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Color Image Simplification by Morphological Hierarchical Segmentation and Color Clustering

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***Abstract*—Morphological hierarchical segmentation of color images may be achieved in a straightforward way by measuring the persistence of regional minima from color gradients and using these measurements as a criterion to select markers for the watershed from markers framework. Since color has an implicit role in the selection of markers, the segmentation process may provide a bad combination of distinct colored regions, and this may lead to a distorted image simplification. This paper proposes a new method to color image simplification in which the importance of color is raised because color information is added to the marker selection process. Such method provides finer control over the final number of regions (n) and the resulting number of colors (c). A color clustering method splits the regional minima in to c minima sets, each of which has a representative color. The most prominent regional minima from each minima set are selected to form the markers for the segmentation framework. In the final segmentation, the color assigned to a region is given by the representative color bound to the marker that points to the region. It leads to an image whose segmented regions are quantized to fewer distinct colors.**

***Keywords*—morphological hierarchical segmentation; color image segmentation; color clustering.**

I. Introduction

The dawn of the cheaper technologies to acquire and store digital images made the waste of precious color information due the need of a better performance, be computational or in the storage, be decreased proportionally with the current computational limitations. Thus, it is natural that color image processing techniques gain popularity, even when some of these techniques borrows solutions from techniques designed to grayscale images. Color image processing techniques have been applied to several frameworks such as image compression, information transmission via Internet, document processing, digital libraries, artistic image edition, etc.

Color image simplification consists of the application of image processing techniques to simplify color images according to spatial and/or color terms. Such simplification techniques may lead to output images which are, for instance, sampled (or partitioned) in a set of disjoint regions (or subsets) [1], [2] or to output images which color set is quantized [3]. Color image simplification often has a major supporting role in several applications such as image display and compression [4], video coding and pattern recognition techniques [5].

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One of the broadly research fields in color image processing is the design of techniques to compute color gradients. Despite graylevel gradients have been extensively studied, they are not straightforwardly extended to the color case: there are cases when intensity information is not enough to detect boundary between regions of different colors [6]. Usually, given a color space model and a chosen similarity metric, color gradients techniques combines color information to produce a graylevel image which areas of color discontinuity are overvalued.

One important purpose of color gradients is its supporting role to color image segmentation [7], since it reduces the color case to the well studied graylevel one. Mathematical Morphology [8] provides a segmentation framework known as watershed from markers [9]: such framework gives the partitioning of an image in function of its filtered gradient and the simulation of a flood process in this gradient, which can be considered as a topological surface.

Watershed from markers is the basis for a way to compute hierarchical segmentation. An image is hierarchically segmented in function of an hierarchy of nested partitions. Through a systematic selection of markers (initially imposed in all regional minima), segmentation may be refined or coarsened. Hierarchical segmentation may lead to the representation of objects in an image in several detail levels.

A straightforward way to perform color image simplification is by a combination of color gradients [10], [6], [11], [12] and morphological hierarchical segmentation [1]: the regional minima from the color gradient are sorted in a decreasing order according to their extinction values [2]. The most prominent regional minima are chosen as markers to the well known watershed from markers segmentation framework [9], [8]. Each subset of the resulting partitioning defines a new simplified region, and the representative color to the subset may be given by the average color inside such region in the input image.

The partitioning of a color image in to n regions by the framework described above takes account of the structural information of the gradient (the measurement of its peak and valleys). Color has an implicit role in the segmentation results - the peaks and valleys in the gradient image are given by local color similarity measurements. The sorting of the regional minima is given only by their extinction values. As a consequence, two quite distinct colored regions may be merged, depending on the value n, and the average color inside this region may be too distorted, compared to the original image. In addition, this approach provides no control over the number of colors in the output image: it may be as high as n.

This paper proposes a new color hierarchical segmentation scheme by combining color gradients, classical hierarchical morphological segmentation and color clustering. In this scheme, the importance of color is raised by adding color information to the sorting of the regional minima. Minima are still sorted according to their extinction values, but not before they are split into c minima sets. The most prominent regional minima from each minima set are applied as markers in the segmentation framework. The proposed approach allows the control of the final number of regions (n) and the final number of colors (c). This is possible because the cited minima sets are computed by clustering the regional minima colors in the c sets.

This paper is organized as follows: Section II reviews some preliminary concepts. Section III introduces the proposal for hierarchical image segmentation by adding the clustering criterion to the regional minima sorting. Experimental results are shown in Section IV and conclusion and further discussion are provided in Section V.

