A Preliminary Approach to Simulating Cyclic Variability in a Port Fuel Injection Spark Ignition Engine

A. Suyabodha*, A. Pennycott and C.J. Brace

Powertrain & Vehicle Research Centre
University of Bath
United Kingdom

1 Abstract

Differences in the combustion process from one cycle to the next, termed cyclic variations, are an important feature of spark ignition engines. These variations cause fluctuations in the work output of the engine and can therefore degrade engine and vehicle performance. In addition, the uneven running caused by cyclic variability of combustion constrains the engine operating range and thus has a direct effect on fuel consumption. Existing one-dimensional engine models typically represent cyclic variability using some form of stochastic behaviour defined by a pre-set normal distribution. This approach does not offer an insight into the mechanisms underlying variability and makes it difficult to include variability when calibrating the engine using simulation. Three-dimensional modelling approaches can offer an insight but are too complex to be used extensively within a calibration exercise.

In this paper, a simple, preliminary approach using empirical functions easily generated using standard engine instrumentation is used to augment a one-dimensional engine model via a co-simulation approach to include a representation of the effects of the air-fuel ratio and residual gas fraction on the efficiency, rate and duration of combustion. These parameters allow the engine model to simulate the effects of deterministic aspects of cyclic variability on heat release, in-cylinder pressure and indicated mean effective pressure.

The model is validated by comparing its prediction of cyclic variability under both rich and lean operation to experimental data. The resulting predictions match experimental results qualitatively and quantitatively. The model can be used to inform subsequent optimisation processes, representing the variability-induced constraints on the operating envelope. This will assist in the generation of fuel efficient calibrations and allow cycle-to-cycle variation effects to be included much earlier in the design process. The model will also aid the development of online control approaches aiming to reduce cycle-to-cycle engine variability.

Keywords: cyclic variability, simulation, gasoline engine, engine model

2 Introduction

Cyclic variability refers to fluctuations in engine performance from one engine cycle to the next in spite of constant engine control settings being used. The mean effective pressure, heat release and

*a.suyabodha@bath.ac.uk

in-cylinder pressure are typical variables used to reflect the degree of cycle-to-cycle variability [1]. Cyclic variability can be a limiting factor in engine performance, affecting fuel economy [2, 3, 4], giving rise to fluctuations in engine speed which degrade drivability [4], and leading to undesirable engine vibrations [5].

One source of cyclic variability is found in the varying amounts of residual gas in the combustion chamber [6], causing differences in burn rates and even engine misfire in extreme cases [7]. Other causes are changes in the overall air-fuel ratio in the cylinder, and local variations in the mixture composition, particularly near the spark plug [5, 8, 9]. Research suggests that variability increases at higher air-fuel ratios (i.e. weak mixtures) [5, 10]. Variations in the turbulence intensity are another source, since turbulence can affect flame development, giving rise to flame wrinkling which can contribute to variability [11, 12, 13, 14]. Turbulence dissipation, flow separation in the intake port, exhaust gas momentum and the influence of incoming pressure waves can affect turbulence intensity [15]. Spark duration and energy, which can vary appreciably from one cycle to the next, can have important effects on flame development and volume [16], with research pointing to an association between poor engine cycles and sparks of low energy and short duration [17]. Mean flow speed and direction at the spark plug and leakage through valves are further sources of cycle-to-cycle variations [8].

Previous approaches to modelling cyclic variability have generally included an artificial stochastic element which produces fluctuations in key aspects such as the air-fuel ratio and the combustion process [18]. While such modelling and simulation can reproduce plausible behaviour in terms of cycle-to-cycle variations of the engine, they offer little insight into the underlying physical processes governing the variability.

In this paper, a model is developed which can reproduce some aspects of engine cyclic variation. A first approach to modelling variability is presented: empirical functions governing combustion are combined with an engine and gas dynamics model, enabling variables such as the heat release rate and indicated mean effective pressure (IMEP) to be predicted. The model is validated using data extracted from an engine dynamometer test.

3 Methods

3.1 Evaluation of Variability

The coefficient of variation (COV) is used to assess variability in the IMEP. This uses the mean (μ) and standard deviation (σ) of the variables under study. The COVs of heat release (HR) and of IMEP are

$$\mathrm{COV}_{\mathrm{HR}} = \frac{\sigma_{\mathrm{HR}}}{\mu_{\mathrm{HR}}} \tag{1}$$

$$\mathrm{COV}_{\mathrm{imep}} = \frac{\sigma_{\mathrm{imep}}}{\mu_{\mathrm{imep}}}. \tag{2}$$

Note that vehicle drivability is impaired when $\mathrm{COV}_{\mathrm{IMEP}}$ is greater than approximately 0.1 [9].

3.2 Engine Details

The engine evaluated in this study is an Audi port fuel injection, 1.8 l spark ignition engine whose characteristics are summarised in table 1. Since the behaviour during catalyst light-off is of interest, the engine is run at idle.

Characteristic	Value
Displacement	1781 cm^3
Bore	81 mm
Stroke	86 mm
Connecting rod length	144 mm
Inlet valve	7.67 mm
Exhaust valve	9.3 mm
Intake valve spread angle	190°
Exhaust valve spread angle	200°
Compression ratio	9.3
Firing order	1-3-4-2

Table 1: Technical data of Audi 1.8 l engine.

