate for this future integration due to its potentially excellent stiffness and strength-to-weight ratio, and the inherent advantage of being a multi-element structure into which actuators can be embedded. Development of these machines will utilise expertise in several fields, involving kinematics, dynamics, actuation and multi-axis motion control.

The research presented in this thesis concerns the study of multi-axis actuated tensegrity structures. A form-finding method has been developed to find stable geometries and determine stiffness properties of the type of tensegrity structure proposed. It has been shown that a tensegrity structure, with practical nodes of finite size, can be designed with actuated members to give shape-changing properties while potentially allowing a good stiffness to mass ratio. An antagonistic multi-axis control scheme has been developed for the tensegrity structure. The describing function technique has been used to analyse the dead band controller in the control scheme, giving a stability criterion.

An experimental actuated tensegrity system has been designed and built incorporating pneumatic muscles controlled by switching valves. Mathematical models for the experimental actuated tensegrity system have been developed in detail, including the pneumatic actuation system and the structure geometry. The dynamic behaviour of the tensegrity system has been investigated via several simulation studies, using the developed models and the proposed control scheme. Experimental validation has been successfully conducted. The multi-axis control scheme can accurately control the tensegrity structure to achieve shape changes while maintaining a desired level of internal pre-load. The mathematical models can be used as a basis for further development.
Acknowledgements

I would like to express my sincere gratitude to my supervisors Professor Andrew Plummer and Dr. David Cleaver for their continuous support and invaluable advice of my Ph.D study as well as their patience, motivation and immense knowledge. I would like to thank technicians Guy Brace, Gary Barter, Nicholas Waywell, and instrumentation specialists Vijay Rajput and Stephen Coombes, for their practical assistance.

I thank my fellow doctoral students for stimulating discussions, useful feedback and of course friendship. My thanks also go to all the staff of the Centre for Power Transmission and Motion Control for the last minute favours.

I am very grateful to my parents Ming Lai and Hua Gao, for supporting me spiritually and giving me unending love and care. Also I wish to thank my other family members and friends who have supported me along the way.

Special thanks to Di Xiao for staying by my side, making my life colourful.

I am also grateful to Moog Inc. for financial support, and to Paul Guerrier, Mike Baker and Ian Brooks of that company for their help and suggestions.

This thesis is dedicated to my beloved mother.
# Contents

Abstract .......................................................................................................................... I

Acknowledgements ......................................................................................................... II

Notation .......................................................................................................................... VII

1 Introduction .................................................................................................................. 1
  1.1 Motivation ................................................................................................................. 1
  1.2 Aims and objectives ................................................................................................. 2
  1.3 Original research contribution ............................................................................... 3
  1.4 Thesis outline .......................................................................................................... 4

2 Literature review ........................................................................................................... 6
  2.1 Introduction .............................................................................................................. 6
  2.2 Introduction to aircraft wing morphing .................................................................... 6
    2.2.1 Wing planform morphing .................................................................................. 9
      2.2.1.1 Span morphing ............................................................................................ 9
      2.2.1.2 Sweep morphing ....................................................................................... 12
      2.2.1.3 Chord morphing ....................................................................................... 13
    2.2.2 Wing out-of-plane transformation ..................................................................... 14
      2.2.2.1 Twisting ...................................................................................................... 14
      2.2.2.2 Dihedral and span-wise bending .................................................................. 16
    2.2.3 Airfoil morphing ............................................................................................... 19
  2.3 Structures integrated with actuation systems .......................................................... 19
  2.4 Tensegrity structures ............................................................................................... 25
    2.4.1 Introduction ....................................................................................................... 25
    2.4.2 Research in form-finding of tensegrity structures ............................................ 28
    2.4.3 Applications of tensegrity structures ................................................................. 29
  2.5 Concluding remarks ............................................................................................... 34
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Geometry and stability of tensegrity structure</td>
<td>36</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>36</td>
</tr>
<tr>
<td>3.2</td>
<td>Finding equilibrium node positions</td>
<td>37</td>
</tr>
<tr>
<td>3.3</td>
<td>Calculating the stiffness matrix</td>
<td>41</td>
</tr>
<tr>
<td>3.4</td>
<td>Extension to finite nodes</td>
<td>42</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Extension for finding equilibrium node positions</td>
<td>42</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Extension for calculating the stiffness matrix</td>
<td>45</td>
</tr>
<tr>
<td>3.5</td>
<td>Kinematics of an example actuated tensegrity structure</td>
<td>47</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Configuration of the example structure</td>
<td>47</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Actuation of example tensegrity structure</td>
<td>48</td>
</tr>
<tr>
<td>3.6</td>
<td>Optimised structure with externally applied load</td>
<td>54</td>
</tr>
<tr>
<td>3.6.1</td>
<td>Structural optimisation with external load</td>
<td>54</td>
</tr>
<tr>
<td>3.6.2</td>
<td>Comparison with cylindrical tube</td>
<td>59</td>
</tr>
<tr>
<td>3.7</td>
<td>Note on structure stability with finite nodes</td>
<td>60</td>
</tr>
<tr>
<td>3.8</td>
<td>Concluding remarks</td>
<td>61</td>
</tr>
<tr>
<td>4</td>
<td>Controller design</td>
<td>62</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>62</td>
</tr>
<tr>
<td>4.2</td>
<td>Antagonistic control of two actuators</td>
<td>62</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Dead band control</td>
<td>63</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Position control</td>
<td>64</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Force control</td>
<td>64</td>
</tr>
<tr>
<td>4.3</td>
<td>Simulation studies for antagonistic control of two actuators</td>
<td>65</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Modelling</td>
<td>65</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Simulation results</td>
<td>68</td>
</tr>
<tr>
<td>4.4</td>
<td>Stability analysis for the dead band controller</td>
<td>72</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Modelling of a simplified system</td>
<td>72</td>
</tr>
<tr>
<td>4.4.2</td>
<td>System behaviour of dead band controller with no velocity feedback</td>
<td>73</td>
</tr>
<tr>
<td>4.4.3</td>
<td>Describing function of the dead band controller</td>
<td>76</td>
</tr>
<tr>
<td>4.4.4</td>
<td>Criterion of guaranteed stable response</td>
<td>79</td>
</tr>
<tr>
<td>4.5</td>
<td>Control of multiple actuators</td>
<td>80</td>
</tr>
<tr>
<td>4.5.1</td>
<td>Transformation for multi-axis control</td>
<td>80</td>
</tr>
<tr>
<td>4.5.2</td>
<td>Multi-axis control for the actuated tensegrity structure</td>
<td>81</td>
</tr>
<tr>
<td>4.6</td>
<td>Concluding remarks</td>
<td>83</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>84</td>
</tr>
<tr>
<td>5.2</td>
<td>Pneumatic system</td>
<td>85</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Connection of pneumatic artificial muscle</td>
<td>85</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Pneumatic artificial muscles</td>
<td>86</td>
</tr>
<tr>
<td>5.2.3</td>
<td>Valve set</td>
<td>89</td>
</tr>
<tr>
<td>5.2.4</td>
<td>Sensors</td>
<td>90</td>
</tr>
<tr>
<td>5.3</td>
<td>Tensegrity structure</td>
<td>92</td>
</tr>
<tr>
<td>5.4</td>
<td>Control system</td>
<td>94</td>
</tr>
<tr>
<td>5.5</td>
<td>Testing of components</td>
<td>97</td>
</tr>
<tr>
<td>5.5.1</td>
<td>Node test</td>
<td>97</td>
</tr>
<tr>
<td>5.5.2</td>
<td>PAM test</td>
<td>98</td>
</tr>
<tr>
<td>5.6</td>
<td>Concluding remarks</td>
<td>100</td>
</tr>
<tr>
<td>6.1</td>
<td>Overview of system modelling</td>
<td>101</td>
</tr>
<tr>
<td>6.2</td>
<td>Modelling of pneumatic actuation system</td>
<td>101</td>
</tr>
<tr>
<td>6.2.1</td>
<td>Modelling of pneumatic artificial muscle</td>
<td>101</td>
</tr>
<tr>
<td>6.2.2</td>
<td>Modelling of on-off solenoid valve</td>
<td>104</td>
</tr>
<tr>
<td>6.2.3</td>
<td>Empirical model for PAM volume</td>
<td>104</td>
</tr>
<tr>
<td>6.3</td>
<td>Lumped volume model for supply line</td>
<td>105</td>
</tr>
<tr>
<td>6.4</td>
<td>PAM filling/discharging flow rate tests</td>
<td>107</td>
</tr>
<tr>
<td>6.4.1</td>
<td>Flow rate measurement</td>
<td>107</td>
</tr>
<tr>
<td>6.4.2</td>
<td>Model performance for supply line</td>
<td>110</td>
</tr>
<tr>
<td>6.4.3</td>
<td>Model performance for discharge line</td>
<td>113</td>
</tr>
<tr>
<td>6.5</td>
<td>Tensegrity structure model</td>
<td>118</td>
</tr>
<tr>
<td>6.6</td>
<td>Simulation results</td>
<td>120</td>
</tr>
<tr>
<td>6.6.1</td>
<td>Arrangement of simulation system</td>
<td>120</td>
</tr>
<tr>
<td>6.6.2</td>
<td>Simulation with small amplitude square wave position demand</td>
<td>123</td>
</tr>
<tr>
<td>6.6.3</td>
<td>Simulation with small amplitude sine wave position demand</td>
<td>127</td>
</tr>
<tr>
<td>6.6.4</td>
<td>Simulation with large amplitude square wave position demand</td>
<td>129</td>
</tr>
<tr>
<td>6.6.5</td>
<td>Simulation with large amplitude sine wave position demand</td>
<td>133</td>
</tr>
<tr>
<td>6.6.6</td>
<td>Simulation with mixed position demand</td>
<td>135</td>
</tr>
<tr>
<td>6.7</td>
<td>Concluding remarks</td>
<td>138</td>
</tr>
</tbody>
</table>
7 Experimental studies ................................................................. 139
  7.1 Introduction .............................................................................. 139
  7.2 Experiments with small amplitude square wave position demand .... 139
    7.2.1 Experimental results .......................................................... 139
    7.2.2 Comparison between simulation and experimental results ........... 146
  7.3 Experiments with small amplitude sine wave position demand .......... 148
  7.4 Experiments with large amplitude square wave position demand ....... 151
  7.5 Experiments with large amplitude sine wave position demand .......... 154
  7.6 Experiments with mixed position demand .................................... 157
  7.7 Concluding remarks ................................................................... 159

8 Conclusions ................................................................................. 161
  8.1 Achievements ........................................................................... 161
  8.2 Further work ............................................................................ 163

References ....................................................................................... 165

Appendix 1 Connectivity matrix and matrix of node coordinates ............... 174
# Notation

## Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0_{n,n}$</td>
<td>$n$-by-$n$ zero matrix with all its entries being zero</td>
</tr>
<tr>
<td>$a$</td>
<td>Number of actuators</td>
</tr>
<tr>
<td>$a_1$</td>
<td>Fundamental component in the Fourier series</td>
</tr>
<tr>
<td>$A$</td>
<td>Tube cross-sectional area</td>
</tr>
<tr>
<td>$A(j\omega)$</td>
<td>Constituent function of $D(j\omega)$</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Opening area of the valve orifice</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Cross-sectional area of the cylindrical tube</td>
</tr>
<tr>
<td>$b_0$</td>
<td>Intercept of the linear function</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Slope of the linear function</td>
</tr>
<tr>
<td>$B$</td>
<td>Level of position tolerance in $G$</td>
</tr>
<tr>
<td>$B_f$</td>
<td>Level of force tolerance in $G_f$</td>
</tr>
<tr>
<td>$c_{al}$</td>
<td>Damping coefficient of the left actuator in the system with $m_p$</td>
</tr>
<tr>
<td>$c_{ar}$</td>
<td>Damping coefficient of the right actuator in the system with $m_p$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Specific heat at constant volume</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Damping coefficient in the system with $m_s$</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Discharge coefficient</td>
</tr>
<tr>
<td>$C_k$</td>
<td>Mass flow ratio</td>
</tr>
<tr>
<td>$C_m$</td>
<td>Mass flow parameter for air</td>
</tr>
<tr>
<td>$C_{n_i}$</td>
<td>Vector for $i^{th}$ row in $C_n$</td>
</tr>
<tr>
<td>$C$</td>
<td>Position transformation matrix to actuator space coordinates</td>
</tr>
</tbody>
</table>
**Cn** Connectivity matrix

\(d\) Number of independent degrees of freedom

\(D\) Tube internal diameter

\(D(j\omega)\) Linear part of the open loop transfer function in the system with \(m_s\)

\(D_c\) Cable diameter

\(D_s\) Strut diameter

**D** Force transformation matrix to actuator space

\(E_{cv}\) Energy of control volume

\(E_s\) Young’s modulus of the strut material

\(f\) Flow friction factor

\(f(P_r)\) Function of the pressure ratio

\(f_{ai}\) Element \(i\) in \(f_m\)

\(F_c\) Compression in the strut

\(F_e\) Euler buckling load

\(F_p\) Force feedback of \(m_p\)

\(F_r\) Force demand of \(m_p\)

\(F_s\) Sheeting buckling load

\(F_t\) Tension in the cable

\(f_a\) Vector of measured forces of actuator

\(f_d\) Virtual force feedback vector

\(f_m\) Vector of member forces

\(f_r\) Force demand vector

\(f_v\) Vector consisting of node force/moment vectors

\(f_w\) Vector transformed from \(f_v\) by \(P_b^{-1}\)

\(F_{mx}\) Vector containing member force vectors in the \(x\)-axis

\(F_{my}\) Vector containing member force vectors in the \(y\)-axis


\( F_{mz} \) Vector containing member force vectors in the z-axis

\( F_{nx} \) Vector containing resultant force vectors at each node in the x-axis

\( F_{ny} \) Vector containing resultant force vectors at each node in the y-axis

\( F_{nz} \) Vector containing resultant force vectors at each node in the z-axis

\( F_e \) Matrix of external force vectors at each node

\( F_m \) Matrix of member force vectors

\( F_n \) Matrix of resultant force vectors at each node

\( g \) Gravitational acceleration

\( g_i \) Stiffness of member \( i \)

\( G \) Dead band controller for position control

\( G_f \) Dead band controller for force control

\( G_{N(M)} \) Describing function

\( G_{N(M)}^{\text{max}} \) Maximum value of the describing function

\( h_0, h_1, h_2, h_3, h_4 \) Coefficients of the polynomial model

\( h_{in} \) Specific enthalpy of compressed air mass inflow

\( h_{out} \) Specific enthalpy of compressed air mass outflow

\( I_3 \) 3-by-3 identity matrix

\( J_i \) Vector for \( i^{th} \) column in \( J \)

\( J_{xi} \) Vector of joint position in the x-axis for member \( i \)

\( J_{yi} \) Vector of joint position in the y-axis for member \( i \)

\( J_{zi} \) Vector of joint position in the z-axis for member \( i \)

\( J \) Jacobian matrix relating the rate of change of member lengths to node positions

\( J_r \) Jacobian matrix relating the rate of change of member lengths to node angles
$J_x$  Matrix of joint positions in the $x$-axis relative to node centres when nodes rotate

$J_{x0}$  Matrix of joint positions in the $x$-axis relative to node centres when there is no node rotation

$J_y$  Matrix of joint positions in the $y$-axis relative to node centres when nodes rotate

$J_{y0}$  Matrix of joint positions in the $y$-axis relative to node centres when there is no node rotation

$J_z$  Matrix of joint positions in the $z$-axis relative to node centres when nodes rotate

$J_{z0}$  Matrix of joint positions in the $z$-axis relative to node centres when there is no node rotation

$k$  Heat capacity ratio

$k_{al}$  Spring stiffness of the left actuator in the system with $m_p$

$k_{ar}$  Spring stiffness of the right actuator in the system with $m_p$

$k_s$  Spring stiffness in the system with $m_s$

$K_e$  Coefficient in the Euler buckling equation

$K_s$  Coefficient in the sheet buckling equation

$K_e$  Matrix of elastic stiffness

$K_g$  Matrix of geometric stiffness

$K_{g11}$  Block of $K_g$ giving the relationship between an applied force and the linear displacement of nodes that the force produces

$K_{g12}$  Block of $K_g$ relating the applied force to the angular displacement of nodes

$K_{g21}$  Block of $K_g$ associating an applied moment with the linear displacement of nodes

$K_{g22}$  Block of $K_g$ giving the relationship between the applied moment and the angular displacement of nodes

$K_m$  Matrix of member stiffnesses

$K_n$  Node stiffness matrix

$l$  Tube length
\( l_{mi} \) Element \( i \) in \( l_m \)

\( L \) Instantaneous length of the PAM

\( L_s \) Strut length

\( l_m \) Vector of member lengths

\( l_{m0} \) Vector of free (unloaded) member lengths

\( m \) Number of structural members

\( m_a \) Mass of air in the control volume of the PAM

\( \dot{m}_a \) Rate of change in mass within the control volume of the PAM

\( \dot{m}_{in} \) Inflow rate of air to the PAM

\( \dot{m}_{out} \) Outflow rate of air from the PAM

\( m_p \) Mass driven by two actuators

\( m_s \) Mass in the simplified system

\( \dot{m}_s \) Mass flow rate into the supply line

\( M \) Amplitude of the excitation waveform

\( m_i \) Coordinates of member \( i \)

\( M_{nx} \) Vector containing resultant moment vectors at each node in the \( x \)-axis

\( M_{ny} \) Vector containing resultant moment vectors at each node in the \( y \)-axis

\( M_{nz} \) Vector containing resultant moment vectors at each node in the \( z \)-axis

\( M \) Matrix of member vectors

\( \dot{M} \) Matrix of member vectors for nodes of finite size

\( M_n \) Matrix of resultant moment vectors at each node

\( n \) Number of structural nodes

\( n_v \) Vector consisting of node linear/angular position coordinates

\( n_w \) Vector transformed from \( n_v \) by \( P_b^{-1} \)

\( N \) Matrix of node coordinates
Na  Matrix of node angles

$p$  Probability factor depending on the wall initial irregularities

$P$  Pressure of air in the PAM

$\dot{P}$  Rate of change in pressure of the PAM

$P_{atm}$  Atmospheric pressure

$P_{cr}$  Critical pressure ratio

$P_d$  Valve downstream pressure

$P_r$  Ratio of downstream to upstream pressure across the valve

$P_s$  Supply pressure

$P_u$  Valve upstream pressure

$P$  Position transformation matrix to work space coordinates

$P_b$  Matrix whose columns are eigenvectors of $K_a$

$q_i$  Element $i$ in $q$

$\dot{Q}$  Rate of heat exchange

$q$  Vector of member tension coefficients

$Q$  Force transformation matrix to work space

$r_e$  Equivalent radius of the PAM

$R^2$  Coefficient of determination

$R_s$  Specific gas constant for air

$Re$  Reynolds number

$R_{li}$  Matrix that converts the rotational movement of nodes into the translational movement of joints for member $i$

$R_{x}(\theta)$  Basic rotation matrix about the $x$-axis

$R_{y}(\theta)$  Basic rotation matrix about the $y$-axis

$R_{z}(\theta)$  Basic rotation matrix about the $z$-axis

$s$  Differential operator

$s_i$  Element $i$ in $\delta_a$
\( S_{1i} \)  
Single-entry vector with the \( i^{\text{th}} \) entry being 1 and the rest of the entries being 0

\( S_{3i} \)  
Single-entry matrix whose entry in the \( i^{\text{th}} \) row and \( i^{\text{th}} \) column is 1

\( t \)  
Time variable

\( t_2 \)  
Transition time

\( t_s \)  
Wall thickness of the tubular strut

\( T \)  
Temperature of air in the PAM

\( T_S \)  
Period

\( \dot{U} \)  
Rate of change in internal energy

\( u \)  
Signal vector for the control of the valve set

\( u_c \)  
Signal vector from the dead band controller in the position loop

\( u_d \)  
Signal vector from the dead band controller in the force loop

\( v \)  
Speed scaling factor

\( \bar{v} \)  
Mean flow velocity

\( v_{\text{in}} \)  
Velocity of air mass inflow

\( v_{\text{out}} \)  
Velocity of air mass outflow

\( V \)  
Volume of air in the PAM

\( \dot{V} \)  
Rate of change in volume of the PAM

\( W \)  
Rate of work done

\( x \)  
Position of the movable base in the system with \( m_s \)

\( x_{\text{al}} \)  
Position of the left base in the system with \( m_p \)

\( x_{\text{ar}} \)  
Position of the right base in the system with \( m_p \)

\( x_i \)  
Element \( i \) in \( x \)

\( x \)  
Vector consisting of \( x \)-coordinates of nodes

\( y \)  
Position of \( m_s \)

\( y_d \)  
Position demand of \( m_s \)

\( y_i \)  
Element \( i \) in \( y \)
\( y_p \)  
Position of \( m_p \)

\( y_r \)  
Position demand of \( m_p \)

\( y \)  
Vector consisting of \( y \)-coordinates of nodes

\( z_i \)  
Element \( i \) in \( z \)

\( z_{in} \)  
Height of air mass inflow

\( z_{out} \)  
Height of air mass outflow

\( z \)  
Vector consisting of \( z \)-coordinates of nodes

**Greek**

\( \alpha \)  
Actuator position difference in bend

\( \beta \)  
Actuator position difference in shear

\( \gamma \)  
Actuator position difference in twist

\( \Delta P \)  
Frictional pressure loss

\( \delta_a \)  
Vector of measured displacements from zero position of actuator

\( \delta_c \)  
Virtual position feedback vector

\( \delta_r \)  
Position demand vector

\( \theta \)  
Rotation angle of a vector about the \( x \)-, \( y \)- or \( z \)-axis operating with \( R_x(\theta) \), \( R_y(\theta) \) or \( R_z(\theta) \)

\( \theta_i \)  
Element \( i \) in \( \theta \)

\( \Theta \)  
Vector consisting of pitch angles of nodes about the \( y \)-axis

\( \kappa \)  
Coefficient for the correction of different end support conditions

\( \lambda_i \)  
Element \( i \) along the diagonal of \( \Omega \)

\( \mu \)  
Air dynamic viscosity

\( \nu \)  
Poisson’s ratio of the strut material

\( \rho \)  
Air density

\( \sigma \)  
Tensile strength of the cable
τ  Switching value
φᵢ  Element i in φ
φ  Vector consisting of roll angles of nodes about the x-axis
ψᵢ  Element i in ψ
ψ  Vector consisting of yaw angles of nodes about the z-axis
ω  Frequency
ωₘₑ  Frequency when the phase of $D(jω)$ is $-180^\circ$.
Ω  Diagonal matrix with eigenvalues of $K_n$ along its diagonal
Ω'  Modified inverse of Ω

Acronyms

DAQ  Data Acquisition
MAV  Micro Air Vehicle
PAM  Pneumatic Artificial Muscle
SAR  Specific Air Range
UAV  Unmanned Aerial Vehicle
VGT  Variable Geometry Truss
VSS  Variable Stiffness Spar
1 Introduction

1.1 Motivation

In the future, dynamic machines will require distributed actuation integrated with load-bearing structures, so that they are lighter, move faster, use less energy, are more human-friendly, and are more adaptable. The integration has great potential to create smart structures that can provide better static and dynamic performance, redundancy and more adaptability than current designs.

An example application is in aerospace. Highly efficient aircraft are desirable for both environmental and economic reasons. A possible solution to further improve efficiency is to allow an aircraft to adapt its aerodynamic shape for different flight regimes. It is not a new concept to change the shape of an aerial vehicle. It can be dated back to the first powered and controlled aircraft which used a wing-warping system to change the twist of the wing (Lawrence and Padfield, 2006; Weisshaar, 2013). Modern commercial aircraft have commonly used active shape-changing devices in several components such as leading-edge slats, spoilers, ailerons, etc. Nevertheless, these are only effective for a limited part of the flight regime and are unable to acclimate to different flight scenarios. A significant disadvantage of these devices is that they rely on the deflection of hinged, discrete control surfaces, which can, even under moderate levels of deflection, set up localised areas of severe adverse pressure gradient (typically along the hinge line) that produce regions of flow separation, and poor wing efficiency (Barbarino et al., 2011).

To reduce the surface discontinuity and sharp edges of an aircraft, a viable solution is to replace part of the conventional wing structures with smart structures to perform distributed actuation, which allows subtle changes in curvature, thereby leading to better aerodynamic performance. With the advent of ‘smart’ materials, e.g. shape memory polymer, elastic memory composite, etc., this target becomes more achievable, as these materials are not only deformable but also load-bearing (Thill et al., 2008).
It is not just in aerospace industry. Many industries require machines which can dynamically control motion or force simultaneously in several linear or rotary axes. The testing and simulation industry is an example, where in-service motion or force usually needs to be replicated; this ranges from automotive durability testing, to earthquake simulation, and motion-based flight training simulators. Increasingly demanding applications are emerging for lightweight dynamic multi-axis motion control systems, such as for high degree-of-freedom robot manipulators.