II. Preliminary Concepts

Let $E \subset \mathbb{Z} \times \mathbb{Z}$ be a rectangular finite subset of points. Let $K = [0, k]$ be a totally ordered set. Denote by $Fun[E, K]$ the set of all functions $f : E \to K$. A *graylevel image* is one of these functions.

Let $Fun[E, \mathbf{C}]$ be the set of all functions $\mathbf{f} : E \to \mathbf{C}$, where $\mathbf{C} = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$ and $\mathbf{c}_i \in \mathbb{R}_+ : 0 \leq \mathbf{c}_i \leq 1$. $Fun[E, \mathbf{C}]$ denotes the set of all *color images*.

Let B_c be the structuring element that defines a connectivity [8]. A *connected subset* of E is a subset $X \subset E$ such that, $\forall x, y \in X$, there is a path $P(x, y) = (p_0, p_1, ..., p_t)$, $p_i \in X$, such that $p_0 = x$, $p_t = y$ and $\forall i \in [0, t-1], \exists b \in B_c : p_i + b = p_{i+1}$.

Let $f \in Fun[E, K]$. A *regional minimum* of f is a connected subset $M \subset E$ such that, $\forall x \in M$, $\forall b \in B_c$, $f(x) \leq f(x+b)$. Let $\mathtt{Min}(f)$ be the set of all regional minima of f. Let $\#\mathtt{Min}(f)$ be the cardinality of $\mathtt{Min}(f)$.

Let $\mathbf{M}$ be a set of markers, denoted by a set of disjoint connected subsets of E. Let $\mathcal{W}_{\mathbf{M}}(f)$ be the watershed from markers [8] operator which segments $f \in Fun[E, K]$ in function of $\mathbf{M}$.

A. Color Gradient

Literature presents several ways to compute color gradients [13], [7]. They are usually designed taking into account the analysis of each color component image under a certain color space model. For instance, color gradients may be designed under RGB [13], HSL [10] or L*a*b* [12].

In this work, the color gradient is computed under the L*a*b* color space model. In this perceptual model, the Euclidean distance may be applied to evaluate the perceptual distance between two colors. Such equivalence does not hold for larger distances [12], but the mentioned distance function suffices as the metric applied to compute the color gradient.

Let $\mathbf{f} \in Fun[E, \mathbf{C}]$ be a color image under the L*a*b* color space model [13]. Let $D(\mathbf{a}, \mathbf{b})$ be the Euclidean distance between two colors $\mathbf{a}, \mathbf{b} \in \mathbf{C}$, under the L*a*b* color space. The color gradient $\nabla_{B_{\mathtt{grad}}} : Fun[E, \mathbf{C}] \to Fun[E, \mathbb{R}_+]$ is given by, $\forall x \in E$,

$$\nabla_{B_{\mathtt{grad}}}(\mathbf{f})(x) = \bigvee_{b \in B_{\mathtt{grad}}} D(\mathbf{f}(x), \mathbf{f}(x+b)) - \\ - \bigwedge_{b \in B_{\mathtt{grad}}} D(\mathbf{f}(x), \mathbf{f}(x+b)),$$

where $B_{\mathtt{grad}} = B_c - \{o\}$ is given by the structuring element that denotes the connectivity minus the origin.

In order to compute the extinction values from the color gradient, it needs to be converted to an image $\overline{\nabla}(\mathbf{f}) \in Fun[E, K]$. Consider a normalization of $\nabla_{B_{\mathtt{grad}}}(\mathbf{f})$ from the interval $[\min\{\nabla_{B_{\mathtt{grad}}}(\mathbf{f})\}, \cdots, \max\{\nabla_{B_{\mathtt{grad}}}(\mathbf{f})\}]$ to $[0, \cdots, 1]$. The gradient $\overline{\nabla}(\mathbf{f})$ is given by multiplying the normalized gradient by k (and rounding it down).

B. Extinction Values

Extinction functions [2] compose a set of persistence measurements applied to the extrema of an image. Such measurements evaluate regional minima and maxima according to structural characteristics such as area, volume and contrast. These values support tasks such removal of irrelevant structures from an image. Extinction values are very useful in achieving hierarchical segmentation and, although both maxima and minima can have roles to play, in his work only the regional minima are considered in the extrema analysis.

