

*Citation for published version:*

De Paola, A, Angeli, D & Strbac, G 2017, 'Price-Based Schemes for Distributed Coordination of Flexible Demand in the Electricity Market', *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 3104 - 3116.  
<https://doi.org/10.1109/TSG.2017.2707121>

*DOI:*

[10.1109/TSG.2017.2707121](https://doi.org/10.1109/TSG.2017.2707121)

*Publication date:*

2017

*Document Version*

Peer reviewed version

[Link to publication](#)

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# Price-based Schemes for Distributed Coordination of Flexible Demand in the Electricity Market

Antonio De Paola, *Member, IEEE*, David Angeli, *Fellow, IEEE*, and Goran Strbac, *Member, IEEE*

**Abstract**—This paper proposes novel distributed control schemes for large-scale deployment of flexible demand. The problem of efficiently coordinating price-responsive appliances operating in the electricity market is tackled within a game-theoretical framework. Adopting the concept of Nash equilibrium and Lyapunov-based techniques, a new iterative control algorithm is designed in order to always converge to a satisfactory solution for the individual customers, which aim at minimizing their energy costs. From the system perspective, it is shown that global quantities such as total generation costs are reduced at each algorithm iteration. These results are achieved for any penetration level of flexible demand and for all types of interruptible electrical appliances. The proposed control scheme can be applied in practice through a one-shot implementation that, at the price of a negligible degradation of the equilibrium performance, ensures faster convergence to a stable solution. Simulation results are also presented, testing the novel schemes in realistic future scenarios of the Great Britain power system with high penetration of flexible demand.

**Index Terms**—Electric power networks, flexible demand, game theory, distributed price-based control, electricity markets.

## NOMENCLATURE

$j \in N$	Index and set of flexible appliances
$t \in T$	Index and set of time instants
$\Delta t$	Time discretization step (h)
$E_j^r$	Energy required by $j$ -th appliance (KWh)
$P_j^r$	Rated power of $j$ -th appliance (KW)
$\mathcal{A}_j$	Time-availability interval of $j$ -th appliance
$u_j(\cdot)$	Power consumption profile of $j$ -th appliance (KW)
$u_j^*(\cdot)$	Power profile of $j$ -th appliance at Nash Eq. (KW)
$u_j^\varepsilon(\cdot)$	Power profile of $j$ -th appliance at $\varepsilon$ -Nash Eq. (KW)
$\mathcal{U}_j$	Set of feasible power profiles for $j$ -th appliance
$\mathcal{U}_j$	Set of feasible ON/OFF power profiles for $j$ -th appliance
$D(\cdot)$	Broadcast demand signal (GW)
$D_i(\cdot)$	Inflexible demand profile (GW)
$D_f(\cdot)$	Flexible demand profile (GW)
$D_a(\cdot)$	Aggregate demand profile (GW)
$D_a^*(\cdot)$	Aggregate demand profile at Nash Equilibrium (GW)
$D_a^\varepsilon(\cdot)$	Aggregate demand profile at $\varepsilon$ -Nash Equilibrium (GW)
$\Pi(D)$	Electricity price function (£/MWh)
$p(t)$	Broadcast price signal (£/MWh)
$p_j^*(t)$	Price broadcast to $j$ -th appliance in Alg. 2 (£/MWh)
$C_j$	Expected energy cost of the $j$ -th appliance (£)
$\bar{C}_j$	Final energy cost of the $j$ -th appliance (£)
$\varepsilon$	Parameter of equilibrium approximation (£)

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## I. INTRODUCTION

One of the defining elements of power systems transition towards the smart grid paradigm is the increasing flexibility of demand. A growing fraction of electric loads installed in private households will accommodate the possibility to reschedule (at least partially) their power consumption during the day. Such developments can lead to significant benefits [1], [2], [3], in the form of reduced operational costs and more efficient integration of renewables in the system. To fulfil this potential, it is necessary to devise robust and efficient control strategies for large-scale deployment of flexible demand, pursuing its efficient integration in the electricity market. A consistent amount of research has investigated this problem, producing a wide array of different solutions [4]. Since centralized schemes [5] may not always be scalable to large systems with many independent agents, several distributed techniques have been proposed. These include adaptive strategies [6], Lagrange relaxation [7], stochastic pricing [8], [9] and the introduction of aggregators as mediating entities in the system [10]. Distributed optimization using the Alternating Direction Method of Multipliers (ADMM) has also received increasing attention [11], [12]. With this approach, both the devices and the system operator follow an iterative procedure that, under certain convexity assumptions, converges to a final result which is optimal for some combination of global and local objective functions.

In this paper, the problem of appliances coordination with distributed schemes is tackled within a game-theoretical framework. Individual customers/devices are modelled as competitive players that receive a price signal and schedule power consumption so as to minimize the energy cost required to complete their task. The main objective is to induce a Nash equilibrium: no device can achieve a reduced energy cost by unilaterally changing its initially scheduled power. By accounting for the global effect of flexible demand on energy prices, new demand peaks are avoided, achieving a flattened aggregate demand profile and reduced generation costs. Distinctive elements of such approach include:

- Full control of the individual agents over their behaviour. Unlike centralized schemes, in our approach the central coordinator only broadcasts electricity prices and does not prescribe a certain power consumption.
- The loads do not need any external incentive and react to the received signals by pursuing their self-interest, in the form of their own cost minimization. This is not always the case in distributed optimization schemes, where a specific power update needs to be followed by each

device, even if it does not always lead to a direct reduction of its energy cost.

- Given the distributed nature of the considered framework, reduced computational times and communication infrastructure can be achieved.

Note that the chosen game-theoretical framework naturally focuses, through the concept of Nash equilibrium, on the objectives of the individual devices. The optimization of global quantities (such as total generation costs) is usually achieved through particular design choices and does not constitute the core element of the coordination strategy, as in centralized schemes. In this respect, distributed optimization approaches usually opt for an intermediate objective, optimizing some combination of global and local cost functions.

Game theory has been extensively applied to the problem of demand response. For instance, [13] determines the energy consumption of the appliances as the best response to external signals. This concept is extended in [14], using a larger time horizon to guarantee fairness for the devices and better global results, and in [15], which approximates the appliances population as infinite and models the problem as a mean field game. It has been shown, mostly in the case of electric vehicles, that distributed control schemes can converge to a Nash equilibrium when additional quadratic terms are introduced in the cost function of the devices [16], [17] or under some conditions on the number of appliances and the considered energy price [18], [19].

The main novelty of our approach is that, with the proposed iterative strategy, convergence to an equilibrium is ensured for any penetration level of flexible demand and for any type of interruptible loads. The case of ON/OFF devices and appliances with partial time availability is also accommodated and no specific knowledge of the electricity price function is required. This result is achieved with Lyapunov techniques, showing that some global functional (e.g. total generation costs) is reduced at each iteration of the proposed algorithm. For faster practical implementations, a one-shot scheme is also designed: at the price of a negligible reduction in the equilibrium performance, the devices can be successfully coordinated through a single broadcast of a price signal (different in general for each device). In addition, it has been proved that the proposed techniques are fair (devices with equal parameters incur equal energy costs) and incentivize flexibility (devices that are available at more time instants pay less to complete their task). The presented strategies are finally evaluated in simulation, considering likely future scenarios of the UK grid, with high penetration of flexible demand.

The rest of the paper is structured as follows: Section II presents the main modelling choices and characterizes the demand response coordination problem as a static competitive game. The description of the iterative control strategy, with a discussion of its theoretical properties and possible practical implementation, is included in Section III while Section IV presents the same results for the alternative one-shot scheme. Simulations of the proposed control strategies are presented in Section V while Section VI contains some conclusive remarks.

## II. COORDINATION OF PRICE-RESPONSIVE APPLIANCES

This section presents the main modelling choices for the flexible appliances and the electricity market. The problem of coordinating the price-responsive demand in a distributed manner is then formulated within a game-theoretical framework.

### A. Flexible Appliances

We consider a population  $\mathcal{N} = \{1, \dots, N\}$  of price-responsive appliances that are required to complete an assigned task over a discrete time interval  $\mathcal{T} = \{1, \dots, T\}$ . The objective of each device is to exploit its flexibility so as to consume power at the cheapest hours of the day, reducing the total energy cost required for task completion. The task of the  $j$ -th device can be described by three quantities: the total amount of required energy  $E_j^r$ , its rated power  $P_j^r$  and its availability window  $\mathcal{A}_j \subseteq \mathcal{T}$ . These parameters unequivocally characterize the set  $\mathcal{U}_j$  of feasible power consumption profiles  $u_j: \mathcal{T} \rightarrow \mathbb{R}_+$  that guarantee task completion for the  $j$ -th device. If one denotes by  $\mathbb{I}_x$  the indicator function, it holds:

$$\mathcal{U}_j := \left\{ u_j(\cdot) : \sum_{t=1}^T u_j(t) \Delta t = E_j^r, \right. \\ \left. 0 \leq u_j(t) \leq P_j^r \cdot \mathbb{I}_{\mathcal{A}_j}(t) \forall t \in \mathcal{T} \right\}. \quad (1)$$

Any feasible power profile  $u_j \in \mathcal{U}_j$  must fulfil two properties. The first condition in (1) states that the total consumed energy (equal to the sum of  $u_j$  multiplied by the time discretization step  $\Delta t$ ) must correspond to  $E_j^r$ , required for task completion. From the second condition, at any time instant  $t \in \mathcal{A}_j$  the power consumed by the  $j$ -th device cannot be greater than  $P_j^r$  and must be equal to zero when  $t \notin \mathcal{A}_j$  as the device is not available to consume power.

