Piezoelectric anisotropy and energy-harvesting characteristics of novel sandwich layer BaTiO$_3$ structures

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Abstract

This paper presents a detailed modelling and experimental study of the piezoelectric and dielectric properties of novel ferroelectric sandwich layer BaTiO$_3$ structures that consist of an inner porous layer and dense outer layers. The dependencies of the piezoelectric coefficients $d_{3j}^*$ and dielectric permittivity $\varepsilon_{33}^*$ of the sandwich structure on the bulk relative density $\alpha$ are analysed by taking into account an inner layer with a porosity volume fraction of 0.5–0.6. The observed changes in $d_{3j}^*$ and $\varepsilon_{33}^*$ are interpreted within the framework of a model of a
laminar structure whereby the electromechanical interaction of the inner porous layer and outer dense layers have an important role in determining the effective properties of the system. The porous layer is represented as a piezocomposite with a 1–3–0 connectivity pattern, and the composite is considered as a system of long poled ceramic rods with 1–3 connectivity which are surrounded by an unpoled ceramic matrix that contains a system of oblate air pores (3–0 connectivity). The outer monolithic is considered as a dense poled ceramic, however its electromechanical properties differ from those of the ceramic rods in the porous layer due to different levels of mobility of 90° domain walls in ceramic grains. A large anisotropy of $d_{3j}^*$ at $\alpha = 0.64–0.86$ is achieved due to the difference in the properties of the porous and monolithic layers and the presence of highly oblate air pores. As a consequence, high energy-harvesting figures of merit $d_{3j}^* g_{3j}^*$ are achieved that obey the condition $d_{33}^* g_{33}^*/(d_{31}^* g_{31}^*) \approx 10^2$ at $d_{33}^* g_{33}^* \approx 10^{-12}$ Pa$^{-1}$, and values of the hydrostatic piezoelectric coefficients $d_{h}^* \approx 100$ pC / N and $g_{h}^* \approx 20$ mV/m / N are achieved at $\alpha = 0.64–0.70$. The studied BaTiO$_3$-based sandwich structures has advantages over highly anisotropic PbTiO$_3$-type ceramics as a result of the higher piezoelectric activity of ceramic BaTiO$_3$ and can be used in piezoelectric sensor, energy-harvesting and related applications.

Keywords: Piezoelectric properties, anisotropy, porous ferroelectrics, energy-harvesting figures of merit, dielectric permittivity.
1. Introduction

Interest in the development of new porous ferroelectric materials and their applications [1–7] has arisen for a number of reasons. Firstly, when porous ferroelectric ceramics (FCs) with a perovskite-type structure are in a poled state, they exhibit piezoelectric properties [4–9] that are of value for piezotechnical and related transducer applications. Secondly, the microgeometry–properties relations in these porous materials are complex [6, 8, 9] and can be affected by a variety of conditions related to their manufacture, poling and mechanical properties [1–4]. Thirdly, the opportunity to manufacture novel ferroelectric materials with a specific microgeometry of air pores [1–3, 10, 11] enables researchers to tailor the effective electromechanical properties and their anisotropy [8, 9]. Fourthly, different methods [8, 11–16] can be applied*) to predict the effective properties of the porous material and features of its poling. Finally, in the last decade, the desire to produce high-performance lead-free FCs and improve their electromechanical properties [17–19] has become an important stimulus to manufacture novel porous

*) Effective electromechanical properties of porous ferroelectric materials can be calculated, for instance, within the framework of the theoretical approach [12] based on the micromechanical model wherein the electromechanical coupling in the piezoelectric medium and the pore shape are taken into account. The full set of electromechanical constants of the porous material can also be evaluated using analytical formulae [11, 15], by consecutively applying the methods [8, 12, 13] concerned with micromechanical models and specifics of the composite microgeometry, and by the finite element method [14, 16]. In work [14] an incomplete poling of porous samples is also taken into account. The finite element multi-inclusion modelling of piezocomposites [16] that contain a few components is also promising for an analysis of porous materials with a complex microgeometry.
piezoelectric materials [5–7] and structures with improved characteristics.

Recently, a novel porous sandwich layer (PSL) BaTiO$_3$ has been put forward [7] as a new kind of porous lead-free material that can be applied in piezoelectric transducers, energy harvesters and sensors. However in Ref. 7 only limited characteristics related to the longitudinal piezoelectric effect are discussed, and no data on the lateral piezoelectric effect, anisotropy of the piezoelectric coefficients $d_{3j}$ and related parameters are reported. The aims of the present paper are (i) to study the piezoelectric and dielectric properties and figures of merit (FOMs) of novel PSL BaTiO$_3$ with an inner layer with a porosity volume fraction of 0.5–0.6 and (ii) to highlight the density ranges whereby a large piezoelectric anisotropy is observed in these structures.

