Network Pricing for Smart Grids considering Customers’ Diversified Contribution to System Peak

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Abstracts—This paper proposes a modified long-run incremental cost pricing (LRIC) method for distribution network pricing considering the diversified contributions of network users to system peak. The Shapley-value method and modified coincident factor method are used to determine network users’ various contributions. The comparison between original LRIC and the Modified LRIC indicates the positive correlation between the contribution to system peak and network charges for different network users. This paper also explores the potential users’ behavior to gain bill reductions according to the cooperative-game theory and the consequential network investment deferral.

Index Terms—Network pricing, Long-run-incremental-cost pricing, Shapley value, coincident factor, Contributions.

I. INTRODUCTION

Network pricing methods are designed to recover network reinforcement cost and certain level revenue for network operators. An advanced pricing method should be able to send end-users clear signals of network congestion, and network users can adjust their network usage behaviors according to financial signals. This can lead to system stresses alleviated and deferral of network reinforcement.

Currently, the mainly used pricing methods for transmission and distribution network are Distribution reinforcement model (DRM), investment-cost-relative pricing (ICRP), long-run-incremental-cost (LRIC) [1]. DRM averages network costs at each voltage level. ICRP sets network charges by using the capacity of lines and distances of supply points. In LRIC, the incremental cost is calculated based on network maximum capacity and annual peak utilization [2]. However, none of aforementioned methods consider that various network users have different peak load characteristics and individual peak load may not coincide with each other or the system peak.

Under current LRIC pricing arrangement, at each nodal level, a demand node of very low level at system peak period is charged according to the same system peak utilization level compared to other larger contributors. Under a demand node, where network users can be categorized into classes with different criteria and have diversified contributions to network components, but various loads are charged at the same unit price. Therefore, it is reasonable to reconsider network pricing methods by respecting diversified contributions of various loads to upstream networks peak utilization that defines future network investment.

Paper [3] presents a new method for determining the contributions of each load to the power flow through each line in an electrical-power system. The main idea of computation algorithm is to determine the extraction factors of each load node from the lines incident to it, then moving backward, propagating these extraction factors to the lines in the upstream direction up to source nodes.

Paper [4] proposes a coincident demand based smart long run incremental cost (LRIC) pricing mechanism. Coincidence factor, which is defined as the fraction of network user’s demand at upstream asset peak usage to user’s individual peak demand, is reflective of user’s contribution to network component usage. The coincident demand is used as power flow input to accommodate actual asset usage. Compared to original LRIC pricing mechanism, all network users will obtain network charges reduction at different extents depending on their contributions to network asset usage. Paper [5] proposes an enhanced LRIC pricing mechanism based on the contribution of generation and load located at various nodes, to network peak of each upstream asset, the contribution is modeled as a load-to-asset contribution factor (LACF) and generation-to-asset contribution factor (GACF) for load and generation users respectively, to assess the impact of user contributions during peak load on network investment.

Shapley value based on cooperative game theory is a well-established concept in common cost allocation and contribution determination [6]. There are several transmission-related allocation methods based on the Shapley Value. Paper [7] provides a transmission loss allocation method based on equivalent injection current. Similar to power flow analysis, the proposed method uses current injection analysis to determine the voltage contribution of each network user on network components. The Shapley Value is used for final transmission loss allocation. Paper[8]-[9] propose similar method using Shapley Value to allocate transmission cost incurred to accommodate all the loads, where the method overcomes the drawback of conventionally used methods such as postage-stamp method and MW-miles method, encouraging the economically optimal usage of transmission facilities. Proposed Shapley value methods distribute the common cost of cooperation based upon the assumption that the cost proportion of a participant in a coalition is determined by the incremental cost that the participant generates by joining the coalition.

