Artificial intelligence and uncertainty in power system operation

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ARTIFICIAL INTELLIGENCE AND
UNCERTAINTY
IN POWER SYSTEM OPERATION

Submitted by K.R.W. Bell B.Eng. (Hons)
for the degree of
Doctor of Philosophy
at the University of Bath
1995

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K.R.W. Bell

Bath, August 31, 1995
Summary

This research is concerned with operation of a large interconnected power system, which can be simply stated as the dispatch of generation and control settings to optimally maintain consumers' supply. The thesis gives a brief overview of current practice in controlling the UK National Grid and discusses reasons why progress in running a power system more efficiently is believed to require the support of artificial intelligence. Sources of uncertainty in the operation of a power system and different forms of artificial intelligence are briefly described. The application of fuzzy logic to reducing the impact of uncertainty where fuzzy logic is a method of reasoning which models uncertainty, is justified.

A further important means of reducing the impact of uncertainty is in the reduction of the uncertainty associated with measurement errors. In power system operation, state estimators are used to reduce measurement error. This thesis presents a discussion of state estimation and describes recent methods which improve their reliability. New enhancements to these are presented.

Uncertainty exists in the enhancement of security, where security limits are uncertain and the ideal ways of meeting are also poorly determined. A prototype power system dispatch facility based on a fuzzy expert system has therefore been developed which performs approximate reasoning as a way of reducing the impact of uncertainty. It performs a co-ordinated active and reactive security-constrained dispatch which allows the operator to choose one or more of a number of objectives including low cost and low number of controllers. Its function and the fuzzy set theory upon which it is built are described. Comparisons are presented with published work on an alternative fuzzy expert dispatch system, a conventional production rule-based expert system and a linear programming based dispatch algorithm.
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<td>$\eta$</td>
<td>Efficiency, severity index</td>
</tr>
<tr>
<td>$T$</td>
<td>Absolute temperature</td>
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<tr>
<td>$\mu$</td>
<td>Degree of membership of fuzzy set</td>
</tr>
<tr>
<td>$X$</td>
<td>Universe of discourse of a fuzzy set</td>
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<tr>
<td>$P(H\mid E)$</td>
<td>Probability of $H$ given $E$</td>
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<td>$O(H)$</td>
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<td>$LS$</td>
<td>Sufficiency measure</td>
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<tr>
<td>$LN$</td>
<td>Necessity measure</td>
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<td>$CF$</td>
<td>Certainty factor</td>
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<tr>
<td>$N(p)$</td>
<td>Degree of necessity of $p$</td>
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<td>$\Pi(p)$</td>
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<td>$\alpha$</td>
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<tr>
<td>$x$</td>
<td>Power system state vector</td>
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<tr>
<td>$V$</td>
<td>Complex voltage</td>
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<tr>
<td>$V$</td>
<td>Voltage magnitude</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Voltage angle</td>
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<td>Quadrature booster phase angle</td>
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<td>$\alpha$</td>
<td>Complex tap ratio</td>
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<td>Active power</td>
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<td>$Q$</td>
<td>Reactive power</td>
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<td>$S$</td>
<td>Apparent power</td>
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<tr>
<td>$R$</td>
<td>MVA rating</td>
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<tr>
<td>$y$</td>
<td>Complex admittance</td>
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<td>$I$</td>
<td>Complex current</td>
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<tr>
<td>$h(x)$</td>
<td>Vector of functions of power system state</td>
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<td>$G$</td>
<td>Vector of power system injections</td>
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<td>$u$</td>
<td>Vector of control variables</td>
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<tr>
<td>$S_P$</td>
<td>Active power sensitivity matrix</td>
</tr>
<tr>
<td>$S_Q$</td>
<td>Reactive power sensitivity matrix</td>
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## List of Symbols

<table>
<thead>
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<th>Symbol</th>
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<tbody>
<tr>
<td>$z$</td>
<td>Vector of measurements</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Error vector, weighting factor</td>
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<tr>
<td>$r$</td>
<td>Residual vector</td>
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<tr>
<td>$J(x)$</td>
<td>Objective function</td>
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<tr>
<td>$W$</td>
<td>Matrix of weightings</td>
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<td>Langrangian</td>
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<td>$\mu, \lambda$</td>
<td>Lagrange multipliers</td>
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<td>$k$</td>
<td>Iteration number</td>
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<tr>
<td>$U$</td>
<td>Control setting</td>
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<tr>
<td>$N_{\text{bar}}$</td>
<td>Number of busbars</td>
</tr>
<tr>
<td>$N_{g}$</td>
<td>Number of generators</td>
</tr>
<tr>
<td>$N_{v}$</td>
<td>Number of voltage controlled reactive sources</td>
</tr>
<tr>
<td>$N_{t}$</td>
<td>Number of transformers</td>
</tr>
<tr>
<td>$N_{b}$</td>
<td>Number of mechanically switched capacitors</td>
</tr>
<tr>
<td>$v$</td>
<td>Control contribution</td>
</tr>
<tr>
<td>$C_{ij}$</td>
<td>Contribution factor</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Maximum sensitivity</td>
</tr>
<tr>
<td>$M$</td>
<td>Margin for control</td>
</tr>
<tr>
<td>$c$</td>
<td>Number of converged solutions</td>
</tr>
<tr>
<td>$n_{c}$</td>
<td>Average number of controls used</td>
</tr>
<tr>
<td>$n_{s}$</td>
<td>Number of controls set to control limits</td>
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1

Introduction

1.1 Electrical Energy in the 20th Century

The industrial revolution which began in the 18th century made possible the use of natural resources that up to that point had been irrelevant to human society and gave birth to the concepts of industrial production and economic growth. Economics became a discipline that dictated the lives of many thousands of people and the need for continually increasing production (engendered by the perceived need for economic growth) dictated that hugely increased amounts of energy were needed [1]. It can be argued that the second phase of the industrial revolution only began with the availability of a form of energy that could be produced in mass quantities, transported simply and quickly, and easily used. This form of energy was electricity.

Electricity has been one of the key components of modern industrialised society, being strongly linked with economic growth and GNP [2]. Almost every facet of contemporary life in the developed world depends on the cornerstone of a cheap, safe and secure supply of electricity through what has become known as the power system.

1.2 Generation of Electrical Energy

Virtually all electrical energy generation concerns the conversion of chemical energy into heat and then kinetic energy and finally electrical. Other forms of conversion that
are common are the direct conversion of the kinetic energy of water into electrical, and the generation of heat by nuclear fission. The efficiency of the heat engines involved in the process can be described by

\[ \eta_{\text{max}} = \frac{T_1 - T_2}{T_1} \]  

(1.1)

where \( T_1 \) is the absolute temperature of the working fluid (usually steam) and \( T_2 \) is the minimum condenser absolute temperature. In power stations, \( T_1 \) is limited to 823 \( K \) and \( T_2 \) to 373 \( K \). The maximum possible efficiency in this case is 55\%, but in practice 40\% is more realistic [3].

### 1.3 Modern Power Systems

Economy of scale dictated earlier in this century that the practice of a local power system supplying local demand was replaced by fewer larger power stations local to primary fuel sources interconnected by a transmission system with a distribution system supplying remote consumers. Benefits of this approach, aside from cost, included the possibility of a more reliable supply (remote power stations being able to supply power lost from some faulted one, or being able to respond to sudden increases in demand) and more diverse use of primary energy sources, whose main ‘benefit’ has been political i.e. reduced vulnerability to the vagaries of industrial relations and primary fuel markets, and increased independence in expertise on nuclear power. With a wide range of sources of electrical energy at the disposal of the power system operator and a relatively cheap and efficient means of interconnecting them, a half-hourly or faster choice of the most cost-effective sources has been possible, expressed by means of the so-called *merit order*.

The early days of electrical energy generation in the U.K. saw different utilities use different frequencies. These were standardised by an Act of Parliament in 1926 to 50Hz, a frequency deemed just high enough for flicker of electric lights to be unnoticeable. The interconnection of the electrical energy sources began at that time at 132kV, but further interconnection since then has been done at higher voltages because the energy loss of transmission of electrical energy is inversely proportional to the transmission
introduction}
voltage. Thus the ‘supergrid’ completed in the UK in 1962 was designed to be operated at 400kV. Higher transmission voltages up to 765kV [4] are used over long distances in other countries. Although technology, particularly in power electronics, would now permit economic transfer of DC electrical power such as across the link between the U.K. and France, most utilities around the world use not only AC generation, but also AC transmission and distribution, usually in three phase form.

At the time of writing, the transmission system in England and Wales consists of 257 substations at 400kV and 275kV, 1300 transmission lines, roughly 11000 switches and circuit breakers and numerous items of protection equipment[5]. There are 64 power stations connected directly onto the national grid.

Almost all of the transmission lines are overhead lines (12,776 km) with very few underground cables (564 km). This choice was made on economic grounds with underground cables about 20 times more expensive to install and maintain than overhead lines [4]. They also give unique operating problems caused by high capacitance. On the other hand, overhead lines are exposed to strikes by lightning which can cause a circuit to be suddenly rendered unavailable. These are some of the issues which affect not only the design of a power system, but also its minute-by-minute operation. Some of these are explored in the next section.

1.4 Power System Operation

Power systems, defined as the means of generating, transmitting and distributing electrical energy, depend on various ‘real-time’ factors for their success, where ‘success’ might be described initially as keeping consumers supplied with power, and, further, as doing so in the most efficient manner possible. Conventionally, energy is converted to AC electrical energy at a certain frequency at a limited number of bulk conversion points, or power stations. Power is distributed to consumers by means of an interconnected network operating at certain voltage levels. For power to continue to be generated and distributed, demand and supply must be continuously balanced, i.e. the system frequency
and nodal voltages must be kept within specified limits and the integrity of the power system plant maintained. The results of a failure to maintain such security can be catastrophic. On July 13th 1977, a relay on a transmission line from Canada into New York mal-operated and caused the cascade tripping of a host of other lines in the New York area resulting in a 5 hour 'blackout' of the city during which looting, murder and general unrest was rife. The blackout has been estimated to have cost in the region of millions of dollars [4]. A similar, though less severe event occurred in Seattle in 1984 during a storm [4]. Here the equipment operated correctly, but a blackout still happened. The principles behind control of the power system to prevent these and less serious events are introduced in this section and are further described in chapter 2.

Such a task of controlling a large non-linear system is enormously complex. Current practice details the breaking down of the task into constituent parts and using numerical analyses designed for each one (figure 1.1).

![Figure 1.1: Basic tasks in control of a power system](image_url)

The first task is the meeting of the demand for active power i.e. power that will do work. This requires the scheduling of enough active power generation to match the MW de-
mand at every point in time. Too much generation will result in the system frequency increasing, too little will result in it decreasing. The Electricity Supply Regulations 1989 and the 'Grid Code' in the UK [6] dictate that system frequency must be maintained within ±1% of the nominal 50 Hz. Because run-up rates of large steam and nuclear generating plant are quite slow, power must be ordered well in advance. Small fluctuations are dealt with by changing the load or generating power at the pumped storage stations which can respond within minutes, and by a few generating sets providing 'spinning reserve' i.e. surplus capacity that is already connected and spinning in synchronism with the system. In scheduling active power, however, it must be ensured that transmission plant is not overloaded. The scheduling of active power in advance is known as unit commitment. In the short-term, it is known as active dispatch.

Most industrial loads require reactive power as well as active power (i.e. power that is in effect exchanged between load and source that is a result of a phase difference between the voltage and current), and the transmission of power down lines also places a demand for reactive power such that lightly loaded lines appear to generate MVArs and heavily loaded lines absorb them. Failure to meet the reactive demand results in a depression of system voltage around the point of the shortfall and can result in voltage falling so far that compensating equipment reaches its limits and the whole system collapses. At times of low demand, voltage can tend to creep up leading to excessive loading of consumers' plant. The Grid Code details that voltage must be maintained at ±10% of nominal on the transmission system. The scheduling of sufficient reactive power local to demand and the co-ordination of transformer tap ratios to control its flow is known as reactive dispatch.

A further consideration in the scheduling of generation and control plant is the stability of generating sets. Should an interconnection between the generator and the grid be lost, the amount of power that can be transferred away from the generator will be reduced. If the input power to the generator is not reduced in time, the rotor will accelerate and may lose synchronism with the rest of the system i.e. the machine pole-slips or becomes transiently unstable. Under certain conditions, oscillations of power and machine rotor angles fail to be damped out even if synchronism is retained. These oscillations can in-
crease, possibly leading to pole-slipping or causing maloperation of protection relays, leading to more serious problems. This is known as oscillatory instability or hunting [7]. A further stability consideration is that of steady state stability where the system can be expected to remain in the vicinity of its current operating point when subjected to some small change. When all the requirements i.e. plant loading, voltage, stability and frequency are met, the system is regarded as secure.

As well as having to ensure that the current state of the system meets the appointed standards, the power system operator (in England and Wales, the National Grid Company) is required to ensure that the standards are still met in the event of credible events such as switching or faults. Such events are known as contingencies. This process is known as contingency analysis and it enables operators to prepare in advance for possible occurrences so that they can take action to prevent insecurity or instability should the event happen, or plan corrective action [8].

Since the number of possible events is enormous and the power system is very complex, the analysis required to be able to ensure the safe operation of the power system is extremely burdensome. A host of computational aids to the process have been developed over the last 25 years, many of which are still impractical for real systems, requiring a certain amount of guesswork or intuition to be able to operate the system conservatively to so-called $N - 1$ or $N - 2$ levels of security i.e. still secure in the event of one or of two concurrent contingencies.

1.5 The Aims of This Work

This project is concerned with enabling improvement in the operation of a large interconnected power system. As such, it examines what the problems are that are associated with the operation of such a system and briefly reviews techniques employed to overcome them. It looks at the uncertainties involved and at a mathematical framework for modelling them or enabling them to be dealt with. Then the co-ordination of results of the various analysis techniques used in power system operation is considered, and finally
an expert system framework is developed to enable flexible, fast and coherent utilisation of such results. These aims are further discussed in section 1.6.

1.5.1 Motivation for Improving Operation of the Power System

Modern power system utilities, whether under private or state ownership, are particularly concerned with reducing cost in order to improve return on investment or gain maximum benefit to the economy. They are being required to operate existing systems closer to the theoretical limits of their abilities to transmit power. In addition, the opportunity to reinforce the transmission system is being made available less often when transmission towers' impact on the countryside is unpopular and the long-term effects of electro-magnetic radiation are still largely unknown. More sophisticated techniques have therefore had to be used to ascertain what the operating limits are, and to be able to schedule actions appropriate to preventing them being exceeded. In the last 5 years or so, however, an extra factor has begun to come into play: energy efficiency [9]. Not only is this important because of the long-term social and environmental impact of the burning of fossil fuels, building, running and de-commissioning of nuclear power plant, flooding of land for hydro-electric schemes, and so on, but also because of medium and long term economics [10]. Future performance of energy management systems will have to be judged on financial cost and on environmental cost. The particular aim of this project is to enable utilities to make better use of existing plant judged on both these criteria, i.e. run the system more cost and energy efficiently.

The cost of security

In an ideal world, a power system utility would be able to receive information on the cost of different items of generating plant and simply order the cheapest until MW demand is met. This not being an ideal world, however, with the bulk of generation, certainly in the UK, being sited away from centres of demand, some generation must be restricted to prevent overloading of transmission plant, violation of voltage requirements or instability. Other generation that is not ideal in terms of cost must be ordered instead because
of these constraints, and the cost in excess of the ideal, or 'uplift', was estimated to be £189 million in 1993-94 in England and Wales [11]. With security constraints costing so much, there is a great desire to be able to establish more precisely what current or future levels of security are, what the restrictions need to be, and even to re-define what is meant by security [12].

1.5.2 Uncertainty in Power System Operation

"As far as the laws of mathematics refer to reality, they are not certain; as far as they are certain, they do not refer to reality." – Albert Einstein

Something that is uncertain might be defined as something which is not known or not known dependably, or which is changeable [13]. Using this definition and recognising that the mathematical modelling used in power system analysis at all stages uses approximations, it can be seen that power systems are controlled under uncertain conditions. The main sources of uncertainty are

Load forecast: predictions of future demand on the power system are by their very nature uncertain.

Weather conditions: as well as forecasts of weather (upon which demand will depend) being uncertain, the current conditions around the power system are not known with any certainty. These affect the dissipation of heat from generation and transmission plant. In addition, weather conditions will affect the extent to which power station emissions are dissipated, and the risk of trip of overhead lines.

State estimation: the process by which an incomplete and error-prone set of SCADA (Supervisory Control and Data Acquisition) measurements is assembled into a complete state vector cannot do so with absolute certainty [14].

Power system parameters: Accurate analysis of the behaviour of a power system depends on accurate knowledge of its parameters. These parameters, while they may
have had detailed design specifications and some of them may be approximately measurable, are not known accurately [14].

**Security analysis:** All mathematical models used are approximate and may have used a number of assumptions to speed the calculation such as [8]

- the coupling between the active and reactive subsystems is small;
- the ratios of $R$ to $X$ in transmission lines are small;
- machine governor responses can be ignored;
- complex regulator actions can be ignored;
- uncertainty in the response times of protection equipment is insignificant;
- transient instability will occur within a certain time-frame if it will occur at all;
- loads are unaffected by small changes in system frequency or voltage;
- only certain machines will contribute to instability.

**Arbitrary nature of limits:** lack of accurate knowledge of plant thermal capacities, generating plant dynamic responses and power system dynamics, particularly concerning voltage stability, have led to the adoption of safe operating limits known from experience to be conservative, but which will not enable the system to be run as cost effectively or energy efficiently as possible for a given level of security [15, 16, 8].

**Operational priorities:** While utilities define certain priorities for certain conditions, the detailed response to specific real operating conditions is a matter of the operator’s judgement, even when comprehensive analytical tools are available. Indeed, many of these tools, in particular optimal power flow or dispatch programs, require tuning to be done by hand which is to say that they, too, are dependent on the operator’s judgement. The judgements concern such variables as sensitivity, control margin, cost, number of controllers, emissions, losses and level of security.
A number of mathematical frameworks have been developed which attempt to model different kinds of uncertainty. One of these, which its proponents claim to be the most comprehensive and which has the largest volume of work, is fuzzy logic. Descriptions will be given both of means of reducing some of the above uncertainties and of fuzzy logic.

1.6 About This Thesis

1.6.1 Summary of research aims

The overall aim of the research is to enable better utilisation of the resources of a large, interconnected power generation and transmission system. This requires the meeting of security criteria in order to reliably maintain supply in as efficient a manner as possible. Such a task involves the use of a range of analytical tools which are co-ordinated by an operator. The thesis, then, aims to achieve this by

- investigating the analysis and maintenance of power system security;
- assessing problems with existing approaches to optimal security enhancement or dispatch;
- investigating artificial intelligence methodologies which may enable improvements in security analysis and enhancement;
- developing a flexible framework for modelling decision-making in security enhancement;
- implementing a prototype security enhancement or dispatch system.

The prototype dispatch expert system which this work presents allows better co-ordination of resources through the fast, reliable and flexible modelling of operators’ judgements based on the results of numerical analysis. The software to be developed for the project includes two numerical analysis routines, a fuzzy inference engine and a fuzzy rule-base.
1.6.2 This work’s contribution

The main contributions of this study are

- to enable some understanding of the nature of uncertainty in power system operation;

- to bring a deeper of understanding of a range of artificial intelligence techniques, and in particular fuzzy logic, into the area of power systems engineering;

- to demonstrate enhancements to an approach to state estimation which enable the observable regions of a power system to be identified and transformer tap ratios to be found, thus reducing uncertainty about the power system state vector;

- to demonstrate a new approach to security constrained dispatch (or security enhancement) which models semantic uncertainty in operators’ judgements, allows flexible and intuitive adjustment of priorities and which reaches a solution near enough to the optimum quickly enough to be of use in an on-line environment;

- to compare the new approach with other published approaches to dispatch i.e. one based on linear programming and two based on expert systems.

1.6.3 Thesis layout

Chapter 2 describes current practice in the operation of large interconnected power systems. It describes the regimes of security analysis and introduces means of enhancing security.

Chapter 3 gives an introduction to artificial intelligence (AI) techniques including expert systems, artificial neural networks and evolutionary techniques. In describing the foundations of expert systems, it gives an introduction to classical logic theory. Finally, the chapter reviews applications of AI methods in power system operation.
Chapter 4 describes different approaches to modelling of uncertainty in expert systems. It then assesses the kinds of uncertainty encountered in power system operation in terms of these approaches bringing a new understanding of the knowledge and information involved in power system operation. It goes on to describe fuzzy logic, a powerful framework for modelling semantic uncertainty in control of engineering systems.

Chapter 5 briefly describes expert system shells used in the development and implementation of expert systems. In particular, it describes the shells specially developed for this project.

Chapter 6 describes the static analysis of power systems which provides the data used by operators. Load flow and sensitivity analysis packages developed in this project are described, in particular a new approach to active power sensitivity.

Chapter 7 describes a new implementation of the state estimation process which assembles the data upon which all the security analysis and enhancement functions are based. The new approach is an enhancement of an existing one and reduces the uncertainty due to errors. It employs an expert system in checking power system’s observability, and has the facility to estimate transformer tap ratios.

Chapter 8 describes an existing implementation of security-constrained re-dispatch, or security enhancement. The method described is one of the best established and is based on linear programming (LP).

Chapter 9 summarises problems encountered with LP-based dispatch and the original motivations for the use of expert systems. It then describes two published expert system approaches to reactive dispatch along with enhancements made to them in this project. It also describes two new expert systems for active dispatch which use the same principles as underlie those described for reactive dispatch.

Chapter 10 summarises problems encountered with existing expert system approaches to dispatch before giving a detailed description of the new fuzzy expert systems developed in this project for different aspects of the security-constrained dispatch problem. Finally, integration of the new expert systems is outlined.
Chapter 11 presents results in the forms of comparisons for corrective actions between the LP dispatch, the two published expert system approaches with the enhancements added in this project, and the new expert system developed in this project. Results are then presented for the new system for the derivation of preventative actions. Case studies illustrating the use of the time-dependent load monitoring and the derivation of corrective action are shown.

Chapter 12 suggests directions in which the work presented in this thesis can be developed.

Chapter 13 presents the main conclusions of the research.

The inter-dependency of the chapters is illustrated in figure 1.2.

1.7 Summary

Power systems are required to supply electrical energy to consumers in as cheap and reliable a way as possible. In order for supply to be maintained, the security of the system must be acceptable, that is, plant loadings, voltages and risks of instability must be
within limits. This study examines a possible basis of deriving actions that will prevent power system insecurity or correct it incurring the least possible cost in terms of money, fuel and emissions within a realistic, on-line time frame. It will use a fuzzy expert system to co-ordinate the results of different numerical analyses, reduce the impact of uncertainty and model operator heuristics or qualitative reasoning in recommending preventive or corrective actions.
The tasks involved in the operation and control of a large inter-connected power system are extremely complex and were introduced in section 1.4. The co-ordination and interlinking of the various software analysis tools used in the process has come to be known as an energy management system or EMS. It performs monitoring of the current operating state and levels of security and carries out contingency analysis and generation dispatch. How an EMS fits in to a power system and the constituent parts of a typical EMS are shown in figure 2.1. The parts are described in the following sections.

![Diagram of a typical energy management system]

Figure 2.1: A typical energy management system
2.1 SCADA and State Estimation

Every function of the EMS relies on up-to-date and accurate information on the current state of the power system. The Supervisory Control and Data Acquisition (SCADA) system sends telemetered line and busbar data such as real and reactive power flow, real and reactive nodal power injections and nodal voltage magnitudes to the control centre. The SCADA measurements themselves are not 100% accurate, being prone to transducer and communication drift and noise [17], and measurements are not available for every line or busbar on the system. This incomplete and inaccurate set of measurements is therefore assembled into a more accurate and complete state vector describing the current state of the whole system. This process is known as state estimation and is often performed as a weighted least squares minimisation of the error. Due to the criticality of the SCADA and state estimation aspect of the EMS, considerable work has been done as part of this project in utilising and improving recent state estimation methods. This is described in chapter 7.

2.2 Security Assessment

Security Assessment concerns the determination of the current level of security of the power system and is shown by the ability of the system to withstand severe and credible faults (contingencies) and maintain supply to consumers. Traditionally, there have only been two levels of security in which operators have been interested, secure and insecure. Modern power system operation, with security made more difficult to maintain by the presence of fewer, larger power stations connected over larger distances and tighter economic constraints, requires more detailed knowledge of security with information given on proximity to insecurity. Stott et al [8] proposed a set of 6 levels of security which are shown in figure 2.2 in answer to the questions

- Is all the load being supplied?
- Are operating limits being violated?
• Is preventive or corrective action needed?

• Are critical power flows within limits?

Such a proximity is often reflected by a severity index for each contingency [18].

Insecurity itself can be thought of as falling within one of three types which are described in the following sections.

LEVEL 1
(Secure)

LEVEL 2
(Correctively Secure)

LEVEL 3
(Alert)

LEVEL 4
(Correctable Emergency)

LEVEL 5
(Correctable Emergency)

LEVEL 6
(Restorative)

Control Actions

Figure 2.2: Levels of power system static security

2.2.1 Static Security

Static security concerns the loading of plant in the steady state i.e. the condition where all the operating quantities that characterize the power system can be considered to be constant. All items of transmission plant have limits on thermal loading and nodes must be maintained within limits of voltage magnitude that are defined in order to guarantee
the quality of supply to the consumer and to provide a crude protection against voltage instability or voltage collapse, a phenomenon where, as the loading on a line increases, the voltage cannot be maintained and falls to zero. Some utilities impose limits on the step change in voltage after a fault. A further aspect of static security is the level of fault current that must be interrupted by a circuit breaker on the system—this must be kept below the rating of the breaker.

The standard way of assessing static security is to find the load flow [19] profile of a given system configuration, where the 'load flow' is the solution of the non-linear algebraic power network equations for a given generation and load profile. A severity index \( \eta \) of the form [18]

\[
\eta = \sum_{i=1}^{N} \omega_i \left( \frac{X_i}{X_i^{\text{max}}} \right)^n
\]

(2.1)
can then be found where \( X_i \) is a line power flow or the deviation from nominal of a busbar voltage magnitude and \( X_i^{\text{max}} \) is the corresponding maximum, \( \omega_i \) is some weighting, \( n \) is some integer, and \( N \) is the total number of quantities considered.

2.2.2 Voltage instability

Figure 2.3 shows a simple single line connection between a load and a generator. The graph shows the relationship of the voltage magnitude at node 2 for different active power loads and different load power factors. While thermal limits may be defined for the line for different weather conditions, it can be seen that under certain circumstances, these are not the lowest limits. For example, for a load lagging power factor of 0.9 and \( P = a \), there is no sustainable voltage. If the load with a lagging power factor of 0.95 started at \( b \) and increased, it can be seen that the voltage will fall dramatically (or 'collapse') when \( P \) reaches \( c \). The adoption of a simple lower limit on voltage provides a crude protection against voltage collapse since it guarantees that the operating point will be to the left of the characteristic, i.e. away from the 'knee point'.

Real power systems are of course more complex than the circuit shown, though some active loads decrease when the voltage decreases, lessening the problem. However, an
added complication may be the presence of variable reactive compensation. If a variable susceptance is added to the circuit of figure 2.3 as in figure 2.4 where some control law determines the variation of $B$ to maintain a constant $V$, the $PV$ characteristic will be of the shape shown. It can be seen that the voltage does not decline gradually with increase in $P$, but collapses suddenly (possibly when still above the pre-defined lower operating limit) when the reactive power ‘generated’ at node 2 reaches an upper limit i.e. when a deficit of reactive power occurs. Further, it is found that on-load tap changing transformers and certain kinds of loads can exacerbate the problem by tending to react to a voltage drop by drawing more current and causing a further voltage drop [20, 21].

Figure 2.3: $PV$ curve showing voltage collapse

Figure 2.4: $PV$ curve showing voltage collapse with SVC present
Considerable attention has been devoted in recent years to study of voltage instability phenomena such as reported in [22, 23, 24, 25, 26]. Much of this attention has been prompted by some notable examples of what has been reported to be voltage instability in France in 1978 [27] and 1987 [28], Belgium in 1982 [29] and Sweden in 1983 [30]. Prevention is mainly concerned with the provision of sufficient reserve of reactive power, while the demand for reactive power is dependent not only on loads but on the transfer of power around an inter-connected system, something which can be understood by considering that transmission lines tend to 'generate MVARs' when lightly loaded and 'absorb' them when heavily loaded.

### 2.2.3 Transient Security

Transient security concerns the stability of the power system in the event of large changes on the system, such as busbar faults, or lines or generating sets tripping (a 'large change' has been defined in [7] as one for which the equations that describe the dynamics of the power system cannot be linearized for the purpose of analysis). The instability is observed as one or more synchronous generators losing synchronism with the rest of the system i.e. pole-slipping [16]. This happens when the kinetic energy of the machine exceeds the electrical energy which can be sunk in the transmission system so that the rotor accelerates, such as when a line or transformer near to the generator terminal trips.

Generating set automatic voltage regulators (AVRs) and power system stabilisers (PSSs) respond to changes seen at the generator's terminal by changing the field voltage which affects the coupling between the rotor and stator magnetic fields, allowing stable operation at higher load angles. In the example in figure 2.5, however, the machine rotor angle has exceeded 180° i.e. it has pole-slipped. Governor response to the increase in rotor speed is too slow to be of use in preventing pole-slipping, though generator protection schemes close down one or more of the steam valves once a fault has occurred near the generator's terminal.

Traditionally, the most reliable way of predicting instability has been to perform a step-by-step numerical integration of a reduced set of power system and machine state equa-
tions, but this is computationally demanding [31, 32, 33]. For a number of years, a rule-of-thumb for assessing the balance of input to output energy has been employed by power system operators known as the 'equal area criterion' [34]. Recent research has made use of the analogy that instability occurs when the potential energy margin of the power system is insufficient to absorb the excess kinetic energy associated with the contingency [35] through the 'transient energy function' method [36, 37, 38, 39, 40].

Prevention of transient instability is achieved by defining the maximum power which should be generated by a machine if enough of it is to be transferred to the transmission system for insufficient to be left to accelerate the machine to the point at which it pole-slips. Since there are often many generating sets at one location, the MW limit set is often that for the group of generators, and this is associated with the collection of lines connecting the group of generators with the rest of the transmission system. The definition of such a MW transfer limit has often been carried out by a trial and error process, though much work is going on to characterise the relationships involved. An example of this work is reference [41].
2.2.4 Oscillatory stability

A subset of transient security concerns the behaviour of the system after an event when no pole-slipping has occurred. In certain cases, the oscillations resulting from the change may take a considerable time to damp out, or may get larger over a period of minutes.

Such problems, known as oscillatory instability or ‘hunting’ [7] are often noticed on the occurrence of changes when excessive amounts of power are transferred across weak boundaries, such as between Scotland and England on the UK’s national grid [42], an example of which is illustrated in figure 2.6. In the figure, some event has caused a change in the rotor angles of a group of machines. This change has stimulated a change of angles within another similarly sized group, and the two groups, equivalent to two large machines, their regulators acting out of phase with each other, have begun to swing against each other causing oscillations in the power transferred between the groups and other knock-on effects. The amplitude of the swing of the rotor angle of the machine shown steadily increases until it loses synchonism. Machines with fast automatic voltage regulators (AVRs) tend to be more vulnerable to this sort of behaviour.

![Rotor Angle DEG](image)

Figure 2.6: Illustration of England-Scotland dynamic instability
2.2.5 Steady state security

This concerns the ability of the power system to remain in the vicinity of the initial operating state after some small change [43, 44, 45] where a small change is a disturbance for which the equations that describe the dynamics of the system may be linearised for the purpose of analysis [7]. Insecurity in this case can give rise to long term oscillations possibly leading to pole-slipping, limit cycles which can stress generation and transmission plant causing long term damage, or maloperation of protection causing further insecurity.

Utilities seek to prevent such problems by limiting the flows across critical boundaries. Analysis of the situation conventionally requires the finding of the eigenvalues of the linearised system, including the machine, governor and AVR equations [46]. Recent work on this subject has used non-linear techniques to locate bifurcation points [47].

2.2.6 Dynamic security assessment

Where once the practice of security assessment was only concerned with the monitoring of branch flows and nodal voltages, increased economic pressures where existing power system infrastructures are required to supply higher loads and where generation tends to be remote from load centres has motivated considerable attention paid to on-line dynamic security assessment. This monitors not only static security, but also transient and steady state stability, providing complete results for the current operating state of the system and contingencies applied on it at a rate commensurate with the rate of change of the state. The most intensive part of the analysis is that of monitoring transient, oscillatory and steady state stability, and various methods, including those listed in section 2.2.3 above, have been tried to provide a reliable short-hand. In addition, a number of researchers are now exploiting the parallel nature of contingency evaluation in using a heterogeneous distributed processing system comprised of different types of workstation which can be added to the system as they become available. Such an approach is adopted in ‘OASIS’, ‘On-line Algorithm for System Instability Studies’ [33]. OASIS
performs time simulations of each contingency from a list of critical faults and ranks the contingencies in order of severity.

### 2.3 Load Forecasting and Unit Commitment

Since the power generated must balance the power absorbed by transmission losses and consumer demand at all times (the only convenient way of storing excess generated energy is by means of a so-called 'pump-storage' scheme where the energy is stored as potential energy in water), the load on the power system must be predicted and generation, since it cannot, in general, be 'switched on' instantaneously, scheduled in advance. Various methods have been used to predict load [48] and unit commitment has been similarly well explored. Unit commitment programs, such as the National Grid Company's 'GOAL' program [49], are principally concerned with generator availability and MW loading. They take account of projected down-times, run-up rates and fuel costs in deriving an optimum (i.e. lowest cost) schedule to meet projected daily loads where the generation will be specified for roughly half-hourly intervals. Since generator sets sometimes take an hour or two to come up to full load, such projections are critical in enabling demand to be met.

![Diagram of planning of a generation and control schedule]

Figure 2.7: Planning of a generation and control schedule
The unit commitment problem has been broken down into sub-tasks at the planning and operation stages. At the planning stage (shown in figure 2.7), unit commitment is carried out on a basis of cost. The generation schedule thus found is given as the input to a load flow program and stability studies are performed to check the security of the power system under those conditions. If any security violations are identified, amendments to the generation schedule are found using some security-constrained dispatch or 'optimal power flow' (OPF) routine, which may itself be divided into reactive and active subsystems. These are introduced in the next section.

![Flowchart](image)

**Figure 2.8: On-line economic scheduling of generation**

### 2.4 Security-Constrained Dispatch

In operational i.e. same day time-scales, an economic dispatch (or unit commitment) is carried out typically every half-hour or so [49]. This proposed schedule is checked for security and amended using a security-constrained dispatch function if necessary (figure 2.8). In the time between economic dispatches, the current state of the power system is monitored for security and controls applied if necessary where the control actions are found from the security-constrained dispatch routine (figure 2.9).

As described above, security problems are generally divided into static and dynamic cat-
gories where the latter includes transient instability, oscillatory instability, small signal instability and voltage instability, although analysis of proximity to voltage instability is often carried out based on static analyses using various assumptions. The sets of actions, too, are generally divided into two categories. These derive from the well-known approximation whereby it is assumed that active power flows depend mostly on voltage angles and reactive flows on voltage magnitudes [19]. The sets of actions are

**Active controls:** MW generation and quadrature booster (i.e. phase-shifting transformer) angles;

**Reactive controls:** Voltage magnitude set-points on generation and static voltage compensators (SVCs) which control MVAr generation, set-points on mechanically switched compensators (MSCs) and transformer tap ratios.

In addition, lines can be switched out, or back in (when available), or load dropped.

Actions to control branch MW flows are generally changes to MW generation and changes to quadrature booster angles. Actions to control nodal voltage magnitudes are concerned with provision and control of reactive power and generally involve changes to voltage set-points, MSC settings or transformer tap ratios. Transient or oscillatory instabilities are also generally prevented by altering MW flows since they are phenomena associated with the transfer of machine kinetic energy as electrical energy. Strategies for preventing such instabilities generally specify maximum MW transfers across defined boundaries which in turn can be related to groups of lines so that the inequality constraints which an OPF or security-constrained dispatch program can respect can be found.

Voltage instability is more difficult to prevent since it may be caused by problems associated with MW flows or reactive generation reserve, or indeed reactive control actions such as those taken by on-load tap-changing transformers (OLTCs) i.e. those which tap up or down automatically in an attempt to meet a voltage schedule on one of the terminals. Large MW flows across long distances have large MVAr losses associated with them resulting in depressed voltage magnitudes at the receiving end and higher currents which may in turn lead to activation of protection (as is believed to have happened in the
Swedish collapse of 1983 [30]). Alternatively, local MVAr reserves to support voltage may have been inadequate.

Security-constrained optimal dispatch or optimal power flow (OPF) programs optimise generation and reactive power controls for cost and security based on 'snap-shots' of the system [50]. However, such a task is computationally extremely demanding with massive problems being encountered in the formulation of the objective function to be minimised and in the incorporation of the time dimension in such items as generator run-up rates, control response times and predicted load [51]. In practice, assumptions are made about the responses of controls, and some degree of optimality in the derivation of a realistic control schedule can then be found. One of the key compromises is that between the finding and planning of actions to correct limit violations and the scheduling of actions to prevent violations. This compromise is discussed in the next section.

![Diagram](image)

Figure 2.9: On-line maintenance of security

2.4.1 Co-ordination of Preventative and Corrective Actions

In some cases, short term violation of operating limits can be tolerated before taking corrective action. However, other forms of insecurity are regarded so seriously that every effort is made to prevent them from occurring such that 'preventative' action is necessary. These include transient instability and voltage instability (or voltage collapse). Under other circumstances where corrective actions will normally suffice, it may be found that the control action required is not available within a satisfactorily short time after
the occurrence of the security violation. In this case, too, in particular where there are no alternative actions, the action may have to be ordered preventatively.

Such a scheduling of preventative actions presents the power system operator with extra problems. The first difficulty occurs in the analysis of marginal cases where an ideal corrective action cannot be carried out quickly enough but where there are alternative courses of action which may be carried out before or after the event giving rise to the violation. Which course of action, the use of the 'ideal' one preventatively or the alternative correctively, offers the greatest benefit in terms of cost and security?

The next difficulty occurs in the context of contingency analysis where the base case (i.e. pre-contingency case) is 'alert' i.e. no operating limits are violated but there are some contingencies which cause non-correctable violations (see figure 2.2). From the contingency analysis and security-constrained dispatch applied to insecure contingencies, a number of actions which would need to be performed preventatively may be found. However, for the analysis to be valid if the actions are indeed taken, the base case should be updated and the analysis repeated. As well as this being time-consuming to perform once, a number of conflicts may arise necessitating a number of iterations.

The remainder of this section will address these problems in more detail.

**Benefit of corrective or preventative alternatives**

Assuming that there are two alternative actions available, both of which will be sufficient to relieve a given violation where one can be carried out quickly after the occurrence of the event giving rise to the violation and the other must be carried out preventatively, a judgement on which one to adopt depends on

- the cost of the action;
- the risk of the event causing the violation taking place;
- the risk of serious damage to the system occurring after the event and before the corrective action has been completed.
The first two of these are related since the cost of the preventative action will depend on over what period of time it should be maintained, and this in turn depends on the future development of the state of the power system (where the severity of the violation in question may be predicted to become higher or lower) and the risk of the event taking place. The last of these is also difficult to judge and depends on the type of violation. A temporary low voltage problem or thermal overload may be tolerable, but if the low voltage condition is allowed to persist in a situation where the system is vulnerable to voltage instability, the result may be catastrophic.

In practice, simplifications of these judgements are commonly used. For example, in the U.K., the statutory regulations specify the sorts of deviations from nominal that can be tolerated before and immediately following faults in planning and operational timescales [6]. For example, planners are required to ensure that

- 275kV voltages are maintained within ±5% and 400kV voltages within ±2.5% before a fault;
- 275kV voltages are maintained within +9.1% and -10% and 400kV voltages within ±5% immediately after a fault.

Operators are required to ensure that voltages on the 275kV network and the 400kV network are maintained within ±10% both before and after a fault.

The significance of the two sets of planning criteria is that preventative action need not be taken as long as the immediate voltages post-fault (i.e. after transients) are all within +9.1% and -10% of nominal on the 275kV system and ±5% on the 400kV system. However, corrective actions should be planned to restore voltages to within ±5% of nominal on the 275kV system and ±3% of nominal on the 400kV system. The system after corrective actions is then regarded as being in a steady state.

Thermal violations are treated in a similar way with post-event loadings tolerated up to the short-term rating, assuming that corrective action will then be taken to reduce the loading to the level permitted over a longer period. Otherwise, preventative action
Risk of transient instability generally requires preventative action. The only circumstance where it does not is when there is an intertrip scheme installed associated with the machine at risk which can automatically carry out what amounts to very fast corrective action. Otherwise, when a machine is about to go unstable, generator protection operates and the power supplied to the grid by the machine is lost. This may result in some portion of the system load being unsupplied. The choice of action, then, depends on the amount of load that would cease to be supplied after loss of generation or the tripping of key circuits, the risk of that happening, the value of the lost load and the cost of the security. Some further doubt exists, however, concerning the sorts of events that machine sets should be secured against preventatively i.e. whether 3-phase to ground, phase to phase or single phase to ground, the fault clearance times (i.e. how quickly a circuit is assumed to trip) and how many concurrent outages are to be secured against [12].
After a full contingency analysis, two sets of actions can be found. One, $P$, is that of actions to relieve security violations which should be taken preventatively. The other, $C$ is that of actions that can be taken correctively. The set $P$ should be applied to the base case so that the base case, pre-contingency, is secure, and the contingencies for which the actions have been taken are secure. Within $P$, however, there may be conflicts between actions for different contingencies to be implemented on one controller. In addition, some may cause insecurity in the pre-contingency state. These may have to be resolved by repeated iterations of the derivation of the preventative actions. Finally, the whole contingency analysis should be repeated since $C$ may no longer be valid as the base-case has changed. The process is shown in figure 2.10.

2.5 Summary

This chapter has outlined various aspects associated with the operation of a large interconnected power system. It has introduced the way data concerning the current state of the power system is obtained and the security analysis function which checks its robustness. The various factors influencing the scheduling of controls and the means of scheduling controls have been briefly outlined.
Of the functions of an EMS concerning operation of a large inter-connected power system described in chapter 2, a number make use of well-developed analytical tools. Others, however, notably 'dynamic security analysis' and dispatch, are less well served. Research in recent years has focused on attempts to utilise artificial intelligence techniques (AI), particularly knowledge-based systems, neural networks, evolutionary optimization and fuzzy expert systems. Knowledge-based systems were developed from attempts to automate logical deductive processes, and fuzzy logic was an extension of that. This chapter therefore briefly outlines two-valued logic and goes on to outline production rule (knowledge-based) expert systems, artificial neural networks and evolutionary techniques (fuzzy logic and fuzzy expert systems are described in chapter 4). The chapter concludes by briefly describing some applications of AI techniques to power system operation.

3.1 AI and Expert Systems

As the possibility of practical digital computers grew through the 1940s, speculation also grew on the potential for the development of systems that would mimic human intelligence. Early work focused on developing artificial intelligence systems that could demonstrate general intelligence, but it soon became apparent that an extensive base of
knowledge was crucial to their success. This led to the development of 'expert systems' with knowledge specialised to one area.