3.3 Engine and Gas Dynamics Model

The engine model was developed using the engine and gas dynamics simulation software package WAVE (Ricardo plc, Shoreham-by-Sea, United Kingdom). The intake manifold is throttled, and each cylinder has a dedicated fuel injector located upstream. The structure of the WAVE model is shown in figure 1.

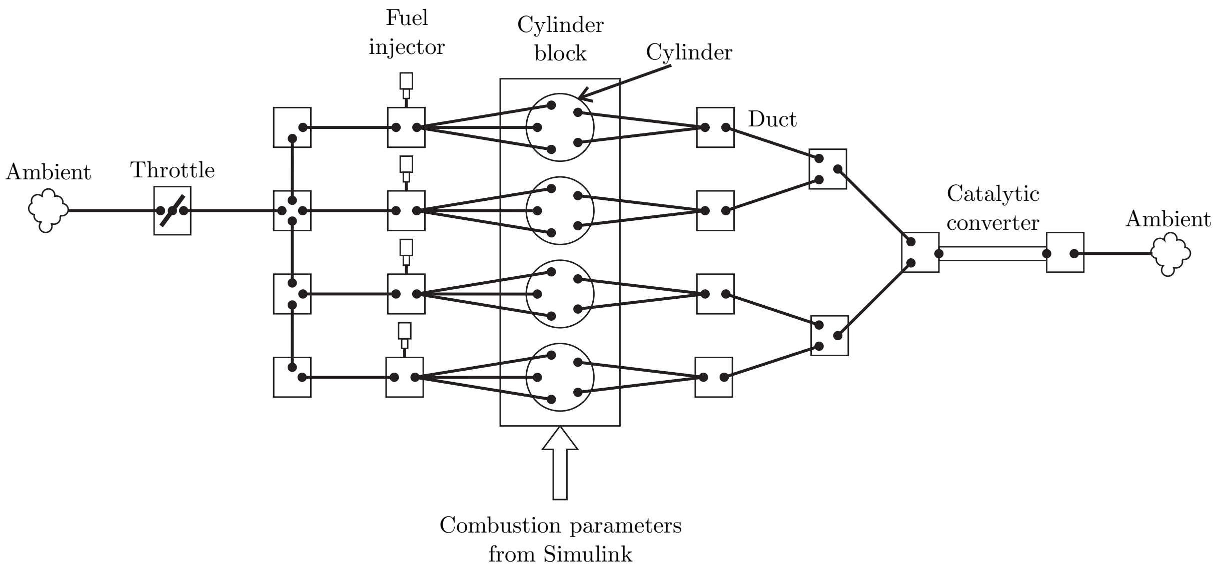


Figure 1: Schematic of WAVE engine model.

The WAVE model includes fuel spray and evaporation. An averaged fuel rate measured during the experiments (12.85 mg per cycle) is used in the WAVE model, while the fuel mass per unit drop is set to 0.05 mg. As a simplification, a constant speed of 1225 rpm (representing the average during the tests) is applied to the model. The mass flow rate of air during the tests is used to set the throttle position in WAVE so as to match the measured and simulated air flow rates.

3.4 Combustion Model

Various aspects of combustion are empirically modelled in this study. The cumulative mass fraction burned strongly depends on the timing of the combustion and can be predicted using the Wiebe function [9, 19, 20], whose basic form is

$$x_b(\theta) = 1 - \exp\left(-a\left[\frac{\theta - \theta_i}{\Delta\theta}\right]^{W+1}\right) \tag{3}$$

where θ denotes the crank angle, θ_i is the angle at the start of combustion and $\Delta\theta$ is the combustion duration - the crank angle range in which the burned mass fraction progresses from 10% to 90%. The constant a is determined from the crank angle corresponding to the end of combustion [20]. W is termed the Wiebe exponent and influences the shape of the response. Typical forms of the cumulative mass fraction burned for different values of W are shown in figure 2; here, the combustion duration is 15° and the start of combustion is set at a crank angle of -20°.

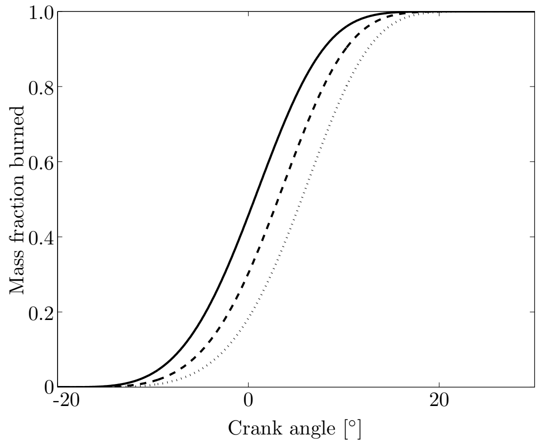


Figure 2: Wiebe combustion model for different Wiebe exponents. Solid, dashed and dotted lines respectively correspond to Wiebe exponents of 3, 3.5 and 4.