*Assumption 1:* The parameters  $(E_j^r, P_j^r, \mathcal{A}_j)$  of each device  $j$  are such that the set  $\mathcal{U}_j$  of feasible power profiles is non-empty. Equivalently, all appliances can complete their tasks by operating at rated power during their time availability window:

$$\sum_{t \in \mathcal{A}_j} P_j^r \cdot \Delta t \geq E_j^r \quad \forall j \in \mathcal{N}.$$

Each device is required to pay for the energy consumed while completing its task. For a certain price signal  $p(t)$  and power profile  $u_j$ , the total cost  $C_j$  sustained by the  $j$ -th device is:

$$C_j = \sum_{t=1}^T p(t) \cdot u_j(t) \Delta t. \quad (2)$$

Each term of the sum in (2) corresponds to the energy cost of the device  $j$  at time  $t$  and it is equal to the product of the electricity price  $p(t)$  by the consumed energy  $u_j(t) \cdot \Delta t$ , where  $\Delta t$  denotes the chosen time discretization step.

### B. Power Demand and Electricity Prices

To quantify the energy costs sustained by the appliances, the electricity market has been abstracted by a price function  $\Pi$  of the aggregate power demand. At a certain time instant  $t \in \mathcal{T}$ , the electricity price  $p(t)$  corresponds to  $p(t) = \Pi(D_a(t))$ ,

where  $D_a(t)$  denotes the total power demand at time  $t$ . This quantity is the sum of two distinct terms: the total power consumption  $D_f$  of the flexible appliances and the power demand  $D_i$  of the other (inflexible) loads. Denoted by  $u_j(\cdot)$  the power consumption profile of the  $j$ -th device, the following expressions can be provided:

$$D_f(t) = \sum_{j=1}^N u_j(t) \quad (3a)$$

$$D_a(t) = D_i(t) + D_f(t) = D_i(t) + \sum_{j=1}^N u_j(t). \quad (3b)$$

*Assumption 2:* The single device  $j$  has no market power and cannot unilaterally modify the electricity price. Such quantity only depends on the total power consumed by the whole population of appliances:

$$\Pi(D_a(t)) = \Pi\left(D_i(t) + \sum_{k=1}^N u_k(t)\right) \simeq \Pi\left(D_i(t) + \sum_{k \in \mathcal{N} \setminus j} u_k(t)\right).$$

This is not only a simplifying hypothesis for the game-theoretical analysis presented next but it is also a crucial technical remedy required to preserve equilibrium existence in the case of high penetration of flexible demand. The proposed approximation is reasonable if one considers domestic loads whose individual power consumption is orders or magnitude smaller than the total power demand in the system.

*Assumption 3:* The price function  $\Pi(D_a)$  is strictly monotone increasing with respect to  $D_a$ .

It is assumed, in a first approximation, that electricity will be more expensive when more power needs to be generated to meet higher demand levels. The current formulation has been chosen for its simplicity but the presented modelling framework can also accommodate more realistic and detailed cases. For example, in scenarios with high penetration of renewables, one can assume that the electricity price is monotone increasing with respect to the generation from traditional sources. If  $\Pi(D_a(t))$  is replaced with  $\Pi(D_a(t) - G_r(t)) = \Pi(D_i(t) + D_f(t) - G_r(t))$ , where  $G_r(t)$  represents the amount of renewable generation, all the results presented in the rest of the paper can still be obtained with minor modifications. More generally, the presented analysis is valid as long as the electricity price can be expressed as  $p(t) = \Pi(\alpha(t) + D_f(t))$ , where  $\alpha(t)$  is an arbitrary function of time and  $\Pi$  is monotone increasing. It is worth mentioning that network constraints could potentially impact the electricity prices. However, it is a common assumption to neglect network models in studies on large-scale coordination of flexible demand [13]–[18].

*Remark 1:* The control schemes in Section III and IV do not require precise knowledge of the price function  $\Pi$ , only its monotonicity with respect to power demand must be verified.

### C. Game-theoretical Formulation

The main objective of the present work is to induce (in a distributed manner) a system configuration which is satisfactory for all the involved agents. In doing so, it is crucial to take into account rebound effects and loss of diversity. For high penetration of flexible demand, if all devices schedule

their power consumption at the same time instants, they will increase total demand (and therefore electricity prices) at those times, causing suboptimality of their operation strategy. To tackle this problem, a game-theoretical framework is adopted, modelling the flexible devices as competing rational players. The elements of the considered static game are the following:

- *Players:* The set  $\mathcal{N}$  of flexible electrical appliances.
- *Strategies:* For each player  $j \in \mathcal{N}$ , the set  $\mathcal{U}_j$  of feasible power profiles guaranteeing task completion.
- *Objective function:* Minimization of the actual energy cost  $\bar{C}_j$  sustained by the individual device  $j$  to complete its task. The considered energy price depends on the final profile of aggregate demand  $D_a$  through the function  $\Pi$ :

$$\bar{C}_j = \sum_{t=1}^T \Pi(D_a(t)) \cdot u_j(t) \Delta t. \quad (4)$$

In this context, a fair and desirable system configuration can be described as a Nash equilibrium, in the sense specified below:

*Definition 1:* Consider the individual power consumption profiles  $u_j^*(\cdot) \in \mathcal{U}_j$  for all  $j \in \mathcal{N}$ . These quantities correspond to a Nash equilibrium in the electricity market if the following condition is fulfilled for all  $j \in \mathcal{N}$ :

$$\sum_{t=1}^T \Pi(D_a^*(t)) \cdot u_j^*(t) \Delta t = \min_{u_j(\cdot) \in \mathcal{U}_j} \sum_{t=1}^T \Pi(D_a^*(t)) \cdot u_j(t) \Delta t \quad (5)$$

s. t.  $u_j(\cdot) \in \mathcal{U}_j$

where the aggregate demand profile  $D_a^*$  is defined as:

$$D_a^*(t) = D_i(t) + D_f^*(t) = D_i(t) + \sum_{j=1}^N u_j^*(t) \quad \forall t \in \mathcal{T}. \quad (6)$$

The proposed definition is conceptually similar to the equilibrium notions that have been recently presented for games with infinite players, also referred to as mean field games [20], [21]. In these works, it is assumed that the (infinitesimal) single agent does not have a significant impact on the global quantities of the system and therefore only the global behaviour of the players' population (the mean field) must be taken into account. In our case, it has been established in Assumption 2 that the single appliance has negligible market power and only the aggregate power consumption  $D_f$  of the flexible load impacts the electricity price. Therefore, at equilibrium, the scheduled power profile  $u_j^*$  of each player is cost-minimizing for a certain aggregate demand  $D_a^* = D_i + D_f^*$  (through (5)) and, at the same time, the whole set of power schedules induces that very same demand profile (as a result of (6)). Similar definitions of Nash equilibrium, in the context of flexible demand, have been presented in [17] and [22].

From a global point of view, it is worth mentioning that the problematic rebound effects associated to large penetration of flexible demand are cancelled at the Nash equilibrium solution. In fact, if new demand peaks were to be created by the operation strategies of the price-responsive loads, new demand valleys (characterized by cheaper electricity) would also appear. It follows that the single appliances could reduce their costs by shifting part of their power consumption to these new valleys. By definition, this cannot occur when a Nash equilibrium has been reached.



### III. ITERATIVE CONTROL STRATEGY

An iterative scheme is initially considered for distributed coordination of flexible appliances with continuous power consumption, incorporating in Section IV the case of ON/OFF loads. The iterative control strategy is formally defined in Section III-A, proving its convergence and equilibrium properties and discussing the results achieved at a global level. Practical implementations are discussed in Section III-B.

#### A. Theoretical Control Scheme

The proposed iterative strategy is described by Algorithm 1, which consists of three main phases.