The motivation leads to the following research question: how can an actuated structure for dynamic machines be designed, built and controlled? In addition, as the research progressed, an open question needed to be resolved: how can the selected pneumatic actuator be stably controlled by using on-off solenoid valves?

1.2 Aims and objectives

It is believed that there is potential for future multi-axis motion systems to achieve exceptional performance through integration of structures, actuation and control, and their mathematical optimisation. The research project aim is to develop a load-bearing structure with shape-changing capability through distributed actuation, and to investigate a better way to integrate structures with actuation systems.

A tensegrity structure is a promising basis for a smart structure that can be deformed in three dimensions through the actuation of some elements. The first objective of the research is to propose a suitable tensegrity structure and assess its properties. More specifically, mathematical methods are to be developed and used to study and analyse the proposed structure.

Other objectives are:

- Design and develop a control scheme that can effectively actuate the proposed structure to realise motion and force control.
- Design and build a physical model and implement the proposed control scheme to validate the overall concept experimentally.
• Model the experimental system to simulate its dynamic behaviour to improve understanding of the system for further development.

1.3 Original research contribution

In this research, original contribution has been made in several aspects. Based on the approach proposed in (Guest, 2006), a form-finding method that can calculate the equilibrium state of tensegrity structures is developed. The method makes progress in terms of practical application, extending the original approach by taking into account the fact that in a real structure with several members meeting at each node, the ends of the members are in fact separated by a finite distance.

The research proposes a tensegrity structure with practical nodes of finite dimension and uses the form-finding method to demonstrate that the geometrical configuration of the proposed tensegrity structure can be designed with actuated members to give shape changing properties. On the basis of the general co-ordinate transformation framework for multi-axis motion control in (Plummer, 2010), a multi-axis control scheme is developed for the proposed tensegrity structure. The scheme uses dead band controllers to achieve both motion and force control of the proposed structure. The controller stability is studied by the describing function technique, and a condition for guarantee stability is derived.

An experimental actuated tensegrity system is designed and built to validate the proposed control scheme and to study the dynamic behaviour of the proposed structure. The experimental system is modelled in detail, including the pneumatic actuation system and the tensegrity structure. The pneumatic actuation system is modelled mathematically from the fundamental physical properties. A multi-body mechanical model implemented in SimMechanics is created for the tensegrity structure. The simulation of the experimental system provides a better understanding of the system itself, giving opportunities to maximise the system performance and further develop the system.
1.4 Thesis outline

Chapter 2 presents a literature review of the smart-structures field focusing on tensegrity and similar structures. Several insightful examples are provided to demonstrate the benefits that future machines can gain from the integration of load-bearing structures with distributed actuation. A detailed review is made for morphing wings, as this area has been researched widely and has great potential for application.

Chapter 3 presents a form-finding method that can find the equilibrium node positions of a tensegrity structure and calculate its stiffness through iteration. The method considers the practical application by including nodes with finite dimension. A tensegrity structure is proposed and is shown that its geometrical configuration can be designed with actuated members to have desired shape changing properties. The proposed tensegrity structure is also compared with a cylindrical tube of the same length in terms of mass and stiffness.

A multi-axis control scheme for the actuated tensegrity structure is designed and described in Chapter 4. The control scheme has both position and force control, using dead band controllers. A simple simulation model with two antagonistic actuators is created to validate the principle of the control scheme. The stability of the dead band controller is investigated using a further simplified system. The controller is further studied by the describing function technique, and a criterion for guaranteed stability is derived.

Chapter 5 presents the design and construction of an experimental actuated tensegrity system. The components are described in detail, including the pneumatic actuation system and tensegrity structure, and some components are tested. The computer system is also described, which includes interfacing module, supporting software and how the computer system is operated.

The modelling of the actuated tensegrity system is the main subject of Chapter 6. The fully developed models, including the pneumatic actuation system and the tensegrity structure, are described in detail. The pneumatic actuation system is modelled mathematically, and the tensegrity structure model is implemented using SimMechanics.
Several simulation studies are conducted to investigate the performance of the actuated tensegrity system.

Chapter 7 presents the experimental implementation of the multi-axis control scheme and the validation of the experimental system. The simulation tests in Chapter 6 are implemented experimentally. The experimental results are compared with the simulation results. The differences are discussed and the possible reasons are suggested. Two additional sets of tests are performed. One, made by using various force demands, is to see the performance under different pre-loads. The other, carried out by removing the top unit cell, is to see the influence of change in mass and validate the criterion of guaranteed stability derived in Chapter 4.

Conclusions and recommendations for further work are covered in Chapter 8.
2 Literature review

2.1 Introduction

The smart-structures field is multidisciplinary. Different topics relevant to this field are covered in this Chapter. Morphing aircraft are reviewed comprehensively in Section 2.2 as the scope and potential of smart-structures applications are expanding rapidly in aerospace systems. According to NASA: “Aircraft of the future will not be built of traditional, multiple, mechanically connected parts and systems. Instead, aircraft wing construction will employ fully-integrated, embedded ‘smart’ materials and actuators that will enable aircraft wings with unprecedented levels of aerodynamic efficiencies and aircraft control.” (Allen, 2008).

Section 2.3 covers some recent research activities in structures integrated with actuation systems. Other applications of smart structures range from civil structures, to space and other systems and are described in Section 2.4.3. A tensegrity structure is deemed to be a very promising candidate for this future integration of structures with actuation systems due to its unique properties, and is reviewed in detail in Section 2.4.

2.2 Introduction to aircraft wing morphing

Although existing civil aircraft have adopted active shape-changing devices in several components, e.g. leading-edge slats, these devices are only effective for a limited part of the flight regime. The reason is that these control surfaces are connected by hinges which make surfaces discontinuous and can cause the loss of lift or even regional stall (Barbarino et al., 2011).

Morphing is a word that is originated from metamorphose. In the aerospace field, it refers to technologies which are able to enhance the performance of an air vehicle by deforming its shape (Weisshaar, 2013). Unlike the active morphing devices, i.e. flight control
surfaces, on conventional aircraft, shape morphing technologies for future air vehicles should reduce design compromises while having the ability to adapt to a changing ambient condition.

However, amongst researchers no consensus on the definition of shape morphing has been reached. Some researchers use a simple criterion to define shape morphing. It should make an aircraft be more efficient and more adaptive (Wlezien et al., 1998). Efficient not only means better fuel efficiency but also refers to creating a mechanically simpler and lighter system. Adaptive means that the aircraft should be versatile to reduce or even eliminate design compromises and conflicts (Weisshaar, 2006).

Due to the close connection with smart materials, shape morphing concepts have often used complicated and expensive approaches to substitute the current mature techniques over the past 20 years, whilst only providing limited improvement. In addition, it is believed that the success of a morphing device can only be achieved if the new device is lighter or simpler than the existing components. All these are misunderstandings of the morphing technology. This is because too much attention has been paid to the component instead of paying attention to the whole system level (Weisshaar, 2013).

Shape morphing techniques have great potential for fixed-wing aircraft because aerodynamic forces are primarily generated by wings, and can be categorised into three major types: wing planform morphing, out-of-plane morphing and airfoil morphing (Barbarino et al., 2011). Planform morphing is a one-dimensional deformation which includes the shape changes in sweep, span and chord. Out-of-plane morphing is a two dimensional transformation. It refers to the changes of a wing in twist, span-wise bending and dihedral angle. Airfoil morphing comprises the changes in camber contour and airfoil thickness. The review of morphing aircraft focuses on shape morphing techniques in these three major types, covers the effects of different morphing techniques on the performance of fixed-wing aircraft, and provides some insight into smart structures and their actuation mechanisms.

Wing performance can be affected by many geometric parameters. It is essential to understand the effects of these parameters before knowing the motivation of each type
of morphing. Table 2.1 summarises the influence of changing these parameters on aircraft performance.

Table 2.1 Summary of the influence of wing geometric parameters on aircraft performance (Jha and Kudva, 2004)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Effects of variability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing plan area</td>
<td>↑ Increased: lift and load factor capability</td>
</tr>
<tr>
<td></td>
<td>↓ Decreased: parasitic drag</td>
</tr>
<tr>
<td>Wing aspect ratio</td>
<td>↑ Increased: lift/drag ratio, loiter time, cruise distance and turn rates; Decreased: engine requirements</td>
</tr>
<tr>
<td></td>
<td>↓ Increased: maximum speed; Decreased: parasitic drag</td>
</tr>
<tr>
<td>Wing dihedral</td>
<td>↑ Increased: rolling moment capability and lateral stability</td>
</tr>
<tr>
<td></td>
<td>↓ Increased: maximum speed</td>
</tr>
<tr>
<td>Wing sweep</td>
<td>↑ Increased: critical Mach number and dihedral effect; Decreased: high-speed drag</td>
</tr>
<tr>
<td></td>
<td>↓ Increased: maximum lift coefficient</td>
</tr>
<tr>
<td>Wing taper ratio</td>
<td>Wing efficiency (span-wise lift distribution) and induced drag</td>
</tr>
<tr>
<td>Wing twist distribution</td>
<td>Span-wise lift distribution and prevention of tip stall behaviour</td>
</tr>
<tr>
<td>Airfoil camber</td>
<td>Zero-lift angle of attack, airfoil efficiency and separation behaviour</td>
</tr>
<tr>
<td>Airfoil thickness/chord ratio</td>
<td>↑ Improved: low-speed airfoil performance</td>
</tr>
<tr>
<td></td>
<td>↓ Improved: high-speed airfoil performance</td>
</tr>
<tr>
<td>Leading edge radius</td>
<td>↑ Improved: low-speed airfoil performance</td>
</tr>
<tr>
<td></td>
<td>↓ Improved: high-speed airfoil performance</td>
</tr>
<tr>
<td>Airfoil thickness distribution</td>
<td>Airfoil characteristics and laminar/turbulent transition</td>
</tr>
</tbody>
</table>
2.2.1 Wing planform morphing

Planform morphing involves length change in span-wise and chord-wise directions, and the angle change in sweep. Take a wing aspect ratio (the ratio of a wing span to its mean chord) as an example. A wing aspect ratio is dominated by wing span and sweep angle. An aircraft can have a longer total flight range and more loiter time if its wing aspect ratio increases. Conversely, for high-speed flight, a low aspect ratio is preferred as it can reduce the drag while increasing the maximum speed (Jha and Kudva, 2004; Anderson, 2012). In consequence, an aircraft is capable for multiple missions if its span and/or sweep can be deformed drastically.

2.2.1.1 Span morphing

Telescopic wing mechanism is one concept to modify wing span. It was developed by the Russian expatriate Ivan Makhonine, where variable wing span and area were achieved by telescoping the outer wing segment into the inner wing. He first introduced this span morphing feature to a low wing monoplane, the MAK-10 (see Figure 2.1), that first flew in 1931. The aircraft could increase the span from 13 up to 21 m for take-off, landing and flights requiring more stability, and retract the wing for high-speed flight (Weisshaar, 2006). The mechanism was powered pneumatically (Barbarino et al., 2011).

Figure 2.1 Mak-10, (a) Wings almost contracted, (b) Wings completely extended (Anon., 1932)
Another example of telescopic wing aircraft designed by Neal et al. (2004) is shown in Figure 2.2. The aircraft could also undergo variable sweep, tail contraction/extension and variable twist of the outer wing segment. The planform variations were achieved by five linear actuators, driving the morphing segments directly. Those for span and tail morphing were pneumatic as for large strokes pneumatic actuators should be lighter than hydraulic and electromechanical actuators, justified by the authors. The two in the sweep system were electromechanical and were non-back-drivable under load. The design had a 38% change in span and 40 degrees change in sweep. Wind tunnel results indicated that low drag was maintained by the planform variations over a range of flight conditions.

![Internal structure of the telescopic mechanism (Neal et al., 2004)](image)

A scissor-like mechanism for the wingbox is another promising concept for span morphing. Ajaj et al. (2013) used a zigzag structure to build a wingbox that could vary the wing span by 44% (22% for both extension and contraction), as shown in Figure 2.3. The concept was to enhance the operational performance of an unmanned aerial vehicle (UAV). The wingbox contained rigid and morphing parts. The rigid parts consisted of ribs which served as the main structure to which the flexible skins were attached. The morphing parts had several compartments. Each compartment contained two spars (solid blue lines) each formed by two beams hinged together. The mechanism had four possible actuator arrangements to actuate the wingbox as illustrated in Figure 2.4. The researchers chose arrangement B as both span and sweep morphing could be realised while allowing the control of the chordwise bending stiffness and enhancing the torsional stiffness of the
wingbox. Again, a pneumatic actuation system was used because it simplified the design and eliminated the concern of oil leakage. The relatively small forces required by the prototype system suited to the pneumatic actuation. A challenge of the mechanism is that flexible skins are required, which is a problem for many shape morphing concepts.

![Diagram of zigzag wingbox concept](image)

Figure 2.3  Top view of the zigzag wingbox concept (Ajaj et al., 2013)

![Diagram of actuator arrangements](image)

Figure 2.4  Actuator arrangements, (A) Vertex to vertex, (B) Crossed, (C) Direct driving, (D) Rib to vertex (Ajaj et al., 2013)

Span morphing could also be used for roll control by creating an asymmetric wing span. It was demonstrated in (Henry and Pines, 2007) that asymmetrical span morphing was effective for roll control. The research also showed that the total damping in the system could be increased by increasing the span due to the conservation of angular momentum, and vice versa.
2.2.1.2 Sweep morphing

A variable-sweep wing, also known as swing wing, allows an aircraft to efficiently fly at low speed, high speed and supersonic speed. The easiest way to build a variable-sweep wing is to allow the wing to pivot at the wing root. The advantages of variable-sweep wing include short take-off distance, higher load-carrying ability, and the good manoeuvrability in the fast and low-level situation. However, it requires a large gearbox which is complex and causes a weight penalty. Moreover, the pivoting device carries all the aerodynamic forces, thus increasing the maintenance requirements. All of these factors could also lead to an increase in fuel consumption (Barbarino et al., 2011).

There are three main reasons to sweep a wing forward or backward. First, it is for the enhancement of the longitudinal stability. Secondly, it can provide both longitudinal and directional stability for a flying wing aircraft (tailless aircraft). Thirdly, it is able to suppress the drag rise at transonic speed (Weisshaar, 2013). Figure 2.5 shows several examples of variable-sweep aircraft.

Figure 2.5   (a) Me P-1101 drawings (Weisshaar, 2013), (b) Bell X-5 in-flight and on the ground (NASA, 2014)
However, with the advent of relaxed stability flight control systems in 1970s, many disadvantages of fix platform was eliminated. Thus, no more variable-sweep aircraft were built after the Tu-160 which first flew in 1981. Today, the focus of recent studies has been shifted to micro air vehicles (MAVs) and UAVs with the aim of having multi-mission capabilities. These studies adopted variable sweep in cooperation with variable span to achieve multi-mission aircraft, using traditional actuation systems (Barbarino et al., 2011; Ivanco et al., 2007).

### 2.2.1.3 Chord morphing

It is a challenging task to modify the length of a chord because spars extend from fuselage to wingtip and need to carry aerodynamic loads when in-flight and weight of a wing after landing (Thom, 1988). As a result, chord morphing has been investigated least and is mainly for helicopter rotor blades.

Chord morphing does provide some benefits for fixed wing aircraft. It could increase wing area. One possible application is in the region with high-lift devices, which is outside the wing box. The high-lift devices could be replaced by a variable camber airfoil with chord elongation capability, which only requires a small amount of the wing chord to be deformed. This makes chord morphing more feasible from a structural perspective (Barbarino et al., 2011).

Figure 2.6 is an example of chord morphing aircraft designed by Bakshaev in 1937 using a telescopic wing mechanism. The telescopic mechanism, changing the wing area by 44%, had six overlapping wing sections that were extended from the fuselage up to two thirds of the wing during take-off and landing (Weisshaar, 2006).
2.2.2 Wing out-of-plane transformation

Out-of-plane morphing consists of the change in twist, dihedral angle and span-wise bending. For instance, adjusting the twist of a wing could prevent the stall of the wingtip and redistribute the lift in the span-wise direction.

2.2.2.1 Twisting

Active aeroelastic structures have the potential to reduce drag and enhance roll and loads control. They can change the aerodynamic shape of a lifting surface by varying their internal configuration, which can produce considerable improvements in performance and control and does not require large planform transformations commonly used in planform morphing. Thus, their mechanisms are potentially relatively simple and lightweight (Barbarino et al., 2011).

Chen et al. (2000) developed an approach that could improve the aircraft roll manoeuvre performance by changing the torsional stiffness of the wing using variable stiffness spars (VSSs). The spar is shown in Figure 2.7. The VSS mechanism divided the spar into several segments, and the segmented spar was connected to ribs by using articulated joints and was rotated by an electrical actuator up to 90 degrees. When the joints are in the horizontal direction, there is no bending stiffness in the spar as all the segments are
uncoupled. The spar begins to gain the stiffness as it is rotated and reaches the maximum stiffness when the segments join completely in the vertical direction of the joints.

![Figure 2.7 VSS mechanism (Chen et al., 2000)](image)

The approach created a torsion-free wing concept as illustrated in Figure 2.8. The wing bending moment was mainly supported by two very strong and stiff spars placed close to each other (spars 3 and 4). Two VSSs were placed near the leading and trailing edges, controlling the stiffness in accordance with Mach number and altitude. Spars 2 and 5 were removed. As a consequence, the wing torsional stiffness was low. The torsion-free wing could amplify the aeroelastic effects and thus improve the roll performance significantly. The approach leaded the wing to reach the roll-rate requirement of 120 deg/s and achieved an enhancement of 29-126% in the roll rate (Chen et al., 2000).

![Figure 2.8 VSS/torsion-free design](image)

Majji et al. (2007) developed a novel morphing wing with twistable sections covered with an elastomeric skin. The design, as shown in Figure 2.9, coupled four concentric and telescoping tubes to the structure of the wing at four locations from the wing root to the wingtip. The tubes were rotated independently by using servomotors. The angle of attack of the whole wing could be changed by rotating the tube at the root location. And each wing section could be twisted by actuating the corresponding tube. It was shown that the twistable sections expanded the operating envelope of the angle of attack of the wing, allowing the tip section to stall later than the inner sections.
Both of the two studies above can create washout. It is a feature of wing that reduces the angle of attack across the span and leads the tip to have the lowest angle of attack as depicted in Figure 2.10. The purpose is to ensure that the wingtip stalls later than the wing root, enabling the aileron in the tip region to be controlled continuously at the stall speed and thereby preventing the loss of control.

2.2.2.2 Dihedral and span-wise bending

For a morphing wing, it usually has more than one type of geometric deformation. A variable dihedral wing is always coupled with span-wise bending. This type of wing is able to improve the performance and control of an aircraft by changing the wing shape according to the environment. It can control the aerodynamic span and can also replace
conventional control devices. Moreover, it can make an aircraft to be more agile. Additionally, by changing the distribution of vorticity, the induced drag could be reduced (Barbarino et al., 2011).

Ursache et al. (2007) presented a study in MORPHLET (normally known as MORPHing wingLET). The original intention of this study was to optimise specific air range (SAR) of narrow body aircraft by using the MORPHLET. The SAR is the distance travelled per unit of fuel. So the greater the SAR, the higher the efficiency an aircraft has. The MORPHLET contained three segments outboard from the wing tip and also included the aileron region as depicted in Figure 2.11. Since the geometry of the wing was kept constant, there was no need to redesign the primary wing structure.

Through computational simulation, results showed a considerable increase in SAR could be achieved. For example, the optimum design at the start of final cruise and the end of final cruise, Figure 2.12, reached an increase in SAR of 5.17% and a reduction of induced drag of approximate 18.5%, with an increase in span of 4.21 m. Another benefit was a canted wing tip with zero cant angle (winglet vertical) could ensure each aircraft complies with the ICAO Code C (a limit on wing span related to the width of gate, taxi-track clearances, etc.).

![Figure 2.11 MORPHLET (Ursache et al., 2007)](image)
Smith et al. (2010) also performed a numerical analysis on a MORPHLET wing system by comparing to a datum aircraft in order to find out the potential gains. Results indicated that the MORPHLET wing system could maintain an enhancement of 4.5 to 5.5% in SAR for all analysed phases, e.g. climb, descent, etc. For fixed winglets, improvements were only around 3.5%. Figure 2.13 gives the optimised wing profiles for different flight phases. In addition, the results also showed considerable improvements in the lift-to-drag ratios during climb as well as take-off and landing distances.
2.2.3 Airfoil morphing

Airfoil morphing is mainly related to change of camber profile, i.e. curvature and thickness. The airfoil morphing could be realised by either deforming the camber partially (e.g. at the leading edge) or letting the camber change globally. The actuation mechanisms suggested are diverse, ranging from conventional actuation systems (i.e. hydraulic, pneumatic and electromagnetic) to novel actuation technologies (i.e. piezoelectric, shape memory alloys, etc.). And the actuation system could be distributed or localised (Barbarino et al., 2011).

Spillman (1992) pointed out that, for subsonic flight, the airfoil shape could be continuously modified for different flight conditions with the intention of increasing the lift-to-drag ratio. Continuous shape control has several merits and is especially important for small air vehicles, principally due to two reasons (Barbarino et al., 2011). Firstly the Reynolds number for small air vehicles is quite low, which could easily result in flow separation thus reducing the effectiveness of the control surfaces at the trailing edge. Secondly UAVs and MAVs cannot afford the energy loss through drag at their control surface due to their very limited propulsion.

Gern et al. (2005) conducted a computational study in airfoil morphing and tried to replace the trailing-edge flaps by morphing airfoils. The paper outlined that a morphing airfoil had the possibility to enhance the roll performance (higher rolling moment) with actuators being distributed. In addition, it could provide smoother pressure distribution on the wing surface and thus increase the aerodynamic quality of the wing.

2.3 Structures integrated with actuation systems

One approach is to use truss structures to create shape morphing structures by replacing some of the truss elements with linear displacement actuators. The concept of variable geometry truss (VGT) structures was first proposed by Miura (1984) as a type of deployable actuated structure, consisting of the repetition of an octahedral truss module longitudinally as illustrated in Figure 2.14. The lateral members in the lateral triangular truss were variable length beams (telescopic beams), while the diagonal members were
fixed length beams. Geometric transformations were achieved by changing the lengths of the lateral members. Based on the concept, Miura et al. (1985) developed a VGT robotic manipulator arm that was arguably the first shape morphing truss structure with three degrees of freedom as shown in Figure 2.15. The variable length members, each connected to an actuator comprising a DC motor with an internal encoder, were telescopic beams, and each beam was actuated by the DC motor through a ball screw. The design used a hinge mechanism to follow the movement of the truss. Other possible applications of the VGT structure included support architectures for space stations and active control systems for large space systems.

Figure 2.14       Configuration of variable geometry truss (Miura, 1984)
Ramrakhyani et al. (2005) developed a compliant cellular truss for wing morphing. The structure used tendons as active members as shown in Figure 2.16. A bending-type deformation was achieved by pulling and releasing appropriate tendons (cables).
Later, Sofla et al. (2009), using tetrahedral truss unit cells, created a shape morphing hinged truss structure capable of bending, twisting and undulating as shown in Figure 2.17. The truss structure shape was changed by actuating shape memory wire actuators in an antagonistic manner as illustrated in Figure 2.18. A novel spherical-pivotal joint, developed by Sofla et al. (2007), was used to connect several struts at a node. The joint, as shown in Figure 2.19, could allow up to 18 struts to be connected at each node.

![Figure 2.17](image1)

Figure 2.17 Deformations of morphing hinged truss (72 tetrahedra), (a) Bending, (b) Twisting, (c) Undulating (Sofla et al., 2009)

![Figure 2.18](image2)

Figure 2.18 Schematic diagram of antagonistically actuating one-way shape memory wire actuators (Sofla et al., 2009)
More recently, Moosavian et al. (2013) presented a novel design of underactuated parallel mechanism for the application of morphing wingtip. The structure of a conventional wingbox was replaced by active and passive linearly adjustable members. The mechanism could let the structure move in all six spatial degrees of freedom. Two sets of three-member truss were used to replace the spars, as shown in Figure 2.20 (a), providing the bending stiffness for the wing. In order to overcome the out-of-plane bending, another two members were added to the system, Figure 2.20 (b).