According to Giarratano [52], the field of expert systems is "a branch of AI that makes extensive use of specialised knowledge to solve problems at the level of a human expert", while an expert is "a person who has expertise in a certain area" i.e. knowledge not known or available to most people. One of the pioneers of expert systems technology, Professor Edward Feigenbaum of Stanford University in the United States has defined an expert system as "...an intelligent computer program that uses knowledge and inference procedures to solve problems that are difficult enough to require significant human expertise for their solution." [53]. An understanding of what expert systems are about is probably better achieved through consideration of some of their characteristics. An expert system differs from a conventional program because it [54]

- reproduces human knowledge;
- has an in-depth focused knowledge;
- can apply certain knowledge and 'rules of thumb';
- is able to explain its behaviour and results;
- has the ability to deal with missing or uncertain data;
- can receive new information and knowledge without being re-programmed.

Among an expert system’s attractive features relative to a human expert are [52]

- increased availability;
- reduced cost of use (though the cost of development and maintenance is likely to be considerably higher);
- reproducability of a number of people’s expertise;
- the reliable reproducability of decisions;
The most common categories of expert system are rule-based systems, artificial neural networks and fuzzy expert systems. Evolutionary techniques are often also included in the field of artificial intelligence. Which paradigm is suitable for which application depends largely on the type of knowledge to be represented. While a detailed set of criteria for deciding which method to use has yet to be developed, a useful first approximate judgement is that where large amounts of empirical data exist that describe a problem but there is no recognised algorithm, an artificial neural network (ANN) is likely to be appropriate, especially when a classification into a small number of categories is to be performed since a ANN is not particularly tolerant of noise. Where knowledge of what constitutes a good solution to a problem exists with no constraint on the time in which it is to be found, but there is no knowledge of how to reach the solution or of initial data, some evolutionary technique may be best. However, where knowledge in the form of heuristics is present, representation in the form of production ‘IF...THEN’ rules in an expert system may be most appropriate.

Rule-based expert systems, ANNs and evolutionary techniques will be briefly described in this section. Fuzzy systems are introduced but will be described in more detail in chapter 4.

3.1.1 Two-valued logic

This section describes some of the foundations of deductive two-valued logic that have underpinned the development of expert systems.

Definitions

When discussing formal two-valued logic, it is generally propositional logic that is being addressed. Also known as “propositional calculus”, “statement calculus” or “sentential calculus”, it deals with propositions represented by symbols or logical variables and as
such is concerned with the truth of the proposition. If two propositions “all men are mortal” and “Socrates is a man” are presented which are true, then a conclusion “Socrates is mortal” can be drawn. This kind of conclusion drawn from two premises is a syllogism. Propositions represented by lower case letters are variables and those represented by upper case letters are constants.

When two statements or propositions are represented by p and q, they can be connected by such observations as “p implies q”, “p only if q”, “p is sufficient for q”, “q if p”, “q is necessary for p” and “if p then q”. All these can be denoted p → q and are conditional statements or “material implications”. Biconditional statements are also possible and these are denoted p ↔ q and have the meaning “p if and only if q”, “q if and only if p” and “if p then q and if q then p”.

There are three other ways of connecting propositions in addition to the conditional and biconditional namely “and”, “or” and “not”. These are denoted by ∧, ∨ and ¬ or ~ respectively.

Since propositional logic allows the use of variables, particular compound statements may or may not be true, depending on the value of the variables. Special cases are tautologies and contradictions. A tautology is a compound statement that is always true whether the individual statements are true or false, e.g. p ∨ ¬p. A contradiction is a compound statement that is always false such as p ∧ ¬p. A contingent, meanwhile, is a statement that is neither a tautology nor a contradiction.

A special case of a conditional statement that is a tautology is an implication and it is denoted by ⇒. A biconditional statement that is a tautology is an equivalence and is denoted by ⇔.

A more sophisticated form of logic that makes use of all the above forms but which also allows the use of quantifiers such as “all”, “some” and “no” is first order predicate logic. All quantifiers in first order predicate logic must be expressed in terms of the universal quantifier, ∀ (interpreted as “for all...”), or the existential quantifier ∃ (interpreted as “there is some...”, “at least one...” or “some...”). Examples of predicate functions are
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\[(\forall x)(\text{dog}(x) \rightarrow \text{animal}(x))\] meaning "for all \(x\), if \(x\) is a dog, \(x\) is an animal", or "all dogs are animals".

**Inference**

Logic systems have found use in forming reasoning systems or systems of proof or denial, and as such they are what underpin expert systems. This section looks at methods of reasoning or *inference*.

There are a number of kinds of inference:

- **deduction** - logical reasoning where a conclusion *must* follow from a set of premises;
- **induction** - inference from a specific case to the general;
- **intuition** - there is no proven theory and an answer appears either by extrapolation or interpolation, or is in some way the 'best answer' (artificial neural networks and fuzzy expert systems may be capable of this, but production rule expert systems are not);
- **heuristics** - 'rules of thumb' based on experience;
- **generate and test** - by a process of trial and error;
- **abduction** - reasoning which works back from a true conclusion to find the premises;
- **default** - an assumption made in the absence of contradictory evidence;
- **analogy** - a conclusion is inferred based on similarities to another situation;
- **nonmonotonic** - where a new conclusion is made upon receipt of new 'knowledge' (previous 'knowledge' may be found to have been incorrect).

In formal logic, deduction is the most significant form. A chain of reasoning based on deductive logic is an *argument*, and an example of an argument is a syllogism such as
Anyone who plays for Arsenal is good
Ian Wright plays for Arsenal
\[ \therefore \text{Ian Wright is good} \]

or IF anyone who plays for Arsenal is good AND Ian Wright plays for Arsenal THEN
Ian Wright is good.

**Modus ponens, modus tollens and resolution**

Two special cases of syllogistic logic are *modus ponens* (also known as "direct reasoning" or "forwards reasoning") and *modus tollens* (or "backwards reasoning"). Modus ponens is of a form such as

\[
\text{If there is power the computer will work.}
\]
\[
\text{There is power.}
\]
\[ \therefore \text{The computer will work.} \]

This can be compared directly to the syllogistic form which is

\[
\text{All computers with power work.}
\]
\[
\text{This computer has power.}
\]
\[ \therefore \text{This computer will work.} \]

In general, modus ponens can be written

\[
p \rightarrow q
\]
\[
p
\]
\[ \therefore q \]

or \( p, p \rightarrow q; q \). For \( N \) premises \( P_1 \ldots P_N \) of the form \( r \) or \( r \rightarrow s \), it can be written \( P_1, P_2, \ldots P_N; C \) where \( C \) is a conclusion.

The argument \( P_1, P_2, \ldots P_N; C \) is only a valid argument if and only if \( P_1 \land P_2 \land \ldots P_N \rightarrow C \) is a tautology. Modus ponens, then, is tautology which can be written \( (p \rightarrow q) \land p \rightarrow q \). The truth table for modus ponens is shown in table 3.1.

Modus tollens can be regarded as the inverse of modus ponens. Its schema is
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<table>
<thead>
<tr>
<th>variables</th>
<th>premises</th>
<th>conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>q</td>
<td>p → q</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
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<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Table 3.1: Truth table for modus ponens

\[
p → q
\]
\[
\neg q
\]
\[
\therefore \neg p
\]

The truth table is shown in table 3.2.

<table>
<thead>
<tr>
<th>variables</th>
<th>premises</th>
<th>conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>q</td>
<td>p → q</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
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<td>T</td>
<td>F</td>
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<td>F</td>
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<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Table 3.2: Truth table for modus tollens

Other “laws of inference” include the “law of the contrapositive” \((p → q; \neg q → \neg p)\), the “chain rule” or “law of the syllogism” \((p → q, q → r; p → r)\) and the “law of disjunctive inference” \((p ∨ q, \neg p; q)\).

A formal logic proof is often based on sets of valid substitutions and modus ponens. However, it is possible to use only one rule, that of resolution. In this, valid statements are expressed in “conjunctive normal form” which is the conjunction of disjunctions of literals where literals are the basic building blocks and are statements which do not use connectives or quantifiers. For example, \(A_1, A_2, \ldots A_N → B\) may be re-expressed as \(A_1 ∧ A_2 ∧ \ldots A_N → B\) or \(\neg (A_1 ∧ A_2 ∧ \ldots A_N) ∨ B\) which is \(\neg A_1 ∨ \neg A_2 ∨ \ldots ∨ \neg A_N ∨ B\).

Once an expression has been resolved, its constituent clauses can be tested to check the validity of the whole statement. Indeed, the validity of a statement is often checked by
looking for a contradiction in the negation of the statement (or theorem). This is known as "reductio ad absurdum". As an example, consider the theorem

\[
A \rightarrow B \\
B \rightarrow C \\
C \rightarrow D \\
\therefore A \rightarrow D
\]

Using the equivalence \( p \rightarrow q \equiv \neg p \lor q \), the conclusion is \( A \rightarrow D \equiv \neg A \lor D \). The contradiction of this is \( \neg(\neg A \lor D) \equiv A \land \neg D \). The premises and the conclusion can be re-expressed as the conjunction of the disjunctive forms as \( (\neg A \lor B) \land (\neg B \lor C) \land (\neg C \lor D) \land A \land \neg D \). Each clause can be expanded as shown at each node of a resolution refutation tree. The tree for the example is shown in figure 3.1. The result follows trivially from the "law of the excluded middle" i.e. \( p \land \neg p \Rightarrow 0 \). As can be seen from the figure, the result is nil thus disproving the negated theorem and proving the theorem.

```

-\neg A \lor B
-\neg B \lor C
-\neg A \lor C
-\neg A \lor D
-\neg C \lor D
-\neg D

nil
```

Figure 3.1: Example of a resolution refutation tree

Such resolution processes are important in proving theorems that can then be connected in inference chains, or decision trees. These will now be briefly introduced before moving on to expert systems.
Shallow (or experiential) and deep (or causal) reasoning

The terms shallow and deep are used to describe chains of inference or decision trees. In shallow (or "experiential") reasoning, the chain is very short while in deep (or "causal") reasoning, it is long i.e. inferred knowledge is used in the premises of other conditional statements. Deep reasoning may be used not just to test conditions but to provide some explanation of the conclusion or provide a model of the process being tested. Whether deep or shallow reasoning is to be used depends on whether lots of facts down the chain can be observed, or if there are only a few at the top which are observable. It is possible to change from deep to shallow reasoning by combining premises from many rules into one where the single rule is a theorem which can be proved by resolution as described above. So it is that different decision trees can be assembled which are the basis for expert or "rule-based" systems. However, an expert system is not simply a decision tree.

Unification

An expert system is distinguished from a decision tree by the possibility of using variables in rules. For example, if there was a fire alarm system decision tree, there might be a number of rules:

\[
\begin{align*}
\text{IF sensor 1 activates THEN sound fire alarm 1} \\
\text{IF sensor 2 activates THEN sound fire alarm 2} \\
\vdots \\
\text{IF sensor N activates THEN sound fire alarm N}
\end{align*}
\]

In an expert system, however, one rule suffices:

\[
\text{IF sensor ?N activates THEN sound fire alarm ?N}
\]

The process of determining the set of variables in a theorem is known as unification [55].
3.1.2 Rule-based systems

Rule-based expert systems, or production systems because they are based on sets of production rules of the form IF A THEN B, are the most popular form of expert system. Their advantages include [56]

- the simplicity of each rule;
- the ease of separation of knowledge and inference;
- modularity of rules in some systems.

However, there are some disadvantages:

- they may be an inappropriate representation of the knowledge;
- the search through the rule-base becomes inefficient with growth in the size of the rule-base;
- verification and validation of large rule-bases is difficult;
- maintenance of a large rule-base is difficult.

![Figure 3.2: The typical structure of a production rule-based system](image)

Figure 3.2: The typical structure of a production rule-based system

The general model for the design of a production rule-based system is shown in figure 3.2. The inference engine infers further knowledge from the knowledge base through
the execution of the rules. The rules are assembled in a decision tree which is searched in order to elicit knowledge by one of two methods:

- forward chaining;
- backward chaining.

*Forward chaining* is what is known as a ‘data-driven’ strategy [57] and is analogous to the modus ponens. Rules are executed based upon an initial set of data which may be changed depending on the actions caused by the firing of certain rules. It tends to direct the search down through the tree. *Backward chaining* is ‘goal-driven’ with a hypothesis formed at the start of the search and all the rules applied to see which of them satisfy the hypothesis. It analogues to the modus tollens. Such a strategy can be used to find a number of factors which contribute to the situation being analysed.

To illustrate forward and backward chaining, consider the inference chain

\[
\text{elephant (x)} \rightarrow \text{mammal (x)}
\]

\[
\text{mammal (x)} \rightarrow \text{animal (x)}
\]

Given a fact “Clyde is an elephant”, can it be concluded that Clyde is an animal? The forward chain which reaches that conclusion is shown in figure 3.3, though what is really happening is shown in figure 3.4 where unifications of variables and implications are being performed.

![Figure 3.3: Example of a forward chain](image)

The backward chain would have started with the hypothesis that Clyde is an animal and would have found that the evidence “Clyde is an elephant” supported the hypothesis.
Forward chaining is more appropriate for expert systems which expect to find one conclusion, classification or course of action. It is the first main task for the designer of an expert system to decide whether the application is more suitable for forward or backward chaining.

Once it has been decided whether the knowledge is more appropriate for a data driven search or a goal driven search, the procedure for moving through the tree is then chosen, in general, as one of

- depth-first search;
- breadth-first search.

In a depth-first search, fired rules are pursued all the way to the end of the tree. In a breadth-first search, each level of the tree is explored fully and then fired rules are followed to the next level. These processes are illustrated in figure 3.5 where the numbers correspond to the orders in which the rules are tested.
3.1.3 Artificial neural networks

Artificial neural networks (ANNs) were first developed in the 1980s as a method of modelling the way in which the brain processes information. An ANN is basically a highly parallel analogue computer [58, 59]. Each processing node or neuron performs some simple function (known as an activation) on its inputs to give one output which may be the input to a neuron on the next layer. Weightings on connections between neurons are not programmed but are initially randomly set and 'trained' by comparing the output of the ANN with the desired output. A neural network can often be a good choice for problems for which there is a lot of empirical data and no algorithm which can be solved quickly enough in a conventional way. Other advantages include

- fault tolerance—a portion of a net can be removed and the net retrained to the original 'skill level' if enough neurons remain;
- graceful degradation—the performance of the net degrades gracefully in proportion to the amount of the net removed; there is no catastrophic loss of performance;
- interpolation from stored information—ANNs can classify data that has not been seen before;
- speed.

Artificial Neural Networks (ANNs) have grown from the perceptron concept [60] where the problem of how a machine might learn by example was first seriously addressed. The internal reorganisation of the machine from its initial state to a final state enables it not only to recognise example patterns that it has 'seen' before but also to recognise similar patterns. Of the various types of ANN, the feed-forward layered model has had most attention paid to it as it is especially suitable for use as a pattern classifier.

A typical feed-forward ANN, is illustrated in figure 3.6, where a layer represents a topological set of neurons.
Patterns are presented to the neurons in the input layer, and the outputs of the input layer neurons are then calculated. These values are then multiplied by connection weights and fed-forward to the inputs of the next layer neurons, and the process repeated until the neurons in the output layer have computed their output values.

There are a variety of learning algorithms suitable for a feed-forward ANN which alter the neuron connection weights. All of these approaches share the same basic method where the ANN is presented with an input pattern and feed-forward propagation is used to calculate the output pattern. This output pattern is then compared to the desired output pattern in the training data set and the error is calculated. Various methods can then be employed to back-propagate the error through the ANN so that when the pattern is propagated through the ANN again, the error has been reduced. One of the most widely used learning algorithms is that of back-propagation with momentum [61] which provides a faster learning algorithm than the basic approach.

Key factors in the successful application of a ANN are the choice and application of training data. Since the efficiency of the network degrades with a higher number of neurons, only items of information which are particularly significant in enabling the classification should be chosen as inputs (known as feature selection). Since the net is to be used to interpolate between training cases to give classification for cases not seen be-
fore, the training set should adequately cover the 'feature space'. In addition, the net should not be over-trained. Once the error has begun to change by a significantly small amount from one training iteration to the next, training should be stopped lest 'memorisation' takes place where the ANN will exactly reproduce the training patterns but will be unable to perform the required interpolation.

While ANNs are excellent at performing classifications, they are less suitable for applications requiring 'number crunching' or optimum solutions. The training can be extremely time-consuming and the results that a net gives are difficult to justify or explain.

3.1.4 Fuzzy logic

Conventional logic assumes that a variable has one precise value i.e. it is crisp. Fuzzy logic attempts to model the vagueness of human reasoning by reflecting uncertainty about a variable's value through the assignment of a set of values to the variable, each of which has a degree of membership of the set which reflects the likelihood of the variable having that value [62]. A membership function defines the degree of membership over the range of possible values or universe of discourse. Such a function can be assigned for an adjective (known as a linguistic value or a fuzzy set) that describes the set of values. It is this property that gives fuzzy logic its power to model qualitative reasoning and to be used in knowledge representation.

When applied in a fuzzy expert system, fuzzy logic can be viewed as providing a hybrid of rule-based systems and artificial neural networks. While, ostensibly, the knowledge is represented in the form of rules, the mechanism for doing so is analogous to an ANN where the membership function of a fuzzy set is comparable to the transfer function of a neuron and where fuzzy sets are connected in parallel with signals fed on to one or more output fuzzy sets. Precedences exist for trained neural networks then being represented by fuzzy expert systems (this is possible where a large amount of empirical data is available to represent the problem), and for fuzzy expert systems being implemented as neural networks [63].
Fuzzy logic and fuzzy expert systems are discussed in more depth in chapter 4.

3.1.5 Evolutionary optimization

Many authors include evolutionary optimization in the category of artificial intelligence, even though it is arguably not intelligent at all as it relies on probabilistic rather than decision-based processes (by the same token, it could be argued that artificial neural networks, or ANNs, are not intelligent either as they can be regarded simply as non-linear interpolators). A brief discussion of the approach is included here for completeness as, like those methods more uniformly associated with AI (including ANNs), it claims to simulate some 'biological' process. Indeed, it could be argued that it belongs in the area of 'knowledge engineering' as it simulates 'knowledge handed down' from one 'generation' to another.

There are two main techniques that come within the category of evolutionary optimization: genetic algorithms and evolutionary programming. They attempt to model evolutionary processes where "generations" of possible solutions to a problem are used to create another 'better' generation until some convergence criterion is met. The main advantage of these methods is that they are said to be able to find to global optima i.e. they do not stop at local optima. Genetic algorithms and evolutionary algorithms will be briefly described in the next two sections.

Genetic algorithms

Genetic algorithms simulate evolution by emphasising chromosomal operators [64]. The general procedure is as follows:

1. An objective function is defined that will demonstrate the fitness of any solution.

2. With certain constraints defined, an initial population of candidate solutions is set up, each trial population being represented as a vector $x$. Each chromosome $x_i$,
\( i = 1, \ldots P \), is a binary string i.e. a string of 1s and 0s. The length of the string is decided by the programmer according to how much precision is needed.

3. Each \( x_i \) is assigned a fitness score \( \mu(x_i) \) which represents the degree to which the objective is satisfied.

4. Each \( x_i \) is assigned a probability of reproduction, \( p_i \), in proportion to its fitness relative to the other chromosomes.

5. A new population of chromosomes is generated from the existing one according to the probabilities of reproduction. “Offspring” are generally created by one of two methods, crossover and bit mutation. In crossover, two parent chromosomes are selected and two offspring the made by splicing the genetic coding of one of the parents at some random distance along it, then splicing the other at the same place and swapping either the left half or the right between the two parents to create the offspring. In bit mutation, individual bits (or genes) are flipped from 1 to 0 or 0 to 1 according to some probability function.

6. When a ‘suitable solution’ has been found or the allotted processing time has expired, the process is stopped, otherwise the cycle is repeated from step 3.

Evolutionary algorithms

The category of evolutionary algorithms is sub-divided into evolution strategies and evolutionary programming. The distinction between the two is subtle [64]. Only the basic approach will be outlined here. Unlike the genetic algorithm, it emphasises behaviour of each individual with respect to some objective in the forming of each new population.

When applied to the optimization of some objective function \( F(x) \) where \( x \) is an \( n \)-dimensional vector of parameters upon which the objective function depends, the general method is:

1. Start with a population of parent vectors \( x_i, i = 1, \ldots P \) with each element \( x_{ij} \),
\[ j = 1, \ldots n \] of each vector \( x_i \) determined as a random number within the feasible range.

2. A family of offspring vectors \( x'_i, i = 1, \ldots P \) is found from the initial population by adding a Gaussian random variable with zero mean and pre-selected standard deviation to each component of each vector.

3. The vectors which survive are chosen as the \( P \) vectors from the initial population \( x_i, i = 1, \ldots P \) and the offspring population \( x'_i, i = 1, \ldots P \) which give the lowest values of the objective function \( F(x) \). These then form the new generation of parents.

4. If some convergence criterion for objective function found from one of the vectors \( x_i \) is reached, this forms the solution, otherwise the process goes back to step 2 until the allotted computation time is exhausted.

### 3.2 AI Methods in Power System Operation

In power system operation, the principle concern is that of maintenance of secure supply. To achieve that, the security status of the system must be assessed and generation and other controllable plant dispatched to meet demand and constraints on security. In the following sections, some applications of AI to these functions are briefly described with some of their relative merits and de-merits. Since this thesis is primarily concerned with the dispatch of controls to meet security constraints, applications of AI methods to protection, planning, the design of power system stabilisers, restoration, load forecasting, unit commitment and alarm processing are not addressed.

#### 3.2.1 AI in Security Assessment

The assessment of static security i.e. that based on the solution of the algebraic power system flow equations where branch flows and nodal voltage magnitudes must be kept within limits, is well-established [18]. However, the assessment of transient, oscillatory
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and steady state stability is more difficult and is therefore less well developed. Oscillatory and steady-state stability are becoming more of interest to power utilities as they attempt to transfer more power over greater distances on existing transmission systems. Transient stability is something has been of concern for some time and large volume of work addressing it exists. This will therefore be the subject of the rest of this section.

As stated earlier, the earliest methods for estimation of transient stability focused on simulation of the generating sets through the solution of the differential equations associated with the machines and AVRs for successive points in time. Such repeated solutions are inevitably time-consuming though a simulator developed for implementation on a workstation such as the DEC Alpha is capable of modelling the behaviour of a 1000-node, 150-machine power system in faster than real-time [33]. While this is fine when security assessment is to be performed for a small number of contingencies, the analysis of the effects of perhaps two or three thousand contingencies seen on a 1000-bus system every 10-15 minutes is impossible without huge investment in computing resources. Some short-cuts would appear to be necessary.

The short-cuts have come about in two ways:

1. to replace the time-domain simulation by some faster approximation and

2. to break the set of contingencies down into those where the effects are known with confidence and those where they are not.

It has been suggested that the time-domain simulation can be replaced by calculation of the so-called ‘transient energy function’ (TEF) [39] and by what has become known as the ‘extended equal area criterion’ [34]. However, these methods are only applicable for analysis of the first swing of rotor angles. In addition, it has recently been recognised that limitations are imposed on both in terms of the complexity of the machine governor and AVR models that can be used, and the sequence of circuit breaker actions that can be modelled [65, 40].

The transient energy function and extended equal area criterion have therefore found use
in quickly providing preliminary assessments of transient stability. Such assessments perform screening of the contingency list enabling judgements to be made about those contingencies which the operators confidently believe will not lead to instability. Those which the approximation shows either to be unstable or close to being unstable then undergo more detailed analysis or evaluation. This process is illustrated in figure 3.7. The area of contingency screening is the one where AI techniques have found most use.

All of the AI approaches to contingency screening have been concerned with pattern classification. ANNs have been of particular interest in this area since they are capable of performing classifications very rapidly [66, 67]. The two cited were trained using the results of calculations of transient energy functions since they provide direct measures of proximity to instability. A similar approach using fuzzy clustering has been reported in [68].

Few of these classification methods have been tested on realistically-sized power systems where the dependence on being trained for specific network topologies appears to be a major drawback. In addition, many published implementations have had a number of inputs to the ANN which is approximately equal to the number of nodes in the power system. Since the efficiency of an ANN falls away rapidly when the number of inputs is much above 20 or 30, this 'curse of dimensionality' is another stumbling block. Recently, however, work has been published which claims to have overcome this problem.
through judicious choice as inputs of key features which correlate well with a measure of stability, or statistical functions of those features [69]. This work also uses time-domain simulations to train the ANN so that the full set of network and machine states is available for use as features for the ANN. The measure of stability is the biggest machine rotor angle found in the 30 seconds after the onset of the contingency. When being used to predict stability, a simulation is performed until all the switching actions associated with the contingency have finished. The output of the ANN is a prediction of the greatest machine rotor angle which would ensue.

Some papers have described expert system classifications, particularly a sequence of work from Liege in Belgium which applies inductive inference to the definition of a decision tree i.e. it reasons from specific cases to find general rules. The work goes on to describe how the system can be self-learning to deal with cases the system has not seen before, thereby appearing to overcome a major drawback of knowledge- or production rule-based systems [70, 71]. Since the expert system uses data from a pre-contingency power system state as its fact base (rather than some training set based on the TEF, for example), this work would appear to be less dependent on power system topology.

The contingencies which have been fully evaluated are then ranked in order of severity so that the most severe may be addressed first by the operators when they come to find preventative or corrective actions. Conventional formulae for finding severity indices follow the pattern of equation 2.1, though most of these are concerned only with static security. Even so, problems can arise due to ‘masking’ when a number of slight violations can mask the effect of one large one. This has led to the work reported in [72] and [73]. The former uses fuzzy sets to classify severity. Recent work by Matos et al [74] uses fuzzy clustering techniques which employ fuzzy sets to describe a multi-dimensional feature space. Fuzzy clustering is not addressed elsewhere in this thesis, but the interested reader can refer to [75].

Classification of transient stability problems has tended to be along the lines of critical fault clearing time which can be obtained directly from transient energy function analysis. Groom et al extended the fuzzy set classification approach of [72] to transient and
oscillatory stability based on measurements taken from a time-domain simulation [76].

3.2.2 AI in Active and Reactive Dispatch

Since ANNs are poor at reaching optimal 'number-crunching' solutions or solutions with a large number of output variables, their applications to dispatch are few and far between. Those that have been published have tended to be along the lines of the application of neural networks designed for numerical optimisation applied to conventional formulations and objective functions [77], i.e. the ANN simply solves the equation. This results in little gain over traditional methods since the principle problem with those methods is the definition of the objective function. Training ANNs to perform dispatch based on past heuristically derived schedules would appear infeasible at present owing to the huge number of output variables and possible permutations.

AI methods which model operator heuristics directly, or model the ways operators respond to analytical solutions (from which the control schedule is then derived), have appeared the most fruitful lines of research in this area, particularly when concerned with reactive dispatch where expert systems have been applied since 1986. Production rule systems which model clean operator decision boundaries and fuzzy logic which models more qualitative co-ordination of sets of priorities are ideally suited. While some of the methods in existence are hybrids between sets of production rules and linear programming, there is a fair amount of published literature on the subject [78, 79, 80, 81, 82, 83]. Some work utilises fuzzy sets in modelling the uncertainty associated with approximations in a linear programming MW dispatch routine [84].

In a recent paper, a genetic algorithm was used for reactive power optimisation [85]. It was claimed that in minimizing a severity index like that of equation 2.1, the global optimum was found, but it was admitted that the complexity of the method grew exponentially with the size of the power system being optimised. Large numbers of populations and 'excessive' CPU were needed.

In conclusion, ANNs, which researchers have tried to use to quickly derive the final
schedule directly from a set of measurable power system parameters have not shown
great advantages over traditional techniques. The use of expert systems to model heur­
istics in the reduction of the problem or in the co-ordination of different priorities has
proved more fruitful.

Security-constrained reactive and active dispatch is further discussed in chapters 8, 9
and 10.

3.3 Summary

This chapter has introduced artificial intelligence and briefly outlined its main fields. It
has also described something of the way formal logic is used in expert systems.

The chapter has then briefly reviewed some of the ways the AI techniques described have
been applied in power system operation, in particular security assessment and dispatch.
4
Uncertainty, Fuzzy Logic and Fuzzy Control

All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life but only an imagined celestial existence. Bertrand Russell (1923)

The idea of fuzzy sets is an extension of conventional set theory formalised by L.A. Zadeh in 1965 [86] in order to deal with uncertainty concerning a statement’s exact meaning. As introduced in section 3.1.4, it attempts to reproduce the qualitative nature of human thinking (for example, a human would not say, “meet you in The Star at 9.03 and 57.56 seconds”, they would say “meet you in The Star at about 9 o’clock”).

![Figure 4.1: Example of a fuzzy membership function](image)

Figure 4.1: Example of a fuzzy membership function
While a crisp variable either is or is not a member of a particular set, a fuzzy variable has a degree of membership or degree of truth which for the range of the variable is described by a membership function. A membership function, generally denoted by $\mu(x)$ where $x$ is the variable whose degree of membership is being described, may look like the one shown in figure 4.1. A fuzzy variable's value may be described not by a number but by an adjective. In this way, a fuzzy variable is also known as a linguistic variable and its value as a linguistic value [62]. It is this property that gives fuzzy logic its power to model human qualitative reasoning. The figure illustrates the degree of truth of qualitative statements such as “person A is tall” which is the mechanism by which numerical meaning is assigned to qualitative statements. The statement “A is tall” is known as a fuzzy proposition where the linguistic variable ‘height’ has the value ‘tall’. If person A has a height of 7 feet, the degree of truth of the statement “A is tall” would be 1.0. If, on the other hand, A was 5 feet 6 inches tall, the truth of the statement would be 0.4. A crisp variable can also have a membership function (fuzzy systems can include crisp functions, but not vice-versa), but there would only be one value for which the degree of membership of the set, or the degree of truth, would be non-zero.

Fuzzy logic is not the only means of modelling uncertainty that has been developed. In order to illustrate its application, other means of modelling different kinds of uncertainty will be described before going on to provide details of the way fuzzy logic can be used to model experts’ judgements in control systems.

### 4.1 Uncertainty

Uncertainty can be considered as having insufficient information to be able to make a decision, or to make a decision with complete confidence. As was hinted at by Russell in the quote that opened this chapter, most real situations are prone to uncertainty as they are very often not crisp or deterministic and cannot be described precisely. Schwarz questioned the ill-considered use of a mathematics formed on systems of axioms justified by the sorts of processes described in section 3.1.1 when he said [87] “An argument, which is only convincing if it is precise, loses all its force if the assumptions on which
it is based are slightly changed, while an argument which is convincing but imprecise may well be stable under small perturbations of its underlying axioms.”. Because of the difficulty of providing a one-to-one mapping from natural language to mathematics, some new form of mathematical description of real world concepts or observations was sought. This provided the basis for “inexact reasoning” (the reasoning described in section 3.1.1 was “exact”).

Uncertainty can arise in different ways, for example from ambiguity when a statement can be interpreted in more than one way, from incompleteness when some information is missing, from imprecision in the measurement of a value, from some incorrectness when some supplied information is wrong, or from some error of reasoning [52]. The uncertainties in power systems listed in section 1.5.2 cover most of these categories.

The oldest way of dealing with uncertainty is classical probability theory. This basis, also known as *a priori* probability, assumes that the systems being measured are ideal i.e. they are precisely reproducible. How reproducible they are often depends on the sample size. For example, the odds of throwing one 6 in 6 throws of a die are unlikely to match the number of times a 6 really is thrown. However, you are likely to be able to predict with relatively greater accuracy how many 6s you will throw in 600 throws. A *posterior* probability, on the other hand, depends on taking measured probabilities from observed events.

### 4.1.1 Bayes’ Theorem

The “Bayesian” approach to uncertainty, named after an 18th century British clergyman and mathematician, resolves the problem of finding the probability of some event having occurred given that some later one has. It relies on probabilistic analysis of domain data.

Normally, conditional probability provides the probability of a later event occurring given an earlier one. If event $B$ is the earlier one and event $A$ the later one, this probability is $P(A|B)$. This would normally be found knowing the probability of $A$ and $B$ occurring (regardless of which came first) which is $P(A \cap B)$, and the probability of $B$
Bayes' Theorem states that, when \( P(A|B) \), \( P(A) \) and \( P(B) \) are known, \( P(B|A) \) can be found from

\[
P(B|A) = \frac{P(A|B)P(B)}{P(A)}
\]  

(4.2)

This is important because, for example, it is easier to find the proportion of people who, if they have measles, have spots than the proportion of people who, if they have spots, have measles.

The theorem is more conventionally stated in terms of a hypothesis \( H \) and evidence \( E \) so that the probability of the hypothesis given the evidence is

\[
P(H|E) = \frac{P(E|H)P(H)}{P(E)}
\]  

(4.3)

\( P(H|E) \) can also be interpreted as a 'degree of belief' or likelihood that \( H \) is true given \( E \). If \( P(H|E) \) is 1, then it can be believed that \( H \) is certainly true. In a practical expert system, such a likelihood will often be some expert's judgement of the likelihood of \( H \).

Expert systems quite often evaluate odds so that the odds in favour of \( H \) (known as the 'prior odds') are

\[
O(H) = \frac{P(H)}{1 - P(H)}
\]  

(4.4)

The odds of \( H \) given \( E \) (the 'posterior odds') are then

\[
O(H|E) = O(H)LR(H|E)
\]  

(4.5)

where \( LR(H|E) \) is the likelihood ratio. This likelihood ratio is also known as the sufficiency measure, \( LS \) and is

\[
LS = LR(H|E) = \frac{P(E|H)}{P(E|H')}
\]  

(4.6)

where \( H' \) is not-\( H \).
There is another likelihood ratio which is used to find the odds of \( H \) given that \( E \) is not present. This is the *necessity measure* \( LN \) and is

\[
LN = LR(H\mid E') = \frac{P(E'\mid H)}{P(E'\mid H')}
\]  

(4.7)

The certainty of the hypothesis, then, can be found using \( LS \) when the evidence \( E \) is present and using \( LN \) when \( E \) is not present. Interpretations associated with different values of \( LS \) and \( LN \) are shown in tables 4.1 and 4.2.

<table>
<thead>
<tr>
<th>( LS )</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( H ) is false when ( E ) is true</td>
</tr>
<tr>
<td>( 0 &lt; LS \ll 1 )</td>
<td>( E ) is unfavourable for concluding ( H )</td>
</tr>
<tr>
<td>( \approx 1 )</td>
<td>( E ) has no effect on belief of ( H )</td>
</tr>
<tr>
<td>( LS \gg 1 )</td>
<td>( E ) is favourable for concluding ( H )</td>
</tr>
<tr>
<td>( \infty )</td>
<td>( E ) is logically sufficient for ( H ), or ( )</td>
</tr>
<tr>
<td></td>
<td>Observation of ( E ) means ( H ) is true</td>
</tr>
</tbody>
</table>

Table 4.1: Interpretation of \( LS \)

<table>
<thead>
<tr>
<th>( LN )</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( H ) is false when ( E ) is absent</td>
</tr>
<tr>
<td>( 0 &lt; LN \ll 1 )</td>
<td>Absence of ( E ) is unfavourable for concluding ( H )</td>
</tr>
<tr>
<td>( \approx 1 )</td>
<td>Absence of ( E ) has no effect on belief of ( H )</td>
</tr>
<tr>
<td>( LN \gg 1 )</td>
<td>Absence of ( E ) is favourable for concluding ( H )</td>
</tr>
<tr>
<td>( \infty )</td>
<td>Absence of ( E ) is logically sufficient for ( H )</td>
</tr>
</tbody>
</table>

Table 4.2: Interpretation of \( LN \)

In reality, not only the hypothesis \( H \), but also the evidence is uncertain. A degree of belief in the complete evidence \( E \) is dependent on some partial evidence \( e \) and is \( P(E\mid e) \). The partial evidence is the portion of \( E \) that is known for certain. \( P(E\mid e) \) is the belief in \( E \) given imperfect knowledge, \( e \), of the complete evidence \( E \). The chain of inference that results is discussed in [52].

Some hypotheses depend on the conjunction of different pieces of evidence, and probability theory provides a mechanism for assessing the odds of the hypothesis under those circumstances. It does, however, depend on having some knowledge of the prior odds.
before any evidence is gathered. In addition the degrees of belief in individual pieces of
evidence are multiplied together, but this is often impractical as few pieces of evidence
are completely independent. The combination of evidence by using the minimum of all
the evidences, as in fuzzy logic, is one approach that could be used instead.

These problems with finding prior odds and independent evidence (problems which are
greatest when the evidence is based on some human judgement rather than measured
data) led to research into other theories of inexact reasoning.

4.1.2 Certainty factors

Uncertainties in rules can arise from errors in the data, uncertainty about evidence or
uncertainty about how different evidences are to be combined. The way rules fit together
may give problems such as contradictions or conflicts.

The problems with modelling uncertainty using Bayes’ Theorem led the researchers who
developed ‘MYCIN’, an expert system for diagnosing bacterial infections of the blood
[88], to associate certainty factors with each fact and each rule. Their suspicions about
the limitations of Bayesian reasoning were first raised when experts expressing their de­
grees of belief in certain statements objected to one minus the degree of belief being
used to express their degree of disbelief, as probability theory would require. The cer­
tainty factor approach attempts to model the expert’s degrees of belief and disbelief in
one number.

A measure of ‘increased belief in \( H \) due to \( E \), \( MB \), is defined based either on prob­
abilities or an expert opinion. A measure of ‘increased disbelief in \( H \) due to \( E \), \( MD \),
is defined similarly. These are then combined into the certainty factor \( CF \) in the hypo­
thesis \( H \) due to evidence \( E \) by

\[
CF = \frac{MB - MD}{1 - \min(MB, MD)}
\]  \hspace{1cm} (4.8)

The value of \( CF \) lies in the range \([-1, 1]\). Different pieces of evidence can be connected
by the operators listed in table 4.3. It will be noted that they resemble standard operators
for fuzzy connectives.

<table>
<thead>
<tr>
<th>Connective</th>
<th>Resulting evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1 \text{ AND } E_2$</td>
<td>$\min(CF(H, E_1), CF(H, E_2))$</td>
</tr>
<tr>
<td>$E_1 \text{ OR } E_2$</td>
<td>$\max(CF(H, E_1), CF(H, E_2))$</td>
</tr>
<tr>
<td>NOT $E$</td>
<td>$-CF(H, E)$</td>
</tr>
</tbody>
</table>

Table 4.3: MYCIN connectives

A convention is also provided in MYCIN for combining certainty factors from different rules that conclude the same hypothesis.

Problems existed with the certainty factors approach due to situations where a certainty factor could contradict a conclusion that would be reached with conditional probability. This is a serious problem if the 'likeliest' conclusion is being sought, such as in medical diagnosis. There were also difficulties with propagating certainty factors through long inference chains, though this has also been said of fuzzy logic [89].

4.1.3 Fuzzy logic and uncertainty

Problems with modelling of uncertainty based on Bayes' Theorem and a lack of formal rigour in certainty factors have led researchers to seek alternative models. This section outlines the way researchers have tried to develop a formal logic from two-valued logic (section 3.1.1) which models uncertainty about a variable's precise value and which generalises multi-valued logics.

In the introduction to this chapter, fuzzy sets were described which characterise ambiguous statements such "$x$ is tall". This sort of uncertainty lies in the semantics, or the meaning of the phrase i.e. it could mean "$x$ is 6ft tall" or it could mean "$x$ is 6ft 5in tall". Such 'semantic uncertainty', though it is the one most commonly associated with fuzzy set theory, is not the sort modelled by Bayes' Theorem or certainty factors. They are concerned with assigning measures of uncertainty caused by incomplete information concerning crisp hypotheses [90]. In the realm of fuzzy sets, this too is possible. This use of fuzzy set theory to model uncertainty about a crisp statement is known as pos-
sibility theory [91]. The two kinds of treatments of uncertainty facilitated by fuzzy set theory are introduced in the next two sections.

Fuzzy reasoning: possibility

Possibility theory concerns the association of a degree of possibility or a degree of necessity to a (crisp) proposition or hypothesis in order that a judgement can be made about whether the proposition is more plausibly true or false. \( \Pi(p) \) denotes the possibility degree of the proposition \( p \) and \( N(p) \) denotes the necessity degree [90]. Both \( \Pi(p) \) and \( N(p) \) fall in the range 0 to 1. Various conventions are defined:

1. \( N(p) = 1 \) meaning that, from the available knowledge, \( p \) is certainly true.
2. \( \Pi(p) = 0 \) meaning that it is impossible for \( p \) to be true.
3. \( \Pi(p) = 1 - N(\neg p) \) meaning that to say \( p \) is impossible is equivalent to saying \( \neg p \) is certainly true.
4. \( \Pi(p) = \Pi(\neg p) = 1 \) and \( N(p) = N(\neg p) = 0 \) meaning that, from the available knowledge, nothing can disprove or confirm \( p \) i.e. there is total ignorance about \( p \).
5. \( \Pi(p \lor q) = \max(\Pi(p), \Pi(q)) \).
6. \( N(p \land q) = \min(N(p), N(q)) \).
7. \( \Pi(p \land q) \leq \min(\Pi(p), \Pi(q)) \).
8. \( N(p \lor q) \geq \max(N(p), N(q)) \).

Rules are also defined for resolution [90].

The last four of these conventions and the resolution rules mean that, when necessities and/or possibilities are defined for the predicates in a knowledge base, refutation can be performed which will give some measure of necessity or possibility as an inequality.
The use of these inequalities in association with crisp hypotheses is important as it means that the laws of excluded middle and contradiction can be maintained [92, 93].

**Fuzzy reasoning: semantic uncertainty**

Uncertainty about the precise meaning of a statement was modelled by Zadeh [86] by associating a set with a statement and ascribing different degrees of membership of the set to different possible interpretations of the statement. Zadeh then built a system of approximate reasoning [62] on top of this, which he called 'fuzzy logic', which was a multi-valued logic i.e. it was not restricted to having statements that were simply true or false.

In crisp (conventional, two-valued) logic, an object is either in a set or out of it. In fuzzy logic, an object has a degree of membership of the set. This fits with 'real-world' descriptions where very often one cannot be sure about whether an object really does belong to a set or not. Another way of thinking of this 'partial membership' is to consider that the degree of membership of the set represents the extent to which the object has the attribute described by the set [75].

Other multi-valued logics have been formulated. The most common are 'trivalent' or 'three-valued' logics which represent TRUE, FALSE and UNKNOWN by 1, 0, and 1/2 respectively. The first $N$-valued logic was developed by Lukasiewicz in the 1930s. Such a logic is called $L_N$ logic where $N \geq 2$. $L_2$ is classical two-valued logic. The set of truth values $T_N$ is assumed evenly divided over the interval $[0, 1]$ so that

$$T_N = \left\{ \frac{i}{N-1} \right\} \text{ for } 0 \leq i < N$$

(4.9)

Some Lukasiewicz logic operators resemble some used in fuzzy logic (see section 4.2) in that $\neg x = 1 - x, x \land y = \min(x, y), x \lor y = \max(x, y)$ and $x \rightarrow y = \min(1, 1+y-x)$. The infinite valued $L$-logic, $L_\infty$, is labelled $L_1$.