2. Manufacturing of porous sandwich layers and experimental results

PSL BaTiO$_3$ samples were manufactured by alternatively layering prepared barium titanate starting powders containing varying fractions of pore forming agent before uniaxial pressing and sintering. For the dense outer layers, the starting powder was prepared from commercial barium titanate powder (Ticon P, Ferro, UK) and 5 wt.% organic binder, polyethylene glycol (PEG) (Sigma, UK), which was ball milled with zirconia milling media and distilled water for 24 h prior to drying, regrinding and sieving through a 150 µm mesh. The starting powder for the porous inner layer was produced from dry mixing barium titanate with PEG in varying quantities (25 and 35 wt. %) so as to achieve different inner layer porosities. The
powders were sequentially layered into a uniaxial tablet die with layers gently flattened before the next was layer added, before being pressed at 185 MPa to form the green pellets. The pellets were sintered in air at 1300°C for 2 h with a heating/cooling rate of 60°C / h, and a two-hour dwell stage was included at 400°C to burn off the organic binder/pore forming agent. The sintered pellets were cleaned, and their densities measured via the Archimedean method. Silver electrodes were painted onto the samples (RS Components, Product No 186-3600) prior to corona poling at 14 kV at a distance of 35 mm from the samples, at a temperature of 115°C.

The piezoelectric properties, namely, piezoelectric coefficients $d_{3j}^*$, were measured using a Take Control Piezometer PM25 at 97 Hz. Impedance spectroscopy (Solartron 1260 and 1296 Dielectric Interface, Solartron Analytical, UK) was used to measure the dielectric properties of the samples allowing calculation of the relative permittivity. Scanning electron microscopy was used to investigate the PSL structure and measure the inner porosity $p_{in}$ in the porous layer. An optical micrograph of the outer surface of the PSL BaTiO$_3$ sample is shown in figure 1(a). Samples produced with high fractions of pore-forming agent in the PSL demonstrated excellent cohesion between layers, although some inter-layer cracks perpendicular to the poling direction were occasionally observed in SEM images. These tended to be smaller than the pore size and therefore not thought to significantly influence the measured properties. Sequential pressing of layers also resulted in the PSL exhibiting a small slant with respect to the poling direction in
some samples, as can be seen in figure 1(a); a processing technique such as tape casting would likely be able to produce PSL materials with more consistent layering.

Experimental data showing the changes in the piezoelectric coefficients $d_{3j}^*$ and dielectric permittivity $\varepsilon_{33}^*$ at mechanical stress $\sigma = \text{constant}$ with the relative density $\alpha$ of the PSL are represented in figure 2 for two levels of the inner layer porosity ($p_{in}$); lower porosity fractions were difficult to achieve using the fabrication method discussed here due to inter-layer cracks forming during pressing and sintering. Hereafter we denote the samples PSL-1 and PSL-2 for $p_{in} \approx 0.5$ and 0.6, respectively. It is seen that the piezoelectric coefficients $d_{3j}^*$ [figure 2(a)] obey the condition

$$\frac{d_{33}^*}{|d_{31}^*|} \approx 10$$

at $d_{33}^* \approx (90\text{--}130) \text{ pC } /\text{N}$ and the relative permittivity level $\varepsilon_{33}^* / \varepsilon_0 \sim 10^2\text{--}10^3$ [figure 2(b)]. The studied PSL material has advantages over the highly anisotropic PbTiO$_3$-type FCs [20] with a large anisotropy of $d_{3j}^*$ due to the lead-free nature of the PSL and the $d_{33}^*$ values being about twice larger than $d_{33}$ of the PbTiO$_3$-type FCs. From the experimental results shown in figure 2, a large piezoelectric coefficient $g_{33}^* = d_{33}^* / \varepsilon_{33}^*$ is achieved due to the relatively small dielectric permittivity $\varepsilon_{33}^*$. For instance, at a relative density $\alpha = 0.653$ for PSL-2, $g_{33}^* = 30.2 \text{ mV} / \text{m} / \text{N}$, and at $\alpha = 0.698$ for PSL-1, $g_{33}^* = 21.3 \text{ mV} / \text{m} / \text{N}$. This is in contrast to
the piezoelectric sensitivity of the monolithic poled BaTiO$_3$ FC [21] which is characterised by a piezoelectric coefficient $g_{33} = 12.6$ mV/m / N at room temperature. Due to the relatively large values of the piezoelectric coefficient $d_{33}^* \sim 10^2$ pC / N of the PSL in a wide $\alpha$ range [figure 2(a)], the order-of-magnitude of the energy-harvesting FOM

$$ (Q_{33}^*)^2 = d_{33}^* g_{33}^* $$  \hspace{1cm} (2)

(or the squared FOM concerned with the longitudinal piezoelectric effect) is typically $10^{-12}$ Pa$^{-1}$. As is known, the FOM from Eq. (2) is proportional to the ‘signal – noise’ ratio [8] for a piezoelectric transducer at the longitudinal oscillation mode. The squared FOM concerned with the transverse piezoelectric effect of the PSL sample is

$$ (Q_{31}^*)^2 = d_{31}^* g_{31}^* $$  \hspace{1cm} (3)

and small in comparison to $(Q_{33}^*)^2$ from Eq. (2) while Eq. (1) is valid for the studied PSL material.