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#### Algorithm 1 Iterative scheme - Flexible demand coordination

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- 1) **Initialization phase.** Starting values for power scheduling of the appliances and flag variables are set:

$$u_j^{(0)}(\cdot) = u_j^0(\cdot) \in \mathcal{U}_j \quad \forall j \in \mathcal{N} \quad l = 0 \quad conv = 0$$

$$D^{(0)}(\cdot) = D_i(\cdot) + \sum_{j=1}^N u_j^{(0)}(\cdot)$$

- 2) **Power scheduling update.** The scheduled power profiles of the individual appliances are iteratively updated :

**WHILE** ( $conv == 0$ )

a)  $conv = 1$ .

b) **FOR**  $j = 1 : 1 : N$

i)  $l = l + 1$ .

ii)  $D^{(l)}(\cdot) = D^{(l-1)}(\cdot) \quad u_j^{(l)}(\cdot) = u_j^{(l-1)}(\cdot) \quad \forall j \in \mathcal{N}$ .

iii) **FIND**  $t_1, t_2 \in \mathcal{A}_j$  such that:

$$D^{(l-1)}(t_1) < D^{(l-1)}(t_2) \quad u_j^{(l-1)}(t_1) < P_j^r \quad u_j^{(l-1)}(t_2) > 0$$

iv) **IF**  $t_1, t_2$  exist:

$$\Delta = \min \left( \left\{ P_j^r - u_j^{(l-1)}(t_1), u_j^{(l-1)}(t_2), \frac{D^{(l-1)}(t_2) - D^{(l-1)}(t_1)}{2} \right\} \right)$$

$$u_j^{(l)}(t_1) = u_j^{(l-1)}(t_1) + \Delta \quad D^{(l)}(t_1) = D^{(l-1)}(t_1) + \Delta$$

$$u_j^{(l)}(t_2) = u_j^{(l-1)}(t_2) - \Delta \quad D^{(l)}(t_2) = D^{(l-1)}(t_2) - \Delta$$

$$conv = 0.$$

- 3) **Final results.** The Nash equilibrium solution is returned:

$$D_a^*(\cdot) = D^{(l)}(\cdot) \quad u_j^*(\cdot) = u_j^{(l)}(\cdot) \quad \forall j \in \mathcal{N}.$$


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As a first step, in the **initialization phase**, preliminary values  $u_j^{(0)}$  are set for the power scheduling of the flexible devices, calculating the resulting aggregate demand  $D^{(0)}$ . In phase two, named **power scheduling update**, the algorithm verifies if the energy cost of each device can be reduced with respect to the current configuration. At each iteration of the **WHILE** cycle, all devices  $j \in \mathcal{N}$  are analysed sequentially. During the  $l$ -th execution of the **FOR** cycle, it is verified whether the single appliance  $j$  can switch a feasible amount  $\Delta$  of its power consumption from time  $t_2$  to another time instant  $t_1$ , characterized by a lower demand value (and therefore lower energy price). If this is the case, the scheduled power  $u_j^{(l)}$  of the  $j$ -th device is updated while all other devices

preserve their previous strategy ( $u_k^{(l)} = u_k^{(l-1)}$  for all  $k \neq j$ ). The corresponding values of the resulting aggregate demand  $D^{(l)}$  are modified accordingly. If, at some iteration of the **WHILE** cycle, there is no device  $j$  that can reduce its energy cost, the variable  $conv$  (set to 1 in step 2.a) preserves its value throughout the **FOR** cycle in step 2.b). This means that a Nash equilibrium has been obtained and the second phase of the algorithm is concluded. The resulting power profiles  $u_j^*$  and aggregate demand  $D_a^*$  (fulfilling (5) and (6) in Definition 1) will correspond to the values of  $u_j^{(l)}$  and  $D^{(l)}$  at the last iteration, as established in the **Final results** step.

Two important properties justify the use of Algorithm 1 for flexible demand coordination. It can be shown that such algorithm always converges to a final configuration that is also a Nash equilibrium. This is true for any penetration level of flexible demand and for any feasible set of devices parameter ( $E_j^r, P_j^r, \mathcal{A}_j$ ) fulfilling Assumption 1. To formally prove these results, a technical lemma is preliminarily presented:

*Lemma 1:* Consider two demand profiles  $D^- : \mathcal{T} \rightarrow \mathbb{R}_+$  and  $D^+ : \mathcal{T} \rightarrow \mathbb{R}_+$ . Assume there exists  $t_1, t_2 \in \mathcal{T}$  such that:

$$D^-(t_1) < D^+(t_1) \leq D^+(t_2) < D^-(t_2) \quad (7a)$$

$$D^-(t_1) + D^-(t_2) = D^+(t_1) + D^+(t_2) \quad (7b)$$

$$D^-(k) = D^+(k) \quad \forall k \in \mathcal{T} \setminus \{t_1, t_2\}. \quad (7c)$$

For any functional  $V(D) = \sum_{t=1}^T f(D(t))$  where  $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is a strictly convex function, it holds:

$$V(D^+) < V(D^-). \quad (8)$$

The convergence and equilibrium properties of the proposed control strategy can now be formally enunciated.

*Theorem 1:* Consider a population  $\mathcal{N} = \{1, \dots, N\}$  of interruptible flexible devices with continuous power consumption. Under Assumptions 1, 2 and 3, Algorithm 1 asymptotically converges to the final result  $u^*$ .

*Proof:* See Appendix A. ■

*Theorem 2:* Under Assumptions 1, 2 and 3, the power scheduling  $u^*$  returned by Algorithm 1 corresponds to a Nash equilibrium in the electricity market, in the sense specified by Definition 1.

*Proof:* See Appendix B. ■

We can conclude that the application of Algorithm 1 guarantees convergence to a Nash equilibrium under very general assumptions and for any penetration level of flexible demand. In the final stable configuration, no individual appliance can change its scheduled power so as to reduce its energy cost.

The proposed control scheme also shows interesting global properties. We wish to emphasize that, in general, the behaviour of competing agents under a market paradigm does not lead to a maximization of the global welfare. In fact, the greedy market participants will seek to maximize their own private pay-off even if this implies suboptimal global solutions. However, in the case of flexible demand deployment with the proposed technique, the stable configuration characterized as a Nash equilibrium corresponds to flattened demand profiles and reduced generation costs, exhibiting strong affinities with the global optimum. As demonstrated by Lemma 1 and the proof

of Theorem 1, each iteration of the **FOR** cycle in Algorithm 1 (where the scheduled power of single devices is updated) reduces some convex functional  $V$  of aggregate demand. Such functional can be chosen as the total generation costs of the system or quantify the achieved flattening of the aggregate demand profile. It follows that the final configuration not only corresponds to a Nash equilibrium but it also represents a “local minimum” of these quantities. At the induced stable solution, any functional  $V$  of the kind described in Lemma 1, quantifying for example total generation costs, cannot be further reduced by modifying the strategy of a single device. A future study will determine if multiple minima of such kind exist, modifying the control scheme so as to ensure converge to a global optimizer.

### B. Practical Distributed Implementation

The control strategy of Algorithm 1 is straightforward to implement in a practical context, adopting a bi-directional communication scheme between some central entity (e.g. the system operator) and the individual flexible appliances. The operations required to implement the different phases of Algorithm 1 are analysed and discussed separately.

1) **Initialization phase:** As a first step, some initial price signal  $p^{(0)}(t)$  is broadcast to the devices. This quantity can be chosen, for example, as the price of inflexible demand, setting  $p^{(0)}(t) = \Pi(D_i(t))$ . In turn, each device  $j \in \mathcal{N}$  schedules its initial power profile  $u_j^{(0)}$  so as to minimize its expected energy cost:

$$u_j^{(0)}(\cdot) \in \arg \min_{u_j(\cdot) \in \mathcal{U}_j} \sum_{t=1}^T p^{(0)}(t) \cdot u_j(t) \Delta t.$$

The scheduled power profiles are then communicated to the central entity that calculates  $D^{(0)}$  using the expression provided in Algorithm 1.

2) **Power scheduling update:** Each iteration of the **WHILE** cycle, summarized by the scheme in Fig. 1, can be implemented through the following steps:

- The system operator keeps track of the logical variable  $conv$  and the iteration counter  $l$ . At each execution of the **FOR** cycle, the considered demand profile  $D^{(l)}$  is initialized as the one obtained at the previous iteration, with  $D^{(l)} = D^{(l-1)}$ . These operations correspond to steps 2.a), 2.b.i-ii) in Algorithm 1.
- The price signal  $p^{(l-1)} = \Pi(D^{(l-1)})$  is broadcast to the  $j$ -th device (Phase  $j.A$  in Fig. 1). On the basis of this information and its previous power scheduling  $u_j^{(l-1)}$ , the  $j$ -th device can determine the time instants  $t_1$  and  $t_2$  at which perform a power swap that reduces its energy cost. Specifically,  $t_1$  and  $t_2$  must fulfil the following conditions:

$$p^{(l-1)}(t_1) < p^{(l-1)}(t_2) \quad u_j^{(l-1)}(t_1) < P_j^r \quad u_j^{(l-1)}(t_2) > 0.$$

Note that, as a result of Assumption 3 and definition of  $p^{(l-1)}$ , these are equivalent to the inequalities in step 2.b.iii) of Algorithm 1.

- The quantity  $\Delta$  (presented in step 2.b.iv) is internally calculated by the  $j$ -th device. While the first two terms in the min function of the provided expression are

straightforward to derive based on the power rating  $P_j^r$  and previous power scheduling  $u^{(l-1)}$  of the device, the third term requires some extra information from the central entity. This can be the demand profile  $D^{(l-1)}$ , as supposed in Phase  $j.A$  in Fig. 1, or directly correspond to the maximum allowed power shift  $\Delta_{MAX} = (D^{(l-1)}(t_2) - D^{(l-1)}(t_1))/2$ .

- The  $j$ -th device broadcasts the values of  $t_1$ ,  $t_2$  and  $\Delta$  to the central entity (Phase  $j.B$  in Fig. 1), which in turn modifies accordingly the new demand profile  $D^{(l)}$  and the flag variable  $conv$ .

3) **Final results:** After some iterations, as proved in Theorem 1 and 2, the **WHILE** cycle described above is concluded and the devices cannot further reduce their energy costs. Their final power profiles  $u_j^*$ , corresponding to a Nash equilibrium, will be equal to  $u_j^{(l)}$  at the last  $l$ -th iteration. In a similar way, the resulting aggregate demand  $D_a^*$  will correspond to  $D^{(l)}$ .