Fuzzy logic, with its system of degrees of membership of fuzzy sets, has been viewed as a generalization of multi-valued logics, although fuzzy logic is concerned with *imprecise* reasoning rather reasoning with a set of *precise* possible outcomes.
To illustrate the concept of degree of membership, consider an example. If there is a set \( H \) which comprises the numbers between 6 and 8, it could be written

\[
H = \{ r \in \mathbb{R} \mid 6 \leq r \leq 8 \}
\]  

(4.10)

Alternatively, the function \( h \) could be described in terms of its membership function \( \mu_H : \mathbb{R} \mapsto [0, 1] \)

\[
\mu_H(r) = \begin{cases} 
1; & 6 \leq r \leq 8 \\
0; & \text{otherwise}
\end{cases}
\]  

(4.11)

Now, consider a fuzzy set \( F \) of real numbers close to 7. The membership function here will not be unique, but will depend on what the user chooses. Fuzzy sets are described by membership functions of the form \( \mu : X \mapsto [0, 1] \) where \( X \) is the universe of objects or universe of discourse describing the range of variables that might have a degree of membership of the set \( F \). The non-uniqueness of the membership function can be a powerful tool, allowing a modeller to change the model very easily.

In the above case, for the set \( F \) of numbers close to 7, the membership function might be triangular, say

\[
\mu_F(r) = \begin{cases} 
\frac{1}{2}r; & r \leq 7 \\
2 - \frac{1}{2}r; & r \geq 7
\end{cases}
\]  

(4.12)

The universe of discourse of a fuzzy set might be finite, in which case the set can be expressed as a summation of singletons (a single point in the universe of discourse which has a positive non-zero degree of membership of the fuzzy set), or infinite (even if over a finite range) so that \( F \) should be described as an integral with the variable \( r \) modulated in each case by the membership function [62]. The support of a fuzzy set is the set of points in the universe of discourse with positive non-zero degrees of membership of the fuzzy set. Like the universe of discourse, the support can be finite or infinite. Note also that the membership function is also sometimes called the 'fuzzy subset' [95].
4.1.4 Uncertainty in this study

In choosing a methodology to represent the uncertainties encountered in power system operation, the kind of uncertainty exhibited by each must be understood. The list of section 1.5.2 is tabulated in table 4.4.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Type of uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>load forecast</td>
<td>inaccuracy or imprecision</td>
</tr>
<tr>
<td>weather conditions</td>
<td>ambiguity or inaccuracy</td>
</tr>
<tr>
<td>state estimation</td>
<td>classification, incompleteness, incorrectness, inaccuracy</td>
</tr>
<tr>
<td>power system parameters</td>
<td>inaccuracy</td>
</tr>
<tr>
<td>security analysis</td>
<td>classification, imprecision, ambiguity</td>
</tr>
<tr>
<td>arbitrary nature of limits</td>
<td>inaccuracy or imprecision</td>
</tr>
<tr>
<td>operational priorities</td>
<td>ambiguity</td>
</tr>
</tbody>
</table>

Table 4.4: Uncertainties in power system operation

Of these uncertainties, errors due to inaccuracy or imprecision are best addressed by refinement of the processes giving rise to them, for example in conducting research into better load forecasting techniques or better weather forecasting. This is the approach adopted in this study towards inaccuracies found in state estimation (chapter 7). However, classification errors are found in topology processing in state estimation i.e. errors where a 'true' or 'false' conclusion is wrongly reached such as to the statement 'this line has been switched out'. This aspect of state estimation relies on binary information being received from circuit breakers, which could be false. However, such information is received from a number of sources in which there could be different degrees of belief depending on knowledge already inferred or knowledge of the characteristics of the measurement device or communication. The type of reasoning system used to derive the power system topology could therefore be implemented using Bayesian probabilities, certainty factors or possibility theory. Possibility theory has a framework defined which deals with nonmonotonicity i.e. the possibility of knowledge being contradicted by new knowledge, and would seem to be ideally suited. This, however, is a very specific aspect of the whole process of power system operation. This study is more concerned with the general procedure from the receipt of the power system state vector to the dispatch of controls. Topology processing is therefore outside the scope of this thesis.
Classification errors emerge in security analysis, also, for example in the statements “the power system is secure” or “the power system is insecure”. In practice, such crisp statements about security are only made with respect to stability. In this case, the operator will wish to know which observation is more plausible. While this would seem to be a suitable area for research, the gathering of information that can be represented in predicate form has only been addressed by Wehenkel et al [70]. Presently, neural network techniques seem more promising (see section 3.2.1), though they may in future include measures of belief, certainty, possibility or necessity. This is an extremely large area by itself and will therefore not be discussed further here.

Static security often has a severity index associated with it which is likely to be imprecise, or has some other statement like “the system is quite secure” or “the system is very secure” [74] both of which are ambiguous.

Ambiguities are errors due to uncertainty about precise meaning of statements i.e. semantic uncertainties. As has been described above, fuzzy logic provides an elegant, intuitive paradigm for modelling such uncertainties. It is possible, though, that the other forms could be used with appropriately comprehensive decision tree structures.

Numerous studies have been performed examining differences and analogies between probability theory, possibility theory, fuzzy set theory and Dempster-Shafer theory [96] of which the system of certainty factors is a special case [97]. Of these, fuzzy set theory in the shape of fuzzy logic has the largest volume of work associated with the formulation of expert systems for control. Many of these works describe elegant, working systems (see [75, 98, 99] among many other texts for examples). Thus, it can be concluded that fuzzy logic is the best established in the domain of engineering and control. In addition, fuzzy logic is more compact in its representation of knowledge than conventional (crisp) rule-based systems (see section 4.3).

At the same time as fuzzy logic appearing ideally suited to representing the semantic uncertainties in operational objectives (in particular), it is claimed that fuzzy logic control provides good results for processes that are very complex as they permit the control space to be broken down qualitatively into regions with fuzzy mathematics interpolating
smoothly and controllably between them [100]. It is also claimed to offer improvements on conventional control techniques when the available sources of information are inexact [101]. Power system operation is a complex task which would benefit from qualitative modelling and the sources of information, in particular state estimation and security analysis, suffer from inexactness.

Fuzzy logic was therefore adopted as the means by which operators' knowledge could be represented in a qualitative reasoning tool in this project. Some more of the detail of fuzzy logic will now be presented.

### 4.2 Fuzzy logic

Conventional notation for a fuzzy singleton is

\[ A = \mu / y \]  \hspace{1cm} (4.13)

where \( \mu \) is the degree of membership of \( y \) in the fuzzy set \( A \) (in a crisp set, the only singleton would be denoted by \( 1 / y \)). A set with a finite support \( y_1, y_2, \ldots, y_n \) is the union (denoted by \( + \)) of its constituent fuzzy singletons and is denoted

\[ A = \mu_1 / y_1 + \cdots + \mu_n / y_n \]  \hspace{1cm} (4.14)

or

\[ A = \sum_{i=1}^{n} \mu_i / y_i \]  \hspace{1cm} (4.15)

A set with an infinite support is

\[ A = \int_X \mu_A(y) / y \]  \hspace{1cm} (4.16)

where \( X \) is the universe of discourse.
4.2.1 Logical operations on fuzzy sets

Set theoretic operations

If $A$ and $B$ are fuzzy sets in the universe $U$ with elements $u \in U$, the following membership functions can be defined\cite{75}

- **Union $A \cup B$, for all $u \in U$** is

$$\mu_{A \cup B} = \max(\mu_A(u), \mu_B(u)) \quad (4.17)$$

- **Intersection $A \cap B$ for all $u \in U$**

$$\mu_{A \cap B} = \min(\mu_A(u), \mu_B(u)) \quad (4.18)$$

- **Complement $\bar{A}$ for all $u \in U$**

$$\mu_{\bar{A}}(u) = 1 - \mu_A(u) \quad (4.19)$$

- **Cartesian product $A_1 \times \ldots A_n$ in the universe $U_1 \times \ldots U_n$ where $A_1, \ldots A_n$ are fuzzy sets in the universes $U_1 \ldots U_n$**

$$\mu_{A_1 \times \ldots A_n}(u_1, \ldots u_n) = \min(\mu_{A_1}(u_1), \ldots \mu_{A_n}(u_n)) \quad (4.20)$$

or

$$\mu_{A_1 \times \ldots A_n}(u_1, \ldots u_n) = \mu_{A_1}(u_1) \cdot \mu_{A_2}(u_2) \ldots \mu_{A_n}(u_n) \quad (4.21)$$

**Connectives**

Logical operations such as NOT, AND, OR and the implication can be performed on fuzzy sets. The negation NOT is most commonly taken as\cite{62}

$$\text{NOT } A = \int_X \mu_A(\bar{x})/x = \int_X (1.0 - \mu_A(x))/x \quad (4.22)$$
The operator used for the conjunction (AND) is chosen from a family of *triangular norms* which include \[101\], for all \( x, y \in [0,1] \),

\[
\begin{align*}
\text{intersection} & \quad x \land y = \min(x, y) \\ 
\text{algebraic product} & \quad x \cdot y = xy \\ 
\text{bounded product} & \quad x \odot y = \max(0, x + y - 1) \\ 
\text{drastic product} & \quad x \sqcap y = \begin{cases} 
    x & y = 1 \\
    y & x = 1 \\
    0 & x, y < 1
\end{cases}
\end{align*}
\]  
(4.23) (4.24) (4.25) (4.26)

If the triangular norm is represented by \( * \), the conjunction \( A \text{ AND } B \) where \( A \) and \( B \) are fuzzy sets in the universes \( U \) and \( V \) and \( u \in U, v \in V \), is

\[
A \text{ AND } B = \int_{U \times V} \mu_A(u) * \mu_B(v)/(u,v)
\]  
(4.27)

The operator used for the disjunction (OR) is chosen from a family of *triangular co-norms*. These include \[101\]

\[
\begin{align*}
\text{union} & \quad x \lor y = \max(x, y) \\ 
\text{algebraic sum} & \quad x \hat{+} y = x + y - xy \\ 
\text{bounded sum} & \quad x \oplus y = \max(0, x + y - 1) \\ 
\text{drastic sum} & \quad x \sqcup y = \begin{cases} 
    x & y = 0 \\
    y & x = 0 \\
    0 & x, y > 1
\end{cases}
\end{align*}
\]  
(4.28) (4.29) (4.30) (4.31)

If the triangular co-norm is represented by \( \dagger \), the disjunction \( A \text{ OR } B \) is

\[
A \text{ OR } B = \int_{U \times V} \mu_A(u) \dagger \mu_B(v)/(u,v)
\]  
(4.32)

In section 3.1.1, it was explained that the implication \( p \rightarrow q \) could be interpreted as "\( p \) implies \( q \)" , "\( p \) if and only if \( q \)" , "\( p \) is sufficient for \( q \)" , "\( q \) if \( p \)" , "\( q \) is necessary for \( p \)" or "\( p \) then \( q \)". If \( p \) and \( q \) are replaced by the fuzzy propositions \( x \ IS \ A \) and \( y \ IS \ B \) where \( A \) and \( B \) are fuzzy sets and \( x \) and \( y \) are linguistic variables, the fuzzy implication can be found. As for the conjunction and disjunction, a number of definitions are possible. If the implication operator is defined in terms of the fuzzy set \( R \) which relates \( A \) to \( B \) (\( R \) is known as a *fuzzy relation*. For more see section 4.3.2), then \( R \) takes the place of the
truth table in two-valued logic and can be regarded as a translation between the universe $U$ and the universe $V$.

The fuzzy implication, characterised by a fuzzy relation, can be used in various tautologies such as modus ponens, modus tollens and syllogism [52]. For example, modus ponens or

\[
p \rightarrow q \\
p \\
\therefore q
\]

where $p$ and $q$ are propositions, which can be interpreted as "if the statement 'if $p$ is true then $q$ is true' is true, and $p$ is true, then we can conclude that $q$ is true" [75], can be generalized to the fuzzy case. The generalized modus ponens can be written [102]

- **Premise:** $x$ is $A'$
- **Implication:** if $x$ is $A$ then $y$ is $B$
- **Conclusion:** $y$ is $B'$

The implication $A \rightarrow B$ here can be represented by the fuzzy relation so that given a fuzzy set $A'$, $B'$ can be found.

The generalized modus tollens is

- **Premise:** $y$ is $B'$
- **Implication:** if $x$ is $A$ then $y$ is $B$
- **Conclusion:** $x$ is $A'$

Some standard definitions, then, for the implication $R$ are [101]

- the min-operation due to Mamdani

\[
R = \int_{U \times V} \mu_A(u) \land \mu_B(v)/(u,v)
\]

\[
R = \int_{U \times V} \mu_A(u) \land \mu_B(v)/(u,v)
\] (4.33)

- the product operation due to Larsen

\[
R = \int_{U \times V} \mu_A(u) \mu_B(v)/(u,v)
\] (4.34)
the arithmetic rule due to Zadeh

\[ R = (\text{NOT}A \times V) \oplus (U \times B) \]  
\[ = \int_{U \times V} 1 \land (1 - \mu_A(u) + \mu_B(v))/(u, v) \]  

the max-min rule due to Zadeh

\[ R = (A \times B) \cup (\text{NOT}A \times V) \]  
\[ = \int_{U \times V} (\mu_A(u) \land \mu_B(v)) \lor (1 - \mu_A(u))/(u, v) \]  

the 'standard sequence' rule

\[ R = A \times V \rightarrow U \times B \]  
\[ = \int_{U \times V} (\mu_A(u) > \mu_B(v))/(u, v) \]  

where

\[ \mu_A(u) > \mu_B(v) = \begin{cases} 1 & \mu_A(u) \leq \mu_B(v) \\ 0 & \mu_A(u) > \mu_B(v) \end{cases} \]  

the Boolean rule

\[ R = (\text{NOT}A \times V) \cup (U \times B) \]  
\[ = \int_{U \times V} (1 - \mu_A(u)) \lor \mu_B(v))/(u, v) \]  

Goguen's rule

\[ R = A \times V \rightarrow U \times B \]  
\[ = \int_{U \times V} (\mu_A(u) \gg \mu_B(v))/(u, v) \]  

where

\[ \mu_A(u) \gg \mu_B(v) = \begin{cases} 1 & \frac{\mu_B(v)}{\mu_A(u)} \leq \mu_B(v) \\ \frac{\mu_B(v)}{\mu_A(u)} & \mu_A(u) > \mu_B(v) \end{cases} \]  

Putting memberships 1 and 0 into the above expressions yield the standard Boolean results, illustrating what was stated earlier, i.e. that fuzzy sets and logic are generalisations of classical set theory [75]. This is known as the extension principle.
4.2.2 Hedges

Hedges in fuzzy logic are terms which qualify the values of linguistic variables and are analogous to the qualifiers in first order predicate logic (see section 3.1.1). For example, the variables low or high could be qualified by very, slightly, quite, fairly and so on. Zadeh [62] quotes some definitions of hedges which are commonly used.

In general, a linguistic variable's value may be composed of a number of atomic terms which may be classified as

1. primary terms or the labels of the specified fuzzy subsets of the universe of discourse;
2. the negation NOT and connectives AND and OR;
3. hedges such as VERY, MUCH, HIGHLY;
4. markers such as parentheses.

A composite term such as 'a very tall person' can be broken down into its constituent parts in the manner of $a = hu$ where $h$ is a hedge and $u$ is a term with a specified meaning. In this case, $h$ is very and $u$ is tall person.

In defining the operations which hedges perform, it is understood that the operation is performed on the degree of membership of the fuzzy set of each singleton in the set. Some common definitions are now listed below:

$$\text{VERY} A = A^2 \quad (4.47)$$

or more precisely

$$\text{VERY} A = \int_U \mu_A^2(x) / x \quad (4.48)$$

$$\text{NOT \text{ VERY}} A = (A^2) \quad (4.49)$$

$$\text{VERY \text{ NOT}} A = (\neg A)^2 \quad (4.50)$$
There are further operations which can be performed on fuzzy sets. CONCENTRATION and DILATION are \( \text{CON}(A) = A^2 \) and \( \text{DIL}(A) = A^{0.5} \) and contrast INTENSIFICATION (which reduces fuzziness) is given by

\[
\text{INT}(A) = \begin{cases} 
2A^2, & \text{for } 0 \leq \mu_A(x) \leq 0.5 \\
-2(-A)^2, & \text{for } 0.5 \leq \mu_A(x) \leq 1 
\end{cases}
\]  

(4.51)

### 4.2.3 Alpha cuts

Some fuzzy expert system designers attempt to limit the number of variables causing changes in the system by filtering out all singletons with membership values less than a chosen amount, known as \( \alpha \). These singletons would have their memberships of the fuzzy set to which the alpha cut is being applied set to zero.

### 4.2.4 Cardinality

For a finite fuzzy set \( A \), the cardinality \( |A| \) is defined as

\[
|A| = \sum_{i=1}^{n} \mu_A(x_i)
\]  

(4.52)

For an infinite fuzzy set \( A \) with a universe of discourse \( X \), \( |A| \) is

\[
|A| = \int_{X} \mu_A
\]  

(4.53)

There are a number of reported examples, e.g. that in [75], for the cardinalities of consequent clauses of a set of fuzzy rules being used in decision support in enabling the finding of precedence in a list of feasible decisions.

### 4.3 Fuzzy Control

Fuzzy control is an area for which there is a wealth of published work and real applications. It is based on using fuzzy sets to model control decisions which are semantically
Uncertainty. In a manner similar to that of conventional rule-based expert systems, the fuzzy sets are combined in sets of rules to represent the knowledge applicable in a decision making process. Such sets of rules are known as fuzzy expert systems.

According to Kickert and Mamdani [103], "the basic idea...was to incorporate the 'experience' of a human process operator in the design of a controller. From a set of linguistic rules which describe the operator's control strategy, a control algorithm is constructed where the words are defined as fuzzy sets. The main advantages of this approach seem to be the possibility of implementing 'rule of thumb' experience, intuition and heuristics, and the fact that it does not need an [exact] model of the process."

Fuzzy expert systems' advantages over conventional production-rule based expert systems have been characterised as including [104, 105]

- fuzzy sets neatly symbolise natural language terms used by experts;
- since knowledge captured in 'IF...THEN' statements is often not naturally true or false, fuzzy sets afford representation of the knowledge in a smaller number of rules;
- fuzzy rules can be tuned on- or off-line;
- a smooth mapping can be obtained between input and output data.

The various procedures and terms are described in the following sections.

4.3.1 Fuzzy conditional statements

A fuzzy expert system executes a series of rules or conditional statements similar in form to

\[
\text{IF } x \text{ is low AND } y \text{ is high THEN } z \text{ is medium}
\]
Since the inputs to fuzzy control systems are crisp and crisp control signals are required, fuzzy expert systems used in control work in four steps (figure 4.2):

1. **Fuzzification.** From knowledge of the crisp value of the variable being fuzzified and the membership function of the linguistic value, the degree of truth of the proposition is found.

2. **Inference.** The truth value for each rule’s premise is computed and related to the conclusion part of the rule.

3. **Composition.** All the fuzzy subsets (membership functions) assigned to each output variable are combined to form a single subset or membership function for each output variable.

4. **Defuzzification.** This converts a fuzzy set to a crisp value.

**Fuzzification**

Fuzzification concerns the finding of the degree of truth of a proposition such as $x$ is *tall* for a given $x$. This can also be viewed as the degree to which the given $x$ is a member of the fuzzy set *tall*. For example, the degree of truth of $x$ is *tall* when $x$ is 6 feet 2 inches and *tall* is as shown in figure 4.1 is 0.8.
Inference methods

Most fuzzy control applications use forward chaining through rules to reach conclusions about suitable control actions from given evidence. As such, they are based on applications of the generalized modus ponens. There are two keys in the execution of the rules. The first concerns the treatment of multiple evidences, and the second concerns the implication.

Multiple evidences, such as in the connection of fuzzy propositions \( x \) is \( A \) and \( y \) is \( B \) through AND where \( A \) and \( B \) are fuzzy sets, are resolved into a single evidence in the manner of Lukasiewicz logic by applying some operator for AND for each pair of singletons in the universes of \( A \) and \( B \) (see section 4.2.1).

The implication operator is chosen from operators such as those given in section 4.2.1. It effectively performs a transition from the universe of the antecedent to that of the consequent so that a conclusion in the universe of the consequent can be found from any premise in the universe of the antecedent.

Composition methods

A general fuzzy expert system for control with \( n \) rules \( \rho_1 \ldots \rho_n \), two input variables \( x \) and \( y \) and one output variable \( z \), may be represented as

\[
\rho_1: \quad \text{IF } x \text{ is } A_1 \text{ AND } y \text{ is } B_1 \text{ THEN } z \text{ is } C_1 \\
\rho_2: \quad \text{IF } x \text{ is } A_2 \text{ AND } y \text{ is } B_2 \text{ THEN } z \text{ is } C_2 \\
\vdots \\
\rho_n: \quad \text{IF } x \text{ is } A_n \text{ AND } y \text{ is } B_n \text{ THEN } z \text{ is } C_n
\]

where \( A_i, B_i \) and \( C_i \) are fuzzy sets in the universes \( U, V \) and \( W \) for \( i = 1 \ldots n \).

The composition process concerns the combination of \( C_1 \ldots C_n \) to find \( z = C' \). This is usually performed by giving the membership function for \( C' \) as the piecewise maximum of \( C_1 \ldots C_n \) over the universe \( W \) [62]. This operation is referred to by some authors as
being performed by the connective ‘also’ [101].

Defuzzification

As with other fuzzy operations, there are many different ways of doing this but the most common are the maximum [62] and centroid methods [106].

The maximum method gives the point in the support of the fuzzy set to be defuzzified which has the greatest degree of membership of the set as the crisp equivalent of a fuzzy set. There are further variations on the maximum method but only in what they do if there is more than one crisp value with the same maximum degree of membership (for example, the average of the maxima might be taken).

The centroid method requires the calculation of the moment of the membership function of the fuzzy set to be de-fuzzified (i.e. the integral of the product of the output membership function and the output variable with respect to the output variable) divided by the area under the membership function.

4.3.2 Fuzzy relations

In traditional two-value logic, the conditional statement IF \( A \) THEN \( B \), where \( A \) and \( B \) are propositions, can be regarded as an implication described by \( A \rightarrow B \) with the connective defined by the standard truth table. As described above, such conditional statements have meaning in fuzzy logic, but the implication will be defined not by a table but by a matrix relating the singletons of fuzzy set \( A \) to fuzzy set \( B \). The matrix is known as a fuzzy relation [62, 107].

As an example, consider the conditional statement IF player is skillfull THEN opposition is aggressive where

\[
\text{skillfull} = 0.4/\text{Adams} + 0.8/\text{Le Tissier} + 0.9/\text{Cantona} \quad (4.54)
\]

\[
\text{aggressive} = 0.2/\text{Swindon} + 0.9/\text{Sheffield Utd} + 1.0/\text{Wimbledon} \quad (4.55)
\]
The fuzzy relation, which might be called likelihood of getting kicked = \( L \), is given by the fuzzy subset of the Cartesian product player is skillfull × opposition is aggressive for the collection of ordered pairs \((p, o), p \in \text{player}, o \in \text{opposition}\).

\[
L = \int_{P \times O} \mu_L(p, o)/(p, o) \tag{4.56}
\]

If the implication of the rule is taken as the minimum degree of membership,

\[
L = 0.2/(\text{Adams, Swindon}) + 0.4/(\text{Adams, Sheffield Utd.}) \tag{4.57}
+0.4/(\text{Adams, Wimbledon}) + 0.2/(\text{Le Tissier, Swindon})
+0.8/(\text{Le Tissier, Sheffield Utd.}) + 0.8/(\text{Le Tissier, Wimbledon})
+0.2/(\text{Cantona, Swindon}) + 0.9/(\text{Cantona, Sheffield Utd.})
+0.9/(\text{Cantona, Wimbledon})
\]

Alternatively, this can be written as a fuzzy relational matrix.

<table>
<thead>
<tr>
<th></th>
<th>Swindon</th>
<th>Sheffield Utd.</th>
<th>Wimbledon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adams</td>
<td>0.2</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Le Tissier</td>
<td>0.2</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Cantona</td>
<td>0.2</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The process of composition described in the preceding section can also be defined in terms of matrices. If \( R \) is a relation from \( X \) to \( Y \) and \( S \) is a relation from \( Y \) to \( Z \), then the composition of \( R \) and \( S \) is a fuzzy relation denoted by \( R \circ S \). If MAX-MIN composition is used,

\[
R \circ S = \int_{X \times Z} \max_y \left[ \min_x (\mu_R(x, y), \mu_S(y, z)) \right] / (x, z) \tag{4.59}
\]

If the domains of \( x, y \) and \( z \) are finite, then the relation matrix for \( R \circ S \) is the max-min product of the relation matrices for \( R \) and \( S \) i.e. the matrix product with the operation of addition replaced by max and that of multiplication replaced by min.

4.3.3 Numerical example

The fuzzy expert system process is best illustrated by an example.
Consider the following membership functions:

\[
\begin{align*}
    l(t) &= 1 - \frac{t}{10} \\
    h(t) &= \frac{t}{10}
\end{align*}
\]  
(4.60) (4.61)

where \(l(t)\) representing low and \(h(t)\) representing high are linguistic variables and are shown in figure 4.3.

Now consider a few rules:

1. IF \(x\) is low AND \(y\) is low THEN \(z\) is high
2. IF \(x\) is low AND \(y\) is high THEN \(z\) is low
3. IF \(x\) is high AND \(y\) is low THEN \(z\) is low
4. IF \(x\) is high AND \(y\) is high THEN \(z\) is high

\(l(t) + h(t) = 1\) for all \(t\). This is not necessary but is common. Although here, the same membership functions are used for each variable, they need not be.

First, the fuzzification is carried out. For each value of \(x\) and \(y\), the degrees of membership or truth of each in the subsets \(l\) and \(h\) should be found. The degree of truth (or 'alpha') of the antecedent of each rule is then found, and if it is non-zero, the rule is said to have 'fired'. Given that the AND operand returns the minimum of the two truths, rule 1 for \(x = 0.0\) and \(y = 3.2\) would give a premise degree of \(\alpha_1 = 0.68\). The set of premise truths \(\alpha_1 \ldots \alpha_4\) for \(x = 0.0\) and \(y = 3.2\) is shown in table 4.5.

The inference is then performed. This modifies the membership function of the consequent by some function of the rule premise. Using MIN inferencing, the membership
<table>
<thead>
<tr>
<th>$\mu_1(x)$</th>
<th>$\mu_2(x)$</th>
<th>$\mu_1(y)$</th>
<th>$\mu_2(y)$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.0</td>
<td>0.68</td>
<td>0.32</td>
<td>0.68</td>
<td>0.32</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4.5: Premises of example rules

function assigned to $z$ for rule 1, i.e. high, would be be clipped off at the degree of truth of the premise to give the membership function for $z$ of

$$r_1(z) = \begin{cases} \frac{z}{10} & \text{if } z \leq 6.8 \\ 0.68 & \text{if } z \geq 6.8 \end{cases} \quad (4.62)$$

PRODUCT inferencing would give

$$r_1(z) = 0.68 \times h(z) \quad (4.63)$$

$$= 0.68 \times \frac{z}{10} \quad (4.64)$$

This represents the membership function $h$ scaled by the degree of truth of the premise i.e. 6.8.

As can be seen from table 4.5, with $x = 0.0$ and $y = 0.0$, only rules 1 and 2 have fired. The outputs from these rules using MIN and PRODUCT inferencing are shown in figure 4.4.

The next stage, that of composition, is often termed along with the inference method e.g. 'MIN-MAX' composition or 'PRODUCT-SUM' [107, 75] composition.
MAX composition involves taking the pointwise maximum over all the fuzzy subsets. SUM involves taking the pointwise sum. This can result in values of over 1.0 so it should only be followed by defuzzification methods that can cope with such values e.g. the centroid method. Otherwise, all the pointwise value in the SUM method should be normalised. MAX composition of the outputs from rules 1 and 2 found by MIN inferencing and SUM composition of the outputs found by PRODUCT are shown in figure 4.5.

\[
\begin{align*}
\text{MAXIMUM composition} & & \text{SUM composition} \\
\mu(z) & & \mu(z) \\
1.0 & & 1.0 \\
0.68 & & 0.68 \\
0.32 & & 0.32 \\
5 & & 5 \\
10 & & 10 \\
z & & z \\
\end{align*}
\]

Figure 4.5: Results of composition

If a crisp value is needed, the output fuzzy set must finally be de-fuzzified. Using MIN-MAX composition, \(z\) can be defuzzified by the CENTROID method to give 5.78 and by the ‘average of maxima’ method to give 8.40. Defuzzification of \(z\) when \(z\) has been found using PRODUCT-SUM composition gives 5.00 using the CENTROID method and 10.00 using ‘average of maxima’.

4.4 Summary

This chapter has described the different kinds of uncertainty that might be found in some decision or control process. It has briefly described different paradigms used to implement inexact reasoning in expert systems and has introduced approximate reasoning. The sorts of uncertainty found in power system operation have then been discussed. A new way of understanding the process of power system operation in terms of the uncertainties encountered has been presented and this has been used to suggest ways in which the operation of a power system can be improved through the inclusion of models of uncertainty in different analysis tools. Fuzzy logic has then been suggested as offering the best means of doing this, particularly when modelling operators’ decisions, and the the-
Theoretical frameworks of fuzzy logic and fuzzy control have been described in some detail.
This chapter offers a brief summary of some commercially available expert system shells and goes on to describe two custom-built ones developed for the study described in this thesis.

5.1 Commercial Shells

The commercial shells that were considered were 'C Language Integrated Production System' or 'CLIPS', 'Knowledge Engineering System' or 'KES', 'Level 5' and 'OPS5'.

CLIPS, as its name implies, is written in 'C' and has a Lisp-like rule syntax. It allows rules which have multiple premises. These can be connected by AND in which case all of them must be true before the rule fires, or OR where any one the premises being false causes the rule not to fire. Inference can only be carried out by forward chaining where conditions on the left-hand side of a rule are matched with facts in the knowledge base. The firing of a rule causes the action specified by the right-hand side to be performed. It is available for IBM PCs, Macintosh and a number of workstations. It has an interactive development environment. A debugger is provided which allows break-points in the rule-base to be defined. The source code is supplied so that CLIPS can be implemented on any machine with a standard 'C' compiler and linked with a user's own source code.

KES is written in 'C' and provides for forward chaining and backward chaining, though the rules must be written differently in each case. It has the facility to handle certainty
factors. The default mode of forward chaining is a depth-first search so that upon firing of a rule the first subsequent action is carried out immediately right through to the end of the tree regardless of whether an earlier fired rule required two or more actions. It has a graphical X-Windows based development environment and is available for a wide variety of hardware platforms. KES routines can be linked with user-defined code.

**Level 5** is written in Pascal and available for PCs, Macintoshes, VAXs and IBM mainframes [108]. Rules are defined using Level 5's own 'Production Rule Language' which uses keywords IF, AND, OR and THEN. Facts must also be represented in an appropriate form. Inference is performed by backward chaining. The user specifies one or more goals and Level 5 attempts to justify them based on the rules and facts it has been provided with. There is no straightforward mechanism which supports forward chaining. Some debugging facilities including break-points are provided.

**OPS5** is descended from a family of production system languages developed at Carnegie Melon University. DEC markets a proprietary version which only runs on DEC machines. The syntax resembles Lisp. It does not support 'wild cards' (i.e. variables representing generic semantics) on the left hand side of rules, nor does it support disjunctions. The system forward chains through the rules and provides for conflict resolution. It allows back-tracing through the inference chain. The OPS5 compiler converts the rule-base into VAX object code which can be linked with other objects.

### 5.2 Specially Developed Inference Engines

Of the commercially available shells listed above, only CLIPS would appear to offer the flexibility needed in this project in terms of it being linkable to existing 'C' sources and available for a number of different hardware platforms. In addition, the problems to be addressed in this study, i.e. ones which derive actions rather than test hypotheses, fit more naturally into a structure using forward chaining. Forward chaining is difficult to implement in some of the above systems. A recent CLIPS development also has the advantage of being the only one that has the facility to handle fuzzy rules. When a li-
ence for CLIPS is purchased, the source code is supplied meaning that it can be adapted. However, it is very large with over 25,000 lines for the version without fuzzy facilities [108]. It also requires the learning of a complex syntax. The rules are interpreted at run-time which suggests some loss of speed, although an option can be invoked which writes them into a module of ‘C’ code. This is then compiled and linked with the rest of the sources, an approach which is also taken by Stallard [54]. This is reported in [108], however, to give only a small speed-up.

It was decided, then, that it would be as easy to develop inference engines specially for this project which are tailored to the needs of the project as it would be to learn use of CLIPS. Two such engines have been written, one for handling straight production rules and the other for fuzzy rules. They have very similar structures and the fuzzy version can include crisp rules as well as fuzzy rules. Both allow procedures to be written that are executed on the firing of rules. They are both written in ‘C’ to allow easy linking with existing sources and execution on a variety of hardware platforms. They will now be described in turn.

5.2.1 Production rule expert system

The production rule inference engine, i.e. that to manipulate ‘IF...THEN...’ rules where the premises and consequent are crisp, has been written to allow both depth- and breadth-first searches of the decision tree. It was written in ‘C’ to ease compatibility with existing sources such as, in the context of this project, a loadflow routine or power system simulator, which act as the ‘on-line’ fact-base. The easy handling of symbols within ‘C++’ made that an attractive alternative which could be implemented alongside other sets of sources written ‘C’ since ‘C++’ compilers are supersets of ‘C’ compilers. This option was not taken up as it was felt that the overhead in learning ‘C++’ was not worth the gain in the aesthetic quality of the source code, which is what the gain would be. This option could, however, be taken up at some time in the future by adapting the existing inference engine sources.

The inference engine is a stand-alone module for which the underlying syntax of the
rules and conditions are defined. The rule-base is defined in terms of states, which represent states of knowledge, either inferred or initial, and rules which query the fact base to infer new knowledge so that a new state can be reached. These are represented internally as 'StateNodes' and 'RuleNodes'.

In general, a state will be associated with some attribute or class of object, or group of attributes or classes, and rules are defined for each of the possible values or combinations of the attributes or objects which are relevant to the decision process. For example, the attribute associated with a state may be the number of wheels on a vehicle and the possible values may be 6, 4, 3 or 2. Four rules would then exist which lead to conclusions dependent on whether there are 6, 4, 3 or 2 wheels.

Each StateNode has a list of RuleNodes each of which may lead to an action, a conclusion or another StateNode whenever the antecedent of the rule is true. By linking particular rules from states to particular other states, the decision tree structure is assembled. This structure can have multiple levels and n-ary branching from nodes, and can form a lattice. The inference engine module also gives the inference mechanism. The default mechanism is forward chaining whereby facts are tested and conclusions are reached so that the current state of 'knowledge' contained within the expert system moves forwards through the decision tree. Since the branches and nodes in the tree (given by the links between states and rules) are defined by double-linked lists, backward chaining is also possible, and justification of conclusions can be achieved by tracing back through the tree to see which rules have fired for which conditions. Depth- or breadth first searches are both possible. The choice is made by the user as an input option.

The rule-base is defined in a separate module. Linguistic labels can be specified for the StateNodes and instances of the 'RuleNodes' which are associated with them. Each RuleNode points to a function associated with it which implements some action to test the condition. This action can be a simple query regarding some fact, or can be some other routine, possibly in another 'C' module, which will eventually return a TRUE or FALSE conclusion. The rule can have one premise or a conjunction of premises. If the premises are true, the function may call another action or may simply
return '1' or '0'. A flag is then set associated with the rule indicating whether it has fired or not. The program then moves on either to the StateNode indicated by the condition, or to a terminal node, in which case, if forward chaining is being used, execution stops.

A schematic of the structure is shown in figure 5.1.

5.2.2 Fuzzy expert system inference engine

A fuzzy logic based expert system inference engine has been written in 'C' that allows the execution of fuzzy rules. Rules and linguistic variables can be defined. Various linguistic values can be defined, each with unique or variable membership functions. These can be given linguistic labels so that the translation between the terms used in the design of the rule base and those used in the expert system code can be easily achieved. Since the code is written in 'C', the inference engine and rule base can both be easily linked with other code which can be interrogated when regarded as part or all of the fact base. In this way, modules for inference, rules and facts can be kept separate and maintained separately.
Membership functions can be triangular, trapezoidal or sigmoid. Each of these shapes is defined by two or three values giving the value or values at which the function is zero and that at which it is 1. Alternatively, these values can be allowed to vary so that a membership function can be made dependent on some parameters only found when the program is run. This is achieved by defining pointers to variables from which the membership function is set.

Rules are implemented as individual functions in much the same way as in the conventional production rule system described above. This allows different procedures to be executed as part of the rule such as those perhaps necessary to derive values which are to be fuzzified. Actions dependent on the firing of the rule can also be easily included.

Fuzzification is performed via a simple 'Fuzzify' statement for which the arguments are the crisp value to be fuzzified and the fuzzy set of which its degree of membership is being found. Two possible operators are given for 'AND' which take either the product of the connected memberships or the minimum. Input and output fuzzy sets can be either finite or infinite.

The rule base is executed in a breadth-first, forward chaining manner. Possible inference operators include the consequent membership function being multiplied by the degree of truth of the antecedent of the rule (a product operator), or being clipped at the degree of truth of the antecedent (a minimum operator). Composition can be carried out using sum or maximum methods. Two choices are again provided to the user for defuzzification, average of maxima or centroid defuzzification.

5.3 Summary

This chapter has provided a brief review of a few commercially available expert system shells and described the reasoning behind the development of two new shells specially for this project. These have been written in 'C' and are used for inferencing using production rules and fuzzy rules. They have been described in some detail.
6

Static Analysis of Power Systems

This chapter describes two basic tools for static analysis of power systems (i.e. concerned with the algebraic rather than differential equations). The first is the load flow tool which solves the non-linear equations concerned with complex nodal voltages, power injections and power flows. The second is sensitivity analysis in which the algebraic load flow equations are linearised to show changes in dependent variables for changes in control variables. The equations are introduced in the first section.

6.1 Basics

A power system can be described by a set of differential equations and a set of algebraic equations to form a differential-algebraic system which can be written

\[ \dot{x} = F(x, u, p) \]

\[ 0 = G(x, u, p) \]  

(6.1)  

(6.2)

where \( x \) is the vector of power system state (dependent) variables, \( u \) is the vector of independent variables and \( p \) is a vector of system parameters. Alternatively, the system can be described in terms of differential equations, algebraic equations and difference equations describing discrete time events such as changes to on-load tap changing transformer tap ratios and changes in switchable shunt susceptances. In this case, the system can be written

\[ \dot{x} = F(x, u, z, p) \]  

(6.3)
where $z$ is the vector of variables which can be changed in discrete steps at time $k$.

In this study, only static controls and their algebraic constraints are being considered as even when constraints are originally defined according to dynamic criteria, standard practice in operation of large power systems determines that the constraints are expressed in terms of static quantities (see section 2.2). Hence, only equation 6.2 is of interest.

The function $G$ is the sum of the set of power injection equations. These equations are functions of the complex nodal voltages and complex branch currents. The total complex power $S$ should be zero for each node. For a node $i$ it is

$$S_i = V_i I_i^*$$  \hspace{1cm} (6.6)$$

where $V_i$ is the complex voltage at node $i$ and $I_i$ is the current at node $i$. For every branch from node $i$ to a node $j$, $j = 1 \ldots N, j \neq i$,

$$S_i = P + jQ = \sum_{j=1}^{N, j \neq i} V_i I_{ij}^*$$  \hspace{1cm} (6.7)$$

where $P$ is the active power and $Q$ is the reactive power. If the complex admittance of the branch connecting nodes $i$ and $j$ is $y_{ij}$,

$$S_i = \sum_{j=1}^{N, j \neq i} V_i (y_{ij}(V_i - V_j))^*$$  \hspace{1cm} (6.8)$$

6.2 Load Flow

Load flow routines have been used to solve the algebraic power flow equations in analysis of static power system conditions for many years and have taken many different forms [109]. In the late 1970s and early 1980s, fast decoupled routines became popular [19], although trends now again favour fully coupled implementations for analysis of systems run under a wider variety of conditions.
The load flow routine implemented as part of this project is based on a standard formulation such as that described in [110]. It has been further developed to provide an interactive shell through which the user can easily change parameters such as transformer tap ratios, generator $P$ and $V$ set points and $Q$ generation limits, and line switched in/out status. Network sensitivity matrices can be calculated from each new system condition (see section 6.3). It is within this environment that the dispatch systems described in chapters 8, 9 and 10 have been placed so that for any load flow solution, violations of static limits can be monitored and corrective actions ordered.

The operating state of an interconnected power system can be described in terms of four sets of quantities which relate to each node of the system. These are the nodal voltage magnitude $V$, voltage angle $\theta$, active power injection $P$ and reactive power injection $Q$. The load flow allows two of these quantities to be found for each node once the others have been defined.

Three different bus conditions are defined depending on which two of the four parameters are pre-defined. These are

**PV or voltage-controlled (regulated) bus.** Such a bus has the facility to maintain a fixed voltage magnitude $V$ which is specified and as such will be a variable source of reactive power. The active power injected is also specified. In practice, this represents a generator or compensator bus, and maximum or minimum limits on the reactive power injected $Q_{\text{max}}$ and $Q_{\text{min}}$ may be set in which case the bus is known as a $PVQ$ bus.

**PQ or unregulated bus.** The total injection $P + Q$ is specified corresponding to the load at a load bus.

**Slack or swing bus.** This bus defines the voltage angle reference and also has the voltage magnitude defined. $P$ and $Q$ are allowed to vary since all powers cannot be defined in advance as system losses are unknown. The slack bus can be regarded as analogous to a generator responsible for maintenance of system frequency.
The set of simultaneous power equations which defines the system's state is non-linear. It is therefore solved via a set of successive linear approximations based on first order Taylor expansions of the power equations. The most common method is that of Newton which is what has been used in this study.

The solution routine requires the assignment of some initial estimate to all the busbar voltage magnitudes and angles (the slack bus is typically assigned to $1\angle 0$) and the calculation of the initial real and reactive power mismatches. Should any of these be above the set tolerance, the Jacobian is formed and solved for updates of voltages and angles upon which new estimates of the power mismatches are obtained. While convergence is not obtained, the Jacobian is again formed and new updates of $V$ and $\theta$ are found.

The power equation for bus $k$ is

$$S_k = P_k + jQ_k = E_k I_k^*$$

$$= E_k \sum_{m \in k} y_{km} E_m^*$$

where $E_k = V_k^{re} + jV_k^{im} = V_k \angle \theta_k$ is the voltage at bus $k$, $y_{km}$ is the admittance between buses $k$ and $m$ and $I_k$ is the current injected at bus $k$. In polar co-ordinates, $P_k$ and $Q_k$ are

$$P_k = \sum_{m \in k} V_k V_m \{G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)\}$$

$$Q_k = \sum_{m \in k} V_k V_m \{G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m)\}$$

Linear relationships for small changes in $V$ and $\theta$ are found so that for a $PQ$ bus

$$\Delta P_k = \sum_{m \in k} \frac{\partial P_k}{\partial \theta_m} \Delta \theta_m + \sum_{m \in k} \frac{\partial P_k}{\partial V_m} \Delta V_m$$

$$\Delta Q_k = \sum_{m \in k} \frac{\partial Q_k}{\partial \theta_m} \Delta \theta_m + \sum_{m \in k} \frac{\partial Q_k}{\partial V_m} \Delta V_m$$

For a $PV$ bus, only equation 6.13 is used since $Q_k$ is not specified, and there are no equations for the slack bus.