The large piezoelectric anisotropy observed in the novel PSL BaTiO$_3$ leads to an increase of the hydrostatic piezoelectric coefficients $d_h^* = d_{33}^* + 2d_{31}^*$ and $g_h^* = g_{33}^* + 2g_{31}^*$ when compared to the $d_h$ and $g_h$ parameters of the monolithic counterpart. Moreover, an increase of $g_h^*$ of the PSL compared to the monolithic $g_h$ is also a result of the dielectric properties that strongly depend on porosity $v_p$ of the PSL structure. Examples of the experimental dependence of the piezoelectric coefficients on the bulk relative density $\alpha = 1 - v_p$ are given in Table 1. According
to experimental data [21], the monolithic poled BaTiO$_3$ FC is characterised by hydrostatic piezoelectric coefficients $d_h = 34$ pC / N and $g_h = 2.26$ mV m / N. As follows from measurements on the monolithic poled BaTiO$_3$ FC samples in the present study, their piezoelectric coefficients are $d_{33} = 124.8$ pC / N and $d_{31} = -47.8$ pC / N and relative dielectric permittivity at 1 kHz is $\varepsilon_{33}^{\sigma} / \varepsilon_0 = 1504$ (average values). This means that the hydrostatic piezoelectric coefficients are $d_h^s = 29.2$ pC / N and $g_h^s = 2.19$ mV m / N, i.e., in agreement with the hydrostatic parameters from Ref. 21. For the studied PSL samples, the typical ratios $d_h^s / d_h \approx 2–3$ and $g_h^s / g_h \approx 4.1–9.5$ are achieved, see experimental values in Table 1.

3. Modelling and interpretation of the piezoelectric and dielectric properties of the PSL BaTiO$_3$

Recently, the modelling of the longitudinal piezoelectric response of the PSL BaTiO$_3$ [7] showed that incomplete poling of the porous ferroelectric material had a strong influence on its piezoelectric activity and related parameters. Our current efforts are concentrated on describing the piezoelectric and dielectric properties of PSL materials by taking into account features of their porosity and domain-wall mobility in their FC grains and to interpret the influence of microgeometric factors on the piezoelectric anisotropy.

The PSL structure is assumed to contain porous and monolithic FC layers poled along the co-ordinate $OX_3$ axis, see figure 1(b). In the porous layer, the FC matrix contains isolated spheroidal air pores, see the inset of figure 1(b). It is
assumed that these pores are regularly distributed in the porous layer of the PSL. The shape of the air pore is described by the equation \((x_1 / a_{1p})^2 + (x_2 / a_{2p})^2 + (x_3 / a_{3p})^2 = 1\) relative to the axes of the co-ordinate system \((X_1X_2X_3)\), where the semiaxes of the pore are \(a_{1p} = a_{2p}\) and \(a_{3p}\), and \(\rho_p = a_{1p} / a_{3p}\) is its aspect ratio. The porous matrix shown in the inset in figure 1(b) is described by 3–0 connectivity in terms of work by Newhnam et al. [22]. This porous matrix is reinforced by a system of monolithic FC rods that are oriented along the \(OX_3\) axis. This particular orientation of the FC rods facilitates their poling under an applied electric field \(E \parallel OX_3\) and promotes an effective electromechanical coupling between the layers that are shown in figure 1(b). We assume that each FC rod is in the form of a long circular cylinder which is characterised by a considerable mobility of the 90° domain walls in each FC grain in the poled state. The porous matrix surrounding the monolithic FC rods leads to a decrease of the mechanical stress during the 90° domain switching and can be regarded as an additional argument for the considerable domain-wall mobility. Changes in the parameters \(m_p\) and \(\rho_p\) of the porous matrix enable us to vary the elastic anisotropy of the matrix and porous layer as a whole. This factor influences [8, 13] the piezoelectric properties and their anisotropy in the PSL structure.

The porous layer shown in figure 1(b) is characterised by 1–3–0 connectivity, and its electromechanical properties depend on the volume fractions \(r\) and \(m_p\) and domain-wall mobility in the FC rods. The outer monolithic layer of the PSL sample is represented by the same FC, but with a lower domain-wall mobility in FC grains.
in comparison to the domain-wall mobility in the FC-rod grains. This lower domain-wall mobility may be a result of the perpendicular orientation of the monolithic layer with respect to the poling direction $OX_3$, see figure 1(b).

The $90^\circ$ domain-wall mobility in the FC grains can be taken into account in terms of work [23]. According to Aleshin [23], it is assumed that during the poling stage of the FC sample, the $180^\circ$ domains are removed. As a result, each grain is assumed to be split into the $90^\circ$ domains that are separated by planar walls, and the $90^\circ$ domain-wall displacements are caused by an external field, which is either electric or stress based. The electromechanical properties of the poled FC depend on $\lg \gamma$ that characterises the mobility of the $90^\circ$ domain walls, where $\gamma = (Hc)^{-1} \cdot 10^{-6}$ Pa, $H$ is the average width of the $90^\circ$ domain, and $c$ links the domain-wall displacement $x$ and thermodynamic pressure $f$ in accordance with the linear relation $f = cx$. Full sets of electromechanical constants of the monolithic poled BaTiO$_3$ FC were calculated by Aleshin [23] in the range $-4 \leq \lg \gamma \leq 4$. The value of $\lg \gamma = -4$ corresponds to almost immovable $90^\circ$ domain walls in the FC grain, and at $\lg \gamma = 4$ the highest mobility of the $90^\circ$ domain walls is expected. At $-4 \leq \lg \gamma \leq 0$, an appreciable increase of elastic compliances $|s_{ab}^E|$, piezoelectric coefficients $|d_{ij}|$ and dielectric permittivities $\varepsilon_{pp}^\sigma$ is observed. For instance, the piezoelectric coefficient $d_{33}$ of the poled BaTiO$_3$ FC monotonously increases from $-75$ to $158$ pC / N. At $0 < \lg \gamma \leq 4$, the properties of the poled BaTiO$_3$ FC undergo smaller changes [23].
It should be added that the concepts [23] were highlighted in work [24] on the 1–3-type FC / auxetic polymer composites and their piezoelectric performance. The full sets of electromechanical constants of the poled BaTiO$_3$ FC at $-4 \leq \lg \gamma \leq 4$ [23] were first applied [24] to find the piezoelectric coefficients $d_{33}^*$ and $g_{33}^*$, and their hydrostatic analogs $g_{33}^*$ and $g_h^*$ of the 1–3-type composites. These parameters undergo minor changes at $0 < \lg \gamma \leq 4$ in FC grains and at constant volume fractions of the FC component in the composite.