Note that, if  $D^{(0)}$  in Algorithm 1 is known, the proposed implementation preserves the privacy of the appliances. These are only required to communicate to the system operator their power variation  $\Delta$  (and the time instant  $t_1$  and  $t_2$  at which occurs) and do not need to reveal their parameters  $(E_j^r, P_j^r, \mathcal{A}_j)$ .

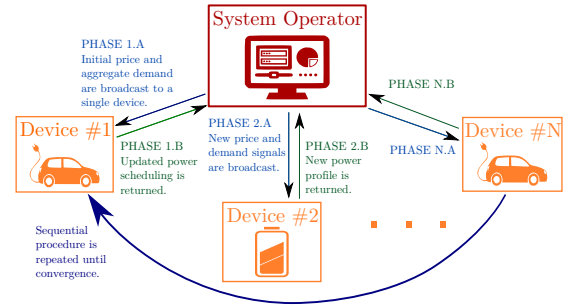


Fig. 1. Practical implementation of the **power scheduling update** in the iterative scheme of Algorithm 1.

**Remark 2:** For a compact and clear representation of its core mechanisms, Algorithm 1 has not been optimized for a practical implementation. To obtain faster convergence, multiple minor changes can be considered. For example, in order to require fewer iterations of its update steps 2.b.i)-2.b.iv), the same device  $j$  could perform multiple updates before passing on to the  $(j+1)$ -th appliance. The order in which devices are contacted could also be modified.

### IV. ONE-SHOT CONTROL STRATEGY

One of the main drawbacks in practical applications of Algorithm 1 is the necessity to sequentially communicate (in principle more than one time) with each flexible appliance. When large populations of agents are considered, this could require significant time and communication resources. For this reason, we propose an alternative coordination scheme that can be implemented through the broadcast of a single price signal (different in general for each appliance). Such scheme not only can be applied to devices with continuous power consumption but it can also accommodate the case of ON/OFF loads. A

(negligible) equilibrium approximation is introduced and the resulting final solution corresponds to an  $\varepsilon$ -Nash equilibrium, defined next.

**Definition 2:** Consider the individual power consumption profiles  $u_j^*(\cdot) \in \mathcal{U}_j$  for all  $j \in \mathcal{N}$ . They correspond to an  $\varepsilon$ -Nash equilibrium in the electricity market if, for some  $\varepsilon > 0$ , the following condition is fulfilled for all  $j \in \mathcal{N}$ :

$$\sum_{t=1}^T \Pi(D_a^*(t)) \cdot u_j^*(t) \Delta t \leq \min_{u_j(\cdot)} \sum_{t=1}^T (\Pi(D_a^*(t)) \cdot u_j(t) \Delta t) + \varepsilon \quad \text{s. t. } u_j(\cdot) \in \mathcal{U}_j \quad (9)$$

where the aggregate demand profile  $D_a^*$  is defined as:

$$D_a^*(t) = D_i(t) + D_j^*(t) = D_i(t) + \sum_{j=1}^N u_j^*(t) \quad \forall t \in \mathcal{T}. \quad (10)$$

When condition (9) holds, each device, by unilaterally changing its power scheduling  $u_j$ , can at most reduce by a small  $\varepsilon$  its energy cost. An expression will be provided for this quantity, showing that in the considered scenarios  $\varepsilon$  is negligibly small.

#### A. Theoretical Control Scheme

The modified control strategy for coordination of flexible demand is defined by Algorithm 2. The only difference with

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#### Algorithm 2 One-shot scheme - Flexible demand coordination

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- **Steps 1) - 2.b.iii)** are equal to Algorithm 1.
- **Step 2.b.iv):** IF  $t_1, t_2$  exist:

$$\begin{aligned} 1) \Delta &= \min \left( \left\{ P_j^r - u_j^{(l-1)}(t_1), u_j^{(l-1)}(t_2) \right\} \right). \\ 2) \Delta_D &= \frac{D^{(l-1)}(t_2) - D^{(l-1)}(t_1)}{2}. \\ 3) \text{ IF } \Delta \leq \Delta_D \end{aligned}$$

$$\begin{aligned} u_j^{(l)}(t_1) &= u_j^{(l-1)}(t_1) + \Delta & u_j^{(l)}(t_2) &= u_j^{(l-1)}(t_2) - \Delta \\ D^{(l)}(t_1) &= D^{(l-1)}(t_1) + \Delta & D^{(l)}(t_2) &= D^{(l-1)}(t_2) - \Delta \\ \text{conv} &= 0. \end{aligned}$$

- **Step 3):** The Nash equilibrium solution is returned

$$D_a^*(\cdot) = D^{(l)}(\cdot) \quad u_j^*(\cdot) = u_j^{(l)}(\cdot) \quad \forall j \in \mathcal{N}.$$


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respect to the previous version is in the calculation, at step 2.b.iv.1), of the swapped power  $\Delta$  between the time instants  $t_2$  and  $t_1$ . In this case,  $\Delta$  corresponds to the maximum feasible amount of power that can be reallocated between the two time instants. This quantity is compared with  $\Delta_D$ , i.e. the maximum amount of power that can be swapped while preserving the following demand inequality:

$$D^{(l)}(t_1) = D^{(l-1)}(t_1) + \delta \leq D^{(l-1)}(t_2) - \delta = D^{(l)}(t_2) \quad \forall \delta \leq \Delta_D.$$

If the condition  $\Delta \leq \Delta_D$  is not verified, no power swap occurs. The main properties of Algorithm 2 are now formalized:

**Theorem 3:** Under Assumptions 1, 2 and 3, Algorithm 2 asymptotically converges to the power profiles  $u^*$ , corresponding to an  $\varepsilon$ -Nash equilibrium as specified by Definition 2, with  $\varepsilon$  defined as:

$$\varepsilon := \max_{j \in \mathcal{N}} \left[ \Pi(D_j^+) - \Pi(D_j^+ - 2P_j^r) \right] E_j^r. \quad (11)$$

Given  $D_a^*$  defined in (10) and using  $\text{supp}$  to denote function support, the quantity  $D_j^+$  is equal to:

$$D_j^+ := \max_{t \in \text{supp}(u_j^*)} D_a^*(t). \quad (12)$$

*Proof:* See Appendix C. ■

**Remark 3:** It can be shown that Algorithm 2 not only converges asymptotically but it is actually completed in a finite number of iterations. Proof is omitted for length reasons.

#### B. Equilibrium Approximation

At the  $\varepsilon$ -Nash equilibrium achieved by Algorithm 2, the single device can at most reduce by  $\varepsilon$  its energy cost. It is important to point out that, in the considered context with large numbers of small flexible appliances, such approximation in the equilibrium condition is theoretically required to establish convergence but it has a negligible effect in practical implementations. This can be verified by considering the value of  $\varepsilon$  which is provided in (11) and quantifies the entity of the equilibrium approximation. In this respect, we remind that our analysis accounts for the demand and price variations introduced by the whole population of price-responsive devices. At the same time, the effect of the single device is assumed to be negligible as its maximum power consumption  $P_j^r$  is orders of magnitude smaller than the total aggregate demand  $D_a^*$  (which determines the energy price). Therefore, the following approximations can be considered for any device  $j \in \mathcal{N}$ :

$$D_j^+ \simeq D_j^+ - 2P_j^r \rightarrow \Pi(D_j^+) \simeq \Pi(D_j^+ - 2P_j^r) \rightarrow \varepsilon \simeq 0.$$

For example, in the scenario analysed in the simulation section, expression (11) returns  $\varepsilon = 1.7 \cdot 10^{-6} \text{€}$ .

More importantly, if one considers the final configuration achieved through Algorithm 2, the maximum price differential that could potentially be exploited by the individual device  $j$  to reduce its energy cost is comparably small. Using algebraic operations similar to the ones in the proof of Theorem 3, one can verify that such price differential corresponds to  $\Pi(D_j^+) - \Pi(D_j^+ - 2P_j^r)$ . For instance, in the proposed case study, its maximum value amounts to  $3.68 \cdot 10^{-5} \text{€}/\text{MWh}$ . This means that in practical implementations, if the final price of aggregate demand is broadcast with reasonable approximation (for example up to the third decimal digit), the devices will not detect any advantageous price differential at the final solution and no power swap will occur.

#### C. Power Consumption as ON/OFF Profile

By using Algorithm 2, it is possible to obtain a stable system configuration where the power consumption  $u_j^*(t)$  of each device  $j$ , at the single time instant  $t$ , is either maximum or zero. This result is obtained also for loads that can modulate their power consumption and it considerably facilitates practical implementations, as discussed in the next subsection. Reminding that  $\mathcal{U}_j$ , defined in (1), denotes the set of feasible power schedules ensuring task completion for device  $j$ , its subset of ON/OFF profiles  $\tilde{\mathcal{U}}_j$  can be defined as follows:

$$\tilde{\mathcal{U}}_j := \{u_j(\cdot) \in \mathcal{U}_j : u_j(t) \in \{0, P_j^r\} \forall t \in \mathcal{T}\}. \quad (13)$$

*Assumption 4:* The subset  $\bar{\mathcal{U}}_j \subseteq \mathcal{U}_j$  is nonempty for all  $j \in \mathcal{N}$ . Equivalently, there exists some constant  $\gamma_j \in \mathbb{N}$  such that the following holds for the parameters of the  $j$ -th device:

$$E_j^r = \gamma_j \cdot P_j^r \cdot \Delta t.$$

Under Assumption 4, each device can always complete its task by only operating at rated power  $P_j^r$  at certain time instants. For this to be case, the required energy  $E_j^r$  must be equal to an integer multiple of  $P_j^r \cdot \Delta t$ . Note that such condition always holds when ON/OFF appliances are considered. In the other cases, it can be fulfilled by choosing a sufficiently small  $\Delta t$  or admitting some approximation in the required energy  $E_j^r$ .