With the intention of efficiently controlling the truss structure, instead of replacing all the members with actuators, only the diagonal members (grey) were replaced by
actuators and the remainder were replaced by lockable passive members as illustrated in Figure 2.21.

In addition, the paper presented the actuation sequences for optimal motion control and identified the optimum sequence based on minimum energy actuation through simulation. They built and tested a prototype to validate the proposed underactuated motion control as shown in Figure 2.22. Electromechanical linear actuators were used for the active members, controlling both speed and position. Hydraulic cylinders, controlled by on-off solenoid valves, were used for the passive members.

Among the above VGT examples, the modular motion control has only been studied in detail in (Moosavian et al., 2013). The example truss structures in (Ramrakhyani et al., 2005; Sofla et al., 2009) are intended to verify the feasibility of shape morphing without much consideration of external load, e.g. aerodynamic forces. So, for high-load applications, the precise control of these structures may be difficult to achieve. Therefore, further studies in these areas are deemed valuable for creating truss structures with high stiffness and desired motion capability.

Figure 2.21 Placement of active and passive members (Moosavian et al., 2013)
2.4 Tensegrity structures

2.4.1 Introduction

A tensegrity structure is a very promising candidate for smart structures due to its potentially excellent stiffness and strength-to-weight ratio, and the inherent advantage of being a multi-element structure into which actuators can be embedded. Under natural selection, many living creatures have evolved biomechanical tensegrity structures, e.g. human bones held by muscles and tendons (Ingber, 1997). Tensegrity was first investigated by D. G. Emmerich, R. B. Fuller and K. Snelson in 1950s. They were regarded as the pioneers in the field of tensegrity. The word “tensegrity” was named by Fuller (Moored and Bart-Smith, 2007). He created the word through the truncation of the phrase “tensional integrity” and described tensegrity structures as “islands of compression in a sea of tension elements” (Fuller, 1962).
The origin of tensegrity can be dated back to the early 1920s. K. Ioganson, a Russian sculptor, built a sculpture similar to a tensegrity system, called “Study in Balance” (Motro, 2003). It is generally believed that the X-piece structure made by Snelson in 1948 is the first reported tensegrity structure (Motro, 1996). Unlike Ioganson’s sculpture, Snelson’s X-piece structure can be self-stabilised without applying external forces. Since then, Snelson has built numerous tensegrity structures that are primarily for art exhibitions. At the same time in France, Emmerich was also committed to the study of self-stabilised systems consisting of struts and cables, and developed tensegrity systems based on prism (Motro, 1992). Initial studies, by Fuller (1962), Emmerich (1964) and Snelson (1965), focused on the use of geometric methods to find the equilibrium form of tensegrity structures, and created a large number of tensegrity configurations. Pugh (1976) classified these configurations into three pattern types: diamond, circuit and zigzag, and provided a variety of tensegrities with detailed schemes and advice on how to build them. Figure 2.23 presents an example of a tensegrity structure.

A tensegrity structure has multiple elements. Some of them are rigid members (struts) and are always in compression. And the rest are in tension at all times, and cables are often used for these. Therefore, the tension members can be lighter as they do not have to resist buckling. In addition, tensegrity structures have two very good properties. One is that the stiffness of a tensegrity structure can be modified without changing its shape, and vice versa. The other is that the stiffness of a tensegrity structure is mainly dictated
by the choice of geometry rather than pre-stress level and mass (Skelton and Oliveira, 2009).

As its name suggests, the whole structure is required for ‘integrity’. In other words, it can only be stabilised by tensile member forces acting on compressive members. All the elements are axially loaded, which means there is no bending moment in any member or torque at any joint. By using the knowledge of parallel kinematics, a tensegrity structure could be designed to achieve good stiffness-to-mass ratio without sacrificing the number of degrees of freedom.

A good definition was given by (Pugh, 1976): “A tensegrity system is established when a set of discontinuous compression components interacts with a set of continuous tensile components to define a stable volume in space.” Motro (2003) suggested a broader definition: “A tensegrity system is a system in a stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components.” Motro’s definition allows compressive members to be interconnected and has been widely accepted. Skelton and Oliveira (2009) made a classification for tensegrity structures based on the number of rigid bodies meeting at a point. According to their classification: “A tensegrity configuration that has no contacts between its rigid bodies is a class 1 tensegrity system, and a tensegrity system with as many as $k$ rigid bodies in contact is a class $k$ tensegrity system.” Figure 2.24 demonstrates an example for the distinction.

![Figure 2.24](image)

Figure 2.24 (a) A class 2 tensegrity structure, (b) A class 3 tensegrity structure (Skelton and Oliveira, 2009)
### 2.4.2 Research in form-finding of tensegrity structures

Research in tensegrity structures has often focussed on the form-finding problem (also known as prestressability problem). This problem is a key issue in designing tensegrity structures, which involves finding a structural equilibrium configuration in which cables are all in tension and struts are all in compression. Researchers have developed analytical (Zhang et al., 2009; Zhang et al., 2010; Koohestani and Guest, 2013) and numerical (Koohestani, 2013; Tran and Lee, 2013; Zhang et al., 2014) methods for the form-finding of tensegrity structures. Analytical methods have been used for tensegrities with relatively simple configuration and tensegrities that have high level of symmetry. For more complicated tensegrities, numerical methods have been used. These methods find the pre-stress level and its corresponding geometric configuration that give a stable structure for a given number of members and their interconnections.

However, these results are limited as just being stable is not adequate from an engineering point of view. The structure, for instance, should also have high strength and stiffness to mass ratios. Skelton and Oliveira (2009) presented useful analytical results for simple tensegrity structures, such as algorithms to optimise strength under compressive and bending loads. As stated by Plummer and Lai (2015), more engineering issues need to be considered. For example, the nodes where members meet have finite size. Each node is a rigid body in its own right, and thus members cannot meet at the same point in reality. An effective method (Guest, 2006) for calculating structural stiffness of general tensegrity structures in the form of a stiffness matrix has only been developed relatively recently.

The kinematics, dynamics and control of actuated tensegrity structures have received little attention because of their mathematical complexity as well as the need to combine dynamics/control expertise with structural optimisation. Experimental studies are particularly limited at present. It was Aldrich et al. (2003) who first proposed a control method for tendon-driven robotic systems that included tensegrity structures. Skelton and Oliveira (2009) investigated the dynamics and presented a control strategy for three-dimensional tensegrity structures, but with limited experimental validation. A difficulty is that cables can only take tension. This is a type of control saturation that complicates control design.
2.4.3 Applications of tensegrity structures

Tensegrities have received significant interest among scientists and engineers in different fields, e.g. architecture and space application. This section includes several examples of tensegrity structures, although these by no means represent the only uses. In architecture, architects have designed tensegrity structures for architectural applications such as bridges, roofs, domes, etc. An example is the Kurilpa Bridge, a multiple-mast, cable-stay structure based on tensegrity principles, built in 2009 for pedestrian and cycle crossing. The City of La Plata Stadium in Argentina, designed by architect Roberto Ferreira, is a very good example of using a tensegrity structure to support a membrane roof (see Figure 2.25). More tensegrity application concepts in architecture can be found in (Jáuregui, 2010).

![Figure 2.25 Tensegrity roof of the City of La Plata Stadium in Argentina (Roth, 2011)](image)

In mechanical engineering, tensegrity has been explored most for space applications, (Furuya, 1992; Benaroya, 1993; Pellegrino, 2001; Caluwaerts et al., 2014), e.g. for large, lightweight truss-type space structures to support antennae and instrumentation, because of the mass efficiency and the capability of being easily stowed and deployed (Toklu et al., 2013). In space, deployable structures have been widely used because they are the only practical ways to build large, lightweight structures for remote locations. Using tensegrity structures has the potential to make existing space structures even lighter. However, achieving the deployment of a tensegrity structure is particularly difficult, since many constraints must be met simultaneously during the process (Sultan, 2009).
The following requirements, summarised in (Sultan, 2009), need to be met in order to effectively deploy a tensegrity structure. Firstly, to guarantee the integrity of structural members, their stress levels must be limited. Secondly, internal collision and contact must be avoided. This can be achieved by ensuring that there is sufficient clearance between the structural members. Thirdly, cables should be in tension throughout the entire motion to prevent cable-entanglement that can reduce the structural stiffness. Additionally, other constraints such as energy consumption and time of deployment should be considered to optimise the system performance. These requirements significantly increase the complexity of the deployment problem. Thus, for a long time, the deployment of tensegrity structures only stayed in the concept stage.

Deployable tensegrity structures appeared conceptually in the early 1990s. Furuya (1992) proposed a concept of deployable tensegrity space structures, and made some conceptual considerations from a geometric point of view. Hanaor (1993) investigated deployable tensegrity grids and presented some analytical results and deployable models. The researcher mentioned that the ability of deployment and pre-stress can be achieved by extending struts, shortening cables or combining the two techniques. Duffy et al. (2000) studied the feasibility of using highly elastic cables for the deployment. This approach prevents cable-entanglement, but can result in very high stowage forces and stiffness creep in the structure. Besides, the controlled deployment of tensegrity structures is expected to be slow. Quick deployment may introduce high shock and vibration to the spacecraft (Knight, 2000).

Major progress occurred in the late 1990s, when a deploy strategy using equilibrium manifolds to meet all of the aforementioned requirements was proposed by Sultan and Skelton (1998) and was expanded in (Sultan and Skelton, 2003). The main idea is to control the motion of the structure, making the deployment path close to an equilibrium manifold. Figure 2.26 is a deployment example of using this strategy. Certain properties (e.g. all tendons are in tension) that these equilibrium configurations have can be passed to the intermediate configurations through which the structure passes; and the successive configurations the structure passes through are similar to the equilibrium ones (Sultan and Skelton, 2003). However, it should be noted that a large number of members need to be controlled in this deploy strategy (Sultan and Skelton, 2003).
An interesting application is to use tensegrity structures for locomotion. Friesen et al. (2014) proposed a duct climbing robot (see Figure 2.27) based on a tetrahedral tensegrity structure with two linked tetrahedral frames. A system of eight actuated cables is used to connect the two frames together, and each frame has a linear actuator. The robot can climb a duct by alternately wedging the linear actuators against the duct wall and then using the actuated cables to move one frame relative to the other.

Another locomotion application is from NASA. NASA, using a six-strut actuated tensegrity structure to provide locomotion, built the Spherical Underactuated Planetary Exploration Robot ball (SUPERball) (Sabelhaus et al., 2015) and its predecessor, the Reservoir Compliant Tensegrity Robot (ReCTeR) (Caluwaerts et al., 2014) for planetary exploration, as shown in Figures 2.28 and 2.29. Both SUPERball and ReCTeR are driven by cables and can effectively roll over the ground. Moreover, SUPERball has 12 actuators for 24 spring-cable assemblies, which is more than ReCTeR with 6 actuators. Their actuators are located at the ends of struts, and rotational DC motors are used for actuation. Control modules, batteries and wireless communication are all integrated with their structures. In addition, ReCTeR was used to validate the NASA Tensegrity Robotics Toolkit, a simulator in a physics-based environment. Control strategies can be quickly developed and tested using this simulator. It can also be used to study the physical properties of complex tensegrities.
Figure 2.27  Duct climbing tetrahedral tensegrity prototype (Friesen et al., 2014)

Figure 2.28  NASA SUPERball tensegrity robot platform (Ackerman, 2015)

Figure 2.29  NASA ReCTeR tensegrity robot (Caluwaerts et al., 2014)
Moored and Bart-Smith (2007) conducted a concept study of using four-strut tensegrity unit cells to construct a morphing wing similar to a cownose ray's wing. Two plate tensegrity structures for wing morphing were investigated. An embedded actuation was used to actuate these structures by replacing passive cables with actuators e.g. active discontinuous cables. Embedded actuations can be used for any type of tensegrity structures. However, as the number of active elements increases, this strategy becomes impractical due to increased controls complexity, energy consumption, added mass, and cost. Another strategy for introducing actuators into a tensegrity system is to use a strut-routed actuation. This strategy allows an actuator to be attached to an active element by running a cable through a series of connected struts that make a path from the active element to the actuator location (Motro, 2003). The advantage of this strategy is that actuators can be placed outside of the structure, thereby reducing added mass and element size constraints in embedded actuation. Nevertheless, this strategy is only applicable to tensegrity structures with strut-to-strut connections, and becomes infeasible for tensegrity structures with large numbers of active elements due to increased spatial constraints for routing the cables. Moored and Bart-Smith (2009) proposed a potential strategy that routed active cables through frictionless pulleys or frictionless loops at nodes. This strategy, known as clustered actuation, has the benefits of strut-routed actuations, and works well for tensegrity structures with many active elements. It can reduce the number of actuators required for complex shape changes and the force requirements of actuators in dynamic structures, justified by the authors.

Actuated tensegrity structures might be useful for aircraft wing morphing. An improvement in aircraft efficiency can be realised if the aircraft can precisely adapt its aerodynamic shape to the ideal form for different flight conditions as explained in Section 2.2.

A potential application example in robotics is human-friendly robots. Future robots must interact safely with humans, e.g. in assistive or medical applications. Conventional industrial arms are too rigid and heavy, making it difficult for them to behave gently and safely (Salisbury et al., 1988; Bicchi and Tonietti, 2004). The most successful design so far, the DLR arm, has been committed to minimising inertia (Haddadin et al., 2010). An actuated tensegrity structure is an ideal option to create an ultra-lightweight arm.
Although never pursued, using tensegrity for human-friendly robots was first proposed in 2003 at the University of California (Aldrich et al., 2003).

Another viable application of actuated tensegrity structures is snake-arm robots. These robots are slender manipulators with many joints, designed to perform remote handling operations in confined and hazardous environments (He et al., 2012). They can squeeze through small openings and crawl around obstacles. The nuclear industry has been driving the development of these manipulators because of the needs of working in radiation and confined spaces (Buckingham and Graham, 2005; Bloss, 2011). Automation can increase productivity and standardise processes. However, it is almost impossible to use conventional industrial robotic-arms in the aerospace industry due to the requirements such as passing through an access hole and conducting work within a wing box. So using snake arm robots to do these tasks are being researched. Other potential applications in confined spaces include repair and maintenance in mining and process industries. Snake-arm robots are driven by tendons (cables). The cables, terminating in different segments along the length of the arm, limit the number of segments. Other challenging issues include cable friction and cable stretch. An actuated tensegrity structure, not having these drawbacks, is an alternative solution for snake-arm robots.

Other potential applications include, but are not limited to, adaptive space structures, and structures requiring distributed vibration isolation/damping. Actuated tensegrity structures provide the opportunity to change the positioning of instruments, and make active damping of the low resonant modes more achievable.

### 2.5 Concluding remarks

The purpose of this Chapter is to review the field of smart-structures, focusing on tensegrity and similar structures. Some insightful examples are provided to show how future machines can benefit from distributed actuation integrated with load-bearing structures. Good examples are shape-changing aircraft wings that can precisely adapt to the ideal aerodynamic form under different flight conditions, and human-friendly robots that can interact safely with humans. The field of morphing wings is a potential application area that has been researched widely, so this has been reviewed in detail.
After reviewing the research activities in structures integrated with actuation systems in Section 2.3, further research on motion control and precise control under high-load applications is considered to be valuable for creating structures with high stiffness and desired motion. A tensegrity structure is a promising candidate for this future integration due to its unique properties. To fully develop an actuated tensegrity structure, many research issues need to be addressed. The following are just a few:

- Developing mathematical techniques to analyse a given structure in terms of measures of specific stiffness, and strength.
- Finding effective approaches to calculate and express shape change forward kinematics (change in length of actuated members to change in node positions).
- Creating techniques to synthesise structures to meet specified requirements.
- Establishing methods to determine dynamic models appropriate for controller design.
- Proposing and validating control schemes to provide closed-loop motion and/or force control, including internal force (pre-stress) control.
- Using systems engineering principles to optimise closed-loop performance, by adjusting the structural parameters in conjunction with the controller parameters.
3 Geometry and stability of tensegrity structure

3.1 Introduction

This Chapter presents a form-finding method for tensegrity structures in order to investigate the properties of tensegrity structures for distributed actuation. The method determines the equilibrium node positions of a tensegrity structure and calculates its stiffness through iterations. The method assumes that the members at each node meet at a single point in Sections 3.2 and 3.3, and includes node rotation in Section 3.4. For the case in Sections 3.2 and 3.3, the equilibrium configuration of a tensegrity structure can be found according to the block diagram in Figure 3.1. For the case in Section 3.4, the block diagram in Figure 3.2 is used to find the equilibrium configuration.

Define \( C_n, l_{m0}, K_m \) & initial \( N \)  
Calculate the force in each member (Equations (3.3) – (3.6))  
Calculate the resultant force vectors at each node (Equations (3.7) & (3.8))  
If one or more termination criteria are met  
Update node positions (Equation (3.14))  
Diagonalise node stiffness matrix and get the inverse of the diagonal matrix (Equations (3.11 – (3.13), (3.15) & (3.16))  
Obtain node stiffness matrix (Equations (3.17) – (3.21))  
End

Figure 3.1  Block diagram for finding the equilibrium configuration of tensegrity structures with members meeting at a single point
The investigation focuses on a specific geometrical configuration of tensegrity structures that is composed of tension/compression triangles. This tensegrity configuration has total triangulation in the tension network, which is important for making a tensegrity structure stiff (Snellson, 2014). It is also a circuit system, in which the compressed members are formed by close circuits. In other words, each closed circuit of struts can be thought of a single compressive unit. As mentioned in Pugh’s discovery, a circuit system is more rigid than a rhombic one with the same number of struts (Pugh, 1976). Its shape changing ability and stability are studied in detail, in preparation for its use in this research.

### 3.2 Finding equilibrium node positions

Figure 3.3 presents an example of a pin jointed tensegrity structure. There are \( n \) nodes (in this case 10) and \( m \) structural members (in this case 27). All the members are axially loaded. There is no bending moment in any member or torque at any joint. Typically, each member is designed to support either a compressive load (in which case it is called
a strut), or a tensile load (in which case it is called a cable). Cables are depicted with red thinner lines. The structure is normally pre-stressed. This is often necessary when the structure is supporting an external load so that the force in the cables and struts remains in the right direction (tension or compression), and may be required to stabilise the structure via its resulting geometric stiffness. In the following, the stiffness of each member is specified, and the form-finding method moves the node positions through iteration until the resultant force at each node reaches zero. The resulting node positions are those when the structure is in static equilibrium, which may not exist. In addition, the method can be used to find the structural equilibrium state with or without external loads applied to nodes in the structure.

Figure 3.3 Example tensegrity structure (struts are in black thick lines)

The connectivity matrix, \( C_n \in \mathbb{R}^{m \times n} \), defines the configuration of the structure. Each row in the matrix \( C_n \) corresponds to a member of the structure and contains a \(-1\) and a \(+1\) entry indicating the two nodes that the member spans, and otherwise the elements are zero. Taking the structure of Figure 3.3 as an example, each member in this example spans from a node with a lower node number to a node with a higher node number. However, the equilibrium structure remains the same, even if the connectivity matrix is allocated differently. The first four rows of the example are:
Let \( N \in \mathbb{R}^{3 \times n} \) be a matrix of node coordinates:

\[
N = \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\]

where \( x = [x_1 \ x_2 \ \cdots \ x_n], y = [y_1 \ y_2 \ \cdots \ y_n] \) and \( z = [z_1 \ z_2 \ \cdots \ z_n] \). Each member is represented by a vector from its ‘\(-1\)’ node to its ‘\(+1\)’ node, such that the matrix of member vectors \( M \in \mathbb{R}^{3 \times m} \) is given by:

\[
M = NC_n^T
\]

A vector \( l_m \in \mathbb{R}^{m \times 1} \) of member lengths can be calculated from \( M \) and each element in \( l_m \) is:

\[
l_{mi} = \sqrt{m_i^T m_i}
\]

where

\[
m_i = [M(1,i) \ M(2,i) \ M(3,i)]^T
\]

i.e. \( m_i \) is a column in \( M \).

Let the vector of free (unloaded) member lengths be \( l_{m0} \), so the force in each member is given by:

\[
f_m = K_m (l_m - l_{m0})
\]

where \( K_m \in \mathbb{R}^{m \times m} \) is a diagonal matrix of the member stiffnesses. Projecting these forces along the member vectors contained in \( M \) gives the matrix of member force vectors \( F_m \in \mathbb{R}^{3 \times m} \):

\[
F_m = M \text{diag}(f_{m1}/l_{m1} \ \cdots \ f_{mu}/l_{mu} \ \cdots \ f_{mn}/l_{mn})
\]

where \( \text{diag}(\cdot) \) is a diagonal matrix with its vector argument providing the sequence of main diagonal elements. Force vectors of positive magnitude in \( F_m \) point from the ‘\(-1\)’ to the ‘\(+1\)’ nodes defined in \( C_n \).

The matrix \( F_n \in \mathbb{R}^{3 \times n} \) of resultant force vectors at each node is given by:
\[ F_n = -F_mC_n + F_e \]  
(3.8)

where \( F_e \in \mathbb{R}^{3 \times n} \) is the matrix of external force vectors at each node. For a structure with no externally applied loads, \( F_e \) is a matrix with all its entries being zero.

\( K_n \in \mathbb{R}^{3n \times 3n} \) is a node stiffness matrix which relates node displacements in each Cartesian direction to the corresponding change in node forces. By expressing the node position coordinates in a column vector \( n_v \in \mathbb{R}^{3n \times 1} \) and the node force vectors in a similar manner in \( f_v \in \mathbb{R}^{3n \times 1} \),

\[ n_v = [x \quad y \quad z]^T \]
\[ f_v = [F_n(1,1) \quad F_n(1,2) \cdots F_n(2,1) \cdots F_n(3,1) \cdots]^T \]

small changes in these two column vectors can be related by \( K_n \) as follows:

\[ \Delta f_v = K_n \Delta n_v \]

(3.10)

The calculation of \( K_n \) is discussed in Section 3.3.

The stiffness matrix \( K_n \) can be transformed into a diagonal matrix \( \Omega \in \mathbb{R}^{3n \times 3n} \) through a similarity transformation. To achieve this, a linear transformation is first made to the displacements and forces by using a matrix \( P_b \in \mathbb{R}^{3n \times 3n} \) whose columns are the vectors of a new basis. The displacements and forces can be changed to the new basis by left-multiplying their elements by the inverse of \( P_b \):

\[ \Delta n_w = P_b^{-1} \Delta n_v \]
\[ \Delta f_w = P_b^{-1} \Delta f_v \]

(3.11)

This gives:

\[ \Delta f_w = P_b^{-1} K_n P_b \Delta n_w \]

(3.12)

If the columns of \( P_b \) are eigenvectors of \( K_n \), the diagonalisation of \( K_n \) is achieved. Hence,

\[ \Delta f_w = \Omega \Delta n_w \]

(3.13)

and the diagonal matrix \( \Omega \) has the eigenvalues of \( K_n \) along its diagonal.

The form-finding method is an iterative method that starts from an initial estimation of node positions and repeatedly updates the node positions until the resultant force at each node is zero. The change in node position at each iteration is to move the nodes to positions where the resultant forces are zero according to the latest stiffness matrix. However, this also causes a change in the stiffness matrix. So iteration is required. The updated node positions are given by:
\[ n_w(i+1) = n_w(i) - \Omega'(i) f_w(i) \]  
\[ (3.14) \]

where \( \Omega' \in \mathbb{R}^{3n \times 3n} \) is a modified inverse of \( \Omega \), with the six rigid body modes being removed and replaced with zeros. The inverse is derived by first sorting the elements on the diagonal of \( \Omega \):

\[ \Omega = \text{diag}([\lambda_1, \lambda_2, \ldots, \lambda_{3n}]) \]  
\[ (3.15) \]

where \(|\lambda_k|>|\lambda_{k+1}|\), and the associated columns in \( P_b \) accordingly. Then,

\[ \Omega' = \text{diag}([1/\lambda_1, \ldots, 1/\lambda_{3n-6}, 0, \ldots, 0]) \]  
\[ (3.16) \]

The six rigid modes that the inverse removes are not relevant to altering the internal forces on the nodes. This is because in these modes, all the nodes translate or rotate together. In other words, there is no relative motion between the nodes.

For a specific implementation of the form-finding method, equation (3.14) can be combined with one or more termination criteria, e.g. the root mean square value of the resultant forces at all nodes is zero to within a chosen tolerance.