e.g. $d_{33} \approx 160$ pC / N.

The prediction of the effective electromechanical properties and energy-harvesting FOMs from Eqs. (2) and (3) is carried out in three stages as follows. In the first stage, the effective properties of the porous 3–0 non-poled FC matrix [see the inset in figure 1(b)] are evaluated as a function of $m_p$ and $\rho_p$ within the framework of the dilute approach [6, 12], and an interaction between the air pores is neglected. The matrix of the effective properties of the medium with spheroidal air pores is written as

$$|| C^{(m)} || = || C^{(1)} || [ || I || - m_p ( || I || - (1 - m_p) || S || )^{-1} ]. \tag{4}$$

In Eq. (4) $|| C^{(1)} ||$ is the $9 \times 9$ matrix that describes the properties of the monolithic FC, $|| I ||$ is the $9 \times 9$ identity matrix, $|| S ||$ is the $9 \times 9$ matrix containing components of the Eshelby electroelastic tensor [8, 25], and $m_p$ is the volume fraction of the air pores, see the inset in figure 1(b). Elements of $|| S ||$ depend [25] on the aspect ratio $\rho_p$ of the air pore and on the properties of the monolithic FC medium that surrounds the pores. The $|| C^{(1)} ||$ matrix from Eq. (4) is given by

$$|| C^{(1)} || = \begin{pmatrix} || C^{(1),E} || & || e^{(1)} ||^t \\ || e^{(1)} || & - || E^{(1),E} || \end{pmatrix}, \tag{5}$$
where \( \| c^{(1),E} \| \) is the \( 6 \times 6 \) matrix of elastic moduli measured at \( E = \text{constant} \), \( \| e^{(1)} \| \) is the \( 6 \times 3 \) matrix of piezoelectric coefficients, and \( \| e^{(1)},\xi \| \) is the \( 3 \times 3 \) matrix of dielectric permittivities measured at mechanical strain \( \xi = \text{constant} \). The superscript \( t \) in Eq. (5) denotes the transposition. Because of our assumption on the non-poled FC matrix in the porous layer of the PSL structure, the \( \| e^{(1)} \| \) and \( \| e^{(1)} \|^t \) matrices from Eq. (5) contain zero elements only. The \( \| C^{(m)} \| \) matrix from Eq. (4) has the form shown in Eq. (5).

In the second stage, the effective (averaged) properties of the porous layer of the PSL structure are evaluated using the effective field method [8]. In this case we consider the electromechanical interaction between the monolithic FC rods, and the effective properties of the porous layer as a whole (1–3–0 composite) depend on the volume fraction \( r \) of the monolithic FC rods and parameter \( m_p \) and \( \rho_p \) of the porous matrix that surrounds these rods. The evaluation of the effective properties of the porous layer is carried out on assumption that the linear sizes of each pore are much smaller than the radius of the FC rod, i.e., the system of FC rods is surrounded by the porous medium with a set of effective constants. The effective properties of the porous layer are represented in accordance with the effective field method as

\[
\| C^{(pl)} \| = \| C^{(m)} \| + r (\| C^{(FC)} \| - \| C^{(m)} \|) \| I \| + (1 - r) \| S^{(m)} \| \| C^{(m)} \|^{-1} (\| C^{(FC)} \| - \| C^{(m)} \|))^{-1}.
\]

In Eq. (6) the matrices of the properties \( \| C^{(FC)} \| \) (poled FC rods) and \( \| C^{(m)} \| \) [porous non-poled FC matrix, see figure 1(b)] have the form shown in Eq. (5), \( r \) is the volume fraction of the poled FC rods in the porous layer of PSL material, \( \| I \| \) is the
identity matrix, and \( \| S^{(m)} \| \) is the matrix that contains components of the Eshelby electroelastic tensor [25] of the non-poled porous FC medium. Elements of \( \| S^{(m)} \| \) depend on the shape of the FC rod and on the properties of the porous FC medium that surrounds each FC rod.