With the voltages in rectangular form, the partial derivatives are

$$H_{km} = \frac{\partial P_k}{\partial V_m} = G_{km} (V_k^{im} V_m^{re} - V_k^{re} V_m^{im}) - B_{km} (V_k^{re} V_m^{im} + V_k^{im} V_m^{re})$$
\[ N_{km} = V_m \frac{\partial P_k}{\partial V_m} = G_{km}(V_k^r V_m^i - V_k^i V_m^r) - B_{km}(V_k^i V_m^r + V_k^r V_m^i) \] (6.16)

\[ J_{km} = \frac{\partial Q_k}{\partial \theta_m} = -N_{km} \] (6.17)

\[ L_{km} = V_m \frac{\partial Q_k}{\partial V_m} = H_{km} \] (6.18)

\[ H_{kk} = \frac{\partial P_k}{\partial \theta_k} = -Q_k - B_{kk} V_k^2 \] (6.19)

\[ N_{kk} = V_k \frac{\partial P_k}{\partial V_k} = P_k + G_{kk} V_k^2 \] (6.20)

\[ J_{kk} = \frac{\partial Q_k}{\partial \theta_k} = P_k - G_{kk} V_k^2 \] (6.21)

\[ L_{kk} = V_k \frac{\partial Q_k}{\partial V_k} = Q_k - B_{kk} V_k^2 \] (6.22)

These are assembled in a matrix equation of the form

\[
\begin{bmatrix}
\Delta P^{p-1} \\
\Delta Q^{p-1}
\end{bmatrix} =
\begin{bmatrix}
H^{p-1} & N^{p-1} \\
J^{p-1} & L^{p-1}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta^p \\
\Delta V^p
\end{bmatrix}
\] (6.23)

which is solved for \( \Delta \theta^p \) and \( \Delta V^p \) at the \( p \)th iteration where \( \Delta P^{p-1} \) are the \( P \) mismatches for all \( PQ \) and \( PV \) busbars, \( \Delta Q^{p-1} \) are the \( Q \) mismatches for the \( PQ \) busbars, \( \Delta \theta^p \) are the \( \theta \) corrections for all \( PQ \) and \( PV \) busbars and \( \Delta V^p \) are the \( V \) corrections for all \( PQ \) buses.

Maximum and minimum limits on reactive generation are often defined for \( PV \) buses. When a limit is reached, \( Q \) at that bus is fixed to the limiting value and the bus type is switched to \( PQ \) with \( V \) to be found. If at subsequent iterations, \( Q \) comes back within limits, the type is switched back to \( PV \).

### 6.2.1 Treatment of transformers

Figure 6.1 shows the basic equivalent circuit of a transformer with the currents \( I_{km} \) and \( I_{mk} \), voltages \( V_k, V_m \) and \( V' \), admittance \( y_{km} \) and turns ratio \( a \) shown.

With \( a = 1 \),

\[ I_{km} = y_{km} (V_k - V_m) \] (6.24)

\[ I_{mk} = y_{km} (V_m - V_k) \] (6.25)

i.e. \( I_{km} = -I_{mk} \).
With $a \neq 1$,

\[
V' = \frac{V_k}{a} \quad (6.26)
\]

\[
I_{mk} = y_{km}(V_m - V') \quad (6.27)
\]

\[
I_{km} = -\frac{I_{mk}}{a} \quad (6.28)
\]

Eliminating $V'$ from (6.27) and replacing $I_{mk}$ in (6.28),

\[
I_{km} = \frac{y_{km}}{a}V_k - \frac{y_{km}}{a}V_m \quad (6.29)
\]

\[
I_{mk} = -\frac{y_{km}}{a}V_k + y_{km}V_m \quad (6.30)
\]

or in admittance matrix form,

\[
\begin{bmatrix}
I_{km} \\
I_{mk}
\end{bmatrix} =
\begin{bmatrix}
\frac{y_{km}}{a} & -\frac{y_{km}}{a} \\
-\frac{y_{km}}{a} & y_{km}
\end{bmatrix}
\begin{bmatrix}
V_k \\
V_m
\end{bmatrix} \quad (6.31)
\]

This corresponds to equivalent circuit shown in figure 6.2.

**On-load tap changing transformers**

On-load tap changing transformers (OLTCs) automatically change their tap ratios in discrete steps while under load. The changes take place in order to try to attain a target
voltage on either the HV or LV side. If the voltage error on the side of the transformer that is scheduled is \( V^0 - V \) at time \( k \), a function \( f(\Delta V) \) [111] can be used to determine the new tap ratio \( a_{k+1} \) where

\[
a_{k+1} = a_k - f(\Delta V)\Delta a
\]

and \( \Delta a \) is the maximum tap step size, \( V^0 \) is the scheduled voltage magnitude, and \( \epsilon \) is a deadband.

Such a function \( f(\Delta V) \) can be implemented in a load-flow routine at the end of each iteration so that a new tap ratio can be found and the solution adjusted. Care should be taken that convergence is not affected as the solution can tend to zig-zag around when tap ratios are over-compensated as is particularly likely when one node has more than one OLTC connected. In such a circumstance, the tap adjustment may be divided by the number of OLTCs. In addition, since the first iteration is likely to have very large mismatches, adjustment of transformer tap ratios may be delayed until the second of third iteration or until the mismatch at the regulated bus is below a threshold.

### 6.2.2 Treatment of quadrature boosters

The basic equivalent circuit for a quadrature booster or phase-shifting transformer is similar to that shown in figure 6.1, but the turns ratio \( a \) is replaced by the complex turns ratio \( \alpha \), as in figure 6.3, where

\[
\alpha = a + jb
\]

and

\[
V_k = \alpha V'
\]

Since power loss in the ideal transformer is negligible,

\[
V_k I_k = -V' I'^*
\]
From (6.35) and (6.36),
\[ I_k = -\frac{I'^*}{a + jb} \]  
(6.37)

or
\[ I_k = -\frac{I'}{a - jb} = \frac{I'}{a^*} \]  
(6.38)

Thus it can be seen that there are two different turns ratios, one for voltage, \( \alpha_v = a + jb \), and one for current, \( \alpha_i = a - jb \).

Solving for terminal currents,
\[
I_k = \frac{I'}{\alpha_i} = \frac{(V' - V_m)Y_{km}}{\alpha_i} = \frac{(V_k/\alpha_v - V_m)Y_{km}}{\alpha_i} = \frac{Y_{km}V_k - Y_{km}V_m}{\alpha_v \alpha_i} \tag{6.39}
\]

or
\[
-I_m = I' = \frac{y_{km}V_k - y_{km}V_m}{\alpha_v} \tag{6.40}
\]

or in admittance matrix form,
\[
\begin{bmatrix}
I_k \\
I_m
\end{bmatrix} = \begin{bmatrix}
y_{km}/\alpha_v \alpha_i & -y_{km}/\alpha_i \\
-y_{km}/\alpha_v & y_{km}
\end{bmatrix} \begin{bmatrix}
V_k \\
V_m
\end{bmatrix} \tag{6.41}
\]

It can be seen that, unlike for a standard transformer whose admittance matrix is described by (6.31), the admittance matrix for a quadrature booster is non-symmetric and the equivalent circuit is not readily available. The expression for the active power entering the booster, however, can be easily found. Considering the power at node \( k \),
\[ S_k = V_kI_k^* \] or
\[ S_k = V_k(V_kY_{kk} + V_mY_{km})^* \]  
(6.42)
If the complex tap ratio \( \alpha \) is written as \( \alpha = a + jb = t \angle \psi \) and the voltages \( V_k \) and \( V_m \) as \( V_k = V_k \angle \theta_k \), \( V_m = V_m \angle \theta_m \), then from equations 6.41 and 6.42

\[
S_k = V_k \left( \frac{y_{km} V_k}{t^2} - \frac{y_{km} V_m \angle \psi}{t} \right) \quad (6.43)
\]

\[
= y_{km}^* \left( \frac{V_k^2}{t^2} - \frac{V_k V_m \angle (\theta_k - \theta_m - \psi)}{t} \right) \quad (6.44)
\]

With \( y_{km} = g + jb \), the active power \( P_k \) is the real part of \( S_k \) and is

\[
P_k = g \left( \frac{V_k^2}{t^2} - \frac{V_k V_m \cos \phi}{t} \right) - b \frac{V_k V_m \sin \phi}{t}
\]

where \( \phi = \theta_k - \theta_m - \psi \).

### 6.3 Power System Sensitivity Analysis

Consider a power system with \( N \) interconnected nodes. If the total power injected into the system is described by the vector \( G \) of nodal power injections and is a function of the vector of \( N \) dependent complex variables \( x \), \( M \) control variables \( u \) and the vector of power system parameters such as line impedances and shunt susceptances \( p \), then for balanced operation [112],

\[
G(x, u, p) = 0 \quad (6.46)
\]

If a small change to the control vector \( \Delta u \) is applied then a small change in the vector of dependent variables \( \Delta x \) will result. For balanced operation to continue

\[
G(x + \Delta x, u + \Delta u, p) = 0 \quad (6.47)
\]

Using a Taylor series expansion and neglecting higher order terms, 6.47 can be re-expressed as

\[
G(x_0, u_0, p) + G_x(x_0, u_0, p) \Delta x + G_u(x_0, u_0, p) \Delta u = 0 \quad (6.48)
\]

where \( x_0 \) and \( u_0 \) are the original \( x \) and \( u \) vectors before the change and \( G_x \) and \( G_u \) are the Jacobians of \( G \) with respect to \( x \) and \( u \) respectively. If the system was balanced before
the change, the first term vanishes leaving

\[ G_x(x_0, u_0, p) \Delta x + G_u(x_0, u_0, p) \Delta u = 0 \]  (6.49)

From this, the change \( \Delta x \) resulting from \( \Delta u \) can be found

\[ \Delta x = -G_x(x_0, u_0, p)^{-1}G_u(x_0, u_0, p) \Delta u \]  (6.50)

so that the sensitivity matrix \( S \) relating \( \Delta x \) to \( \Delta u \) is

\[ S = -G_x(x_0, u_0, p)^{-1}G_u(x_0, u_0, p) \]  (6.51)

Once \( S \) is known, the change \( \Delta x_i \) resulting from the change in control \( \Delta u_j \) can be estimated simply as \( S_{ij} \Delta u_j \).

### 6.3.1 Relation of reactive power controls to voltage magnitudes

If the well-known principle of decoupling the active and reactive power equations of a power system is utilised, equation 6.49 can be re-expressed for the reactive subsystem as

\[ G_{Qx}(x_{Q0}, u_{Q0}, p) \Delta x_Q + G_{Qu}(x_{Q0}, u_{Q0}, p) \Delta u_Q = 0 \]  (6.52)

where the Jacobians \( G_{Qx} \) and \( G_{Qu} \) are

\[ G_{Qx} = \frac{\partial G_Q}{\partial x_Q} \bigg|_{x=x_{Q0}} \]  (6.53)

\[ G_{Qu} = \frac{\partial G_Q}{\partial u_Q} \bigg|_{u=u_{Q0}} \]  (6.54)

and \( G_Q \) comprises only the reactive power equations, \( x_Q \) contains dependent voltage magnitudes \( V \) and reactive powers \( Q \). \( u_Q \) contains controllable voltage magnitudes \( V \), reactive powers \( Q \), adjustable transformer tap ratios \( t \) and adjustable shunt susceptances \( B \).

The \( N \times M \) reactive sensitivity matrix \( S_Q \) is found by

\[ S_Q = -G_{Qx}(x_{Q0}, u_{Q0}, p)^{-1}G_{Qu}(x_{Q0}, u_{Q0}, p) \]  (6.55)
In practice, $S_Q$ is found by re-expressing equation 6.55 as
\[
G_{Qx}(x_{Q0}, u_{Q0}, p)S_Q = -G_{Qu}(x_{Q0}, u_{Q0}, p)
\] (6.56)

and factorising $G_{Qx}$ into lower and upper triangular factors so that
\[
LU S_Q = -G_{Qu}(x_{Q0}, u_{Q0}, p)
\] (6.57)

If the matrices $S_Q$ and $G_{Qu}$ are described in terms of column vectors $s_1 \ldots s_M$ and $g_1 \ldots g_M$ where
\[
S_Q = [s_1 \ldots s_M]
\] (6.58)
\[
G_{Qu} = [g_1 \ldots g_M]
\] (6.59)

then each equation
\[
LU s_i = -g_i
\] (6.60)

for $i = 1 \ldots M$ can be solved for $s_i$ by forward and backward substitution.

Finally, $S_Q$ can be represented as
\[
\begin{bmatrix}
\Delta V_l \\
\Delta Q_g
\end{bmatrix} =
\begin{bmatrix}
S_{Vl} & S_{VB} & S_{Vv} \\
S_{Ql} & S_{QB} & S_{Qv}
\end{bmatrix}
\begin{bmatrix}
\Delta t \\
\Delta B \\
\Delta V_g
\end{bmatrix}
\] (6.61)

where $V_l$ is the vector of $N_l$ load bus voltage bus magnitudes, $Q_g$ is the vector of $N_g$ reactive injections at generator or compensation buses, $V_g$ is the vector of $N_g$ generator or static voltage compensator (SVC) voltage set-points, $B$ is the vector of $N_b$ shunt susceptances and $t$ is the vector of $N_t$ transformer tap ratios.

### 6.3.2 Relation of active power controls to active power flows

Most references describing the formulation of a matrix of sensitivities of active power flows to active controls have required some pseudo-inverse matrix to be found, for example [113]. The approach adopted in this study and described in this section, however, avoids that and allows fast sparse matrix techniques to be used. This is achieved by
finding the (sparse) Jacobian of derivatives of active power flow with respect to voltage angles and pre-multiplying it with an inverted square matrix.

If \( u_P \) is defined as the vector of controllable active power injections and quadrature booster angles, buses 1 to \( L \) are load buses and buses \( L + 1 \) to \( N - 1 \) are generation buses (the slack bus is excluded), a sensitivity matrix \( A_P \) can be found relating small changes in \( u_P \) to small changes in the nodal voltage angles \( \theta \) where

\[
\Delta \theta = A_P \Delta u_P \tag{6.62}
\]

and

\[
A_P = -G_{P\theta}^{-1} \partial G_{P\theta} \tag{6.63}
\]

where

\[
u_P = \{P_{L+1}, \ldots, P_N\}^T \tag{6.64}
\]

\[
G_{P\theta} = \left. \frac{\partial G_P}{\partial \theta} \right|_{\theta=\theta_0} \tag{6.65}
\]

\[
G_{Pu} = \left. \frac{\partial G_P}{\partial u_P} \right|_{u_P=u_P_0} \tag{6.66}
\]

\( G_P \) comprises only the active power equations.

The changes in nodal voltage angles can be related to changes in active power transmitted along each transmission line or through each transformer \( P_{km} \) from general node \( k \) to node \( m \) by

\[
\Delta P_{km} = \left. \frac{\partial P_{km}}{\partial \theta} \right|_{\theta=\theta_0} \Delta \theta \tag{6.67}
\]

Hence, a matrix directly relating change in active power generation and quadrature booster angles (the vector of quantities which can be set) and change in active power flow (which is to be controlled) is found from

\[
\Delta P_{km} = \left. \frac{\partial P_{km}}{\partial \theta} \right|_{\theta=\theta_0} A_P \Delta u_P \tag{6.68}
\]

It is denoted \( S_P \) such that

\[
S_P = \left. \frac{\partial P_{km}}{\partial \theta} \right|_{\theta=\theta_0} A_P \tag{6.69}
\]
If equation 6.63 is re-expressed as

\[
\frac{\partial G_P}{\partial \theta} A_P = -\frac{\partial G_P}{\partial u_P}
\]  \hspace{1cm} (6.70)

then \( A_P \) can be found and pre-multiplied by \( \partial P_{km}/\partial \theta \) to obtain \( S_P \). Since \( \Delta P_{km} = -\Delta P_{mk} \), \( S_P \) can then be used to determine the effects of changes in active power generation and quadrature booster angles on active power flows measured at each end of items of transmission plant.

### 6.3.3 Example of Jacobians for voltage control

Consider the small test power system shown in figure 6.4 with \( N = 6 \) nodes, \( n_0...5 \) where there is generation at \( n_4 \) and \( n_5 \), static compensation at \( n_3 \) and a tap-changing transformer between \( n_0 \) and \( n_3 \) so that \( M = 4 \). Using standard loadflow analysis, \( n_4 \) and \( n_5 \) are designated as 'PV' nodes, i.e. nodes with controllable active power injection \( P \) and voltage magnitude \( V \). Load nodes \( n_0...2 \) are designated 'PQ' nodes with fixed \( P \) and \( Q \) injections and have voltage magnitudes which are to be controlled. Node \( n_3 \) can be regarded in one of two ways, either as a load 'PQ' node with a shunt capacitor, or a 'PV' node with variable set voltage magnitude \( V \) [110]. These will be considered in turn.

![Figure 6.4: Example 6 bus system](image)
Static compensator node treated as a ‘PV’ node

For the test system described and node $n_3$ treated as a ‘PV’ node, $x_Q$ is a column vector of order $N$ and $u_Q$ is a column vector of order $M$ such that,

$$
x_Q = \{V_0, V_1, V_2, Q_3, Q_4, Q_5\}^T \quad (6.71)
$$

$$
u_Q = \{t_{03}, V_3, V_4, V_5\}^T \quad (6.72)
$$

where $t_{03}$ is the tap ratio of the transformer between nodes $n_0$ and $n_3$.

\[
G_{Qx} = \begin{bmatrix}
\frac{\partial Q_0}{\partial V_0} & \frac{\partial Q_1}{\partial V_1} & \frac{\partial Q_1}{\partial V_2} \\
\frac{\partial Q_2}{\partial V_1} & \frac{\partial Q_2}{\partial V_2} & \\
\frac{\partial Q_3}{\partial V_1} & \frac{\partial Q_3}{\partial V_2} & -1 \\
\frac{\partial Q_4}{\partial V_1} & \frac{\partial Q_4}{\partial V_2} & -1 \\
\frac{\partial Q_5}{\partial V_2} & & \\
\end{bmatrix}
\]

\[
G_{Qx} = \begin{bmatrix}
\frac{\partial Q_0}{\partial t_{03}} & \frac{\partial Q_0}{\partial V_3} \\
\frac{\partial Q_1}{\partial V_3} & \frac{\partial Q_1}{\partial V_4} \\
\frac{\partial Q_2}{\partial V_3} & \frac{\partial Q_2}{\partial V_4} & \frac{\partial Q_2}{\partial V_5} \\
\frac{\partial Q_3}{\partial V_3} & \frac{\partial Q_3}{\partial V_4} & \frac{\partial Q_3}{\partial V_5} \\
\frac{\partial Q_4}{\partial V_4} & \frac{\partial Q_4}{\partial V_5} & \frac{\partial Q_4}{\partial V_5} \\
\end{bmatrix}
\]

(6.73)

(6.74)

Static compensator node treated as a ‘PQ’ node

With node $n_3$ treated as a ‘PQ’ node and the control variable regarded as the variable shunt susceptance at the node $B_{s3}$,

$$
x_Q = \{V_0, V_1, V_2, V_3, Q_4, Q_5\}^T \quad (6.75)
$$

$$
u_Q = \{t_{03}, B_{s3}, V_4, V_5\}^T \quad (6.76)
$$

and

\[
G_{Qx} = \begin{bmatrix}
\frac{\partial Q_0}{\partial V_0} & \frac{\partial Q_6}{\partial V_0} \\
\frac{\partial Q_1}{\partial V_0} & \frac{\partial Q_1}{\partial V_2} & \frac{\partial Q_1}{\partial V_3} \\
\frac{\partial Q_2}{\partial V_0} & \frac{\partial Q_2}{\partial V_2} & \frac{\partial Q_2}{\partial V_3} \\
\frac{\partial Q_3}{\partial V_0} & \frac{\partial Q_3}{\partial V_2} & \frac{\partial Q_3}{\partial V_3} \\
\frac{\partial Q_4}{\partial V_0} & \frac{\partial Q_4}{\partial V_2} & \frac{\partial Q_4}{\partial V_3} \\
\frac{\partial Q_5}{\partial V_2} & \frac{\partial Q_5}{\partial V_3} & -1 \\
\frac{\partial Q_6}{\partial V_3} & \frac{\partial Q_6}{\partial V_3} & -1 \\
\end{bmatrix}
\]

(6.77)
6.4 Summary

This chapter has introduced some of the details of static analysis of power systems. It has described the load-flow problem and the solution method adopted in this study.

Power systems sensitivity analysis has been discussed with formulations described which relate small changes in independent (control) variables to small changes in dependent (state) variables. Two such formulations implemented in this study, one for the reactive subsystem and the other for the active, have been outlined. In the active subsystem, the need to find a pseudo-inverse matrix has been replaced by the inverse of a square matrix pre-multiplied by another sparse matrix.
The ability of an energy management system (EMS) to find a consistent set of states that describes the current condition of the power system from a static set of SCADA information is crucial to its entire function. This state estimation task is the basic building block upon which all EMS functions are based.

Recent attention directed towards the improved performance of state estimators has concentrated on the robust utilisation of ‘zero-injection’ busbar ‘measurements’ to reduce the uncertainty in the derived state. Among the methods employed is the so-called ‘Hachtel’s Augmented Matrix’ method. A separate direction of research has been the determination of the observability of an interconnected power system. Two basic approaches exist, numerical and topological.

This chapter draws together work done in both these areas of work on state estimation, describing a published blocked augmented matrix method which allows the matrix to be solved in a positive-definite manner, and a new development of a topological observability algorithm which utilises a simple expert system and provides matrix ordering information. A new enhancement to blocked augmented matrix method which enables transformer tap estimation and the matrix ordering issues that it raises are also described. Results are presented which compare solutions of the IEEE 57 bus system and 60 and 811 bus models of the UK National Grid for a fully coupled blocked augmented matrix approach and a fast decoupled augmented matrix solution. Results for the IEEE 57 bus system and the 60 bus reduced U.K. system are also shown for Hachtel’s method.
7.1 Introduction

State estimation is the process by which telemetered data on busbar voltage magnitudes and injections and branch (line) active and reactive power flows are assembled into an estimate of the current system state, taken in most instances to be the vector of complex nodal voltages i.e. magnitudes and angles. The methods employed must allow for the existence of errors in the analogue metering such as drift and offset, in the analogue to digital conversion and in the communication of the data. It must also be able to recognise items of data that are incorrect due to meter or communication failure or due to the existence of transients on the system (such data is called bad data), and whether there is enough metered information available to be able to formulate an estimate of the system states. This latter function is known as observability analysis. The different components of the state estimation process are shown in figure 7.1.

![Figure 7.1: The stages in state estimation](image)

State estimators have been implemented successfully in many different energy management systems over the last 15 years. Improvements have been made concerning their speed and numerical stability. These have been made necessary by the increasing complexity of subsequent functions of the EMS which depend on a rapidly and accurately assembled network model.

As energy management systems are built that include computationally intensive operations such as contingency analysis addressed to all regimes of security and optimal power flow, the time needed within the available ‘real time’ window for those functions increases. Less time is therefore available for state estimation hence recent attention has been focused on improvements to existing algorithms to give faster results.
7.2 State Estimation

The state vector $x$ of an interconnected power system is described by the vector of all observable busbar voltage magnitudes ($V$) and angles ($\theta$) for $b$ busbars such that

$$x = [V_1, \ldots, V_b, \theta_1, \ldots, \theta_{b-1}]$$

(7.1)

where $\theta_s$ is the fixed voltage angle reference.

Not all the measurable quantities of a power system are in practice available, and those that are are subject to errors. The vector of measured quantities $z$ may be modelled as the sum of the vector of non-linear functions of the actual power system state $h(x)$ and the measurement error vector $\omega$:

$$z = h(x) + \omega$$

(7.2)

A 'best fit' for the state vector $x$ is derived by minimising the residual vector $r$ where

$$r = z - h(x)$$

(7.3)

Of a number of methods adopted for doing this, the most common and most successful has been the weighted least squares minimization.

7.3 The Basic Formulation

The weighted least squares function

$$J(x) = \frac{1}{2}[z - h(x)]^T W^{-1} [z - h(x)]$$

(7.4)

where $W$ is a diagonal matrix of weightings is minimized, and the resulting estimate of $x$, denoted $\hat{x}$ is found.

The so-called normal equation referred to in [114] is formed from the differentiation of $J(x)$ with respect to $x$ and setting

$$\frac{\partial J(x)}{\partial x} \bigg|_{x=\hat{x}} = -H(\hat{x})^T W^{-1} [z - h(\hat{x})] = 0$$

(7.5)
where

\[ H(\hat{x}) = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\hat{x}} \]  

(7.6)

is the measurement Jacobian matrix.

Since the normal equation is non-linear, an iterative method, usually that of Newton, is employed based on an approximation of \( h(\hat{x}_{k+1}) \):

\[ h(\hat{x}_{k+1}) = h(\hat{x}_k) + H(\hat{x}_k)\Delta \hat{x} \]  

(7.7)

If \( H(\hat{x}_{k+1}) \) is assumed to be \( H(\hat{x}_k) \) then the iterative sequence results:

\[ H(\hat{x}_k)^T W^{-1} H(\hat{x}_k)\Delta \hat{x} = H(\hat{x}_k)^T W^{-1} [z - h(\hat{x}_k)] \]  

(7.8)

\[ \hat{x}_{k+1} = \hat{x}_k + \Delta \hat{x} \]  

(7.9)

It can be seen that a unique solution for \( \Delta \hat{x} \) can only be obtained if \( H(\hat{x}_k) \) is full rank i.e. the network is observable. For the estimate to be reliable, the number of measurements needs to be greater than the rank of \( H \), i.e. there is redundancy.

The weighting matrix \( W \) has often been composed with the diagonal elements \( \sigma_{ii} \) chosen as measurement error variances, but it is found that certain busbars on an electric power system have identically zero injections i.e. they have no generation or load so all the power into the bus must equal the power out. The injections at these buses are perfect measurements. The question arises as to how to treat these zero injection buses in the state estimation formulation. The earliest methods could not treat them as perfect measurements as this would result in zeroes on the diagonal of \( W^{-1} \). Instead, they were treated as very accurate measurements with correspondingly low elements in \( W^{-1} \). However, this was found to lead to ill-conditioning and non-convergence of the state estimate under certain circumstances.

As a consequence, a number of papers, of which [115] is the most significant, proposed the inclusion of zero injections in the least squares formulation as equality constraints so that

\[ J(x) = \frac{1}{2} [z - h(x)]^T W^{-1} [z - h(x)] \]  

(7.10)
is minimized subject to
\[ c(x) = 0 \quad (7.11) \]
where \( c(x) \) is a vector of constraint equations. The constrained minimization problem may then be solved by the method of Lagrange multipliers in which the Lagrangian \( L(x) \) is
\[ L(x) = \frac{1}{2} [z - h(x)]^T W^{-1} [z - h(x)] + \lambda^T c(x) \quad (7.12) \]

This formed the basis for further investigation of the state estimation problem with a view to improving on the basic 'Normal Equations with Constraints' approach in terms of both numerical robustness and computational speed. Methods proposed included solution of the problem by Orthogonal Transformation [116, 117, 118], a Hybrid Method [119], the Method of Peters and Wilkinson and Hachtel's Method [120]. A paper in 1988 by Holten et al [121] compared the standard Normal Equations methods with and without constraints with the Orthogonal Transformation method, the Hybrid method and Hachtel's Method (the Method of Peters and Wilkinson was felt to be too computationally intensive) and concluded that Hachtel's method offered the best trade-off between speed and reliability.

### 7.4 Hachtel's Augmented Matrix Method

The method is founded on the same weighted least squares formulation with constraints as above. Those equations are, as the method's name suggests, augmented by an additional equation [120]. This larger set of equations is, however, sparser so the computation time is competitive with other methods.

The Lagrangian of (7.12) is augmented by treating the residual \( r \) as an unknown.
\[ r = z - h(x) \quad (7.13) \]

Later versions of the method have treated the residual \( r \) as a further equality constraint
to be included in the Lagrangian which may then be written

\[ L(x) = \frac{1}{2} r^T W^{-1} r + \lambda^T c(x) + \mu^T [r - z + h(x)] \quad (7.14) \]

The solution for the least squares estimate must satisfy

\[ \frac{\partial L(x)}{\partial r} \bigg|_{x = \hat{x}} = W^{-1} [z - h(\hat{x})] - \mu = 0 \quad (7.15) \]
\[ \frac{\partial L(x)}{\partial x} \bigg|_{x = \hat{x}} = -C(\hat{x})^T \lambda - H(\hat{x}) \mu = 0 \quad (7.16) \]
\[ \frac{\partial L(x)}{\partial \lambda} \bigg|_{x = \hat{x}} = -c(\hat{x}) = 0 \quad (7.17) \]
\[ \frac{\partial L(x)}{\partial \mu} \bigg|_{x = \hat{x}} = r + z - h(\hat{x}) = 0 \quad (7.18) \]

where

\[ H(\hat{x}) = \frac{\partial h(x)}{\partial x} \bigg|_{x = \hat{x}} \quad (7.19) \]

and

\[ C(\hat{x}) = \frac{\partial c(x)}{\partial x} \bigg|_{x = \hat{x}} \quad (7.20) \]

The linearised version is then found:

\[
\begin{bmatrix}
0 & H^T(\hat{x}_k) & C^T(\hat{x}_k) \\
H(\hat{x}_k) & W & 0 \\
C(\hat{x}_k) & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \hat{x} \\
\mu_{k+1} \\
\lambda_{k+1}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
\Delta z_k \\
-c(\hat{x}_k)
\end{bmatrix}
\quad (7.21)
\]

where

\[ \Delta z_k = z - h(x_k) \quad (7.22) \]

Some versions of the above formulation include the scalar factor \( \alpha \) in the above expression to improve conditioning, replacing \( W \) by \( \alpha W \), \( \lambda_{k+1} \) by \( -\alpha^{-1} \lambda_{k+1} \) and \( \mu_{k+1} \) by \( \alpha^{-1} \mu_{k+1} \). This would be the more general version of the above expression, which could be thought of as having \( \alpha = 1 \). Indeed, Gjelsvik et al’s original paper found good results with \( \alpha = 1 \).
The main problem with Hachtel's Method as described in this section is that the coefficient matrix, though symmetric, is not positive-definite. This requires that ordering of the equations employs a numerical test for the candidate pivot elements.

Much attention has been given in recent years to the issues of making the coefficient matrix positive-definite and therefore open to manipulation using standard sparse matrix methods, and generalising the method to include procedures for addressing observability and bad data analysis. A scheme for ordering the coefficient matrix to make it positive definite is discussed in the next section.

7.5 Blocked Sparse Matrix Formulations

In 1991, two papers offering improvements on Hachtel's Method appeared simultaneously. They both allowed triangular factorization of the coefficient (gain) matrix to proceed in a positive definite manner while preserving the robustness of Hachtel's Method and maximizing diagonal block dominance in order to properly handle ill-conditioned systems. Nucera and Gilles [122] also included aspects of the Normal Equations with Constraints approach to reduce the order of the augmented matrix while Alvarado and Tinney [123] performed a partial factorization to eliminate branch flows and measured injections.

Nucera and Gilles use the same basic matrix formulation as in Hachtel's Method, but partition the residual vector $r$ into parts corresponding to 'squared in' terms and 'unsquared' terms, $r_B$ and $r_A$ respectively. The second equality constraint which augments the matrix in Hachtel's Method is defined to contain all the injections, squared terms are included in the coefficient matrix and the dimensions of the matrix are reduced with respect to the basic Hachtel's Method, improving the speed of solution.

It could be seen from the description of Hachtel's Method in section 2.2 and in [120] that there are no 'squared in' terms at all in the coefficient matrix. This avoids the problem of ill-conditioning by not squaring any terms and treating zero injections as equality constraints. It does, though, increase the dimensions of the coefficient or gain matrix.
7.5.1 Hybrid Blocked Augmented Formulation

In the blocked augmented matrix method described in [122], all the injection measurements which are believed to lead to ill-conditioning are left unsquared, while the others are squared. Thus

\[ r = [r_A, r_B]^T \]  

(7.23)

where \( r_A \) relates to the ‘unsquared’ injection measurements and \( r_B \) to the other measurements that will be ‘squared in’. With the vector \( r \) defined as \( W^{\frac{1}{2}}[z - h(x)] \), it follows that the diagonal matrix of weights \( W \), the vector of measurements \( z \) and the vector of equations \( h(x) \) must be similarly partitioned. The problem can then be characterized as the minimization of

\[ J(x) = \frac{1}{2}[z_B - h_B(x)]^T W_B^{-1} [z_B - h_B(x)] \]  

(7.24)

subject to

\[ c(x) = 0 \]  

(7.25)

and

\[ r_A = W_A^{-\frac{1}{2}}[z_A - h_A(x)] \]  

(7.26)

With both constraints treated by the method of Lagrange multipliers, the Lagrangian is

\[ L(x, r, \lambda, \mu) = \frac{1}{2}[z_B - h_B(x)]^T W_B^{-1} [z_B - h_B(x)] + \frac{1}{2} r_A^T r_A \]

\[ -\lambda^T c(x) - \mu^T \{ r_A - W^{-\frac{1}{2}}[z_A - h_A(x)] \} \]  

(7.27)

The estimated state obtained \( \hat{x} \) must satisfy the optimality conditions

\[ \frac{\partial L}{\partial x} = -H_B^T(\hat{x}) W_B^{-1} [z_B - h_B(\hat{x})] - C^T \lambda - H_A^T(\hat{x}) W_A^{-\frac{1}{2}} \mu = 0 \]  

(7.28)

\[ \frac{\partial L}{\partial r_A} = r_A - \mu = 0 \]  

(7.29)

\[ \frac{\partial L}{\partial \lambda} = c(\hat{x}) = 0 \]  

(7.30)

\[ \frac{\partial L}{\partial \mu} = r_A - W_A^{\frac{1}{2}}[z_A - h_A(\hat{x})] = 0 \]  

(7.31)
These are solved iteratively in the following set of linear equations:

\[
\begin{bmatrix}
-H_B^T W_B^{-1} H_B & H_A^T W_A^{-\frac{1}{2}} & C^T \\
W_A^{-\frac{1}{2}} H_A & I & 0 \\
C & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\mu_{k+1} \\
\lambda_{k+1}
\end{bmatrix}
= 
\begin{bmatrix}
-H_B^T W_B^{-1} \Delta z_B \\
\Delta z_A \\
-c(x_k)
\end{bmatrix}
\]  
(7.32)

where, at the \(k\)th iteration,

\[
\Delta z_A = W_A^{-\frac{1}{2}} [z_A - h_A(x_k)]
\]  
(7.33)

\[
\Delta z_B = [z_B - h_B(x_k)]
\]  
(7.34)

and

\[
x_{k+1} = x_k + \Delta x
\]  
(7.35)

\[
h(x_{k+1}) = h(x_k) + H(x_k) \Delta x_k
\]  
(7.36)

\[
c(x_{k+1}) = c(x_k) + C(x_k) \Delta x_k
\]  
(7.37)

The order of the hybrid matrix is less than that of the equivalent Hachtel's version. Note that if all measurements except zero injections are included in \(H_B\), the formulation reduces to that of the Normal Equations with Constraints approach, and if all measurements are in \(H_A\), the formulation reduces to Hachtel's.

### 7.5.2 Blocking of the Gain (Co-Efficient) Matrix Through a Topological Method

Nucera and Gilles [122, 124] proposed using a blocking scheme based on a topological algorithm previously used in observability analysis (see section 7.8 and reference [125]).

The purpose of the scheme is to avoid there being any zero diagonal elements or singular \(2 \times 2\) blocks. This is achieved by pairing every injection, measured and zero, with a node or bus and therefore its corresponding state variables. In addition, a bus which does not have a measured or zero injection and does not have a flow measurement on one of the lines connected to it must be paired with a remote injection measurement i.e. one from a different bus.
It should be noted that if a bus has an adjacent flow measurement, then it does not need to be paired with an injection, so its injection becomes available for use in a remote pairing. It is always preferable, though, to pair a bus with an injection at that bus, even if there is an adjacent flow measurement.

The algorithm proposed by Nucera and Gilles to select the pairings in a near optimal manner bears close relation to the observability algorithm suggested by Krumpholz, Clements and Davis [125]. It involves passing along the observable tree, starting at the selected reference bus (which is always available for pairing even with an angle pseudo-measurement).

Each branch is first assigned a measurement, either a flow or injection, by the observability algorithm.

Now, assume that a bus $m$ is reached from a bus $k$. Let the respective bus injection measurements or zero injections be $P_m$ and $P_k$ (the decoupled active power case is being considered). The decision of interest (i.e. that which is required for the matrix blocking) is to which buses to assign the injections. In reaching $m$ from $k$, one of three scenarios might be experienced.

1. The branch $km$ is assigned to a measurement of flow through $km$. $m$ is not paired—it does not need to be since buses ‘seeing’ a flow measurement do not need to be paired.

2. $km$ is assigned to injection $P_m$. The pairing $(P_m, m)$ is chosen since an injection is best paired with its incident bus.

3. $km$ is assigned to injection $P_k$. Here there are three more possibilities.

   (a) $m$ is not paired because it sees a flow on a subsequent branch $mn$.

   (b) There is an unpaired injection $P_m$ at $m$. This ‘redundant injection’ is paired with $m$ to form $(P_m, m)$.

   (c) Bus $m$ is paired with $P_k$ to form the remote pairing $(P_k, m)$. 
The process is continued until the whole observable tree has been traversed. Finally, all injections that have not been paired are assigned to their local buses.

The gain matrix is re-assembled with the pairings and with all injections that have an assignment in the observable forest ordered so they are eliminated first. This ensures a positive-definite factorization. The matrix will now be block diagonally dominant. \(2 \times 2\) diagonal blocks will have different \((2, 2)\) elements depending on whether they correspond to measured or zero injections. Measured injections will give 1 in the \((2, 2)\) position, and zero injections will give 0.

### 7.6 Transformer Tap Estimation and Matrix Ordering

Although [122] mentioned the possibility of including the estimation of transformer tap ratios in the formulation described, no results were presented. This section describes the addition of transformer tap estimation to the blocked formulation and discusses some of the further issues then raised.

Transformer tap estimation is most conveniently carried out by adding the set of tap ratios to be estimated to the state vector. In the blocked augmented matrix method, the active and reactive power flows associated with the transformers must be available. They can be included either in \(H_B\) or \(H_A\). In the former case, a \(1 \times 1\) diagonal sub-block will result corresponding to the row associated with the tap ratio to be estimated. With the transformer flow measurements included in \(H_A\), they can be paired with the transformer tap ratio in the manner in which injection measurements are paired with states, described above.

In common with other positive definite sets of sparse linear equations, solution speed can be greatly increased by careful ordering of the matrix. In the blocked augmented method, the blocks are re-ordered, but positive definite factorisation without the need for pivot testing can only be guaranteed by the partitioning of the matrix into paired states and unpaired with the paired rows eliminated first [122]. This places a constraint on
the re-ordering possible. Work in this project has been done in re-ordering each of the partitions separately using 'Minimum Degree/MNP' ordering [126]. The topologies of the blocks place no constraints on how the blocks are ordered within each partition i.e. there is no reason why $4 \times 4$ pivots should be ordered before $3 \times 3$ or $2 \times 2$ (a proof is given in [122]), with alteration of pivots in the second partition during the elimination of the first partition guaranteeing non-singular pivots in all cases tested.

The ordering algorithm chosen (which gives a minimum number of fill-ins, an important consideration where some of the benefit of sparsity is lost by the storage of zero elements within blocks) shows that the transformer tap states, which only have two off-diagonal blocks, are likely to be ordered near the beginning of the relevant partition. It follows that the total number of fill-ins during the factorisation of the matrix is likely to be reduced with the transformer tap ratios included in the first, paired partition. This suggests that the power flows through the transformers should be included in $H_A$. Results for different ordering schemes are given in section 7.9.

### 7.7 Fast Decoupled Formulations

It is found in power system analysis that active powers and voltage magnitudes are loosely coupled, that is active power flows or injections are only very loosely dependent on $V$. Similarly, reactive powers and voltage angles are only very loosely coupled. This realisation has been made use of in separating $P-\theta$ equations from $Q-V$ equations in network analysis [19]. Such decoupled solutions, which tend to be considerably faster than full ones, are much used in load flow solution. A natural development of the idea was its application to state estimation.

It is found in the literature that there are three basic approaches to fast decoupled state estimation: algorithm decoupled [127, 128, 129]; model decoupled [128, 129]; and what might be described as two-step pseudo-decoupling [130].

All of these approaches may be implemented with any of a set of standard approximations, the most common of which is the use of a constant (decoupled) gain matrix. This
is evaluated at a ‘flat start’ i.e. voltage magnitudes all equal to 1 p.u. and voltage angles equal to 0. Much time in the solution is saved by negating the need to compute and re-factorise the gain matrix at each iteration. According to [129], the model decoupled approach is the most reliable.

**7.7.1 Model Decoupled Method**

When applied to any state estimation formulation which includes equality constraints, such as Hachtel’s method, the model decoupled approach requires decoupling of the Jacobians $H(x)$ and $C(x)$ and the equations $h(x)$ and $c(x)$ partitioned into those corresponding to real and reactive powers. With $H_{PV}, H_{Q\theta}, C_{PV}$ and $C_{Q\theta}$ set to zero, equation (7.21) can be re-expressed in decoupled form

$$
\begin{bmatrix}
0 & H_{Pz}(\hat{z}^k) & C_{Pz}(\hat{z}^k) \\
H_{Pz}(\hat{z}^k) & W_p & 0 \\
C_{Pz}(\hat{z}^k) & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \hat{\theta} \\
\mu_{z}^{k+1} \\
\lambda_{z}^{k+1}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
\Delta z_p^k \\
-c_P(\hat{z}^k)
\end{bmatrix}
$$

(7.38)

$$
\begin{bmatrix}
0 & H_{Qz}(\hat{z}^k) & C_{Qz}(\hat{z}^k) \\
H_{Qz}(\hat{z}^k) & W_Q & 0 \\
C_{Qz}(\hat{z}^k) & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \hat{V} \\
\mu_{z}^{k+1} \\
\lambda_{z}^{k+1}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
\Delta z_Q^k \\
-c_Q(\hat{z}^k)
\end{bmatrix}
$$

(7.39)

**7.8 Observability Analysis**

The first function of the state estimator is to establish whether the power network being analysed is in fact observable given the set of measurements available. This function is known as observability analysis. Since the inception of state estimator technology and its formulation as a weighted least squares problem, observability has been characterised as depending on the rank of the measurement Jacobian, $H(x)$, which needs to be full. Researchers have attempted to check this in one of two ways: by numerical algorithms or by topological algorithms.

In 1980, Krumpholz, Clements and Davis proposed a topological algorithm [125] that not only determined whether a network was observable or not, but also determined which parts of it were unobservable. In 1991, Nucera and Gilles published work on
observability that claimed to have near-optimized the topological algorithm and which was considerably faster than the numerical approach [124]. Since a topological observability algorithm forms the basis of the blocked augmented matrix state estimation formulation, the method takes on extra significance. A newer version of the approach has been presented in [131]. This chapter offers a further variation which elegantly utilises a simple expert system.

The algorithm tries to find a directed graph based on the lines and measurements in the system which connects as many busbars as possible. The final directed graph will not, in general, be unique, but once it is found, the observable buses are known. The direction of each edge of the graph and which line of the system it lies on is determined by the busbar injection measurements, each of which may only be associated with one edge.

Since flow measurements are themselves unique to one line, they are assigned first and the direction of the edge is unimportant. The assignment of injections is more problematic since for each measurement there is usually more than one line to which the directed edge can be assigned.

Each bus with an injection measurement (including zero injections) is visited in turn. A directed edge lying along a line of the power system must be assigned for each. For example, in the 8 bus example system shown in figure 7.2, bus 1 has lines connecting it with 2, 4 and 6. The directed edge to which it is assigned could lie along 1-2 in the direction of 2, 1-4 in the direction of 4 or 1-6 in the direction of 6. Once one of those has been chosen, the algorithm moves on to the next unassigned bus. The difficulty is in deciding which edge to choose such that no loops are created. A sequence of actions, assigning or de-assigning edges is derived and then implemented when appropriate. It is found that three simple rules suffice.