*Remark 4:* The theoretical results and the distributed implementation presented in the rest of this section still hold when Assumption 4 is not considered. In this case a more complicated framework is required, defining  $\bar{\mathcal{U}}_j$  as the set of feasible  $u_j$  that are always equal to 0 and  $P_j^r$ , with the exception of a single time  $\bar{t}$  where  $u_j(\bar{t}) = \left( E_j^r - \left\lfloor E_j^r / (P_j^r \Delta t) \right\rfloor P_j^r \Delta t \right) \Delta t^{-1}$ . The following property of Algorithm 2 can now be introduced:

*Lemma 2:* If  $u_j^{(0)}(\cdot) \in \bar{\mathcal{U}}_j$  in step 1) of Algorithm 2, for the final power profiles  $u_j^*$  it holds:  $u_j^*(\cdot) \in \bar{\mathcal{U}}_j$ .

*Proof:* Given step 3) in Algorithm 2, it is sufficient to prove the same inclusion for any temporary profile  $u_j^{(l)}$ :

$$u_j^{(l)}(\cdot) \in \bar{\mathcal{U}}_j \quad \forall l. \quad (14)$$

This can be verified by induction. Since  $u_j^{(0)}(\cdot) \in \bar{\mathcal{U}}_j$ , we only need to show that (14) holds when  $u_j^{(l-1)}(\cdot) \in \bar{\mathcal{U}}_j$ . From the update equations at step 2.b.iv.3), it is sufficient to observe that in the present case  $\Delta$  is always equal to  $P_j^r$ . ■

#### D. Practical Distributed Implementation

The  $\varepsilon$ -Nash equilibrium returned by Algorithm 2 can be induced in realistic contexts by broadcasting a single price signal (different in general for each appliance). This result can be achieved, both for loads that can modulate their power consumption and for ON/OFF devices, by applying the scheme presented in Fig. 2. The individual implementation steps are

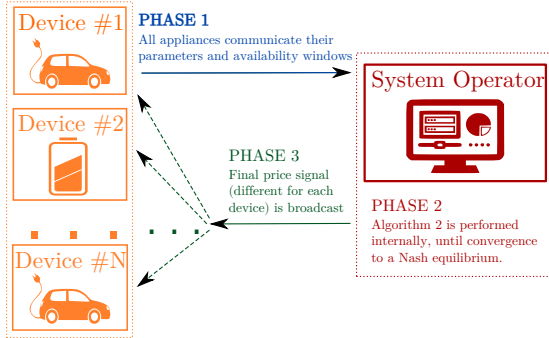


Fig. 2. Implementation of the one-shot scheme of Algorithm 2.

performed as follows:

- *Phase 1:* All appliances communicate their parameters ( $E_j^r, P_j^r, \mathcal{A}_j$ ) to the system operator.
- *Phase 2:* The central entity, using the received information, is able to internally perform all the calculations

required by Algorithm 2. In doing so, it sets  $u_j^{(0)} \in \bar{\mathcal{U}}_j$  so that Lemma 2 holds. This is always possible under Assumption 4, also in the case of devices that can modulate their power consumption. For instance, one can set  $u_j^{(0)}(t) = P_j^r$  in the first  $\gamma_j = E_j^r / (P_j^r \cdot \Delta t)$  time instants of the availability interval  $\mathcal{A}_j$  and zero elsewhere. The obtained final results are the desired power profiles  $u_j^*$  for the individual appliances and the resulting demand  $D_a^*$ , defined in (10). As established in Theorem 3, these quantities correspond to an  $\varepsilon$ -Nash equilibrium in the system.

- *Phase 3:* The power schedules  $u_j^*$  are induced, in a distributed manner, to the population of flexible appliances. A different price  $p_j^*$  is broadcast to each appliance  $j$  which, on the basis of this signal, independently schedules its own cost-minimizing power consumption, corresponding to  $u_j^*$ .

We now characterize in detail the price signals  $p_j^*$  broadcast in Phase 3. In order to induce an  $\varepsilon$ -Nash equilibrium, these must be designed so that  $u_j^*$ , calculated in Phase 2, is the unique minimizer of the energy cost minimization problem for the  $j$ -th device:

$$u_j^*(\cdot) = \arg \min_{u_j(\cdot)} \sum_{t=1}^T p_j^*(t) \cdot u_j(t) \Delta t \quad (15)$$

s. t.  $u_j(\cdot) \in \mathcal{U}_j$

We remind that, from Lemma 2, all  $u_j^*$  are ON/OFF power profiles belonging to the set  $\bar{\mathcal{U}}_j$ , defined in (13). Note that this property is crucial for the calculation of a price signal  $p_j^*$  satisfying (15), since in this case  $p_j^*$  needs only to fulfil the following property:

$$p_j^*(t_1) < p_j^*(t_2) \quad \forall (t_1, t_2) : t_1 \in \text{supp}(u_j^*(\cdot)), t_2 \notin \text{supp}(u_j^*(\cdot)) \quad (16)$$

It is generally not possible to simply choose  $p_j^* = \Pi(D_a^*)$  in order to induce the  $\varepsilon$ -Nash equilibrium. In fact, if  $D_a^*$  takes constant values over a certain time interval, there may exist  $t_1 \in \text{supp}(u_j^*(\cdot))$  and  $t_2 \in \mathcal{A}_j \setminus \text{supp}(u_j^*(\cdot))$  such that:

$$p_j^*(t_1) = p_j^*(t_2)$$

thus violating condition (16). In other words, the device  $j$  could potentially choose multiple power schedules yielding the same energy cost. For this reason, the price signals  $p_j^*$  are chosen so as to incentivize consumption at specific time instants over a flat demand valley. For example, given  $u^*$  and  $D_a^*$  returned by the resolution of Algorithm 2, they can be designed as follows:

$$p_j^*(t) := \begin{cases} \Pi(D_a^*(t)) & \text{if } t \in \text{supp}(u_j^*(\cdot)) \\ \lambda \cdot \Pi(D_a^*(t)) & \text{if } t \notin \text{supp}(u_j^*(\cdot)) \end{cases} \quad (17)$$

where  $\lambda > 1$  is some positive design constant. It is straightforward to verify that  $p_j^*$  as defined in (17) fulfils (16) and therefore (15), thus inducing the sought  $\varepsilon$ -Nash equilibrium. Examples of broadcast price signals for different devices and power schedules are presented in Fig. 4, in the simulation section.

*Remark 5:* The actual electricity price, once the final configuration is reached, will be equal for the whole population



and will correspond to  $\Pi(D_a^*)$ . As discussed in Section IV-B, at the  $\varepsilon$ -Nash equilibrium the price differentials that could potentially be exploited by the individual device to reduce its energy cost become negligible. Therefore, no agent will change its power scheduling when  $p_j^*$  is replaced by  $\Pi(D_a^*)$ . Moreover, by choosing  $p_j^*$  as in (17), the energy cost sustained by each load will remain the same when either  $p_j^*$  or  $\Pi(D_a^*)$  are considered.

*Remark 6:* There is a clear trade-off between computational time/communication resources and agents' privacy in the two presented coordination schemes. In fact, differently from the iterative control strategy presented in Section III, implementation of Algorithm 2 does not require repeated exchange of information between the system operator and the agents. Since all the iterative calculations are internally performed by the central entity, lower communication requirements and computational times are achieved. On the other hand, in order to obtain this result, the privacy of the devices is reduced as they are required to divulge their parameters  $E_j^r$ ,  $P_j^r$  and  $\mathcal{A}_j$ . The one-shot coordination scheme presented in this section can potentially be applied in a receding horizon context, allowing to implicitly account for uncertainties and appliances' requirements that dynamically change over time. In particular, on the basis of the parameters initially received by the price-responsive loads, the central entity could run Algorithm 2 over a 24-hour interval and broadcast the resulting prices to the loads. After a short amount of time (for example 30 minutes), the devices could communicate their updated parameters to the system operator which, in turn, could repeat its calculations and broadcast updated price signals. This whole procedure can then be iteratively repeated over time. Future work will investigate the equilibrium and convergence properties of this approach.

### E. Fairness and flexibility incentives

The pricing scheme and the resulting power allocation  $u^*$  induced by Algorithm 2 have some important fairness properties, summarized by the propositions presented next.