The third stage is concerned with averaging of the properties of the monolithic and porous layers on the volume fraction \( m \) [see figure 1(b)], and during this averaging, we take into account the electromechanical interaction between the piezo-active layers and boundary conditions [8] for mechanical and electric fields at the interface \( x_3 = \text{constant} \) [see figure 1(b)]. The boundary conditions at \( x_3 = \text{constant} \) imply a continuity of components of mechanical stress \( \sigma_{13}, \sigma_{23} \) and \( \sigma_{33} \), strain \( \xi_{11}, \xi_{12} \) and \( \xi_{22} \), electric displacement \( D_3 \), and electric field \( E_1 \) and \( E_2 \). Thus, the effective electromechanical properties of the PSL sample shown in figure 1(b) are given by the \( \| C^* \| \) matrix that depends on \( m_p, \rho_p, r, m, \lg \gamma_1, \) and \( \lg \gamma_2 \), where \( \gamma_1 \) refers to the domain-wall mobility in grains of the FC rod, and \( \gamma_2 \) refers to the domain-wall mobility in grains of the monolithic FC layer. The \( \| C^* \| \) matrix has the form shown in Eq. (5) so that the PSL FC is characterised by the following electromechanical constants: \( e^{*E}_{ab}, \ e^*_{ij} \) and \( \varepsilon^{*s}_{vv} \). Taking the parameters from figure 1(b) into account, we represent the relative bulk density of the PSL sample as \( \alpha = 1 - (1 - r)mm_p \). A transition from the effective piezoelectric coefficients \( e^*_{ij} \) to \( d^*_{ij} \) and \( g^*_{ij} \) is carried out by using conventional formulae [26] for the piezoelectric medium.
Results of our evaluations are shown in figures 3–5 and Tables 1 and 2. In general, the graphs from figure 3 are consistent with the experimental $d_{3j}^\gamma(\alpha)$ dependence shown in figure 2(a). Hereby the porous layer of PLS is characterised by an inner layer porosity $p_{in} = (1 - r)m_p$, see figure 1(b). At $m_p = 0.6$ we obtain $p_{in} = 0.54, 0.51$ and $0.48$ for $r = 0.10, 0.15$ and $0.20$, respectively, and these $p_{in}$ values can correspond to PSL-1. At $m_p = 0.7$ we have $p_{in} = 0.63, 0.595$ and $0.56$ for $r = 0.10, 0.15$ and $0.20$, respectively, and these $p_{in}$ values can correspond to PSL-2. The graphs in figure 3 suggest that even relatively small changes in the volume fraction $r$ of the poled FC rods in the porous layer can lead to the distinct changes in the piezoelectric coefficients $d_{3j}^\gamma$ of the PSL BaTiO$_3$ and also influence the anisotropy of $d_{3j}^\gamma$. On increasing $r$, the anisotropy of $d_{3j}^\gamma$ becomes smaller, and this trend is caused by the small anisotropy of $d_{3j}$ of the monolithic BaTiO$_3$ FC at various values of $\gamma$. In the present study, we consider the situation when the $90^\circ$ domain-wall mobility in the grains of the FC rods is more than that in the monolithic layer of PSL, i.e., the condition $\gamma_1 > \gamma_2$ holds. Comparing figure 3(a) to figure 3(c) and figure 3(b) to figure 3(d), we see that the anisotropy of $d_{3j}^\gamma$ becomes smaller on decreasing the aspect ratio $\rho_p$ of the air pore. This decrease strongly influences the elastic properties of the porous FC matrix shown in the inset of figure 1(b) and leads to changes in the piezoelectric coefficients $d_{3j}^\gamma$ of the PSL BaTiO$_3$. The mutual arrangement of curves 1–3 (piezoelectric coefficient $d_{3j}^\gamma$) in figure 3(b) differs from the arrangements in the remaining graphs of figure 3. This
can be accounted for by a specific competition of the piezoelectric properties of the porous and monolithic layers at a relatively small difference between the piezoelectric coefficients $d_{33}$ of these layers. The competition is related to the PSL where the matrix porosity level is high ($m_p = 0.7$) and the pores are highly oblate ($\rho_p = 100$). Such a porous structure leads to a large elastic anisotropy that also influences the piezoelectric properties of the porous layer and PSL structure as a whole. A simple comparison of figure 3(a) and (b) enables us to state that for the smaller matrix porosity $m_p = 0.6$, a crossing of the curves 1–3 is also observed, but at higher relative densities $\alpha$ of the PSL structure compared to $m_p = 0.7$. We observe minor changes of the $d_{33}^*$ values [cf. curves 1–3 in figure 3(a) and (b)], and therefore, the aforementioned competition of the piezoelectric properties of the layers is effective.

