Optimal guidelines for restricted tendering

ANTONIO NICOLO *  Elnaz Bajoori †

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Abstract

In this paper we study a delegated search model in a principal agent framework without monetary transfers. The agent is delegated to buy an object that can have either low or high quality and high quality objects are costlier. The agent knows which quality is needed while the principal only knows the price distributions for each quality, and cannot observe the quality of the object purchased by the agent. Since the principal pays the price of the object, the agent is only interested in minimizing the search cost that she sustains. The principal can only decide which search rule to adopt, without having the possibility to use contingent monetary transfers to incentivize the agent to search optimally. We characterize the optimal search rule within a class in which the principal may impose a minimum number of searches to the agent.

JEL Codes. C70, D82, D86.

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*antonio.nicolo@manchester.co.uk. School of Social Sciences, U. of Manchester, Manchester, UK.
†e.bajoori@bath.ac.uk. University of Bath, Dept. of Economics, Bath, UK.
1 Introduction

Public procurement represents an important share of the economy (about 12.1% of GDP and 29% of total general government expenditures in the OECD (OECD, 2015)). Moreover, the government is the largest buyer in many countries. Several procurement methods are used to purchase goods and services from auctions to restricted tendering and direct contracting, which involves different degree of competition. Procurement methods play a key role both for the quality and cost of government purchases. Public procurement does not only represent an important share of the government expenditure, but it also involves a relevant cost for public authorities that have to run the procurement process, evaluate tenders correctly against the relevant criteria, or select a number of suppliers, contractors or service providers. The optimal design of procurement methods should also take into account these indirect costs for the public administration.

In this paper we focus on the optimal design of a widespread method of public procurement, named restricted or selective tendering. Typically, in order to use this method, public entities need to provide a justification, explaining and documenting the need to do so. Possible justifications include cases in which only few suppliers exist, such as emergencies and cases in which organizing more competitive method like auctions would represent a disproportionate cost. Procurement officers are usually required to get a minimum number of quotes from suppliers, before awarding the contract and this number varies often depending on the value of the purchase. The Internal Purchasing Policy of the EBRD (European Bank for Reconstruction and Development), for instance, requires that purchasing for contracts estimated to cost 50,000 or more and less than 150,000 should be executed through selective tendering by issuing a written solicitation to a minimum of five suppliers. The Procurement & Supplier Diversity Services of the University of Virginia has to solicit a minimum of six quotations for any purchase of 50,000$ or above, and a minimum of three for purchases from 5,000$ to 50,000$. Gerardino et al (2017) show that in Chile where a government agency (Chilecompra) is in charge of managing the entire public procurement system (representing about 4% of the country GDP) restricted tendering is still used for a share equal to 17% of total value of purchase, with about 60% of contracts
requiring only one quote and 40% requiring three quotes.

A common interpretation of the rationale for the requirement of a “minimum number of quotes”, is that it aims to fight corruption or favouritism imposing a minimum level of competition and transparency in the tendering process. However, to impose a minimum number of quotes can hardly be considered an effective way to avoid corruption, if the procurement officer can strategically select which other quotes to submit when he wants to favour a specific contractor.

Our paper offers a different and novel explanation to rationalize the constraint of a minimum number of quotes, focusing on the agency problem within the public administration that runs the procurement process. We present a model of delegated search with moral hazard and adverse selection in which the agent (the public entity that has to purchase some goods or services) and the principal (the government) have correlated preferences. The principal can impose a search rule, but cannot use monetary transfers to incentivize or punish the agent. This model perfectly fits the problem of public procurement but it could also be applied in other contexts, like hiring in public administration. In our model there are two types of agents, (High and Low) who differ in the quality of the good, high quality and low quality, respectively, they need to purchase. Goods of different qualities have different price distributions, with high type goods being more expensive than low type goods. While high type agents need a high quality good, low quality agents are indifferent with respect the quality of the good they purchase. Principal and agent have aligned interests except as concerning the price of the good, which is fully paid by the principal (which again is an assumption that fits the case of public procurement) Since the agent bears a constant (non-monetary) search cost for each quote she asks for, but she does not pay the purchasing price, she always prefers to stop searching as soon as possible.

In the symmetric information case, in which the principal observes agent’s type, and knows the price distribution for each quality, she can impose to each type to buy the quality she really needs and to follow the optimal (for the principal) search rule. If the search cost is constant, the optimal rule is a stopping rule with a threshold: the agent can stop searching when she finds a price lower
than a given threshold. Under the assumption that high quality goods are on average more expensive than low quality goods, the optimal threshold for high type agents is higher than the optimal threshold for low type agents.

In the asymmetric information case, when the principal does neither observe the agent’s type nor the quality of the purchased good, a low type agent could find it profitable to report to be a high type agent, to be allowed to buy an expensive low quality good so that to save on her search cost. It follows that the first best rule that assigns a different threshold to each type is not incentive compatible. To solve this problem, we consider a class of search rules that consists of a minimum number of searches and a threshold. The possibility of a minimum number of searches helps the principal to restore the incentive compatibility at the cost of inducing an overprovision of effort to the agents. We characterize the optimal search rule within this class. We show that under some conditions it is optimal for the principal to propose a menu of incentive compatible search rules that imposes to a high type agent a minimum number of searches. Namely, a low type agent can stop searching when she finds a price lower than her first best threshold, while the high type agent can stop searching if she finds a price lower than her first best threshold, and she has performed a minimum number of searches.

Since the 80’s economists have studied procurement as an agency problem between the buyer and the supplier who has private information about production costs or quality of the goods (see for an excellent survey Laffont and Tirole 1993). More recently, a series of papers beginning with Bajari and Tadelis (2001) following an approach that blends agency theory with transaction cost, study the optimal design of procurement contracts when contractual specifications are inherently incomplete. In this paper we focus on a previously unexplored agency problem within the public administration that runs the public procurement process, and we study how a specific procurement method, the restrictive tendering, can be designed to solve this problem.