The extent of combustion may be characterised in terms of the combustion efficiency, C; this has been shown to depend strongly on the equivalence ratio, ϕ [18]. In this paper, the combustion efficiency is modelled using a simple look-up table with the reciprocal of ϕ, $\lambda = \frac{1}{\phi}$, as an argument. This is represented as

$$C = f_1(\lambda). \tag{4}$$

Similarly, the Wiebe exponent, W, is governed by a look-up table dependent on the exhaust gas residual, EGR. This latter variable is defined as

$$\mathrm{EGR} = \frac{m_{egr}}{m_a + m_{egr}} \tag{5}$$

where m_a is the trapped mass of air and m_{egr} is the trapped mass of exhaust in the cylinder [21]. The empirical function for the Wiebe exponent is

$$W = f_2(\mathrm{EGR}). \tag{6}$$

The combustion duration is dependent on the Wiebe exponent and is given by

$$\Delta\theta = f_3(W). \tag{7}$$

The empirical functions f_1, f_2 and f_3 were iteratively tuned by comparing the experimental values of heat release rate, IMEP and combustion duration to those yielded by simulation. This process is shown in figure 3.

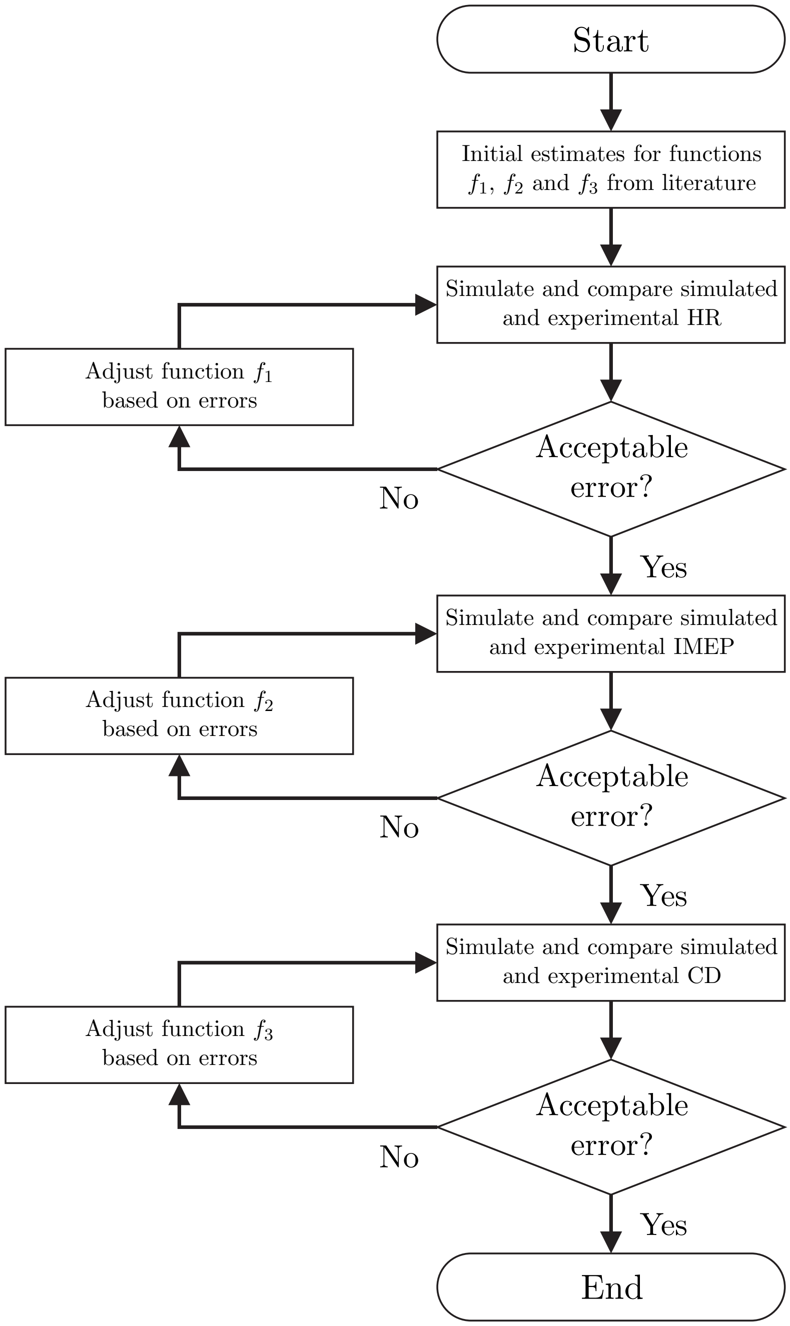


Figure 3: Flow chart for iterative determination of functions f_1, f_2 and f_3 based on simulated heat release (HR), indicated mean effective pressure (IMEP) and combustion duration (CD).

3.5 Co-Simulation

A co-simulation approach is used to combine the gas dynamics and engine model with the functions governing combustion. The combustion parameters described in section 3.4 are evaluated in a Simulink model (Mathworks, Natick, United States) and subsequently used in the WAVE model, as depicted in figure 4.