An anti-granulometry [8] is an increasing and extensive family of operators $\Lambda = \{\varphi_\lambda : \lambda \geq 0\}$ such that

$$\forall \mu \geq 0, \varphi_\lambda(\varphi_\mu(f)) = \varphi_\mu(\varphi_\lambda(f)) = \varphi_{\max(\lambda,\mu)}(f).$$

Anti-granulometry may be applied to sift structures from an image: as the value of λ increases, the number of filtered structures in an image also increases. In this work, Λ is composed by a family of closings by reconstruction.

Let $M \in \mathtt{Min}(f)$, $f \in Fun[E, K]$. It is possible to measure the persistence of M under the sequential application of Λ. The extinction function under a anti-granulometry Λ is formally given by

$$\mathcal{E}_\Lambda(M) = \max\{\lambda \geq 0 : \forall 0 \leq \mu \leq \lambda, M \subseteq \mathtt{Min}(\varphi_\mu(f))\}.$$

This means that the anti-granulometry is applied incrementally to f, from $\lambda = 0$, until the regional minimum of $\varphi_\lambda(f)$ related to M is removed. If $M \nsubseteq \mathtt{Min}(\varphi_{\lambda+1}(f))$, the extinction value assigned to M is λ. In this work, Λ is composed by a family of closings by reconstruction.

Literature presents several criteria for designing Λ [14], [15]. This work uses the volume extinction criterion [16], since it has provided experimentally better qualitative results compared to other criterion such as area or dynamics.

C. Hierarchical Segmentation

A partition $\mathcal{X}$ of E is a family of subsets $X_i \in E$, such that $X_i \cap X_j = \emptyset$, $i \neq j$, and $\bigcup_i X_i = E$. Let $\boldsymbol{\mathcal{X}} = (\mathcal{X}_i)$ be a family of partitions such that $\mathcal{X}_1 = E$ and

$$X_j \in \mathcal{X}_i \Rightarrow \exists X_k \in \mathcal{X}_{i-1} : X_j \subseteq X_k,$$

$\forall i > 1$. The family of partitions $\boldsymbol{\mathcal{X}}$ defines a hierarchy of nested partitions [1], and the hierarchical segmentation of an image may be given by a partition $\mathcal{X} \in \boldsymbol{\mathcal{X}}$.

Watershed from markers and extinction functions may be combined to achieve hierarchical segmentation of an image $f \in Fun[E, K]$. A partition $\mathcal{X} \in \boldsymbol{\mathcal{X}}$ may be computed by applying an extinction function to each regional minima $M \in \texttt{Min}(f)$, followed by selection of marker according to given criteria and strategies.

For instance, the regional minima with the n highest extinction values may be chosen as markers to be imposed in the watershed from markers application. Alternatively, all regional minima which extinction values are higher than a thresholding value may be selected as markers.

The criterion to select markers in this work is detailed in Section III.

D. Color Clustering

Clustering consists in divide the elements of a data set into groups in a way that, given a criterion of similarity or relationship, the elements of a group must be close to each other, while being distant from the elements of other different groups [17]. It has been successfully applied to image analysis, pattern recognition and information retrieval.

In this work, clustering is done by the application of the k-means algorithm [17], a well known clustering technique which aims to find $\texttt{k}$ clusters, such that $\texttt{k}$ is chosen by the user. The representation of each cluster is given by its centroid; all elements of a cluster must be closer to its centroid than to other representative centroids.

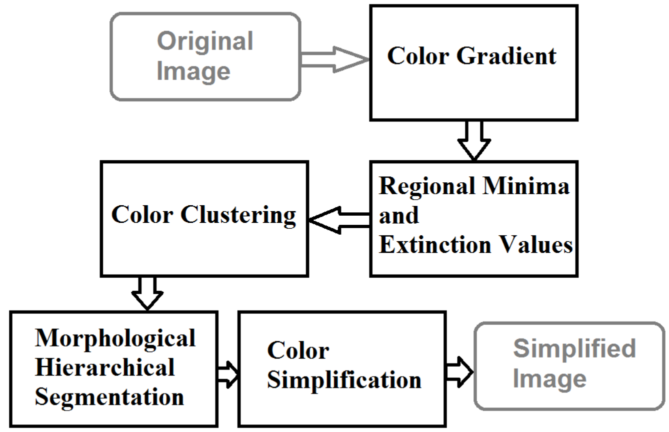
Basically, k-means is an iterative algorithm where, starting from a set of $\texttt{k}$ points as centroids, each iteration is given by the building of $\texttt{k}$ clusters by assigning each data to its closed centroid and, in the following, by the update of the centroids for each cluster. This algorithm runs until an acceptable convergence in the centroids computation [17].