There is a growing literature on delegated search to which this paper offers a contribution. In Armstrong and Vickers (2010) [2], the principal delegates an
agent to select a project and can only decides the characteristics of the admissible projects. Mauring (2016) [9] instead, focuses on the agent’s optimal policy in the case the principal cannot affect the search process directly. In her model the agent selects a set of alternatives and the principal picks her preferred one, so she analyzes the optimal stopping rule for the agent, knowing which final alternative the principal will pick from the set she proposes. Kováč et al. (2013) [7] study optimal stopping rules when the principal lacks relevant information but can consult with a better informed agent, while the exchange of contingent monetary transfers is infeasible; principal’s utility from stopping depends on the state that can take only two values, either H or L: conflict of interests, as in our paper, arises because the principal prefers to stop only in state H, while the agent has always interest in stopping. In their setting, a stopping rule is such that the principal commits to a deadline. Until the deadline, the agent can make at most one proposal to stop. If the agent makes a proposal, it is accepted with some probability.

Other papers (Postl (2004) [10], Lewis (2012)[8], and Ulbricht (2016) [11]), assume transferable utility and focus on the principal’s optimal contracts. Ulbricht (2016) [11] in particular shows that when the distribution of search revenues is unknown to the principal, and the search process is unobservable, search is almost surely inefficient (it is stopped too early) and second best remuneration is shown to optimally utilize a menu of simple bonus contracts.

Finally, another recent and related strand of literature (Albrecht et al.(2010) [1], Bergemann andVälimäki (2011) [3], Compte and Jehiel (2010) [4], Guler et al (2012) [5] among others), study non-homogeneous search committees that jointly decide over the continuation of a search process.

## 2 Model

We study a principal-agent model in which the agent needs an object of a certain quality paid fully by the principal. The principal delegates to the agent the search for the object. There are two types of agents, high type (H-agent) and low type (L-agent), depending on the quality of the object they need to carry on their task, high (H) quality and low (L) quality object, respectively. We model the
situation as follows: There are two different groups of sellers to whom the agent may ask for a quote. Each seller sells an object at a given price. High (low) quality sellers sell high (low) quality objects; for simplicity of exposition and generality, we refer to the two groups of sellers as two different boxes from which the agent draws prices. Each box contains an infinite number of objects of the same quality, and, abusing notation, we denote $H$-box and $L$-box, the two boxes containing high quality and low quality objects, respectively. Let $F^a(p)$ denote the probability distribution of prices in box $a \in \{H, L\}$, which is common knowledge. For an $H$-type agent, objects in the $H$-box have a positive value $\bar{V}$, while objects in the $L$-box have a value equal to zero. For an $L$-type agent all objects, those in the $H$-box and those in the $L$-box have a positive value equal to $V$, where $\bar{V} \geq V > 0$.

Let $c$ be the cost of a search (a draw from the box), and $k$ be the number of searches. An agent of type $a \in \{H, L\}$ has the utility

$$U^a = V^a(\hat{a}) - ck,$$

where $V^a(\hat{a})$ denote the value of an object of type $\hat{a}$ for an agent of type $a$, and therefore, by assumption, $V^H(L) = 0, V^H(H) = \bar{V}$ and $V^L(L) = V^L(H) = V$ with $\bar{V} \geq V > 0$.

It is immediate to notice that the agent does not pay the price so she aims to minimize the number of searches\(^1\). The principal’s utility when agent is type $a$ is

$$U^P_a = V^a(\hat{a}) - ck - p,$$

where $p$ is the price of the object, which is entirely paid by the principal. Notice that the principal is "benevolent" because she internalizes the agent’s utility, but, differently than the agent, also cares about the price paid to buy the object.

We assume that the principal cannot offer any contingent monetary transfer to the agent, and she can only propose a search rule to the agent. Moreover, by assumption only high quality objects are valuable for an $H$-type agent, and therefore an $H$-type agent needs to search in the $H$-box. We assume that $F^H(p)$

\(^1\)The model can be easily generalized to the case in which the agent pays a fraction $\lambda < \frac{1}{2}$ of the price and the principal the remaining fraction $1 - \lambda$. 


first order stochastically dominates \( F^L(p) \), therefore, the principal prefers that an \( L \)-type agent searches in the \( L \)-box, given that both types of objects are equally valuable for this type of agent, but objects in the \( L \)-box are in expectation cheaper.

As customary in the mechanism design approach, we assume that the principal specifies a search rule for each type and the agent announces her type.

First, we briefly analyze the standard case in which the principal knows agent’s type and can observe in which box the agent searches.

### 3 Symmetric Information

Suppose that the principal can observe agent’s type and the box from which the agent draws prices. The optimal stopping rule is such that the principal imposes to each type \( a \) to search in box \( a \), and to stop the search when the sampled price is lower than a given threshold, which is constant over time. Hence, the optimal stopping rule for type \( a \) fixes a threshold \( y_a^* \) such that

\[
\int_0^{y_a^*} (y_a^* - p) dF^a(p) = c.
\]

The optimal mechanism with symmetric information imposes on each agent \( a \) to draw prices from box \( a \) and stop searching at time \( t \) if and only if agent \( a \) finds a price lower or equal than \( y_a^* \). We call this search rule a stopping rule with a threshold.

**Proposition 3.1** If \( F^H \) first order stochastically dominates \( F^L \), then \( y_H^* \geq y_L^* \).

**Proof.** See the appendix.

Let \( E^a(n|y) \) denote the expected number of searches in box \( a \) \( \in \{L, H\} \) given a threshold \( y \).

**Proposition 3.2** Given a threshold \( y \), we have \( E^a(n|y) = \frac{1}{F^a(y)} \).

**Proof.** See the appendix.

One can easily conclude that, if \( F^H \) first order stochastically dominates \( F^L \), given a threshold \( y \), we have \( E^L(n|y) \leq E^H(n|y) \).
4 Asymmetric Information

We look now at the more interesting case in which the principal does not observe agent’s type, or monitor agent’s behavior. Specifically, we assume that the agent can search in any of the two boxes and the principal does not observe the box from which the price is drawn.