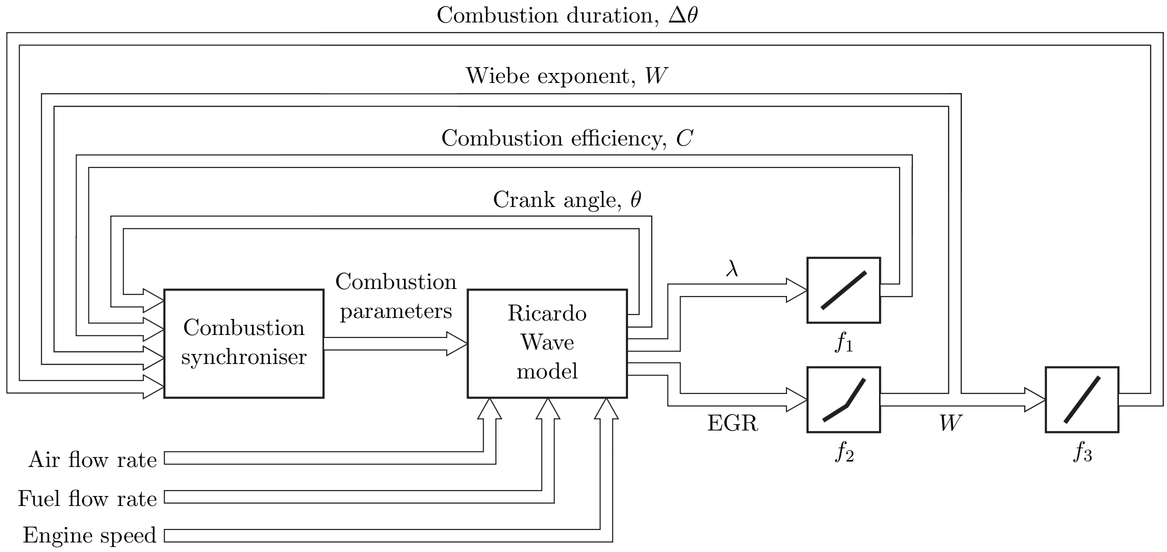


Figure 4: Co-simulation of the WAVE and combustion models in the Simulink environment.

The combustion synchroniser is triggered when the crank angle corresponds to the point in the cycle at which the intake value is closed. When this occurs, the various combustion parameters are held constant and fed into the engine model.

3.6 Cyclic Variability in Simulation

The principal source of combustion variability in the one-dimensional model used in this study arises from pressure waves from the inlet and exhaust manifolds, leading to fluctuations in the overall air-fuel ratio in the cylinder, variations in the residual gas concentration, and thus variations in the burn characteristics.

The main purpose of this work is to examine the extent to which a deterministic, 1D modelling approach can qualitatively and quantitatively reproduce cyclic variability engine behaviour. While the deterministic sources of variability described above are included in the model, the 1D model is unable to represent more complex flow effects, turbulence, and local variations in the air-fuel ratio in the cylinder. Furthermore, variations in the spark energy and duration are not included in the model.

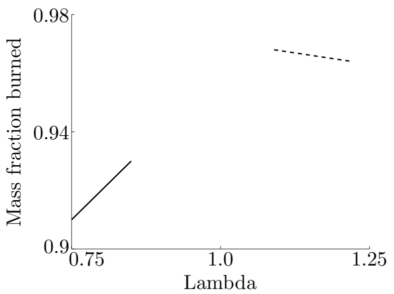
3.7 Experimental Procedure and Scenarios

During testing, the engine was directly coupled with an air-cooled AC-dynamometer in order to measure engine torque. In-cylinder pressure was measured with the ThermoComp Quartz 6061B pressure sensor (Kistler Group, Winterthur, Switzerland) installed in every cylinder, whilst exhaust gases were fed into a MEXA 7000 emission measurement system (Horiba Ltd, Kyoto, Japan) for later analysis. Various parameters including camshaft position and catalytic heating time were adjusted in order to allow the engine to achieve its target conditions. During the tests, a number of variables were controlled via the engine control units (ECU), including engine speed and λ. Furthermore, spark angle was controlled according to engine speed and air charge via a look-up table.

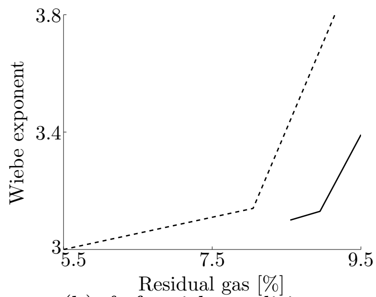
Two distinct scenarios are evaluated in this paper. The first case corresponds to operation at a rich air-fuel ratio (i.e. the air-fuel ratio was less than stoichiometric). In the second scenario, the engine is run at a lean air-fuel ratio. For each scenario, variations in the heat release and indicated mean effective pressure were examined. 500 engine cycles were measured and simulated in each case, allowing probability density functions (PDF) and the coefficient of variation to be generated in order to evaluate the ability of the model to generate realistic cyclic variation. The PDFs were determined through kernel density estimation [22] using the function *ksdensity* of MATLAB (Mathworks, Natick, United States). The development of the in-cylinder pressure was also determined in the experimental and simulated cases to allow comparison between the simulated and measured results.

4 Results

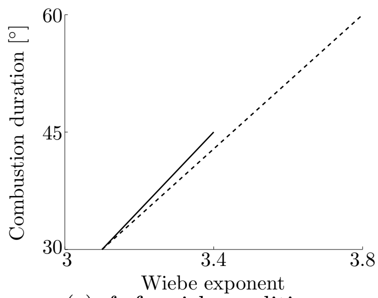
The empirical relationships (f_1, f_2 and f_3) used to model the combustion process are shown in figure 5.



