Modelling the impact of urbanisation on flood frequency relationships in the UK

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Abstract
This paper investigates the effect of urbanisation on the three key statistics used to establish flood frequency curves when combining the index flood method with the method of L-moments for estimating distribution parameters, i.e. the median annual maximum peak flow (the index flood), L-CV and L-SKEW. Using an existing procedure for estimating the three statistics at ungauged sites in the UK using catchment descriptors, as-rural estimates of the three statistics were obtained in 200 urban catchments and compared with the corresponding values obtained from observed data. The (log) differences of these estimates were related to catchment descriptors relevant to the urbanisation process using linear regression. The results show that urbanisation lead to a reduction in L-CV but an increase in L-SKEW. A jackknife leave-one-out experiment showed that the adjustment factors developed here were generally better at predicting the effect of urbanisation on the flood frequency curve than the existing adjustment factor currently used in the UK.

Key Words: flood frequency estimation, urbanisation, index flood, L-moments, FEH

INTRODUCTION
The UK standard method for establishing flood frequency relationships (or curves) is based on statistical analysis of annual maximum series (AMS) of instantaneous peak flow, and was first described in the Flood Studies Report (FSR) (NERC, 1975), and later updated in the Flood Estimation Handbook (FEH) (Institute of Hydrology, 1999). It allows estimation of T-year peak flow values at any gauged or ungauged catchment larger than 0.5 km$^2$. Recently, the FEH method has again been updated by the Environment Agency (2008) as documented by Kjeldsen and Jones (2009a, 2009b).

The method is based on regional frequency analysis using L-moment ratios, and is an adaptation of the index flood method as presented by Stedinger et al. (1993), Hosking and Wallis (1997), and the Institute of Hydrology (1999). The objectives of this study are to investigate the effect of urbanisation on flood frequency relationships, and to use this information to develop a new set of procedures for adjusting the FEH flood frequency curve for the effect of urbanisation when applied in an ungauged catchment.

Urbanisation is a radical form of land-use change, and the construction of impervious surfaces (roads, pavements, roofs) inhibits the natural infiltration capacity, while the increased conveyance capacity reduces the catchment response times. It is well established in the literature that the effect of urbanisation can be detected in the magnitude of individual annual maximum series of peak flow (Packman, 1980; Sheng and Wilson, 2009), and thereby lead to changes in the flood frequency characteristics. It is also generally considered that the effect of urbanisation is to increase the low return period floods more than the high return period floods. These effects have been accepted qualitatively for several decades (Hall, 1973), but the ability to predict the effect in an ungauged catchments is still limited. Summarising data from published literature, Hollis (1975) found that, compared with the pre-urban flood response, 35%
impervious area would lead to an increase of the mean annual flood of about 275%, whereas the 100-year flood would increase by about 80%. Comparable effects were reported by Beighley and Moglen (2003). Analysing annual maximum series from 115 urban catchments in the UK, Robson and Reed (1999) developed a model to predict the ratio between the median annual maximum peak flow as estimated from a catchment in an urban or rural state, respectively, and found that this ratio could vary between no effect (one) and up to a factor of about 20, depending on the degree of urbanisation and the underlying soil type. However, the factor 20 was largely a result of extrapolation from observed data, and the effect from observed data was confined to an increase of 100% and less; a more dramatic effect of urbanisation is expected when an urban area is built on a permeable, non-responsive, soil type than when built on less permeable soils, e.g. clay. The FEH (Robson and Reed, 1999) also suggested that the effect would gradually diminish as the return period increases, and at very high return period, no effect could be detected. This latter assumption was not verified by evidence derived from observed data.

A substantive issue when attempting to study the effect of urbanisation on flood characteristics is the need to consider the temporal development of urbanisation. A number of studies have suggested an approach based on naturalisation of flood series from urbanised catchments, either through statistical methods (McCuen, 1998; Moglen and Shivers, 2006) or through more detailed hydrological modelling using rainfall-runoff models (Beighley and Moglen, 2003). Both methods require substantial knowledge of the temporal development of urbanisation. However, no such systematic data on temporal urban development are readily available in the UK which rules out such detailed adjustments of individual catchments. Instead, this study has taken a
different approach, where a value of the urban extent is sought which is representative of the period spanned by the observed record. In practice, the extent of urban development in each catchment is back-dated from a level recorded in a national survey around the year 2000 to a level corresponding to the mid-record level. For example, for a record spanning the period 1980-2000, the urban extent is back-dated to a level representing the mid-level at year 1990. The actual back-dating itself is based on a national model of urban development which is described in a subsequent section. Next, a set of urban adjustment factors for the median, and high order L-moment ratios (L-CV and L-SKEW) of the annual maximum series have been developed by comparing estimates in urban catchments obtained directly from observed data with best estimates of the as-rural values of the flow statistics obtained using FEH procedures as if the catchment was rural and ungauged. The difference between the observed flow characteristics and the as-rural estimates can then be related to the data on urban extent available at each site. This methodology has some similarities with the adjustment procedures presented by Sauer et al. (1983) and Moglen and Shivers (2006) and allows the resulting models to be used in conjunction with existing UK models used for prediction of flow statistics in rural catchments. The following sections provide information on the FEH procedures used to obtain as-rural estimates, details on the urban adjustment procedures, the data used in this study, including the procedure used for back-dating the urban extent for each catchment. Finally, a set of urban adjustment procedures are derived and their predictive ability assessed to alternative existing procedures. The results suggest that the procedures developed in this study are better at predicting the effect of urbanisation on the flood frequency curve than the existing methods.
As-rural estimates in urban catchments in the UK

The as-rural estimates are obtained using the latest development of the FEH index flood methodology as presented by the Environment Agency (2008). A key element of the FEH is the use of the index flood method, where the flood frequency curve is defined as a product of a site specific index flood, $\xi$, (in the FEH defined as the median annual maximum flow) and a dimensionless growth curve which describes the relationship between the dimensionless flood and the exceedance probability (often expressed as the return period, T) and is denoted $z_r$. The FEH recommends the three parameter Generalised Logistic (GLO) distribution for flood frequency estimation in the UK. Using the GLO distribution, the flood frequency curve (or the quantile function, or inverse cumulative distribution function) for estimating the T-year peak flow, $Q_T$, is given as

$$Q_T = \xi + \frac{\alpha}{\kappa} \left[ 1 - (T - 1)^{\kappa} \right] = \xi \left[ 1 + \frac{\beta}{\kappa} \left( 1 - (T - 1)^{\kappa} \right) \right] = \xi z_r \tag{1}$$

where $\xi$, $\alpha$, $\beta$ and $\kappa$ are model parameters, and the growth curve is defined as the term within the square brackets. Note that according to the definition in Eq. (1), for a return period of two years, the growth curve takes a value of one, thus the 2-year peak flow value is equal to the median annual maximum flood, i.e. the flood exceeded on average every other year.