Under asymmetric information the principal cannot propose the first best menu of stopping rules that provides a different threshold for each type of agents, because this menu violates incentive compatibility. The agent’s best response is to announce to be the type with the highest threshold in order to minimize the expected search cost. To fix the incentive compatibility problem, we give the possibility of imposing a minimum number of searches to agents. We consider a class of rules that consists of a minimum number of searches and a threshold for each type of agents. Let

$\mathcal{R} = \{ (k_a, y_a) \mid k_a \geq 0, y_a \geq 0 \text{ for } a \in \{L, H\} \}$

be the class of such rules. In other words, the agent $a \in \{L, H\}$ is asked to do at least $k_a$ searches and if he could find a price below the threshold $y_a$ in the first $k_a$ searches, then the minimum of those will be used to make the purchase. Otherwise, she has to continue the search and find a price below the threshold $y_a$. Also, denote $R_L = (k_L, y_L)$ and $R_H = (k_H, y_H)$.

We focus on this class of rules because they are simple to adopt and, as we argued in the introduction, are largely used in practice. A simple alternative for the principal to overcome the incentive problem, is to fix a common threshold for both types of agents. We first characterize the optimal rule within the class of rules with a minimum number of searches that satisfies the incentive compatible and individual rationality conditions. Then we characterize the optimal common threshold rule, and, finally, provide conditions under which the menu of incentive compatible rules outperforms the common threshold rule.

We point out again that in this paper we constrain ourselves to not allow any

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2We assume $k_a \in \mathbb{R}_+$. When $k_a$ is a non-integer number, one can use a random mechanism that imposes $\lfloor k_a \rfloor$, that is the floor of $k_a$, searches with probability one, and an additional search with probability $k_a - \lfloor k_a \rfloor$, which does not depend on the outcome of the previous searches.
money transfer from the principal to the agent.

4.1 Rules with minimum number of searches

In this section we consider the class of rules where \( y_L \neq y_H \), denoted by \( \tilde{R} \). Throughout the paper, by \( y_L^* \) and \( y_H^* \) we mean the first best thresholds for \( L \)-type and \( H \)-type agents respectively. In the following, we denote \( \mathbb{E}^a(n|R_{\hat{a}}) \) the expected number of searches in box \( a \in \{L, H\} \) when the agent has to follow the rule \( R_{\hat{a}} \) where \( \hat{a} \in \{L, H\} \).

Let \( \rho \in (0, 1) \) be the probability that the agent is \( L \)-type and \( 1 - \rho \) be the probability that the agent is \( H \)-type. We would like to find \( \{(k_L, y_L), (k_H, y_H)\} \) that maximizes the principal’s expected utility which is

\[
\rho \left( V - c\mathbb{E}^L(n|R_L) - \mathbb{E}^L(p|R_L) \right) + (1 - \rho) \left( \bar{V} - c\mathbb{E}^H(n|R_H) - \mathbb{E}^H(p|R_H) \right)
\]

subject to IC conditions:

1. \( \mathbb{E}^L(n|R_L) \leq \mathbb{E}^H(n|R_L) \),
2. \( \mathbb{E}^L(n|R_L) \leq \mathbb{E}^L(n|R_H) \),
3. \( \mathbb{E}^L(n|R_L) \leq \mathbb{E}^H(n|R_H) \),
4. \( \mathbb{E}^H(n|R_H) \leq \mathbb{E}^H(n|R_L) \).

The above four constraints clarify the two sources of asymmetry of information. An agent can misreport his type and search in the box of the other type, or simply misreport his type and using the corresponding threshold to search in his “true box”. The next theorem shows that under a mild condition one can find the optimal search rule within the class \( \tilde{R} \). This condition intuitively says that the difference between \( y_H^* \) and \( y_L^* \) should not be too small. Under this incentive compatible optimal rule the \( L \)-type agent is given her fist best threshold \( y_L^* \) and she does not need to perform a minimum number of searches. The optimal rule for the \( H \)-type agent consists of her fist best threshold \( y_H^* \) and an optimal minimum of number of searches \( k_H^* \). The number \( k_H^* \) is the smallest number that prevents the \( L \)-type agent announcing to be \( H \)-type and still searching in \( L \)-box.
**Theorem 4.1** Suppose $E^L(n|y^*_L) + E^H(n|y^*_H) \leq E^H(n|y^*_L)$. Then, there is a value $k^*_H > 0$ that makes the search rule $R^* = \{(0,y^*_L),(k^*_H,y^*_H)\}$ optimal for the principal in the class $\tilde{R}$. Moreover, $k^*_H$ solves

$$E^L(n|y^*_L) = E^L(n|R^*_H).$$

**Proof.** See the appendix.

To illustrate the condition of the theorem, consider the following example. Let the $L$-box be $L = [0,V]$, and the $H$-box to be $H = [0,\eta V]$ for $\eta > 1$ with uniform price distributions on both intervals. Then, the condition in the theorem above boils down to having $\eta \geq 2.62$. \(^3\)

### 4.2 Common threshold

In this section we analyze optimal rule if the principal chooses a stopping rule with a common threshold for both types of agents.