(a) f_1 for rich condition



(b) f_2 for rich condition



(c) f_3 for rich condition

Figure 5: Empirical functions of the combustion model, with functions for rich conditions plotted as solid lines, and those for lean conditions as dashed lines.

Figure 6 provides an example of simulated flow conditions in terms of pressure and flow rate in the intake and exhaust valves. Fluctuations in the variables are apparent which can lead to variations in the air-fuel ratio and therefore cause cyclic variability in the heat release rate and the indicated mean effective pressure.

Estimates of the probability density functions for the cumulative heat release and indicated mean effective pressures for the experimental data and simulated engine are shown in figure 7. Results for the engine in the rich and lean air-fuel engine conditions are plotted separately. The corresponding values for the coefficients of variation for the measured and simulated variables are given in table 2.

Lag plots for the cumulative heat release and indicated mean effective pressure are shown in Figure 8. These plot the value of a variable at cycle i against the value at the next cycle (cycle $i+1$).

Figure 9 shows the maximum and mininum cylinder pressures developed in the experiments and simulations. The model is able to represent the maximum and minimum cases of cylinder pressure with good accuracy. Note the two pressure peaks characteristic of late combustion, the first corresponding to the pumping pressure, and the second to the actual combustion event. Furthermore, the peak in-cylinder pressure is low since the engine is running at idle.

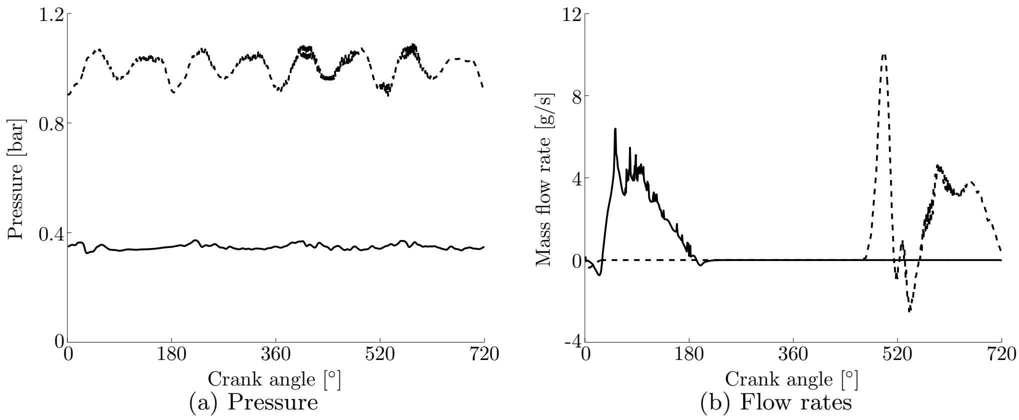


Figure 6: Example of pressure and flow rate variations in the intake and exhaust valves; values corresponding to the inlet are plotted as solid lines, while those of the exhaust are plotted as dashed lines.

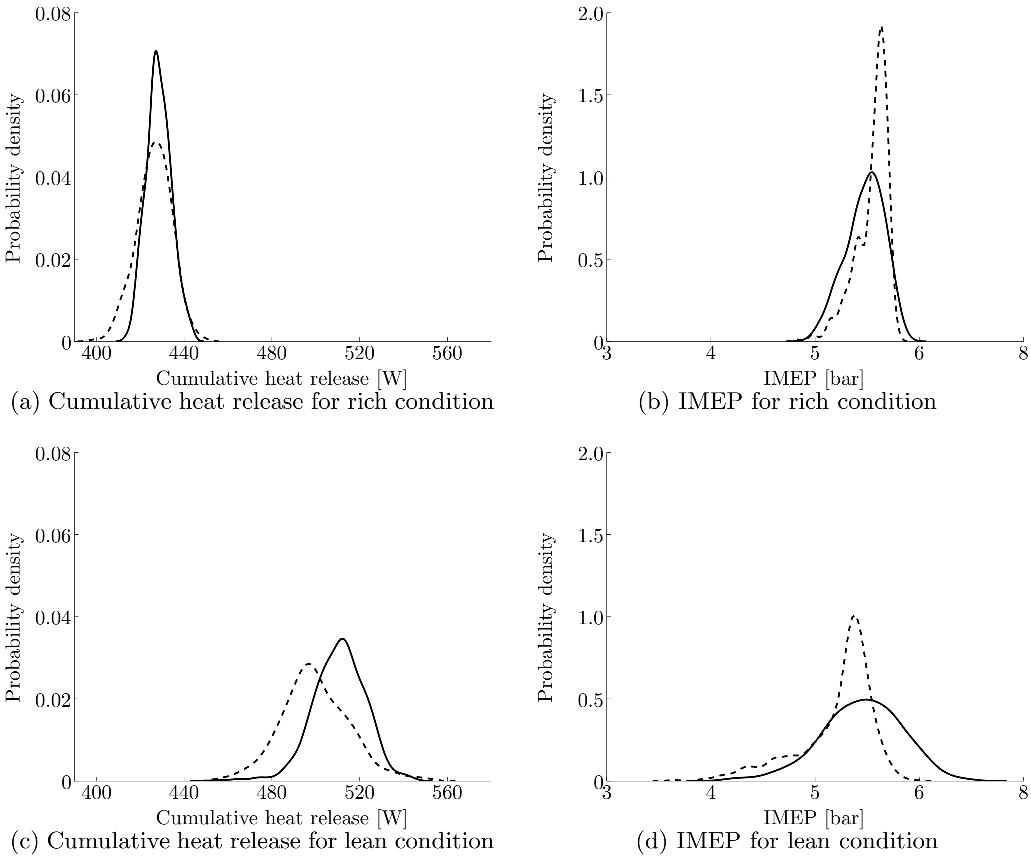


Figure 7: Estimates for the probability density functions (PDF) for heat release rates and indicated mean effective pressure (IMEP). PDFs for the experimental data are shown as solid lines while those for the simulated data are shown as dashed lines.