In the context of the index flood method, the disproportional effect of urbanisation on low and high return period flood, as documented by Hollis et al. (1975) and discussed
in the previous section, is expected to result in higher values of the index flood but
flatter growth curves in urban catchments than in corresponding rural catchments.

The GLO model parameters are estimated using a variant of the method of L-moments
(Robson and Reed, 1999) where the location parameter $\xi$ is defined as the median
annual maximum flood and the two parameters controlling the growth curve ($\beta = \alpha/\xi$
and $\kappa$) are estimated using L-CV and L-SKEW. Specifically, when an estimate of the
T-year peak flow is required in an ungauged catchment, the as-rural estimates of the
median, L-CV, and L-SKEW can be obtained through the improved FEH

The as-rural estimate of the median annual maximum flood (m$^3$s$^{-1}$) is estimated from a
set of catchment descriptors as

$$\xi = 8.3062\text{AREA}^{0.8510} \cdot 0.1536 \left(\frac{\text{SAAR}}{1000}\right)^{3.4451} \cdot 0.0460^{\text{FARL}^2}$$  (2)

where $\xi$ is the median (denoted QMED by Robson and Reed, 1999), $\text{AREA}$ is the
catchment area (km$^2$), $\text{SAAR}$ is the standard average annual rainfall is measured in the
reference period 1961-90 (mm), $\text{FARL}$ is an index of flood attenuation due to
upstream reservoirs and lakes and can take values between zero (strong attenuation)
and one (no attenuation). Values of FARL are based on lakes, reservoirs, ponds and
other static water bodies as digitised from a 1:50000 scale map and made available on
a 50m×50m grid, thus excluding water bodies less than 50m across. Finally
$\text{BFIHOST}$ is an index of baseflow as defined by HOST soil classes (Boorman et al.,
1995) and can take values between zero (very impermeable soils) and one (very
permeable soils). More details on each catchment descriptor is provided by Bayliss (1999).

Next, as-rural estimates of L-CV and L-SKEW are obtained using a regional statistical method known as pooled analysis, where estimates at the ungauged site are weighted averages of L-moment ratios from a collection of other sites considered to be hydrologically similar to the site of interest. Hydrological similarity is defined in terms of catchment descriptors, including AREA, SAAR and FARL (all defined above), and FPEXT which is an indicator of the extent of floodplains in the catchment. A summary of the pooling-group method is provided in Appendix A and a comprehensive description is provided by Kjeldsen and Jones (2009b).

Developing models for urban adjustments

In a study of urbanised catchments in the US, Sauer et al. (1983) considered the difference between estimates of flood statistics in urban catchments obtained directly from data with the corresponding as-rural estimates of the same statistics, and related this difference to a set of catchment descriptors. Sauer et al. (1983) based their method on models linking the T-year peak flow directly to catchment descriptors rather than statistical moments (as in this study). Here the effect of urbanisation on the median, L-CV and L-SKEW (the three summary statistics used for estimating the GLO model parameters) is investigated by comparing i) estimates of these statistics obtained directly from observed data in the urban catchments, $y^{(A-r)}$, with ii) the corresponding as-rural estimates as obtained from the FEH method outlined above and denoted $y^{(A-R)}$. The difference between the two log-transformed statistics (here represented in a
vector form containing data from all sites used in the model fitting) are related to a set
of catchment descriptors through an ordinary least squares regression model as
\[ \ln[y_{\text{obs}}^{(A-s)}] - \ln[y_{\text{cds}}^{(A-R)}] = X\theta + \varepsilon. \]  
(3)
where \( X \) is a matrix of catchment descriptors, \( \theta \) is a vector of regression model
parameters, and \( \varepsilon \) is a vector of random and independent regression errors. The
subscripts \( \text{obs} \) and \( \text{cds} \) have been added to emphasise that the estimates are obtained
from observed data (\( \text{obs} \)) and from catchment descriptors (\( \text{cds} \)), respectively. As the
sample estimated of L-SKEW can take negative values, a constant of one was added to
all estimates of L-SKEW to allow log-transformation. The catchment descriptors
included as explanatory variables in the \( X \) matrix in Eq. (3) should ideally be
describing aspects of urbanisation in each of the considered catchments.

The FEH (Robson and Reed, 1999) provided a calibrated version of Eq. (3) for
adjusting the median for the impact of urbanisation where the urban adjustment factor
(UAF) is applied to the as-rural estimate of the median to get the corresponding
median for the urban catchment. No similar model was developed for the L-moment
ratios or the growth curve. Instead, as part of the FEH, Robson and Reed (1999)
presented a non-parametric adjustment factor, assuming that for a very large flood
(arbitrarily defined as having a return period of 1000 years) the degree of urbanisation
would have no influence on the growth curve. The adjustment factor was defined as
\[ z_T^{(u)} = UAF \left[ \frac{\ln T - \ln 2}{\ln 1000 - \ln 2} \right] z_T^{(A-R)}, \quad 2 \leq T \leq 1000 \]  
(4)
where \( z_T^{(A-R)} \) is the as-rural estimate of the growth curve for the T-year return period as
defined in Eq. (1), and \( z_T^{(u)} \) is the resulting estimate of the growth curve in the urban
catchment. Note that the superscript \( (u) \) represents a predicted value of the growth
curve rather than an observed value, which was indicated with the superscript \((A-S)\) in Eq. (3). When applying an automated version of the FEH procedure to the entire UK, Morris (2003) found the growth curve adjustment to be inconsistent on a small number of catchments that are both heavily urbanised and permeable at the same time. On these catchments T-incoherence could occur, defined as cases where \(z_{U}^{(U)} < z_{T}^{(U)}\).