Abusing notation let $EU_P(y)$ denote principal’s expected utility when she proposes a stopping rule with a common threshold $y$. Also, let $U_a^P(y)$ be the principal’s utility from agent $a \in \{L,H\}$ under the threshold rule with the threshold $y$. Note that the principal’s expected utilities from the $L$-type agent and the $H$-type agent are maximized at $y^*_L$ and $y^*_H$ respectively. To avoid non-interesting cases, we assume that $y^*_L < y^*_H$, and that $U_a^P(y^*_a) > 0$ for both $a \in \{L,H\}$. Principal’s expected utility is

$$EU_P(y) = \rho \left( V - cE^L(n|y) - E^L(p|y) \right) + (1 - \rho) \left( \bar{V} - cE^H(n|y) - E^H(p|y) \right) = \rho U^P_L(y) + (1 - \rho)U^P_H(y).$$

Also, let $y^m_H$ be the smallest threshold such that the $H$-type agent participates (if a smaller threshold is proposed to an $H$-type agent, his expected cost of search are higher than $\bar{V}$). By assumption $U_a^P(y^*_a) > 0$ for both $a \in \{L,H\}$, so it follows that $y^m_H < y^*_H$. Two cases must be considered:

First, suppose that $y^*_L < y^m_H$. In this case, if the principal proposes to both types of agents a stopping rule with threshold equal to $y^*_L$, an $H$-type agent would

\(^3\)The calculations are available upon request.
prefer to refuse to search. The principal then has to choose between imposing the threshold \( y^*_L \) excluding \( H \)-type agents, or a threshold that allows \( H \)-type agents’ participation, which of course, will induce \( L \)-type agent to search less than optimally.

**Proposition 4.2** Assume \( y^*_L < y^*_H \). If

\[
(1 - \rho)U^P_H(y^*) < \rho (U^P_L(y^*_L) - U^P_H(y^*)) ,
\]

then the principal weakly prefers to propose a stopping rule with threshold \( y^*_L \) such that only \( L \)-type agent accepts to search, to a stopping rule with a threshold \( y^* \in [y^*_H, y^*_H] \) such that both types of agents accept to search. If

\[
(1 - \rho)U^P_H(y^*) \geq \rho (U^P_L(y^*_L) - U^P_H(y^*)) ,
\]

then the principal weakly prefers to propose the latter stopping rule to the former.

**Proof.** See the appendix.

It is clear that a principal prefers to adopt a stopping rule with threshold \( y^*_L \) when \( \rho \) is large enough. In fact, if the agent is of \( L \)-type with sufficiently high probability, then the principal prefers to exclude \( H \)-type participation to distort the first best rule for the \( L \)-type.

When the first best threshold for the \( L \)-type agent does not prevent \( H \)-type from participating, the optimal common threshold rule induces some distortion in the search of both types.

**Proposition 4.3** Assume \( y^*_L \geq y^*_H \). If the principal proposes a stopping rule with a common threshold for both types, then the optimal common threshold is \( y^* \in (y^*_L, y^*_H) \).

**Proof.** See the appendix.
4.3 The menu of IC rules vs. common threshold

In this section we investigate under which conditions the principal prefers to propose the optimal menu of IC rules to the optimal stopping rule with a common threshold. In section 4.2 we see that the optimal common threshold is either \( y^*_L \), in which the \( H \)-type agent does not participate, or the threshold \( y^* \in (y^*_L, y^*_H) \).

Suppose first, that we are under the conditions stated in Proposition 4.3 such that \( y^*_L \) is the optimal common threshold. It follows immediately that if principal’s expected utility from having an \( H \)-type agent who follows the rule \( R^*_H = (k^*_H, y^*_H) \) is non-negative, then offering the menu \( R^* = \{(0, y^*_L), (k^*_H, y^*_H)\} \) is weakly better than offering a stopping rule with common threshold \( y^*_L \).

We now present some sufficient conditions that guarantees that \( H \)-type agent participation is beneficial for the principal when she proposes the menu of IC rules \( R^* \). Let \( f(k) \) be the difference in number of searches under the search rule \( (k, y^*_H) \) and the first best. If we denote \( q = F^H(y^*_H) \), then \( f(k) = k + \frac{(1-q)k}{q} - \frac{1}{q} \). Also, let \( U^P_H(y^*_H) = \bar{V} - \mathbb{E}^H(p|y^*_H) - c(\mathbb{E}^H(n|y^*_H)) \). Now, we have the following result.

**Proposition 4.4** Suppose \( y^*_L \) is the optimal common threshold rule. If \( cf(k^*_H) \leq U^P_H(y^*_H) \), then principal gets higher payoff offering the menu of IC rules \( R^* \) than proposing the stopping rule with common threshold \( y^*_L \).

**Proof.** See the appendix.

The intuition behind this result is straightforward. If the increase in the expected search cost imposed to the \( H \)-type agent is lower than principal’s first best utility from the \( H \)-type agent, then the menu of IC rules outperforms the common threshold rule. Obviously, we can also state this condition as a threshold on the minimum number of searches imposed to the \( H \)-type agent, i.e. \( k^*_H \leq f^{-1}\left(\frac{U^P_H(y^*_H)}{c}\right) \).

Now, consider the case where \( y^* \) is the optimal common threshold. In the next proposition we provide a sufficient condition under which offering the menu \( R^* \) is more beneficial for the principal than offering the common threshold rule with
the threshold $y^*$. Principal’s payoff from $L$-type agent is higher under rule $R^*$ when compared to common threshold rule $y^*$, as this agent is offered the first best rule in $R^*$. Therefore, if the increase in the search cost imposed to the $H$-type agent by the rule $R^*$ compared to the first best, is lower than the increase in the principal’s payoff from the $H$-type compared to the common threshold rule, then the rule $R^*$ outperforms the common threshold rule.

**Proposition 4.5** Suppose $y^*$ is the optimal common threshold rule. If $cf(k_H^*) \leq U_P^H(y_H^*) - U_P^H(y^*)$, then principal gets higher payoff offering the menu of IC rules $R^*$ than proposing the stopping rule with common threshold $y^*$.

**Proof.** See the appendix.

Before concluding we present two examples to illustrate two cases in which the sufficient condition of the Proposition 4.4 holds. In the first example prices are drawn from an interval, while in the second example prices are drawn from a finite set. For simplicity of the calculations in these examples, we consider the rule $\{(0, y_L^*), (\bar{k}, y_H^*)\}$ in which $\bar{k} = \lceil k_H^* \rceil$. This will serve the purpose of the section, where we provide conditions under which the menu of IC search rules outperforms the optimal common threshold rule. In other words, if the rule $\{(0, y_L^*), (\bar{k}, y_H^*)\}$ is preferred to the common threshold $y_L^*$ by the principal, then the rule $\{(0, y_L^*), (k_H^*, y_H^*)\}$ will be too.