5 Discussion

The model presented here is a preliminary approach to simulating cyclic variability in that various stochastic sources of cyclic variability are omitted. Nevertheless, the deterministic modelling ap-

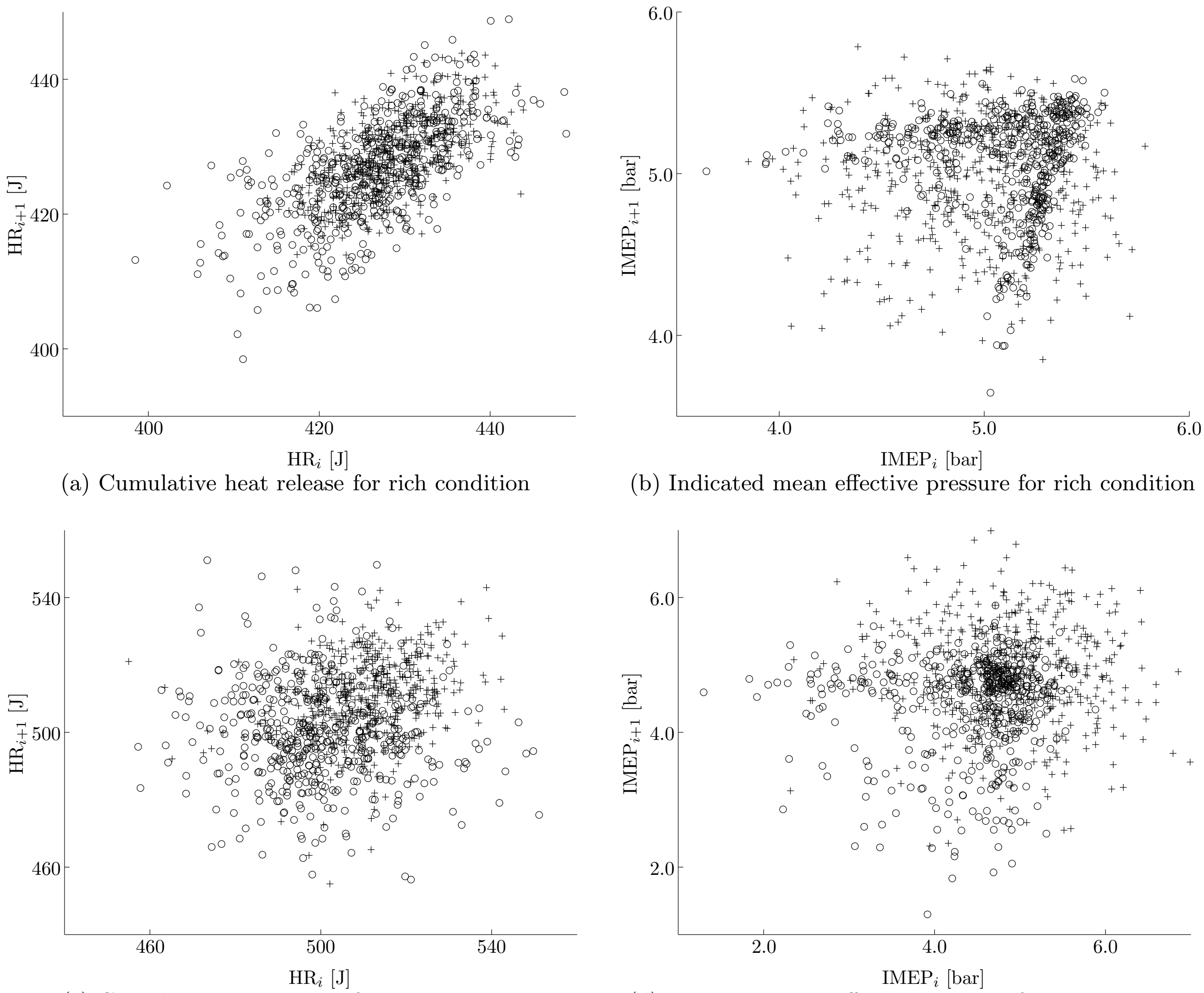
Variable	Experimental COV	Simulated COV
HR	0.0131	0.0186
IMEP	0.0751	0.0611

(a) Rich condition

Variable	Experimental COV	Simulated COV
HR	0.0235	0.0308
IMEP	0.159	0.166

(b) Lean condition

Table 2: Coefficients of variation (COV) for measured and simulated cumulative heat release (HR) and indicated mean effective pressure (IMEP).