Morris (2003) suggested that the adjustment to the rural growth factor should be defined as

\[
1 + \frac{\left( z_{T}^{(A-R)} - 1 \right) \left( z_{1000}^{(A-R)} - 1 \right)}{UAF} \quad 2 \leq T \leq 1000
\]  

rather than through Eq. (4) to avoid this T-incoherence.

DATA

**Annual maximum series of peak flow**

The hydrological dataset used in this study consists of annual maximum series instantaneous peak flow data from 602 rural catchments used to develop the improved FEH methods for producing as-rural estimates, and a corresponding dataset of 206 annual maximum series from urbanised catchments not included in the development of the improved FEH tools. A summary of the two datasets is shown in Table 1.

**TABLE 1**

**Catchment descriptors**

Digital catchment descriptors are available for all catchments in the UK larger than 0.5 km\(^2\) (CEH, 2007). The number of different catchment descriptors that could potentially be included to explain the difference between the at-site and as-rural
estimates is large, but only a subset of variables previously found to have links to the
effect of urbanisation has been included in this analysis.

A key catchment descriptor is the proportion of the spatial extent of urbanisation,
available in all UK catchments larger than 0.5 km² and derived from digital land-cover
data (Bayliss et al., 2006). This index is referred to as $URBEXT_{2000}$, where the
subscript 2000 indicates that the land-cover data represent the catchment state as
observed between 1998-2000. The underlying land-cover map uses two classifications
of urbanisation, urban and suburban, made available on a national 50m grid. The
urban class contains large areas of concrete and tarmac typically found in city centres
and major industrial and commercial sites. The suburban class describes grid squares
where a mixture of build-up area and permanent vegetation is found such as city
suburbs and small towns and villages. The $URBEXT_{2000}$ index is a composite index of
urban and suburban extent, and is defined as the fraction of the urban class plus half
the fraction of the suburban class, assuming that half of a grid square defined as
suburban is covered by vegetation (Bayliss et al., 2006).

Packman (1980) argued that the effect of urbanisation on the flood frequency
relationship should be related to separate changes in runoff volume (or percentage
runoff) and catchment lag-time. It is generally accepted that the catchment lag-time is
related to the proportion of urbanisation in a catchment (NERC, 1975; Packman 1980;
Sheng and Wilson, 2009). Based on work by Packman (1980), an updated version of
an index quantifying the effect of urbanisation on percentage runoff, the percentage
runoff urban adjustment factor (PRUAF), was defined by Kjeldsen (2009) as
where both $BFIHOST$ and $URBEXT_{2000}$ have been defined above.

Other possible catchment descriptors related to the urban development describe the relative location and the urban areas (URBLOC) and the concentration of the urban areas (URBCONC). More details on both descriptors are provided by Bayliss (2006), but they were found not to improve the description of the median or the L-moment ratios in this study.

Adjusting observed records for urbanisation

The lack of systematic and comparable data on the temporal development of urban extent covering the period of most gauged records rule out a detailed adjustment of each individual data series. Instead the values of the descriptor $URBEXT_{2000}$ were backdated for all catchments to coincide with the midpoint of the observed record of each individual data series using a general UK urban expansion factor (UEF). The underlying model describing UEF was developed by Bayliss et al. (2006) by combining different official dataset on the total area of land in UK under development. The UEF is defined to have a value of one at the year 2000 and is given as

$$UEF(\text{year}) = 0.7851 + 0.2124\frac{\text{arctan}\left(\frac{\text{year} - 1967.5}{20.32}\right)}{\text{year}}$$

where the evaluation of the arctan function is based on radians. The UEF was developed to cover the period 1935-2000, thus the constant 1967.5 in Eq. (7) represent the mid-point of this period. The UEF model is illustrated in Figure 1.
RESULTS

The effect of urbanisation was investigated separately for the median, the L-CV and the L-SKEW using ordinary linear regression models. Before the regression models were evoked, an exploratory analysis was conducted for each of the two L-moment ratios to investigate if an urbanisation effect could be expected, and to compare the differences between the at-site and as-rural estimates with the corresponding estimates obtained from the 602 rural catchments. The latter comparison of residuals was undertaken to ensure that the FEH methods can provide reasonable as-rural estimates of the L-moment ratios in the urban catchments. Of course, this assumption can only be tested indirectly as no as-rural estimates can be obtained from data in the urban catchments.

THE MEDIAN

The regression model for predicting the median from catchment descriptors shown in Eq. (2) was developed by Kjeldsen and Jones (2009a) as a log-linear regression model. Thus, this investigation will be based on the residuals obtained as the difference between the log-transformed at-site and the FEH as-rural estimates of the median in the urban catchments. Note that six of the 206 catchments were excluded from this analysis. These catchments were all located in an area north-west of London and the as-rural estimates, Eq. (2), of the median were significantly larger than the observed at-site values. The reasons for these discrepancies are not fully understood but are likely to be related to the complex hydrology of the area dominated chalk.
A first assessment of the effect of urbanisation on the median is shown in Figure 2 where histograms of (log) residuals obtained from the 602 rural catchments from Kjeldsen and Jones (2009a) are compared to the corresponding residuals obtained from the 200 urban catchments. To further assess the impact of urbanisation, two subsets of the urban dataset were used classified according to whether $URBEXT_{2000}$ is smaller (155) or larger (45) than 0.150.

The resemblance of the two sets of residuals (urban and rural) in Figure 2 indicates that the effect of urbanisation on the median can be expected to be limited. However, while still scattered around zero, the urban residuals have a slight tendency for more positive values than the rural residuals, and that this tendency is more pronounced for the more urbanised catchments, which indicates that the urban residuals contain some structural information describing the variation in flood statistics between catchments not found in the rural dataset. It should be noted that even for the very urbanised catchments, the at-site median can still be smaller than the predicted as-rural value, showing that the effect of urbanisation is not necessarily unidirectional, and that anecdotal evidence of reduction of peak flow values as a result of attenuation from hydraulic infrastructure appears evident in the data analysed here.