This paper uses Shapley value of cooperative game theory to calculate the expected marginal contribution of diversified loads on the utilization of each network asset, then determines contribution coefficients based on coincident factor concept, and finally uses modified utilization levels as input to calculate Long-run-incremental cost from each network component to
load. Comparison between original LRIC pricing method and the proposed method illustrates that large contributors to system peak is responsible for higher network charges while small contributors will be rewarded by lower charges. The strong incentive to network users can guide them to modify demand pattern. The analysis of users’ strategies proves that by cooperative demand pattern adjustment, all active participants will obtain network charges reduction and system utilization is alleviated, resulting in network reinforcement deferral.

II. MATHEMATICAL FORMULATION OF MODIFIED LRIC

a) LRIC pricing method

In electricity networks, the time to reinforcement horizon \( n_j \) can be determined from the actual power flow peak level \((P_j)\) and the capacity \((C_j)\) of network asset \(j\). The present value \((PV_j)\) of future reinforcement can be calculated according to asset cost and the reinforcement horizon:

\[
n_j = \frac{\log C_j - \log P_j}{\log(1+r)}
\]

\[
PV_j = \frac{\text{Asset}_j}{(1+d)^n}
\]

Where \(r\) is the load growth rate; \(d\) is the discount rate.

Resulting from the incremental power withdraw \((\Delta P_N)\) at node \(N\), the incremental cost of network asset \(j\) can be determined by the power flow change along the network asset \((\Delta P_j)\):

\[
n_j \text{new} = \frac{\log C_j - \log(P_j+\Delta P_j)}{\log(1+r)}
\]

\[
PV_j \text{new} = \frac{\text{Asset}_j}{(1+d)^{n \text{new}}}
\]

Where \(n \text{new}\) and \(PV \text{new}\) are new asset investment horizon and the present value with additional power withdrawn or injection.

The change in the present value as a result of the nodal injection or withdrawn is given by

\[
\Delta PV_j = PV_j \text{new} - PV_j
\]

The annualized incremental cost \((IC_j)\) of the network component \(j\) is the difference in the present value of the future investment as a result of \(\Delta P_N\) at node \(N\) multiplied by an annuity factor

\[
IC_j = \Delta PV_j \times \text{annual factor}
\]

Therefore, for the network, the long-run incremental cost to support node \(N\) \((LRIC_N)\) can be determined by

\[
LRIC_N = \frac{\sum IC_j}{\Delta P_N}
\]

b) Shapley value

A cooperative game theoretic approach based on Shapley Value is used here to determine the expected marginal contribution of diversified loads to the utilization of network components in the distribution system. The Shapley value is a method which divides the value of a coalition between the players of that coalition based on their contribution to the value of coalition. Since the contribution of players on the value of a coalition depends on the selection order of players, some situations may appear that symmetric players in a game acquire different values. To handle this problem, Shapley uses averages over all possible permutations of players to calculate the value, which is called Shapley value. Let \(\Pi_N\) denotes the set of all possible permutations of network users: \(\{1, ..., N\}\). For a permutation \(\pi \in \Pi_N\), \(S_{ik}\) denotes the set of users that are predecessors of user \(i\) in the \(\pi\). In a cooperative game \(G\) with \(N\) players and the characteristic function \(\sigma\), Shapley value for user class \(i\) is calculated as:

\[
SV_i = \frac{1}{n!} \sum_{\pi \in \Pi_N} \sigma(S_{ik} \cup \{i\}) - \sigma(S_{ik})
\]

Where the characteristic function \(\sigma\) here is the peak demand level of a network asset.

The summation over different permutations considers all possible coalitions in the game. This Shapley value indicates the expected marginal contribution of each network user.

c) Modified Coincident Factor (Contribution Coefficient)

Coincident factor \((CF)\) indicates that how the peak demand of an individual load coincides to the peak utilization of an upstream network asset, calculated as:

\[
CF_{ij} = \frac{d_{ij}}{D^i_j}
\]

Where \(D^i_j\) is the demand level of load \(j\) at network asset \(i\) peak time, \(D^i_j\) is the peak demand level of individual load \(i\).