### 3.3 Calculating the stiffness matrix

When moving a single node in any direction, not only the member forces in the members attached to the single node will be changed but also the resultant force acting on that node and all nodes to which it is connected will be changed. These changes can be attributed to both the change in magnitude of the member forces and their change in direction. The extent of these changes is dependent upon the node stiffness. A conventional way is to consider the node stiffness (also known as tangent stiffness) as being composed of two parts, one of which is the elastic stiffness that depends on the axial stiffness of the members, the other is the geometric stiffness that depends on the size of member forces and their orientation.

The node stiffness matrix \( K_n \) is given by the following equations described in (Guest, 2006). \( J \in \mathbb{R}^{3n \times m} \) is the Jacobian matrix that describes the rate of change of member lengths with respect to node positions and is arranged as follows:

\[
J = \begin{bmatrix}
C_n^T \text{diag}(C_n x^T) \text{diag}([1/l_{m1}, \ldots, 1/l_{m2}]) \\
C_n^T \text{diag}(C_n y^T) \text{diag}([1/l_{m1}, \ldots, 1/l_{m2}]) \\
C_n^T \text{diag}(C_n z^T) \text{diag}([1/l_{m1}, \ldots, 1/l_{m2}])
\end{bmatrix}
\]  
\[ (3.17) \]
remembering that \( l_{mi} \) is the length of member \( i \). The product of the two diagonal matrices in each term of equation (3.17) gives the components of the unit vectors of the members in that axis. For the complete structure, the elastic stiffness \( \mathbf{K}_e \in \mathbb{R}^{3n \times 3n} \) is given by:

\[
\mathbf{K}_e = \mathbf{J} \text{diag}([g_1 \cdots g_m]) \mathbf{J}^T
\]  

(3.18)

where \( g_i \) is the axial stiffness of member \( i \). \( \mathbf{q} \in \mathbb{R}^{m \times 1} \) is a vector of member tension coefficients (sometimes called force densities) and is defined as:

\[
\mathbf{q} = \begin{bmatrix}
    f_{m1}/l_{m1} & \cdots & f_{mn}/l_{mn}
\end{bmatrix}^T
\]  

(3.19)

The geometric stiffness \( \mathbf{K}_g \in \mathbb{R}^{3n \times 3n} \) can then be derived from (Guest, 2006),

\[
\mathbf{K}_g = \mathbf{I}_3 \otimes (\mathbf{C}_n^T \text{diag}(\mathbf{q}) \mathbf{C}_n) - \mathbf{J} \text{diag}(\mathbf{q}) \mathbf{J}^T
\]  

(3.20)

where \( \mathbf{I}_3 \in \mathbb{R}^{3 \times 3} \) is an identity matrix and \( \otimes \) is the Kronecker product (direct matrix product) that forms a block matrix by multiplying the \( \mathbf{C}_n^T \text{diag}(\mathbf{q}) \mathbf{C}_n \) block with each of the elements of the \( \mathbf{I}_3 \). And the final node stiffness can be derived from the sum of equations (3.18) and (3.20),

\[
\mathbf{K}_n = \mathbf{K}_e + \mathbf{K}_g
\]  

(3.21)

### 3.4 Extension to finite nodes

#### 3.4.1 Extension for finding equilibrium node positions

For practical application of a structure, engineering issues need to be considered such as the fact that several struts and cables cannot meet at a single point. A physically realisable arrangement is to have nodes of finite size and connect members to nodes via joints that are spatially separated. An example is given in Figure 3.4. The members are connected together by nodes of finite size. The circles indicate pin joints and the bodies of the nodes are not shown. A node with finite dimensions has additional 3 degrees of freedom in rotation. To consider this practical situation, it is necessary to take into account the torsional stiffness dependent on member stiffnesses and forces. The iterative method is extended as follows by including the orientation of nodes in the calculation of member lengths and considering not only the resultant force of member forces on each node but also their resultant moment. The node material itself is assumed to be rigid.
Consider $J_0 \in \mathbb{R}^{m \times n}, J_y \in \mathbb{R}^{m \times n}$ and $J_z \in \mathbb{R}^{m \times n}$. as matrices that in turn give the joint spatial positions in the $x-, y-$ and $z-$axes relative to node centres when there is no node rotation, and $N_a \in \mathbb{R}^{3 \times n}$ as a matrix of node angles,

$$N_a = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \quad (3.22)$$

where the three orthogonal rotations are roll $\phi = [\phi_1 \ \phi_2 \ \cdots \ \phi_n]$, pitch $\theta = [\theta_1 \ \theta_2 \ \cdots \ \theta_n]$ and yaw $\psi = [\psi_1 \ \psi_2 \ \cdots \ \psi_n]$ about the $x-, y-$ and $z-$axes, respectively. When the nodes rotate, the spatial positions of a joint on node $k$ and with member $i$ attached can be related by:

$$\begin{bmatrix} J_x(i, k) \\ J_y(i, k) \\ J_z(i, k) \end{bmatrix} = R_x(\psi_k)R_y(\theta_k)R_z(\phi_k) \begin{bmatrix} J_{x0}(i, k) \\ J_{y0}(i, k) \\ J_{z0}(i, k) \end{bmatrix} \quad (3.23)$$

where $J_x \in \mathbb{R}^{m \times n}, J_y \in \mathbb{R}^{m \times n}$ and $J_z \in \mathbb{R}^{m \times n}$ are matrices of joint positions in the $x-, y-$and $z-$axes relative to node centres when the nodes rotate,

$$J_x = \begin{bmatrix} J_{x1} \\ J_{x2} \\ \vdots \\ J_{xn} \end{bmatrix}; \quad J_y = \begin{bmatrix} J_{y1} \\ J_{y2} \\ \vdots \\ J_{yn} \end{bmatrix}; \quad J_z = \begin{bmatrix} J_{z1} \\ J_{z2} \\ \vdots \\ J_{zn} \end{bmatrix} \quad (3.24)$$

and $R_x(\theta), R_y(\theta)$ and $R_z(\theta)$ are rotation matrices that rotate vectors by an angle $\theta$ about the $x-, y-$ and $z-$axes and are given by:
Now, member vectors and member lengths (equations (3.3) and (3.4)) have to be calculated including the orientation of the nodes as well as the deviations of the members from the node centres. The matrix of member vectors \( \tilde{\mathbf{M}} \) can be derived in two steps, first using equation (3.3) to give a matrix of vectors between node centres with the \( i \)th column \( \mathbf{m}_i \) of the matrix as indicated in equation (3.5), and then including the effect of joint offset and node orientation on member \( i \) as follows:

\[
\tilde{\mathbf{M}} = \sum_{i=1}^{m} \left( \mathbf{m}_i + \begin{bmatrix} \mathbf{J}_x \\ \mathbf{J}_y \\ \mathbf{J}_z \end{bmatrix} \mathbf{C}_n \mathbf{S}_i \right) \mathbf{S}_i^T \mathbf{S}_i
\]

(3.26)

where \( \mathbf{S}_i \in \mathbb{R}^{1 \times m} \) is a single-entry vector with the \( i \)th entry being 1 and the rest of the entries being 0. Each member length can be calculated in the same way as before by substituting the corresponding column in \( \tilde{\mathbf{M}} \) into equation (3.4).

The member forces are combined to give not only a resultant force on each node (equation (3.8)) but also a resultant moment. The matrix \( \mathbf{F}_m \) of member force vectors is written as:

\[
\mathbf{F}_m = \begin{bmatrix} F_{mx} \\ F_{my} \\ F_{mz} \end{bmatrix}
\]

(3.27)

The matrix \( \mathbf{M}_n \in \mathbb{R}^{3 \times n} \) of resultant moment vectors at each node is then given by:

\[
\mathbf{M}_n = \begin{bmatrix} F_{my} (\mathbf{C}_n \circ \mathbf{J}_x) - F_{mx} (\mathbf{C}_n \circ \mathbf{J}_y) \\ F_{mx} (\mathbf{C}_n \circ \mathbf{J}_x) - F_{my} (\mathbf{C}_n \circ \mathbf{J}_z) \\ F_{mx} (\mathbf{C}_n \circ \mathbf{J}_y) - F_{my} (\mathbf{C}_n \circ \mathbf{J}_z) \end{bmatrix}
\]

(3.28)
where $\odot$ is the Hadamard product (entrywise product) that produces another matrix by multiplying two matrices of the same dimensions element by element.

The two matrices $\bm{F}_n$ and $\bm{M}_n$ are written as:

$$
\bm{F}_n = \begin{bmatrix} F_{nx} \\ F_{ny} \\ F_{nz} \end{bmatrix} \quad \text{and} \quad \bm{M}_n = \begin{bmatrix} M_{nx} \\ M_{ny} \\ M_{nz} \end{bmatrix}
$$

(3.29)

The node linear/angular position coordinates are expressed in one vector $\bm{n}_v \in \mathbb{R}^{6n \times 1}$, and the node force/moment vectors are expressed similarly in $\bm{f}_v \in \mathbb{R}^{6n \times 1}$. Thus,

$$
\bm{n}_v = [x \ y \ z \ \varphi \ \theta \ \psi]^T \\
\bm{f}_v = [F_{nx} \ F_{ny} \ F_{nz} \ M_{nx} \ M_{ny} \ M_{nz}]^T
$$

(3.30)

Apart from dimensions, the diagonalisation and iteration equations are just the same as before (equations (3.11) to (3.16)).

### 3.4.2 Extension for calculating the stiffness matrix

When rotating a single node, it will also produce a series of changes that are similar to those when moving a node. The effect of node rotation can be considered by resolving the rotational movement into an equivalent translational movement of the joints. $\bm{J}_r \in \mathbb{R}^{3n \times m}$ is the Jacobian matrix that relates the rate of change of member lengths to the node angles, and is given by:

$$
\bm{J}_r = \sum_{i=1}^{m} \bm{R}_i^T \bm{S}_{3i}
$$

(3.31)

where $\bm{S}_{3i} \in \mathbb{R}^{m \times m}$ is a single-entry matrix whose entry in the $i^{th}$ row and $i^{th}$ column is 1, and $\bm{R}_i \in \mathbb{R}^{3n \times 3n}$ is a matrix for member $i$ that converts the rotational movement of nodes into the equivalent translational movement of joints and is given by:

$$
\bm{R}_i = \begin{bmatrix}
0_{n,n} & \text{diag}(\bm{J}_x) & -\text{diag}(\bm{J}_y) \\
-\text{diag}(\bm{J}_x) & 0_{n,n} & \text{diag}(\bm{J}_z) \\
\text{diag}(\bm{J}_y) & -\text{diag}(\bm{J}_z) & 0_{n,n}
\end{bmatrix}
$$

(3.32)

where $0_{n,n} \in \mathbb{R}^{n \times n}$ is a zero matrix. Including the effect of node rotation, now the elastic stiffness $\bm{K}_e \in \mathbb{R}^{6n \times 6n}$ is given by:
\[ \mathbf{K}_e = \begin{bmatrix} \mathbf{J} \\ \mathbf{J}_r \end{bmatrix} \text{diag}[g_1, \ldots, g_m] \begin{bmatrix} \mathbf{J} \\ \mathbf{J}_r \end{bmatrix}^T \]  
(3.33)

The matrices \( \mathbf{C}_n \) and \( \mathbf{J} \) and the vector \( \mathbf{q} \) are written as:

\[
\mathbf{C}_n = \begin{bmatrix} \mathbf{C}_{n1} \\ \mathbf{C}_{n2} \\ \vdots \\ \mathbf{C}_{nm} \end{bmatrix}; \quad \mathbf{J} = [J_1, J_2, \ldots, J_m]; \quad \mathbf{q} = [q_1, q_2, \ldots, q_m] 
(3.34)

Now the geometric stiffness \( \mathbf{K}_g \in \mathbb{R}^{6n \times 6n} \) is a block partitioned matrix, written as:

\[
\mathbf{K}_g = \begin{bmatrix} \mathbf{K}_{g11} & \mathbf{K}_{g12} \\ \mathbf{K}_{g21} & \mathbf{K}_{g22} \end{bmatrix} 
(3.35)

where the block \( \mathbf{K}_{g11} \in \mathbb{R}^{3n \times 3n} \) gives the relationship between an applied force and the linear displacement of nodes that the force produces and can be calculated using equation (3.20), the block \( \mathbf{K}_{g12} \in \mathbb{R}^{3n \times 3n} \) relates the applied force to the angular displacement of nodes, given by:

\[
\mathbf{K}_{g12} = \sum_{i=1}^{m} (\mathbf{I}_3 \otimes (\mathbf{C}_w^T q_i \mathbf{C}_w) - \mathbf{J}_i \mathbf{q}_i \mathbf{J}_i^T) \mathbf{R}_h 
(3.36)
\]

the block \( \mathbf{K}_{g21} \in \mathbb{R}^{3n \times 3n} \) associates an applied moment with the linear displacement of nodes, given by:

\[
\mathbf{K}_{g21} = \mathbf{K}_{g22}^T 
(3.37)
\]

and the block \( \mathbf{K}_{g22} \in \mathbb{R}^{3n \times 3n} \) gives the relationship between the applied moment and the angular displacement of nodes, and can be found by:

\[
\mathbf{K}_{g22} = \sum_{i=1}^{m} \mathbf{R}_h^T (\mathbf{I}_3 \otimes (\mathbf{C}_w^T q_i \mathbf{C}_w) - \mathbf{J}_i \mathbf{q}_i \mathbf{J}_i^T) \mathbf{R}_h 
(3.38)
\]

The node stiffness \( \mathbf{K}_n \in \mathbb{R}^{6n \times 6n} \) can be derived in the same way as before (equation (3.21)) by adding the elastic and geometric stiffnesses given by the two equations (3.33) and (3.35) together.
3.5 Kinematics of an example actuated tensegrity structure

3.5.1 Configuration of the example structure

The geometrical configuration of an example tensegrity structure is depicted in Figure 3.5. The example structure has 13 struts (black thick lines) and 20 cables (red thinner lines), and is built using three basic tensegrity unit cells of the same configuration. The members are connected together by 12 nodes of finite size, and thus do not meet at a point. The circles indicate pin joints, the bodies of the nodes are not shown. Assuming that four tensile members (cables) in the centre part of the structure are actuated (members 15, 18, 21 and 24), it is possible to produce three independent motions by controlling these members antagonistically, whilst also controlling the internal force of the structure.

In preparation for the use of the example structure in this research, the stability of the structure under different situations is studied using the form-finding method. Some of its physical properties are listed in Table 3.1. Its connectivity matrix $C_n$ and matrix $N$ of node coordinates are given in Appendix 1. Note that the joints are positioned at the specified offsets away from the node centre in the direction of the corresponding member when the structure is in its neutral position.

<table>
<thead>
<tr>
<th>Table 3.1</th>
<th>Physical properties of the example tensegrity structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial strut length in $l_{m0}$</td>
<td>0.2284 m</td>
</tr>
<tr>
<td>Initial cable length in $l_{m0}$</td>
<td>0.1786 m</td>
</tr>
<tr>
<td>Strut stiffness in $K_m$ based on 6 mm diameter carbon fibre solid rod with Young’s modulus of 181 GPa</td>
<td>$2.3262 \times 10^7$ N/m</td>
</tr>
<tr>
<td>Cable stiffness in $K_m$ based on 1 mm diameter steel wire with Young’s modulus of 200 GPa</td>
<td>$8.7947 \times 10^5$ N/m</td>
</tr>
<tr>
<td>Offset of strut joint from node centre in $J_{x0}$, $J_{y0}$ and $J_{z0}$</td>
<td>0.005 m</td>
</tr>
<tr>
<td>Offset of cable joint from node centre in $J_{x0}$, $J_{y0}$ and $J_{z0}$</td>
<td>0.008 m</td>
</tr>
</tbody>
</table>
3.5.2 Actuation of example tensegrity structure

It is shown in this section that the example tensegrity structure, with practical nodes of finite dimensions, can be engineered to have shape changing properties by embedding actuated members in the structure. Figures 3.6 to 3.9 demonstrate the case where the four tensile members are unactuated (neutral position). The force levels are illustrated in Figure 3.6 in the equilibrium state with no external forces. Orthogonal views of the same structure are shown in Figures 3.7 to 3.9. The form-finding method finds an equilibrium state in which the member forces are in the right direction, i.e. the struts are all in
compression and the cables are all in tension, and are at a reasonable force level. The member forces are adjusted by altering the unloaded lengths of the members by trial and error.

Figures 3.10 to 3.15 illustrate a set of actuation examples. The top view in Figure 3.10 shows the structural equilibrium state when initial lengths of members 15 and 18 are contracted by 14.8% and members 21 and 24 are extended by 20.2%. This causes the structure to bend in the $xy$-plane (compare with top view in Figure 3.7) and an increase in the member forces (compare Figure 3.11 with the unactuated structure in Figure 3.6). The front view in Figure 3.12 is an example of shear motion in the $yz$-plane, achieved by contracting members 15 and 24 by 21.5% and extending members 18 and 21 by 17.5%. The third motion approximates to a twist about the $y$-axis and is caused by the contraction of members 15 and 21 (21.5%) and the extension of members 18 and 24 (17.9%), as shown in Figure 3.14. By adjusting the relative contraction and extension of the actuated members, the structure preload could be changed.

Figure 3.6 Member forces in the example structure when unactuated
Figure 3.7  Top view of the example structure when unactuated

Figure 3.8  Front view of the example structure when unactuated
Figure 3.9  Side view of the example structure when unactuated

Figure 3.10  Bend mode, top view (contracting 15 and 18, extending 21 and 24)
Figure 3.11  Member forces in the bend mode (see Figure 3.10)

Figure 3.12  Shear mode, front view (contracting 15 and 24, extending 18 and 21)
Figure 3.13  Member forces in the shear mode (see Figure 3.12)

Figure 3.14  Twist mode, side view (contracting 15 and 21, extending 18 and 24)
3.6 **Optimised structure with externally applied load**

3.6.1 **Structural optimisation with external load**

In this section, the struts and cables are resized so that the example structure is just able to withstand an external load (compression) of 9 kN without failure and the member forces are just in the right direction. This is to optimise the mass of the structure. The example structure should be lightweight while being able to support the loading condition. A comparison between the example structure and a cylindrical tube of the same length is made in terms of mass and stiffness to evaluate the performance of the structure.

The limiting factors concerned in the optimisation are the buckling load of the strut and the tensile strength of the cable. It is assumed that there is no node compliance or node failure in the example structure. The example loading condition is just a simple case to start with. The external load is evenly applied to the structure through six nodes at both
ends, compressing the structure in its longitudinal direction (y-direction). Now the struts in the example structure have a tubular section and are resized by taking both the sheet buckling load \( F_s \) and the Euler buckling load \( F_e \) into account.

The sheet buckling, that is dependent on wall initial irregularities, is a local buckling of a strut, given in ESDU 83034 (ESDU International plc, 1988):

\[
F_s = K_s t_s^2
\]  
(3.39)

with the coefficient,

\[
K_s = \frac{2\pi\kappa E_s \rho}{\left[3(1-\nu^2)\right]^{3/2}}
\]  
(3.40)

where \( \kappa \) is a coefficient for the correction of different end support conditions, \( E_s \) is the Young’s modulus of the strut material, \( \rho \) is the probability factor that depends on the wall initial irregularities (normally ranging from 0.2 to 0.4), \( \nu \) is the Poisson’s ratio of the strut material and \( t_s \) is the wall thickness of the tubular strut. The Euler buckling can be found from:

\[
F_e = K_e t_s D_s^3
\]  
(3.41)

with the coefficient,

\[
K_e = \frac{\pi^3 E_s}{8L_s^2}
\]  
(3.42)

where \( L_s \) is the strut length and \( D_s \) is the strut diameter. It is assumed that the struts are thin-walled in the above four equations.

Before the structural optimisation, consider a case where a cylindrical tube of 0.2 m long needs to be made by aluminium of 0.2 kg. In this case, the cross-sectional area \( A_s \) of the cylindrical tube is a constant, the aluminium density is 2700 kg/m\(^3\), and \( E_s = 69 \) GPa. Substituting \( t_s = A_s/(\pi D_s) \) into equations (3.39) and (3.41) gives:

\[
F_s = K_s \left(\frac{A_s}{\pi D_s}\right)^2
\]  
(3.43)

\[
F_e = K_e \frac{A_s D_s^2}{\pi}
\]

Figure 3.16 shows the variation of the sheet buckling and Euler buckling forces with diameter. The optimum diameter of the tube and the corresponding wall thickness are
found to be 35.6 mm and 3.3 mm by equating the sheet buckling to the Euler buckling in
equation (3.43), giving the cylindrical tube a maximum buckling load of 1 MN.

Now consider the structural optimisation of the example tensegrity structure. The struts
should be capable of carrying the external load of 9 kN without failing due to either type
of buckling, while only using the minimum amount of material. This requires,

\[ F_s = F_c = F_c \]  (3.44)

where \( F_c \) is the compression in the strut. By substituting the force \( F_c \) into equations (3.39)
and (3.41) and rearranging the equations, the optimum wall thickness and diameter of
each strut can be calculated:

\[ t_s = \left( \frac{F_c}{K_s} \right)^{1/2} \]  (3.45)

\[ D_s = \left( \frac{F_c K_s}{K_e} \right)^{1/6} \]  (3.46)

With the same consideration, the cables are optimised by considering the tensile strength
\( \sigma \) as follows:

\[ D_c = \left( \frac{4F_1 \pi \sigma}{\pi \sigma} \right)^{1/2} \]  (3.47)

where \( D_c \) is the cable diameter and \( F_1 \) is the tension in the cable.

The struts and cables in the example tensegrity structure are optimised according to the
maximum compression and tension found in the structure. Two structural equilibrium
states need to be considered. One is the state where no external load is applied. The other
is the state with the external load of 9 kN. The forces in each member for the two states
are then compared, and the larger values are used for the structural optimisation. Every
cable and strut is the same based on the largest force for all struts and all cables
respectively. It would be possible to minimise the mass of the structure further if this was
not the case. To withstand the external load, the pre-stress level of the structure needs be
increased. This can be achieved by shortening the initial cable length.

The members are optimised by iterations until the maximum compression and tension
between two consecutive iterations are convergent and are within an acceptable tolerance
of 50 N. To begin with, the parameters in Table 3.1 are used as initial values. The
parameters for the structural optimisation are listed in Table 3.2. It is found that the strut thickness of 0.14 mm is sufficient. Such a thin wall is impractical and is only used for comparison purposes. The initial cable length is now 0.1780 m which is 0.6 mm shorter than before (see Table 3.1). The full optimised dimensions of the example structure as well as the maximum compression and tension of the last two iterations are summarised in Table 3.3. The two equilibrium states of the example structure in the last iteration are shown in Figures 3.17 and 3.18.

![Graph showing variation of sheet buckling and Euler buckling with diameter](image)

**Figure 3.16** Variation of the sheet buckling and Euler buckling with diameter

<table>
<thead>
<tr>
<th>Table 3.2 Parameters for the structural optimisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient (for simply-supported end conditions) $\kappa$</td>
</tr>
<tr>
<td>Young’s modulus of the strut material (carbon fibre) $E_s$</td>
</tr>
<tr>
<td>Probability factor $p$</td>
</tr>
<tr>
<td>Poisson’s ratio of the strut material $\nu$</td>
</tr>
<tr>
<td>Strut length $L_s$</td>
</tr>
<tr>
<td>Strength $\sigma$ of the cable (carbon fibre) after multiplying a safety factor of 0.8 to its tensile strength</td>
</tr>
<tr>
<td>Optimised dimensions of the example structure</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>Strut diameter based on the last iteration $D_s$</td>
</tr>
<tr>
<td>Strut wall thickness based on the last iteration $t_s$</td>
</tr>
<tr>
<td>Cable diameter based on the last iteration $D_c$</td>
</tr>
<tr>
<td>Initial cable length in $l_{n0}$</td>
</tr>
<tr>
<td>Unloaded dimensions of the example structure in x-, y- and z-directions</td>
</tr>
<tr>
<td>Maximum compression and tension in the second last iteration (found when the structure is loaded)</td>
</tr>
<tr>
<td>Maximum compression and tension in the last iteration (found when the structure is loaded)</td>
</tr>
</tbody>
</table>

![Figure 3.17 Structural equilibrium state without the external load](image-url)
3.6.2 Comparison with cylindrical tube

The cylindrical tube of the same unloaded length (0.5043 m) is also made by carbon fibre and is sized in the same way as the strut by substituting the load of 9 kN into equations (3.45) and (3.46). It is found that the optimum diameter and wall thickness of the cylindrical tube are 31.29 mm and 0.23 mm. The comparison between the example structure and the cylindrical tube in terms of mass and stiffness and the associated physical properties are summarised in Table 3.4. The stiffness to mass ratio of the example structure is 431.50 MN/(m·kg), which is higher than that of the tube of 331.74 MN/(m·kg).
<table>
<thead>
<tr>
<th></th>
<th>Example structure</th>
<th>Cylindrical tube</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material</strong></td>
<td>Carbon fibre</td>
<td>Carbon fibre</td>
</tr>
<tr>
<td><strong>Density</strong></td>
<td>1600 kg/m$^3$</td>
<td>1600 kg/m$^3$</td>
</tr>
<tr>
<td><strong>Unloaded length</strong></td>
<td>0.5043 m</td>
<td>0.5043 m</td>
</tr>
<tr>
<td><strong>Loaded length</strong></td>
<td>0.5038 m</td>
<td>/</td>
</tr>
<tr>
<td><strong>Axial stiffness</strong></td>
<td>$18.75 \times 10^6$ N/m (equivalent in y-direction)</td>
<td>$5.97 \times 10^6$ N/m</td>
</tr>
<tr>
<td><strong>Mass</strong></td>
<td>0.0435 kg</td>
<td>0.0180 kg</td>
</tr>
<tr>
<td><strong>Stiffness to mass ratio</strong></td>
<td>431.50 MN/(m·kg)</td>
<td>331.74 MN/(m·kg)</td>
</tr>
</tbody>
</table>

### 3.7 Note on structure stability with finite nodes

It is stated in (Plummer and Lai, 2015) that member forces of a static tensegrity structure can always be engineered to meet at a point at each node, and any small dimensional errors can be accommodated by having built-in rather than pin-jointed members. Nevertheless, shape-changing tensegrity structures do not have this luxury. Figure 3.19 shows a tensegrity structure model with two unit cells actuated using McKibben ‘pneumatic muscles’. The physical model replicates the tensegrity configuration in Figure 3.3.