The final form of the regression model linking the effect of urbanisation to a set of catchment descriptors was the result of an iterative process where not every step is reported here. Throughout the process the existing FEH model was used as a benchmark against which to measure other potential models. Note that the variable
selection is constrained by the need for the urban adjustment factor to produce a value of one for \( URBEXT_{2000} \) equal to zero, i.e. no adjustment for a completely rural catchment. The exploratory analysis found only a connection between the effect of urbanisation and two variables; \( (1 + URBEXT_{2000}) \) and \( PRUAF \). Other transformations of \( URBEXT_{2000} \) were attempted, such as \( (1 + URBEXT_{2000}^2) \) but were found not to improve the description of the data. A summary of the regression statistics for the considered models is shown in Table 2.

TABLE 2:
The last of the models in Table 2 (model 6) is the most comprehensive model and includes both explanatory variables plus a term representing the interaction between the two variables. Despite having a smaller residual standard error than any of the other models, the \( p \)-values for the coefficients on \( (1 + URBEXT_{2000}) \) and the interaction terms are relatively large suggesting that the these explanatory variables are not contributing significantly to the description of the data. Considering both model-simplicity and descriptive ability, the results in Table 2 points towards either Model 2 or Model 4 as the preferred model.

Model 5 could provide a reasonable compromise between model complexity and performance. However, this particular model structure will result in very high values of urban adjustment when applied to catchments with high values of \( BFIHOST \) (permeable) as well as a high degree of urbanisation. Note here that the dataset contains few catchments which combines high \( BFIHOST \) values with high values of
URBEXT$_{2000}$; thus extrapolation is likely to be necessary for practical use. For extrapolation to such catchments, the estimates from Model 5 will be an order of magnitude larger than the corresponding estimates from the existing FEH model. Finally it was decided to adopt Model 4 as it provides a reasonable model and is consistent with the existing FEH model.

\[ \xi^{(a)}_{\text{urb}} = \xi^{(A-R)}_{\text{urb}} (1 + \text{URBEXT}_{2000})^{0.37} \text{PRUAF}^{2.16} \]  

The results in Table 2 suggest that the term \((1 + \text{URBEXT}_{2000})\) in Model 4 add little to the ability of the model to describe the data. Thus, an alternative choice of model could have been Model 2, describing the effect of urbanisation using \(\text{PRUAF}\) only, i.e.

\[ \xi^{(a)}_{\text{urb}} = \xi^{(A-R)}_{\text{urb}} \text{PRUAF}^{2.51} \]  

This model was not chosen based on Model 4 having a closer resemblance to the existing FEH model.

THE L-MOMENT RATIOS

A generalised method for adjusting growth curves for the effect of urbanisation was presented by Packman (1980) who stressed that extrapolation beyond return periods of 50-years should be considered ‘largely intuitive’. The adjustment method later published by the FEH went one step further, hypothesising that for very extreme flood of a 1000-year return period, the effect of urbanisation on the peak flow magnitude is negligible, thus the growth factor of the urban catchment is equal to what it would have been if the catchment was not impacted by urbanisation. In this study, the effect of urbanisation on growth curves will be investigated primarily by examining the
effect on each of the L-moment ratios (L-CV and L-SKEW, which control the growth curve according to Eq. 1), rather than the growth curve itself.

Investigating applicability of generalised rural models in urban catchments

Using the recently developed improved FEH pooling-group method (Kjeldsen and Jones, 2009b), pooled L-moment ratios (as-rural estimates) can be derived for each of the urban catchments. By considering the urban catchment to be ungauged, the pooled estimates represent the best available estimate of what the L-moment would be at the site if it was not influenced by urbanisation, i.e. as-rural. It should be noted that the pooled estimates of L-moment ratios are estimated as if the site of interest is ungauged and thus these estimates are associated with a higher uncertainty than the corresponding at-site estimates obtained directly from the data at each site (Kjeldsen and Jones, 2006).

No compelling evidence was found that the L-moment ratios from the six catchments initially excluded from the analysis of the median were outliers, and thus they were retained in this analysis. Using only catchments with a record length in excess of 20-years (177 catchments), a first tentative assessment of the impact of urbanisation on the L-moment ratios is shown in Figure 3 where the difference in L-moment ratios between the at-site estimate and the as-rural estimate obtained from the pooling-group method is plotted for two subsets of the urban data defined according to the level of urbanisation. The first subset consists of 150 catchments, which according to the classification scheme by Bayliss et al. (2006) are categorised as being slightly to moderately urbanised (0.030 < $URBEXT_{2000}$ ≤ 0.150). The second subset includes 27
catchments categorised as being heavily to very heavily urbanised

\( (0.150 < URBEXT_{2000} \leq 0.600) \).

A comparison of the histograms in Figure 3 indicates that the effect of urbanisation manifests itself in lower values of L-CV and higher values of L-SKEW than would be expected for rural catchments. The figures also suggest that this effect is more pronounced for higher values of \( URBEXT_{2000} \) than at lower values. The effect of urbanisation is generally considered to be a larger proportional increase in more frequent floods than the more rare floods. Packman (1980) argued that this effect would lead to a reduction in the standard deviation (thus L-CV) but did not extend the argument to include the coefficient of skewness (or L-SKEW). However, it seems reasonable to assume that the effects of the disproportional increase would lead to samples with a greater tendency for positive skewness. Thus, the lowering of L-CV found in this study supports the previous findings that urbanisation results in a flatter growth curve (e.g. Packman, 1980), whereas the effect of urbanisation on L-SKEW to the author’s knowledge has not been reported elsewhere.

A straightforward comparison of the at-site and pooled L-moment ratios is complicated by the fact that the pooling-group method was developed using the rural dataset, but did not include the urban dataset. As a result, the residuals (at-site minus as-rural estimates) from the urban catchments are expected to have a slightly higher degree of variability than the residuals from the rural catchments. Also, the observed difference between the at-site estimate from an urban catchment and the corresponding
pooled estimate will be caused by different factors including: i) the effect of urbanisation, ii) bias in the pooling-group method because a particular urban catchment might not be well represented with regard to its catchment descriptors in the dataset of rural catchments available for pooled analysis, and iii) sampling uncertainties in the estimates due to limited record lengths. An implicit assumption of this analysis is that the last two factors have an insignificant influence compared with the effect of urbanisation itself.