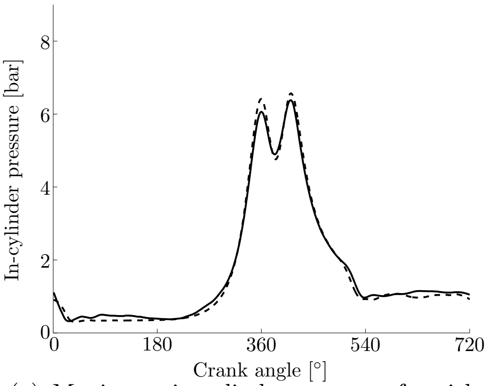
(a) Cumulative heat release for rich condition

(b) Indicated mean effective pressure for rich condition

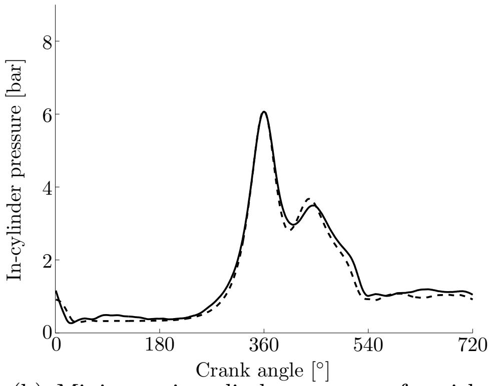
(c) Cumulative heat release for lean condition

(d) Indicated mean effective pressure for lean condition

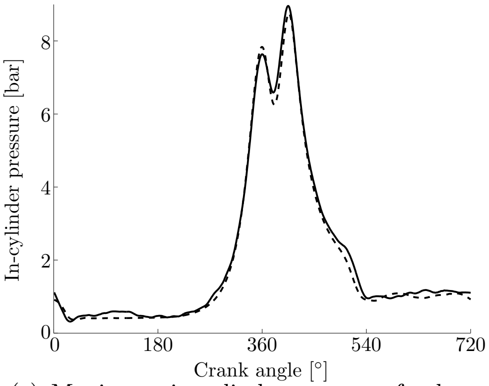
Figure 8: Lag plots for the heat release and indicated mean effective pressure. Measured results are shown as plus signs (+) while simulated data are shown as open circles (o).



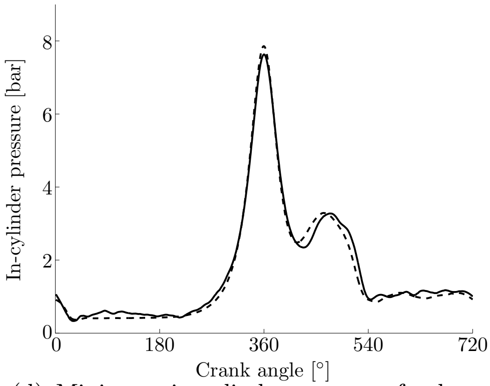
(a) Maximum in-cylinder pressure for rich condition



(b) Minimum in-cylinder pressure for rich condition



(c) Maximum in-cylinder pressure for lean condition



(d) Minimum in-cylinder pressure for lean condition

Figure 9: Variation of in-cylinder pressure for rich and lean conditions, with experimental data shown as solid lines and simulated data shown as dashed lines.

proach is able to represent a large degree of the experimental variability. The variability observed here is somewhat high due to the fuel injection technology used (port fuel injection). Moreover, the control parameters were not optimised for the engine, and therefore it is likely that the degree of variability observed here could be reduced through a more thorough engine calibration. The variability recorded during lean conditions is higher than during rich air-fuel ratios, corroborating previous results [5, 10].

The linear expressions developed for the mass fraction and other combustion characteristics are very simplistic due to the uncertainty in and scarcity of the data, making it unrealistic to fit a more complex expression. Extreme values of air-fuel ratio, rather than compositions close to the stoichiometric ratio, were used in this study. It was elected to study the engine at low air-fuel ratios since the behaviour during catalyst light-off was of interest, where the engine is often operated at rich conditions; on the other hand, lean conditions are beneficial for fuel economy but are known to be problematic with regard to cyclic variability [5]. Note that the empirical curves governing the mass fraction burned for lean and rich conditions (figure 5) would not intersect one another at stoichiometric conditions. This arises from the linear relationship chosen due to scarcity of data, whereas in reality, a higher order curve would be expected. Further tests are needed to enable the development of more realistic functions and to allow the model to represent the engine's behaviour at a wider range of operating conditions.

Nevertheless, these simple expressions give a good prediction of the effects of operating conditions on cylinder pressure as shown in figure 9, where the extremes of early and late combustion from nominally similar operating conditions are shown. The simulation is able to differentiate between these cases and successfully predicts the fast, early burn and the slow, late burn. As such, the cumulative heat release predictions for the sample of cycles used here are relatively close to the experimental results, as shown in figure 7. Any errors in the heat release prediction are compounded when the indicated mean effective pressure is calculated since this variable is sensitive to errors in both the timing and magnitude of the predicted pressure trace.

It is due to these factors that the probability density functions presented in figure 7 show that the heat release is predicted with a higher fidelity than the IMEP. There is a small offset in the mean for cumulative heat release in the lean case, but the shape of the distribution is similar to the observed data. Interestingly, the IMEP predictions from the simulations suggest that it is slightly more skewed towards the lower end of the scale than the experimental data. It may well be that the empirical functions used to model combustion tend to exaggerate the effects of the residual gas fraction and the air-fuel ratio on heat release, in which case further refinement of the functions could be proposed as a future exercise if the heat release data from a large set of cycles could be analysed.