**Example 4.6** Let $L = [0, 1]$ and $H = [0, 8]$ be the two boxes, with uniform price distributions on both intervals: $F^L(y) = y$ and $F^H(y) = \frac{y}{8}$. Also, let $V = \bar{V} = 2$, $c = 0.2$, and $\rho = 0.7$. From equation (1), one can derive $y_L^* = 0.63$ and $y_H^* = 1.79$. We have $\mathbb{E}^L(n|y_L^*) = \frac{1}{F^L(y_L^*)} = 1.58$ and $\mathbb{E}^H(n|y_H^*) = \frac{1}{F^H(y_H^*)} = 4.47$. Similarly, we have $\mathbb{E}^H(n|y_L^*) = \frac{1}{F^H(y_L^*)} = 12.65$. An $H$-type agent refuses to search given the threshold $y_L^*$, because $\bar{V} - c\mathbb{E}^H(n|y_L^*) = -0.53$.

To check whether the principal gets a higher utility proposing a stopping rule with threshold $y_L^*$ than a stopping rule with a higher common threshold that allows $H$-type participation, first we need to find the optimal common threshold $y^* \in (y_L^*, y_H^*)$. By using equation (4) in the appendix, we can derive $y^* = 1.11$. 
Then, principal’s expected utility by proposing a stopping rule with common threshold \( y^* \) is
\[
EU(y^*) = 0.7(1.26) + 0.3(0.006) = 0.88
\]

while her expected utility using \( y^*_L \) is
\[
EU(y^*_L) = 0.7(1.367) = 0.957.
\]

Therefore, proposing the common threshold \( y^*_L \) is better than the common threshold \( y^* \) for the principal.

Now, consider the search rule \( R^* = \{(0, y^*_L), (k^*_H, y^*_H)\} \). We know that for any \( k, y \in \mathbb{R}^+ \), the expected number of searches under the rule \((k, y)\) is
\[
\mathbb{E}(n|(k, y)) = k + (1 - F(y))k\mathbb{E}(n|y).
\]

By using Theorem 4.2, we can find \( k^*_H = 1.58 \). For simplicity of the calculations let \( \bar{k} = \lceil k^*_H \rceil = 2 \), and for \( \bar{k} = 2 \) we get
\[
\mathbb{E}^H(n|(2, y^*_H)) = 2 + (1 - F^H(y^*_H))^2\mathbb{E}^H(n|y^*_H)
\]

Also, for any \( k \in \mathbb{R} \) we have
\[
\mathbb{E}^H(p|(k, y)) = \left(\frac{y}{8}\right)^k\left(\frac{y}{k+1}\right) + k\left(\frac{y}{8}\right)^{k-1}(1 - \frac{y}{8})\left(\frac{y}{k}\right)
\]
\[
+ \binom{k}{2}\left(\frac{y}{8}\right)^{k-2}(1 - \frac{y}{8})\left(\frac{y}{k-1}\right) + \ldots + (1 - \frac{y}{8})^k\left(\frac{y}{2}\right)
\]

And for \( \bar{k} = 2 \):
\[
\mathbb{E}^H(p|(2, y)) = \left(\frac{y}{8}\right)^2\left(\frac{y}{3}\right) + 2\left(\frac{y}{8}\right)(1 - \frac{y}{8})\left(\frac{y}{2}\right) + (1 - \frac{y}{8})^2\left(\frac{y}{2}\right)
\]

Now, it is easy to see that, first, an \( H \)-type agent participates under the rule \( R_H = (\bar{k}, y^*_H) \), as \( \bar{V} - c\mathbb{E}^H(n|R_H) = 1.06 > 0 \). Second, the principal’s expected utility from the \( H \)-type agent is strictly positive, as
\[
\bar{V} - c\mathbb{E}^H(n|R_H) - \mathbb{E}^H(p|R_H) = 0.18 > 0
\]
Moreover, the principal’s expected utility offering the menu \( \{ y_L^*, (2, y_H^*) \} \) is

\[
\mathbb{E}U((0, y_L^*), (2, y_H^*)) = 0.7(0.957) + 0.3(0.18) = 1.011,
\]

which is larger than \( \mathbb{E}U(y_L^*) \), principal’s utility offering the common threshold \( y_L^* \).

In the next example the price set for the low quality and high quality objects are finite.

**Example 4.7** Let box \( L = \{5, 10, 20\} \) , with probability distribution \( \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \) and \( H = \{20, 100\} \) with probability distribution \( \left( \frac{1}{100}, \frac{99}{100} \right) \). Assume that the cost of search is \( c = 2 \). Finally let \( V = 25, \bar{V} = 104 \), and the probability that agent is type \( a \in \{H, L\} \) is \( \frac{1}{2} \). It is easy to check that at the first best, \( y_L^* = 10 \) and \( y_H^* = 100 \), and if the principal proposes a threshold strictly lower than 100, then H-type agent does not participate, because \( 104 - 100(2) < 0 \). Under the first best rule the expected number of draws for the \( L \)-type is 1.5 and her expected utility is \( 25 - 1.5(2) = 22 \); the expected utility of the principal when only \( L \)-type agent accepts the contract is \( \frac{1}{2}(25 - 1.5(2) - 7.5) = 7.25 \). If the principal proposes a stopping rule with common threshold equal to 100 she gets

\[
\frac{1}{2}(25 - \frac{35}{3} - 2) + \frac{1}{2}(104 - (\frac{20}{100} + 99) - 2) \simeq 7.07.
\]