The graphs in figure 4 show that the evaluated $\varepsilon_{33}^{*\sigma}(\alpha)$ dependence correlates with the experimental data from figure 2(b), however there are differences between the calculated and experimental $\varepsilon_{33}^{*\sigma}$ values. The main reason for these differences is the regular porous structure at $m_p = \text{constant}$ and $\rho_p = \text{constant}$, see the inset in figure 1(b). The next reason may be concerned with features of the monolithic poled FC rods and their regular arrangement in the porous matrix: in the present study we consider the system of the long cylindrical FC rods oriented parallel to the poling axis only. The interfaces that separate the monolithic and porous layers in the PSL sample are planar [see figure 1(b)], however any deviation from the
ideal planar interface could lead to changes in the electromechanical coupling between the layers and influence the piezoelectric and dielectric properties of the PSL sample as a whole. It should be added that recently, Wang et al. [27] studied sandwich-structured polymer BaTiO$_3$-based composites wherein the interfaces between adjacent layers strongly influence the dielectric properties, breakdown strength and other characteristics of these materials. A minor reason for the differences between the calculated and experimental $\varepsilon^{x\sigma}_{33}$ values [see figures 2(b) and 4] consists in the poling degree of the porous and monolithic layers within the PSL material. We assume that its porous matrix is fully non-poled, and each grain of the monolithic FC rod is characterised by the domain-wall mobility $\gamma_1$ that is constant over the whole porous layer. By analogy with the FC rod, the domain-wall mobility of the monolithic FC layer is $\gamma_2 = \text{constant}$. Despite the aforementioned reasons, we state that the columnar FC structure in the porous layer represents reliable polarisation paths though the porous inner layer and strongly influences its piezoelectric and dielectric properties (figures 3 and 4).

To compare the calculation results, we used analytical formulae for effective electromechanical properties of the 1–3 [28] and series-connected 2–2 [15] composites based on FCs. The formulae from work [28] were applied to find the full set of electromechanical constants of the porous layer of the PSL on assumption that the properties of the porous matrix were previously determined by using Eq. (4). The formulae from monograph [15] were applied to the laminar structure [figure 1(b)] wherein the electromechanical interaction between the
monolithic and porous layers was taken into consideration. Hereby we state good agreement between the effective properties found for the PSL by different methods: a difference between the similar parameters calculated in different ways is less than 3%.

Based on the piezoelectric and dielectric properties of the PSL BaTiO$_3$, we show the dependence of its squared FOMs $(Q^*_{3j})^2$ on the relative density $\alpha$ (figure 5). Due to validity of Eq. (1), the condition

$$(Q^*_{33})^2 \gg (Q^*_{31})^2$$

holds. On increasing the volume fraction $r$ and on decreasing the volume fraction $m_p$, we observe decreasing $(Q^*_{3j})^2$ that is caused by the considerable influence of the piezoelectric coefficients $g^*_{3j}$ on the squared FOMs, see Eqs. (2) and (3). Based on the electromechanical constants of the poled BaTiO$_3$ FC [21], we evaluate its squared FOMs $(Q_{33})^2 = 2.40$ and $(Q_{31})^2 = 0.404$ (in $10^{-12}$ Pa$^{-1}$). The squared FOMs evaluated from experimental data on the monolithic FC samples used in the present study are $(Q_{33})^2 = 1.17$ and $(Q_{31})^2 = 0.172$ (in $10^{-12}$ Pa$^{-1}$). Thus, the studied PSL structure enables us to increase $(Q^*_{33})^2$ by a few times in comparison to the poled monolithic BaTiO$_3$ FC, and the obtained $(Q^*_{3j})^2$ values obey the condition (7) in the wide $\alpha$ range.

In Table 2 we show the performance of the PSL BaTiO$_3$ with a small difference between $\gamma_1$ and $\gamma_2$, i.e., when the domain-wall mobility in grains of the monolithic FC components undergoes small changes. It is seen that the
piezoelectric coefficient $d_{33}^*$ is overestimated even at a volume fraction $r \leq 0.1$ of the FC rods, and no $d_{33}^*(\alpha)$ dependence is observed among the calculated values. An average value of $d_{31}^*$ related to $r = 0.10$ and 0.05 is in good agreement with the experimental $d_{31}^*$ value [see figure 2(a)] when $\alpha$ is constant. The dielectric permittivity $\varepsilon_{33}^{*\sigma}$ is smaller than that measured on the PSL sample [see figure 2(b)]. The large $d_{33}^*$ values related to PSL BaTiO$_3$ at $r \leq 0.1$ (see Table 2) suggest that the domain-wall mobility $\gamma_2$ of the monolithic layer is to be decreased at the unchanged characteristics of the porous layer.

4. Conclusions

This paper describes a detailed experimental and modelling analysis of the piezoelectric properties, dielectric properties and energy-harvesting parameters of novel porous sandwich layer (PSL) structures based on BaTiO$_3$ containing dense outer layers and inner layer with a porosity of $p_{in} \approx 0.5–0.6$. The experimental study of the microgeometry and physical properties of this lead-free ferroelectric material demonstrate that conditions in Eqs. (1) and (7) are valid in a wide range of the relative density ($\alpha$). The ability of the structure to achieve a large piezoelectric anisotropy with a high $d_{33}^* \approx 100$ pC / N suggests that this lead-free piezoelectric material has obvious advantages over conventional highly anisotropic PbTiO$_3$-type FCs. The $d_{3j}^*(\alpha)$ and $\varepsilon_{33}^{*\sigma}(\alpha)$ dependences are interpreted in terms of a model of a laminar structure [figure 1(b)] whereby the electromechanical interaction of the
porous and monolithic layers play a key role in forming the beneficial electromechanical properties and their anisotropy. The porous layer is represented as a piezocomposite with 1–3–0 connectivity, and the system of the monolithic FC rods strongly influences the electromechanical properties of this layer. The difference between the 90° domain-wall mobility levels in grains of the FC components of PSL structures and the presence of the heavily oblate air pores are to be taken into account at the prediction of the piezoelectric and dielectric properties, hydrostatic parameters and squared FOMs. The studied PSL BaTiO₃ is characterised by the squared FOM \((Q_{33}^*)^2 \sim 10^{-12} \text{ Pa}^{-1}\) under longitudinal excitation and by the hydrostatic piezoelectric coefficients \(d_{h}^* \approx 100 \text{ pC} / \text{ N}\) and \(g_{h}^* \approx 20 \text{ mV m} / \text{ N}\) at \(\alpha = 0.64–0.70\). The new type of ferroelectric structure proposed here that is based on a lead-free material provides a fascinating route for the design of ferroelectric structures for piezoelectric sensor, energy-harvesting and related transducer applications.