There are three tensile members (two McKibben ‘pneumatic muscles’ and a chain) and four compressive members meeting at each of the two nodes pointed to by the arrows. These two nodes are twisted because an angular component of their geometric stiffness is dominated by the compressive members, giving negative stiffness in one angular direction. Therefore, a general rule for actuated tensegrity structures is that cables (tensile members) must be designed to dominate struts (compressive members) at all times in terms of geometric angular stiffness. This is because cables stabilise a finite node by contributing positive geometric stiffness, while struts destabilise, causing twisting of the node (buckling the load path).
Figure 3.19 Tensegrity structure with two unit cells actuated using McKibben ‘pneumatic muscles’ that has been built at the University of Bath

3.8 Concluding remarks

A form-finding method that can calculate the equilibrium state of tensegrity structures has been presented in this Chapter. The method extends the original approach by including nodes with finite dimensions, so that members meeting at a node are spatially separated rather than impractically meeting at a single point. A node with finite dimensions has additional three degrees of freedom in rotation, and member stiffnesses and forces both contribute to the torsional stiffness in these directions. To this end, the orientation of nodes is included in the calculation of member lengths and both the resultant force of member forces at each node and their resultant moment are considered. Besides, the stiffness matrix considers not only the relationship between linear node displacements and resultant node forces but also the effect of resultant node forces and moments on linear node displacements and angular node displacements.

It is shown that the geometrical configuration of the example tensegrity structure can be designed with actuated members to give shape changing properties. The example structure can achieve three independent motions (bend, shear and twist) as well as the control of its pre-stress level by antagonistically controlling the four tensile members in the centre part. The comparison with a cylindrical tube shows that the example tensegrity structure has a higher stiffness to mass ratio. The mass of the structure can be further minimised if each member is individually optimised. The studies in the subsequent chapters are carried out based on this geometrical configuration.
4 Controller design

4.1 Introduction

The preceding Chapter has demonstrated a set of deformation examples of the example actuated tensegrity structure. The actuated tensegrity structure needs to be controlled antagonistically to realise three degrees of freedom (bend, shear and twist) and is required to maintain an appropriate level of pre-stress in the course of deformation, which means a combination of position and force control is necessary.

In this Chapter, a multi-axis control scheme is created to realise the motion and force control of the actuated tensegrity structure. The control scheme is developed according to considerations of practical application. It has been suggested in (Sterk, 2003) that actuated tensegrity structures can be controlled by pneumatic actuation to create ‘responsive architecture’. Pneumatic artificial muscles are adopted for the experimental system described later in this thesis. Therefore, the control scheme is designed by considering pneumatic actuation using on-off solenoid valves. In order to control the on-off valves, a relay controller is adopted. More details about the relay controller are discussed later in Section 4.2.1. A simplified model, analogous to the actuated tensegrity structure, is used to investigate the stability of the relay controller. And the stability of the relay controller is studied using the describing function technique.

4.2 Antagonistic control of two actuators

Prior to the development of the multi-axis control scheme for the actuated tensegrity structure, antagonistic control of two actuators is investigated. The control scheme for the two actuators consists of a relay controller to achieve both position and force control. The overview of the control scheme is as depicted in Figure 4.1.
The control scheme contains a position loop and a force loop. As the two actuators act against one another antagonistically, it is not feasible to individually control the position and the force of each actuator to reach the desired motion and the pre-stress level. Hence, the position loop and the force loop are combined together.

**Figure 4.1** Overview of the control scheme for the antagonistic control of two actuators

### 4.2.1 Dead band control

There are three types of relay controller, which are bang-bang controller, dead band controller and hysteresis controller (Dougherty, 1995). It is the interval of the dead zone that determines the type of the relay controller. For a dead band controller, the interval of the dead zone has the same level of tolerance for both the positive and negative limits. If the dead zone reduces to zero, the relay controller becomes a bang-bang controller. And the controller turns into a hysteresis controller if the levels of tolerance change depending on direction of travel (Dougherty, 1995). Relay control is appropriate as on-off valves are to be used rather than proportional valves.

These three controller can all be used to realise the antagonistic control. However, the bang-bang controller is very energy inefficient. It has only two operating modes, i.e. positive fully open and negative fully open, which means it switches the valves continually and the air is always either flowing into or out of the actuators. The other two
relay controllers have an extra operating mode, i.e. shut, in which the compressed air can be held within the actuators. These two controllers continuously switch the valves until the actuating signal lies between the dead zone limits where no actuation occurs. The hysteresis controller requires greater understanding of the dynamics of the system to optimise the tolerance setting. Hence, the dead band controller is chosen.

4.2.2 Position control

The two actuators act against one another antagonistically and are connected in series. Figure 4.2 shows an aligned connection of the two actuators. The position is controlled by considering the deviation of the two actuators from a predefined neutral position where the two actuators are of the same length. In other words, the position is defined as the difference in length of the actuators. Starting from the neutral position (zero position difference), a positive position demand will control both actuators to move towards one side and a negative position demand will make them to move in the opposite direction.

![Schematic diagram of an aligned connection of the two actuators](image)

**Figure 4.2** Schematic diagram of an aligned connection of the two actuators

4.2.3 Force control

The pre-stress level needs to be maintained during the antagonistic motion of the two actuators. For the connection shown in Figure 4.2, the tension in the two actuators is identical and can be controlled independently. However, for other cases, e.g. Figure 4.3, the two actuators are not aligned, so the actuator forces are not the same any more when they displace away from the neutral position. For this reason, the force is controlled by taking the average of the actuator forces.
4.3 Simulation studies for antagonistic control of two actuators

4.3.1 Modelling

A simulation model is created using MATLAB Simulink to validate the antagonistic control scheme for two actuators. It is assumed that the two actuators are in a straight line pulling against each other as in Figure 4.2. Figure 4.4 shows a mechanical representation of the system.

A mass represents the inertia driven by the two actuators. At each side of the mass, there is a spring and a damper which represent the damped stiffness of the actuator. A movable base is attached to the other end of the spring and the damper and is controlled by the dead band controller to move left or right at a constant speed to reach different demands.
of the position and force. The constant speed is equivalent to the actuator motion when a valve opens.

A completed free body diagram at position $y_p$ is as depicted in Figure 4.5. Since the speed of each movable base is defined as an input to the system, only the free body diagram at position $y_p$ needs to be drawn.

Summing all these forces gives:

$$m_{p} \ddot{y}_p = k_{al} (x_{al} - y_p) + c_{al} (\dot{x}_{al} - \dot{y}_p) + k_{ar} (x_{ar} - y_p) + c_{ar} (\dot{x}_{ar} - \dot{y}_p)$$  \hspace{1cm} (4.1)

where $m_p$ is the mass, $y_p$ is the position of the mass, $x_{al}$ is the position of the left base, $x_{ar}$ is the position of the right base, $k_{al}$ and $c_{al}$ are the spring stiffness and the damping coefficient for the left actuator respectively, and $k_{ar}$ and $c_{ar}$ are those for the actuator on the right side.

The general arrangement of the control system is illustrated in block diagram form in Figure 4.6. The position and force (tension) demands are denoted as $y_t$ and $F_t$ respectively, and $y_p$ and $F_p$ are feedbacks corresponding to them. The output of the dead band controller for the position loop is determined by the error between the demand and the feedback and is:

$$G = \begin{cases} 
1 & \text{if error} > B \\
0 & \text{if } -B \leq \text{error} \leq B \\
-1 & \text{if error} < -B 
\end{cases}$$  \hspace{1cm} (4.2)

where $B$ is the level of position tolerance. The dead band controller $G_t$ for the force loop is defined in the same way, but with a tolerance of $B_t$. The outputs from the two dead band controllers are combined together and then pass through a sign detection block. Its
output is +1 for a positive input signal or −1 when the input signal is negative, and zero otherwise.

The sign detection block is followed by a speed scaling factor $v$ before entering each actuator. This gives the two velocities $\dot{x}_{al}$ and $\dot{x}_{ar}$ which are the inputs to the plant. A positive position demand will cause both actuators at the neutral position with zero pre-stress to move to the left (the positive direction), while a positive force demand will cause the left actuator to move left and the other one to move in the opposite direction. The plant model for the two actuators is created according to equation (4.1). Figure 4.7 demonstrates details of connection in the plant model.

![Figure 4.6 General arrangement of the control system](image)

![Figure 4.7 Details of connection in the plant model for the two actuators](image)
4.3.2 Simulation results

The antagonistic control scheme of two actuators is verified and studied using the plant model presented above. Table 4.1 lists the values of controller and model parameters for the preliminary simulation. The speed scaling factor in the table is calculated based on the results in the actuator filling test in Section 6.4.1. The actuator stiffness and damping coefficient are estimated from the actuator material (rubber).

The time step of the numerical integration is fixed at $1 \times 10^{-3}$ s. Initially, the control system is at its neutral position, i.e. $y_p = 0$, and the pre-stress level is set to 100 N with $x_{al} = 0.01$ m and $x_{ar} = -0.01$ m. Square wave position demand signals of 5 and 30 mm amplitude are used for the preliminary simulation, while the force demands are 150 and 500 N respectively.

Simulation results with the square wave position demand signal of 5 mm and the force demand of 150 N are shown in Figure 4.8. The dead band controller successfully tracks position and force as required. Similar results are obtained when the position and the force demands are increased to 30 mm and 500 N as illustrated in Figure 4.9. The dead band controller can effectively control the system to achieve different positions and forces.

Table 4.1 Values of controller and model parameters for the preliminary simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td>Speed scaling factor</td>
<td>0.014</td>
<td>m/s</td>
</tr>
<tr>
<td>$k_{al}$ and $k_{ar}$</td>
<td>Actuator stiffness</td>
<td>$1 \times 10^4$</td>
<td>N/m</td>
</tr>
<tr>
<td>$c_{al}$ and $c_{ar}$</td>
<td>Actuator damping coefficient</td>
<td>50</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$B$</td>
<td>Level of position tolerance</td>
<td>$5 \times 10^{-4}$</td>
<td>m</td>
</tr>
<tr>
<td>$B_f$</td>
<td>Level of force tolerance</td>
<td>0.5</td>
<td>N</td>
</tr>
</tbody>
</table>
Figure 4.8 Response to square wave position demand of 5 mm with force demand maintained at 150 N ($m_p = 10$ kg)
Figure 4.9 Response to square wave position demand of 30 mm with force demand maintained at 500 N ($m_p = 10$ kg)

In the first two simulations, the mass $m_p$ is kept at 10 kg. Figure 4.10 presents the behaviour of the model when the mass is increased to 15 kg. The other parameters and the position and force demands are the same as the second simulation. After the first half period, the system becomes very oscillatory, entering a limit cycle. This indicates that the system is not always stable, and the conditions for stability are studied in the following section.
Figure 4.10  Response to square wave position demand of 30 mm with force demand maintained at 500 N ($m_p = 15$ kg)
4.4 Stability analysis for the dead band controller

4.4.1 Modelling of a simplified system

The stability of the system is related to the nonlinearity of the dead band controller. A simplified model, analogous to the position control of the antagonistic actuators, is used to analyse the stability of the dead band controller. A diagram of the simplified system is shown in Figure 4.11. A damper and a spring are placed in parallel with a mass attached at one end and a movable base at the other end. The dead band controller can control the base to move left or right at a constant speed to reach different position demands for $y$. The input speed is equivalent to the actuator motion when a valve opens, the spring and the damping represent the damped stiffness of the two actuators, and the mass $m_s$ represents the inertia driven by them. The spring stiffness and the damping coefficient are represented by $k_s$ and $c_s$ respectively.

The simplified system can be used to study position control stability. $y_d$ is the position demand signal. The dead band controller $G$ as shown in equation (4.2) is used for the position control. With only the position loop in this control system, the sign detection block is not required. The product of $G$ and $v$ gives the input velocity $\dot{x}$ to the plant. Applying Newton’s Second Law, the mathematical model of the simplified system is:

$$m_s \ddot{y} = k_s (x - y) + c_s (\dot{x} - \dot{y})$$

(4.3)

And the arrangement of the control system is illustrated in block diagram form in Figure 4.12.

![Figure 4.11](image-url) Simplified system analogous to the antagonistic position control of the two actuators

72
4.4.2 System behaviour of dead band controller with no velocity feedback

The parameters that remain constant in the simulation are listed in Table 4.2. The dead band controller is investigated using different position tolerances $B$ and masses $m_s$. The position demand is a square wave signal of 10 mm. The investigation is carried out in the following sequence. Initially, the tolerance and the mass are set to 1 mm and 6 kg respectively. Secondly, the mass is doubled to 12 kg to see its effect while maintaining the tolerance. Then, based upon the second setting, the tolerance is increased to 1.2 mm.

The time step of the numerical integration is also fixed at $1 \times 10^{-3}$ s. In the beginning, the control system is at its neutral position with $y = 0$, and there is no force in the system by setting $x = 0$. Time responses and phase plane trajectories are plotted in Figures 4.13 to 4.15 to illustrate the system behaviour. In all the phase plane plots, only part of the trajectory is presented and solid arrow heads are used to indicate the direction that a trajectory runs through. Two dash lines are drawn to represent the two dead band limits in each plot.

Table 4.2 Parameters for the simulation of the simplified system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td>Speed scaling factor</td>
<td>0.014</td>
<td>m/s</td>
</tr>
<tr>
<td>$k_s$</td>
<td>Spring stiffness</td>
<td>$1 \times 10^3$</td>
<td>N/m</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Damping coefficient</td>
<td>50</td>
<td>Ns/m</td>
</tr>
</tbody>
</table>
Figure 4.13 shows the system behaviour with \( m_s = 6 \text{ kg} \) and \( B = 1 \text{ mm} \). The system is stable with the trajectory ending at zero velocity and lying between the dead zone limits. Figure 4.14 illustrates the system behaviour when the mass is doubled while keeping the position tolerance the same. Oscillations are observed intermittently after the first half demand cycle. The unstable system oscillates back and forth and spirals into a steady limit cycle.

![Graph showing system behaviour](image)

Figure 4.13  Time response and phase plane trajectory between 5 and 10 s with \( m_s = 6 \text{ kg} \) and \( B = 1 \text{ mm} \)
Figure 4.14  Time response and phase plane trajectory between 10 and 20 s with $m_s = 12$ kg and $B = 1$ mm

The system becomes stable again as depicted in Figure 4.15, when the position tolerance is increased to 1.2 mm. The simulation results indicate that increasing the mass can create instability. And an unstable system can be stabilised by increasing the tolerance.
Figure 4.15 Time response and phase plane trajectory between 5 and 10 s with $m_s = 12$ kg and $B = 1.2$ mm

### 4.4.3 Describing function of the dead band controller

To analyse the control system, the describing function technique described in (Dougherty, 1995) is used. The technique approximates the non-linear part, i.e. the dead band controller, of the control system by considering the fundamental component of a Fourier series. The describing function is defined as:

$$G_N(M) = \frac{a_1}{M}$$  \hspace{1cm} (4.4)
where $a_1$ is the amplitude of the fundamental component in the Fourier series and $M$ is the amplitude of the excitation waveform.

To simplify the Fourier transform, the excitation and the non-linear output are assumed to be even. So sine terms in the Fourier series can be eliminated. The integrand has a quarter wave symmetry which implies not only does the wave of the integrand have half wave symmetry but each of its half waves is also symmetrical about its mid-point (Dougherty, 1995). Consequently, the integration range is reduced to one quarter of the period. And the fundamental component can be calculated by quadrupling the result.

$$a_i = 4 \cdot \frac{2}{T_S} \left[ \int_0^{t_2} G \cos(\omega t) dt + \int_{t_2}^{\frac{T_s}{4}} G \cos(\omega t) dt \right]$$

(4.5)

where $T_S$ is the period, $t_2$ is the transition time from $G = 1$ to 0, $\omega$ is the frequency and $t$ is the time variable. At the transition time $t_2$, the value of the excitation waveform is equal to the tolerance $B$ which gives:

$$B = M \cos(\omega t_2)$$

(4.6)

By combining equations (4.4) to (4.6), the describing function of the dead band controller for position is derived and for $M > B$, otherwise the function is zero.

$$G_{N}(M) = \frac{4}{\pi M} \sqrt{1 - \left(\frac{B}{M}\right)^2}$$

(4.7)

with its maximum value,

$$G_{N}(M)_{\text{max}} = \frac{2}{\pi B}$$

(4.8)

As an example, the describing function against $M$ at $B = 1$ and 1.2 mm is plotted in Figure 4.16. The stability of the non-linear control system can be examined on a Nyquist plot of the product of the describing function and the linear part of the open loop transfer function. However, as the describing function is a function of amplitude and the linear part is a function of frequency, it is more convenient to conduct the examination by plotting the frequency response of the linear part and the negative reciprocal of the describing function. There is a chance of limit cycle behaviour if they intersect (Dougherty, 1995). From Figure 4.12, the linear part of the open loop transfer function of the simplified system is:
\[ D(j\omega) = \frac{k_s + j\omega c_s}{j\omega(k_s - m_s\omega^2 + j\omega c_s)} \]  

(4.9)

Figure 4.17 is an example of the analysis when the control system is unstable \((m_s = 12 \text{ kg and } B = 1 \text{ mm})\). It is clear that the two plots intersect.
4.4.4 Criterion of guaranteed stable response

Therefore, for guaranteed stability of the control system, there should be no intersection. This requires:

\[ D(j\omega) > \frac{1}{vG_s(M)_{\text{max}}} \]  \hspace{1cm} (4.10)

where \( \omega_c \) is the frequency at which the phase of \( D(j\omega) \) is \(-180^\circ\).

Equation (4.9) can be decomposed into the product of an integrator and an expression \( A(j\omega) \) which is:

\[ A(j\omega) = \frac{k_s + j\omega c_s}{(k_s - m_s\omega^2) + j\omega c_s} \] \hspace{1cm} (4.11)

The frequency at which \( \angle A(j\omega) = -90^\circ \) can be found from the right-angle triangle formed by numerator and denominator complex numbers plotted on a phasor diagram, giving \( \omega_c \) as:

\[ \omega_c = \sqrt{\frac{k_s^2}{m_s k_s - c_s^2}} \] \hspace{1cm} (4.12)

By using equations (4.8) to (4.10) and (4.12), the requirement for stability is given by:

\[ \frac{m_s k_s - c_s^2}{c_s k_s} < \frac{\pi B}{2v} \] \hspace{1cm} (4.13)

This stability criterion is consistent with the simulation results in Figures 4.13 to 4.15, indicating that increasing the mass can make the system unstable and increasing the position tolerance helps to stabilise the system. It also indicates that the control system is more likely to be stable with higher damping. The criterion for guaranteed stable response is used as a general rule to achieve stable control of the tensegrity prototype in the simulation and experimental studies described in Chapters 6 and 7.
4.5 Control of multiple actuators

4.5.1 Transformation for multi-axis control

The multi-axis control scheme for the actuated tensegrity structure is developed according to a general co-ordinate transformation framework for multi-axis motion control (Plummer, 2010) and modified from (Plummer and Lai, 2015) as shown in Figure 4.18. This control scheme can be applied to any number of actuators, from two and above.

The multi-axis control scheme has the same principle of the antagonistic control for two actuators. A combination of position and internal force control is necessary in order to achieve the desired motion whilst maintaining an appropriate level of pre-stress. Actuators are divided into different combinations to achieve multiple degrees of freedom. There are \(d\) position loops and \(a-d\) force loops in the multi-axis control scheme, where \(d\) is the number of independent degrees of freedom and \(a\) is the number of actuators.

For the position control, the position demands are defined as the deviation of the structure in \(d\) degrees of freedom from the nominal neutral position and are represented by vector \(\delta_r \in \mathbb{R}^{d \times 1}\). The actuator displacements away from their own zero positions, e.g. mid stroke, are measured giving vector \(\delta_a \in \mathbb{R}^{a \times 1}\). For the force control, when the structure displaces away from its neutral position, the structure is no longer symmetrical. The actuator forces will no longer be the same as each other as would be expected in a symmetrical pre-loaded structure. So a virtual force demand considering the average force of actuators is used, forming vector \(f_r \in \mathbb{R}^{(a-d) \times 1}\). It is assumed that the actuator forces are measured, which gives vector \(f_a \in \mathbb{R}^{a \times 1}\).

The feedback signals \(\delta_a\) and \(f_a\) need to be transformed to virtual feedbacks \(\delta_c \in \mathbb{R}^{d \times 1}\) and \(f_d \in \mathbb{R}^{(a-d) \times 1}\) as shown in Figure 4.18. Therefore, transformation \(P\) defines the workspace position coordinates. \(P\) and the other signal transformations in Figure 4.18 could be based on full kinematic models, or they could be constant linear matrix transformations if displacements are small. The latter is considered in the multi-axis control scheme. Then, \(P \in \mathbb{R}^{d \times a}\), \(C \in \mathbb{R}^{a \times d}\), \(D \in \mathbb{R}^{a \times (a-d)}\), and \(Q \in \mathbb{R}^{(a-d) \times a}\). The compensators in the position and force loops have different options, which might be
proportional-integral controllers or other types of controllers, and are implemented in actuator co-ordinate space. In this research, the corresponding dead band controllers are used in the compensators.

\[
\begin{bmatrix}
s_1 \\
s_2 \\
s_3 \\
s_4 \\
\end{bmatrix}
\]

(4.14)

Figure 4.18 Detailed arrangement of the multi-axis control scheme for the actuated tensegrity structure modified from (Plummer and Lai, 2015)

4.5.2 Multi-axis control for the actuated tensegrity structure

The choice of \( P \), defining the workspace control coordinates, depends on user requirements. It could simply be a set of actuator position differences. For the multi-axis control scheme of the experimental actuated tensegrity structure discussed in Chapter 5, \( a = 4 \) and \( d = 3 \), as there are three degrees of freedom (bend, shear and twist) to be implemented by embedding four actuators into the structure. So for this application the vector \( \delta_a \) is:
where $s_i$ is the measured displacement from the neutral position of actuator $i$.

The four actuators are of the same length and pre-stress level when the tensegrity structure is at the neutral position. The four actuators are divided into three different combinations and are controlled by considering the deviation from the neutral position, which give:

\[
\begin{align*}
(s_1 + s_2) - (s_3 + s_4) &= 2\alpha \\
(s_1 + s_3) - (s_2 + s_4) &= 2\beta \\
(s_2 + s_3) - (s_1 + s_4) &= 2\gamma
\end{align*}
\]  

(4.15)

where $\alpha$, $\beta$, and $\gamma$ are the actuator position differences in bend, shear and twist respectively. Thus, the position demand, $\delta_r$, is a column vector containing 3 elements. Each element simply represents an actuator length difference in that specific degree of freedom. And the position transformation matrix is:

\[
P = \frac{1}{2} \times \begin{bmatrix}
1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 \\
-1 & 1 & 1 & -1
\end{bmatrix}
\]  

(4.16)

The conversion of the position loop control signal components back to actuator coordinate space can be achieved by $C = P^T$.