Hence, it is more profitable for the principal to propose a search rule with threshold \( y_L^* = 10 \) than the one with threshold \( y_L^* = 100 \). One can derive \( k_H^* = 1.5 \) and \( k = \lceil k_H^* \rceil = 2 \). If the principal proposes the search rule \( R_H = (2, 100) \) to the \( H \)-type agent, the \( H \)-type agent is going to stop with probability one after two searches (and therefore she accepts to search under this rule). The probability that the minimum of the two draws is 20 is

\[
\frac{1}{100} \cdot \frac{1}{100} + 2(\frac{1}{100} \cdot \frac{99}{100} \cdot \frac{99}{100} \cdot \frac{99}{100} = 0.0199
\]

and the probability that the minimum of the two draws is 100 is

\[
\frac{1}{100} \cdot \frac{99}{100} = 0.9801.
\]

Therefore,

\[
\mathbb{E}^H(p|R_H) = 20(0.0199) + 100(0.9801) = 98.408,
\]
Finally, the expected utility of the principal offering the IC menu \{ (0, 10), (2, 100) \} is

\[
\frac{1}{2} (25 - 1.5(2) - 7.5) + \frac{1}{2} (104 - 4 - 98.408) = 8.046.
\]

which is higher than the expected utility of the principal when she offers the search rule with common threshold \( y^*_L = 10 \) and only \( L \)-type agents accept to search.

\[
\diamondsuit
\]

5 Discussion

This paper provides a theoretical explanation to the practice of imposing a minimum number of searches to agents who are delegated to make a choice on behalf of the principal. Delegation opens the door to problems of moral hazard and adverse selection, which are especially severe when the principal cannot offer monetary incentives to the agent. We consider a set of simple incentive compatible rules that a principal may adopt and we characterize the optimal rule among this class. We leave as an open question for further research the characterization of an optimal rule in the broader class of incentive compatible rules without any further restriction.

A couple of final remarks; In this paper we assumed that the principal pays the entire price of purchasing, but our results can be easily extended to the case in which the agent partially cover the expenses. We also assume that the principal is benevolent and internalizes the agent’s payoff, but this seems a reasonable assumption. Again the results can be extended to the case in which the cost of search is divided among the principal and the agent; nonetheless, for the case of procurements in public organizations the assumption of a benevolent principal who internalize agent’s utility, seems more appropriate.

6 Appendix

Proof of Proposition 3.1 : Let \( \phi^a(y) \equiv \int_0^y (y - p) dF^a(p), \) for \( a \in \{ H, L \} \).

First we show that for every \( y > 0, \phi^L(y) \geq \phi^H(y) \). Using integration by parts
we have
\[ \phi^a(y) = (y - p)F^a(p) \bigg|_0^y + \int_0^y F^a(p) \, dp = \int_0^y F^a(p) \, dp \]

Since \( F^H \) first order stochastically dominates \( F^L \), for every \( y > 0 \) we have
\[ \int_0^y F^H(p) \, dp \leq \int_0^y F^L(p) \, dp \]

Therefore,
\[ \phi^L(y) \geq \phi^H(y). \]

We know for \( y^*_a \) we have \( \phi^a(y^*_a) = c \) for \( a \in \{L, H\} \). As \( \phi^L(y) \geq \phi^H(y) \) for every \( y > 0 \), we can conclude that \( y^*_H \geq y^*_L \).

Proof of Proposition 3.2: We know
\[ E^a(n|y) = F^a(y) + 2(1-F^a(y))F^a(y) + 3(1-F^a(y))^2F^a(y) + 4(1-F^a(y))^3F^a(y) + \ldots \]

\[ = F^a(y) \left( 1 + 2(1-F^a(y)) + 3(1-F^a(y))^2 + 4(1-F^a(y))^3 + \ldots \right) \]

\[ = F^a(y) \left( \frac{1}{F^a(y)^2} \right) = \frac{1}{F^a(y)} \]

To prove Theorem 4.1, first we prove the following lemma.

Lemma 6.1 Suppose there is only one type agent and the principal offers a search rule \( R = (k, y) \) with \( k > 0 \) fixed. Then it is optimal for the principal to offer the rule \( R = (k, y^*) \) in which \( y^* \) is the first best threshold.

Proof. Suppose the agent has already searched \( k \) times. Suppose the minimum price among them is higher than \( y^* \): the principal can buy the good at that price or ask the agent to search to find a price lower than \( y^* \). Clearly, the principal’s expected utility is larger in the latter case. Suppose now that there exists a price \( \tilde{p} \leq y^* \). The principal’s payoff if she buys the good at price \( \tilde{p} \) is higher than the expected payoff if the agent continues to search. Therefore the optimal threshold for \( y \) is equal to \( y^* \).
**Proof of Theorem 4.1:** Let $\rho \in (0, 1)$ be the probability that the agent is $L$-type and $1 - \rho$ be the probability that the agent is $H$-type. We would like to find $\{(k_L, y_L), (k_H, y_H)\}$ that maximizes the principal’s expected utility which is

$$
\rho \left( V - c E^L(n|R_L) - E^L(p|R_L) \right) + (1 - \rho) \left( \bar{V} - c E^H(n|R_H) - E^H(p|R_H) \right)
$$

subject to IC conditions:

1. $E^L(n|R_L) \leq E^H(n|R_L)$,
2. $E^L(n|R_L) \leq E^L(n|R_H)$,
3. $E^L(n|R_L) \leq E^H(n|R_H)$,
4. $E^H(n|R_H) \leq E^H(n|R_L)$,

It is easy to see that the principal’s expected utility without the constraints above is maximized at $\{(0, y_L^*), (0, y_H^*)\}$. Therefore, first we find the minimum values for $k_L$ and $k_H$ that satisfy the constraints above. One can derive

$$
E^a(n|R_b) = k_b + \frac{(1 - F^a(y_b))^{k_b}}{F^a(y_b)},
$$

in which $a, b \in \{L, H\}$. Condition (1) holds, because we assume that $F^H$ stochastically dominates $F^L$. Also, due to this assumption, it is easy to see that condition (2) implies condition (3). Therefore, for IC conditions it is enough to have (2) and (4). From (2) we have

$$
k_L + \frac{(1 - F^L(y_L))^{k_L}}{F^L(y_L)} \leq k_H + \frac{(1 - F^L(y_H))^{k_H}}{F^L(y_H)},
$$

which implies

$$
k_H - k_L \geq \frac{(1 - F^L(y_L))^{k_L}}{F^L(y_L)} - \frac{(1 - F^L(y_H))^{k_H}}{F^L(y_H)},
$$