In this work, the data set will be composed by color information, under the $L^*a^*b^*$ color space model. Centroids were initialized from data set samples and the chosen proximity function was the squared Euclidean distance.

III. The Proposed Method

This Section describes the color simplification method proposed in this paper (Fig. 1). It receives as input a color image $\mathbf{f} \in Fun[E, \mathbf{C}]$, under the $L^*a^*b^*$ color space model (Fig. 2 (a)), and two numerical parameters n and c, which define the final number of regions and the final number of colors, respectively. The output is a color image $\mathbf{sf} \in Fun[E, \mathbf{C}]$, simplified to n regions, which each region was filled with one of the c computed colors (Fig. 2 (b)).

The proposed method is given by the following steps:



(a)

Fig. 1. The proposed method.

A. Color Gradient

The first step of the proposed method is the computation of the color gradient. Gradient $\overline{\nabla}(\mathbf{f}) \in Fun[E, K]$, proposed in Section II-A, is applied to $\mathbf{f}$.

B. Regional Minima and Extinction Values

In the following, the regional minima $\texttt{Min}(\overline{\nabla}(\mathbf{f}))$ are found and their extinction values calculated. Consider that all regional minima in $\texttt{Min}(\overline{\nabla}(\mathbf{f}))$ are indexed. Let $\mathcal{E}_\Lambda(M_i)$ be the extinction value computed to the regional minimum $M_i \in \texttt{Min}(\overline{\nabla}(\mathbf{f}))$.

C. Color Clustering

In this step, a data set containing $\#\texttt{Min}(\overline{\nabla}(\mathbf{f}))$ elements is constructed. Each element of the data set is a tuple $\langle M_i, \mathbf{c}_i \rangle$: $M_i \in \texttt{Min}(\overline{\nabla}(\mathbf{f}))$ and $\mathbf{c}_i$ is given by the average color from the set $\{\mathbf{f}(x) : x \in M_i\}$. Such average is computed because more than one unique color may be observed inside the region denoted by M_i in $\mathbf{f}$.

In this work, clustering task is performed by the k-means algorithm [17], which receives c and $\langle M_i, \mathbf{c}_i \rangle$ as input data. Centroids were initialized with c samples of colors $\mathbf{c}_i$ from the data set and the chosen proximity function was the squared Euclidean distance. The clustering algorithm outputs a set of c clusters $\texttt{Clu}_j = \langle \texttt{c}_j, \texttt{RM}_j, \texttt{n}_j \rangle$, such that:

- $\texttt{c}_j$ is the centroid of the j-th cluster;
- $\texttt{RM}_j$ is the set of all regional minima $M \in \texttt{Min}(\overline{\nabla}(\mathbf{f}))$ assigned to the j-th cluster. Let $\#\texttt{RM}_j$ be the cardinality of $\texttt{RM}_j$; and
- $\texttt{n}_j = \left\lfloor \frac{\#\texttt{RM}_j}{\#\texttt{Min}(\overline{\nabla}(\mathbf{f}))} \cdot n + 0.5 \right\rfloor$. This is the number of regional minima taken from $\texttt{RM}_j$ to be part of the set of markers in the segmentation step. The value $\texttt{n}_j$ is proportional to n as $\#\texttt{RM}_j$ is to $\#\texttt{Min}(\overline{\nabla}(\mathbf{f}))$. Note that, since $\texttt{n}_j$ is a rounded number, it will be eventually valued zero for some clusters when they are too small.

(a)

(b)

Fig. 2. The proposed method. (a) Original image. (b) Simplified image.

D. Morphological Hierarchical Segmentation

Let $\mathtt{S}_j = \langle M_r, M_s, M_t, \cdots M_u \rangle$ be a sequence composed by all markers in $\mathtt{RM}_j$ such that

$$\mathcal{E}_\Lambda(M_r) \geq \mathcal{E}_\Lambda(M_s) \geq \mathcal{E}_\Lambda(M_t) \geq \cdots \geq \mathcal{E}_\Lambda(M_u).$$

The length of sequence $\mathtt{S}_j$ is equal to $\#\mathtt{RM}_j$. Let $\mathtt{S}_{j,l}$ be the l-th element of sequence $\mathtt{S}_j$.