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Systems (NEMESIS). Mr. J. I. Roscow acknowledges funding from Engineering and Physical Sciences Research Council (EPSRC), UK. In the present paper, the results on the research project no. 11.1627.2017/4.6 PCh have been represented within the framework of the state task in the scientific activity area at the Southern Federal University, and Prof. Dr. V. Yu. Topolov acknowledges relevant funding. This research has been performed using the equipment of the Centre of Collective Use ‘High Technologies’ at the Southern Federal University.
References


Appendix. List of symbols and abbreviations

\( a_{1p}, a_{2p} \) and \( a_{3p} \), semiaxes of the pore

\( c \), coefficient that links the \( 90^\circ \) domain-wall displacement and thermodynamic pressure

\( \| c^E \| \) is the matrix of elastic moduli measured at electric field \( E = \) constant

\( \| C^{(FC)} \| \), matrix of electromechanical properties of poled ceramic rods

\( \| C^{(m)} \| \), matrix of electromechanical properties of the poled ceramic medium with spheroidal pores

\( \| C^{(1)} \| \), matrix of electromechanical properties of the monolithic ceramic

\( D \), electric displacement

\( d_h \), hydrostatic piezoelectric coefficient

\( d_{ij} \), piezoelectric coefficient

\( E \), electric field

\( \| e^{(1)} \| \), matrix of piezoelectric coefficients of the monolithic ceramic

\( e_{ij} \), piezoelectric coefficient

\( g_h \), hydrostatic piezoelectric coefficient

\( g_{ij} \), piezoelectric coefficient

\( H \), average width of the \( 90^\circ \) domain

\( \| I \| \) identity matrix

\( m \), volume fraction of the porous layer, see figure 1(b)

\( m_p \) volume fraction of air pores in the ceramic medium, see figure 1(b)
\( p_{in}, \) inner layer porosity

\((Q_{31})^2,\) squared figure of merit concerned with the transverse piezoelectric effect

\((Q_{33})^2,\) energy-harvesting figure of merit, squared figure of merit concerned with the longitudinal piezoelectric effect

\( r,\) volume fraction of poled ceramic rods in the porous layer, see figure 1(b)

\( \| S \|,\) matrix containing components of the Eshelby electroelastic tensor of the poled ceramic medium

\( \| S^{(m)} \|,\) matrix containing components of the Eshelby electroelastic tensor of the non-poled porous ceramic medium

\( s_{ab}^E,\) elastic compliance at electric field \( E = \text{constant} \)

\( t,\) superscript that denotes the matrix transposition

\( v_p,\) porosity of the porous sandwich layer structure, see figure 1(b)

\( \alpha,\) bulk relative density of the porous sandwich layer structure

\( \gamma,\) factor that characterises the \( 90^\circ \) domain-wall mobility in ceramic grains; \( \gamma_1 \) refers to the domain-wall mobility in grains of the ceramic rod, and \( \gamma_2 \) refers to the domain-wall mobility in grains of the monolithic ceramic layer [see figure 1(b)]

\( \varepsilon_0,\) dielectric permittivity of free space \( (\varepsilon_0 = 8.854 \times 10^{-12} \text{ F} \text{ m}^{-1}) \)

\( \| \varepsilon^{(1),\xi} \|,\) matrix of dielectric permittivities of the monolithic ceramic at mechanical strain \( \xi = \text{constant} \)

\( \varepsilon_{pp}^\sigma,\) dielectric permittivity at mechanical stress \( \sigma = \text{constant} \)

\( \rho_p,\) aspect ratio of the air pore
σ, mechanical stress

ξ, mechanical strain

1–3–0, 1–3, 3–0, and 2–2, connectivity indexes of composites

Asterisk (*) is used to show effective properties and parameters of the porous sandwich layer or composite

FC, ferroelectric ceramic

FOM, figure of merit

PEG, polyethylene glycol

PSL, porous sandwich layer; PSL-1 and PSL-2 are related to inner layer porosity

$p_{in} \approx 0.5$ and 0.6, respectively

SEM, scanning electron microscopy
Table 1. Experimental and calculated values of piezoelectric coefficients $d_{33}^*$, $d_h^*$ (in pC / N), $g_{33}^*$, and $g_h^*$ (in mV m / N) of PSL BaTiO$_3$ at $\alpha = \text{const}$