Systematic variation in residuals of L-CV and L-SKEW related to catchment descriptors other than urbanisation was investigated by plotting the residuals against each of the catchment descriptors used for defining hydrological similarity as shown in Figures 4 and 5. The polylines in each figure represent the outermost convex hull as defined by the rural dataset.

From Figures 4 and 5 little or no systematic variation with any of the four catchment descriptors can be readily identified for either L-CV or L-SKEW. Also, the spread of the residuals for the urban catchments for the vast majority falls within the region defined by the rural residuals, thereby adding confidence that the pooling-group method can be assumed to provide as-rural estimates of the L-moment ratios in the urban catchments with a degree of uncertainty comparable to that of the rural catchments.
Initially, an exhaustive search for the best subsets of explanatory variables in Eq. (3) for predicting the difference between the urban and rural L-moment ratios was undertaken based on linear regression. Both log-transformed and non-transformed catchment descriptors were considered, but the only significant explanatory variable to be identified for both L-CV and L-SKEW was $URBEXT_{2000}$. Similar to the investigation into the effect of urbanisation on the median, the variable $PRUAF$ was also included, but no relationship between the L-moment ratios was identified. The summary statistics of selected regression models for L-CV and L-SKEW are shown in Tables 3 and 4.

From the results in Tables 3 and 4 it can be seen that the relationship between the difference of the (log) at-site (urban) estimates of the L-moment ratios and the corresponding (log) as-rural estimates is generally weak for L-CV, and even weaker for L-SKEW, and in both cases weaker than in the corresponding results obtained for the median (Table 2). For both L-CV and L-SKEW there is little evidence that using the pooled estimate as a predictor in combination with $URBEXT_{2000}$ has any benefits over a model relating the difference directly to $URBEXT_{2000}$.

For L-CV the best model relates the difference directly to $URBEXT_{2000}$ without any transformation of $URBEXT_{2000}$ which performs slightly better than the version relating
the difference to \( \ln[1 + URBEXT_{2000}] \). Thus, for L-CV it is recommended that the urban adjustment procedure for L-CV is given as

\[
L - CV^{(u)} = L - CV^{(A-R)} \times 0.5547^{URBEXT_{2000}}
\]  

(10)

For L-SKEW there is little difference between a model relating the difference to either a log-transformation of \( URBEXT_{2000} \) or to \( URBEXT_{2000} \) directly. Thus, to ensure consistency, the urban adjustment factor for L-SKEW is defined as

\[
L - SKEW^{(u)} = [(L - SKEW^{(A-R)} + 1) \times 1.1545^{URBEXT_{2000}}] - 1.
\]  

(11)

In the next section the predictive ability of these adjustment procedures will be compared to the urban adjustment procedures suggested by the FEH (Robson and Reed, 1999) and Morris (2003).

**COMPARISON OF PREDICTIVE CAPABILITY**

A cross-validation experiment based on the leave-one-out technique (Efron and Tibshirani, 1993) was carried out to assess and compare the ability of different adjustment procedures to predict the T-year growth factor in urbanised catchments. Only the dimensionless growth factors were considered in this experiment as no competing procedures for adjusting the median were suggested in this study. The leave-one-out procedure was considered necessary in order to compare the L-moment ratio adjustment developed in this study with the calibration-free adjustment procedures suggested by the FEH (Robson and Reed, 1999) and Morris (2003). In this study four different procedures were considered:

1. No adjustment (estimate growth curve as if it was a rural catchment)
2. Adjust both L-CV and L-SKEW (method developed in this study)
At some gauging stations the observed annual maximum series include one or two flood events that are very large (eight to ten times the median annual maximum runoff) compared with the bulk of the observations in that series. For such catchments the at-site sample estimates of L-CV and L-SKEW are much higher than the typical average values predicted by the pooling-group method. The annual maximum series of peak flow from catchment 40012 located in South-East England, shown in Figure 6, is an example of such a catchment where the at-site L-CV is 1.89 times the corresponding pooled estimate. If the large event in water year 1967 (September 18, 1968) is removed from the series, then the at-site estimate of L-CV is reduced to 0.27 and the ratio between the at site and the pooled estimate is reduced to 1.03.

It would be tempting to remove the catchments from the dataset where the at-site and as-rural estimates are very different. Unfortunately, it is not generally known what causes the difference between the at-site and the as-rural (or pooled) estimate, and it could be caused by a number of factors such as: i) oddities in the at-site samples (as discussed above), ii) failure of the pooling-group method to accurately represent the at-site L-moment ratios, iii) the residual effect of urbanisation, or iv) any combination of the three first reasons. Therefore, any censoring of the dataset will involve some arbitrary decisions. To reduce, but unfortunately not remove, the influence of these catchments it was decided to use the absolute difference between at-site and predicted
growth factors rather than the squared difference for assessing predictive ability. The

cross-validation statistic adopted in this study and based on observations is thus
defined as

$$\frac{1}{m} \sum_{i=1}^{m} |z_i - \hat{z}_i^{(-i)}|$$  \hspace{1cm} (12)

where $z_i$ is the observed quantity (here growth factor) at the i’th site and $\hat{z}_i^{(-i)}$ is the
corresponding estimate of the same quantity from a model fitted to the observations
with the i’th observation omitted from the dataset. Eq. (12) is also known as the cross-
validation estimate of prediction error. Table 5 compares the cross-validation statistic
in Eq. (12) for each of the five methods listed above.