Here this paper proposes a novel method to calculate the contribution of diversified loads to system peak, which is called contribution coefficient \((CC)\). The contribution coefficient is the fraction of the actual contribution to the expected marginal contribution:

\[
CC_{ij} = \frac{\delta_{ij}^{ap}}{\delta_{ij}^P}
\]

Where \(CC_{ij}\) is the contribution coefficient of load \(i\) to network asset \(j\). \(\delta_{ij}^{ap}\) is the actual power flow induced by load \(i\) at peak time of network asset \(j\), presenting the actual contribution of load \(i\) to peak utilization of network asset \(j\). \(\delta_{ij}^P\) is the expected marginal contribution of load \(i\) to peak utilization of network asset \(j\) which is calculated via Shapley method.

If \(CC>1\), the load is large contributor, vice versus, and if \(CC=1\), the load is average contributor.

d) Modified LRIC

To determine the LRIC with diversified load classes under various nodes, firstly power flow analysis is used to determine the utilization of network components and the actual power flows along networks induced by each load, then contribution coefficient matrix is obtained by using equation (8) and (10). Changes of power flows \(\Delta P_N\) cross each network asset due to the incremental demand \((\Delta P_N)\) of the node \(N\) is calculated by power flow analysis. Then the contribution coefficients as weights are multiplied by actual power flow at the peak level on each network asset:

\[
P_{ij} = P_j \times CC_{ij}
\]

Where \(P_j\) is actual power flow peak level across network asset \(j\), \(P_{ij}\) is the modified power flow peak level across network asset \(j\) considering the contribution of the load \(i\) under.

The reinforcement horizon of network asset \(j\) and new reinforcement horizon for various loads due to incremental demand are determined as:
\[ n_j = \frac{\log C_j - \log P_{ij}}{\log (1+r)^{-1}} \]  
\[ n_{j\text{new}} = \frac{\log C_j(1 + \Delta P_j) - \log P_{ij}}{\log (1+r)^{-1}} \]  
Overall LRIC for load \( i \) is calculated by using equations (12)-(13) and equations (5)-(7)

### III. DEMONSTRATION

#### a) System description

A distribution system with five buses and five branches is used to demonstrate. For simplicity, here only distribution lines as sole network component on this distribution system are considered and the rest of network assets can be analysed by same method proposed in this paper. System topology is shown is fig.1. There are three demand nodes in system. Node 1 and node 2 comprise singular class of load, representing industry user (load 1) and small business user (load 2) respectively. Node 3 has two differed classes of load, where class 1 is categorized as domestic unrestricted user and class 2 is domestic user with smart meters [10]. Load profiles in the system are shown in fig.2.

![Demo distribution system](image1)

![Load profiles at various nodes](image2)

Demand peaks of loads in node 1 and node 2 occur in the mid of the day while load in node 3 class 1 peaks at early evening and load in node 4 class 2 reaches peak level at late night.

#### b) LRIC implementation

From available load data at various nodes, the power flow cross each distribution lines can be determined and power flow contribution from each load can be calculated by power flow analysis. DC power flow is applied to the demonstration.

The expected marginal contributions of each load at each distribution line is calculated from equation (8), and the contribution coefficients are calculated by using equation (10).

Table.1 shows the contribution coefficients of each load at each distribution line.

<table>
<thead>
<tr>
<th>Node 1</th>
<th>Node 2</th>
<th>class 1</th>
<th>class 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>1.0070</td>
<td>0.9612</td>
<td>1.0976</td>
</tr>
<tr>
<td>D2</td>
<td>1.0343</td>
<td>1.0381</td>
<td>0.8350</td>
</tr>
<tr>
<td>D3</td>
<td>0.7077</td>
<td>0.8843</td>
<td>1.1713</td>
</tr>
<tr>
<td>D4</td>
<td>0.9216</td>
<td>0.9947</td>
<td>0.9493</td>
</tr>
<tr>
<td>D5</td>
<td>0.7984</td>
<td>0.9209</td>
<td>1.2030</td>
</tr>
</tbody>
</table>