*Proposition 1:* Consider two devices  $j_1, j_2 \in \mathcal{N}$  and denote by  $\bar{C}_{j_1}^*$  and  $\bar{C}_{j_2}^*$  their energy cost at the  $\varepsilon$ -Nash equilibrium induced by Algorithm 2:

$$\bar{C}_{j_1}^* = \sum_{t=1}^T \Pi(D_a^*(t)) \cdot u_{j_1}^*(t) \Delta t \quad (18a)$$

$$\bar{C}_{j_2}^* = \sum_{t=1}^T \Pi(D_a^*(t)) \cdot u_{j_2}^*(t) \Delta t. \quad (18b)$$

If the devices are identical ( $E_{j_1}^r = E_{j_2}^r$ ,  $P_{j_1}^r = P_{j_2}^r$  and  $\mathcal{A}_{j_1} = \mathcal{A}_{j_2}$ ) then it holds:

$$|\bar{C}_{j_1}^* - \bar{C}_{j_2}^*| \leq \varepsilon. \quad (19)$$

*Proof:* If the proposition statement is not true we can assume, without loss of generality, that  $\bar{C}_{j_1}^* > \bar{C}_{j_2}^* + \varepsilon$ . Given that  $u_{j_2}^* \in \mathcal{U}_{j_1}$  (devices are identical and  $\mathcal{U}_{j_1} = \mathcal{U}_{j_2}$ ), it holds:

$$\begin{aligned} \min_{u_{j_1}(\cdot)} \quad & \sum_{t=1}^T \Pi(D_a^*(t)) \cdot u_{j_1}(t) \Delta t + \varepsilon \leq \bar{C}_{j_2}^* + \varepsilon < \bar{C}_{j_1}^* \\ \text{s. t.} \quad & u_{j_1}(\cdot) \in \mathcal{U}_{j_1} \end{aligned} \quad (20)$$

This implies that (9) should not hold for  $j = j_1$ , contradicting the initial hypothesis of  $\varepsilon$ -Nash equilibrium and thus concluding the proof. ■

*Proposition 2:* Consider two devices  $j_1, j_2 \in \mathcal{N}$  and assume the following relationships between their parameters:

$$\begin{aligned} E_{j_1}^r &= E_{j_2}^r \\ P_{j_1}^r &= P_{j_2}^r \\ \mathcal{A}_{j_1} &\subseteq \mathcal{A}_{j_2}. \end{aligned}$$

Let  $\bar{C}_{j_1}^*$  and  $\bar{C}_{j_2}^*$  in (18) denote their energy costs at the  $\varepsilon$ -Nash equilibrium induced by Algorithm 2. If  $\varepsilon = 0$ , then it holds:

$$\bar{C}_{j_1}^* \geq \bar{C}_{j_2}^*.$$

*Proof:* Since  $\mathcal{U}_{j_1} \subseteq \mathcal{U}_{j_2}$  under the current assumptions, we have:

$$\begin{aligned} \min_{u_j(\cdot)} \sum_{t=1}^T \Pi(D_a^*(t)) \cdot u_j(t) \Delta t &\geq \min_{u_j(\cdot)} \sum_{t=1}^T \Pi(D_a^*(t)) \cdot u_j(t) \Delta t \\ \text{s. t. } u_j(\cdot) \in \mathcal{U}_{j_1} &\quad \text{s. t. } u_j(\cdot) \in \mathcal{U}_{j_2} \end{aligned} \quad (21)$$

As we are assuming  $\varepsilon = 0$ , at the considered  $\varepsilon$ -Nash equilibrium the condition (9) becomes an equality. This implies that the left and right-hand side of (21) correspond respectively to  $\bar{C}_{j_1}^*$  and  $\bar{C}_{j_2}^*$ , thus concluding the proof. ■

Two important considerations can be made from the results presented above. It follows from Proposition 1 that the energy costs of two identical devices, at equilibrium, differ at most by a negligible quantity  $\varepsilon$ . This is true even if the price signals they receive (and their resulting power profiles) are different, showing that the proposed technique guarantees a fair cost distribution among the devices. Moreover, from Proposition 2, we can conclude that the considered pricing scheme incentivizes the devices to be flexible: the larger is their availability time window, the lower will be their energy costs. This is true when  $\varepsilon$  can be approximated as zero, as discussed in Section IV-B.

## V. SIMULATION RESULTS

The performance of the proposed control strategies is now evaluated in a simulation context, considering a time interval of 24 h and a time discretization step  $\Delta t = 0.25$  h. The UK power system is chosen for our case study, assuming that the profile of inflexible demand  $D_i$  is equal to historical data of total power consumption in the system [23]. A significant penetration of flexible demand has been considered. Specifically, we analyse a population of  $N = 2 \cdot 10^6$  electric vehicles (EVs) aiming at fully charging their batteries during night-time. It is assumed that the rated power is equal for the whole population, with  $P_1^r = \dots = P_N^r = 12$  KW. The energy  $E_j^r$  required for task completion is different in general for each vehicle and depends on the state of charge of its battery when it is connected to the grid. In the chosen scenario,  $E_j^r$  follows a gaussian distribution with mean  $\mu_E = 30$  KWh and standard deviation  $\sigma_E = 1.5$  KWh, resulting in a total energy requirement by the EVs of about 60 GWh. Each vehicle also specifies its time availability window, i.e. the time instants during which it is connected to the grid and available for charging. We assume that a vehicle  $j \in \mathcal{N}$  is available in the continuous

time interval  $[t_j, t_j + d_j]$ , where  $t_j$  and  $d_j$  also follow gaussian distributions, with the following mean and standard deviation:

$$\mu_t = 20:00 \text{ h} \quad \sigma_t = 1 \text{ h} \quad \mu_d = 10 \text{ h} \quad \sigma_d = 1 \text{ h}.$$

It is straightforward to determine the availability interval  $\mathcal{A}_j$  for the  $j$ -th vehicle in the chosen discrete time variable:

$$\mathcal{A}_j = \{t \in \mathcal{T} : t_j \leq t \cdot \Delta t \leq t_j + d_j\}.$$

Note that the sets  $\mathcal{A}_j$  characterize the driving patterns of the considered EVs, specifying at which time instants the  $j$ -th vehicle is not being driven and it is therefore available for charging. Different driving scenarios also lead to different values of  $E_j^r$ : the longer a car is driven, the higher will be its required energy when plugged into the grid. We wish to emphasize that the proposed coordination schemes guarantee convergence to a Nash equilibrium for all driving patterns and corresponding parameter values  $\mathcal{A}_j$  and  $E_j^r$ , as long as Assumption 1 is verified and each vehicle is connected at least for the minimum time required to complete its charge. However, the parameters  $\mathcal{A}_j$  have an impact on the convergence speed of the algorithm. If their values are sufficiently diversified, fewer updates of the EVs charging profiles will be required. On the other hand, if all vehicles tend to be available at similar time instants, a larger number of iteration steps are necessary to avoid demand peaks and efficiently spread power consumption over time.

The coordination of the electric vehicles has been performed with Algorithm 2, choosing  $D^{(0)}$  as the profile of aggregate demand obtained if the price  $\Pi(D_i)$  were to be broadcast to the appliances. To speed up calculations, it is assumed that the power update of steps 2.b.iv.1)-2.b.iv.3) can be performed multiple times on the device  $j$  before moving on to the agent  $j+1$ . This modification does not alter the equilibrium results presented in Theorem 3. The simulation has been performed in MATLAB on a computer with a 4-core 2.40 GHz Intel(R) Xeon(R) E5620 processor and 12 GB of RAM. The calculations have been completed in about 2 hours but, in real applications, reduced computational times can be easily obtained by further optimization of the algorithm, utilization of more powerful machines and possibly aggregation of the flexible loads. It follows that the computational complexity of the proposed algorithm does not constitute a practical limit to its implementation. The profiles  $D^{(l)}$ , corresponding to the aggregate power demand  $D_a$  obtained at the  $l$ -th iteration of the coordination algorithm, are shown in Fig. 3 for different values of  $l$ . It is immediate to verify that  $D_a = D^{(0)}$  does not correspond to a Nash equilibrium. In this case, most of the appliances are operating between  $t = 2:00\text{h}$  and  $t = 5:00\text{h}$ , increasing demand (and therefore prices) in such interval. They could reduce their costs by shifting power consumption in the two valleys that appear in  $D^{(0)}$ . By a gradual update of the power scheduling, the algorithm converges to a flat profile of aggregate demand. When  $l = 1.25 \cdot 10^6$  and approximatively half of the devices have updated their strategy, significant peak-shaving/valley-filling has already been introduced. The final demand profile  $D_a^* = D^{(l)}$  with  $l = 4 \cdot 10^6$  (magenta trace), obtained when the algorithm has converged to equilibrium, is completely flat. It should be emphasized that, when Algorithm

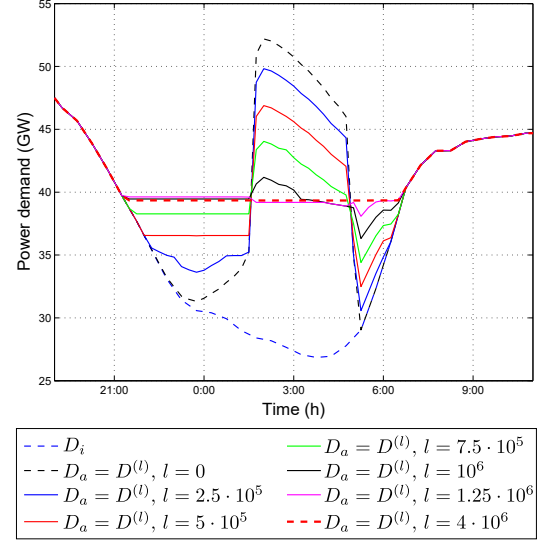


Fig. 3. Aggregate demand profile at different iterations  $l$  of Algorithm 2.