Similarly, from (4) we have

$$
k_H + \frac{(1 - F^H(y_H))^{k_H}}{F^H(y_H)} \leq k_L + \frac{(1 - F^H(y_L))^{k_L}}{F^H(y_L)},
$$
and implies
\[ k_H - k_L \leq \frac{(1 - F^H(y_L))^{k_L}}{F^H(y_L)} - \frac{(1 - F^H(y_H))^{k_H}}{F^H(y_H)}. \]

Therefore, the IC condition is:
\[ \frac{(1 - F^L(y_L))^{k_L}}{F^L(y_L)} - \frac{(1 - F^L(y_H))^{k_H}}{F^L(y_H)} \leq k_H - k_L \leq \frac{(1 - F^H(y_L))^{k_L}}{F^H(y_L)} - \frac{(1 - F^H(y_H))^{k_H}}{F^H(y_H)}. \]

Now we need to find the minimum values for \( k_H \) and \( k_L \) that satisfies the above inequalities. Clearly, the best is to choose \( k_L = 0 \). Having this, the IC condition is
\[ \frac{1}{F^L(y_L)} - \frac{(1 - F^L(y_H))^{k_H}}{F^L(y_H)} \leq k_H \leq \frac{1}{F^H(y_L)} - \frac{(1 - F^H(y_H))^{k_H}}{F^H(y_H)}. \]

The minimum value for \( k_H \) comes form solving the equation below:
\[ \frac{1}{F^L(y_L)} - \frac{(1 - F^L(y_H))^{k_H}}{F^L(y_H)} = k_H. \]

As \( k_L^* = 0 \), the optimal threshold \( y_L \) is \( y_L^* \), i.e. the first best threshold for \( L \)-type agent. Moreover, by Lemma 6.1 we conclude that the optimal threshold for \( y_H \) is \( y_H^* \). Therefore, the minimum value for \( k_H \) is denoted by \( k_H^* \) and solves
\[ \frac{1}{F^L(y_L^*)} = k_H^* + \frac{(1 - F^L(y_H^*))^{k_H^*}}{F^L(y_H^*)}. \]

If we write \( R_H^* = (k_H^*, y_H^*) \), then \( k_H^* \) solves
\[ \mathbb{E}^L(n|y_L^*) = \mathbb{E}^L(n|R_H^*). \]

Now, we show that if \( \mathbb{E}^L(n|y_L^*) + \mathbb{E}^H(n|y_H^*) \leq \mathbb{E}^H(n|y_L^*) \), then the existence of the optimal value \( k_H^* \) is guaranteed. To this end, it is enough to show
\[ \frac{1}{F^L(y_L^*)} - \frac{(1 - F^L(y_H^*))^{k_H^*}}{F^L(y_H^*)} \leq \frac{1}{F^H(y_L^*)} - \frac{(1 - F^H(y_H^*))^{k_H^*}}{F^H(y_H^*)}, \]
which comes from equation (3). We have

\[
\begin{align*}
\mathbb{E}^L(n|y^*_L) + \mathbb{E}^H(n|y^*_H) &\leq \mathbb{E}^H(n|y^*_L) \\
\Rightarrow \frac{1}{F^L(y^*_L)} + \frac{1}{F^H(y^*_H)} &\leq \frac{1}{F^H(y^*_L)} \\
\Rightarrow \frac{1}{F^L(y^*_L)} &\leq \frac{1}{F^H(y^*_L)} - \frac{1}{F^H(y^*_H)} \\
\Rightarrow \frac{1}{F^L(y^*_L)} - \frac{(1 - F^L(y^*_H))^{k_H}}{F^L(y^*_H)} &\leq \frac{1}{F^H(y^*_L)} - \frac{(1 - F^H(y^*_H))^{k_H}}{F^H(y^*_H)} \\
\leq \frac{1}{F^H(y^*_L)} - \frac{1}{F^H(y^*_H)} &\leq \frac{1}{F^H(y^*_L)} - \frac{(1 - F^H(y^*_H))^{k_H}}{F^H(y^*_H)}
\end{align*}
\]

Therefore,

\[
\frac{1}{F^L(y^*_L)} - \frac{(1 - F^L(y^*_H))^{k_H}}{F^L(y^*_H)} \leq \frac{1}{F^H(y^*_L)} - \frac{(1 - F^H(y^*_H))^{k_H}}{F^H(y^*_H)}.
\]

The proof is complete.

**Proof of Proposition 4.2:** We want to maximize \( \mathbb{E}U^P(y) \) on \([0, \infty)\). We know that \( U^P_a(y) \) increases for \( y < y^*_a \) and decreases for \( y > y^*_a \) where \( a \in \{L, H\} \). Also, we know that \( \mathbb{E}U^P(y) \) is differentiable on \([0, \infty)\). To maximize \( \mathbb{E}U^P(y) \) we find all the points in which the first derivative of this function is zero. We call such points critical points. Then, the maximum point would be the point that has the largest value. Therefore, we have

\[
\frac{d\mathbb{E}U^P(y)}{dy} = 0 \Rightarrow \rho \frac{dU^P_L(y)}{dy} + (1 - \rho) \frac{dU^P_H(y)}{dy} = 0
\]

\[
\Rightarrow \rho \frac{dU^P_L(y)}{dy} = -(1 - \rho) \frac{dU^P_H(y)}{dy}
\]