1. IF the chosen edge does not form a loop
   THEN add the assignment of that edge to the sequence
   AND implement the sequence.

2. IF the chosen edge forms a loop
THEN choose another edge.

3. IF the chosen edge $E$ forms a loop
   AND no other edges remain for that bus
   AND $E$ does not lead to another bus at one end of any edge already in the sequence
   AND there is an injection bus $B_l$ in the loop not involved in the current sequence
   THEN add the assignment of $E$ to the sequence
   AND add the de-assignment of $B_l$ to the sequence
   AND re-assign $B_l$.

This procedure is best illustrated by example. Consider the eight bus network shown in figure 7.2. The lines 1-4, 2-3 and 4-5 are included in the graph first as they have flow measurements. The injection buses to be assigned are buses 1, 2 and 8. Bus 1 is considered first, and line 1-2 is arbitrarily chosen for edge 1 → 2. The assignment of edge 1 → 2 is added to the current sequence, and by rule 1 the sequence is implemented. The graph at this stage is as shown in figure 7.3.

![Figure 7.2: 8 bus example system](image)

Next, bus 2 is considered. Edge 2 → 1 is considered but rejected since it forms a loop. Rule 2 says that another edge must be considered, but the only other one available is 2 → 4, and this, too, forms a loop. Rule 3 now dictates that assignment of edge 2 → 4 is added to the sequence. This is the first action in the sequence. The second must be the de-assignment of a directed edge in the loop whose root is not already in the sequence. This results in the de-assignment of edge 1 → 2 being added to the sequence. Bus 1
must now be re-assigned. Edge 1 \rightarrow 2 fails on three counts: it forms a loop; it leads to a bus already featured in the sequence (bus 2); and edge 1 \rightarrow 2 has already been in the sequence through its de-assignment. Edge 1 \rightarrow 6 is therefore considered. This does not form a loop so its assignment is added to the sequence, which, according to rule 1, is then implemented, i.e. 2 \rightarrow 4 is assigned, 1 \rightarrow 2 is de-assigned and 1 \rightarrow 6 is assigned. The forest at this stage is as shown in figure 7.4.

Finally, bus 8 is assigned. Edge 8 \rightarrow 6 is arbitrarily selected, and rule 1 allows its assignment. The final forest is shown in figure 7.5.

The maximal forest can be used then to identify the observable subnetworks. The rule for determining whether a bus is observable states that an assigned injection associated
with a bus that has at least one incident line not in the span of the forest cannot be used in the solution and must be removed. This implies that all the buses with injections must be revisisted to check if any of them have connections which such lines.

In the example under consideration, the forest of figure 7.5 is compared with the original network shown in figure 7.2. It can be seen that there is a line linking nodes 8 and 7 where node 7 is outside the maximal forest. Injection bus 8 must be de-assigned. Hence, nodes 8 and 7 are unobservable. They do not themselves form a separate observable island because of lines from those nodes to observable nodes. The observable island is that shown in figure 7.6.
State Estimation

<table>
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<tr>
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<th>m20b60</th>
<th>b811</th>
</tr>
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<td></td>
<td>Flows</td>
<td>46</td>
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</tr>
</tbody>
</table>

Table 7.1: Test systems

7.9 Results

Five methods, Hachtel's method and 4 blocked methods, fast decoupled (blocked method A), fully coupled without transformer tap estimation (B), and fully coupled with transformer tap estimation and transformer power flows included in $H_B$ (C) and transformer flows in $H_A$ (method D) were tested on SCADA measurements modelled on a real-time power system simulator [31]. The actual states from the original simulation were available for comparison with the states derived by the state estimators and enabled the recording of $V$ and $\theta$ r.m.s errors in table 7.2.

The test systems are those listed in table 7.1. ieee57 is the IEEE 57 bus test system, m20b60 is a 60 bus reduction of the U.K. national grid, and b811 is an 811 bus reduction of the U.K. national grid. The solution for b811 with transformer flows in $H_B$ was terminated after the generation of over 100,000 fill-in elements. The tests were performed on a Silicon Graphics Iris Indigo R4000 SPECmark 60 machine. It should be noted that the matrix solution routines for Hachtel's method were those given in [132] while those which did not require pivot testing were specially formulated.

7.9.1 Discussion of results

The results for the IEEE 57 bus test system show all the formulations' power, while those for b811 demonstrate that a large system can be feasibly solved accurately and quickly. The addition of 453 extra states through the estimation of transformer tap ra-
tios undoubtedly puts considerable extra demand upon the solution method, but with transformer power flow measurements included in the unsquared part of the gain matrix, the result is thought to be acceptable, particularly when recognizing the unusual and potentially problematic features of a numerically reduced network with lines with very different $X/R$ ratios in close proximity. Such line characteristics have meant that the decoupled approach is unreliable. The coupled blocked hybrid method should be suitable for further development with the addition of a good data filter.

7.10 Summary

This chapter has looked at the state estimation process which assembles data received from a SCADA system into a vector of power system states. A simple new expert system has been developed which identifies observable regions of the power system based on a topological algorithm. Two robust state estimation formulations which overcome the problem of ill-conditioned gain matrices have been examined, implemented and compared. The effects of matrix re-ordering have been addressed and the estimation of transformer tap ratios newly added to the formulation. Results have been presented for three test systems, the IEEE 57 bus test system, a 60 bus reduction of the UK national grid system, and a 811 bus reduction of the UK system.
Dispatch by Linear Programming

Dispatch primarily concerns the scheduling of controls in order to meet active and reactive demand and losses most cost-effectively. It may also be formulated to do so while respecting security limits such as upper and lower limits on nodal voltage magnitudes and loading of transmission plant, in which case it is often described as 'security-constrained' dispatch. This latter form will henceforth be the concern of this thesis and will be termed simply as 'dispatch'.

The well-known principle under which the active and reactive sub-systems of a large inter-connected power system are regarded as being decoupled, much used for the fast solution of load flow problems, is also often applied to dispatch. Active dispatch concerns the scheduling of power system MW generation in order to maintain all line and transformer power flows within pre-specified limits which are most usually determined by the thermal capacity of the transmission plant but may instead be set lower for transient, steady-state or voltage stability [11]. Reactive dispatch concerns the setting of bus-bar voltage set-points, transformer tap ratios and shunt susceptances to maintain nodal voltage magnitudes within limits given constraints on generation and control plant.

Since the analysis can be carried out on a linearised model of the power system, linear programming (LP) is one method by which the dispatch can be done, particularly as it offers the facility of maximizing or minimizing some other objective. It is a method that has been extensively used in the past by power system engineers [133, 134] and can be regarded as a benchmark technique.
This chapter describes separate LP formulations implemented for active and reactive dispatch in order that they can be compared with other approaches. Only the derivation of corrective actions for restoring load bus voltages to within limits and alleviating line overloads are considered, though application to preventative actions is discussed briefly in section 8.4.2. Results are given in chapter 11.

8.1 Introduction to Linear Programming

An optimization problem that is suitable for solution by linear programming will generally be of a form where there are \( N \) independent variables \( x_1, \ldots, x_N \) and where the function to be \textit{maximized}, \( z \), is a linear combination of the \( N \) variables such that

\[
z = a_{01}x_1 + a_{02}x_2 + \cdots + a_{0N}x_N \tag{8.1}
\]

There is a set of primary constraints where

\[
x_1 \geq 0, x_2 \geq 0, \ldots, x_N \geq 0 \tag{8.2}
\]

The problem can then be described with respect to a further set of \( M = m_1 + m_2 + m_3 \) constraints of the form

\[
a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{iN}x_N \leq b_i \tag{8.3}
\]

for \( b_i \geq 0 \) and \( i = 1, \ldots, m_1 \),

\[
a_{j1}x_1 + a_{j2}x_2 + \cdots + a_{jN}x_N \geq b_j \geq 0 \tag{8.4}
\]

for \( j = m_1 + 1, \ldots, m_1 + m_2 \) and

\[
a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kN}x_N = b_k \geq 0 \tag{8.5}
\]

for \( k = m_1 + m_2 + 1, \ldots, m_1 + m_2 + m_3 \).

\( M \) can be greater than, less than or equal to \( N \) and the various \( a_{ij} \)'s need not be non-negative or non-zero.
The solution process, illustrated in figure 8.1, involves first finding a set of values for $x_1, \ldots, x_N$ that satisfies all the constraints. This is the feasible basic vector and it is then optimized by moving the $x_n$ with the most negative co-efficient in the objective function until it hits a constraint limit (effectively moving a variable along a constraint vector), and then moving the others until the optimal feasible vector is found. The optimal feasible vector could fail to be found either because there are no feasible vectors or because the set of constraints fails to bound one or more of the variables (thereby permitting them to increase to infinity).

Since a number of standard linear programming methods exist, of which the most established is the simplex method [135], all an engineer needs to do to make use of them is describe the problem in such a way that the objective function is linear and should be maximized, the variables are all non-negative and the constraints are all linear and non-negative. The next two sections describe the formulations adopted for the reactive and active dispatch sub-problems.
8.2 Reactive Dispatch Formulation

This concerns the search for an optimal re-scheduling of MVAr sources to restore system nodal voltages to within pre-determined limits [136].

8.2.1 Variables and constraints

Since all the variables must be non-negative, the whole set of control variables for the \( N_g \) generating sets, \( N_v \) static voltage compensators (SVCs), \( N_b \) mechanically switched capacitors (MSCs) and \( N_t \) variable tap transformers must be resolved into variables for positive changes and negative changes so that

\[
\begin{align*}
\Delta V_i &= \Delta V_i^+ - \Delta V_i^- \quad \text{for } i = 1 \text{ to } N_g \\
\Delta V_j &= \Delta V_j^+ - \Delta V_j^- \quad \text{for } j = 1 \text{ to } N_v \\
\Delta B_k &= \Delta B_k^+ - \Delta B_k^- \quad \text{for } k = 1 \text{ to } N_b \\
\Delta t_l &= \Delta t_l^+ - \Delta t_l^- \quad \text{for } l = 1 \text{ to } N_t
\end{align*}
\]

where all \( \Delta V_i^+, \Delta V_i^-, \Delta V_j^+, \Delta V_j^-, \Delta B_k^+, \Delta B_k^-, \Delta t_l^+ \) and \( \Delta t_l^- \) are non-negative.

The constraints on the \( m = 1 \ldots N_t \) load bus voltages are of the form

\[
\begin{align*}
\Delta V_m^\text{max} &\geq N_g + N_v \sum_{i=1}^{N_g} S_{Q_m}(\Delta V_i^+ - \Delta V_i^-) + \sum_{k=1}^{N_b} S_{Q_mk}(\Delta B_k^+ - \Delta B_k^-) + \sum_{l=1}^{N_t} S_{Q_ml}(\Delta t_l^+ - \Delta t_l^-) \quad (8.10) \\
-\Delta V_m^\text{min} &\geq N_g + N_v \sum_{i=1}^{N_g} S_{Q_m}(\Delta V_i^- - \Delta V_i^+) + \sum_{k=1}^{N_b} S_{Q_mk}(\Delta B_k^- - \Delta B_k^+) + \sum_{l=1}^{N_t} S_{Q_ml}(\Delta t_l^- - \Delta t_l^+) \quad (8.11)
\end{align*}
\]

where \( S_{Q_mj} \) is the sensitivity of the voltage at bus \( m \) to a change in the \( j \)th control,

\[
\Delta V_m^\text{max} = V_m^\text{max} - V_m^0 \quad \text{and} \quad \Delta V_m^\text{min} = V_m^\text{min} - V_m^0.
\]

If \( V_m^0 > V_m^\text{max} \), equation (8.10) is replaced by

\[
\begin{align*}
-\Delta V_m^\text{max} &\geq N_g + N_v \sum_{i=1}^{N_g} S_{Q_m}(\Delta V_i^- - \Delta V_i^+) + \sum_{k=1}^{N_b} S_{Q_mk}(\Delta B_k^- - \Delta B_k^+) + \\
\end{align*}
\]
\[ \sum_{i=1}^{N_t} S_{Q_m t} (\Delta t_i^- - \Delta t_i^+) \] (8.12)

and if \( V_m^0 < V^\text{min} \), equation (8.11) is replaced by

\[ \Delta V_m^{\text{min}} \geq \sum_{i=1}^{N_g + N_v} S_{Q_m i} (\Delta V_i^+ - \Delta V_i^-) + \sum_{k=1}^{N_c} S_{Q_m k} (\Delta B_k^+ - \Delta B_k^-) + \sum_{i=1}^{N_c} S_{Q_m t} (\Delta t_i^+ - \Delta t_i^-) \] (8.13)

The control constraints are

\[ \Delta V_i^- - \Delta V_i^+ \leq -\Delta V_i^{\text{min}} \quad (-\Delta V_i^{\text{min}} \geq 0) \] (8.14)
\[ \Delta V_i^+ - \Delta V_i^- \leq \Delta V_i^{\text{max}} \quad (\Delta V_i^{\text{max}} \geq 0) \] (8.15)

for all the \( N_g + N_v \) voltage control set-points where \( \Delta V_i^{\text{min}} = V_i^{\text{min}} - V_i^0 \) and \( \Delta V_i^{\text{max}} = V_i^{\text{max}} - V_i^0 \),

\[ \Delta B_k^- - \Delta B_k^+ \leq -\Delta B_k^{\text{min}} \quad (-\Delta B_k^{\text{min}} \geq 0) \] (8.16)
\[ \Delta B_k^+ - \Delta B_k^- \leq \Delta B_k^{\text{max}} \quad (\Delta B_k^{\text{max}} \geq 0) \] (8.17)

for the \( N_c \) banks of capacitors where \( \Delta B_k^{\text{min}} = B_k^{\text{min}} - B_k^0 \) and \( \Delta B_k^{\text{max}} = B_k^{\text{max}} - B_k^0 \), and

\[ \Delta t_i^- - \Delta t_i^+ \leq -\Delta t_i^{\text{min}} \quad (-\Delta t_i^{\text{min}} \geq 0) \] (8.18)
\[ \Delta t_i^+ - \Delta t_i^- \leq \Delta t_i^{\text{max}} \quad (\Delta t_i^{\text{max}} \geq 0) \] (8.19)

for the \( N_t \) transformers where \( \Delta t_i^{\text{min}} = t_i^{\text{min}} - t_i^0 \) and \( \Delta t_i^{\text{max}} = t_i^{\text{max}} - t_i^0 \).

In addition, the voltage controlled reactive sources are limited by maximum and minimum reactive power generation such that

\[ \Delta Q_i^- - \Delta Q_i^+ \leq -\Delta Q_i^{\text{min}} \quad (-\Delta Q_i^{\text{min}} \geq 0) \] (8.20)
\[ \Delta Q_i^+ - \Delta Q_i^- \leq \Delta Q_i^{\text{max}} \quad (\Delta Q_i^{\text{max}} \geq 0) \] (8.21)

The changes in reactive generations \( \Delta Q_i \) are related to the changes in voltage set points through

\[ \Delta Q_i = S_{Q V} \Delta V_i \] (8.22)

where \( S_{Q V} \) is the sensitivity of a change in reactive generation to a change in voltage at node \( i \) found from equation (6.61).
8.2.2 Objective function

The objective function adopted in this study is the minimization of the weighted sum of the control moves and the cost. Since only the variable costs need be modelled in the LP formulation, and the cost of reactive generation from a power station is the only variable cost, the function to be minimized is

\[ z_Q = \sum_{i=1}^{N_g} W_i |\Delta U_i^+ - \Delta U_i^-| + \sum_{j=1}^{N_g} f_c (\Delta V_j^+ - \Delta V_j^-) \]  

(8.23)

where \( \Delta U \) is a change of \( V, B \) or \( t \) and \( W_i \) is a weighting. \( f_c \) is the cost of a change of voltage reference.

Cost of reactive compensation

Reactive power is not currently included in the U.K.'s pool pricing system, so on the basis of British practice, marginal prices of reactive compensation need not be modelled. However, a number of proposals are under discussion which may be implemented in the near future [11].

In a low load scenario, not all generating plant will be operating. On occasions, some plant will be ordered on for voltage support, and the cost in this situation will be readily available as that of the minimum stable MW generation i.e. the cost of coming onto the busbar. Some plant on the National Grid is under contract to provide whatever reactive generation is requested between pre-defined leading and lagging power factors if it is already generating MWs.

New proposals for costing of voltage support on the National Grid include

- payment for MVAr hours produced by generating units at a simple fixed rate per MVAr hour;
- payment for bidded lagging and leading reactive capability.

Other possible methods of costing reactive generation are
on a basis of what equivalent compensation would cost to install

according to an 'opportunity cost' proportional to the reduction in MW capability
ensuing from the change in reactive generation.

The costs of changing transformer tap ratios, switching of shunt capacitors and changing
of set points on static compensation may be determined based on some reflection of de­
preciation, the cost of implementing the change, or as having zero cost. Alternatively,
the cost of using an SVC may be modelled as reflecting the MW loss incurred by a given
MVAr output of the device, such as shown in figure 8.2 where the cost is directly pro­
portional to the loss. Modelling of the cost in this way requires the addition of two extra
variables to the linear programming formulation for each SVC. \( \Delta V^+0 \) describes the in­
crease in the voltage magnitude of the voltage reference setting which is equivalent to
the MVAr output being increased to zero; \( \Delta V^{++} \) corresponds to the increase in voltage
setting equivalent to an increase in MVAr output above zero; \( \Delta V^-0 \) is the change in
voltage setting which brings about a decrease in MVAr output down to zero; and \( \Delta V^{--} \)
is the decrease in MVAr output down from zero.

\[
\begin{align*}
\Delta V^{min} & \leq \Delta V^{++} + \Delta V^+0 - \Delta V^-0 - \Delta V^{--} \leq \Delta V^{max}
\end{align*}
\]  

Figure 8.2: Cost of reactive compensation

Under different circumstances, these variables will have different constraints. For ex­
ample, if the current MVAr output \( Q^0 \) is positive, then \( \Delta V^+0 \) is constrained as being
equal to zero and \( \Delta V^-0 \) must be less than or equal to \( Q^0/SqV \). If the current MVAr out­
put is \( -Q^0 \), then \( \Delta V^+0 \) must be less than or equal to \( Q^0/SqV \) and \( \Delta V^-0 \) is constrained
to zero. As in the original formulation,
With reactive generation modelled as having a cost of $C_g$ per MVAr, SVC reactive output above zero having a cost of $C^+_v$ per MVAr and $C^-_v$ where $C^-_v$ is less than zero per MVAr less than zero, and transformer and shunt susceptance changes having zero cost, the objective function to be minimized will be of the form

$$z_Q = \sum_{i=1}^{N_t} W_i(\Delta V_i^+ + \Delta V_i^-) + C_g(\Delta V_i^+ - \Delta V_i^-)$$

$$+ \sum_{j=1}^{N_u} (C^+_v + W_j)\Delta V_j^{++} + (C^-_v + W_j)\Delta V_j^{+0}$$

$$- (C^+_v - W_j)\Delta V_j^{-0} - (C^-_v - W_j)\Delta V_j^{-+}$$

$$+ \sum_{k=1}^{N_t} W_k(\Delta B_k^+ + \Delta B_k^-)$$

$$+ \sum_{l=1}^{N_t} W_l(\Delta t_l^+ + \Delta t_l^-)$$

(8.25)

The function to be maximized is $z'_Q = -z_Q$.

### 8.2.3 Implementation

The optimization was performed using a standard simplex routine [132]. The number of variables $N$ was $2 \times (N_g + N_b + N_t) + 4 \times N_v$. The number of constraints, $M$, equalled $2 \times (N_{\bar{b}} + N_g + N_b + N_t) + 4 \times N_v$ where $N_{\bar{b}}$ is the total number of busbars in the system.

### 8.3 Active Dispatch Formulation

The aim is to find an optimal combination of control actions, i.e. the MW output of available generation and the angles of quadrature boosters, to relieve plant thermal overloads.

#### 8.3.1 Variables and constraints

As given in [133], the set of control variables for the $n_c$ controllers $\Delta U_1 \ldots \Delta U_{n_c}$ is split into those for positive changes in control setting $\Delta U_i^+$ and negative changes $\Delta U_i^-$ such
that

\[ \Delta U_i = \Delta U_i^+ - \Delta U_i^- \]  

(8.26)

where all \( \Delta U_i^+ \geq 0 \) and all \( \Delta U_i^- \geq 0 \). In practice, the \( n \)-th controller is not available for control as it is the swing bus.

The generation constraints are described as

\[ \Delta P_i^- - \Delta P_i^+ \leq -\Delta P_i^{\text{min}} \quad (-\Delta P_i^{\text{min}} \geq 0) \]  

(8.27)

\[ \Delta P_i^+ - \Delta P_i^- \leq \Delta P_i^{\text{max}} \quad (\Delta P_i^{\text{max}} \geq 0) \]  

(8.28)

where \( \Delta P_i^{\text{min}} = P_i^{\text{min}} - P_i^0 \), \( \Delta P_i^{\text{max}} = P_i^{\text{max}} - P_i^0 \) and \( P_i^{\text{min}} \) is the minimum MW output from the \( i \)-th generator, \( P_i^{\text{max}} \) is the maximum MW output from the \( i \)-th generator and \( P_i^0 \) is the current output.

The constraints on quadrature booster angles are

\[ \Delta \phi_i^- - \Delta \phi_i^+ \leq -\Delta \phi_i^{\text{min}} \quad (-\Delta \phi_i^{\text{min}} \geq 0) \]  

(8.29)

\[ \Delta \phi_i^+ - \Delta \phi_i^- \leq \Delta \phi_i^{\text{max}} \quad (\Delta \phi_i^{\text{max}} \geq 0) \]  

(8.30)

where \( \Delta \phi_i^{\text{min}} = \phi_i^{\text{min}} - \phi_i^0 \), \( \Delta \phi_i^{\text{max}} = \phi_i^{\text{max}} - \phi_i^0 \) and \( \phi_i^{\text{min}} \) is the minimum angle at the \( i \)-th quadrature booster, \( \phi_i^{\text{max}} \) is the maximum angle from the \( i \)-th quadrature booster and \( \phi_i^0 \) is the current angle.

The transmission constraints are of the form

\[ \Delta P_{km} \leq P_{km}^{\text{max}} - P_{km}^0 \quad (P_{km}^0 \leq P_{km}^{\text{max}}) \]  

(8.31)

\[ \Delta P_{km} \geq P_{km}^0 - P_{km}^{\text{min}} \quad (P_{km}^{\text{min}} \leq P_{km}^0) \]  

(8.32)

\[ \Delta P_{mk} \leq P_{mk}^{\text{max}} - P_{mk}^0 \quad (P_{mk}^0 \leq P_{mk}^{\text{max}}) \]  

(8.33)

\[ \Delta P_{mk} \geq P_{mk}^0 - P_{mk}^{\text{min}} \quad (P_{mk}^{\text{min}} \leq P_{mk}^0) \]  

(8.34)

For the small change model, \( \Delta P_{km} = -\Delta P_{mk} \). Since a maximum power flow is readily obtainable from \( P_{km}^{\text{max}} = \sqrt{R_{km}^2 - Q_{km}^2} \) where \( R_{km} \) is the rated MVA limit and \( Q_{km} \) is the current MVAr flow, it is convenient to reduce the set of constraints in equations
(8.31)–(8.34) to

\[ \Delta P_{km} \leq P_{km}^{\text{max}} - P_{km}^0 \quad (P_{km}^0 \leq P_{km}^{\text{max}}) \] (8.35)

\[ -\Delta P_{km} \leq P_{mk}^{\text{max}} - P_{mk}^0 \quad (P_{mk}^0 \leq P_{mk}^{\text{max}}) \] (8.36)

To maintain the non-negativity of the constraint, equation (8.35) should be replaced by

\[ -\Delta P_{km} \geq P_{km}^0 - P_{km}^{\text{max}} \] (8.37)

when \( P_{km}^0 > P_{km}^{\text{max}} \) and equation (8.36) replaced by

\[ \Delta P_{km} \geq P_{mk}^0 - P_{mk}^{\text{max}} \] (8.38)

when \( P_{mk}^0 > P_{mk}^{\text{max}} \).

The solution routine requires that each \( \Delta P_{km} \) is expressed in terms of \( \Delta U_1^+, \ldots, \Delta U_{nc}^+, \Delta U_1^-, \ldots, \Delta U_{nc}^- \). This is achieved by use of the sensitivity matrix described in section 6.3.2 since, for the linearised system,

\[ \Delta P_{km} = \sum_{i=1}^{nc} S_{kmi}(\Delta U_i^+ - \Delta U_i^-) \] (8.39)

where \( S_{kmi} \) is sensitivity of a change in \( P_{km} \) to a change in control setting \( \Delta U_i \).

### 8.3.2 Balancing of load and generation

When control actions are applied on a system with \( nc \) active power sources with the \( nc \)-th bus given as the swing bus, there will be some change in the net active generation \( \sum_{j=1}^{nc-1} \Delta P_j \) which on the load-flow would be taken up by the swing bus. In the LP formulation, it would be possible to define a further constraint where the sum of all the suggested changes to MW outputs must equal zero in order to maintain the balance of generation and load with the swing bus left to only pick up any change in system losses. However, it may be desirable for this change to be compensated for by machines across the power system on free governor operation. Such an approach follows that of Ekwue [137] where corrections are applied after the set of control moves has been derived which
are distributed across the set of generators in proportion to individual machines' MVA ratings. This could be thought of as mimicking free governor responses.

If the $j$th generator has a change in MW output of $\Delta P_j$, a total correction $\Delta P_c$ should be applied to re-impose power balance, and this should be

$$\Delta P_c = - \sum_{j=1}^{n_g-1} \Delta P_j$$

(8.40)

where $n_g$ is the number of generators.

The individual corrections are therefore

$$\Delta P_j = \frac{\Delta P_c R_j}{\sum_{i=1}^{n_g} R_i} \text{ for } j = 1 \text{ to } n_g$$

(8.41)

where $R_j$ is the rated MVA of the $j$th generator and the corrections are applied to all the generators, including the slack bus.

Such an application of corrections will mean that the profile of active power injections, and therefore of active power flows, will not be as predicted by the LP routine. If any violations remain, the routine should be run again.

8.3.3 Objective function

The objective function for the formulation used in this study is defined so as to minimize the extent of the control moves suggested to relieve the violated security constraints and the cost of doing so. This leads to a minimization of the form

$$z_P = \sum_{i=1}^{n_c} W_i |\Delta U_i| + f_c(\Delta U_i)$$

(8.42)

where $W_i$ is a weighting factor and $f_c(\Delta U_i)$ is the cost of a change in control setting $U_i$.

With each $\Delta U_i$ resolved into $\Delta U_i^+$ and $\Delta U_i^-$, $z_P$ is re-written as

$$z_P = \sum_{i=1}^{n_c} W_i (\Delta U_i^+ + \Delta U_i^-) + f_c(\Delta U_i)$$

(8.43)
Modelling of cost

The cost function used for generation is the much simplified piece-wise linear approximation to a quadratic shown in figure 8.3 [134]. $C_i^+$ is defined as the gradient of the straight line for a non-zero positive change in MW generation $\Delta P_i^+$ (i.e. $\Delta P_i > 0$) and $C_i^-$ is the gradient for a non-zero $\Delta P_i^-$ (i.e. $\Delta P_i < 0$). The fixed cost element can be neglected since for a given scenario the basic variables in the LP solution (i.e. those which can be non-zero) will be fixed, so the contribution to the objective function of that cost element is constant. This means that the choice of the magnitude of each variable to optimize objective function is unaffected by the fixed cost, though the final value of the objective function may be different since some of the basic variables can be zero (hence contributing no cost). $z_P$ is then

$$z_P = \sum_{i=1}^{n_c} W_i (\Delta U_i^+ + \Delta U_i^-) + C_i^+ \Delta U_i^+ + C_i^- \Delta U_i^- \quad (8.44)$$

$C_i^+$ and $C_i^-$ are zero for quadrature boosters.

![Figure 8.3: MW generation cost](image)

The LP routine requires an objective function to be maximized, so $z'_P = -z_P$ is used.

It should be noted that the level of accuracy of the approximation to the quadratic cost curve can have a marked effect on the speed of the optimisation [138].
8.3.4 Implementation

The optimization was performed using a standard simplex routine [132]. The number of variables \( N \) is twice the number of controls, i.e. all the generators except the slack bus plus the number of quadrature boosters, while the total number of constraints \( M \) is twice the number of controllers plus twice the number of lines. There are \( m_1 \) constraints of the form of equation (8.3). \( m_1 \) equals twice the number of controllers plus twice the number of lines less the number of overloaded lines; there are \( m_2 \) constraints of the form of equation (8.4) and \( m_2 \) equals the number of overloaded lines. There are no constraints of the form of equation (8.5).

If no constraint of the form \( \sum_{i=1}^{n} \Delta P_i = 0 \) is used, a correction must be applied to the MW generation after the solution in the manner described in section 8.3.2 in order to maintain the balance of MW generation and load. This requires that the LP routine is performed again as violations may remain after the MW corrections.

8.4 Further Issues in Dispatch by Linear Programming

Although they have not been implemented as part of the research described in this thesis, some other issues in dispatch by linear programming are worthy of some consideration. They are briefly discussed in the next two sections.

8.4.1 Alternative objectives

Alternatives to the objectives of reducing cost and the sum of the absolute control moves outlined above include [134] minimizing the square of the control moves and minimizing active power losses [139]. Any terms which are not linear require piecewise linear approximation. It is shown in [138] that excessive detail in the modelling of quadratic functions can slow the solution down. In [138], different approximations are used at different stages of the process so that the optimum found by the routine becomes progressively closer to the real optimum.
8.4.2 Derivation of preventative actions

As outlined in section 2.4.1, a viable dispatch routine should be capable of deriving preventive as well as corrective actions. The standard procedure in LP-based dispatch involves the execution of a load-flow on all studied contingency cases, the linearisation of each of those cases and an augmentation of the base case set of constraints which is then optimised [138]. This process is shown schematically in figure 8.4.

![Figure 8.4: Optimisation of preventative control by linear programming](image)

The constraint equations to be added include coefficients from the row vectors of the sensitivity matrices found for each post-contingency operating point which correspond to violations [140]. For the voltage dispatch cycle, the added constraints will be of the form of equation (8.12) for over-voltage violations and equation (8.13) for under-voltage violations. This has the effect of limiting the feasible solution space for the base-case.

For the active dispatch cycle, the added post-contingency constraints will be of the form of equation (8.37) or equation (8.38) with the required changes in the branch loading $P_{km}$ expressed in terms of post-contingency sensitivities in the manner of equation (8.39).
Once the LP has reached a solution, a base-case load-flow is updated with the new control values and the contingency analysis repeated to ensure that no new violations have been created. If any have, they provide new constraints to the LP which is repeated on a linearisation of the latest base-case load-flow.

8.5 Summary

This chapter has described the dispatch function in power system operation and has introduced the variables associated with it. It has then described one of the most common means of implementing the dispatch function, linear programming, and outlined the theory behind it.

Two implementations of LP-based dispatch have been described, one for dispatch of reactive power for control of nodal voltage magnitudes, the other for dispatch of active power for control of branch flows.
A number of authors have reported findings from investigations into the applications of expert systems to dispatch. Two such approaches used for reactive dispatch, one using production rules based on a sensitivity tree and the other using fuzzy rules, are described here. They have been implemented so that they can be compared with other methods and have had the facility to schedule transformer tap ratios added. The underlying principles have then been extended to two new expert systems addressing the active dispatch problem.

9.1 Expert Systems for Voltage Control

Before numerical optimisation techniques were widely available for security-constrained re-dispatch, operators chose remedial or preventative actions based on intuitive assessments of a number of criteria. Firstly, each controller’s effect on the voltage to be corrected was considered, namely in which direction the control should be moved and by how much to remove the voltage limit violation (generally, those controls nearest to the target bus will have the greatest effectiveness). Secondly, an estimate had to be made of the amount by which the control should be moved, and a check carried out to ensure that enough control margin existed at that controller for that change.

The problem with this approach, so heavily reliant on operator experience and intuition, was that effective co-ordination of different controllers was very difficult. Controls
sometimes ended up being moved more than they needed to be (with associated costs), or secondary effects may have been caused where in correcting the voltage at one bus, a violation at another may have been created. These difficulties motivated the provision of numerical optimization tools, and the linear programming approach quickly became popular. However, while such techniques do quite reliably provide optimal solutions whenever a solution is available, they do not always find solutions, nor do they always converge within a given time-scale [141]. Some linear programming-based approaches reconfigure the constraints and/or the objective function whenever a solution is not found [138], but this, too, is often slow and relies heavily on efficient programming. In addition, it is reported [142] that “the complexity of optimal power flow (OPF) often discourages the user” who “has to make decisions on the input and output from the OPF” while the “OPF can solve so many different problems that it is often a challenge to focus OPF on the problem of interest”.

The difficulty of defining the problem for some numerical optimisation routine and of guaranteeing a feasible solution has prompted investigation of whether expert systems might be able to reliably model operators’ actions, and be more rigorous than operators. Some investigators have used linear programming methods combined with expert systems, while others have used expert systems exclusively to derive solutions.

Since 1986, expert systems have been extensively applied to the control of the voltages at \( L \) load buses \( i = 1, 2, \ldots, L \) using \( j = 1, 2, \ldots, N \) controls on buses with generation and reactive compensation and on transformers [78, 79, 80, 81, 82, 83]. In restoring violated load bus voltages back to within limits, the first two requirements, to determine the effectiveness of each control upon specified target buses and to estimate how much the control needs to be moved by, are achieved by use of sensitivity analysis, described in section 6.3. The expert system then chooses which controls to move.

Liu and Tomsovic [78] and subsequently Barruncho et al [143] have presented work on a voltage control expert system in which the expert system is used first to determine in which of three levels of insecurity the system and its violations lie within. For the two most severe, a linear programming routine is called. The usual constraints on load bus
voltages and control settings are applied for the less severe scenario with the objective of minimizing controller movement, and only the constraints are applied for the more severe. For the least severe of the three conditions, the expert system itself derives the control action and its movement to remove the single most severe constraint, then moving on to the most severe of any remaining violations.

The expert system of Cheng et al [79] adopts a similar procedure to the expert system dispatch of Liu and Tomsovic, taking the most severe voltage violation first and applying the single most sensitive controller (found from a sensitivity matrix) to try to relieve it without creating new violations. This is achieved by a depth-first search of a sensitivity tree and is described in more detail in the next section.

9.2 Sensitivity Tree Based Reactive Dispatch

An outline of the procedure is shown in figure 9.1. Once the current state of the system has been determined, all the nodal voltage magnitudes are checked for violations and the worst one, $V_i$, is flagged. This determines the starting node in the sensitivity tree (figure 9.2).

From the sensitivity matrix, the most effective available controller is found i.e. that to which the violated load bus voltage is most sensitive. In [79], the sensitivities of different types of controllers are compared directly. This does not, however, give a good indication of the relative efficiencies of different controllers since the control ranges are different. Therefore, in the implementation adopted in this project, the sensitivities are divided by the respective control ranges $U^{max} - U^{min}$.

By finding the most 'efficient' controller from the scaled sensitivity, the expert system moves to the next level of the tree. The control action necessary to correct the violated voltage is determined from the sensitivity matrix and rounded to the nearest valid setpoint. The action is checked against its control limits, maximum and minimum voltage set points and maximum and minimum reactive generation for generation and static compensation, maximum and minimum capacitance for static compensation, and max-
imum and minimum tap ratios for transformers. If a limit is exceeded, the control move is set to the limit. The control move is then tested on every other load bus in the network to ensure that it does not aggravate existing violations or create new ones (this is a search of the bottom level of the tree), and limited further if necessary.

In the implementation used in this project, the control move suggested is applied in a load flow study, the results of which then provide the input data for the expert system to be run again until no further control moves are possible or the violations have been removed.
9.3 Fuzzy Expert System for Reactive Dispatch

Yokoyama et al [82] approached the problem of voltage control from the point of view of operator heuristics which are modelled by means of a set of fuzzy linguistic variables and membership functions, and four fuzzy rules. A number of control settings are moved simultaneously rather than moving them one at a time as in [79]. The outline of the method is illustrated in figure 9.3.
9.3.1 The rule base

The judgements made by the operator are modelled by four linguistic variables, six linguistic values (fuzzy sets) and four rules $\rho = 1 \ldots 4$. The rules are:

1. IF voltage IS low AND sensitivity IS significantly positive AND control margin IS enough to raise THEN raise setting IS $C_{ij} \Delta U_{Rj}^{max}$

2. IF voltage IS high AND sensitivity IS significantly positive AND control margin IS enough to lower THEN lower setting IS $C_{ij} \Delta U_{Lj}^{min}$

3. IF voltage IS low AND sensitivity IS significantly negative AND control margin IS enough to lower THEN lower setting IS $C_{ij} \Delta U_{Lj}^{min}$

4. IF voltage IS high AND sensitivity IS significantly negative AND control margin IS enough to raise THEN raise setting IS $C_{ij} \Delta U_{Rj}^{max}$

The variables voltage, sensitivity and control margin are linguistic variables i.e. they take values which are characterised by fuzzy sets labelled by adjectives. Voltage is the voltage at a load bus $V_i$, control margin is the margin available at a candidate controller compared with the required output $\Delta U_j$ to correct the voltage at the load bus, and sensitivity is the sensitivity of the load bus voltage to a change in the candidate controller setting $S_{ij}$.

The terms low, high, positive, negative, enough to raise and enough to lower, shown in figure 9.4, correspond to fuzzy sets and concern the extent of the voltage violation, the sensitivity of voltage at a load bus to a control and the available control margin. They are defined as trapezoidal functions in the manner shown. The variables lower setting and raise setting denote the suggested negative and positive controller adjustments respectively which must be defuzzified and added together to give a single crisp value for each controller.

The degrees of truth of the fuzzy propositions "voltage IS high", "sensitivity IS significantly positive" and so on are found from fuzzification of the crisp values of $V_i$, $S_{ij}$ and...
In this way, the voltage propositions with high and low do not just give a measure of whether the load bus voltage is violated or not, but give a scaling of the extent of the violation which in turn will set the priority of action for each controller. A larger violation will then be weighted more highly in the derivation of a controller's new setting than a smaller one.

The fuzzy sets for the sensitivity of controller $j$ controlling a violation at bus $i$ are defined with $\min(S^-)$ as the most negative element in the $i$th row of the sensitivity matrix and $\max(S^+)$ as the most positive. Thus, the truth of sensitivity is set according to the effectiveness of the controller rectifying the violation at bus $i$ relative to the other controllers. The deadband is set to eliminate the least effective controllers from the suggested control
strategy with

\[ S_d^+ = a \max(S^+) \quad (9.1) \]
\[ S_d^- = b \min(S^-) \quad (9.2) \]

The scaling factors \( a \) and \( b \) can be adjusted to eliminate a greater or smaller number of controllers.

In contrast to the original implementation in [82], the version implemented in this project is an extended one which has the facility to use variable transformer tap ratios and switchable banks of capacitors as controls. To give properly related sensitivities of different types of control, the sensitivities used in the definitions of significantly positive and significantly negative are scaled by the respective control ranges, as are the sensitivities which are fuzzified.

\( \Delta U_{Lj} \) and \( \Delta U_{Rj} \) in figure 9.4 are the amounts by which the controller setting must be lowered or raised to relieve the violation. \( U_j^{\text{min}} - U^0 \) and \( U_j^{\text{max}} - U^0 \) the margins available at \( j \). The deadband set by \( M_d \) can be adjusted to eliminate trivially small control moves.

### 9.3.2 Inferencing

The degree of truth of each rule's antecedent clause \( x \text{ AND } y \text{ AND } z \) is found by multiplying the degrees of truth of the propositions \( x, y \) and \( z \) together. The implication or inference operation determines the effect of the degree of truth of the antecedent on the consequent. In this case, with the original degree of truth of each consequent proposition set for raise setting and lower setting equal to 1.0, it is given as the product of the degree of truth of raise setting or lower setting and the degree of truth of the antecedent [101].

Each suggested control adjustment is modified by a 'contribution factor' in order to prevent a combination of controllers over-compensating for the same voltage violation. The suggested control adjustment is the margin limited positive control change at controller
for rules 1 and 4 and by the margin limited negative change $\Delta U_{Lj}^{\text{min}}$ to relieve the violation for rules 2 and 3 such that

\[
\begin{align*}
\Delta U_{Rj}^{\text{max}} &= \min(\Delta U_{Rj}, U_j^{\text{max}} - U_j^0) \\
\Delta U_{Lj}^{\text{min}} &= \max(\Delta U_{Lj}, U_j^{\text{min}} - U_j^0)
\end{align*}
\] (9.3) (9.4)

where $\Delta U_{Rj}$ and $\Delta U_{Lj}$ are the positive and negative changes necessary to relieve a voltage violation at busbar $i$, $U_j^{\text{max}}$ and $U_j^{\text{min}}$ are the maximum and minimum control settings and $U_j^0$ is the current control setting. $\Delta U_{Rj}$ and $\Delta U_{Lj}$ are found from the $ij$th element of the sensitivity matrix (described in section 6.3) and the necessary load bus voltage change $\Delta V_i$ such that $\Delta U_{Rj}$ equals $\Delta U_j$ when $\Delta U_j$ is positive and $\Delta U_{Lj}$ equals $\Delta U_j$ when it is negative, where

\[
\Delta U_j = \frac{\Delta V_i}{S_{ij}}
\] (9.5)

and

\[
\Delta V_i = \begin{cases} 
1.05 - V_i^0 & \text{for } \rho = 2 \text{ and } \rho = 4 \\
0.95 - V_i^0 & \text{for } \rho = 1 \text{ and } \rho = 3
\end{cases}
\] (9.6)

where $\rho$ is the rule number and $V_i^0$ is the current voltage at bus $i$ to be corrected.

The contribution factor $C_{ij}$ for controller $j$ applied to a voltage violation at bus $i$ is proportional to the sensitivity of that voltage to an action at that controller and is

\[
C_{ij} = \frac{S_{ij}}{\chi S_{k_j}}
\] (9.7)

where

\[
\chi S_{k_j} = \max(|S_{kj}|) \text{ for } k = 1, 2, \ldots, L
\] (9.8)

### 9.3.3 Output composition

After execution of the rules, there will be finite fuzzy sets assigned to the variables lower setting and raise setting for each controller. These are found from composition of the outputs of each rule.
The method of composition used is the 'sum' [106] method, and the composition process is illustrated in figure 9.5. In the figure, a, b, c and d are the output control signals ΔU assigned for one controller by different rules. The implications of the rules which fired to give these settings are μ(a), μ(b), μ(c) and μ(d). These are combined by taking the piecewise sum of the separate fuzzy functions over the range of ΔU to give the final single membership function for setting shown.