<table>
<thead>
<tr>
<th>Sample</th>
<th>PLS-1</th>
<th>PLS-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.70</td>
<td>0.72</td>
</tr>
<tr>
<td>$d_{33}^*$, experimental</td>
<td>110</td>
<td>122</td>
</tr>
<tr>
<td>$d_h^*$, experimental</td>
<td>84.1</td>
<td>89.3</td>
</tr>
<tr>
<td>$g_{33}^*$, experimental</td>
<td>19.6</td>
<td>18.6</td>
</tr>
<tr>
<td>$g_h^*$, experimental</td>
<td>15.0</td>
<td>13.6</td>
</tr>
<tr>
<td>$d_{33}^*$, calculated</td>
<td>132</td>
<td>131</td>
</tr>
<tr>
<td>$d_h^*$, calculated</td>
<td>91.4</td>
<td>89.8</td>
</tr>
<tr>
<td>$g_{33}^*$, calculated</td>
<td>26.1</td>
<td>25.1</td>
</tr>
<tr>
<td>$g_h^*$, calculated</td>
<td>18.1</td>
<td>17.1</td>
</tr>
</tbody>
</table>

Note. Calculations of the piezoelectric coefficients were performed within the framework of the model shown in figure 1(b) and by using the averaging procedures described in Section 3. Hereby the following parameters were used: $m_p = 0.6$, $\rho_p = 100$, $r = 0.20$, $\lg \gamma_1 = 0$, and $\lg \gamma_2 = -4$ (for PSL-1), and $m_p = 0.7$, $\rho_p = 100$, $r = 0.20$, $\lg \gamma_1 = 0$, and $\lg \gamma_2 = -4$ (for PSL-2).
Table 2. Experimental and calculated values of piezoelectric coefficients $d_{3j}^*$ (in pC / N) and relative dielectric permittivity $\varepsilon_{33}^{*\sigma}$ of PSL-2 BaTiO$_3$ at $\alpha = \text{const}$, $\lg \gamma_1 = 0$, and $\lg \gamma_2 = -1$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$d_{33}^*$</th>
<th>$d_{31}^*$</th>
<th>$\varepsilon_{33}^{*\sigma} / \varepsilon_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experimental</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.66</td>
<td>121</td>
<td>−10.0</td>
<td>534</td>
</tr>
<tr>
<td>0.72</td>
<td>114</td>
<td>−11.8</td>
<td>631</td>
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<tr>
<td>0.82</td>
<td>109</td>
<td>−17.9</td>
<td>886</td>
</tr>
<tr>
<td><strong>Calculated at $m_p = 0.7$, $\rho_p = 100$ and $r = 0.10$ (inner layer porosity $p_{in} = 0.63$)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.66</td>
<td>136</td>
<td>−13.2</td>
<td>374</td>
</tr>
<tr>
<td>0.72</td>
<td>136</td>
<td>−15.1</td>
<td>431</td>
</tr>
<tr>
<td>0.82</td>
<td>134</td>
<td>−19.8</td>
<td>568</td>
</tr>
<tr>
<td><strong>Calculated at $m_p = 0.7$, $\rho_p = 100$ and $r = 0.05$ (inner layer porosity $p_{in} = 0.57$)</strong></td>
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<td>0.66</td>
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<tr>
<td>0.72</td>
<td>124</td>
<td>−8.1</td>
<td>252</td>
</tr>
<tr>
<td>0.82</td>
<td>124</td>
<td>−11.3</td>
<td>345</td>
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</table>
Figure 1. Optical micrograph of PSL BaTiO$_3$ (a) and schematic (b) that is used to describe the electromechanical properties and related parameters of the PSL structures. In the micrograph (a), the porous layer is located between the monolithic FC layers. The PSL sample as a whole has been poled in the vertical direction. In the schematic (b), ($X_1X_2X_3$) is the rectangular co-ordinate system, $m$ and $1 - m$ are volume fractions of the porous and monolithic layers, respectively, $r$ is the volume fraction of monolithic FC rods in the porous layer, and $m_p$ is the volume fraction of air pores in the matrix (see the inset) that surrounds the monolithic FC rods in the porous layer.
Figure 2. Room-temperature piezoelectric coefficients $d_{3j}^*$ and relative dielectric permittivity $\varepsilon_{33}^*/\varepsilon_0$ as functions of the bulk relative density $\alpha$ of PSL BaTiO$_3$ at the the inner layer porosity $p_{in} \approx 0.5$ and 0.6. The relevant layer is shown in the middle part of figure 1(a) and termed ‘porous layer’ in figure 1(b).
Figure 3. Evaluated piezoelectric coefficients $d^*_{3j}$ (in pC / N) as functions of the bulk relative density $\alpha$ of PSL BaTiO$_3$ at fixed volume fractions $r$ of the poled FC rods in the porous layer [see the schematic in figure 1(b)].
Figure 4. Evaluated relative dielectric permittivity $\varepsilon_{33}^{*}/\varepsilon_0$ as a function of the bulk relative density $\alpha$ of PSL BaTiO$_3$ at fixed volume fractions $r$ of the poled FC rods in the porous layer [see the schematic in figure 1(b)].
Figure 5. Evaluated squared FOM $(Q_{33}^*)^2$ (in $10^{-12}$ Pa$^{-1}$) as a function of the bulk relative density $\alpha$ of PSL BaTiO$_3$ at fixed volume fractions $r$ of the poled FC rods and $m_p$ of air pores in the porous layer [see the schematic in figure 1(b)].