In this step, the morphological segmentation of $\overline{\nabla}(\mathbf{f})$ is given by $\mathcal{W}_\mathbf{M}(\overline{\nabla}(\mathbf{f}))$. The set of markers $\mathbf{M}$ is given by

$$\mathbf{M} = \bigcup_{j=1}^{c} \bigcup_{l=1}^{\mathtt{n}_j} \mathtt{S}_{j,l}.$$

It means that $\mathbf{M}$ is composed by the union of the most valued regional minima from each cluster. Each cluster j contributes with their $\mathtt{n}_j$ regional minima which have the greatest extinction values.

Instead of composing $\mathbf{M}$ by selecting the n most valued regional minima from $\overline{\nabla}(\mathbf{f})$ in a straightforward and classical way, the proposed approach adds the color clustering criterion to the minima selection. It still takes the most valued markers, but not before clustering them in c clusters; the number of regional minima taken from a cluster is proportional to the size of the cluster.

E. Color Simplification

The application of $\mathcal{W}_\mathbf{M}(\overline{\nabla}(\mathbf{f}))$ results in a partitioning $\mathcal{X} \in E$ such that each subset $X_i \in E$ is related to a regional minima $M_i \in \mathbf{M}$.

Consider the set of c clusters $\mathtt{Clu}_j = \langle \mathtt{c}_j, \mathtt{RM}_j, \mathtt{n}_j \rangle$ as described at Section III-C, the output image $\mathbf{sf} \in Fun[E, \mathbf{C}]$ is finally given by, $\forall x \in E$,

$$\mathbf{sf}(x) = \mathtt{c}_j : (x \in X_i) \text{ and } (M_i \in \mathtt{RM}_j).$$

Thus the color value of $\mathbf{sf}$ at x is given by the centroid $\mathtt{c}_j$ such that x belongs to the subset $X_i \subseteq \mathcal{X}$ and the marker M_i that points to X_i belongs to $\mathtt{RM}_j$.

IV. Experimental Results

The proposed techniques is applied to three images from the Berkeley Segmentation Dataset and Benchmark [18] (Fig. 3). The left column (Fig. 3 (a), (d) and (g)) presents the original images and also gives the number of distinct colors contained in each image, as color information. In the experiments, all images were reduced to 1000 regions.

The segmentation results achieved by the proposed approach using parameters $n = 1000$ and $c = 256$ are presented in the middle column of Fig. 3. The number of distinct colors each resultant image contains is also provided, see the captions for Fig. 3 (b), (e) and (h). The number of colors is typically lower than the target of $c = 256$ because some clusters were too small to provide markers for the segmentation step.