As we assume \( y^*_L < y^*_H \), there will be two critical points: \( y = y^*_L \) and \( y = y^* \in (y^*_L, y^*_H) \). Basically, \( y^* \) is the threshold that maximizes \( \mathbb{E}U^P(y) \) over interval \((y^*_L, y^*_H)\) and it is the optimal common threshold for the stopping rule that both types of agents participate. Also, we have \( y^* \in [y^*_H, y^*_H] \), because for any \( y < y^*_H \), the \( H \)-type agent does not participate so the choice will only distort the threshold for the \( L \)-type. Therefore, any \( y \in (y^*_L, y^*_H) \) is dominated by \( y^*_L \).
Now, if $\mathbb{E}U^P(y^*) < \mathbb{E}U^P(y^*_L)$, then $y^*_L$ is the optimal common threshold for the stopping rule and this is true when

$$\mathbb{E}U^P(y^*) < \mathbb{E}U^P(y^*_L) \iff \rho U_L^P(y^*) + (1 - \rho) U_H^P(y^*) < \rho U_L^P(y^*_L)$$

$$\iff (1 - \rho) U_H^P(y^*) < \rho (U_L^P(y^*_L) - U_L^P(y^*))$$

The proof is complete.

**Proof of Proposition 4.3**: We want to maximize $\mathbb{E}U^P(y)$ on $[0, \infty)$. We know that $U^P_a(y)$ increases for $y < y^*_a$ and decreases for $y > y^*_a$ where $a \in \{L, H\}$. Also, we know that $\mathbb{E}U^P(y)$ is differentiable on $[0, \infty)$. To maximize $\mathbb{E}U^P(y)$ we find all the points in which the first derivative of this function is zero. We call them critical points. Then, the maximum point would be the point that has the largest value. Therefore, we have

$$\frac{d\mathbb{E}U^P(y)}{dy} = 0 \Rightarrow \rho \frac{dU_L^P(y)}{dy} + (1 - \rho) \frac{dU_H^P(y)}{dy} = 0$$

$$\Rightarrow \rho \frac{dU_L^P(y)}{dy} = -(1 - \rho) \frac{dU_H^P(y)}{dy} \quad (4)$$

As we assume $y^*_L \geq y^*_H$, there is only one critical point. Since in this case $\frac{dU_L^P(y^*_H)}{dy} \neq 0$ and $\frac{dU_H^P(y^*_L)}{dy} \neq 0$, we conclude $\mathbb{E}U^P(y)$ is maximized when $U_L^P(y)$ and $U_H^P(y)$ have opposite slopes, that is, when $U_L^P(y)$ is strictly decreasing and $U_H^P(y)$ is strictly increasing. Therefore, in this case $\mathbb{E}U^P(y)$ is maximized at some $y^*$ such that $y^*_L < y^* < y^*_H$.

**Proof of Proposition 4.4**: Suppose $cf(k^*_H) \leq U^P_H(y^*_H)$, then we show that

$$\bar{V} - \mathbb{E}^H(p|R^*_H) - c(\mathbb{E}^H(n|R^*_H)) \geq 0.$$  

Under the rule $R^*_H$ when compared to the first best case for the $H$-type agent, the expected price is lower. Therefore, it is enough to show that

$$\bar{V} - \mathbb{E}^H(p|y^*_H) \geq c(\mathbb{E}^H(n|R^*_H)),$$
or equivalently to show
\[ V - \mathbb{E}^H(p|y_H^*) - c(\mathbb{E}^H(n|y_H^*)) \geq c(\mathbb{E}^H(n|R_H^*) - \mathbb{E}^H(n|y_H^*)), \]
which is the same as showing
\[ \frac{U_P^H(y_H^*)}{c} \geq \mathbb{E}^H(n|R_H^*) - \mathbb{E}^H(n|y_H^*). \]

We know that \( f(k) = k + \frac{(1-q)^k}{q} - \frac{1}{q} \). Therefore, we have
\[ \mathbb{E}^H(n|R_H^*) - \mathbb{E}^H(n|y_H^*) = k_H^* + \frac{(1-q)^{k_H^*}}{q} - \frac{1}{q} = f(k_H^*). \]

One can easily see that \( f'(k) > 0 \), for every \( k \in \mathbb{R} \). Therefore, \( f(k) \) is a continuous and strictly increasing function. Also, \( f(0) = 0 \). Then, there is a \( \hat{k} > 0 \) such that \( f(\hat{k}) = \frac{U_P^H(y_H^*)}{c} \). Hence, for every \( k \leq \hat{k} \) we have \( f(k) \leq \frac{U_P^H(y_H^*)}{c} \).

The proof is complete.

**Proof of Proposition 4.5:** The principal’s expected utility under the common threshold rule \( y^* \) is
\[ \mathbb{E}U_P^L(y^*) = \rho U_L^P(y^*) + (1 - \rho) U_H^P(y^*). \]

Moreover, the principal’s expected utility given the menu \( R^* \) is
\[ \mathbb{E}U_P^L(R^*) = \rho U_L^P(R_H^*) + (1 - \rho) U_H^P(R_H^*). \]

Clearly, \( U_L^P(y_L^*) \geq U_L^P(y^*) \). We also know that the expected price under \( R_H^* \) is less than the expected price in the first best. Now, assume \( c f(k_H^*) \leq U_H^P(y_H^*) - U_H^P(y^*) \), then
\[
\begin{align*}
&c(\mathbb{E}(n|R_H^*) - \mathbb{E}(n|y_H^*)) \leq U_H^P(y_H^*) - U_H^P(y^*) \\
\implies &c(\mathbb{E}(n|R_H^*) - c\mathbb{E}(n|y_H^*)) \leq V - c\mathbb{E}(n|y_H^*)) - \mathbb{E}(p|y_H^*)) - U_H^P(y^*) \\
\implies &U_H^P(y^*) \leq V - \mathbb{E}(p|y_H^*)) - c\mathbb{E}(n|R_H^*) \leq V - \mathbb{E}(p|R_H^*)) - c\mathbb{E}(n|R_H^*) \\
\implies &U_H^P(y^*) \leq U_H^P(R^*)
\end{align*}
\]

This shows that the principal’s expected profit under the rule \( R^* \) is higher than under the common threshold \( y^* \). ■
References