9.3.4 Defuzzification

In order that meaningful signals can be sent to the controllers, the output of the expert system should present a set of crisp control changes, one for each controller. This requires that the fuzzy sets assigned to lower setting and raise setting for each controller are defuzzified to find the crisp control signal ΔUj. The defuzzification method used is the 'centroid' method [106] where, for controller j,

$$\Delta U_j = \frac{\sum_{i=1}^{L} C_{ij}(\alpha_{1ij}\Delta U_{1R}^{max} + \alpha_{2ij}\Delta U_{2L}^{min} + \alpha_{3ij}\Delta U_{3L}^{min} + \alpha_{4ij}\Delta U_{4R}^{max})}{\sum_{i=1}^{L} \sum_{\rho=1}^{4} \alpha_{\rho ij}}$$

(9.9)

and $\alpha_{\rho ij}$ is the implication of the $\rho$th rule executed for control $j$ and a violation at bus $i$, $C_{ij}$ is the contribution factor and $\Delta U_{\rho R}$ and $\Delta U_{\rho L}$ are the raise and lower settings suggested by rule $\rho$ for controller $j$. The defuzzified value is rounded to the nearest valid set-point.
9.4 Active Dispatch

For the purposes of offering reasonable comparisons, the principles behind the expert system approaches to reactive dispatch described above have been adapted for application to the alleviation of transmission overloads. The controls available in this case are the MW outputs of generators on the system and quadrature booster angles. The two new expert systems will now be described.

9.4.1 Sensitivity tree approach

A sensitivity tree similar to that in figure 9.2 is assembled with the top node corresponding to the most overloaded line. The controller to which the power flow down that line is most sensitive is found and the control action necessary to relieve the overload calculated from

\[ \Delta U_j = \frac{\Delta P_{km}}{S_{kmj}} \]  

(9.10)

where \( S_{kmj} \) is sensitivity of the active power entering the line connecting buses \( k \) and \( m \) at bus \( k \) to a change in the setting of the \( j \)th controller. The necessary correction \( \Delta P_{km} \) is

\[ \Delta P_{km} = \sqrt{R_{km}^2 - (Q_{km}^0)^2} - P_{km}^0 \]  

(9.11)

where \( R_{km} \) is the line MVA rating, \( Q_{km}^0 \) is the reactive power currently entering the line at bus \( k \) flowing in the direction of bus \( m \) and \( P_{km}^0 \) is the active power entering the line at bus \( k \).

If any change has been suggested for MW generation, either corrections to generation must be applied in the manner described in section 8.3.2 or some other compensating change in generation must be ordered so that there is a net change of zero in order to maintain the load/generation balance. If the latter approach is used, if the change chosen by the expert system is an increase in generation then a corresponding decrease in generation must be found. This is chosen as a reduction in MWs at the most expensive gen-
erator. If the change chosen by the expert system is negative, the compensation should be positive and it is ordered at the generator at the top of the merit order.

The ordered control change or changes are then checked against the control limits and restricted if necessary, then checked through the next level of the sensitivity tree for whether the control moves aggravate other violations or create new ones. If any violations remain after application of the move, the worst violation is again found and the expert system applied again.

9.4.2 Fuzzy approach

The fuzzy overload alleviation system is based on ideas presented in section 9.3 describing the fuzzy reactive dispatch expert system. It uses the sensitivity analysis described in section 6.3.2.

The modelling of operator heuristics in the re-scheduling of generation to alleviate overloads is achieved through two fuzzy rules with fuzzy sets defined for high loading, significantly positive sensitivity, significantly negative sensitivity, enough margin to raise and enough margin to lower:

1. IF loading IS high AND sensitivity IS significantly positive AND control margin IS enough to lower THEN setting IS $C_{kmj} \Delta U_{l_j}^{min}$

2. IF loading IS high AND sensitivity IS significantly negative AND control margin IS enough to raise THEN setting IS $C_{kmj} \Delta U_{R_j}^{max}$

The sensitivity related terms significantly positive and significantly negative are defined as for the voltage control expert system (see figure 9.4) but with the parameters based upon the active sensitivity matrix. Again, the deadband can be varied to restrict the number of controllers moved. The margin is also defined in a similar way to that for the voltage control system (figure 9.4). The raise and lower control actions $\Delta U_{R_j}$ and $\Delta U_{L_j}$
necessary to relieve the violation in question are

\[ \Delta U_{Lj} = \frac{\Delta P_{km}}{S_{kmj}} \] for rule 1 \hspace{1cm} (9.12) \\
\[ \Delta U_{Rj} = \frac{\Delta P_{km}}{S_{kmj}} \] for rule 2 \hspace{1cm} (9.13) 

where \( \Delta P_{km} \) is the necessary change in active power flow from node \( k \) to node \( m \) found from equation (9.11) and \( S_{kmj} \) is the sensitivity of \( P_{km} \) to a change \( \Delta U_j \). \( \Delta U_{Lj}^{\text{max}} \) and \( \Delta U_{Lj}^{\text{min}} \) are determined from

\[ \Delta U_{Rj}^{\text{max}} = \min(\Delta U_{Rj}, U_j^{\text{max}} - U_j^0) \hspace{1cm} (9.14) \]
\[ \Delta U_{Lj}^{\text{min}} = \max(\Delta U_{Lj}, U_j^{\text{min}} - U_j^0) \hspace{1cm} (9.15) \]

The maximum control setting \( U_j^{\text{max}} \) is given by the maximum stator MVA, maximum generator transformer loading or maximum governor setting, whichever implies the lowest MW generation, for generation, or the maximum angle for a quadrature booster. The minimum setting \( U_j^{\text{min}} \) is determined by the minimum stable MW generation or the minimum booster angle.

The fuzzy set for high loading is shown in figure 9.6. \( P_h \) is some co-efficient chosen by the user.

![Figure 9.6: Linguistic variable relating to loading](image)

To prevent excessive compensation through the combined action of more than one controller, a 'contribution factor' \( C_{kmj} \) is again applied where

\[ C_{kmj} = \frac{S_{kmj}}{\chi S_{akj}} \hspace{1cm} (9.16) \]
where

\[ X_{s_{ab}} = \max (|S_{ab}|) \text{ for all lines } ab \]  

(9.17)

The final crisp control setting \( \Delta U_j \) is found from

\[
\Delta U_j = \frac{\sum_{k=1}^{b} \sum_{m=1}^{b} C_{kmj} (\alpha_{1kmj} \Delta U_{1l_{kj}}^{\text{max}} + \alpha_{2kmj} \Delta U_{2l_{kj}}^{\text{min}}) \exists l_{km}}{\sum_{k=1}^{b} \sum_{m=1}^{b} (\alpha_{1kmj} + \alpha_{2kmj}) \exists l_{km}}
\]

(9.18)

where \( \alpha_{p_{ij}} \) is the implication of the \( p \)th rule executed for control \( j \) and a violation on line \( l_{km} \), \( C_{kmj} \) is the contribution factor, \( \Delta U_{p_{R}} \) and \( \Delta U_{p_{L}} \) are the raise and lower settings suggested by rule \( p \) for controller \( j \) and \( b \) is the number of busbars. The defuzzified value is rounded to the nearest valid set-point.

After the selection of the control actions, corrections must be applied in the manner described in section 8.3.2 in order to maintain the load/generation balance.

### 9.5 Summary

This chapter has introduced the application of expert systems to security-constrained dispatch. Two such expert systems for reactive dispatch, one based on a ‘sensitivity tree’ [79] and the other on fuzzy rules [82], have been described with the facility to set transformer tap ratios added. Sensitivities have also been scaled by control ranges to better reflect relative efficiencies.

The basic principles of each expert system have then been developed for application in two new MW dispatch expert systems for alleviation of overloads.
The linear programming approach to dispatch outlined in chapter 8 has been in use for a number of years, but the difficulty of defining the objective function for a range of objectives and of ensuring that a solution is reached led to the research into the use of expert systems that is described in chapter 9. These approaches, too, have some disadvantages.

The sensitivity tree based expert system of section 9.2 does not optimize the solution reached, and tends to use up the control margin at one controller before moving on to another leaving the system in the vicinity of that controller vulnerable to voltage instability due to shortage of reactive reserve. This strategy is also reported to lead to poor voltage profiles and higher losses [144]. As it only addresses one violation and one controller at a time, a large number of iterations may be needed. In addition [144], no guidance is given in [79] on how to compare sensitivities of different types of control device.

The disadvantage of the fuzzy expert system described in section 9.3 is that it recommends adjustments on a large number of controllers, although this effect can be reduced by adjustment of the sensitivity and margin deadbands so that only effective controllers are moved and they are not moved by trivially small amounts. In addition, the use of ‘contribution factors’ to ensure that violations are not over-corrected turns out to be fairly crude as the possibility of aggravating existing violations or creating new ones remains, though it is less. It is also possible that controls actions are reduced by too much so that the violations are still present and further iterations are required. In this sense, the method is ‘blind’, i.e. it does not look to see if the scaling factor that has been applied is appropriate to the case in question.
A new fuzzy approach has been developed which reduces the number of controllers used, co-ordinates use of one controller for rectification of more than one violation where possible and chooses control moves which have low cost. The controllers are ranked for suitability for correcting the whole set of violations, and the first \( n_c \) are chosen for application at each step, where \( n_c \) can be defined by the operator or found using some measure of overall system security. The use of fuzzy sets allows the control decision to be broken down into different regions of interest [100] which can be adapted easily and intuitively. These regions of interest include a measure of the severity of the violations to be corrected, consideration of control sensitivity relative to other controllers and preservation of control margin. The framework developed allows the straightforward inclusion of other factors such as stability indices, losses or emissions.

10.1 Modelling of Cost, Availability and Emissions

Among the aims set out for this project in chapter 1 was the more efficient control of an interconnected power system, where efficiency can be interpreted as concerning low monetary cost, low energy cost or low emissions. This latter interpretation can be justified in terms of current understanding on the global environmental effects of electricity generation and anticipated new environmental laws. One of the attractions of a fuzzy expert system is the ease with which a number of apparently contradictory objectives can be programmed [101], and the ease with which they can be understood by an operator. This feature is utilised in aiming to bring the objectives of minimum cost, minimum losses and minimum emissions into the solution routine.

If a cost is defined for each available controller, a new antecedent clause of cost IS low can be added to each rule of an expert system such as that described in section 9.3. The cost value which is fuzzified to give a degree of membership of the fuzzy set low will be the cost of increasing or decreasing the controller setting to the desired position for relief of the violation in question.

In practice, this will give a measure of control availability, too, since a controller that is
less available for a change of setting will have a high marginal cost. The membership function for low cost is shown in figure 10.1 where $c$ is an over-estimate of the average marginal cost. A discussion of different models of the cost of reactive compensation has been presented in section 8.3.3.

Emissions can be modelled similarly, so that an additional antecedent clause of emission IS low is added to each rule through the AND operator. The emission value whose degree of membership of low is to be found is the change in emissions as a result of the suggested control action. For a generator being used for MW flow control, if a relationship between generator MW output and emissions is defined, this can be easily found from the change in MW generation.

The linguistic variable for low emissions is shown in figure 10.2 where $-e$ is some estimate of a reasonable expectation of decrease in emissions.

In the same way, a fuzzy set for low losses can be defined. If a relationship is found
between a control action and active power losses, then the change in losses for a suggested action can be found and the degree of membership of low losses obtained.

### 10.2 Termination Conditions

The two expert systems described in sections 9.2 and 9.3 perform further iterations until either no further control actions are possible or the violations have been removed. Where removal of all the violations is not possible, the final power system state can end up being less secure than the initial one. To overcome this, the severity index \( \eta \) of the contingency under consideration is determined both before application of any control actions and after each iteration of the expert system. It may be expected that this will help to prevent actions being ordered which cause divergences in the load flow because as the 'knee-point' in the \( PV \)-curve is approached, reactive sources reach limits and the reactive demand from the transmission system increases, voltages will tend to drop giving an increased severity index.

The severity index used is based on that quoted by Ejebe and Wollenberg [18] (equation (2.1)). It is restated here and is

\[
\eta = \sum_{i=1}^{L} \left( \frac{X_i}{X_{\text{max}}} \right)^n
\]

(10.1)

where \( X_i \) is the deviation of the voltage magnitude at bus \( i \) from nominal or a line power flow, \( X_{\text{max}} \) is the maximum voltage deviation or line power flow, and \( n \) is some chosen integer.

If the severity index at the end of an iteration is higher (i.e. the system is less secure) than at the end of the previous iteration, then the previous set of control actions is chosen as the final one and the expert system exited. This ensures that the expert system finds a more secure system state, though there is a risk of the system stopping at a local minimum of severity index.
10.3 Decisions Based on the Rule Consequents

The inclusion of the new antecedents for cost, emissions and losses described above would give different implications for the rules. If the approach described in section 9.3 was used, the cost antecedent clause would prioritise movement of the controller for an action that is less costly for each individual controller where there is more than one violation. However, it would still be difficult to make judgements about one controller's benefit with respect to another as rules fire to give an indication not of which controllers are best, but simply which are possible to be used.

If the secondary aim of reducing the number of controllers used is also considered, it can be seen that there is a need to set up the rule base or co-ordinate the inferences of the rules to rank the controllers in order of suitability. Suitability includes consideration of the relative severity of the violation being addressed, the sensitivity of the controller, the control margin available and the cost. Two possible ways of weighing up controllers' relative suitability for relieving violations will now be considered. One is the application of an alpha-cut to the rule implications, and the other is the use of the cardinality of setting for each controller.

10.3.1 Controller selection by application of an alpha-cut

An alpha-cut can be applied to the fuzzy set assigned to the linguistic variable setting on the execution of each rule so that any implication (or degree of membership of setting) below the alpha-cut value \( \alpha_c \) is set to zero. In this way, controllers which give poor fits to the conditions described by the antecedent clauses of the rules for a particular violation are excluded from the set of control actions. A controller which still gives a good fit for a number of different violations will have its final set point found in the usual way.

There is one problem with the alpha-cut method: that of deciding where the alpha-cut should be put. A situation can be envisaged where a violation exists but for which every rule's implication falls below the alpha-cut. The violation should still be removed, and
can still be removed, but all the controllers are less than ideal. For example, if there are two violations $i$ and $ii$, and rules for two controllers, $a$ and $b$ fire, the fuzzy sets for setting$(a)$ and setting$(b)$ may be as shown in figure 10.3. If the initial alpha cut is at $\alpha_{c1}$, no action will result. One way to overcome this is to artificially lower the alpha-cut level and re-execute the rules until some control action results, though the number of controllers finally needed will not be directly set and a number of iterations may be needed. In the example, a reduction of the alpha-cut to $\alpha_{c2}$ would give actions on both controllers, one for each violation.

10.3.2 Controller selection based on cardinality

The alternative approach is to use the cardinality of a controller's fuzzy setting to assess its overall suitability. It will be recalled from section 4.2.4 that the cardinality of a fuzzy set is the integral of its membership function across its universe of discourse. For setting, this would be the sum of the degrees of membership for each singleton since the set is finite. A fuzzy controller is described in [75] which uses the cardinalities of rule outputs as measures of their suitability for the control action. It is useful to apply this concept in this case, too.

The use of the cardinality approach gives a direct measure of relative suitability is at each iteration. However, having such a measure, a decision still needs to be made about
which controllers to use. In doing this, the potential for setting a limit on the number of controllers used exists. This is discussed in the next section.

10.3.3 Limiting the number of controllers

One of the concerns of an operator taking advice from expert systems such as those described is that they may require the movement of a large number of control set points. The operator is likely to want to restrict the movement of many controller settings because of the logistical difficulties of communicating a large number of changes (though with future automatic dispatch of instructions this will be less of a problem), and due to possible transient problems caused by the largely unpredictable interaction of the changes over the immediate post-action period.

The disadvantage of allowing the user to restrict the number of controllers is that the execution of the expert system may take longer when too few are allowed, or the controllers allowed may be insufficient to relieve the violations. As an alternative, the user may be allowed to select different limits on the number of controllers depending on the severity of the system insecurity which is being tackled as a severe condition is likely to require a larger number of actions. This is itself a fuzzy decision which would lend itself well to expression in the form of a set of fuzzy rules, executed as a ‘pre-processing’ step. The rules may take the following form

1. IF severity index IS high THEN permissible number of controllers IS high

2. IF severity index IS medium THEN permissible number of controllers IS medium

3. IF severity index IS low THEN permissible number of controllers IS low

Controllers are ranked according to the cardinality of each controller’s fuzzy variable setting. Once a limit on the number of controllers $n_c$ has been established, only the first $n_c$ most suitable controllers are moved at each iteration, the extent of the movement determined as before from the de-fuzzification of setting. If, between one iteration and
the next, the system severity index has decreased by an amount less than a threshold set by the user, the expert system is permitted at the next iteration to use one extra controller it has not used before. In this way, the limit on the number of controllers to be used is relaxed only when strictly necessary.

10.3.4 Prevention of new violations

As has been described previously, the fuzzy approach of section 9.3 uses a ‘contribution factor’ to prevent a number of controllers from interacting to over-compensate a violation, or from creating a new one. It has been found, however, that it does not reliably achieve this, meaning that there is sometimes no improvement in overall system security. In addition, on occasions, a single control action has been scaled down so much that its effect is almost negligible, and further iterations of the expert system are called.

Instead, the effects of the complete set of defuzzified controller settings can be tested using the sensitivity matrix on all the monitored quantities to ensure that the combined actions do not aggravate existing violations or create new ones. It is assumed that the effects of each controller are super-imposed and the change in a given dependent variable $\Delta x_i$ for a set of changes in control variables $\Delta U_1 \ldots \Delta U_N$ is

$$\Delta x_i = \sum_{j}^{N} S_{ij} \Delta U_j$$

(10.2)

where $S_{ij}$ is the $ij$th element of the sensitivity matrix.

Some control actions $\Delta U_j$ will tend to push $x_i$ up, while others will push it down. If the net effect $\Delta x_i$ is to push $x_i$ above its nominal limit, then all the control actions $\Delta U_j$ which have positive effects are reduced in proportion in order to keep $x_i$ within limits. Likewise, if $x_i$ is predicted to fall below the nominal lower limit, all $\Delta U_j$ which have the effect of reducing $x_i$ are reduced. Where $x_i$ is already violated, the controls actions are adjusted if necessary so as not to make the violation any worse.
10.4 Run-Up Rates and the Time Dimension

The expert system described gives only an instantaneous ‘snap-shot’ solution of the dispatch problem where the maximum control settings are the highest settings that could be achieved after some unknown period of time. They would need to be adapted to incorporate information on changing system load and generator response times or ‘run-up rates’. A simple way of doing this would be for the expert system to be run every \( t \) minutes, where \( t \) relates to some suitable dispatch window (for example, National Gird Company’s ‘GOAL’ program produces updates every 30 minutes [49]), with the maximum setting for each controller found from a look-up table for each controller which relates the current setting to the maximum (or minimum) possible after a further \( t \) minutes. After \( t \) minutes have elapsed and the control schedule for that period has been implemented, the expert system is run again on the new set of current system data. In this way, a near-optimal solution is found for every \( t \) minute period. If a plot of a control schedule is wanted for some period of time in advance, the expert system is run repeatedly off-line with projected data.

If the system load is projected to have increased by the time any ordered actions have taken place, then, for the active dispatch system which must maintain a balance of load and generation, the net change in generation ordered should equal the projected change in load.

10.5 The New Fuzzy Expert System for Reactive Dispatch

The overall scheme is illustrated in figure 10.4

Once the system state has been determined and the list of violations assembled, the sensitivity matrix is found. If the user has not set a limit to the number of controllers to be used, a limit \( n_c \) is calculated based on the severity index (equation 2.1) and the fuzzy rules listed in section 10.3.3. The rules of the dispatch system are executed for each vi-
oration and each controller and the first $n_c$ controls are selected.

10.5.1 The new rule base for reactive dispatch

If the objectives of low cost and low power are included in a reactive dispatch expert system, the following four rules are executed for each controller and each violation:

1. **IF** voltage IS low AND sensitivity IS significantly positive AND control margin IS enough to raise AND cost IS low **THEN** setting IS $\Delta U^{max}$

2. **IF** voltage IS high AND sensitivity IS significantly positive AND control margin IS enough to lower AND cost IS low **THEN** setting IS $\Delta U^{min}$
3. IF voltage IS low AND sensitivity IS significantly negative AND control margin IS enough to lower AND cost IS low THEN setting IS $\Delta U^{\text{min}}$

4. IF voltage IS high AND sensitivity IS significantly negative AND control margin IS enough to raise AND cost IS low THEN setting IS $\Delta U^{\text{max}}$

The linguistic variables and values are those defined in sections 9.3.1 and 10.1. Alternative ‘s-function’ definitions of the membership functions to those given in figure 9.4 are shown in figure 10.5. Execution of the rules is made more efficient than those in section 9.3 by use of one output fuzzy variable setting which combines raise and lower settings. The values $\Delta U^{\text{max}}$ and $\Delta U^{\text{min}}$ of setting are the margin limited changes in control setting found from sensitivity analysis so that for the $j$th controller and a violation at the $i$th bus

$$\Delta U_j^{\text{max}} = \min(\Delta U_j, U_j^\text{max} - U_j^0) \quad (10.3)$$

$$\Delta U_j^{\text{min}} = \max(\Delta U_j, U_j^\text{min} - U_j^0) \quad (10.4)$$

where $U_j^\text{max}$ and $U_j^\text{min}$ are the maximum and minimum control settings and $U_j^0$ is the current control setting, and

$$\Delta U_j = \frac{\Delta V_i}{S_{ij}} \quad (10.5)$$

with

$$\Delta V_i = \begin{cases} 1.05 - V_i^k & \text{for rules 2 and 4} \\ 0.95 - V_i^k & \text{for rules 1 and 3} \end{cases} \quad (10.6)$$

The selected controllers’ settings $\Delta U$ are found from defuzzification of the respective setting fuzzy sets where the crisp setting for the $j$th controller is

$$\Delta U_j = \frac{\sum_{i=1}^L \sum_{\rho=1}^4 \alpha_{\rho ij} \Delta U_{ij}}{\sum_{i=1}^L \sum_{\rho=1}^4 \alpha_{\rho ij}} \quad (10.7)$$

and $\rho$ is the rule number.

The actions are then tested using the sensitivity matrix for whether they would aggravate any existing violations or create new ones and reduced if necessary. They are then implemented in a load flow study to predict more accurately their combined effects.
If any violations remain, the new severity index is found from the load flow. If the improvement from the last iteration is below a threshold, an extra control in addition to those used at previous iterations is allowed to be used. If, however, the severity index has increased, the solution from the last iteration is retained and the expert system exited. Otherwise, using the same sensitivity matrix as before, the expert system is re-executed and the controls used before plus any extra that are allowed are selected and tested.
10.5.2 'One-at-a-time' fuzzy control

If the number of controllers to be moved at each iteration is limited to one and only one violation (the worst) is addressed at a time, the approach reduces to a fuzzy version of that of Cheng et al (section 9.2 and reference [79]).

The 'best' controller is determined after the firing of the fuzzy rules by the 'rule of the preponderant alternative' [62] whereby the controller which has the highest setting membership value (equal to the implication of the fired rule) is selected. This is consistent with power system operator heuristics where a judgement on the 'best' controller is made considering the sensitivity of the violated bus voltage to adjustment of the controller, the available margin and the cost.

10.5.3 Prevention of Voltage Collapse

As has been described in section 2.2.2, voltage collapse as a phenomenon occurs when there is a deficiency in generated reactive power causing such a large decline in voltage that more current is drawn and the voltage declines further. Simple protection against it concerns the planning and scheduling of reactive power and reserves so that nodal voltages are kept within limits. Further protection can be afforded by defining the fuzzy sets concerning margin in such a way that reactive sources with less margin are less favoured in the dispatch.

10.6 A New Fuzzy Expert System for Active Dispatch

The ideas presented in the preceding sections of this chapter have been applied to overload alleviation as part of an expert system for active dispatch. This system is discussed in the following sections.
10.6.1 The rule base

The linguistic variables are combined in the following way:

1. **IF** loading IS *high* AND sensitivity IS *significantly positive* AND control margin IS *enough to lower* AND cost IS *low* **THEN** setting IS *lower*

2. **IF** loading IS *high* AND sensitivity IS *significantly negative* AND control margin IS *enough to raise* AND cost IS *low* **THEN** setting IS *raise*

The fuzzy set related to loading is shown in figure 9.6. $p_{max}$ is determined by the line MVA rating multiplied by a ‘temperature factor’ which shows the change in effective short-term line rating for a change in ambient temperature while $P_h$ is chosen by the user. The maximum loading is inversely proportional to the ambient temperature [145]. If the ‘book’ summer line rating is quoted for an ambient temperature of 20°C and the winter rating of 120% of the summer rating for an ambient temperature of 5°C [11], then the rating increase factor $f_r$ can be related to the ambient temperature in degrees Celsius $T_a$ by

$$f_r = 0.0133 \times T_a + 1.2667 \quad (10.8)$$

After the selection of the control actions, corrections may be applied in the manner described in section 8.3.2 in order to maintain the load/generation balance. The effects of the actions and simulated governor responses are then checked using the sensitivity matrix to verify that no existing violations are aggravated or new ones created. If they are, the actions (and corresponding corrections) are reduced.

In practice, only a limited number of generating sets will be on free governor operation, in which case the corrections are applied only to these. If free governor response is not to be modelled, some other means of re-establishing the balance between load and generation is needed. In this instance, one method would be to find two equivalent sets of actions, one in which the control actions increase MW generation and the other in which
an equal decrease is implemented. This can be achieved by assembling two separate ranked lists, one for positive actions, the other for negative.

The first controller to be selected will be the one from either list whose linguistic variable setting has the highest cardinality. The next to be chosen will be the highest ranked from the other list, and the third will be the highest ranked remaining controller from the first list, and so on until the maximum number of controllers is reached. Changes to quadrature booster settings are added to the final list of actions to be carried out whenever the cardinality of its fuzzy setting is higher than that of the next positive MW change and negative MW change. Clearly, if some change to MW generation has been ordered, the maximum number of controllers cannot be set below two.

Once all the controllers have been selected, the control actions found are tested using the sensitivity matrix for whether they aggravate existing violations or create new ones, and reduced if necessary. Finally, if neither of the sets of actions for positive or negative changes is empty, that with the highest absolute total of change in generation will have all its actions reduced in proportion to the magnitude of the suggested action until the two sets' combined change in generation is zero. Otherwise, the total generation from the non-empty set is matched by equal and opposite generation chosen from the merit order of generation currently spinning. If the set which is not empty is that for increases in generation, then the merit order will be used to reduce generation on the most expensive sets. If the non-empty set corresponds to that for reductions in generation, then the extra generation ordered to compensate for the change will the cheapest available.

10.6.2 Extension of the rule base to address continuous loading

Thermal limits on transmission line loading are generally set according to a projected increase in temperature of the conductor and ensuing expansion leading to greater droop, or according to the amount of heat that can be sunk by a cooling system. Since extra energy entering the line above that which is lost will result in an increase in temperature over time, the loading of the line over a period of time should be considered. This is often achieved by defining, for example, two loading limits, a higher one which is regarded
as being appropriate for the short term, and a lower one for the longer term.

Study of relevant data \([146]\) reveals that the time for which a line can be loaded to a particular level is inversely proportional to the square of the loading level and is of the form shown in figure 10.6. A fuzzy expert system has been developed which looks at average loadings over the last 1, 2, 3 and 4 state estimation intervals at times \(t_{k-1}, t_{k-2}, t_{k-3}\) and \(t_{k-4}\), and applies appropriate limits. These intervals are described by finite fuzzy sets with one singleton each: very short \(= 1/t_{k-1}\), short \(= 1/t_{k-2}\), quite long \(= 1/t_{k-3}\) and long \(= 1/t_{k-4}\).

![Figure 10.6: Relationship of loading and maximum time](image)

The limits are applied by means of 12 rules with loadings modelled over very short, short and quite long periods by very high, high and quite high. The fuzzy sets for loadings are shown in figure 10.7. The settings found by the rules will depend on the extent of the load and the time over which it has been at or around that level.

1. **IF** loading **IS** very high **AND** time **IS** very short **AND** sensitivity **IS** positive **AND** control margin **IS** enough to lower **AND** cost **IS** low **THEN** setting IS lower a bit
2. IF loading IS very high AND time IS very short AND sensitivity IS negative AND control margin IS enough to raise AND cost IS low THEN setting IS raise a bit

3. IF loading IS very high AND time IS short AND sensitivity IS positive AND control margin IS enough to lower AND cost IS low THEN setting IS lower

4. IF loading IS very high AND time IS short AND sensitivity IS negative AND control margin IS enough to raise AND cost IS low THEN setting IS raise

5. IF loading IS high AND time IS short AND sensitivity IS positive AND control margin IS enough to lower AND cost IS low THEN setting IS lower a bit

6. IF loading IS high AND time IS short AND sensitivity IS negative AND control margin IS enough to raise AND cost IS low THEN setting IS raise a bit

7. IF loading IS very high AND time IS quite long AND sensitivity IS positive AND control margin IS enough to lower AND cost IS low THEN setting IS lower a lot

8. IF loading IS very high AND time IS quite long AND sensitivity IS negative AND control margin IS enough to raise AND cost IS low THEN setting IS raise a lot

9. IF loading IS high AND time IS quite long AND sensitivity IS positive AND control margin IS enough to lower AND cost IS low THEN setting IS lower
10. IF loading IS high AND time IS quite long AND sensitivity IS negative AND control margin IS enough to raise AND cost IS low THEN setting IS raise

11. IF loading IS quite high AND time IS quite long AND sensitivity IS positive AND control margin IS enough to lower AND cost IS low THEN setting IS lower a bit

12. IF loading IS quite high AND time IS quite long AND sensitivity IS negative AND control margin IS enough to raise AND cost IS low THEN setting IS raise a bit

The meaning of “loading IS very high AND time IS very short” is that the average loading over a very short period of time is very high. Since the loadings between one sample time and the next are assumed to be constant with any changes happening in a step at the next sample time, the loading to be fuzzified is that recorded by the last state estimation i.e. at the last sample time.

The phrase “loading IS very high AND time IS short” concerns the average loading over two sample periods i.e. \((P_k + P_{k-1})/2\). The degree of membership of this loading of the fuzzy set very high is to be found. In the same way, the phrase “loading IS high AND time IS short” requires the degree of membership of the fuzzy set high to be found for the average loading over the last two sample periods. Likewise, the phrase “time IS quite long” requires the average loading over the last three sample periods to be considered.

The control changes are those necessary to be taken before or at the next sample time in order to reduce the loading to the level required for the average specified by the rule to be within its limit. For example, when the average loading over three sample periods (“time IS quite long”) is very high, the instantaneous loading at the next sample time should be dramatically lower in order that the average for four sample periods is within limits.

Application of the expert system on the receipt of every new state estimator output with limits being set on the number of controllers to move will result in actions only being taken where necessary. Where a particular loading level can be tolerated for a greater
time, no action will be called. If some event has occurred since the last sample which has caused certain loadings to suddenly become very high, these will be reduced on the next control cycle as a matter of priority.

A case study of the use of this facility is given in chapter 11.

10.7 Scheduling of Preventative Actions

All the dispatch formulations described up to now have been mainly concerned with the finding of corrective (emergency) actions. However, as described in section 2.4.1, there may be circumstances in which actions have be taken pre-emptively in order to prevent certain violations from occurring. Such violations on the U.K.'s national grid system include post-event voltages which are more than 5% away from nominal, branch loadings which exceed the short-term rating and branch loadings which exceed limits set for stability [6].

A brief description was presented in section 8.4.2 of one approach to the derivation of preventative actions within a linear programming dispatch program. This approach places great demands on computing resources both in terms of memory and processing speed. Although these problems can be reduced by applying only relevant constraints and variables at each stage of the process, the routine relies on repeated executions with different constraint sets. In addition, the routine described only finds preventative actions—a separate routine with different definitions of what constitutes a violation has to be run to find corrective actions.

The fuzzy method outlined in this chapter finds corrective and preventative actions simultaneously and does not increase the size of the base-case problem by introducing extra constraints. This is achieved through the definitions of the deadbands in the fuzzy sets and the application of a limit to the number of controllers to be used for contingency as it allows only the best actions (rather than the worst violations) to be considered. The deadbands vary explicitly with the system scenario being studied and so will always be correctly set. In comparison with the LP approach, these factors restrict the set of can-
The procedure followed is illustrated in figure 10.8. Each contingency is applied in turn. The set of actions which would be necessary under those conditions (i.e. post-contingency) to relieve any violations present is derived. Any actions carried out for violations which would have to be prevented (see section 2.4.1 for a discussion on what those violations are likely to be) are then composed with a fuzzy set for preventative settings for that controller to form a new fuzzy set.

When all the contingencies have been tested, each fuzzy set for preventative actions is
defuzzified. The crisp actions are then all tested using the base-case sensitivity matrix to check that no new violations are created on the base-case. They are then implemented to form a new base-case. The process is then repeated until all violations have either been prevented or are correctable. Where they are correctable, the necessary actions will already have been found.

Further issues particular to the reactive and active cases are discussed in the next two sections.

10.7.1 Preventative and corrective actions for voltage

In the U.K., the grid code [6] specifies the tolerable deviations in voltage from nominal. For voltage levels of 275kV and below, these are different in planning and operational timescales. When the dispatch tool is being used by planners to examine some projected system configuration, therefore, some voltage violations will require preventative action while, for others, corrective action will suffice, depending on the size of the violation. Application of the procedure described above would then yield two sets of actions for each contingency - actions to correct smaller violations on busbars at 275kV and below, and actions to prevent large violations on busbars at 275kV and below with those to prevent any violations on 400kV busbars. The preventative actions are then composed with the relevant fuzzy sets and defuzzified at the end of the contingency analysis process.

10.7.2 Treatment of limits imposed for transient stability

While short term violation of thermal limits can be generally tolerated under emergency conditions before corrective action is taken, limits on transfer imposed for stability must not be violated pre- or post-fault. Hence, action to alleviate violations of such limits shown up by contingency analysis should be taken preventatively. In addition, as the limits cannot be known with absolute certainty, action should be taken to prevent loading approaching the limit too closely. The type of expert system described above is particularly suitable for application to this problem due to its treatment of the gradual truth of
the statement ‘loading is high’ and its execution speed. The result will be a new schedule of generation.

The de-fuzzification of the preventative actions automatically makes a compromise between different actions with those relieving higher overloads being more heavily weighted. When relief of the violation is especially critical, the fuzzy sets used in the expert system can be changed so as to relax the priority given to cost or emissions or a low number of controllers.

Group transfer limits can be easily addressed by examining the effects of individual actions on all the lines associated with MW transfer across the boundary. In this instance, then, the loading to be fuzzified is transfer which is the sum of the loadings on lines across the boundary i.e.

\[
\text{transfer} = \sum_{i=1}^{N} P_i
\]  

(10.9)

where \( N \) is the number of lines across the boundary and \( P_i \) is the MW loading on the \( i \)th line.

The suggested action is found from

\[
\Delta U_g = \frac{\Delta \text{transfer}}{\sum_{i=1}^{N} S_{ig}}
\]

(10.10)

where \( \Delta U_g \) is the suggested change in control setting for the \( g \)th controller, \( \Delta \text{transfer} = \text{transfer}^{\text{max}} - \text{transfer}^0 \) and \( S_{ig} \) is sensitivity of the MW loading on line \( l \) to a change in \( U_g \). \( \text{transfer}^{\text{max}} \) is the maximum group transfer and \( \text{transfer}^0 \) is the current group transfer.

10.8 Complete Fuzzy Expert System for Dispatch

This section describes how the reactive and active expert systems are incorporated into a single security enhancement system.

The reactive and active dispatch expert systems are integrated into a single static security enhancement expert system in a manner similar to the solution of the active and
reactive sub-systems in a decoupled load-flow i.e. active and reactive sub-systems are solved alternately. The flow chart for the process executed at each time step is shown in figure 10.9. The reactive sub-system is executed first as changes in MVAr flows will tend to have significant effects on the thermal loading of transmission plant, while small changes in MW flows will have little influence on busbar voltage magnitudes.

10.9 Summary

This chapter has described new approaches to both reactive and active dispatch making use of fuzzy sets within a fuzzy expert system to model operators' decisions. Ways of incorporating judgements on cost, losses, emissions and the number of controls to use have been introduced. Procedures have been discussed for using the expert system to derive actions necessary to prevent excessive post-fault deviations of load bus voltage magnitude from nominal and to prevent loading limits set for transient stability from being exceeded. Further, a means of controlling line or transformer MW loadings over a period of time has been developed.
Finally, the framework for integrating reactive and active dispatch into a single complete dispatch routine has been described.
Results are presented in this chapter which illustrate the use of the new fuzzy expert systems developed for restoring load bus voltages back to within pre-determined limits and for alleviating overloads. Results for the new expert systems based on principles presented in [79] and [82] developed for overload alleviation are also presented. Comparisons are made between these approaches, those reported in [79] and [82] and a linear programming based approach for reactive control. The results for the state estimator developed in chapter 7 are given in that chapter.

The comparisons are made in two ways. In the first, for two different test systems and different loading levels, the overall performance of each dispatch routine is shown for the whole contingency list. This includes average execution time, a measure of the average cost of the chosen actions and a measure of the resulting improvement in security. The second allows the comparison of specific actions chosen for individual cases by the different dispatch routines.

To illustrate the use of the fuzzy expert system which monitors line loadings over periods of time, two scenarios are shown. A scenario is also given to illustrate the use of the fuzzy overload expert system for preventing group transfer limits from being exceeded.

The performance of the new fuzzy expert for reactive dispatch with different AND operators, different fuzzy membership functions and different weightings in the derivation of the initial limit on the number of controllers to be used is also shown.
The test systems used are an adapted IEEE 57 bus test system and a 100 bus reduced model of the UK national grid. The parameters of these are listed in appendix B. Different scenarios are presented for the 100 bus system corresponding to different loading levels where loads and active generation are increased by 20%, 40%, 60% and 80%.

The tests on the expert systems were carried out on a DEC Alpha, SPECfp92 162 and SPECint92 114.

11.1 Overall Performance

11.1.1 Reactive dispatch

In the following tables, the NEW method is that originated in this study and described in chapter 10, the OLD FUZZY method is that of [82], the TREE method is that of [79] and the LP method is a linear programming approach. The \( \text{ave} \Delta \eta \) figure is the average improvement in severity index after application of the dispatch routine for every contingency causing a voltage violation. \( \text{ave} t \) is the average execution time in seconds needed for each contingency. The times given exclude the derivation of the sensitivity matrix since that part of the process is common to all the methods. \( c \) is the percentage of cases for which the derived solution produces a convergence when implemented in a load flow. \( \text{ave} n_c \) is the average number of controllers used. \( n_c \) is the total number of controllers over the whole contingency cycle which were moved to their control limits. \( \text{ave} \Delta P \) shows the average percentage change in system MW losses after the control actions have been taken, and \( \text{cost} \) is the average cost of the chosen control actions. The costs are worked out based on a fixed amount per MVAr output from a generator, zero for transformer tap changes and changes to MSC settings, and based on some cost per unit MW loss for SVCs (see section 8.2.2 for a discussion of these costs).

Three different scenarios are shown for the 100 bus reduced national grid system (table 11.1). That for which the load has been increased from the base case by 40% produced voltage violations on 7 contingencies. The scenario where the load has been increased
by 60% had 12 contingencies with voltage violations, and the 80% case had 24 contingencies with voltage violations. Single line contingencies applied to the IEEE 57 bus system gave 38 contingencies with voltage violations (table 11.2).

\( \omega_u \) is a weighting factor used in the new fuzzy expert system in the derivation of the initial limit on the number of controllers to be used. For the results in table 11.1, \( \omega_u = 0.1 \). The PRODUCT operator is used for the connective AND in both fuzzy expert systems. The objective in the LP routine is minimum sum of control moves and minimum cost.

The timings are averaged across those contingencies for which all the methods reach a solution.

<table>
<thead>
<tr>
<th>Load</th>
<th>Method</th>
<th>ave ( \Delta \eta )%</th>
<th>ave ( t(s) )</th>
<th>c%</th>
<th>( n_c )</th>
<th>( n_s )</th>
<th>( \Delta P_l )%</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>NEW</td>
<td>37.93</td>
<td>0.030</td>
<td>85.7</td>
<td>2.19</td>
<td>1</td>
<td>0.1</td>
<td>0.106</td>
</tr>
<tr>
<td>1.4</td>
<td>OLD FUZZY</td>
<td>12.38</td>
<td>0.033</td>
<td>42.9</td>
<td>4.33</td>
<td>0</td>
<td>-0.067</td>
<td>2.286</td>
</tr>
<tr>
<td>1.4</td>
<td>TREE</td>
<td>16.06</td>
<td>0.043</td>
<td>71.4</td>
<td>1.80</td>
<td>4</td>
<td>0.1</td>
<td>0.672</td>
</tr>
<tr>
<td>1.4</td>
<td>LP</td>
<td>13.29</td>
<td>0.380</td>
<td>85.7</td>
<td>1.17</td>
<td>1</td>
<td>0.4</td>
<td>-0.014</td>
</tr>
<tr>
<td>1.6</td>
<td>NEW</td>
<td>26.50</td>
<td>0.040</td>
<td>91.7</td>
<td>2.45</td>
<td>3</td>
<td>0.4</td>
<td>1.659</td>
</tr>
<tr>
<td>1.6</td>
<td>OLD FUZZY</td>
<td>27.81</td>
<td>0.035</td>
<td>75.0</td>
<td>5.67</td>
<td>2</td>
<td>-0.04</td>
<td>4.790</td>
</tr>
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<td>1.6</td>
<td>TREE</td>
<td>9.14</td>
<td>0.070</td>
<td>75.0</td>
<td>2.33</td>
<td>8</td>
<td>0.12</td>
<td>10.621</td>
</tr>
<tr>
<td>1.6</td>
<td>LP</td>
<td>19.23</td>
<td>0.399</td>
<td>100.0</td>
<td>0.92</td>
<td>0</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>1.8</td>
<td>NEW</td>
<td>29.08</td>
<td>0.035</td>
<td>86.4</td>
<td>2.13</td>
<td>8</td>
<td>0.04</td>
<td>0.078</td>
</tr>
<tr>
<td>1.8</td>
<td>OLD FUZZY</td>
<td>23.91</td>
<td>0.036</td>
<td>77.3</td>
<td>4.83</td>
<td>2</td>
<td>0.00</td>
<td>3.806</td>
</tr>
<tr>
<td>1.8</td>
<td>TREE</td>
<td>9.10</td>
<td>0.071</td>
<td>72.7</td>
<td>2.00</td>
<td>11</td>
<td>0.52</td>
<td>3.259</td>
</tr>
<tr>
<td>1.8</td>
<td>LP</td>
<td>19.18</td>
<td>0.411</td>
<td>95.5</td>
<td>1.00</td>
<td>0</td>
<td>0.04</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 11.1: Results for reactive dispatch on a 100 bus reduction of the UK national grid

<table>
<thead>
<tr>
<th>Method</th>
<th>ave ( \Delta \eta )%</th>
<th>ave ( t(s) )</th>
<th>c%</th>
<th>( n_c )</th>
<th>( n_s )</th>
<th>( \Delta P_l )%</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEW</td>
<td>28.359</td>
<td>0.023</td>
<td>84.2</td>
<td>2.247</td>
<td>1</td>
<td>-0.07</td>
<td>-0.226</td>
</tr>
<tr>
<td>FUZZY</td>
<td>36.019</td>
<td>0.019</td>
<td>78.9</td>
<td>9.079</td>
<td>1</td>
<td>-0.29</td>
<td>0.203</td>
</tr>
<tr>
<td>TREE</td>
<td>22.796</td>
<td>0.027</td>
<td>68.4</td>
<td>2.902</td>
<td>3</td>
<td>-0.11</td>
<td>0.325</td>
</tr>
<tr>
<td>LP</td>
<td>29.106</td>
<td>0.024</td>
<td>94.7</td>
<td>1.380</td>
<td>0</td>
<td>-0.12</td>
<td>0.063</td>
</tr>
</tbody>
</table>

Table 11.2: Results for reactive dispatch on a modified IEEE 57 bus test system
Variation of control parameters in new fuzzy expert system

Table 11.3 shows the variation in the results for the new fuzzy expert system with different $\omega_v$. The higher $\omega_v$, the higher the initial limit. The timings are averaged across the contingencies for which all the implementations of the expert system reach a solution. The systems are tested on contingencies applied to the 100-bus reduction of the UK national grid. The PRODUCT operator is used for the AND connective (see chapter 4).