![Power flow on (a) D2 and (b) D5](image3)

The modified power flow peak levels of distribution lines varies from load contributions, rather than are the same for all network users, calculated by using equation (11). The branch incremental cost and nodal LRIC then can be calculated by equations (12)-(13) and equations (5)-(7). Annual factor, load growth rate and discount rate are assumed to be 0.074, 1.6% and 6.9% respectively. Network asset cost are assumed to be D1 £ 12m, D2 & D5 £ 6m and D3 & D4 £ 3m. Results are presented in table 2.

<table>
<thead>
<tr>
<th>Incremental charges (E/yr/MW)</th>
<th>Node 1</th>
<th>Node 2</th>
<th>class 1</th>
<th>class 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>14051.2</td>
<td>12105.5</td>
<td>18514.9</td>
<td>11061.3</td>
</tr>
<tr>
<td>D2</td>
<td>8070.4</td>
<td>4824.6</td>
<td>829.3</td>
<td>480.6</td>
</tr>
<tr>
<td>D3</td>
<td>-277.6</td>
<td>1527.1</td>
<td>1293.3</td>
<td>399.1</td>
</tr>
<tr>
<td>D4</td>
<td>743.9</td>
<td>2713.3</td>
<td>-753.0</td>
<td>-482.3</td>
</tr>
<tr>
<td>D5</td>
<td>1162.9</td>
<td>5236.1</td>
<td>19905.1</td>
<td>7917.3</td>
</tr>
<tr>
<td>LRIC</td>
<td>23750.9</td>
<td>26406.6</td>
<td>39789.6</td>
<td>19376.0</td>
</tr>
</tbody>
</table>

The result indicates that network charges of D1 and D5 is highest to load in node 3 class 1 while charges of D2 and charges of D3 and D4 are highest to load 1 and load 2 respectively. Negative charges occurs when the demand of loads alleviate network utilizations.

#### c) Result analysis
Table 3 presents the branch and total LRIC based on the original algorithm.

### Table 3 Branch IC and nodal LRIC under original algorithm

<table>
<thead>
<tr>
<th>Incremental charges (£/yr/MW)</th>
<th>Node 1</th>
<th>Node 2</th>
<th>Node 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>13740.9</td>
<td>13740.9</td>
<td>13740.9</td>
</tr>
<tr>
<td>D2</td>
<td>7244.5</td>
<td>4280.1</td>
<td>1477.6</td>
</tr>
<tr>
<td>D3</td>
<td>-840.9</td>
<td>2262.9</td>
<td>779.5</td>
</tr>
<tr>
<td>D4</td>
<td>966.1</td>
<td>2759.8</td>
<td>-889.7</td>
</tr>
<tr>
<td>D5</td>
<td>2391.9</td>
<td>6817.3</td>
<td>11014.9</td>
</tr>
<tr>
<td>LRIC</td>
<td>23502.5</td>
<td>29661.0</td>
<td>26123.2</td>
</tr>
</tbody>
</table>

In the original method, LRIC prices defer at nodal level however diversified load classes under one demand node share a unit price. The comparison indicates that considering diversified loads have different contributions on each network asset, large peak contributors will be responsible for higher network charges, vice versa. For load 1, network charges of D1 and D2 increase 2.5% and 11.4%, charges of D4 and D5 decrease by 23% and 51.4%. Since load 2 does small contribution to D1, D3, D4 and D5 and network charges only increase at D2 by 12.7%, overall network charge reduces by 11.6%. For load class 1, contribution coefficients at D1, D3 and D5 are larger than 1, network charges on those branches rise up 34.7%, 65.9% and 80.7% compared to those from the original LRIC pricing method. For load class 2, the contribution coefficients are all less than 1 and therefore the users at load class 2 will receive a large network charge reduction because of the less system contribution, where overall reduction is 25.8%.