TABLE 5

The results in Table 5 suggest that the adjustment procedure developed in this study
provides better predictions of the growth curve than both the FEH (Robson and Reed,
1999) and Morris (2003) procedures. However, for return periods in excess of 50-
years, the unadjusted as-rural growth curve appears to provide an overall better
prediction of the urban growth curve. It is worth remembering that the L-moment ratio
will have been estimated using annual maximum series with an average record length
of 36 years, i.e. the behaviour at long-return periods is mainly a result of extrapolation
from the observed data based on the GLO distribution. For comparison, the cross-
validation statistic defined as the root sum of squares are also shown in Table 5 for
each method and return period, and the results shown in brackets. The root sum of
squares is defined equivalently to Eq. (12) as

$$\sqrt{\frac{1}{m} \sum_{i=1}^{m} (z_i - \hat{z}_i^{(-i)})^2}$$  \hspace{1cm} (13)
The growth curve for the catchments with short records and extraordinary large singular events (see for example, Figure 6) are generally much steeper than the pooled growth-curve. Any further reduction in growth curve factors, such as imposed by any of the urban adjustments, is therefore likely to indicate that no adjustment is the preferred option. This effect is further amplified when using the sum of squares rather than the absolute value as the basis for the cross-validation statistics. In Table 5, the root sum of squares values suggest that the no-adjustment is the preferred option at a return period of 25-years, whereas the sum of absolute differences, Eq. (12), suggests that no adjustment is preferable for the 50-year return period and beyond. To further assess how much the results in Table 5 are affected by the presence of the catchments discussed above, an additional experiment was conducted where these catchments were removed from the dataset. Figure 7 shows the prediction residuals for the 25-year growth factor for each individual catchment plotted against the corresponding at-site estimate of L-CV.

By repeating the cross-validation experiment outlined above, but using only a subset of the data where the at-site sample values of L-CV are less than 0.33 (points to the left of the vertical dashed line in Figure 7), a new set of average prediction errors have been derived and are shown in Table 6.
The results in Table 6 confirm the results reported using the entire dataset that the adjustment to L-CV and L-SKEW developed in this study generally will provide a better prediction of the effect of urbanisation on the growth curve than the adjustments suggested by the FEH and by Morris (2003). Again, the use of the sum of squares rather than absolute values reduces the return period for which no adjustment is the preferred option from 1000-years to 100-years (based on the return periods represented in Table 6) but does not change the overall recommendation that the adjustment procedure developed in this study is preferable to the alternative adjustment procedures. From both Table 5 and 6 it can be observed that the relative benefit of the growth-curve adjustment procedure is reduced as the return period increases. For a return period of 1000-years, the no-adjustment option is the preferred choice, which is consistent with the existing FEH and Morris (2003) methods (Eq. 4 and 5), though both these methods were found not to perform well at lower return periods and, thus, should not in general be used.

CONCLUSION

Results presented in this paper allow users of the existing FEH procedure for flood frequency estimation in rural catchments to adjust flood frequency curves for the impact of urbanisation when estimated in ungauged and urbanised catchments. Following the comparison of several procedures, the recommended adjustment procedure is based on a set of regression equations, Eq. (8), (10) and (11), linking a set of catchment descriptors to the difference between (log) estimates of the median, L-CV, and L-SKEW obtained from at-site data in urban catchments and the corresponding as-rural estimates obtained from the FEH procedures.
For adjusting the growth-curve, the approach taken in this study was to investigate directly the impact of urbanisation on the relevant L-moment ratios: L-CV and L-SKEW. It was found that increased urbanisation has a tendency to reduce L-CV, i.e. cause a flattening of the growth curve when compared to the as-rural estimate. This effect was supported by the findings of other published studies (Hollis, 1975). With regard to L-SKEW, the results indicated a slight tendency of increased urbanisation to cause an increase in L-SKEW, which will result in more upward curved growth-curves. This effect was statistically less significant than the effect on L-CV, but has not been reported previously.

Acknowledgement

The author wishes to thank the UK measuring authorities for making available the peak flow data through HiFlows-UK. The author would also like to thank the two anonymous reviewers for their helpful comments on an earlier version of the manuscript.

REFERENCES


Appendix A

The as-rural estimates of the L-moment ratios, L-CV or L-SKEW (both denoted \( t^{(a-r)} \)) for convenience in the following), at an ungauged site are obtained by forming a weighted average of L-moment ratios from a collection of gauged catchments considered hydrologically similar to the site of interest. This collection of sites is also known as a pooling group. The as-rural estimate is defined as:

\[
t^{(a-r)} = \sum_{i=1}^{M} \omega_i t_i,
\]

where \( M \) is the number of hydrologically similar gauged sites, \( t_i \) is the L-moment ratios at the i’th site, and \( \omega_i \) is the weight assigned at the i’th site. Hydrological similarity, \( d \), is here defined in terms of catchment descriptors as

\[
d_{ij} = 3.2\left(\ln[\text{AREA}_i] - \ln[\text{AREA}_j]\right)^2 + 0.5\left(\ln[\text{SAAR}_i] - \ln[\text{SAAR}_j]\right)^2
\]

\[
+ 0.1\left(\frac{\text{FARL}_i - \text{FARL}_j}{0.05}\right)^2 + 0.2\left(\frac{\text{FPEXT}_i - \text{FPEXT}_j}{0.04}\right)^2
\]

Where AREA is the catchment area (km²), SAAR is standard annual average rainfall as measured between 1961-90 (mm), FARL is an index of flood attenuation due to upstream reservoirs and lakes and can take values between zero (strong attenuation) and one (no attenuation), and FPEXT is an indicator of the extent of floodplains in the catchment and can take values between one (all floodplain) and zero (no floodplain).

The number of sites to be used is determined by the total number of annual maximum events, which has to exceed 500.
The weights assigned to each gauged site depend on the sampling variability, \( c_i \), and distance in catchment descriptor space from the target site, \( d_i \), and is defined as

\[
\omega_i = \frac{(c_i + b_i)^{-1}}{\sum_{i=1}^{M} (c_i + b_i)^{-1}}, \quad i = 1, \ldots, M
\]

where the quantity \( b_i \) is defined separately for L-CV and L-SKEW as

- **L-CV**: \( b_i = 0.0047 \sqrt{d_i} + 0.0023/2 \)
- **L-SKEW**: \( b_i = 0.0219(1 - \exp[-d_i/0.2360]) \)

The sampling variance is defined for L-CV and L-SKEW, respectively, as

- **L-CV**: \( c_i = 0.02609/(n_i - 1) \)
- **L-SKEW**: \( c_i = 0.2743/(n_i - 2) \)

Where \( n_i \) is the record length at the i’th site.
Table 1: summary of AMS of instantaneous peak flow from the rural and urban dataset

<table>
<thead>
<tr>
<th></th>
<th>Rural</th>
<th>Urban</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of gauges</td>
<td>602</td>
<td>206</td>
</tr>
<tr>
<td>Shortest record length (years)</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Longest record length (years)</td>
<td>117</td>
<td>120</td>
</tr>
<tr>
<td>Average record length (years)</td>
<td>32.7</td>
<td>35.9</td>
</tr>
<tr>
<td>Number of annual maximum events</td>
<td>19679</td>
<td>7401</td>
</tr>
</tbody>
</table>
Table 2: Six different regression models linking the (log) difference between at-site and as-rural estimates of the median annual maximum to catchment descriptors.