In both the rich and lean cases, the IMEP has a higher variance experimentally that the simulation is able to capture, evident from the flatter and wider probability density function of the experimental data. This suggests that the other mechanisms of cyclic variability are having some impact experimentally, but that the simulation is nevertheless able to represent a significant proportion of the variability.

The purpose of the phase lag plots (figure 8) is to aid the visualisation of prior cycle effects within the data. For example, if the current cycle is unusually strong due to the presence of unburnt fuel in the residual gas left over from a previous poor combustion event, then the resulting point will appear towards the top left of the plot. Points which are broadly similar to the proceeding ones will appear along the '$y = x$' line. A phase lag of one is selected here to allow the relationship between directly adjacent events to be visualised. Alternative embedding dimensions may be chosen to show other phenomena such as firing order effects.

Figure 8 reveals that the experimental results show some evidence of structural variability. The characteristic boomerang shape previously observed [18] is evident in the IMEP data for both lean and rich events, although the stochastic variability is large and obscures the deterministic relationship to some extent. Conversely, the simulated data show clearer evidence of deterministic variability, with a more defined arc of points in both IMEP plots. The lean plot does have a significant number of points in the lower left region of the plot, suggesting a high degree of highly chaotic behaviour rather than a simple bifurcated, bimodal behaviour. It seems clear, though, that the simulation is capable of representing cyclic variability in a similar manner to the real engine, which was one of the main aims of the study. Methods that assign a stochastic distribution to the variability will not give insight into its underlying mechanisms.

Note that the heat release in the rich condition exhibits a spread along the $y = x$ line; this is mostly caused by a slight drift of the operating conditions during the test. This behaviour is reflected in both the experimental and simulated data in a similar fashion, though exaggerated somewhat in the latter.

To further develop this work, a first step would be to enlarge the set of data used to generate the empirical functions and ideally, expanding the range of cycle-by-cycle data in order to include a measure of the in-cylinder air-fuel ratio and residual gas fraction. These measures would require more intrusive instrumentation than would be desirable for a routing operation but a specific exercise to generate the sensitivity functions would be beneficial. In the same exercise, data could be gathered to examine early flame growth and thus gain greater insight into firstly, the impact of boundary conditions on this early flame growth and secondly, the relationships between early flame growth and the subsequent heat release profile. It must be stressed that there is already a large body of research that deals with these topics; however, what is needed for this work are data that allow an empirical model to link boundary conditions with the heat release via the mechanism of small scale turbulence. To be of benefit in a one-dimensional cycle simulation, these relationships must be observed between parameters that are routinely simulated within 1D code, and not be dependant on 3D simulations of the engine or heavily intrusive instrumentation.

Future investigations will concentrate on how to use the modelling approach to guide real-time control applications aimed at reducing cycle-to-cycle variability. The source of variability focused on in this paper is the overall in-cylinder air-fuel ratio. Fuel injection pressure and rate may be feasible variables which can be manipulated online in order to reduce variability on a cycle-to-cycle basis. The data in this paper have implied a deterministic element to engine variability. Indeed, other research has highlighted that as the air-fuel ratio becomes increasingly weak, sequences of strong and weak cycles emerge [23]. Accurate, real-time measurement of the air-fuel ratio may thus allow such sequences to be detected early, and corrective control action subsequently taken to reduce the magnitude of the fluctuations.

Underlying patterns of in-cylinder variables can be detected via a number of methods [23]. A Shannon Entropy measure reveals the existence of different lengths of sequences within data [24, 25], though some work suggests that this measure may not have the necessary sensitivity to detect patterns of in-cylinder data [23]. An alternative is autocorrelation, which has been applied to in-cylinder measurements and has revealed patterns in heat release for lean air-fuel mixtures [18].

There are a number of challenges inherent to online control and minimisation of cyclic variability. Firstly, as discussed above, a robust, but sufficiently sensitive means is required to detect the onset of strong and weak engine cycle sequences. Knowledge concerning how engine variables such as fuel injection parameters can affect variability is required in order for a control strategy to be devised. This will require further modelling, and will incorporate aspects of the 1D model presented in this paper. Finally, assuming that such a relationship between engine parameters and variability can be determined, the controlled variables (most likely connected with the fuel injection mechanism) must respond quickly once a pattern has been detected, in order to reduce the magnitude of cycle-to-cycle fluctuation due to deterministic sources of variability. However, cycle-to-cycle variations cannot be completely eliminated using this approach since some of the underlying causes of variability are stochastic in nature.

6 Conclusions

Using a model that empirically incorporates elements of the combustion process such as duration and efficiency in combination with an engine and gas dynamics model, engine variability in terms of heat release and indicated mean effective pressure is predicted with good accuracy. The results indicate that changes in the air-fuel ratio and the residual gases in the cylinder prior to combustion are important constituents of the mechanisms underlying engine variability. The simulation method can now be used to determine the degree of variation in the heat release during future engine design, verifying that cycle-to-cycle fluctuations are within acceptable limits, and can also be used in online engine control approaches aimed at reducing variability.

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