2 is implemented, the single  $l$ -th iteration corresponds to internal calculations of the coordinating entity and does not require actual communication between the flexible loads and the system operator.

As a final step, the price signals  $p_j^*$  that need to be broadcast to the devices in order to induce the Nash equilibrium are calculated according to (17). Some representative examples of their values are shown in Fig. 4. In all three cases, it can be

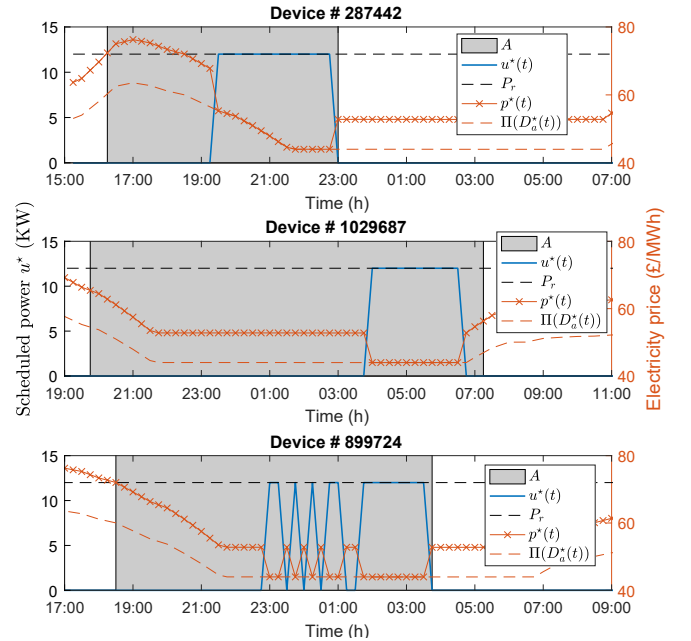


Fig. 4. Examples of scheduled power consumption profiles  $u^*$  for different devices (subscript  $j$  is neglected), together with corresponding availability interval  $\mathcal{A}$ , broadcast demand price  $p^*$  and final price  $\Pi(D_a^*)$  of aggregate demand.

seen that  $u^*$  corresponds to the unique minimizer of the energy cost when the price signal  $p^*$  is considered. Note that each

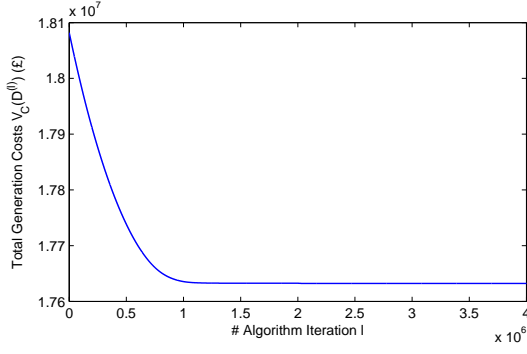


Fig. 5. Total generation costs  $V_C(D^{(l)})$  of the system, at the  $l$ -th iteration of Algorithm 2.

device operates at maximum power during the time instants (within the grey area of its availability interval) with the lowest price values. Furthermore, optimality of  $u^*$  is preserved when the final price  $\Pi(D_a^*)$  is considered, as there is no time instant with cheaper electricity in the grey availability interval, where power consumption can be shifted. To evaluate the global performance of the final stable solution, we consider the total generation costs of the system at different iterations of the coordination algorithm. Assuming a marginal generation cost  $C_M(G) = a \cdot G + b$ , the total generation costs for the aggregate demand  $D_a$  can be rewritten as:

$$V_C(D_a(\cdot)) = \sum_{t=1}^T \frac{a}{2} D_a^2(t) + b \cdot D_a(t).$$

Note that  $V = V_C$  fulfils the functional properties of Lemma 1, implying from the proof of Theorem 3 that the total generation costs are reduced at each algorithm iteration, until convergence to the Nash equilibrium. The decreasing values of  $V_C$  in the current case study are shown in Fig. 5. The asymptotic behaviour of the curve is a result of the specific way in which the power scheduling of the individual device is updated at each iteration. The feasible power swap is performed between the time instants with higher price differential, corresponding to higher reductions in  $V_C$ . As the demand profile is gradually flattened, the price differentials are reduced, implying diminished reductions of the generation costs.

The proposed control scheme (hereby denoted as “*Nash-equilibrium*” - **NE**) has been compared to other strategies, assuming that devices aim at completing their tasks as soon as possible (“*time-greedy*” - **TG**) or minimize their energy cost on the basis of the price of inflexible demand (“*price-greedy*” - **PG**). The corresponding profiles of aggregate demand are compared in Fig. 6. It is evident that the **TG** and **PG** strategies do not introduce any valley-filling and, on the contrary, lead to new peaks in the demand profile. It can also be verified that an equilibrium is not reached: in both cases, the individual devices could reduce their energy cost by shifting part of their power consumption to the new valleys of the aggregate demand (with lower prices). The performance of the three considered schemes is also evaluated according to some quantities of interest, presented in Table I. We compare the total generation costs  $V_C$ , the average energy cost  $C_{av}$  sustained by the individual device and its average time of task completion

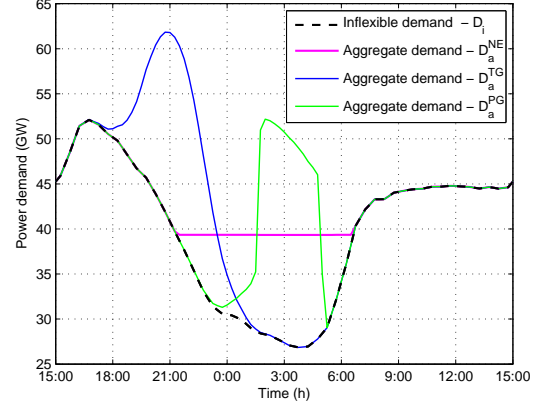


Fig. 6. Inflexible demand profile  $D_i$  and aggregate power demand for the **NE** policy (magenta trace), **TG** policy (blue trace) and **PG** policy (green trace).

TABLE I  
TOTAL GENERATION COST  $V_C$ , AVERAGE ENERGY COST  $C_{av}$  AND AVERAGE FINAL TIME OF OPERATION  $t_{av}^{fin}$  FOR INDIVIDUAL DEVICES.

	$V_C$	$C_{av}$	$t_{av}^{fin}$
NE	$1.7632 \cdot 10^7 \text{ £}$	$1.76 \text{ £}$	5:10 h
PG	$1.8082 \cdot 10^7 \text{ £}$	$2.31 \text{ £}$	4:54 h
TG	$1.9176 \cdot 10^7 \text{ £}$	$2.71 \text{ £}$	23:20 h

$t_{av}^{fin}$ . As expected, the proposed distributed control scheme **NE** allows to achieve lower generation costs for the system and considerably reduces the price paid by the individual devices (24% reduction with respect to the **PG** policy). This is obtained with an average time of task completion  $t_{av}^{fin}$  which is slightly higher in the **NE** scenario. In this case, since the appliances cannot all operate at the same time, it is necessary to distribute their power consumption over a larger time interval.

## VI. CONCLUSIONS

This paper presents novel distributed control schemes for efficient deployment of large populations of price-responsive loads in the power system. Within a game-theoretical framework, an iterative distributed algorithm is initially proposed, theoretically proving its capability to induce a Nash equilibrium in the electricity market for any penetration level of flexible demand. This is used as a starting point for the formulation of a one-shot algorithm that guarantees a faster and easier practical implementation, at the price of a minimum degradation in the equilibrium results (theoretically quantified). Case studies of future scenarios of the UK-power system, with large penetration of flexible devices, are used to assess the performance of the proposed control scheme.

Our ongoing work is focused on the inclusion of network constraints and on the assessment of ancillary services provision by the flexible loads. Moreover, the modeling framework is being adjusted for the expected future energy market paradigm, designing the broadcast prices at a household level. Extensions of the proposed technique to a receding horizon

framework will also be evaluated, determining the resulting convergence and stability properties. Finally, the global optimality of the sought Nash equilibrium will be investigated, in order to obtain an improved control scheme which also induces maximum total welfare.