<table>
<thead>
<tr>
<th>Model no.</th>
<th>Variables</th>
<th>Parameter</th>
<th>Std. dev.</th>
<th>t-value</th>
<th>p-value</th>
<th>$r^2$</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\ln[1 + URBEXT_{2000}]$</td>
<td>1.67</td>
<td>0.21</td>
<td>7.99</td>
<td>1.09 $10^{-17}$ (***)</td>
<td>0.24</td>
<td>0.382</td>
</tr>
<tr>
<td>2</td>
<td>$\ln[PRAF]$</td>
<td>2.51</td>
<td>0.24</td>
<td>10.42</td>
<td>$&lt; 2 \times 10^{-18}$ (***)</td>
<td>0.35</td>
<td>0.353</td>
</tr>
<tr>
<td>3 (FEH)</td>
<td>$\ln[1 + URBEXT_{2000}]$</td>
<td>1.07 (fixed)</td>
<td>0.20</td>
<td>5.43</td>
<td>1.65 $10^{-17}$ (***)</td>
<td>0.35</td>
<td>0.352</td>
</tr>
<tr>
<td>4</td>
<td>$\ln[1 + URBEXT_{2000}]$</td>
<td>0.37</td>
<td>2.16</td>
<td>0.29</td>
<td>5.98</td>
<td>0.197</td>
<td>0.36</td>
</tr>
<tr>
<td>5</td>
<td>$\ln[1 - URBEXT_{2000}]$</td>
<td>9.89</td>
<td>0.91</td>
<td>10.90</td>
<td>$&lt; 2 \times 10^{-16}$ (***)</td>
<td>0.37</td>
<td>0.347</td>
</tr>
<tr>
<td>6</td>
<td>$\ln[1 + URBEXT_{2000}]$</td>
<td>0.32</td>
<td>0.57</td>
<td>0.29</td>
<td>1.11</td>
<td>0.269</td>
<td>0.38</td>
</tr>
<tr>
<td>6</td>
<td>$\ln[PRAF]$</td>
<td>0.68</td>
<td>2.45</td>
<td>2.78</td>
<td>0.006</td>
<td>0.404</td>
<td></td>
</tr>
</tbody>
</table>

*Sign. levels: $p < 0.01$ (***) , $0.01$ (**), $0.05$ (*). No asterisk indicate a significance level larger than 0.05 (not significantly different from zero)*
Table 3: Models for describing L-CV in urban catchments

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Explanatory Variable</th>
<th>Parameter</th>
<th>Std.dev</th>
<th>t-value</th>
<th>p-value</th>
<th>$R^2$</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln($L - CV^{[A-R]}$) - ln($L - CV^{[A-R]}$)</td>
<td>ln[$1 + URBEXT_{2000}$]</td>
<td>-0.6695</td>
<td>0.1476</td>
<td>-4.57</td>
<td>1.06 $10^{-5}$ (***)</td>
<td>0.10</td>
<td>0.263</td>
</tr>
<tr>
<td>ln($L - CV^{[A-R]}$) - ln($L - CV^{[A-R]}$)</td>
<td>ln[$1 + URBEXT_{2000}$] ln[PRUA]</td>
<td>-0.9177</td>
<td>0.2200</td>
<td>-4.17</td>
<td>4.74 $10^{-5}$ (***)</td>
<td>&lt; $2.10^{-16}$ (***)</td>
<td>0.11</td>
</tr>
<tr>
<td>ln($L - CV^{[A-R]}$)</td>
<td>ln[$1 + URBEXT_{2000}$] ln[$L - CV^{[A-R]}$]</td>
<td>-0.9070</td>
<td>0.2161</td>
<td>-4.20</td>
<td>4.30 $10^{-5}$ (***)</td>
<td>0.97</td>
<td>0.262</td>
</tr>
<tr>
<td>ln($L - CV^{[A-R]}$) - ln($L - CV^{[A-R]}$)</td>
<td>URBEXT</td>
<td>-0.5893</td>
<td>0.1286</td>
<td>-4.58</td>
<td>8.64 $10^{-6}$ (***)</td>
<td>0.11</td>
<td>0.263</td>
</tr>
<tr>
<td>ln($L - CV^{[A-R]}$)</td>
<td>URBEXT ln[$L - CV^{[A-R]}$]</td>
<td>-0.7470</td>
<td>0.1795</td>
<td>-4.16</td>
<td>4.97 $10^{-5}$ (***)</td>
<td>0.97</td>
<td>0.262</td>
</tr>
</tbody>
</table>

* Sign. levels: $p < 0.01$ (***) , 0.01 (**), 0.05 (*). No asterisk indicate a significance level larger than 0.05 (not significantly different from zero)