The force is controlled by taking the average of the four actuators. Therefore, in this case,

\[
Q = \frac{1}{4} \times [1 \quad 1 \quad 1 \quad 1]
\]  

(4.17)

which gives the virtual force feedback $f_d$ by first adding the measured actuator forces up and then dividing the sum by 4 to obtain the average force of the four actuators. A suitable choice for $D$ is $D = 4 \times Q^T$.

The multi-axis control scheme for the actuated tensegrity structure contains three position loops and a force loop. Each position loop is to control one of the three degrees of freedom. The signals $u_c$ and $u_d$ from the dead band controllers are combined together. The combined vector contains 4 elements and passes through a sign detection block, yielding $+1$ for a positive element of the combined vector or $-1$ when the element is negative, and zero otherwise. This forms signal vector $u$ for the control of the valve set. Each actuator is controlled by two on-off valves. The high pressure supply valve is opened if the corresponding element in the signal vector $u$ is $+1$. The discharge valve is
opened if the element is $-1$. Otherwise, both valves are closed to maintain a fixed amount (mass) of air in the actuator.

In this application, it is clear how the actuators act antagonistically against one another to produce the three degrees of freedom. However, in a more complex structure this may not be so. And not all actuators will contribute to all degrees of freedom.

### 4.6 Concluding remarks

A multi-axis control scheme for actuated tensegrity structure has been designed and will be applied to the tensegrity prototype in the simulation and experimental studies described in Chapters 6 and 7. Before the development of the multi-axis control scheme, a control scheme for two antagonistic actuators was demonstrated using a simple simulation model. The control scheme adopts dead band controllers for the position and force control. Stable responses are achieved with different position and force demands. However, the system enters a limit cycle when a larger mass is used.

A further simplified system analogous to the antagonistic position control of the two actuators is used to investigate the stability of the dead band controller. The simulation results indicate that increasing the mass can cause instability. And an unstable system can be stabilised by increasing the dead band tolerance. The describing function technique is used to analyse the dead band, giving a criterion for guaranteed stability that conforms to the simulation results. It indicates that the system under the control of a dead band controller is more likely to be stable with higher damping coefficient, larger dead band, or lower mass. The criterion for guaranteed stable response is used as a general rule to achieve stable control of the prototype in Chapters 6 and 7.
5 Experimental system

5.1 Introduction

The multi-axis control scheme for the actuated tensegrity structure has been presented in Chapter 4. In this Chapter, an experimental actuated tensegrity system is designed and built in order to validate and study the control scheme and the behaviour of the structure. The experimental system incorporates pneumatic actuation using on-off solenoid valves. As described in Chapter 3, the example tensegrity structure with practical nodes of finite dimension has shown that it can be designed with actuated members to give shape changing properties. Therefore, the tensegrity structure in the experimental system basically replicates the geometry of the example tensegrity structure and is scaled according to the size of the selected pneumatic actuator.

The experimental actuated tensegrity system has three constituents which are the pneumatic actuation system, the tensegrity structure and the control system, shown schematically in Figure 5.1. The requirement is to antagonistically actuate the tensegrity structure to achieve motion in three degrees of freedom while maintaining the internal force at a certain level.

The tensegrity structure has two unit cells with 9 compressive members and 14 tensile members. Among the tensile members, four are pneumatic actuators embedded in the bottom unit cell. The four pneumatic actuators are pneumatic artificial muscles (PAMs) each controlled by two on-off solenoid valves. The supply pressure is kept at 6 bar gauge pressure during the experiment. Short lengths of polyurethane tubing with 4 mm internal diameter are used for the connection of the pneumatic hardware. An xPC system is set up for the implementation of the real-time control and data acquisition (DAQ) for the experimental studies. The xPC system has a host PC for the user interface and a target environment (a target PC and an interface module) running controller code in real-time.
5.2 Pneumatic system

5.2.1 Connection of pneumatic artificial muscle

The connection of one PAM is shown diagrammatically in Figure 5.2. Each PAM is controlled by two 3/2 on-off solenoid valves. One solenoid valve is connected to the air pressure regulator keeping the supply pressure at 6 bar gauge pressure and is dedicated to pressurise the PAM. The other solenoid valve is connected to the atmosphere for the discharge of the PAM. When both valves are off, the compressed air will be trapped in the PAM, thereby maintaining the position if the force does not change. A pressure transducer with a measuring range of 0-10 bar is connected at the PAM inlet to take the measurement of the air pressure within the muscle. A draw-wire sensor with a resolution of 0.1 mm is used for the displacement measurement. A specially designed 3D printed mounting is used to hold the displacement sensor at one end of the PAM with the wire pulled out and attached at the other end.
5.2.2 Pneumatic artificial muscles

The PAMs in this research are type DMSP-20-290N-RM-CM produced by Festo. They have several advantages over conventional cylinder-type pneumatic actuators, e.g. they are frictionless and have a high force to weight ratio (Tondu, 2012). However, the characteristics of PAMs are complicated due to their intrinsic non-linear characteristics. Detailed modelling of the PAM can be found in Chapter 6. The muscle is constructed by wrapping a pressure-tight rubber tube with inextensible high strength fibres. The fibres made of aramid yarns are orientated to create a rhomboidal pattern and are layered to build a three-dimensional grid structure. When air flows into the PAM, the rubber tube expands in its circumferential direction, which generates a pulling force and a contraction movement in the longitudinal direction. The chosen PAM has an internal diameter of 20 mm and a nominal length of 290 mm, when the muscle is unpressurised. The maximum contraction of the PAM is 25% of its nominal length and its operational range is between 0 and 6 bar gauge pressure.

A rod eye (type SGS-M10x1.25 also from Festo) with spherical bearing is screwed on the male thread at each end of the PAM and attached to a clevis mount. Two plastic spacers are used to keep the rod eye in the middle of the clevis pin. The clevis mount is secured to the node with an ultra-low head hexagon socket head cap screw. The PAM
and the accessories are assembled as shown in Figures 5.3 and 5.4. The clevis linkage allows a certain amount of misalignment of the attached part permitting a restricted rotation of the PAM in three dimensions. The specifications for the PAM and the accessories are summarised in Table 5.1.

Figure 5.3   CAD drawing of the PAM assembly in top view (components described in Table 5.1)

Figure 5.4   Close-up photo of the PAM assembly (components described in Table 5.1)
<table>
<thead>
<tr>
<th></th>
<th>Specifications for the PAM and the accessories</th>
</tr>
</thead>
</table>
| **1** | **PAM** | Type DMSP-20-290N-RM-CM from Festo  
Stroke 25% of nominal length  
Internal diameter when unpressurised 20 mm  
Operating pressure 0 to 6 bar gauge pressure  
Mounting male thread M10×1.25  
Supply port G½ |
| **2** | **Rod eye** | Type SGS-M10×1.25 from Festo  
Bore size of spherical bearing 10 mm |
| **3** | **Clevis mount** | Bespoke  
Material steel |
| **4** | **Clevis pin without head** | Bespoke  
Material aluminium alloy  
Diameter 10 mm  
Held by two R-clips with wire diameter of 1.2 mm |
| **5** | **Spacer** | Laser cut  
Material clear plastic sheet  
Thickness 2 mm |
| **6** | **Ultra-low head hexagon socket head cap screw** | Type CBSS10-25 from MiSUMi  
Thread M10  
Length of 25 mm |
5.2.3 Valve set

The valve set comprises eight 3/2 on-off solenoid valves and two 4 station manifold bases as illustrated in Figure 5.5. The valves are high speed switching valves from SMC with response time of no more than 5 ms. Among them, four are series V114A and the other four are series V124A. The valves of the same series are mounted on the same manifold base. The two manifold bases are of the same type. The compressed air is supplied to the PAMs via port 1 on the manifold base for the V114A valves and discharged to the atmosphere also through port 1 on the manifold base for the V124A valves. The inlet of the PAM is connected to port 2 on both manifold bases using a T connector. For both manifold bases, channels to port 3 are blocked by epoxy resin plugs so that the air is trapped within the PAM when both valves are off (zero voltage). The control signals for the eight valves are wired together into a single D-type connector (see Figure 5.10). Table 5.2 contains some specifications for the valve set.

![Figure 5.5 Valve set](image-url)
Table 5.2  Valve set specification

<table>
<thead>
<tr>
<th>Valve V114A</th>
<th>Type V114A-5LU from SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Response time no more than 5 ms</td>
</tr>
<tr>
<td></td>
<td>Maximum operating frequency 20 Hz</td>
</tr>
<tr>
<td></td>
<td>Operating pressure 0 to 7 bar gauge pressure</td>
</tr>
<tr>
<td></td>
<td>Sonic conductance ports 1 to 2, 0.07 dm$^3$/s-bar</td>
</tr>
<tr>
<td></td>
<td>Critical pressure ratio ports 1 to 2, 0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Valve V124A</th>
<th>Type V124A-5LU from SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Response time no more than 5 ms</td>
</tr>
<tr>
<td></td>
<td>Maximum operating frequency 20 Hz</td>
</tr>
<tr>
<td></td>
<td>Operating pressure 0 to 7 bar gauge pressure</td>
</tr>
<tr>
<td></td>
<td>Sonic conductance ports 2 to 1, 0.085 dm$^3$/s-bar</td>
</tr>
<tr>
<td></td>
<td>Critical pressure ratio ports 2 to 1, 0.16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Manifold base</th>
<th>Type VV100-S41-04-M5 from SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of station 4</td>
</tr>
<tr>
<td></td>
<td>Port size M5×0.8</td>
</tr>
</tbody>
</table>

5.2.4 Sensors

The pressure transducer from Festo is plugged in a push-in T fitting at the inlet of the PAM. It can measure the air pressure from 0 to 10 bar, giving an analogue output of 0 to 10 V. The draw-wire displacement sensor from Micro-Epsilon also gives an analogue output ranging from 0 to 10 V with a measuring range of 0 to 250 mm, and has a linearity of ± 0.1% in the full scale output with the resolution of 0.1 mm. The CAD drawing and photo of the assembly of the sensors are in turn shown in Figures 5.6 and 5.7. Table 5.3 provides some information on these components. For each PAM, the two analogue outputs from the sensors and their power supplies are grouped in to a single D-sub connector (see Figure 5.10).
Figure 5.6  CAD drawing of the assembly of sensors (see Table 5.3 for the description of components)

Figure 5.7  Close-up photo of the assembly of sensors (see Table 5.3 for the description of components)
### Table 5.3  Sensor specification

<table>
<thead>
<tr>
<th></th>
<th>Pressure transducer</th>
<th>Draw-wire sensor</th>
<th>Mounting</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Type SPTE-P10R-S6-V-2.5K from Festo</td>
<td>Type WPS-250-MK30-P10 from Micro-Epsilon</td>
<td>3D printed</td>
</tr>
<tr>
<td></td>
<td>Measuring range 0 to 10 bar gauge pressure</td>
<td>Measuring range 0 to 250 mm</td>
<td>Material ABS</td>
</tr>
<tr>
<td></td>
<td>Analogue output 0 to 10 V</td>
<td>Analogue output 0 to 10 V</td>
<td></td>
</tr>
</tbody>
</table>

### 5.3 Tensegrity structure

The structure has two unit cells with a total of 23 members as illustrated in Figure 5.8. The nine compressive members are struts made from aluminium alloy tube. In addition to the four PAMs, the remaining tensile members are stainless steel cables. There is a left hand swage stud and a right hand swage stud at each end of the stainless steel cable, which means the effective cable length can easily be adjusted. The 23 members are connected together through nine finite nodes.

The structure has a unit cell configuration that is equivalent to the example structure presented in Chapter 3. It is sized according to the mid-stroke position of the PAM, i.e. the length when the PAM contracts 12.5%. So the range of motion is ± 12.5% from the mid-stroke position of the PAM. At the neutral position, the distances from one centre of the node to the other for each compressive member and tensile member are 654.4 mm and 566.8 mm, respectively. Due to the finite size of the node, the length of either the compressive member or the tensile member is shorter than the corresponding distance.

The specifications of the main constituents in the tensegrity structure are listed in Table 5.4 when the structure is at the neutral position.
Figure 5.8  Tensegrity structure with four PAMs embedded in the bottom unit cell
Table 5.4 Specifications of the main constituents in the tensegrity structure

<table>
<thead>
<tr>
<th>Strut assembly</th>
<th>Material aluminium alloy tube with outside diameter of 12.8 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>End fitting M8 threaded 13 mm diameter ball studs</td>
</tr>
<tr>
<td></td>
<td>Length 614.4 mm from one centre of the ball stud to the other</td>
</tr>
<tr>
<td></td>
<td>Quantity 9</td>
</tr>
<tr>
<td>Cable assembly</td>
<td>Material 3 mm stainless steel cables from TECNI CABLE</td>
</tr>
<tr>
<td></td>
<td>End fitting M4 swage studs with left hand thread stud at one end and right hand thread stud at the other end</td>
</tr>
<tr>
<td></td>
<td>Length 524.8 mm from one end to the other</td>
</tr>
<tr>
<td></td>
<td>Quantity 10</td>
</tr>
<tr>
<td>PAM assembly</td>
<td>End fitting clevises screwed in the node</td>
</tr>
<tr>
<td></td>
<td>Length 524.8 mm from one end to the other</td>
</tr>
<tr>
<td></td>
<td>Quantity 4</td>
</tr>
<tr>
<td>Node</td>
<td>3D printed</td>
</tr>
<tr>
<td></td>
<td>Material ABS</td>
</tr>
<tr>
<td></td>
<td>Dimensions 54×54×54 mm cube</td>
</tr>
<tr>
<td></td>
<td>13 mm socket in the centre of each face</td>
</tr>
<tr>
<td></td>
<td>22 mm deep hole at each corner with left hand and right hand threads alternately</td>
</tr>
<tr>
<td></td>
<td>Quantity 9</td>
</tr>
</tbody>
</table>

5.4 Control system

The xPC system for the control of the experimental actuated tensegrity system is illustrated in Figure 5.9. This allows a controller to be developed as a Simulink model and the model parameters to be tuned on the host PC while running the model in real-time on the target environment. The target environment consists of an interface module and a target PC with a DAQ board. The arrangement of the target environment is depicted in Figure 5.10. The xPC system provides real-time monitoring of the experimental signals which include the pressure within the PAM, the displacement of the PAM, and both the position and force demands and their corresponding feedbacks. The force is
estimated from the pressure and displacement of the PAM by using a lookup table (see Section 5.5.2).

The DAQ board from National Instruments is type NI PCI-6251. It is a high-speed multifunction DAQ board with 16 channels of analogue input, 2 channels of analogue output and 24 channels of digital I/O. All the analogue channels are 16-bit with a maximum voltage range of ±10 V. And all the digital channels have a maximum voltage range of 0 to 5 V. Eight channels of the analogue input are used to read voltages from the four pressure transducers and the four draw-wire sensors. Eight channels of the digital I/O are used to control the on-off valves.

There is a power supply unit, a valve drive board, a shielded I/O connector block and eight momentary toggle switches in the interface module. The digital signals are not directly connected to the valves, as the rated voltage of the valve (24 V) is higher than the maximum voltage of the digital I/O. Instead, the digital signals are connected to transistors on the valve drive board which drive the valves powered by the power supply unit. The eight momentary toggle switches enable the tensegrity system to be controlled manually if required. The specifications for the control system interface are summarised in Table 5.5.
Figure 5.10  
Arrangement of target environment

Table 5.5  
Control interface specifications

<table>
<thead>
<tr>
<th>DAQ board</th>
<th>Type NI PCI-6251 from National Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>16-bit resolution, 16 analogue input channels, 2 analogue output channels, maximum $\pm10$ V range</td>
</tr>
<tr>
<td></td>
<td>24 digital I/O channels, maximum 0 to 5 V range</td>
</tr>
<tr>
<td>Connector block</td>
<td>Type NI SCB-68 from National Instruments</td>
</tr>
<tr>
<td></td>
<td>2 general-purpose breadboard areas</td>
</tr>
<tr>
<td>Cable to DAQ board</td>
<td>Type NI SHC68-68-EPM from National Instruments</td>
</tr>
<tr>
<td></td>
<td>Length 2 m</td>
</tr>
<tr>
<td>Pressure transducer</td>
<td>1 V/bar</td>
</tr>
<tr>
<td>Draw-wire sensor</td>
<td>0.04 V/mm</td>
</tr>
</tbody>
</table>
5.5 Testing of components

Prior to the experimental studies of the actuated tensegrity system, some components have been investigated using an Instron materials testing machine to evaluate their mechanical characteristics. The node is tested to ensure that the tensegrity structure is sufficiently strong. The PAM is tested in order to find the relationship between force, pressure and displacement.

5.5.1 Node test

The main concern is the strength of the ABS 3D printed node. A tensile test is performed on the node to make sure it will not fail under the normal operating conditions, and to find its failure load. Average PAM force demand used is never more than 300 N, but individual PAM forces are expected to reach a maximum of about 400 N. The arrangement of the test is as shown in Figure 5.11. Two M10 threaded rods are fully screwed into two holes (22 mm engagement length) on the diagonal of the node. The two rods are clamped and pulled by the Instron machine. The test results are presented in Figure 5.12. The node can sustain a maximum tension of 780 N before it delaminates, which gives a safety factor of 1.95 for the node.
5.5.2 PAM test

Because of its complex non-linear characteristics, it is difficult to obtain the tensile force exerted by the PAM from measurements of pressure and displacement using an analytical model. Instead, a 2-D lookup table is used to determine the force based on the pressure and displacement measurements. Figure 5.13 is a 3-D graphic representation of the data in the 2-D lookup table. The data is derived by testing the PAM on the Instron machine at different pressures.

For each test, the PAM is pressurised to a desired pressure and allowed to contract freely. Then, the PAM is pulled by the Instron machine until it is 2 mm longer than the nominal length (290 mm). When the PAM is pulled by the Instron machine, the air pressure increases gradually if air is trapped in the PAM. A dead band controller is used to maintain the pressure during the test. The controller is defined in the same way as equation (4.2) with a tolerance of 0.05 bar. Figure 5.14 shows a comparison of the test data to the data provided by the Festo product configuration software. There is a significant discrepancy between the two sets of data when the displacement becomes large. This is likely to be due to the approximations used in the manufacturer’s modelling equations. The lookup table obtained through the test is used for the simulation and experimental studies in Chapters 6 and 7.
Figure 5.13 3-D graphic representation of the data in the 2-D lookup table derived from test on Instron machine

Figure 5.14 Comparison of the test data to manufacturer’s data
5.6 Concluding remarks

The design and construction of an experimental actuated tensegrity system is described in this Chapter. There are three main constituents of the experimental system which are the pneumatic actuation system, the tensegrity structure and the control system. The pneumatic actuation system adopts four PAMs to realise motion of the tensegrity structure in three degrees of freedom. The tensegrity structure has two unit cells with a total of 23 members and its cell configuration is the same as the example tensegrity structure described in Chapter 3. An xPC system is used as the control system for the implementation of the real-time control and data acquisition.

The node and the PAM have been tested under load. It is found that the node is capable of sustaining a maximum tension of 780 N, which is well above the forces expected under normal operating condition. Through the PAM test, a 2-D lookup table is built giving the relationship of the tensile force generated by the PAM to pressure and displacement. Experimental results are presented and discussed in Chapter 7.
6 Modelling and simulation of actuated tensegrity system

6.1 Overview of system modelling

This chapter presents models that are used for the simulation studies of the dynamic behaviour of the actuated tensegrity system. The pneumatic actuation system is modelled mathematically using a model of the valve set, the fluid dynamics, an empirical model of the PAM proposed by Wickramatunge and Leephakpreeda (2013) and the 2-D lookup table from the PAM test to determine the pulling force of the PAM with a given pressure and displacement. The pneumatic actuation system is modelled in Simulink and embedded in a multi-body mechanical model of the tensegrity structure implemented using SimMechanics. Combining these models with the multi-axis control scheme (see Section 4.5.2), the dynamic behaviours of the tensegrity structure are investigated.

6.2 Modelling of pneumatic actuation system

6.2.1 Modelling of pneumatic artificial muscle

Figure 6.1 is a diagram with labels for the modelling in Sections 6.2 and 6.3. The dynamic behaviour of the airflow in the PAM are mainly governed by three equations (Wickramatunge and Leephakpreeda, 2013). They are the mass continuity equation of the compressed air, the ideal gas law, and the energy change equation for an open system. Three assumptions are made for the PAM model with the consideration of the actual circumstances:

1. Air behaves like an ideal gas.
2. Pressure and temperature of the compressed air within the PAM are homogeneous.
3. Heat transfer, kinetic energy and potential energy are neglected.

The changes in pressure and volume of the PAM are dictated by the mass flow rate of the compressed air thus resulting in the change of tensile force and displacement of the muscle. The rate of change in mass $\dot{m}_a$ within the control volume of the PAM could be expressed by the subtraction of the inflow rate $\dot{m}_\text{in}$ and the outflow rate $\dot{m}_\text{out}$. The mass continuity equation of the compressed air is given by:

$$\dot{m}_a = \tau \dot{m}_\text{in} - (1 - \tau) \dot{m}_\text{out} \quad (6.1)$$

where $\tau$ is the switching value and equal to either 0 or 1 depended on the actuation signal of the PAM. When $\tau = 1$, it means the compressed air flows into the muscle. When $\tau = 0$, it means the compressed air flows out of the muscle.

According to the ideal gas law, the equation of the state of the hypothetical ideal gas is:

$$PV = m_a R_s T \quad (6.2)$$

where $P$ is the pressure of the air in the PAM, $V$ is the volume of the air in the PAM, $m_a$ is the mass of the air, $R_s$ is the specific gas constant for the air, and $T$ is the temperature of the air.

The energy change for an open system is as follows. Since the mass varies continuously, the inflow and outflow rates of the air mass need to be considered. The equation contains 3 forms of energy which are internal energy, kinetic energy and potential energy.
\[
\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \tau \dot{m}_{in}(h_{in} + \frac{V_{in}^2}{2} + g z_{in}) - (1 - \tau) \dot{m}_{out}(h_{out} + \frac{V_{out}^2}{2} + g z_{out}) \tag{6.3}
\]

where \(E_{cv}\) is the energy of a control volume, \(\dot{Q}\) is the rate of the heat exchange between the system and the surroundings, \(\dot{W}\) is the rate of the work done by the system to the surroundings, \(h_{in}\) and \(h_{out}\) are the specific enthalpies of the compressed air mass inflow and outflow from the PAM, \(V_{in}\) and \(V_{out}\) are the corresponding velocities, \(z_{in}\) and \(z_{out}\) represent the height of the mass flow, and \(g\) is the gravitational acceleration.

According to the 1st assumption, the air behaves like an ideal gas. Hence, the rate of the change in the internal energy could be written as a function that depends only on the temperature:

\[
\dot{U} = \frac{d(m_c c_v T)}{dt} \tag{6.4}
\]

where \(\dot{U}\) is the rate of the change in the internal energy and \(c_v\) is the specific heat at constant volume.

Mayer’s relation gives the following relationship between the specific gas constant and specific heats for a perfect gas:

\[
R = c_v (k - 1) \tag{6.5}
\]

\[
k = \frac{c_p}{c_v} \tag{6.6}
\]

where \(k\) is the heat capacity ratio and \(c_p\) is the specific heat at constant pressure.

Based on the 2nd assumption, the following two relations could be derived:

\[
h_{in} = h_{out} = c_p T \tag{6.7}
\]

\[
\dot{W} = p \dot{V} \tag{6.8}
\]

where \(\dot{V}\) is the rate of change in volume of the PAM.

In accordance with the 3rd assumption, the heat transfer and the changes in both the kinetic energy and the potential energy are neglected. So the energy change equation of the open system is, in fact, the energy change equation of the internal energy with \(\dot{Q} = 0\), i.e. adiabatic process,

\[
\dot{U} = -\dot{W} + [\tau \dot{m}_{in} h_{in} - (1 - \tau) \dot{m}_{out} h_{out}] \tag{6.9}
\]
By substituting equation (6.1), equation (6.2) and equations (6.4) – (6.8) into equation (6.9), the dynamic behaviour of the air pressure in the PAM is derived:

\[
\dot{P} = -\frac{kP}{V} \dot{V} + \frac{kRT}{V} \dot{m}_a
\]

(6.10)

The 2-D lookup table (see Figure 5.13) derived from the test on the Instron machine is used to estimate the force, giving the mechanical behaviour of the PAM at different displacements and pressures. The minimum force is 0 in the lookup table, as the PAM can only take a tensile load.