<table>
<thead>
<tr>
<th>Load</th>
<th>$\omega_v$</th>
<th>ave $\Delta \eta$ %</th>
<th>ave $t(s)$</th>
<th>c%</th>
<th>$n_c$</th>
<th>$n_s$</th>
<th>$\Delta P_i$ %</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.08</td>
<td>32.42</td>
<td>0.055</td>
<td>85.7</td>
<td>1.45</td>
<td>1</td>
<td>0.17</td>
<td>0.000</td>
</tr>
<tr>
<td>1.4</td>
<td>0.10</td>
<td>37.93</td>
<td>0.054</td>
<td>85.7</td>
<td>2.33</td>
<td>1</td>
<td>0.10</td>
<td>0.106</td>
</tr>
<tr>
<td>1.4</td>
<td>0.12</td>
<td>37.93</td>
<td>0.054</td>
<td>85.7</td>
<td>2.26</td>
<td>1</td>
<td>0.10</td>
<td>0.106</td>
</tr>
<tr>
<td>1.6</td>
<td>0.08</td>
<td>19.35</td>
<td>0.050</td>
<td>91.7</td>
<td>1.64</td>
<td>1</td>
<td>0.40</td>
<td>0.104</td>
</tr>
<tr>
<td>1.6</td>
<td>0.10</td>
<td>26.50</td>
<td>0.053</td>
<td>91.7</td>
<td>2.45</td>
<td>3</td>
<td>0.40</td>
<td>1.659</td>
</tr>
<tr>
<td>1.6</td>
<td>0.12</td>
<td>26.50</td>
<td>0.052</td>
<td>91.7</td>
<td>2.45</td>
<td>3</td>
<td>0.40</td>
<td>1.659</td>
</tr>
<tr>
<td>1.8</td>
<td>0.08</td>
<td>21.28</td>
<td>0.044</td>
<td>86.4</td>
<td>1.39</td>
<td>6</td>
<td>0.04</td>
<td>0.070</td>
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<tr>
<td>1.8</td>
<td>0.10</td>
<td>29.08</td>
<td>0.042</td>
<td>86.4</td>
<td>2.11</td>
<td>8</td>
<td>0.04</td>
<td>0.078</td>
</tr>
<tr>
<td>1.8</td>
<td>0.12</td>
<td>29.98</td>
<td>0.041</td>
<td>86.4</td>
<td>2.22</td>
<td>7</td>
<td>0.08</td>
<td>0.078</td>
</tr>
</tbody>
</table>

Table 11.3: Results for new fuzzy expert system with different $\omega_v$

Table 11.4 shows results for the new fuzzy expert system with the MINIMUM operator being used for the AND connective. In these cases, $\omega_v = 0.1$.

<table>
<thead>
<tr>
<th>Load</th>
<th>ave $\Delta \eta$ %</th>
<th>ave $t(s)$</th>
<th>c%</th>
<th>$n_c$</th>
<th>$n_s$</th>
<th>$\Delta P_i$ %</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>38.77</td>
<td>0.063</td>
<td>100</td>
<td>2.29</td>
<td>2</td>
<td>0.03</td>
<td>0.280</td>
</tr>
<tr>
<td>1.6</td>
<td>27.91</td>
<td>0.050</td>
<td>91.7</td>
<td>2.45</td>
<td>2</td>
<td>0.28</td>
<td>1.524</td>
</tr>
<tr>
<td>1.8</td>
<td>29.69</td>
<td>0.039</td>
<td>86.4</td>
<td>2.24</td>
<td>5</td>
<td>0.00</td>
<td>0.070</td>
</tr>
</tbody>
</table>

Table 11.4: Results for new fuzzy expert system with MIN used for AND

Table 11.5 shows the new fuzzy expert system being used with fuzzy membership functions being defined as S-functions such as shown in figure 10.5 rather than as given in figure 9.4. The deadbands, crossover points and points at which the membership function gives a degree of membership of 1 are all the same as with the trapezoidal membership functions used before. The expert systems are used for contingencies applied to the 100-bus reduction of the UK national grid. $\omega_v$ is set to 0.1 and the PRODUCT operator is used for AND.
### 11.1.2 Active dispatch

Table 11.6 shows results for contingencies applied to different system loading levels on the 100-bus reduction of the UK national grid (the IEEE 57 bus test system is not considered as line ratings are unavailable). Various of the contingencies applied cause items of transmission plant to be overloaded. The dispatch routines are used to suggest corrective actions to alleviate the overloads.

The results are compared for the new fuzzy expert system originated in this study ('NEW'), a new expert system for overload alleviation based on methods described in [79] but adapted in this project for application to overload alleviation ('TREE'), and a linear programming approach. Table 11.7 shows results for a new fuzzy expert system developed in this study from methods described in [82] for overload alleviation. This method is labelled 'OLD FUZZY'. It is included in table 11.7 as it maintains the load / generation balance in a different way from the other methods.

For the base loading, 6 contingencies caused overloads. For the scenario with load increased by 20%, 10 contingencies caused overloads. For the 40% case, 24 contingencies caused overloads and for the 60% case, 36 contingencies.

For both fuzzy systems, the NEW one and the OLD one, the operator used for AND is MINIMUM.

Table 11.7 shows results for the new fuzzy expert system originated in this study and a fuzzy expert system based upon ideas presented in [82] adapted for overload alleviation (see section 9.4.2). This is labelled 'OLD FUZZY' in the table. In this table, both expert systems are used with corrections applied to MW generation in order to balance the

<table>
<thead>
<tr>
<th>Load</th>
<th>ave ( \Delta \eta ) %</th>
<th>ave ( t(s) )</th>
<th>c%</th>
<th>( n_c )</th>
<th>( n_s )</th>
<th>( \Delta P_i ) %</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>37.70</td>
<td>0.044</td>
<td>85.7</td>
<td>2.10</td>
<td>1</td>
<td>0.1</td>
<td>0.106</td>
</tr>
<tr>
<td>1.6</td>
<td>30.06</td>
<td>0.053</td>
<td>91.7</td>
<td>2.27</td>
<td>3</td>
<td>0.2</td>
<td>0.381</td>
</tr>
<tr>
<td>1.8</td>
<td>28.36</td>
<td>0.049</td>
<td>86.4</td>
<td>2.12</td>
<td>8</td>
<td>0.0</td>
<td>0.073</td>
</tr>
</tbody>
</table>

Table 11.5: Results for new fuzzy expert system with S-membership functions
11.1.3 Integrated Active and Reactive Dispatch

Table 11.8 shows results for combined reactive and active dispatch applied to different scenarios on the 100 bus reduced national grid system. $n_{cp}$ is the number of active controls used per contingency causing some violations, $n_{ct}$ is the average number of reactive controls used, $n_{cp}$ is the total number of active controls moved to their limits and $n_{sq}$ is the total number of reactive controls moved to their limits. For the system with the load increased by 20%, there were 13 contingencies which caused violations. For that
Results

with the load increased by 40% there were 30 contingencies which caused violations, and where the load had been increased by 60%, there were 43. Since the performance of the 'OLD FUZZY' method for overload alleviation was poor, it was not considered for use in combined active and reactive dispatch.

<table>
<thead>
<tr>
<th>Load</th>
<th>Method</th>
<th>ave $\Delta \eta$ %</th>
<th>ave $t(s)$</th>
<th>c%</th>
<th>$n_{cp}$</th>
<th>$n_{eq}$</th>
<th>$n_{sp}$</th>
<th>$n_{aq}$</th>
<th>$\Delta P_i$ %</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>NEW</td>
<td>30.80</td>
<td>0.035</td>
<td>100</td>
<td>2.15</td>
<td>0.46</td>
<td>2</td>
<td>1</td>
<td>-17.9</td>
<td>-0.850</td>
</tr>
<tr>
<td>1.2</td>
<td>TREE</td>
<td>17.44</td>
<td>0.037</td>
<td>92.3</td>
<td>2.17</td>
<td>0.17</td>
<td>4</td>
<td>0</td>
<td>-10.7</td>
<td>0.882</td>
</tr>
<tr>
<td>1.2</td>
<td>LP</td>
<td>10.90</td>
<td>0.113</td>
<td>84.6</td>
<td>1.64</td>
<td>0.36</td>
<td>4</td>
<td>0</td>
<td>-0.012</td>
<td>0.279</td>
</tr>
<tr>
<td>1.4</td>
<td>NEW</td>
<td>26.40</td>
<td>0.038</td>
<td>96.7</td>
<td>2.31</td>
<td>0.53</td>
<td>7</td>
<td>1</td>
<td>-14.0</td>
<td>-0.771</td>
</tr>
<tr>
<td>1.4</td>
<td>TREE</td>
<td>16.62</td>
<td>0.049</td>
<td>93.3</td>
<td>2.57</td>
<td>0.32</td>
<td>22</td>
<td>3</td>
<td>-8.9</td>
<td>-0.509</td>
</tr>
<tr>
<td>1.4</td>
<td>LP</td>
<td>9.40</td>
<td>0.125</td>
<td>90</td>
<td>2.37</td>
<td>0.26</td>
<td>20</td>
<td>1</td>
<td>-2.5</td>
<td>-0.057</td>
</tr>
<tr>
<td>1.6</td>
<td>NEW</td>
<td>20.57</td>
<td>0.041</td>
<td>95.3</td>
<td>2.51</td>
<td>0.44</td>
<td>13</td>
<td>2</td>
<td>-10.6</td>
<td>-0.592</td>
</tr>
<tr>
<td>1.6</td>
<td>TREE</td>
<td>13.29</td>
<td>0.053</td>
<td>86.0</td>
<td>2.68</td>
<td>0.54</td>
<td>32</td>
<td>6</td>
<td>-6.7</td>
<td>-0.290</td>
</tr>
<tr>
<td>1.6</td>
<td>LP</td>
<td>9.44</td>
<td>0.140</td>
<td>81.4</td>
<td>2.94</td>
<td>0.29</td>
<td>46</td>
<td>0</td>
<td>-2.2</td>
<td>-0.134</td>
</tr>
</tbody>
</table>

Table 11.8: Results for combined active and reactive dispatch for 100 bus reduced national grid system

11.2 Case Studies

In this section, a number of case studies are presented to illustrate the use of the reactive and active dispatch expert systems.

11.2.1 Reactive dispatch

Example of corrective actions

With the system load increased by 40% over the base load on the 100 bus reduction of the UK national grid, the line connecting busbars WHSO4Q and WHSO2 was taken out. This resulted in an under-voltage of 0.9419 p.u. at WHSO2. The corrective actions suggested by the different dispatch routines are shown in table 11.9. All the approaches removed the violation. The LP approach achieved an improvement in the severity index of 24.8% at a cost of 0.0. The sensitivity tree method gained an improvement in the severity index of 30.5% at a cost of 0.739, the old fuzzy method an improvement of 50.5% at
a cost of 1.468 and the new fuzzy method an improvement of 44.4% at a cost of 0.739. The controls designated ‘uVr’ are generators or SVCs where the voltage reference is varied, and those labelled ‘uTap’ are transformers where the transformer tap ratio is to be varied.

<table>
<thead>
<tr>
<th>Method</th>
<th>Control</th>
<th>$\Delta U$</th>
<th>Limit hit?</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>uTapMELK4WHS02:1</td>
<td>-0.017</td>
<td>No</td>
<td>0.0</td>
</tr>
<tr>
<td>TREE</td>
<td>uVrCILF4</td>
<td>0.0151</td>
<td>No</td>
<td>0.739</td>
</tr>
<tr>
<td>NEW FUZZY</td>
<td>uTapMELK4WHS02:1</td>
<td>-0.017</td>
<td>No</td>
<td>0.0</td>
</tr>
<tr>
<td>NEW FUZZY</td>
<td>uVrCILF4</td>
<td>0.0151</td>
<td>No</td>
<td>0.739</td>
</tr>
<tr>
<td>OLD FUZZY</td>
<td>uTapMELK4WHS02:1</td>
<td>-0.017</td>
<td>No</td>
<td>0.0</td>
</tr>
<tr>
<td>OLD FUZZY</td>
<td>uTapABTH2JCILF4:1</td>
<td>0.015</td>
<td>No</td>
<td>0.0</td>
</tr>
<tr>
<td>OLD FUZZY</td>
<td>uVrMELK4</td>
<td>0.0077</td>
<td>No</td>
<td>1.029</td>
</tr>
<tr>
<td>OLD FUZZY</td>
<td>uVrCILF4</td>
<td>0.0090</td>
<td>No</td>
<td>0.439</td>
</tr>
</tbody>
</table>

Table 11.9: Actions taken to alleviate under-voltage at busbar WHS02

The actions chosen by the new fuzzy method were the most suitable from the list of candidates shown in order in Table 11.10. $S_V$ is the sensitivity of the violation to a change at the candidate controller and $\mu_s$ is the degree of truth of the premise sensitivity IS positive or sensitivity IS negative. Margin is the margin available at the controller and $\mu_m$ is the degree of truth of margin IS enough to raise or margin IS enough to lower. $\mu_c$ is the degree of truth of cost IS low. $\Delta U$ is the suggested control move and $\mu_{\text{rule}}$ is the degree of truth of the consequent or the extent to which the rule fires. This is found from the product of the degrees of truth of the premises. The degree of truth of voltage IS low for the voltage at WHS02 is 0.405.

<table>
<thead>
<tr>
<th>Control</th>
<th>$S_V$</th>
<th>$\mu_s$</th>
<th>Margin</th>
<th>$\mu_m$</th>
<th>Cost</th>
<th>$\mu_c$</th>
<th>$\Delta U$</th>
<th>$\mu_{\text{rule}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>uTapMELK4WHS02:1</td>
<td>-1.180</td>
<td>1.000</td>
<td>-0.200</td>
<td>1.000</td>
<td>0.000</td>
<td>0.500</td>
<td>-0.017</td>
<td>0.202</td>
</tr>
<tr>
<td>uVrCILF4</td>
<td>5.367</td>
<td>1.000</td>
<td>0.019</td>
<td>1.000</td>
<td>0.739</td>
<td>0.481</td>
<td>0.015</td>
<td>0.195</td>
</tr>
<tr>
<td>uVrMELK4</td>
<td>4.773</td>
<td>0.877</td>
<td>0.008</td>
<td>0.420</td>
<td>1.029</td>
<td>0.474</td>
<td>0.008</td>
<td>0.071</td>
</tr>
<tr>
<td>uTapABTH2JCILF4:1</td>
<td>0.887</td>
<td>0.072</td>
<td>0.200</td>
<td>1.000</td>
<td>0.000</td>
<td>0.500</td>
<td>0.023</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 11.10: Candidate control actions for alleviation of under-voltage at WHS02

Table 11.11 shows the degrees of membership of the premises with the point in the margin fuzzy sets for reactive sources at which the degree of membership is 1 halved and
the point at which the fuzzy set for low cost becomes non-zero set to 1.5 (it was 20). It can be seen that \( \mu_m \) and \( \mu_c \) are now quite different with more emphasis given to cheaper controls and those with greater margin in reactive power. The \( \mu_{\text{rule}} \) therefore different. As it happens, the two controls chosen (in order of \( \mu_{\text{rule}} \)) are the same, but uTap-ABTH2JCILF4:1 has moved up to third in the list.

<table>
<thead>
<tr>
<th>Control</th>
<th>( S_V )</th>
<th>( \mu_s )</th>
<th>Margin</th>
<th>( \mu_m )</th>
<th>Cost</th>
<th>( \mu_c )</th>
<th>( \Delta U )</th>
<th>( \mu_{\text{rule}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>uTapMELK4WHSO2:1</td>
<td>-1.178</td>
<td>1.000</td>
<td>-0.200</td>
<td>1.000</td>
<td>0.000</td>
<td>0.070</td>
<td>-0.017</td>
<td>0.0282</td>
</tr>
<tr>
<td>uVrCILF4</td>
<td>5.367</td>
<td>1.000</td>
<td>0.019</td>
<td>0.589</td>
<td>0.739</td>
<td>0.035</td>
<td>0.015</td>
<td>0.0084</td>
</tr>
<tr>
<td>uTapABTH2JCILF4:1</td>
<td>0.887</td>
<td>0.072</td>
<td>0.200</td>
<td>1.000</td>
<td>0.000</td>
<td>0.070</td>
<td>0.023</td>
<td>0.0020</td>
</tr>
<tr>
<td>uVrMELK4</td>
<td>4.773</td>
<td>0.877</td>
<td>0.008</td>
<td>0.178</td>
<td>1.029</td>
<td>0.022</td>
<td>0.008</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

Table 11.11: Candidate control actions for alleviation of under-voltage at WHSO2 with different membership function for margin

In some contingency cases, it can be seen that the new fuzzy expert system achieves an improvement in severity index even when the violations cannot be removed. For example, in the situation where, for the same loading conditions, the line between HARK2 and STHA2 is outaged, under voltages result at GALA1 (0.8807), ECCL2Q (0.9176), MAYT1T (0.9387) and COCK2 (0.9400). The new fuzzy expert system moved the transformer tap ratio between HARK2 and GALA1 to its lowest setting and moved that between ECCL2Q and GALA1 down by 0.019 to achieve an improvement in security of 75% even though violations remained at ECCL2Q (0.9215) and COCK2 (0.9460). No further control moves after this point were ordered as the severity index for the scenario with the set of actions suggested by the next iteration of the expert system was higher. The other methods all chose actions which led to diverged load flows. This was because various reactive sources in the vicinity of the outage reached their reactive limits when the methods went on choosing more actions in attempts to relieve all the violations, even when previously selected actions had been set to their control limits.

Preventative action

With the limits outside which violations require preventative actions set as in [6], i.e. ±5\% for the 400kV system and ±10\% for voltage levels lower than that, preventative...
actions were found for the 100 bus reduction of the UK national grid with a 60% increase in load. The outage of the line between HARK2 and STHA2 resulted in a voltage of 0.8579 at GALA1. The actions chosen and the cardinalities or grades of suitability of the setting fuzzy sets were

\[
\begin{align*}
\text{uTapECCL2QGALAl:} & \quad \Delta U = -0.130 \quad \text{grade 0.5000} \\
\text{uTapHARK2GALAl:} & \quad \Delta U = -0.140 \quad \text{grade 0.4605} \\
\text{uTapCOCK2GALAl:} & \quad \Delta U = -0.200 \quad \text{grade 0.2944} \\
\text{uVrHARK2:} & \quad \Delta U = 0.00356 \quad \text{grade 0.0107} \\
\text{uVrKILS2:} & \quad \Delta U = 0.02359 \quad \text{grade 0.0005} \\
\text{uVrKINT2:} & \quad \Delta U = 0.01205 \quad \text{grade 0.0002} 
\end{align*}
\]

The concurrent outage of MELK4-WHS04Q and MELK4-WHS02 required the following actions to achieve an improvement in severity index of 49%. They were chosen to reduce under-voltage violations at WHS04Q, SWAN4, PEMB4, WHSO2 and WALH4.

\[
\begin{align*}
\text{uTapWHS04QWHS02:} & \quad \Delta U = -0.045 \quad \text{grade 4.4559} \\
\text{uTapABTH2JSWAN4:} & \quad \Delta U = -0.060 \quad \text{grade 2.1546} \\
\text{uTapPENN2FECK4:} & \quad \Delta U = -0.200 \quad \text{grade 0.0754} \\
\text{uVrCOWL4:} & \quad \Delta U = 0.005 \quad \text{grade 0.0268} \\
\text{uTapWHS02SWAN4:} & \quad \Delta U = -0.060 \quad \text{grade 0.0140} \\
\text{uTapABTH2JCILF4:} & \quad \Delta U = 0.085 \quad \text{grade 0.0139} \\
\text{uVrMELK4:} & \quad \Delta U = 0.0158 \quad \text{grade 0.0044} \\
\text{uVrRUGE4:} & \quad \Delta U = 0.0132 \quad \text{grade 0.0013} 
\end{align*}
\]

The final chosen actions are those listed below. Some of the actions have been reduced in order to prevent violations in the base case. The violations resulting from the concurrent outage of MELK4-WHS04Q and MELK4-WHS02 are now less severe, and those resulting from the outage of HARK2-STHA2 have been removed.
11.2.2 Active dispatch

Monitoring of loadings over time

This example illustrates the use of the overload alleviation expert system as an on-line
line loading monitoring system (see section 10.6.2 for a full description). The example
is shown for the 100 bus reduction of the UK national grid. For the initial base-load case
there are no violations. Between the receipt of one set of data from the state estimator
and the next, a pick-up of 60\% of load has occurred, matched by spinning generation
reserve and pumped storage generation. At the receipt of the next set of data, a violation
is flagged:

\[
\text{Line CLYM2} \rightarrow \text{STHA2} \quad S = 757.3 \text{ MVA} \quad \text{Rating} = 750.0 \text{ MVA} \quad 100.98\%
\]

This is only flagged at this stage as, although the loading can be regarded as having been
at that level for two sample periods, the average over that time has only now exceeded
the medium-term rating. At the previous sample, it was below the short-term rating.

The fuzzy expert system is called to suggest actions to alleviate the overload. The actions
suggested are

\[
\begin{align*}
\text{uTapWHSO4QWHSO2:1} & \quad -0.028 \\
\text{uTapABTH2JCILF4:1} & \quad 0.052 \\
\text{uTapABTH2JSWAN4:1} & \quad -0.037 \\
\text{uTapWHSO2SWAN4:1} & \quad -0.037 \\
\text{uTapPENN2FECK4:1} & \quad -0.200 \\
\text{uTapHARK2GALA1:1} & \quad -0.086 \\
\text{uTapCOCK2GALA1:1} & \quad -0.123 \\
\text{uTapECCL2QGALA1:1} & \quad -0.080 \\
\text{uVrRUGE4} & \quad 0.0081 \\
\text{uVrCOWL4} & \quad 0.0031 \\
\text{uVrMELK4} & \quad 0.0097 \\
\text{uVrHARK2} & \quad 0.0022 \\
\text{uVrKINT2} & \quad 0.0074 \\
\text{uVrKILS2} & \quad 0.0144
\end{align*}
\]
When these have been completed, a long-term loading violation alarm is flagged up.

Line CLYM2 —► STHA2  S = 646.8 MVA  Rating = 750.0 MVA  86.25%

At the end of the next state estimation period, however, this alarm has gone as the average loading over 3 time periods is now below the long-term rating.

Suppose that at this point some event occurs which causes one of the lines between DINO4 and PENT4 to be tripped out. No alarm is flagged on the receipt of the first set of data after the event, but a medium-term overload alarm comes up on the receipt of the next set.

Line DINO4 —► PENT4  S = 1807.5 MVA  Rating = 1560.0 MVA  115.87%

The fuzzy expert system is activated and the following actions are suggested:

uPgDINO4  \( \Delta U = -343.4 \) MW  cost -2.097
uPgDUNG4  \( \Delta U = 249.1 \) MW  cost 1.661
uPgHINP4  \( \Delta U = 94.3 \) MW  cost 0.629

Once these actions have been implemented, a violation still remains:

Line DINO4 —► PENT4  S = 1464.6 MVA  Rating = 1560.0 MVA  93.89%

The expert system is activated again and suggests further actions:

uPgDINO4  \( \Delta U = -329.1 \) MW  cost -2.009
uPgHINP4  \( \Delta U = 329.1 \) MW  cost 2.194

After these actions have been carried out, the loading on DINO4-PENT4:2 is 72.85% of the rating. At the receipt of the next set of data, all violations, for long, medium and short-term, have been removed.
Example of corrective actions

This example shows the different actions suggested by the new fuzzy expert system, the sensitivity-tree based expert system and the linear program. The scenario used is one implemented on the 100 bus reduction of the national grid described in appendix B. The load on the system has been increased by 60% from that listed in appendix B, and generation increased accordingly. The double circuit between buses LEGA4 and IRON4 is then tripped. An overload of 9.25% on the line between DAIN4 and CELL4 results with 1627 MW flowing from DAIN4 to CELL4, -41 MVAr and 1627 MVA. The rating of the line is 1490 MVA.

The different responses of the dispatch systems are shown in table 11.12. The controls designated ‘uPg’ are generators where the MW output is varied, and that labelled ‘uQb’ is a quadrature booster where the booster angle in radians is to be varied.

<table>
<thead>
<tr>
<th>Method</th>
<th>Control</th>
<th>$\Delta U$</th>
<th>Limit hit?</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>LP</td>
<td>uQbWBUR4-WALP4</td>
<td>0.0524 rads</td>
<td>Yes</td>
<td>0.0</td>
</tr>
<tr>
<td>LP</td>
<td>uPgTRAW2</td>
<td>-239.04 MW</td>
<td>No</td>
<td>-2.154</td>
</tr>
<tr>
<td>LP</td>
<td>uPgHINP4</td>
<td>239.4 MW</td>
<td>No</td>
<td>1.594</td>
</tr>
<tr>
<td>TREE</td>
<td>uQbWBUR4-WALP4</td>
<td>0.0524 rads</td>
<td>Yes</td>
<td>0.0</td>
</tr>
<tr>
<td>TREE</td>
<td>uPgTRAW2</td>
<td>-261.67 MW</td>
<td>No</td>
<td>-2.358</td>
</tr>
<tr>
<td>TREE</td>
<td>uPgDUNG4</td>
<td>261.67 MW</td>
<td>No</td>
<td>1.744</td>
</tr>
<tr>
<td>NEW FUZZY</td>
<td>uPgTRAW2</td>
<td>-173.54 MW</td>
<td>No</td>
<td>-1.564</td>
</tr>
<tr>
<td>NEW FUZZY</td>
<td>uPgFFES2</td>
<td>-316.13 MW</td>
<td>No</td>
<td>-2.817</td>
</tr>
<tr>
<td>NEW FUZZY</td>
<td>uPgDUNG4</td>
<td>489.67 MW</td>
<td>Yes</td>
<td>3.264</td>
</tr>
</tbody>
</table>

Table 11.12: Actions taken to alleviate overload on line DAIN4-CELL4

All the methods succeeded in removing the violation. The sensitivity tree expert system first chose a change in MW generation at TRAW2 as being the control to which the violations was most sensitive. The necessary action was a decrease in generation of 261.67 MW. To maintain the load / generation balance on the system, a matching increase in generation at the cheapest available plant, DUNG4 was ordered. An improvement in security of 9.9% was achieved at a cost of -0.613. The linear program achieved an improvement in security of 9.7% at a cost of -0.560.
The fuzzy expert system found ten actions for which rules fired. The initial limit on the number of controllers to be used was set, based on the severity index, at 3. The degree of truth of high loading on DAIN4-CELL4 was 0.9615. The two most suitable controls from the list of candidates were those with the highest degrees of truth of the consequents of the rule that fired. These were the generators at FFES4 and TRAW4. Since these actions both gave decreases in MW output, some other control had to be chosen to maintain the load/generation balance. This was chosen from the top of the merit order and was DUNG4. Since the maximum number of controllers was 3, the sizes of the changes at FFES4 and TRAW4 were reduced to match the positive change at DUNG4 which was limited by the machine MVA limit. An improvement in security of 18.1% was achieved at a cost of -1.116.

Table 11.13 shows the degrees of the truth of each premise in each rule that fired. They are shown in order of suitability. $S_p$ is the sensitivity of the violation to a change at the candidate controller and $\mu_s$ is the degree of truth of the premise sensitivity is positive or sensitivity is negative. Margin is the margin available at the controller and $\mu_m$ is the degree of truth of margin is enough to raise or margin is enough to lower. $\mu_c$ is the degree of truth of cost is low. $\Delta U$ is the suggested control move and $\mu_{\text{rule}}$ is the degree of truth of the consequent or the extent to which the rule fires. This is found from the minimum of the degrees of truth of the premises.

<table>
<thead>
<tr>
<th>Control</th>
<th>$S_p$</th>
<th>$\mu_s$</th>
<th>Margin</th>
<th>$\mu_m$</th>
<th>Cost</th>
<th>$\mu_c$</th>
<th>$\Delta U$</th>
<th>$\mu_{\text{rule}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>uPgFFES2</td>
<td>0.289</td>
<td>1.000</td>
<td>-621.4</td>
<td>1.000</td>
<td>-2.123</td>
<td>0.712</td>
<td>-476.7</td>
<td>0.712</td>
</tr>
<tr>
<td>uPgTRAW2</td>
<td>0.289</td>
<td>1.000</td>
<td>-261.7</td>
<td>0.550</td>
<td>-1.179</td>
<td>0.618</td>
<td>-261.7</td>
<td>0.550</td>
</tr>
<tr>
<td>uPgCRUA2Q</td>
<td>0.100</td>
<td>0.444</td>
<td>-798.7</td>
<td>0.581</td>
<td>-3.494</td>
<td>0.849</td>
<td>-798.7</td>
<td>0.444</td>
</tr>
<tr>
<td>uPgWYL4F</td>
<td>0.289</td>
<td>0.350</td>
<td>-565.6</td>
<td>1.000</td>
<td>-1.920</td>
<td>0.692</td>
<td>-476.2</td>
<td>0.350</td>
</tr>
<tr>
<td>uPgDINO4</td>
<td>0.289</td>
<td>0.239</td>
<td>-3761.4</td>
<td>1.000</td>
<td>-1.454</td>
<td>0.645</td>
<td>-476.4</td>
<td>0.239</td>
</tr>
<tr>
<td>uPgHEYS4</td>
<td>0.214</td>
<td>0.217</td>
<td>-773.4</td>
<td>1.000</td>
<td>-2.567</td>
<td>0.757</td>
<td>-641.7</td>
<td>0.217</td>
</tr>
<tr>
<td>uPgFOYE2</td>
<td>0.090</td>
<td>0.194</td>
<td>-980.6</td>
<td>0.641</td>
<td>-3.995</td>
<td>0.899</td>
<td>-980.6</td>
<td>0.194</td>
</tr>
<tr>
<td>uPgFIDF2J</td>
<td>0.265</td>
<td>0.157</td>
<td>-1175.4</td>
<td>1.000</td>
<td>-1.883</td>
<td>0.688</td>
<td>-519.4</td>
<td>0.157</td>
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<tr>
<td>uQbWBUR4-WALP4</td>
<td>-2.443</td>
<td>1.000</td>
<td>0.052</td>
<td>0.093</td>
<td>0.000</td>
<td>0.500</td>
<td>0.052</td>
<td>0.093</td>
</tr>
<tr>
<td>uPgPEHE2</td>
<td>0.093</td>
<td>0.032</td>
<td>-762.5</td>
<td>0.516</td>
<td>-3.050</td>
<td>0.805</td>
<td>-762.5</td>
<td>0.032</td>
</tr>
</tbody>
</table>

Table 11.13: Candidate control actions for alleviation of DAIN4-CELL4 overload
Prevention of violation of group transfer limits

The following example is given for the 100 bus UK reduction with load increased by 60%. There is a limit set for the transfer between Scotland and England (the boundary is shown as a broad broken line in figure A.2) of 1650 MW. The initial base case shows the following violations:

\[
\text{Line CLYM2} \rightarrow \text{STHA2:1} \quad S = 757.3 \text{ MVA} \quad \text{Rating} = 750 \text{ MVA} \quad 101.0\% \\
\text{Scottish Import} \quad P = 1763.3 \text{ MW} \quad \text{Limit} = 1650 \text{ MW} \quad 106.9\%
\]

These are removed by the following actions:

\[
\begin{align*}
\text{uPgFOYE2} & \quad \Delta U = -83.4 \text{ MW} \quad \text{cost} -0.679 \\
\text{uPgCRUA2Q} & \quad \Delta U = -82.1 \text{ MW} \quad \text{cost} -0.718 \\
\text{uPgDUNG4} & \quad \Delta U = 165.5 \text{ MW} \quad \text{cost} 1.103
\end{align*}
\]

Each contingency from the contingency list is then applied in turn. Violations of the Scottish Import limit and line loadings over the short-term rating of 130% of the 'book' rating require preventative action.

The only contingency requiring further action is

\[
\text{Line CLYM2} \rightarrow \text{STHA2:1} \quad S = 1207.37 \text{ MVA} \quad \text{Rating} = 750.00 \text{ MVA} \quad 160.98
\]

The action taken is

\[
\begin{align*}
\text{uPgFOYE2} & \quad \Delta U = -206.7 \text{ MW} \\
\text{uPgCRUA2Q} & \quad \Delta U = -314.2 \text{ MW} \\
\text{uPgDUNG4} & \quad \Delta U = 374.6 \text{ MW} \\
\text{uPgHINP4} & \quad \Delta U = 146.3 \text{ MW}
\end{align*}
\]

This succeeds in preventing the violation from occurring and causes no new violations either on the base-case or for any contingency.
11.3 Discussion of Results

The tables of results for overall performance, table 11.1 for the reactive dispatch, table 11.6 for the active dispatch and table 11.8 for the combined reactive and active dispatch, show that the new method originated in this project has an overall best performance of the methods studied. It is in almost all cases the fastest and often achieves the best improvement in severity index.

It can be seen that improvements in severity index are higher for the more heavily loaded scenarios. This is because the dispatch methods are set to re-dispatch controls only while violations exist, and the violations tend to be more severe on the more heavily loaded system scenarios. This means that in order to remove the violations, more significant improvements in severity index are necessary. The actions suggested by the different dispatch routines are corrective actions that would need to be taken if the contingency being studied were actually to occur. This is like moving the system operating state from level 4 in figure 2.2 up to level 3 or above. Once the actions have been carried out, it would be necessary to carry out further contingency analysis on the new system state. This is analogous to ensuring that the system is either in a level 1 or level 2 state in figure 2.2.

The results for the IEEE 57 bus test system do not show the new method to be the obvious leader, but its results nevertheless represent the best compromise between all the criteria. The priorities within the expert system can be easily changed meaning that if one criterion is regarded by the operator as more important than another, extra emphasis can be given to it. Indeed, if the LP program is regarded as giving the benchmark improvement in severity index (the OLD FUZZY method over-compensates at the expense of using a great many more controllers), the new method proves satisfactory as the improvement is as good. However, the cost is vastly better.

The execution time of the LP method increases greatly when the system studied becomes larger. This is as expected as there are many more constraints in the LP tableau. The expert systems, however, show no significant slow-down as in the first instance they con-
The 'old fuzzy' approach described in [82] often achieves a good improvement in severity index, but this is at the expense of moving a large number of controls.

The linear programming approach, while it may be regarded as a bench-mark technique, fails to give good average improvements in severity index for the more stressed systems i.e. the 100 bus reduction at higher loadings. This is because for a number of cases no feasible basic vector is found as not all the constraints can be met simultaneously. It finds no solution and gives no improvement in severity index. The expert system approaches therefore have an advantage as some feasible initial solution is always found and can be tested to see if some improvement in security is gained even if some violations remain. Commercial LP packages for dispatch reproduce this behaviour by modifying the constraint set if no feasible vector exists and re-running the the solution algorithm. It can be seen, though, that this is likely to be more time-consuming and that control of the algorithm at a lower level, i.e. internally, would be more desirable.

The 'free governor' approach to restoring the load/generation balance in the overload alleviation expert system has not proved successful with a large number of cases in table 11.7 producing divergences in the load flow. This may be due to movement of a large number of controls making the linearisation upon which the control decision is based less valid. The approach, certainly for the expert system adapted in this project for overload alleviation from that described in [82], also proves slow, again because a large number of adjustments have been made.

All the methods are vulnerable to choosing actions which lead to divergences of the load-flow. This happens when the non-linear characteristics of the transmission system are
such that transfer of power cannot be supported, or reactive demand from the transmission system increases with respect to that predicted from the linearisation, leading to reactive sources reaching their limits. It would be expected that the methods which preserve control margin would be less vulnerable to this behaviour, and generally speaking this is true with the sensitivity tree expert system causing most divergences. It is evident that some further control of each solution algorithm is necessary to prevent divergences, or provide clearer, more detailed diagnoses of their causes. This is discussed further in section 12.2.

The variation of different control parameters in the new system developed in this study shows how priorities in the dispatch can be controlled by the user. In particular, the variation of \( \omega_v \) to control the initial number of controls used shows that with fewer controls, less improvement is gained in severity index. However, the use of more controls is more expensive. Little is gained from changing the AND operator (table 11.4) or using S-functions in the fuzzy membership functions (table 11.5).

It can be seen from tables 11.10, 11.11 and 11.13 that the membership functions of the linguistic values used in the antecedents of the rules can be changed so that different degrees of truth will result for the same crisp quantities. This in turn will affect the degree of truth of the consequent and will change the cardinalities of the \( \text{setting fuzzy sets} \) associated with each controller. With different cardinalities, the ranked list of controllers will change. In this way, the operator can change the level of priority attached to different considerations in the control decision.

The example of the use of the new fuzzy expert system for on-line monitoring of transmission plant loading shows the value of the approach in that actions are only called at the time when they are necessary. A delay even of only 10 or 20 minutes to a change in the MW generation pattern away from what might be assumed the optimum for cost could bring a cost saving in thousands of pounds.
11.4 Summary

This chapter has presented results illustrating the use of 4 different approaches to dispatch. The methods compared for reactive dispatch were a linear programming based routine, two published expert system approaches and a new one originated in this study. The methods compared for active dispatch were a linear programming routine, two expert systems newly developed from published approaches to reactive dispatch and a new one originated in this study. They were compared for the IEEE 57 bus test system and a number of scenarios implemented on a 100 bus reduction of the UK national grid. In addition, results have been presented for the LP method and an expert system developed from published work compared with a new expert system originated in this study for combined reactive and active dispatch.

A number of case studies have been presented illustrating the use of the different dispatch routines listed above. In addition, use of the new systems developed in this project has been shown for derivation of preventative actions and monitoring of line loadings over a period of time.
A fast and flexible new method for implementing re-dispatch for security has been demonstrated in this study. A number of areas of further work suggest themselves. These include the adaptation of the approach to prevention of voltage instability, the use of the system in deriving load shedding or line switching actions, and the inclusion in the integrated active and reactive version of the facility to change MW generation to relieve voltage violations. A graphical user interface would need to be provided for any version of the new expert system going into production. In addition, the understanding and use of fuzzy theory could be applied to topology processing or alarm processing. These will now be briefly discussed in turn.

12.1 Variation of MW Generation for Voltage Control

Under extreme circumstances, variation of generator terminal voltages, SVC set-points, MSC settings and transformer tap ratios will fail to relieve voltage violations. ‘Active power’ controls must then be used to adapt the MW flows such that the change in line loadings will produce changes in the voltage profile. This requires the derivation of a matrix of sensitivities of load bus voltages to active controls such as generator MW outputs and quadrature booster angles. An additional set of rules where these sensitivities are used instead of the sensitivities of load bus voltages to reactive controls would then be set up and executed when the reactive controls have not been successful. Where such an addi-
tional rule set would fit into an integrated active and reactive dispatch system is shown in figure 12.1.

Figure 12.1: Combined reactive and active dispatch with dispatch of active controls to control voltage

12.2 Prevention of Voltage Instability

With the flexibility of the new approach to dispatch outlined in this thesis, it is possible to envisage its use as part of a voltage instability prevention function. Where the current implementation uses a severity index to monitor whether successive iterations of the expert system are achieving improvements in global system security, a static voltage sta-
bility index [147] could be used. Also, where individual busbar voltages are monitored in the rules for the deviation of the voltage from nominal to determine the corrective or preventative action needed, some measure of the vulnerability of a busbar to voltage instability could be used. One such index is reported in [148]. A matrix of sensitivities of the vulnerability index of a busbar to control moves could be derived [149] such that the rules which now concern voltage deviation from nominal, control margin and sensitivity of voltage to a control move would then concern vulnerability index, control margin and sensitivity of the index to a control move.

Such application of the expert system to the prevention of voltage instability would require a more sophisticated load flow package, in particular one including better modelling of generator reactive power limits. Such limits are determined by limits to field voltage, field current and stator current. Reference [26] describes the modelling of these in a load flow package.

Of more pressing concern, however, may be the need to construct some framework within the expert system to deal with divergences of the load flow. Since these are associated with shortage of reactive power, they can be regarded as representing voltage instabilities. It can be seen from table 11.1 that all the methods used for reactive dispatch are prone to ordering actions which cause instability. This is due to the linearisation of the power system used in the sensitivity analysis being less reliable as the 'knee-point' of the PV curve, where instability occurs, is approached. When a divergence is found, a separate diagnostic routine could be invoked that determines from the load flow

- which PV buses have reached reactive limits
- which busbars have the largest mismatches.

The PV sources which have reached their limits and which have had control moves ordered at the last iteration can be forced to remain with the same control setting from the previous iteration of the dispatch routine. In particular, some change in reactive power output resulting from the change may have been of the opposite sign to that anticipated from the sensitivity analysis [150]. The linearisation can be re-performed around the
Suggestions For Further Work

operating point found from the last converged load flow solution. The expert system can then be re-applied, this time preserving greater reactive power margins at reactive sources in the vicinity of the busbars with the largest mismatches. This can be easily achieved by adjusting the membership functions of the fuzzy sets *enough margin to raise* and *enough margin to lower*.

If such an approach can be made to work efficiently, it may be at least as effective a protection against voltage instability as the more theoretically rigorous methods mentioned above.

12.3 Load Shedding and Line Switching

Among the actions available to power system operators not considered in this study are load shedding and line switching. Load shedding is often regarded as a last resort as it means that the aim of maintaining supply to all consumers has not been met. In this regard it is almost always used correctively. In some circumstances, however, it is necessary to shed some load in order to maintain the integrity of the whole system. In others, agreements exist between a large consumer and a utility whereby the consumer receives power at a lower tariff in return for being disconnected on occasions. Where such an agreement exists, the cost of shedding such a consumer's load can be weighed against the cost of alternative remedial actions. This could be easily modelled in the expert system by including the sensitivities of MW flows or voltages to changes in MW injections at the designated buses.

The most common occurrence of line switching for control is at times of low system load where load bus voltages can tend to creep up. This is because lightly loaded lines tend to be capacitive i.e. they appear to generate MVArS, and these excess MVArS can be understood as leading to increased voltages. Some lines would then be switched out. In other circumstances, lines which had been taken out in preparation for maintenance may still be available for switching back in. This action may be used to relieve low voltages or to alleviate overloads. Alternatively, situations can be envisaged where the removal of an
overloaded line will force power to flow down other lightly loaded lines. All the modelling of such switching actions requires is the derivation of the sensitivities of load bus voltages and active power flows to such actions, perhaps through simulating the switching action as changes in power injections at the busbars at either end of the line in question.

12.4 On-line Monitoring of Transmission Plant Loading

An expert system to check transmission plant loading levels every time a new set of state estimated data is received has been described in chapter 10 and demonstrated in chapter 11. The system has the advantage of considering the time for which an item of plant has been loaded to a certain level so that, while high loadings can be tolerated for a short period of time, actions are only carried out when strictly necessary. The system described does not currently take account of realistic rates of change of MW generation and a production tool would need to do so. This could be modelled by restricting the maximum change in MWs considered by the expert system to that possible in the period between one state estimation and the next. In this way, at the receipt of the next set of data, changes ordered at the last interval will have been completed and more changes can be ordered if necessary.