# Test if coefficient significantly different from 1
Table 4: Models for describing L-SKEW in urban catchments

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Explanatory Variable</th>
<th>Parameter</th>
<th>Std.dev</th>
<th>t-value</th>
<th>p-value</th>
<th>$r^2$</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln[L - SKEW^{(A-S)} + 1] - \ln[L - SKEW^{(A-R)} + 1]$</td>
<td>$\ln[1 + URBEXT_{2000}]$</td>
<td>0.1686</td>
<td>0.0704</td>
<td>2.39</td>
<td>0.018 (*)</td>
<td>0.03</td>
<td>0.126</td>
</tr>
<tr>
<td>$\ln[L - SKEW^{(A-S)} + 1] - \ln[L - SKEW^{(A-R)} + 1]$</td>
<td>$\ln[1 + URBEXT_{2000}]$</td>
<td>0.1014</td>
<td>0.1082</td>
<td>0.96</td>
<td>0.337</td>
<td>0.04</td>
<td>0.126</td>
</tr>
<tr>
<td>$\ln[L - SKEW^{(A-S)} + 1]$</td>
<td>$\ln[1 + URBEXT_{2000}]$</td>
<td>0.1754</td>
<td>0.0826</td>
<td>2.12</td>
<td>0.035</td>
<td>0.58</td>
<td>0.13</td>
</tr>
<tr>
<td>$\ln[L - SKEW^{(A-S)} + 1] - \ln[L - SKEW^{(A-R)} + 1]$</td>
<td>$URBEXT$</td>
<td>0.1436</td>
<td>0.0615</td>
<td>2.34</td>
<td>0.021</td>
<td>0.03</td>
<td>0.126</td>
</tr>
<tr>
<td>$\ln[L - SKEW^{(A-S)} + 1]$</td>
<td>$URBEXT$</td>
<td>0.1754</td>
<td>0.0826</td>
<td>2.12</td>
<td>0.035</td>
<td>0.58</td>
<td>0.126</td>
</tr>
</tbody>
</table>

* Sign. levels: p < 0.01 (***) , 0.01 (**), 0.05 (*). No asterisk indicate a significance level larger than 0.05 (not significantly different from zero)
# Test if coefficient significantly different from 1
Table 5: Comparison of cross-validation statistics (absolute difference) for urban growth curve adjustment factors for \( T = 5, 10, 25, 50, 100 \) and 1000-year return periods. The numbers in brackets are the root sum of square validation statistics.

<table>
<thead>
<tr>
<th>Method</th>
<th>Return period [years]</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No adjustment (as-rural)</td>
<td></td>
<td>0.094/(0.126)</td>
<td>0.185/(0.261)</td>
<td>0.358/(0.545)</td>
<td>0.543/(0.887)</td>
<td>0.796/(1.405)</td>
<td>2.550/(5.980)</td>
</tr>
<tr>
<td>2. Adjust L-CV and L-SKEW (this study)</td>
<td></td>
<td>0.090/(0.124)</td>
<td>0.182/(0.261)</td>
<td>0.356/(0.548)</td>
<td>0.543/(0.893)</td>
<td>0.799/(1.411)</td>
<td>2.560/(5.984)</td>
</tr>
<tr>
<td>3. The FEH adjustment procedure</td>
<td></td>
<td>0.094/(0.129)</td>
<td>0.189/(0.276)</td>
<td>0.368/(0.582)</td>
<td>0.564/(0.945)</td>
<td>0.833/(1.488)</td>
<td>2.647/(6.173)</td>
</tr>
<tr>
<td>4. The Morris (2003) procedure</td>
<td></td>
<td>0.094/(0.131)</td>
<td>0.195/(0.286)</td>
<td>0.382/(0.597)</td>
<td>0.581/(0.963)</td>
<td>0.848/(1.504)</td>
<td>2.644/(6.170)</td>
</tr>
</tbody>
</table>

Table 6: Comparison of cross-validation statistics (absolute difference) for urban growth curve adjustment factors for \( T = 5, 10, 25, 50, 100 \) and 1000-year return periods derived by not including the 14 catchments with highest at-site L-CV values. The numbers in brackets are the root sum of square validation statistics.

<table>
<thead>
<tr>
<th>Method</th>
<th>Return period [years]</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. No adjustment (as-rural)</td>
<td></td>
<td>0.080/(0.102)</td>
<td>0.151/(0.190)</td>
<td>0.274/(0.342)</td>
<td>0.397/(0.498)</td>
<td>0.555/(0.705)</td>
<td>1.488/(2.044)</td>
</tr>
<tr>
<td>2. Adjust L-CV and L-SKEW (this study)</td>
<td></td>
<td>0.076/(0.096)</td>
<td>0.146/(0.183)</td>
<td>0.270/(0.338)</td>
<td>0.396/(0.498)</td>
<td>0.554/(0.710)</td>
<td>1.493/(2.068)</td>
</tr>
<tr>
<td>3. The FEH adjustment procedure</td>
<td></td>
<td>0.079/(0.102)</td>
<td>0.151/(0.197)</td>
<td>0.277/(0.370)</td>
<td>0.410/(0.548)</td>
<td>0.581/(0.784)</td>
<td>1.560/(2.266)</td>
</tr>
<tr>
<td>4. The Morris (2003) procedure</td>
<td></td>
<td>0.077/(0.096)</td>
<td>0.155/(0.197)</td>
<td>0.289/(0.375)</td>
<td>0.424/(0.556)</td>
<td>0.593/(0.791)</td>
<td>1.555/(2.258)</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

**Figure 1:** Urban expansion factor (UEF) defined in Eq. (7). Note that the model is defined to return a value of one for the year 2000.

**Figure 2:** Histogram representing the residuals from estimates of the median for i) 602 rural catchments, and ii) 200 urban catchments.

**Figure 3:** Histograms representing the residuals from the rural catchments (grey) and the corresponding residuals for the urban catchments (black lines) for L-CV and L-SKEW.

**Figure 4:** Comparison of L-CV residuals from the rural (polylines) and urban (‘+) datasets.

**Figure 5:** Comparison of L-SKEW residuals from the rural (polylines) and urban (‘+) datasets.

**Figure 6:** Annual maximum series for catchment 40012. The extreme event occurring on the 16 September 1968 (17 times larger than QMED) is easily identified.

**Figure 7:** As-rural (pooled) estimates of L-CV plotted against at-site estimates of L-CV for 202 urban catchments.
Kjeldsen

Figure 1
Figure 2

Kjeldsen

URBEXT_{2000} < 0.150

URBEXT_{2000} > 0.150
Figure 3
Figure 4
Kjeldsen

Figure 5
Kjeldsen

Figure 6

<table>
<thead>
<tr>
<th></th>
<th>At-site</th>
<th>Pooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-CV</td>
<td>0.50</td>
<td>0.27</td>
</tr>
<tr>
<td>L-SKEW</td>
<td>0.68</td>
<td>0.20</td>
</tr>
</tbody>
</table>

annual maximum peak flow (m$^3$/s)

Water year

Figure 7

Kjeldsen

Absoloute prediction error (T=25)

$\text{L-CV}_{\text{at-elle}}$