### 6.2.2 Modelling of on-off solenoid valve

The dynamic behaviour of the PAM is mainly due to the rate of change in mass within the PAM which results in the changes in pressure and volume of the PAM. The airflow into or out of the PAM is regulated by two solenoid valves. The inflow or outflow rate \( \dot{m}_a \) across the solenoid valve in equation (6.10) can be calculated using,

\[
\dot{m}_a = C_d C_m A_o f(P_t)
\]

(6.11)

with the function of the pressure ratio,

\[
f(P_t) = \frac{P_u}{\sqrt{T}} \left\{ \begin{array}{ll}
1 & \text{if } P_{atm}/P_u \leq P_t \leq P_{cr} \\
C_k \left[ P_t^{2/k} - P_t^{(k+1)/k} \right]^{1/2} & \text{if } P_{cr} < P_t \leq 1
\end{array} \right.
\]

(6.12)

\[
P_t = \frac{P_d}{P_u}
\]

(6.13)

where \( C_d \) is the discharge coefficient, \( C_m \) is the mass flow parameter for the air, \( A_o \) is the opening area of the valve orifice, \( P_u \) is the valve upstream pressure, \( P_d \) is the valve downstream pressure, \( P_{atm} \) is the atmospheric pressure, \( P_{cr} \) is the critical pressure ratio, and \( C_k \) is the mass flow ratio.

### 6.2.3 Empirical model for PAM volume

The PAM approximately maintains a cylindrical shape under different pressures, and an empirical model is used to determine the relationship between the radius and the length of the PAM. The volume of the PAM can be calculated by using the equation below:

\[
V = \pi r^2 L
\]

(6.14)
where \( r_e \) is the equivalent radius of the PAM, \( L \) is the instantaneous length of the PAM, \( b_1 \) and \( b_0 \) are the slope and intercept of the linear function which is derived from an experiment measuring volume flow rate. The flow rate experiment is presented in Section 6.4.1.

### 6.3 Lumped volume model for supply line

A number of factors can cause a pressure drop through a tube, e.g. friction between the fluid and the wall of the tube, change in elevation of the fluid, etc. To further improve the model, the frictional pressure loss down a tube with a length \( l \) and an internal diameter \( D \) is considered in the valve model. Other forms of pressure loss, e.g. losses in fittings and bends, are neglected. As the discharge line is short, only the loss in the supply line is modelled.

The supply line is treated as a lumped volume and is illustrated schematically in Figure 6.1. The pressure drop due to friction results in a difference in the inflow and outflow rates of the air mass across the supply line, thus changing the pressure at the inlet port (upstream pressure) of the high pressure valve. So the dynamic behaviour of the air pressure across the supply line is:

\[
\dot{P}_s = \left( \dot{m}_s - \dot{m}_{in} \right) \frac{RT}{Al}
\]  

(6.16)

where \( \dot{m}_s \) is the mass flow rate into the supply line, and \( A \) is the tube cross-sectional area and equal to \( \pi D^2/4 \). The airflow into the supply line can be determined by,

\[
\dot{m}_s = \rho \bar{v} A
\]

(6.17)

where \( \rho \) is the air density given by \( P_s/R_sT \), and \( \bar{v} \) is the mean flow velocity.

Two methods are used to calculate the mean flow velocity \( \bar{v} \). The air can be either assumed to be incompressible for simplification or treated as a compressible fluid under isothermal process. Both circumstances are modelled and compared with the measured values in the flow rate experiment.

The air is first assumed to be incompressible. For most situations, it is more likely to have turbulent flow conditions. Therefore, the model is further developed for the
frictional pressure loss incurred in turbulent flow. According to the Darcy-Weisbach equation, the frictional pressure loss in turbulent flow for circular tube is given by,

\[ \Delta P = P_s - P_u = \frac{2 f \frac{lv^2 \rho}{D}}{\frac{1}{1}} \]  \tag{6.18}

where \( P_s \) is the supply pressure and \( f \) is the flow friction factor.

The air can also be considered compressible under isothermal process. The pressure drop due to friction from the entrance of the pressure supply to the inlet port of the high pressure valve is:

\[ \Delta P = P_s - P_u = P_s \left( 1 - \frac{4 f \frac{lv^2}{D R T}}{1} \right) \]  \tag{6.19}

The mean flow velocity can be derived using either of the two equations above. For both circumstances, the flow friction factor \( f \) depends on the relative roughness ratio and Reynolds number \( Re \) of the tube. The Reynolds number is defined as \( Re = \frac{\rho v \bar{D}}{\mu} \) and rearranging the equation gives,

\[ Re = \frac{4 \bar{m}_n}{\pi \mu D} \]  \tag{6.20}

where \( \mu \) is the air dynamic viscosity and equal to \( 18.36 \times 10^{-6} \) Pa·s at \( T = 20^\circ C \).

The tube is made by polyurethane. This material can achieve an absolute roughness of a half micrometre (Grzesiak et al., 2015). With \( D = 4 \) mm, the relative roughness ratio of the tube is 0.000125. The tube is smooth enough for the friction factor to be calculated by using the Blasius power law correlation,

\[ f = \frac{0.079}{Re^{\frac{1}{4}}} \]  \tag{6.21}

For a smooth tube with \( Re \) less than \( 10^5 \), this expression yields results for pressure loss to \( \pm 5\% \) (Douglas et al., 2005).

On the basis of the flow rate experiment in Section 6.4.1, the maximum mass inflow rate is \( 4.7176 \times 10^{-4} \) kg/s, giving the maximum value of \( Re \) to be 8179. By substituting the maximum \( Re \) into equation (6.21), it gives a friction factor of 0.0083 which is used for the simulation. Based on the aforementioned equations in this section, a block diagram
for the calculation of the change in the valve upstream pressure caused by the frictional loss in the supply line is shown in Figure 6.2.

![Figure 6.2 Block diagram for the calculation of the air mass flow rate using a lumped volume model for the supply line](image)

### 6.4 PAM filling/discharging flow rate tests

#### 6.4.1 Flow rate measurement

The coefficients $b_1$ and $b_0$ in equation (6.15) are obtained from the flow rate experiment according to the approach proposed by Wickramatunge and Leephakpreeda (2013). The experiment is conducted by using the experimental setup as illustrated in Chapter 5. Initially, the PAM is placed horizontally on a smooth surface and opened to the atmosphere. Then, the discharge line is shut and the PAM is pressurised to the supply pressure, i.e. 6 bar gauge pressure, by keeping the supply line open.

During the whole process, there is no external load applied to the PAM. A flow sensor is placed between the high pressure solenoid valve unit and the PAM actuator to measure the volume flow rate. The flow sensor (PFM550-C6-1-R) is from SMC with a measurement range of 1 to 50 l/min. Apart from the volume of the inflow, the air pressure at the inlet of the PAM and the instantaneous length of the PAM are recorded during the experiment. The volume flow rate is based on standard conditions, i.e. $20^\circ$C, atmospheric pressure and 65% RH, and is converted to mass flow rate by $\dot{m}_a = \rho \dot{V}$. 

107
In Figure 6.3, the mass flow rate, the air pressure and the instantaneous length are measured values shown in the column on the left hand side. The corresponding mass and volume of the compressed air along with the equivalent radius are in the right column and calculated from equations (6.2) and (6.14). The initial volume ($0.911 \times 10^{-4}$ m$^3$) in the PAM at atmospheric pressure is determined by using its nominal length (290 mm) and nominal diameter (20 mm). The corresponding initial mass of the air within the PAM is then calculated by applying the atmospheric pressure and the initial volume to equation (6.2).

The square of the equivalent radius is plotted against the square of the instantaneous length in Figure 6.4. A linear function with coefficient of determination $R^2$ of 0.9929 is used to approximate the relationship between the two variables. The linear function is given by equation (6.15) with $b_1$ and $b_0$ of $-0.00568$ and 5.666 cm$^2$ respectively. As can be seen from the experimental data, the linearity of the two variables is high when the square of the instantaneous length is greater than 530. When further contracting the PAM, it can be viewed that the slope of the experimental data gradually becomes steeper. It means the goodness of fit of the linear regression reduces in this range of the length.
Figure 6.3  Plots of measured values of mass flow rate, air pressure and PAM length in left column along with calculated values of air mass, volume and equivalent radius in right column against time

Figure 6.4  Linear relationship between equivalent radius and PAM length

$r_e^2 = -0.0056827I^2 + 5.6662$

$R^2 = 0.99291$
6.4.2 Model performance for supply line

To validate the model of the pneumatic actuation system and see the performance of the two models for the supply line, several simulation studies are conducted and compared with the measured values. The simulation replicates the flow rate experiment of pressurising the PAM from atmospheric pressure to 7 bar absolute pressure and is calculated according to the block diagram in Figure 6.5. The PAM is unloaded during the flow rate experiment. Its no-load length can be related to the pressure using a polynomial model (Wickramatunge and Leephakpreeda, 2013). The polynomial model can therefore be used to determine the length for a given pressure in simulation. Coefficients for the polynomial model are listed in Table 6.1 and are derived from the data in Figure 6.3.

![Block diagram of supply line model](image)

Figure 6.5 Arrangement for the simulation of the flow rate experiment

<table>
<thead>
<tr>
<th>Term</th>
<th>( h_4 )</th>
<th>( h_3 )</th>
<th>( h_2 )</th>
<th>( h_1 )</th>
<th>( h_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>-0.0219</td>
<td>0.3793</td>
<td>-2.0976</td>
<td>2.7791</td>
<td>28.0692</td>
</tr>
</tbody>
</table>

Table 6.1 Coefficients for the polynomial model

\[ L = h_4 P^4 + h_3 P^3 + h_2 P^2 + h_1 P + h_0 \]
Table 6.2 presents the parameters for the simulation studies of supply line. The parameters are set based on the experimental setup and adjusted slightly. In the lumped volume model, a scaling factor of 50 is multiplied to the frictional pressure loss to take other forms of pressure loss into account, which is found to make the simulation results fit better to the experimental results.

Table 6.2 Parameters for the model of pneumatic actuation system

| General parameters | | |
|-------------------|-------------------|
| **Supply pressure** $P_s$ | 7 bar absolute |
| **Atmospheric pressure** $P_{atm}$ | 1.01325 bar |
| **Air temperature** $T$ | 293.15 K |
| **Specific gas constant for air** $R_s$ | 287 J kg$^{-1}$K$^{-1}$ |
| **Heat capacity ratio** $k$ | 1.4 |

| Mass flow rate across valve | | |
|-----------------------------|------------------|
| **Discharge coefficient** $C_d$ | 0.82 |
| **Mass flow parameter for air** $C_m$ | 0.0405 |
| **Mass flow ratio** $C_k$ | 3.864 |
| **Orifice opening area of high pressure valve** $A_o$ | 0.37 mm$^2$ |
| **Orifice opening area of discharge valve** $A_o$ | 0.435 mm$^2$ |
| **Valve response time** | 0.01 s |
| **Critical pressure ratio** $P_{cr}$ | 0.528 |

| Lumped volume model | | |
|---------------------|------------------|
| **Tube internal diameter** $D$ | 0.004 m |
| **Tube cross-sectional area** $A$ | $1.2566 \times 10^5$ m$^2$ |
| **Tube length** $l$ | 3 m |
| **Flow friction factor** $f$ | 0.0083 |
| **Scaling factor to take other forms of pressure loss into account** | 50 |
In the first simulation study, the supply line model is not included and the valve upstream pressure is maintained at 7 bar absolute pressure. The experimental and simulation results are compared in Figure 6.6. It can be seen that for both the mass flow rate and the PAM pressure the simulation results are different to the measured values. In terms of the PAM length, the simulation results give reasonable agreement to the experimental results.

In the second simulation study, the supply line model is used. The simulation results derived from the two methods are compared to the experimental results as shown in Figure 6.7. In both methods, a similar spike of mass flow rate is observed at the very beginning of the simulation. This indicates that the spike observed in the experiment is caused by the volume in the supply line. The method considering the air as compressible fits the experimental results slightly better and is used for later simulation.

Compared to the first simulation study, the discrepancy between the simulation and experimental results is reduced with supply line model activated but is still obvious. It can be seen that the mass flow rate decreases significantly after 2.2 s. The mass flow rate is mainly due to the ratio of upstream and downstream pressures. The rate of change of air pressure is higher in the simulation than in the experiment. According to equation (6.10), an increase in volume can result in a decrease in $\dot{P}$. To achieve a better agreement to the experimental results, the coefficient $b_0$ in equation (6.15) is adjusted from 5.666 to 6.735.

As depicted in Figure 6.8, the simulation yields better agreement to the experimental results after the adjustment. This indicates that the method of using nominal length and nominal diameter for the calculation of initial volume in the PAM at atmospheric pressure is inaccurate. The actual initial volume should be greater than the initial volume of $0.911 \times 10^{-4} \text{ m}^3$ calculated from the nominal length and diameter. Therefore, the coefficient $b_0$ is replaced with 6.735 for subsequent simulation.

On the other hand, even with the increase of $b_0$, it can be seen that the simulation does not agree well with the experimental results at the end of the supplying process, particularly the mass flow rate prediction (4.5 s onwards). This is probably resulted from the reducing goodness of fit of the linear function with the decrease of the PAM length as shown in Figure 6.4. The slope of the experimental data becomes steeper so that the
calculation of the PAM volume using the linear function is smaller than the volume that should be at the end of the supplying process.

### 6.4.3 Model performance for discharge line

The discharge line is also examined using the same block diagram in Figure 6.5. In the simulation, both lines are closed in the first 0.5 s. Then, the discharge line is kept open and the supply line remains closed. The air pressure in the PAM is initially set to 7 bar absolute pressure. The parameters are listed in Table 6.2. One difference to the supply line is the orifice opening area. According to the guide from SMC, the area is 5 times of the sonic conductance (see Table 5.2). So for the discharge valve, the orifice opening area is set to 0.435 mm$^2$, whereas it is set to 0.37 mm$^2$ for the high pressure valve. The other difference is that there is no lumped volume model because the discharge line is short.

The same experiment was also carried out for the discharge line. The experimental and simulated results are compared in Figure 6.9. In terms of the mass flow rate and the pressure, the simulation results well agree with the experimental results. However, the discrepancy is observed in the PAM length. This may be due to the elastic hysteresis of rubber in contraction and extension. By using a different polynomial model for extension, better fit to the experimental results may be achieved.
Figure 6.6 PAM filling simulation without supply line model compared to experiment
Figure 6.7: PAM filling simulation with supply line model compared to experiment
Figure 6.8 PAM filling simulation with supply line model and $b_0$ increased to 6.735, compared to experiment
Figure 6.9 PAM emptying behaviour
6.5 Tensegrity structure model

Figure 6.10 is the multi-body mechanical model of the tensegrity structure implemented using the MATLAB/Simulink multibody simulation toolbox SimMechanics (Lai et al., 2016). The tensegrity structure model mainly consists of body element, joint and sensor blocks. The body element block defines a geometry, an inertia and mass of a rigid body. The joint block defines the degrees of freedom between two adjoining bodies. The sensor block provides sensing of linear and angular position, force and torque.

In the tensegrity structure model, a spherical joint with three rotational degrees of freedom is used for all the end fittings in the compressive and tensile members. The PAM is modelled as two cylinders connected together by a cylindrical joint. The cylindrical joint provides one translational and one rotational degree of freedom that are coincident along the longitudinal direction of the PAM between the two cylinders. The cylindrical joint is also the interface between the model of the pneumatic actuation system and the tensegrity structure model. The tensile forces from the 2-D lookup table for the PAM (see Figure 5.13) are applied to the tensegrity structure model through each cylindrical joint. Then, according to the received tensile forces, the SimMechanics tensegrity structure model calculates the motion of the structure and sends the updated position back to the model of the pneumatic actuation system again via the four cylindrical joints.

As an approximation to the real structure, the tensegrity structure model is built by considering all the compressive and tensile members meeting at the node centre. In Figure 6.10, the struts are represented by grey cylinders, and the cables are green. The four PAMs are represented by blue cylinders. The parameters in the tensegrity structure model are set based on the properties of the physical tensegrity structure and tabulated in Table 6.3. Unlike the real structure, these components are considered solid in the model, so their densities are scaled down accordingly.
<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strut (grey)</td>
<td>Length</td>
<td>654.4 mm</td>
</tr>
<tr>
<td></td>
<td>Diameter</td>
<td>12.8 mm</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>2025 kg/m³</td>
</tr>
<tr>
<td>Cable (green)</td>
<td>Length</td>
<td>566.8 mm</td>
</tr>
<tr>
<td></td>
<td>Square side length</td>
<td>3 mm</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>1000 kg/m³</td>
</tr>
<tr>
<td></td>
<td>Cable pretension</td>
<td>200 N</td>
</tr>
<tr>
<td>PAM (blue)</td>
<td>Blue cylinder diameter</td>
<td>40 mm</td>
</tr>
<tr>
<td></td>
<td>Length of blue cylinder 1</td>
<td>200 mm</td>
</tr>
<tr>
<td></td>
<td>Length of blue cylinder 2</td>
<td>245 mm</td>
</tr>
<tr>
<td></td>
<td>Density of blue cylinder</td>
<td>400 kg/m³</td>
</tr>
<tr>
<td></td>
<td>Damping coefficient in cylindrical joint along</td>
<td>150 Ns/m</td>
</tr>
<tr>
<td></td>
<td>longitudinal direction</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Short grey cylinder diameter</td>
<td>14 mm</td>
</tr>
<tr>
<td></td>
<td>Length of short grey cylinder</td>
<td>100 mm</td>
</tr>
<tr>
<td></td>
<td>Density of short grey cylinder</td>
<td>2000 kg/m³</td>
</tr>
<tr>
<td>Node</td>
<td>Cube side length</td>
<td>54 mm</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>800 kg/m³</td>
</tr>
</tbody>
</table>
6.6 Simulation results

6.6.1 Arrangement of simulation system

Based on the models developed in this Chapter, a block diagram for the simulation of the dynamic behaviours of the actuated tensegrity system is shown in Figure 6.11. The flow chart demonstrates the detailed arrangement of the plant model in Figure 4.18. The four PAMs in the pneumatic actuation system modelled in Simulink are embedded in the multi-body mechanical model of the tensegrity structure implemented using SimMechanics. The simulation studies for the actuated tensegrity system are conducted by combining this plant model with the multi-axis control scheme (see Figure 4.18).
According to the position and force demands and corresponding feedbacks, the multi-axis control scheme sends the control signal vector $\mathbf{u}$ to the valve set in the plant model, switching the solenoid valves. This causes the variation in the mass flow rate which in turn changes the pressure and the volume of the PAM and thus the tensile force in the PAM. The tensegrity structure model displaces accordingly based upon the received tensile forces and feeds the updated position back to the model of the pneumatic actuation system.

For all the simulation studies, the configuration parameters of solver remain unchanged and are listed in Table 6.4. The dynamic performance of the actuated tensegrity system is tested with different position and force demands. Square and sine waves are used for the position demand while different constants are used as the force demand throughout the simulation studies. Initially, the pressures in the PAMs are set to 3 bar absolute pressure. As presented in Chapter 4, the criterion for guaranteed stable response (equation 4.13) is used as a general rule for the adjustment of the tolerances of the dead band controller to achieve stable control of the actuated tensegrity system.

<table>
<thead>
<tr>
<th>Table 6.4 Configuration parameters of Simulink solver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solver type</td>
</tr>
<tr>
<td>Solver</td>
</tr>
<tr>
<td>Solver maximum step size</td>
</tr>
<tr>
<td>Solver relative tolerance</td>
</tr>
<tr>
<td>Zero-crossing detection algorithm</td>
</tr>
<tr>
<td>Zero-crossing signal threshold</td>
</tr>
<tr>
<td>Number of consecutive zero crossings</td>
</tr>
</tbody>
</table>
Mass flow rate across valve (Equations (6.11) – (6.13))

Dynamic behaviour of air pressure in PAM (Equation (6.10))

Mechanical behaviour of PAM (2-D lookup table)

Empirical model (Equations (6.14) & (6.15))

Lumped volume model (Equations (6.16), (6.17) & (6.19))

Offset for length of PAM

Offset for displacement from full length

Actual displacement

Force

SimMechanics model of Tensegrity structure

Plant for actuated tensegrity

Figure 6.11 Plant model flow chart for the simulation of the dynamic behaviours of the actuated tensegrity system
6.6.2 Simulation with small amplitude square wave position demand

In the first simulation study, the three degrees of freedom (bend, shear and twist) are explored individually. A square wave with an amplitude of 10 mm and a period of 3 s is input to the actuated tensegrity system as the position demand, while the force demand is maintained at 200 N. The position tolerance $B$ and force tolerance $B_f$ are set to 0.2 mm and 3 N, respectively. For the first 3 s, the position demand is 0 mm. This allows the structure to first stabilise to the neutral position with the given force demand, before moving in the specific degree of freedom.

Based on the above inputs, the simulated responses in bend, shear and twist are derived as shown in Figures 6.12 to 6.14. The responses show that the system keeps the force within the level of the force tolerance during the motion. In each degree of freedom, the actuated tensegrity system can follow the paths of the position demand. However, undesired periodic oscillations are observed in most demanded positions. The periodic oscillations are similar to those found in Sections 4.3 and 4.4. The system, forming a steady limit cycle around the demanded position. The periodic oscillations result from the nonlinearity of dead band controller. In addition, for the motion in bend, the periodic oscillations only occur in one direction of the motion. This is probably due to the asymmetry of the mass distribution in the tensegrity structure.

According to the stability criterion described in Chapter 4, a wider dead band tolerance can help to stabilise the system. The force tolerance $B_f$ remains unchanged as the system acts in compliance with the force demand. The position tolerance $B$ is doubled to 0.4 mm through analysing the error. As shown in the zoomed in plots, absolute position errors in bend, shear and twist can reach 0.54, 0.65 and 0.75 mm, respectively, at the first few oscillations. The errors are in the workspace position coordinates. By converting back to the actuator co-ordinate space by $C = P^T$ (see equation (4.16)), it gives 0.27, 0.325 and 0.375 mm in turn. Therefore, doubling the position tolerance that is in the actuator co-ordinate space to 0.4 mm should be able to eliminate the oscillations. The simulated responses with the double position tolerance are illustrated in Figures 6.15 to 6.17. The
periodic oscillations are eliminated as expected. The system achieves stable responses and relatively small steady-state errors across all the three degrees of freedom.

Figure 6.12 Simulated responses with square position demand of 10 mm in bend, force demand of 200 N, $B = 0.2$ mm and $B_f = 3$ N
Figure 6.13  Simulated responses with square position demand of 10 mm in shear, force demand of 200 N, $B = 0.2 \text{ mm}$ and $B_t = 3 \text{ N}$

Figure 6.14  Simulated responses with square position demand of 10 mm in twist, force demand of 200 N, $B = 0.2 \text{ mm}$ and $B_t = 3 \text{ N}$
Figure 6.15  Simulated responses with square position demand of 10 mm in bend, force demand of 200 N, $B = 0.4 \text{ mm}$ and $B_t = 3 \text{ N}$

Figure 6.16  Simulated responses with square position demand of 10 mm in shear, force demand of 200 N, $B = 0.4 \text{ mm}$ and $B_t = 3 \text{ N}$
6.6.3 Simulation with small amplitude sine wave position demand

In this series of simulations, sine waves with the same amplitude (10 mm) and period (3 s) and the same constant force of 200 N are in turn input to the system as the position and force demand. The system adopts 0.4 mm as the position tolerance $B$, and the force tolerance $B_f$ is again set to 3 N. In Figures 6.18 to 6.20, the simulation results in bend, shear and twist are presented. The system satisfactorily tracks the sinusoidal position demand while keeping the force within its level of tolerance in each motion. Phase lags in bend, shear and twist are visible and are approximately 0.054, 0.03 and 0.043 s, respectively. And amplitude ratios are about 0.95, 0.98 and 0.94, correspondingly. During the motion in a certain degree of freedom, small oscillations occur in the other two degrees of freedom, which is due to the use of a switching control system. In addition, the position changes in a stepwise fashion. These small steps are expected due to the dead zone in the dead band controller.

Figure 6.17 Simulated responses with square position demand of 10 mm in twist, force demand of 200 N, $B = 0.4$ mm and $B_f = 3$ N
Figure 6.18  Performance with sinusoidal position demand of 10 mm in bend, force demand of 200 N, $B = 0.4$ mm and $B_f = 3$ N

Figure 6.19  Performance with sinusoidal position demand of 10 mm in shear, force demand of 200 N, $B = 0.4$ mm and $B_f = 3$ N
Figure 6.20  Performance with sinusoidal position demand of 10 mm in twist, force demand of 200 N, $B = 0.4$ mm and $B_f = 3$ N

6.6.4  Simulation with large amplitude square wave position demand

To see the performance under a larger position demand, a square wave with amplitude of 40 mm and period of 10 s is used. The difference in forces between the PAMs becomes greater at higher displacements. And so to keep the highest actuator force to a reasonable level, the force demand is reduced to 100 N in this series of tests. The tolerance settings are once more at $B = 0.4$ mm and $B_f = 3$ N. Figures 6.21 to 6.23 give the simulation results. The tensegrity structure at the neutral position and the corresponding deformations in bend, shear and twist are demonstrated in Figures 6.24 to 6.27.