#### APPENDIX A PROOF OF THEOREM 1

Asymptotic convergence of Algorithm 1 is proved with Lyapunov techniques. A functional  $V(D)$ , lower bounded and such that Lemma 1 can be applied, is chosen (e.g.  $V(D) = \sum_{t=1}^T D^2(t)$ ). We show that, when the **IF** condition in step 2.b.iv) is verified, we have  $V(D^{(l)}) < V(D^{(l-1)})$ . This implies that  $V(D)$  asymptotically converges to some minimum value. At such minimum, the **IF** condition is never fulfilled and step 2.b.iv) is not performed anymore. It follows that the variable *conv*, initialized at step 2.a), will remain equal to 1 throughout the **FOR** cycle of step 2.b), ensuring that step 3) is reached. To prove the reduction of  $V(D)$  and therefore the theorem statement, we apply Lemma 1 with  $D^-(\cdot) = D^{(l-1)}(\cdot)$  and  $D^+(\cdot) = D^{(l)}(\cdot)$ , when the **IF** condition in step 2.b.iv) is verified. In this case,  $D^-$  and  $D^+$  only differ at  $t_1$  and  $t_2$ , implying that (7c) is fulfilled. Moreover, since  $D^+(t_1) = D^-(t_1) + \Delta$  and  $D^+(t_2) = D^-(t_2) - \Delta$ , also (7b) holds. To check that also (7a) is verified, note that its first and last inequalities follow from  $\Delta > 0$ . In fact, from the properties of  $t_1$  and  $t_2$  in step 2.b.iii),  $\Delta$  is the minimum of three positive quantities. It follows:

$$\begin{aligned} D^+(t_1) &= D^{(l)}(t_1) = D^{(l-1)}(t_1) + \Delta = D^-(t_1) + \Delta > D^-(t_1) \\ D^-(t_2) &= D^{(l-1)}(t_2) = D^{(l)}(t_2) + \Delta = D^+(t_2) + \Delta > D^+(t_2). \end{aligned}$$

The second inequality of (7a) holds since, from the expression of  $\Delta$  in step 2.b.iv) of Algorithm 1, we have:

$$2\Delta \leq D^{(l-1)}(t_2) - D^{(l-1)}(t_1) = D^+(t_2) - D^+(t_1) + 2\Delta.$$

Given that (7) is fulfilled by the current choice of  $D^-$  and  $D^+$ , we have  $V(D^{(l)}) < V(D^{(l-1)})$  at each iteration of step 2.b.iv), concluding the proof.

#### APPENDIX B PROOF OF THEOREM 2

To verify the theorem statement, it is sufficient to show that  $u^*$  returned at step 3) of Algorithm 1 fulfils condition (5). To this end, we point out that any  $u_j \in \mathcal{U}_j$  can be expressed as the sum of  $u_j^*$  and  $P$  distinct power swaps:

$$u_j(\cdot) = u_j^*(\cdot) + \sum_{p=1}^P \delta_p(\cdot) \quad (22)$$

Each term  $\delta_p$ , for some  $t_1^p, t_2^p \in \mathcal{A}_j$  and  $\Delta_p > 0$ , has the following structure:

$$\delta_p(t) = \begin{cases} \Delta_p & \text{if } t = t_1^p \\ -\Delta_p & \text{if } t = t_2^p \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

This is true since all feasible power profiles in  $\mathcal{U}_j$  (including  $u_j$  and  $u_j^*$ ) have equal total sum. Without loss of generality,

one can also assume that each  $\delta_p$  is a feasible power swap between two time instants, implying:

$$0 < \Delta_p \leq \min(P_j^r - u_j^*(t_1^p), u_j^*(t_2^p)). \quad (24)$$

The cost function in the right-side of (5) can be evaluated as:

$$\begin{aligned} \sum_{t=1}^T \Pi(D_a^*(t)) \cdot u_j(t) \Delta t &= \sum_{t=1}^T \Pi(D_a^*(t)) \cdot u_j^*(t) \Delta t \\ &+ \sum_{p=1}^P [\Pi(D_a^*(t_1^p)) - \Pi(D_a^*(t_2^p))] \cdot \Delta_p \Delta t. \end{aligned} \quad (25)$$

If  $u_j^*$  does not fulfil (5), there is at least one  $(t_1^p, t_2^p)$  such that:

$$\Pi(D_a^*(t_1^p)) < \Pi(D_a^*(t_2^p)). \quad (26)$$

As  $\Pi$  is strictly monotone increasing, this is equivalent to:

$$D_a^*(t_1^p) < D_a^*(t_2^p). \quad (27)$$

Moreover, since  $\Delta_p$  must be positive, from (24) we have:

$$u_j^*(t_1^p) < P_j^r \quad (28a)$$

$$u_j^*(t_2^p) > 0. \quad (28b)$$

Note that (27) and (28) are equivalent to the conditions of step 2.b.iii) in Algorithm 1 for  $t_1 = t_1^p$  and  $t_2 = t_2^p$ , when  $D^{(l-1)} = D_a^*$  and  $u_j^{(l-1)} = u_j^*$ . Since step 3) of Algorithm 1 has been reached, they are never fulfilled, for any  $j \in \mathcal{N}$  and  $t_1^p, t_2^p \in \mathcal{T}$ . This implies that (26) never holds, (5) is fulfilled and the theorem is verified.

#### APPENDIX C PROOF OF THEOREM 3

Convergence of Algorithm 2 can be verified as in the proof of Theorem 1, showing that each iteration of steps 2.b.i)-2.b.iv) reduces the value of some functional  $V$  of aggregate demand. If  $V$  is chosen to be lower bounded (e.g.  $V(D) = \sum_{t=1}^T D^2(t)$ ), it will asymptotically converge to some minimum value, at which further iterations of steps 2.b.i)-2.b.iv) are not performed and the algorithm is completed. It is now proved that the resulting power profiles  $u_j^*$  correspond to an  $\varepsilon$ -Nash equilibrium, with  $\varepsilon$  fulfilling (11). From the proof of Theorem 2, any feasible  $u_j(\cdot) \in \mathcal{U}_j$  can be expressed as  $u_j(\cdot) = u_j^*(\cdot) + \sum_{p=1}^P \delta_p(\cdot)$ , with  $\delta_p$  as in (23). To reduce the cost of the  $j$ -th device with respect to the case  $u_j = u_j^*$ , it is necessary to find  $t_1^p, t_2^p \in \mathcal{A}_j$  such that:

$$D_a^*(t_1^p) < D_a^*(t_2^p) \quad (29a)$$

$$u_j^*(t_1^p) < P_j^r \quad (29b)$$

$$u_j^*(t_2^p) > 0 \quad (29c)$$

When the final  $u^*$  is reached, further reductions of  $V(D)$  cannot be achieved through steps 2.b.i)-2.b.iv). As a result, if there exist  $t_1^p$  and  $t_2^p$  as specified above, the **IF** condition in step 2.b.iv.3) is not fulfilled for  $D^{(l-1)}(\cdot) = D_a^*(\cdot)$ :

$$P_j^r \geq P_j^r - u_j^*(t_1^p) \geq \Delta > \frac{D_a^*(t_2^p) - D_a^*(t_1^p)}{2} = \Delta_D. \quad (30)$$



As  $D_j^+$  in (12) is the maximum value of  $D_a^*$  over the support of  $u_j^*$ , if condition (29a) is ever fulfilled than it must also hold for  $D_a^*(t_2^p) = D_j^+$ . Applying this substitution in (30) yields:

$$D_a^*(t_1^p) > D_j^+ - 2P_j^r. \quad (31)$$

Denote now by  $C^*$  and  $C$  the total costs of the individual device when  $u_j^*$  and an arbitrary feasible  $u_j = u_j^* + \sum_{p=1}^P \delta_p$  are applied, respectively. Suppose also that, for the chosen  $u_j$ , we have  $C < C^*$ . Similarly to (25), the following expression can be provided:

$$C = C^* + \sum_{p=1}^P [\Pi(D_a^*(t_1^p)) - \Pi(D_a^*(t_2^p))] \Delta_p \Delta t = C^* + \Delta C \quad (32)$$

where each pair  $(t_1^p, t_2^p)$  fulfils (29). The following chain of inequalities is now considered:

$$\begin{aligned} \Delta C &= \sum_{p=1}^P [\Pi(D_a^*(t_1^p)) - \Pi(D_a^*(t_2^p))] \Delta_p \Delta t \\ &\stackrel{a}{\geq} \sum_{p=1}^P [\Pi(D_a^*(t_1^p)) - \Pi(D_j^+)] \Delta_p \Delta t \\ &\stackrel{b}{\geq} \sum_{p=1}^P [\Pi(D_j^+ - 2P_j^r) - \Pi(D_j^+)] \Delta_p \Delta t \\ &\stackrel{c}{\geq} [\Pi(D_j^+ - 2P_j^r) - \Pi(D_j^+)] E_j^r = \Delta C_{\min}. \end{aligned} \quad (33)$$

Since the price function  $\Pi$  is monotone increasing, inequality  $a$  is fulfilled because  $D_a^*(t_2^p) \leq D_j^+$  by definition of  $D_j^+$  in (12). Inequality  $b$  can be proved in a similar manner, considering that in this case (31) holds. To verify that  $c$  holds, note that  $\Pi(D_j^+ - 2P_j^r) - \Pi(D_j^+) < 0$ . Moreover, since the shifted power cannot be greater than the total power consumption, we have:

$$\sum_{p=1}^P \Delta_p \Delta t \leq \sum_{t=1}^T u_j^*(t) \Delta t = E_j^r.$$

From (32) and (33),  $C^* = C - \Delta C \leq C - \Delta C_{\min}$ . From (11) we have  $\varepsilon \geq -\Delta C_{\min}$  and therefore  $C^* \leq C + \varepsilon$ . Such inequality holds for any  $C$  and it is equivalent to (9) when  $C$  is minimum, concluding the proof.

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