![Fig.4 Branch LRIC at node 3 for example](image)

IV. NETWORK USER’ ENERGY MANAGEMENT STRATEGY

Under the corporate game theory, users’ strategy will significantly affect the final payoff of each network user. The proposed pricing method offers network users the opportunities to adjust their usage pattern to acquire bill reduction and benefits.

From table 1, users fall in load class 1 under node 3 would be the first to change their usage pattern trigged by high LRIC charges on network asset D1 and D5. Assume users in class 1 shift their peak time to one hour later, the LRIC then is shown in table 4.

![Fig.5 Power flow on D1 (a) with and (b) without peak time shift](image)

In corporate game theory, users in load 1 obtain the information that users in load 3 class 1 would conduct 1 hour peak shift, which will significantly increase the network charges to load 1, and then countermeasure would be taken by users in load 1. Assuming users in load 1 shave peak duration by 7%, the resultant LRIC is shown in table 5.

### Table 4 Branch IC and nodal LRIC with load class 1 peak time shift

<table>
<thead>
<tr>
<th>Incremental charges (£/yr/MW)</th>
<th>Node 1</th>
<th>Node 2</th>
<th>Node 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>10517.8</td>
<td>9415.1</td>
<td>16992.4</td>
</tr>
<tr>
<td>D2</td>
<td>7039.9</td>
<td>4045.3</td>
<td>745.1</td>
</tr>
<tr>
<td>D3</td>
<td>-325.3</td>
<td>1578.9</td>
<td>1387.8</td>
</tr>
<tr>
<td>D4</td>
<td>654.1</td>
<td>2400.0</td>
<td>-693.8</td>
</tr>
<tr>
<td>D5</td>
<td>1298.8</td>
<td>5133.0</td>
<td>18881.7</td>
</tr>
<tr>
<td>LRIC</td>
<td>19185.3</td>
<td>22572.4</td>
<td>37313.2</td>
</tr>
</tbody>
</table>

### Table 5 Branch IC & nodal LRIC with load class 1 peak time shift and load 1 peak load shave

<table>
<thead>
<tr>
<th>Incremental charges (£/yr/MW)</th>
<th>Node 1</th>
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</tr>
</tbody>
</table>

From results, it can be observed that users under load 3 class 1 have a significant branch incremental charge reduction (63%) on D1 and moderate reduction (4%) on D5 due to peak hour shift by load 3 class 1, and the overall LRIC is largely reduced (31.2%). Meanwhile the branch incremental charge of load 1 on D1 increases by 122.4% and overall LRIC surges to £ 41023.8, which is because D1 peak time is shifted from hour 34 to hour 23 where load 1 becomes the largest contributor to D1 peak.
Compared with the results in table 2, both active demand users who conduct demand pattern adjustment benefit from lower network charges, overall network charges reduce by 19.2% and 6.2% to load 1 and load class 1, but the rest of network users are not guaranteed for benefits or losses because their contribution coefficients are related to active users’ energy management strategies. Based on the strategies from users in node 1 and node 3 class 1, peak utilizations of most of network components decrease, which indicates most network assets reinforcement can be deferred.

![Network utilization with and without users' energy management](image)

In reality, diversified loads have various demand response limitations to conduct demand adjustment, and the optimal demand adjustment for each network user should be analysed among all network users cooperatively. Minimal network utilization level with minimal network charges can be used as the objective function to determine the optimal demand strategies in terms of Nash-equivalent based on game theory. The comprehensive optimization process is beyond this paper and will be explored in the future work.

V. CONCLUSION

This paper proposes a new network pricing method based on LRIC pricing method combining the diversified load contributions to upstream systems peak demand levels. Shapley value method and modified coincident factor concept are used to determine the contribution of diversified loads. A distribution system with 3 demand nodes and 4 load clusters is tested. The numerical results clearly indicate that larger contributors at network utilization peak receive strong incentives to change their usage patterns by a high pricing signal, and small contributors will benefit from lower network charges.

REFERENCES