The tensegrity system responds well to the square wave position demand of large amplitude in all the three degrees of freedom, while having relatively small steady-state errors (less than 0.4 mm) at demanded positions. And the force is controlled well to be close to the 100 N demand. Compared with the results with 10 mm square position demand, the force variation is more significant during movement in this series of tests.
Figure 6.21  Simulated responses with square position demand of 40 mm in bend, force demand of 100 N, $B = 0.4$ mm and $B_f = 3$ N

Figure 6.22  Simulated responses with square position demand of 40 mm in shear, force demand of 100 N, $B = 0.4$ mm and $B_f = 3$ N
Simulated responses with square position demand of 40 mm in twist, force demand of 100 N, $B = 0.4$ mm and $B_f = 3$ N

Tensegrity structure at the neutral position: (a) Front view, (b) Right view and (c) Top view
Figure 6.25  Tensegrity structure at −40 mm bend: (a) Front view, (b) Right view and (c) Top view

Figure 6.26  Tensegrity structure at +40 mm shear: (a) Front view and (b) Top view
6.6.5 Simulation with large amplitude sine wave position demand

Tests similar to the previous section are performed using a 40 mm sinusoidal position demand and keeping the remaining settings unchanged. Figures 6.28 to 6.30 illustrate the performance. The tensegrity system follows the position demand signal and maintains the required pre-load. The phase lags in bend, shear and twist are approximately 0.04, 0.04 and 0.05 s, respectively. And the amplitude ratios are all about 0.99. Small oscillations in the other two degrees of freedom are again observed during the motion in a specific degree of freedom. As before, the position changes in a stepwise fashion, but is less obvious because of the larger amplitude demand signal. All these behaviours are similar to those found with the 10 mm sinusoidal position demand, and are due to the use of the dead band controller and its dead zone.

Figure 6.27 Tensegrity structure at +40 mm twist: (a) Front view, (b) Right view and (c) Top view
Figure 6.28 Performance of tracking sinusoidal position demand of 40 mm in bend with force demand of 100 N, $B = 0.4$ mm and $B_f = 3$ N

Figure 6.29 Performance of tracking sinusoidal position demand of 40 mm in shear with force demand of 100 N, $B = 0.4$ mm and $B_f = 3$ N
6.6.6 Simulation with mixed position demand

With the same settings of position and force tolerance, a final demonstration of the performance under mixed position demands is given in Figures 6.31 and 6.32. The position demands in bend, shear and twist are given by 14, 12 and 10 mm, respectively. For the results of square position demand, variations of force, pressure and displacement (from 290 mm nominal length) in the PAMs during the motion are also provided in Figures 6.33 to 6.35 for demonstrative purposes.

The system performs well in both types (square and sine) of position demand and retains the force close to the 150 N demand. For the mixed square position demand, the steady-state errors are 0.01, 0.11 and 0.21 mm in bend, shear and twist. For the mixed sinusoidal position demand, the phase lags in bend, shear and twist are approximately 0.08, 0.04 and 0.01 s, respectively, while the amplitude ratios are in turn around 0.96, 0.99 and 1.00.
Figure 6.31 Performance for mixed square position demand and force demand of 150 N with $B = 0.4$ mm and $B_f = 3$ N

Figure 6.32 Performance for mixed sinusoidal position demand and force demand of 150 N with $B = 0.4$ mm and $B_f = 3$ N
Figure 6.33 Force variations in the PAMs during the motion with mixed square position demand

Figure 6.34 Pressure variations in the PAMs during the motion with mixed square position demand

Figure 6.35 Displacement variations in the PAMs during the motion with mixed square position demand
6.7 Concluding remarks

The dynamic behaviours of the actuated tensegrity system are simulated by using the plant model described in this Chapter and the multi-axis control scheme presented in Chapter 4. The model includes the pneumatic actuation system and the tensegrity structure. The pneumatic actuation system model implemented in Simulink consists of the lumped volume model for the supply line, which calculates the upstream pressure at the inlet port of the high pressure valve by considering the frictional pressure loss, the solenoid valve set model, which controls the amount of the compressed air within the PAMs, the models for the dynamic behaviour of the PAM, which calculate the rates of change in the air pressure in the four PAMs, and the 2-D lookup table, which gives the tensile forces of the PAM. The tensegrity structure model implemented using SimMechanics displaces according to the tensile forces from the lookup table and feeds the updated position back to the pneumatic actuation system.

The models in the pneumatic actuation system are examined before the simulation of the tensegrity system. This is done by comparing with experimental data from tests of pressurising/discharging a PAM. By slightly adjusting the parameters, the simulation achieves reasonable agreement with the experimental data. The performance of the actuated tensegrity system is then investigated via simulation. With a tight position tolerance of 0.2 mm, limit cycles are observed. Learning from the stability criterion of Chapter 4, the position tolerance is increased to 0.4 mm, which stabilises the system. The limit cycle is not observed in the subsequent simulation studies. The simulation results show that, in all the three degrees of freedom, the tensegrity system is able to maintain the pre-load, while achieving stable responses and relatively small steady-state errors under different square position demands and satisfactorily tracking sinusoidal position demands with small and large amplitudes. Due to the dead zone in the dead band controller, the position changes in a stepwise fashion and is slightly behind the sinusoidal demand signal. Experimental validation is demonstrated in the next Chapter.
7 Experimental studies

7.1 Introduction

In the previous Chapter, the dynamic behaviours of the actuated tensegrity system have been simulated. The simulation results show that, under the position and force tolerances of 0.4 mm and 3 N, the tensegrity system can stably respond to the square position demands and satisfactorily track the sinusoidal position demands, while maintaining the required pre-load.

In this Chapter, similar tests are conducted experimentally by implementing the multi-axis control scheme on the test rig described in Chapter 5, and comparisons are made between the simulation and experimental results. In Section 7.2, two additional sets of tests are performed. The first set is made by using various force demands to see the performance under different pre-loads. The second set is carried out by removing the unactuated top unit cell to see the influence of change in mass and experimentally validate the criterion of guaranteed stability derived in Chapter 4. For all the experimental tests, the tensegrity structure is initially in the neutral position and the sample time is fixed at 10 ms.

7.2 Experiments with small amplitude square wave position demand

7.2.1 Experimental results

In this series of experimental tests, the tensegrity system is tested by using a square wave of 10 mm amplitude with a period of 3 s and a constant force demand of 200 N. The tests are performed by separately inputting the square wave position demand to the three degrees of freedom.
Firstly, the tolerance settings are $B = 0.2$ mm and $B_f = 3$ N to see the performance of the experimental actuated tensegrity system under a tight position tolerance. The experimental results in bend, shear and twist are shown in Figures 7.1 to 7.3. It can be seen that the system oscillates intermittently in the three degrees of freedom. For the motion in shear and twist, the oscillations only occur in one direction of the motion. This is probably due to both the intrinsic asymmetry of the mass distribution in the tensegrity structure and the imperfect assembly of the structure.

To stabilise the system, the position tolerance $B$ is increased to 0.4 mm while the force tolerance $B_f$ remains unchanged. The results are demonstrated in Figures 7.4 to 7.6. The experimental tensegrity system achieves stable responses across all the three degrees of freedom. Steady-state errors in the position control are acceptable and the pre-load is well kept to be close to 200 N during the motion.

The third set of tests in this Section is made by inputting different force demands to the experimental system and adopts the tolerance settings in the previous set. The results are compared in Figures 7.7 to 7.9. It can be seen that when the force demand is small, the system responds faster. This is probably due to the air compressibility, as more air mass is required to flow into the muscle when the pre-load is higher. Another possible reason is that, with a small force demand, the tensegrity structure is less stiff and thus has less friction in the nodes.

To see the influence of change in mass, the experimental system is tested by removing the unactuated top unit cell. The position tolerance is set to 0.2 mm for these tests. The results are shown in Figures 7.10 to 7.12. With less inertia to be driven, the system is able to respond well at the tight position tolerance of 0.2 mm. This is consistent with the stability criterion (equation 4.13) which shows that the system with a dead band controller is more likely to be stable with lower mass.
Figure 7.1  Experimental responses with square position demand of 10 mm in \textit{bend}, force demand of 200 N, $B = 0.2 \text{ mm}$ and $B_f = 3 \text{ N}$

Figure 7.2  Experimental responses with square position demand of 10 mm in \textit{shear}, force demand of 200 N, $B = 0.2 \text{ mm}$ and $B_f = 3 \text{ N}$
Figure 7.3  Experimental responses with square position demand of 10 mm in twist, force demand of 200 N, $B = 0.2 \text{ mm}$ and $B_f = 3 \text{ N}$

Figure 7.4  Experimental responses with square position demand of 10 mm in bend, force demand of 200 N, $B = 0.4 \text{ mm}$ and $B_f = 3 \text{ N}$
Figure 7.5  Experimental responses with square position demand of 10 mm in shear, force demand of 200 N, $B = 0.4$ mm and $B_f = 3$ N

Figure 7.6  Experimental responses with square position demand of 10 mm in twist, force demand of 200 N, $B = 0.4$ mm and $B_f = 3$ N
Figure 7.7  Experimental responses of various force demands under square position demand of 10 mm in **bend**, $B = 0.4$ mm and $B_f = 3$ N

Figure 7.8  Experimental responses of various force demands under square position demand of 10 mm in **shear**, $B = 0.4$ mm and $B_f = 3$ N

Figure 7.9  Experimental responses of various force demands under square position demand of 10 mm in **twist**, $B = 0.4$ mm and $B_f = 3$ N
Figure 7.10  Experimental responses of one cell with square position demand of 10 mm in bend, force demand of 200 N, $B = 0.2$ mm and $B_f = 3$ N

Figure 7.11  Experimental responses of one cell with square position demand of 10 mm in shear, force demand of 200 N, $B = 0.2$ mm and $B_f = 3$ N
7.2.2 Comparison between simulation and experimental results

When the position tolerance is at $B = 0.2$ mm, the behaviours of the experimental system are slightly different from the corresponding simulation results in Section 6.6.2. The oscillations only occur intermittently in the experimental tests but they have been observed more regularly in the simulation. For the stable responses at $B = 0.4$ mm, the experimental and simulation results are similar in the position feedback as illustrated in Figures 7.13 to 7.15. The experimental system takes longer to reach the demanded positions and has slightly larger steady-state errors. In terms of the force feedback, the simulation results are less noisy. These differences are probably due to modelling error in the pneumatic actuation system and node friction that has not been considered in the tensegrity structure model.
Figure 7.13  Comparison between simulation and experiment for square position demand of 10 mm in bend, $B = 0.4$ mm and $B_f = 3$ N

Figure 7.14  Comparison between simulation and experiment for square position demand of 10 mm in shear, $B = 0.4$ mm and $B_f = 3$ N

Figure 7.15  Comparison between simulation and experiment for square position demand of 10 mm in twist, $B = 0.4$ mm and $B_f = 3$ N
7.3 Experiments with small amplitude sine wave position demand

To see the performance with a sinusoidal position demand, a sine wave position demand of 10 mm amplitude and 3 s period is used in turn for each degree of freedom with the force demand fixed at 200 N. The tolerance settings are $B = 0.4$ mm and $B_f = 3$ N in all cases. The experimental results are illustrated in Figures 7.16 to 7.18. It can be seen that the experimental system performs well in both position tracking and force keeping. The comparisons of the simulation and experimental position feedbacks are presented in Figures 7.19 to 7.21. The simulation results agree well with the experimental results. Small oscillations similar to those found in the simulation are observed. The position also changes in a stepwise fashion and is slightly behind the position demand.

Figure 7.16 Performance with sinusoidal position demand of 10 mm in bend and force demand of 200 N
Figure 7.17  Performance with sinusoidal position demand of 10 mm in shear and force demand of 200 N

Figure 7.18  Performance with sinusoidal position demand of 10 mm in twist and force demand of 200 N
Figure 7.19  Comparison between simulation and experiment for sinusoidal position demand of 10 mm in **bend**

Figure 7.20  Comparison between simulation and experiment for sinusoidal position demand of 10 mm in **shear**

Figure 7.21  Comparison between simulation and experiment for sinusoidal position demand of 10 mm in **twist**
7.4 Experiments with large amplitude square wave position demand

The experimental tensegrity system is also tested with a square wave of 40 mm amplitude. To be consistent with the simulation in Section 6.6.4, the force demand is set to 100 N and the position and force tolerances are $B = 0.4$ mm and $B_f = 3$ N. The experimental results are shown in Figures 7.22 to 7.24. The experimental system responds well to the square wave position demand of large amplitude while maintaining the required pre-load.

The comparisons with simulation are illustrated in Figures 7.25 to 7.27. The main discrepancy is that the experimental system takes more time to reach the demanded positions. The rate of change of the position in the simulation is greater than in practice and remains almost constant before reaching the demanded positions. However, in practice, the rate of change of position feedback reduces gradually. This could result from the decreasing agreement of the pneumatic model to the actual situation at the end of the contracting and extending process of the PAM (see Figures 6.8 and 6.9).
Figure 7.23  Experimental responses with square position demand of 40 mm in shear

Figure 7.24  Experimental responses with square position demand of 40 mm in twist
Figure 7.25  Comparison between simulation and experiment for square position demand of 40 mm in bend

Figure 7.26  Comparison between simulation and experiment for square position demand of 40 mm in shear

Figure 7.27  Comparison between simulation and experiment for square position demand of 40 mm in twist
7.5 Experiments with large amplitude sine wave position demand

To see the performance of tracking a large sinusoidal position demand, a sine wave of 40 mm amplitude is input to the experimental system. The force demand is set to 100 N. The tolerance settings are adopted from the previous section with $B = 0.4$ mm and $B_f = 3$ N. The experimental results and the comparisons are shown in Figures 7.28 to 7.30 and Figures 7.31 to 7.33, respectively. The experimental system satisfactorily tracks the large sinusoidal position demands. The experimental responses are slightly slower compared with the simulation, especially when reaching the crests and troughs of the sinusoidal position demand.

Figure 7.28 Performance of tracking sinusoidal position demand of 40 mm in bend
Figure 7.29 Performance of tracking sinusoidal position demand of 40 mm in shear

Figure 7.30 Performance of tracking sinusoidal position demand of 40 mm in twist
Figure 7.31  Comparison between simulation and experiment for sinusoidal position demand of 40 mm in **bend**

Figure 7.32  Comparison between simulation and experiment for sinusoidal position demand of 40 mm in **shear**

Figure 7.33  Comparison between simulation and experiment for sinusoidal position demand of 40 mm in **twist**
7.6 Experiments with mixed position demand

A final demonstration is given in Figures 7.34 and 7.35 for mixed position demands of 14, 12 and 10 mm in bend, shear and twist, respectively. The force demand is set to 150 N. For the results of square wave position demand, variations of force, pressure and displacement (from 290 mm nominal length) in the PAMs are also provided in Figures 7.36 to 7.38 for demonstrative purposes. The experimental system performs well in these two tests. The main difference from the simulation in Section 6.6.6 is in the force variations of the PAMs. This may be due to the imperfect assembly of the tensegrity structure and are acceptable as things are not perfect in practice. Besides, the PAM force is very sensitive to the pressure and displacement. Any small error in pressure or displacement may cause a significant change in the PAM force.

Figure 7.34 Performance under mixed square position demand with force demand of 150 N, $B = 0.4$ mm and $B_f = 3$ N
Figure 7.35  Performance under mixed sinusoidal position demand with force demand of 150 N, $B = 0.4$ mm and $B_f = 3$ N

Figure 7.36  Force variations in the PAMs during the motion with mixed square position demand
Figure 7.37  Pressure variations in the PAMs during the motion with mixed square position demand

Figure 7.38  Displacement variations in the PAMs during the motion with mixed square position demand

7.7  Concluding remarks

The experimental actuated tensegrity system has been tested with different square and sinusoidal position demands. The experimental system is unstable with a tight position tolerance of 0.2 mm, which is consistent with the simulation results. By doubling the position tolerance to 0.4 mm, the system is stabilised and no more oscillation is found in the later tests, which is again consistent with the simulation. It is found that the experimental system responds slower at a higher force demand. This is probably due to the air compressibility, as more air mass is required to flow into the muscle when the pre-load is higher. The increased friction in the nodes may also cause a slower response at a higher force demand. Besides, with the top unit cell removed, the system becomes
lighter and is stable with a tighter position tolerance. This is consistent with the stability criterion derived in Chapter 4.

Comparisons are made between the simulation and experimental results. It is found that the experimental system takes longer to reach the demanded positions and has slightly larger steady-state errors. In terms of the force feedback, the simulation results are less noisy. These differences are probably due to modelling error in the pneumatic actuation system and node friction that has not been considered in the tensegrity structure model.

In addition, when the experimental system is tested with a large amplitude square wave of 40 mm, it is found that the motion slows nearing the final position. This is different from the simulation where the slope hardly changes before reaching the demanded positions. A possible reason is that the agreement of the pneumatic model to the actual situation decreases at the end of the contracting and extending process of the PAM (see Figures 6.8 and 6.9). Furthermore, when the experimental system is tested by using sinusoidal position demands, the position changes in a stepwise fashion and small phase lags are observed at the same time. These behaviours are due to the dead zone in the dead band controller.

Overall, the motion control of the experimental actuated tensegrity system is good. The multi-axis control scheme successfully controls the tensegrity structure to track motion demands in three degrees of freedom. The experimental system is able to maintain the pre-load, while achieving stable responses under different square wave position demands and satisfactorily tracking small and large sinusoidal position demands.
8 Conclusions

8.1 Achievements

The research concerns the study of a proposed multi-axis system configuration that is a type of actuated tensegrity structure. A form-finding method has been developed for the study of this type of tensegrity structure. The method considers the engineering issue that members meeting at a real node cannot meet at a single point by including nodes with finite dimensions.

Several kinematic example of the tensegrity structure have been investigated using the form-finding method. It has been shown that, with the antagonistic control of the four tensile members in the centre part, the geometrical configuration of the proposed tensegrity structure can achieve three independent motions (bend, shear and twist) along with the control of its pre-stress level. A comparison between the example structure and a cylindrical tube of the same length has been made, showing that the structure has a higher stiffness to mass ratio. The study has justified the hypothesis that a tensegrity structure can be designed with actuated members to give shape changing properties while potentially allowing a good stiffness to mass ratio. As a general rule, cables (tensile members) must be designed to dominate struts (compressive members) at all times so as to stabilise a node of finite dimensions by contributing positive geometric stiffness.

A multi-axis control scheme with dead band controllers has been developed for the proposed tensegrity structure, including both motion and force control. Two mathematical models have been created to study the control scheme. The principle of the control scheme has been validated using a model with two antagonistic actuators. The behaviour of the dead band controllers has been investigated using a simplified model, showing that the controller nonlinearity can make the system unstable. Then, the describing function technique has been used to analyse the dead band controller, and a criterion for guaranteed stability has been derived. The criterion indicates that the system
under the control of a dead band controller is more likely to be stable with higher
damping coefficient, larger dead band, or lower mass and can be used as a general rule
to achieve stable control in systems of such type.

An experimental actuated tensegrity system has been designed and built, containing three
main constituents which are the pneumatic actuation system, the tensegrity structure and
the control system. Four PAMs are used in the pneumatic system because they are
frictionless and have a high force to weight ratio compared to both conventional cylinder-
type pneumatic actuators and electromagnetic actuators. Each PAM is controlled by two
on-off solenoid valves. This arrangement, allowing the compressed air to be trapped in
the PAM, can maintain the desired position and pre-load, when both valves are off. A 2-
D lookup table has been established, giving the relationship of the force generated by the
PAM to measured pressure and displacement.

Models for the experimental actuated tensegrity system have been developed in detail,
including the pneumatic actuation system and tensegrity structure. The pneumatic
actuation system model consists of a supply line model, a solenoid valve set model,
models for the dynamic behaviour of the PAM and the 2-D lookup table. The models in
the pneumatic actuation system have been examined by comparing with the experimental
data from tests of pressuring/discharging a PAM. The simulation results show good
agreement with the experimental data. The tensegrity structure model is implemented
using SimMechanics, a Simulink toolbox for multibody simulation.

The dynamic behaviours of the actuated tensegrity system have been investigated via
several simulation studies, using the developed models and the multi-axis control scheme.
Limit cycles are observed, when a tight position tolerance is used. Learning from the
stability criterion, the tensegrity system has been stabilised by slightly increasing the
position tolerance. The simulation studies have shown that, in all the three degrees of
freedom, the tensegrity system can maintain the pre-load, while achieving stable
responses and relatively small steady-state errors.

The simulation results have been validated experimentally. The experimental system has
been found to be unstable with the same tight position tolerance as in the simulation.
After the same increase in the position tolerance, the experimental system is stable and
no limit cycle behaviour is found in the later tests. It has been found that the experimental system responds slower at higher force demand. This could be due to the fact that friction causes slower response. The stability criterion has also been experimentally validated.

Comparisons have been performed between the simulation and experimental results. It has been shown that the experimental system takes longer to reach the demanded positions and has slightly larger steady-state errors. Another difference is that the motion of the experimental system slows near the final position. The possible reasons have been discussed, including modelling error in the pneumatic actuation system and node friction that has not been considered in the structure model.

The overall concept of using a tensegrity structure to develop a load-bearing structure with shape-changing capability has been successfully proved. The performance of the experimental actuated tensegrity system is good. The multi-axis control scheme can effectively control the tensegrity structure to achieve shape changes while maintaining the pre-load. The fully developed models can be used for the further development of the system.

### 8.2 Further work

It has been found that due to its inherent non-linearities, the PAM force is very sensitive to the pressure and displacement. Any small error in pressure or displacement may cause a significant change in the PAM force, thereby reducing the accuracy of the force control. In future studies, it would be useful to develop methods that can accurately obtain the force in PAM to enhance the system performance, using load cells may be preferable.

The multi-axis control scheme can effectively control the experimental actuated tensegrity system. However, in the control scheme, the non-linearity of the dead band controller may cause problems in practical applications, resulting in system instability. Further research to develop the dead band controller to be adaptive would be helpful to improve the robustness of such systems.

It would be valuable to extend the research on practical applications. A suitable area is for aircraft morphing as the shape-changing properties (bend, shear and twist) of the
The proposed tensegrity structure are close to some shape morphing requirements of the aircraft. The morphing could involve any part of the aircraft. It could be for wing, tail and even the fuselage. The active winglet seems to be a potential application for aircraft morphing as the primary wing structure does not need to be redesigned. For aircraft morphing, consideration should also be given to external load conditions and the requirement for a flexible skin. The basic approach could be tailored for other applications. To fit a specific application, the basic tensegrity configuration may need to be resized and its aspect ratio may need to be adjusted. It may be necessary to redistribute actuators and/or re-stack tensegrity unit cells to achieve different desired motions.

Several mathematical problems still need to be solved for practical applications, including kinematic analysis. It would be important to understand the relationship between actuator position changes and structure shape changes. Calculating the forward kinematics would be a vital first step to successfully using the basic approach in any application, particularly for robotic manipulators. For actual control, inverse kinematics would be usually required. It would also be useful to investigate the optimal path for actuation with minimal energy or fastest response.
References


Conference on Robotics and Automation (ICRA), 31 May-7 June Hong Kong, China. IEEE, pp. 4222-4228.


Appendix 1 Connectivity matrix and matrix of node coordinates

The connectivity matrix $C_n$ (equation (3.1)) of the example tensegrity structure and its matrix $N$ (equation (3.2)) of node coordinates are shown in Figures A1.1 and A1.2, respectively.

Figure A1.1  Connectivity matrix $C_n$ of the example tensegrity structure

$C_n = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Figure A1.2  Matrix $N$ of node coordinates of the example tensegrity structure

$N = \begin{pmatrix} -0.115 & 0.115 & 0.115 & 0.345 & 0 & 0 & 0.23 & 0.23 & -0.115 & 0.115 & 0.115 & 0.345 \\ -0.115 & -0.115 & -0.115 & -0.115 & 0 & 0 & 0 & 0 & 0.115 & 0.115 & 0.115 & 0.115 \\ 0 & 0 & 0.23 & 0.23 & -0.115 & 0.115 & 0.115 & 0.345 & 0 & 0 & 0.23 & 0.23 \end{pmatrix}$