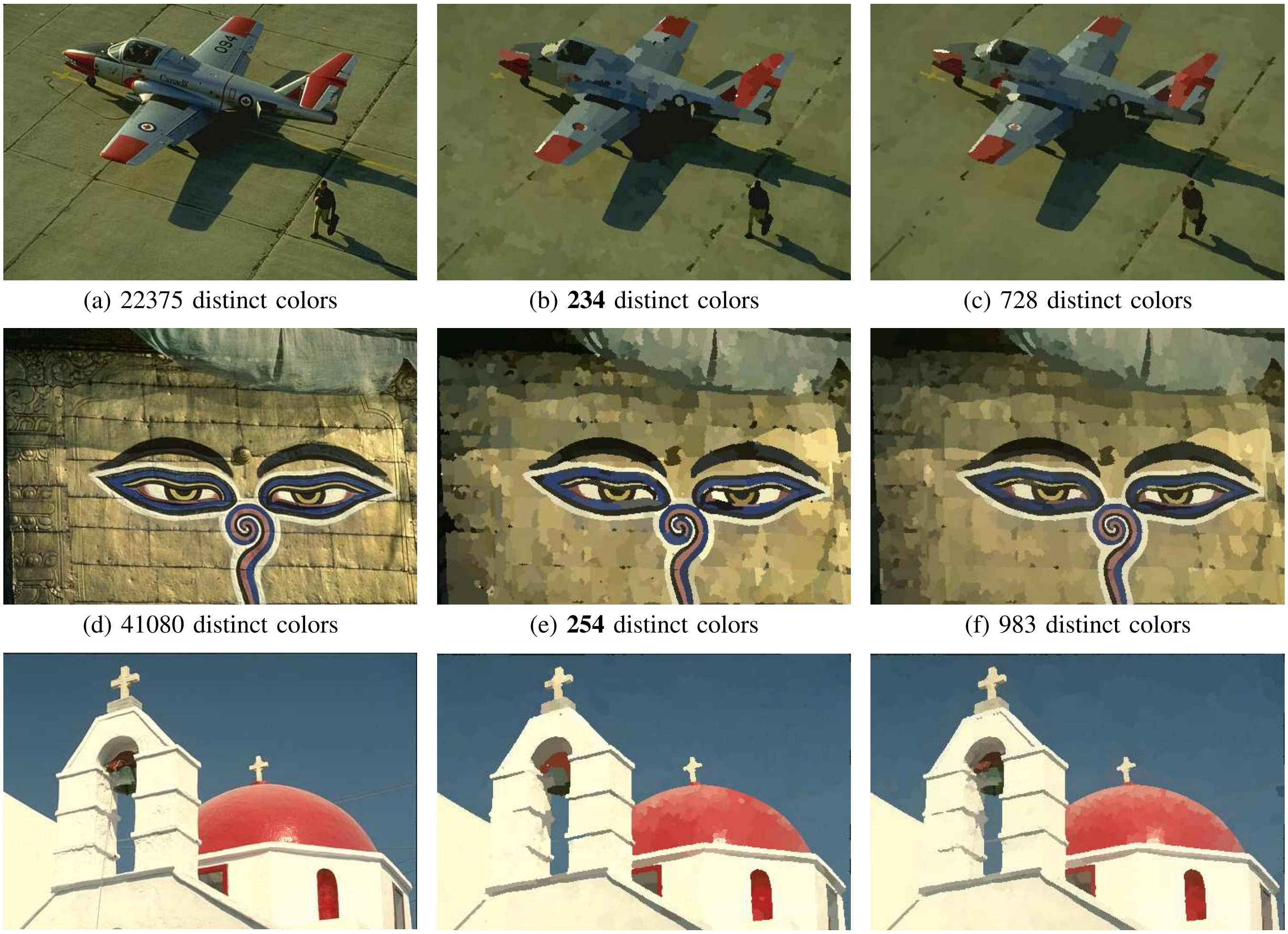
For comparison, a classical approach to morphological hierarchical segmentation, by combination of watershed from markers and extinction functions, was also implemented. The color assigned to each segmented region was taken as the average color inside such region. Segmentation results are provided in the right column of Fig. 3, followed by the number of colors they were reduced to.

According to a subjective visual criterion, both results are quite similar. However, the proposed method has quantized the input images to fewer of distinct colors and provides finer control over the resulting number of colors.

V. Conclusion

This paper introduces a method to color image simplification by combination of morphological hierarchical segmentation and color clustering. In this approach, regional minima are extracted from a color gradient and divided in c clusters. In the following, regional minima are sorted according to both structural and color informations. The n most valued markers are picked from the c clusters, proportionally to the size of each cluster, according to the color distribution of the data set.

The set of selected markers is applied to in the watershed segmentation step, in order to obtain the hierarchical segmentation. The output image is constructed by assigning, to each subset of the hierarchical partitioning, one of the cluster centroids as the color information. A color $\mathtt{c}_j$ is assigned to subset $X_i \in E$ if marker $M_i \subset X_i$ belongs to cluster j.



(a) 22375 distinct colors (b) **234** distinct colors (c) 728 distinct colors

(d) 41080 distinct colors (e) **254** distinct colors (f) 983 distinct colors

(g) 20831 distinct colors (h) **212** distinct colors (i) 861 distinct colors

Fig. 3. Segmentation results for 3 images from the Berkeley Segmentation Dataset and Benchmark. *Left Column :* input images and number of distinct colors contained. *Middle Column :* proposed approach ($n = 1000$ and $c = 256$). *Right Column :* Classical hierarchical segmentation ($n = 1000$). The captions for the middle and left columns contain the number of distinct colors they were reduced to.

As a result, marker selection provides both an hierarchical segmentation and image quantization. Thus, the contribution of the paper is a method which leads to a coarse segmentation and a reduced number of distinct colors, in which both the number of regions and colors are controllable.

Future work includes new ways to sort regional minima and the exploitation of alternative color gradients and color data processing.

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