12.5 Topology Processing and Alarm Processing

As briefly discussed in section 4.1.4, topology processing and alarm processing are both areas in which uncertainty exists. Generally speaking, the uncertainty in such diagnostic tools is that concerning some crisp hypothesis such as ‘line \( x \) is switched out’. Such a hypothesis may be formed on the basis on ‘binary’ SCADA measurements which give a breaker’s status (it is either open or closed), and may be augmented by measures of power flows (if power is flowing down a line that has been switched out i.e. the breakers are apparently open at each end, then either the flow measurement is rubbish, or one of the breakers is not open). The different evidences which would be used to justify such a
hypothesis may have different levels of certainty associated with them. For example, it may be known that one transducer is more reliable than another. A conclusion such as 'line $x$ is switched out' would be reached knowing that the combination of 'the breaker at end $a$ is open' and 'the breaker at end $b$ is open' is more likely than any other combination of $a$ and $b$'s statuses. The use of certainty factors or possibility theory would seem an ideal way of deciding upon a most plausible conclusion.

### 12.6 User Interface

Much work would need to go into the design and implementation of a production tool based on the new ideas outlined in this thesis. Of prime importance would be the clear reporting of the actions suggested by the expert system. The facility to justify the decisions by reference to the degree of membership of the fuzzy sets and control limits should be provided in order to help build an operator's confidence in the system. A rudimentary facility is already provided and the information it gives is shown in some of the case studies shown in section 11.2.

A graphical user interface enabling the operator to easily and intuitively change the fuzzy membership functions in order to adjust the control decisions should be provided. Such a facility may allow the user to change a membership function by dragging parts of its characteristic with a mouse, with the interface then interpreting the move so that the function can be modified internally. A graphical tutorial may also be provided.
Conclusions

The overall aim of the research described in this thesis has been to enable better utilisation of the resources of a large, interconnected power generation and transmission system. Vast amounts of such resources are utilised in the operation of a power system. This study has looked at power system operation and the uncertainties associated with it. It has examined the aims which determine operational policy and decisions and has looked at new ways of meeting those aims. The aims include keeping nodal voltage magnitudes within limits, keeping branch power flows below the thermal ratings of transmission plant and preventing instability. Attempts are made to do this while supplying demand at the lowest cost.

The co-ordination of these objectives is a complex task, and a number of analysis tools have been developed dedicated to sub-tasks. Increasingly over the last few years, these tools have made use of artificial intelligence techniques. This study has therefore described some of these applications with a view to finding some methodology that would allow the operation of a power system to be enhanced through the better control of cost, both in terms of finance and natural resources.

13.1 Artificial Intelligence and Uncertainty

A major difficulty in operating a power system is the presence of uncertainty. This study has examined the different uncertainties encountered with a view to finding ways to model them such that the operation of the power system can be enhanced.
Various artificial intelligence techniques have been introduced including artificial neural networks, expert systems, fuzzy logic and evolutionary techniques. Their application to power system operation has been described and discussions have been presented concerning their efficacy in further improving the operation of a power system. Expert systems techniques which model uncertainty through probabilities, certainty factors or fuzzy sets, have been described in more detail, and possible future avenues of research in power systems have been identified. Uncertainties can be modelled in these so that decisions can be arrived at in which the operator can have more confidence.

13.2 State Estimation

It has been argued that state estimation is the fundamental building block upon which all power system operation functions depend, so this study has looked into ways of reducing the errors associated with it. A robust blocked augmented matrix method has been presented in which the facility to estimate transformer tap ratios has been newly added to a previously published formulation. It has been shown that this enhanced approach achieves more accurate results than a popular, alternative published approach.

Different matrix orderings have been studied so that with the transformers included, sparse matrix techniques can be made best use of in reducing the solution time. Some ordering information comes from a topological observability algorithm which, given measurements obtained from the power system, identifies which portions of the system can have their states estimated. A simple, new expert system has been written which performs this task.

13.3 Dispatch for Security Enhancement

Even with enhanced state estimation techniques, uncertainties are still present in the data used in power system analysis. In addition, operational decisions, expressed by operators, have uncertain meaning and are difficult to co-ordinate. Conventional techniques
Conclusions

for implementing such decisions focus on rectifying situations where system states, such as voltage magnitudes or power flows, have gone outside pre-determined limits i.e. enhancing security. The most common of these techniques has been based on a linear program optimisation. Due to the difficulty of defining the objective function for numerical optimisation routines and of guaranteeing feasible solutions, expert systems have recently been developed to perform the security enhancement function. All these methods make use of linear approximations of the power system in deriving corrective actions. Such analysis using linear approximations, or sensitivity analysis, has been described and implemented in this project. A new approach has been used in the derivation of relationships between active power controls and active power flows which avoids the calculation of a pseudo-inverse matrix and makes use of sparse matrix methods. An implementation of the load flow routine which solves the non-linear algebraic power system equations has also been described.

Dispatch of controls to enhance security by linear programming methods and two published expert systems for control of voltage have been looked at. These expert systems, originally described for control of voltage, have been adapted in this project for application to alleviation of overloads.

This study has investigated the potential for performing security enhancement better using some other technique which allows more intuitive control of the process, models approximations made about data and achieves better results in terms of system security and execution speed. It has been argued that fuzzy logic in a fuzzy expert system represents the most promising way of doing this.

A variety of new fuzzy expert systems have been developed in this project which are comprised of rule-bases built on expert system inference engines specially developed for this work. These tackle the control of voltage magnitudes, the alleviation of overloads and the prevention of instability where transfer limits have been defined. Operators' decisions are modelled by means of linguistic variables and fuzzy sets, and approximations about data are modelled similarly. The facility has been included to limit the number of controls used. It has been shown that the new fuzzy method allows easy, intuitive ad-
justment of operational priorities.

Results have been presented comparing a linear programming approach and the two earlier expert system approaches to voltage control plus their adaptations to overload alleviation with the new system developed in this study. It has been shown that the new approach achieves the best overall performance in terms of execution speed, reliability, enhancement in security, preservation of control margin and cost. Allied with the flexibility of the approach where membership functions modelling operators' judgements can be easily changed and where different measures of system security can be easily incorporated, it would seem that the method would provide a good basis for on-line use in operating a power system. The new facility could be easily controlled by operators and can gain their confidence by demonstrating the means by which decisions have been reached. It offers to provide additional facilities not currently available, such as the monitoring of line loading over time. It also promises to provide an efficient framework for preventing voltage instability.
It's unusual for authors to put some personal comment, unjustified by reference to the literature or experiments, in at the end of an academic work. In fact, I'm not sure I've ever seen it. However, I do feel quite strongly that no academic work should exist in a 'social vacuum', that is to say, no academic work should be carried out in ignorance of the social consequences of what is being described. It would appear that engineers and scientists have been particularly guilty of neglecting the possible social impact of their work. The best example which has shaped world history in the last 50 years has been the development of nuclear weapons. The work on the Manhattan Project may have been carried out in a time of war with what the protagonists believed to be justification in terms of guaranteeing freedom in the world, but many other documentary reports have made it clear that the military tension and the sacrifice of economies (with ensuing hardship and heightened tension) at the altar of nuclear superiority was foreseeable, and therefore avoidable. More recently, the Strategic Defence Initiative (or 'Star Wars') threatened to do the same with engineers fully aware of the folly of the strategy choosing not to inform their political masters of what they knew. Perhaps the greatest challenge now facing scientists' and engineers' ethical understanding is genetic engineering.

I cannot for one moment pretend that the work described in this thesis has such gravitas. Indeed, I will have to accept that this thesis, if I am honest about it, will probably languish untouched on some library shelf for eternity or until the library is knocked down to make way for a car park. However, should, by some small chance, someone pick up this
work and find that the directions it has suggested appear to be worth pursuing, I would like to offer some advice.

As the opening chapter outlined, energy is crucial to an industrialised society, and its efficient use is therefore of great importance. However, ‘efficient use’ currently has a very limited meaning, and is taken to signify only that which gives an adequate return in the short-term on financial investment. The social consequences of this should be obvious. We are likely to leave a legacy to our children of an earth that has been pillaged and a lifestyle that can be sustained no longer. Further, we may leave a bequest of skin diseases and respiratory illnesses that will become forever a fact of life. This is not the only way. Instead, we need to bring into consideration efficient use of natural resources, and if some technical development offers the facility to do so, then I believe we should do so.

Another social consequence concerns employment. As the labour movement became strong in the first half of this century, it rightly pointed out that the disparity between owners of industry and employees where owners received, and expected to receive, constantly growing financial rewards fuelled by new technologies which made workers dispensable, was unjust. As time went on, though, the labour movement came to be seen as the opponent of ‘progress’, of technological development. The ‘Luddites’ were dragged through the streets as peasants and charlatans. But unemployment, too, was dragged through the streets, and it took with it prisoners in terms of peace of mind, opportunity, envy and crime. Even the factory owners couldn’t escape the presence of these on the streets.

No, what I want to present, albeit simplistically, over-optimistically, romantically, even, is an alternative where technological advancement is not hindered, not stopped, but considered. And it should be considered in terms of its sustainable benefit to all. In this way, where expert systems technology appears to be able to replace human operators in performing the same task, we shouldn’t allow it to as there is no benefit to the operators or to society when there are many more unemployed. Instead, we should consider if the advancement in technology couldn’t be used to enhance operating practice. This
would indeed be possible with a production tool based on the methods outlined in this thesis. The expert system would be used to operate the power system more efficiently (in terms of finance and resources) with the operators not being put out of work but being re-skilled to perform the essential maintenance and development of the expert system. It is in this way that real benefit would be found.
Bibliography


[40] EPRI, On-Line Transient Stability Assessment, 6-7 December 1993.


Bibliography


The main test systems used, IEEE-57 [151] and m20b100, a reduced U.K. national grid network [152], are illustrated in this section.

IEEE 4 machine, 57 busbar model

Figure A.1: IEEE-57 bus test system
20 machine, 100 busbar model

Figure A.2: m20b100, 20 machine, 100 bus reduced model of U.K. national grid
## Test system Parameters

Table B.1: IEEE 57 bus test system busbar data

<table>
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<tr>
<th>Name</th>
<th>$P_g$ (pu)</th>
<th>$Q_{g\min}$ (pu)</th>
<th>$Q_{g\max}$ (pu)</th>
<th>$P_i$ (pu)</th>
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Table B.2: IEEE 57 bus test system: line parameters

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Table B.3: IEEE 57 bus test system: transformer parameters

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Table B.5: 100 bus reduction of the UK national grid: line parameters

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Test system Parameters

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Table B.6: 100 bus reduction of the UK national grid: transformer parameters

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Table B.7: 100 bus reduction of the UK national grid: quadrature booster parameters

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The following pages include papers published by the author based on work contained in this thesis. They are

- "A Fast New State Estimator" presented at the 29th Universities' Power Engineering Conference (UPEC), Galway, Ireland, September 1994


A Fast New State Estimator
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School of Electronic and Electrical Engineering, University of Bath, Bath BA2 7AY, U.K.

Abstract
Recent attention directed towards the improved performance of state estimators has concentrated on the robust utilisation of 'zero-injection' busbar 'measurements'. A separate direction of research has been the determination of the observability of an interconnected power system where basic approaches exist, numerical and topological.

This paper draws together work done in both these areas describing a blocked augmented matrix solution which allows the matrix to be solved in a positive-definite manner, and a topological observability algorithm which utilises a simple expert system and provides matrix ordering information. Transformer tap estimation is also described. Results are presented for the IEEE 57 bus system and an 811 bus model of the UK National Grid.

Keywords: Power system state estimation, least squares estimation, Hachtel's method, blocked augmented matrices, observability, expert systems.

1 Introduction
The state vector \( x \) of an interconnected power system is described by the vector of all observable busbar voltage magnitudes \( V \) and angles \( \theta \) for \( b \) busbars such that
\[
\begin{bmatrix}
V_1 \\
\vdots \\
V_b \\
\theta_1 \\
\vdots \\
\theta_b
\end{bmatrix}
\]
where \( \theta_b \) is the fixed voltage angle reference.

Not all the measurable quantities of a power system are in practice available, and those that are are subject to errors. The vector of measured quantities \( z \) may be modelled as the sum of the vector of non-linear functions of the actual power system state \( h(x) \) and the measurement error vector \( \omega \):
\[
z = h(x) + \omega
\]
where \( \omega \) is a vector of constraint equations and the Lagrangian \( L(x) \) is
\[
L(x) = \frac{1}{2}[z - h(x)]^TW^{-1}[z - h(x)] + \lambda^Tc(x)
\]

Because of the increasing complexity of subsequent functions of energy management systems, further improvements in terms of numerical robustness and speed have been sought. In 1988, Holten et al [1] compared a number of different methods and concluded that the so-called Hachtel's method offered the best trade-off between speed and reliability.

2 Hachtel's Method
The method is founded on the same weighted least squares formulation with constraints as above. Those equations are augmented [1] by an additional equation. This larger set of equations is, however, sparser so the computation time is competitive with other methods.

The Lagrangian of (6) is augmented by treating the residual \( r \) (equation 3) as an equality constraint. The Lagrangian is then
\[
L(x) = \frac{1}{2}r^TW^{-1}r + \lambda^Tc(x) + \mu^T[r - z + h(x)]
\]

The linearized system of equations
\[
\begin{bmatrix}
0 & H^T(\hat{x}_k) & C^T(\hat{x}_k) \\
H(\hat{x}_k) & W & 0 \\
C(\hat{x}_k) & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \hat{x} \\
\mu_{k+1} \\
\lambda_{k+1}
\end{bmatrix}
= \begin{bmatrix}
0 \\
\Delta x_k \\
-c(\hat{x}_k)
\end{bmatrix}
\]
is solved where
\[
\begin{align*}
\Delta \hat{x}_{k+1} &= \hat{x}_k + \Delta \hat{x} \\
\Delta x_k &= z - h(\hat{x}_k) \\
H(\hat{x}) &= \frac{\partial h(x)}{\partial x} \\
C(\hat{x}) &= \frac{\partial c(x)}{\partial x} \\
h(\hat{x}_{k+1}) &= h(\hat{x}_k) + H(\hat{x}_k)\Delta \hat{x} \\
c(\hat{x}_{k+1}) &= c(\hat{x}_k) + C(\hat{x}_k)\Delta \hat{x}
\end{align*}
\]

The main problem with Hachtel's Method is that the coefficient matrix, though symmetric, is not positive-definite. This requires that ordering of the equations employs a numerical test for the candidate pivot elements.

3 Blocked Matrix Formulations
factorization of the co-efficient (gain) matrix to proceed in a positive definite manner while preserving the robustness of Hachtel's Method and maximizing diagonal block dominance in order to properly handle ill-conditioned systems.

3.1 Hybrid Blocked Formulation

In the blocked augmented matrix method described in [2], only the injection measurements which cause ill-conditioning are left unsquared, while the others are squared, reducing the order of the matrix. Thus the residual is partitioned so that

$$r = [r_A, r_B]^T$$

where $r_A$ relates to the 'unsquared' injection measurements and $r_B$ to the other measurements that will be 'squared in'.

With the vector $r$ defined as $W^T [x - h(x)]$, it follows that the diagonal matrix of weights $W$, the vector of measurements $r$ and the vector of equations $h(x)$ must be similarly partitioned.

The problem can then be characterized as the minimization of

$$J(x) = \frac{1}{2} [x_B - h_B(x)]^T W_B^{-1} [x_B - h_B(x)]$$

subject to $c(x) = 0$ and $r_A = W_A^{-1} [x_A - h_A(x)]$.

With both constraints treated by the method of Lagrange multipliers, the Lagrangian is

$$L(x, r, \lambda, \mu) = \frac{1}{2} [x_B - h_B(x)]^T W_B^{-1} [x_B - h_B(x)] + \frac{1}{2} r_A^T \lambda + \lambda^T c(x) - \mu^T \left(r_A - W^{-1} [x_A - h_A(x)]\right)$$

The estimated state obtained $\hat{x}$ must satisfy the optimality conditions

$$\frac{\partial L}{\partial x} = -H_A^T [x_B - h_B(x)] - C^T \lambda - H_B^T [x_B - h_B(x)] W_b^{-1} \mu = 0$$

$$\frac{\partial L}{\partial r_A} = r_A - \mu = 0$$

$$\frac{\partial L}{\partial \lambda} = c(x) = 0$$

$$\frac{\partial L}{\partial \mu} = r_A - W_A^{-1} [x_A - h_A(x)] = 0$$

These are solved iteratively in the following set of linear equations:

$$\begin{bmatrix} -H_B^T W_B^{-1} H_B & H_B^T W_B^{-1} C & \Delta x \\ W_A^{-1} H_A & I & 0 \\ 0 & 0 & \lambda_A + 1 \end{bmatrix} \begin{bmatrix} \Delta x \\ \mu_{k+1} \\ \lambda_{k+1} \end{bmatrix} = \begin{bmatrix} \Delta x \\ 0 \\ -c(x) \end{bmatrix}$$

where, at the $k$th iteration, $\Delta x_A = W_A^{-1} [x_A - h_A(x_A)]$ and $\Delta x_B = [x_B - h_B(x_B)]$.

3.2 Blocking of the Gain (Co-Efficient) Matrix Through a Topological Method

In order to avoid there being any zero diagonal elements or singular $2 \times 2$ blocks, the gain matrix of equation (22) is blocked based on a topological algorithm [2, 3]. This is achieved by pairing every injection, measured and zero, with a node or bus and therefore its corresponding state variables. In addition, a bus which does not have a measured or zero injection and does not have a flow measurement on one of the lines connected to it must be paired with a remote injection measurement, i.e. one from a different bus. It should be noted that if a bus has an adjacent flow measurement, then it does not need to be paired with an injection, so its injection becomes available for use in a remote pairing. It is always preferable, though, to pair a bus with an injection at that bus, even if there is an adjacent flow measurement.

The algorithm passes along the observable tree, starting at the selected reference bus (which is always available for pairing even with an angle pseudo-measurement). Each branch is first assigned a measurement, either a flow or injection.

Now, assume that a bus $m$ is reached from a bus $k$. Let the respective bus injection measurements or zero injections be $P_m$ and $P_k$ (the decoupled active power case is being considered).

One of these scenarios might be experienced.

1. The branch km is assigned to a measurement of flow through km. $m$ is not paired—it does not need to be since buses 'seeing' a flow measurement do not need to be paired.
2. km is assigned to injection $P_m$; pairing $(P_m, m)$ chosen since an injection is best paired with its incident bus.
3. km is assigned to injection $P_m$. Here there are three more possibilities:
   i. $m$ is not paired because it sees a flow on a subsequent branch mm.
   ii. there is an unpaired injection $P_m$ at m. This 'redundant injection' is paired with m to form $(P_m, m)$.
   iii. bus m is paired with $P_k$ to form the remote pairing $(P_k, m)$.

The process is continued until the whole observable tree has been traversed. Finally, all injections that have not been paired are assigned to their local buses.

The blocked matrix will now be block diagonally dominant. In the fully coupled formulation where pairs of measurements (active and reactive) exist, $4 \times 4$ diagonal blocks will have different lower right sub-blocks depending on whether they correspond to measured or zero injections. Measured injections will give a $2 \times 2$ identity matrix and zero injections the null matrix.

4 Transformer Tap Estimation and Matrix Ordering Issues

Transformer tap estimation is most conveniently carried out by adding the set of tap ratios to be estimated to the state vector. In the blocked augmented matrix method, the active and reactive power flows associated with the transformers must be available. They can be included either in $H_B$ or $H_A$.

In the former case, a $1 \times 1$ diagonal sub-block will result corresponding to the row associated with the tap ratio to be
estimated. With the transformer flow measurements included in $H_A$, they can be paired with the transformer tap ratio in the same way as injection measurements are paired with states. 

In common with other positive definite sets of sparse linear equations, solution speed can be greatly increased by careful ordering of the matrix. In the blocked augmented method, the blocks are re-ordered, but positive definite factorisation without the need for pivot testing can only be guaranteed by the partitioning of the matrix into paired states and unpaired with the paired rows eliminated first [2]. This places a constraint on the re-ordering possible. Work in this project has been done in re-ordering each of the partitions separately using 'Minimum Degree/Minimum Number Predecessors' ordering [4]. The topologies of the blocks place no constraints on how the blocks are ordered within each partition (a proof is given in [2]), with alteration of pivots in the second partition during the elimination of the first partition guaranteeing non-singular pivots in all cases tested.

The ordering algorithm chosen (which gives a minimum number of fill-ins, an important consideration where some of the benefit of sparsity is lost by the storage of zero elements within blocks) shows that the transformer tap states, which only have voltage magnitudes are loosely coupled. Similarly, reactive equations corresponding to the active and reactive subsystems are solved alternately. This approach is speeded up with the benefit of sparsity is lost by the storage of zero elements within two off-diagonal blocks, are likely to be ordered near the beginning of the relevant partition. It follows that the total number of fill-ins during the factorisation of the matrix is likely to be reduced with the transformer tap ratios included in the first, paired partition. This suggests that the power flows through the transformers should be included in $H_A$.

5 Fast Decoupled Formulation

It is found in power system analysis that active powers and voltage magnitudes are loosely coupled. Similarly, reactive powers and voltage angles are only very loosely coupled allowing separation of $P$-$Q$ equations from $V$-$V$ equations in loadflow and state estimation [5].

The Jacobians $H(x)$ and $C(x)$, and $H(x)$ and $C(x)$ are partitioned into parts corresponding to real and reactive powers. With $H_p$, $H_q$, $C_p$, and $C_q$ are set to zero, two sets of linear equations corresponding to the active and reactive subsystems are solved alternately. This approach is speeded up with the use of constant (decoupled) gain matrices. These are evaluated at a 'flat start' i.e. voltage magnitudes all equal to 1 p.u. and voltage angles equal to 0. Voltage magnitudes in the left and right hand-side vectors are normalized by the corresponding estimate of voltage magnitude at each iteration. The blocking algorithm of section 3.2 will result in gain matrices with $2 \times 2$ and $1 \times 1$ sub-blocks.

6 Observability Analysis

Observability has been characterised as depending on the rank of the measurement Jacobian, $H(x)$, which needs to be full. This has been checked in one of two ways: by numerical algorithms or by topological algorithms. In 1991, work was published on observability which claimed to be faster than the numerical approach and identified separate observable islands where they existed [3]. Since a topological observability algorithm forms the basis of the blocked augmented matrix state estimation formulation, the method takes on extra significance. A newer version of the approach has been presented in [6]. This paper offers a further variation which elegantly utilises a simple expert system.

The algorithm tries to find a directed graph based on the lines and measurements in the system which connects as many buses as possible. The final forest will not, in general, be unique, but once it is found, the observable buses are known. The direction of each edge of the graph and which line of the system it lies on is determined by the busbar injection measurements, each of which may only be associated with one edge. Since flow measurements are themselves unique to one line, they are assigned first and the direction of the edge is unimportant.

Each bus with an injection measurement (including zero injections) is visited in turn. A directed edge lying along a line of the power system must be assigned for each. For example, bus 1 has lines connecting it with 2, 4 and 6. The directed edge to which it is assigned could lie along 1-2 in the direction of 2, 1-4 in the direction of 4, or 1-6 in the direction of 6. Edges must be chosen so that no loops are created, meaning that some buses may have to be re-visited. A sequence of actions, assigning or de-assigning edges is derived and then implemented. It is found that three simple rules suffice.

1. IF the chosen edge does not form a loop THEN add the assignment of that edge to the sequence AND implement the sequence.
2. IF the chosen edge forms a loop THEN choose another edge.
3. IF the chosen edge $E$ forms a loop AND no other edges remain for that bus AND $E$ does not lead to another bus at one end of any edge already in the sequence AND there is an injection bus $B_t$ in the loop not involved in the current sequence THEN add assignment of $E$ to sequence AND add de-assignment of $B_t$ to sequence AND re-assign $B_t$.

This procedure is best illustrated by the example of the 7-bus network shown in figure 1. The lines 1-4, 2-3 and 4-5 are included in the graph first as they have flow measurements. The three injection buses to be assigned are buses 1, 2 and 8. Bus 1 is considered first, and line 1-2 is arbitrarily chosen for edge 1 $\rightarrow$ 2. The assignment of edge 1 $\rightarrow$ 2 is added to the current sequence, and by rule 1 the sequence is implemented. The graph at this stage is as shown in figure 2.

Figure 1: 8 bus example system

![Diagram of a 8 bus example system](image)

Figure 2: Example system with flow measurements and injection at bus 1 assigned

Next, bus 2 is considered. Edge 2 $\rightarrow$ 1 is considered but rejected since it forms a loop through 1 $\rightarrow$ 2. Rule 2 says that another edge must be considered, but the only other one available is 2 $\rightarrow$ 4, and this, too, forms a loop. Rule 3 now dictates...
that assignment of edge 2 → 4 is added to the sequence. This is the first action in the sequence. The second must be the de-assignment of a directed edge in the loop whose root is not already in the sequence. This results in the de-assignment of edge 1 → 2 being added to the sequence. Bus 1 must now be de-assigned. Edge 1 → 2 fails on three counts: it forms a loop; it leads to a bus already featured in the sequence (bus 2); and edge 1 → 2 has already been in the sequence through its de-assignment. Edge 1 → 6 is therefore considered. This does not form a loop so its assignment is added to the sequence, which, according to rule 1, is then implemented, i.e. 2 → 4 is assigned, 1 → 2 is de-assigned and 1 → 6 is assigned. Finally, bus 8 is assigned. Edge 8 → 6 is arbitrarily selected, and rule 1 allows its assignment.

The maximal forest can be used then to identify the observable subnetworks. The rule for determining whether a bus is observable states that an assigned injection associated with a bus that has at least one incident line not in the span of the forest cannot be used in the solution and must be removed. It can be seen that there is a line linking nodes 8 and 7 where node 7 is outside the maximal forest. Injection bus 8 must therefore be de-assigned so nodes 8 and 7 are unobservable. They do not themselves form a separate observable island because of lines from those nodes to observable nodes. The observable island is that shown in figure 3.

7 Results
The 4 methods, fast decoupled (method A), fully coupled without transformer tap estimation (B), and fully coupled with transformer tap estimation (C) and transformer flows in \( H_A \) (method D) were tested on SCADA measurements modelled on a real-time power system simulator [7]. The actual states from the original simulation were available for comparison with the states derived by the state estimators and enabled the recording of \( V \) and \( \theta \) r.m.s errors in table 2. The solution for \( b811 \) with transformer flows in \( H_A \) was terminated after the generation of over 100,000 fill-in elements. The tests were performed on a Silicon Graphics Iris Indigo R4000 SPECmark 60 machine.

The test systems are those listed in table 7. \( iee57 \) is the IEEE 57 bus test system, and \( b811 \) is an 811 bus reduction of the U.K. national grid.

8 Conclusions
A method of performing observability analysis has been developed and different robust approaches to state estimation have been implemented. The results for the IEEE 57 bus test system show all the formulations' power, while those for \( b811 \) demonstrate that a large system can be feasibly solved accurately and quickly. The addition of 453 extra states through the estimation of transformer tap ratios undoubtedly puts considerable extra demand upon the solution method, but with transformer power flow measurements included in the unsquared part of the gain matrix, the result is thought to be acceptable, particularly when recognizing the unusual and potentially problematic features of a numerically reduced network with lines with very different \( X/R \) ratios in close proximity. The method should be suitable for further development with more work done on matrix ordering and the addition of a bad data filter.

Acknowledgement
The continuing technical and financial support of the National Grid Company plc is gratefully acknowledged, particularly the help of Dr A.O. Ekwue.

References
A Fuzzy Expert System for

Low-Cost Security-Constrained Reactive Dispatch

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Abstract—This paper examines expert system approaches to voltage control on large interconnected power systems. Two published systems are described and compared to a new one which models operator heuristics using fuzzy rules and reaches a low-cost solution for which a limit on the number of controls used can be set.

A brief description of fuzzy set theory is given and models of reactive generation cost are discussed. Results are shown for dispatch applied to restore load bus voltage magnitudes to within limits after line outages on a modified IEEE 57 bus test system and a 20 machine, 100 bus reduction of the U.K. national grid.

Keywords: reactive dispatch, fuzzy sets, expert systems.

INTRODUCTION

Control of voltage is one of the primary duties of an electric power utility, both for quality of supply to consumers and security of the system. It is generally viewed as the need to maintain the voltage magnitudes at all the nodes in the system within predetermined limits, particularly under fault conditions, using such controls as generator voltage set points, transformer tap ratios and banks of reactive compensators.

When taking action to correct voltage violations, operators consider the sensitivity of the voltage to be corrected to a control move, a controller's margin and secondary effects where existing violations might be aggravated or new ones created. An attempt is made to minimise cost, fuel and the number of controls used. Since the co-ordination of these criteria is extremely complex, computer analysis tools have been under development since the early 1980s [1]. However, the definition of an analytical objective function that enables each of these aims to be balanced while guaranteeing a viable solution within a given timescale is non-trivial. Human operators, on the other hand, are extremely capable of reaching judgements on an ideal balance, hence more recent developments in the field have focused on the application of expert systems [2]. This paper makes use of fuzzy logic to model the operator's decision-making process while using sensitivity analysis [3] to derive the necessary movements in control settings.

IL FUZZY SETS

Conventional logic assumes that a variable has one precise value i.e. it is crisp. Fuzzy logic attempts to model the vagueness of human reasoning by reflecting uncertainty about a variable's value through the assignment of a set of values to the variable each of which has a degree of membership of the set which reflects the likelihood of the variable having that value [4]. A membership function defines the degree of membership over the range of possible values or universe of discourse, and such a function can be assigned for an adjective (known as a linguistic variable or a fuzzy set) that describes the set of values. It is this property that gives fuzzy logic its power to model qualitative reasoning.

The universe of discourse of a fuzzy set may be finite in which case the set can be expressed as a summation of singletons (a single point in the universe of discourse which has a non-zero degree of membership), or infinite (even if over a finite range). The support of a fuzzy set is the set of points in the universe of discourse with positive non-zero degrees of membership of the fuzzy set. A set with a finite support $y_1, y_2, \ldots, y_n$ is the union (denoted by $+$) of its constituent fuzzy singletons and is

$$A = \sum_{i=1}^{n} \mu_i / y_i$$

where $\mu$ is the degree of membership of $y$ in the fuzzy set $A$.

Logical operations such as NOT, AND and OR can be performed on fuzzy sets [4]. The operation for AND will be of the form

$$A \text{ AND } B = \sum_{i=1}^{n} \mathcal{F} (\mu_A (x_i), \mu_B (x_i))$$

where $\mu_A (x)$ is the degree of membership of $x$ in $A$. $A \text{ AND } B$ is equivalent to the standard set operation $A \cap B$.

A. Cardinality

For a finite fuzzy set $A$, the cardinality $|A|$ is defined as

$$|A| = \sum_{i=1}^{n} \mu_A (x_i)$$

There are a number of reported examples, e.g. that in [5], for the cardinalities of consequent clauses of a set of fuzzy rules being used in decision support in enabling the finding of precedence in a list of feasible decisions.
B. Fuzzy Expert Systems

A fuzzy expert system executes a series of 'IF...THEN' production rules where the antecedents and consequents are fuzzy statements such as $x$ is low where low is a fuzzy set. It can be thought of as working in 4 steps:

1. **Fuzzification.** The membership functions defined for each input variable are applied to the actual variable values to determine their degrees of membership of the fuzzy set.

2. **Inference.** The implication of the rule is derived from application of the degree of membership of the antecedent to that of the consequent.

3. **Composition.** All the membership functions assigned to each output variable are combined to form a single membership function for each one.

4. **Defuzzification.** This converts a fuzzy set to a crisp value.

III. SENSITIVITY ANALYSIS

Sensitivity analysis provides a linear approximation of the effects of control moves on variables to be controlled [3]. If a vector of dependent variable increments $\Delta x_q$ consists of load bus voltage magnitude changes $\Delta V_i$, and $N_q$ reactive injection changes at buses with generation or synchronous reactive compensation $\Delta Q$ and the vector of independent variable increments $\Delta x_q$ consists of $N_q$ voltage set point changes $\Delta V_i$. $N_g$ changes of variable shunt susceptance $\Delta B$ and $N_t$ transformer tap ratio changes $\Delta t$, then using the well-known procedure of decoupling the active and reactive power equations, a sensitivity matrix $S_q$ can be found which relates $\Delta x_q$ to $\Delta u_q$ through

$$\Delta x_q = S_q \Delta u_q$$  \hspace{3cm} (4)

or

$$\begin{bmatrix} \Delta V_i \\ \Delta Q \\ \Delta \Delta \\ \Delta V_g \\ \Delta \Delta \\ \Delta \Delta \\ \Delta \Delta \\ \Delta \Delta \end{bmatrix} = \begin{bmatrix} S_{v1} & S_{v2} & S_{v3} & S_{v4} \\ S_{q1} & S_{q2} & S_{q3} & S_{q4} \\ \Delta B & \Delta B & \Delta B & \Delta B \\ \Delta V_g & \Delta V_g & \Delta V_g & \Delta V_g \end{bmatrix} \begin{bmatrix} \Delta \Delta \\ \Delta \Delta \\ \Delta \Delta \\ \Delta \Delta \end{bmatrix}$$ \hspace{3cm} (5)

IV. EXPERT SYSTEMS FOR REACTIVE DISPATCH

Most approaches to reactive dispatch have been based on linear programming and have been well-proven over a number of years [1,6], but the difficulty of defining the objective function for a range of objectives and of ensuring that a solution is reached have led to research into the use of expert systems. They began to be applied in the mid-'80s and a number of years [2,7,8].

The expert system dispatches of Chong et al [2], which takes the most severe voltage violation first and chooses control moves using a sensitivity tree, and of Yokoyama et al [8], which uses fuzzy rules to select the control actions, are described in the next two sections.

A. Sensitivity Tree Based Reactive Dispatch

The starting node in the sensitivity tree (fig. 1) is determined from the most violated busbar voltage $V_i$.

From the sensitivity matrix, the most effective available controller is found i.e. that to which the violated voltage is most sensitive with the sensitivity weighted, in the implementation adopted for this paper, by dividing it by the control range $U^{\max} - U^{\min}$. This determines which node in the next level of the tree to move to. The control action $\Delta U_j$ necessary to correct the voltage violation $\Delta V_i$ is found from

$$\Delta U_j = \frac{\Delta V_i}{S_{ij}}$$ \hspace{3cm} (6)

and is rounded to the nearest valid set-point. The action is checked against its control limits and limited if any are exceeded. The control move is then tested on every other load bus in the network to ensure that it does not aggravate existing violations or create new ones by searching the bottom level of the tree, and further limited if necessary.

In this implementation, the control move suggested is applied in a load flow study, the results of which then provide the input data for the expert system to be run again until no further control moves are possible or the violations have been removed.

B. Fuzzy Expert Reactive Dispatch System

Yokoyama et al [8] approached the problem of voltage control from the point of view of operator heuristics which are modelled by means of a set of fuzzy linguistic variables and four fuzzy rules. All four of the rules are executed for each combination of the controls and load buses with violated voltage magnitudes. The control actions are implemented in a load-flow study to check whether they have been sufficient to remove the violations, and the expert system is run again if necessary.

B.1 Formulation of rules

The six linguistic variables are combined in the following four rules $p=1 \ldots 4$:

1. IF voltage IS low AND sensitivity IS positive AND control margin IS enough to raise THEN setting IS $C_i \Delta J^{\max}$
2. IF voltage IS high AND sensitivity IS positive AND control margin IS enough to lower THEN setting IS $C_i \Delta J^{\min}$
3. IF voltage IS low AND sensitivity IS negative AND control margin IS enough to lower THEN setting IS $C_i \Delta J^{\min}$
4. IF voltage IS high AND sensitivity IS negative AND control margin IS enough to raise THEN setting IS $C_i \Delta J^{\max}$

The variables voltage, sensitivity and control margin are crisp variables. Voltage is the voltage at a load bus $V_i$, control margin is the margin available at a candidate controller.
compared with the output $\Delta U_j$ required to correct the voltage at the load bus, and sensitivity is the sensitivity $S_{ij}$ of the load bus voltage to a change in the candidate controller setting. The variables high, low, positive, negative and enough to lower are fuzzy sets and are shown in fig. 2. The fuzzy set setting is the suggested controller adjustment which must be defuzzified to give a single crisp value.

Each suggested control adjustment is modified by a "contribution factor" in order to prevent a combination of controllers over-compensating for the same voltage violation, and is determined by the margin limited positive control change at controller $j$ $\Delta U^+_{ij}$ for rules 1 and 4 and by the margin limited negative change $\Delta U^-_{ij}$ to relieve the violation for rules 2 and 3 such that

$$\Delta U^+_{ij} = \min(\Delta U_i, U^\text{max} - U^j_i)$$

(7)

$$\Delta U^-_{ij} = \max(\Delta U_i, U^\text{min} - U^j_i)$$

(8)

where $U^\text{max}$ and $U^\text{min}$ are the maximum and minimum control settings and $U^j_i$ is the current control setting. $\Delta U_j$ is found from equation (6).

The contribution factor $C_{ij}$ for controller $j$ applied to a voltage violation at bus $i$ is proportional to the sensitivity of that voltage to an action at that controller and is

$$C_{ij} = \frac{S_{ij}}{\chi S_j}$$

(9)

where

$$\chi S_j = \max(|S_{pj}|)$$

for $k = 1, 2, \ldots, L$.

B.3 Output composition

After execution of the rules, there will be finite fuzzy sets setting for each controller found from composition of the outputs of each rule.

B.4 Defuzzification

The method used to convert a fuzzy set setting into a crisp control signal for a particular controller is the "centroid" method [10] where, for controller $j$,

$$\Delta U_j = \frac{\sum_{i=1}^{L} \sum_{p=1}^{4} C_{ij} \alpha_{pij} v_{pij}}{\sum_{i=1}^{L} \sum_{p=1}^{4} \alpha_{pij}}$$

(11)

and $\alpha_{pij}$ is the implication of the rule $p$ executed for control $j$ and a violation at bus $i$, $C_{ij}$ is the contribution factor and $v_{pij}$ is the setting suggested by rule $p$ for controller $j$ and violation $i$. The defuzzified value is rounded to the nearest valid set-point.
V. A NEW FUZZY EXPERT SYSTEM

Although positive results have been obtained for the approaches described in section IV, they would appear to have some weaknesses.

The sensitivity tree approach of section IV does not optimize the solution reached, and tends to use up control margin at one controller before moving on to another leaving the system in the vicinity of that controller vulnerable to further contingencies.

The disadvantage of the fuzzy approach described in section IV is that it recommends adjustments on a large number of controllers, although this effect can be minimized by adjustment of the sensitivity and margin deadbands. In addition, although use of "contribution factors" negates the need for another loop of specific checks, it may not be successful in preventing over-compensation or creation of new violations, or it may reduce actions by too much.

A new fuzzy approach has been developed which chooses controllers which have low cost, reduces the number of controllers used and co-ordinates use of one controller for rectification of more than one violation where possible.

A. Modelling of Cost

In the last year or two, a number of utilities have come under pressure to cost reactive compensation more accurately. This is particularly true of the U.K. National Grid Company [11]. Among the proposals for costing of reactive power mentioned in [11] are payment for MVAr hours produced by generating units at a simple fixed rate per MVAr hour and payment for bidded lagging and leading reactive capability. Other possible methods of costing reactive generation include doing so on a basis of what equivalent compensation would cost to install and according to an "opportunity cost" proportional to the reduction in MW capability ensuing from the change in reactive generation.

When the cost of increasing or decreasing the control setting is defined for each available controller, a new antecedent clause of cost IS low can be added to each rule of an expert system such as that of [8]. The cost value of the required change is fuzzified to give a degree of membership of the fuzzy set low.

In this paper, a simple price per MVAr hour is adopted for generation. Changes to transformer tap ratios are modelled as having zero cost, while the costs of changes to the reactive output of compensation reflect changes in losses on the devices (as shown in fig. 4). The membership function for low cost is shown in fig. 5.

![Fig. 4. Cost of reactive compensation](image)

B. The Rule Base

If the objective of low cost is included in a reactive dispatch fuzzy expert system, the following four rules are executed for each controller and each violation:

1. IF voltage IS low AND sensitivity IS positive AND control margin IS enough to raise AND cost IS low THEN setting IS \( \Delta U^{\text{max}} \)

2. IF voltage IS high AND sensitivity IS positive AND control margin IS enough to lower AND cost IS low THEN setting IS \( \Delta U^{\text{min}} \)

3. IF voltage IS low AND sensitivity IS negative AND control margin IS enough to lower AND cost IS low THEN setting IS \( \Delta U^{\text{min}} \)

4. IF voltage IS high AND sensitivity IS negative AND control margin IS enough to raise AND cost IS low THEN setting IS \( \Delta U^{\text{max}} \)

The linguistic variables are those shown in figs. 2 and 5. The values \( \Delta U^{\text{max}} \) and \( \Delta U^{\text{min}} \) of setting are the margin limited changes in control setting (equations (7) and (8)).

C. Termination Conditions

The two expert systems described in section IV perform further iterations until either no further control actions are possible or the violations have been removed. Where removal of all the violations is not possible, this can result in the final power system state being less secure than the initial one. In the new approach, the severity index \( \eta \) of the contingency under consideration is determined [12] both before application of any control actions and after each iteration of the expert system where

\[
\eta = \sum_{i=1}^{L} \left( \frac{\Delta V_i}{\Delta V^{\text{max}}} \right)^n
\]

and \( \Delta V_i \) is the deviation of the voltage magnitude at bus \( i \) from nominal, \( \Delta V^{\text{max}} \) is the maximum permissible deviation and \( n \) is some chosen integer.

If the severity index at the end of an iteration is higher (i.e. the system is less secure) than at the end of the previous iteration, then the previous set of control actions is chosen as the final one and the expert system exited. This ensures that the expert system finds a more secure system state, though there is a risk of the system stopping at a local minimum of severity index.

D. Decisions Based on the Rule Consequents

The inclusion of the new antecedent for cost described above would give different implications for the rules. If the approach used in [8] were followed, the cost antecedent clause would prioritise movement of one controller for an action that is less costly for that controller where there is more than one violation. However, it would still be difficult to make judgements about one controller's benefit with respect to another as rules fire to give an indication not of which controllers are best, but simply which are possible to be used.
If a secondary aim of reducing the number of controllers used is also considered, it can be seen that there is a need to rank the controllers in order of suitability.

D.1 Limiting the number of controllers

The operator is likely to want to restrict the number of control moves suggested by a dispatch routine to limit the number of communications needed and prevent any possible transient problems caused by the largely unpredictable interaction of the changes over the immediate post-action period. Since the limit on the number of moves may be too low to enable the removal of all the violations, only the initial number of controller actions is restricted and freedom is given for additional controllers to be selected by further iterations of the expert system where necessary to remove remaining violations. As an alternative to the user setting the limit, a function $n_c = f(\omega_v, N, \eta)$ may be defined where $n_c$ is the initial maximum number of controllers, $N$ is the total number of controllers, $\eta$ is the contingency severity index and $\omega_v$ is some weighting. In this way, the initial limit on the number of controllers is appropriate to the severity of the system insecurity which is being tackled as a more severe condition is likely to require a larger number of controllers.

The controllers are ranked according to the cardinality of each controller’s fuzzy variable setting i.e. the sum of the degrees of membership for each singleton. The overall suitability $s$ of the controller whose fuzzy set setting is shown in fig. 3 is given by

$$s = \sum_{i} \mu(a) + \mu(b) + \mu(c) + \mu(d)$$

D.2 Prevention of new violations

The effects of the complete set of defuzzified controller settings are tested using the sensitivity matrix on all the load bus voltage magnitudes to ensure that the combined actions do not aggravate existing violations or create new ones.

Some control actions $\Delta U_i$ will raise $V_i$, while others will lower it. If the net effect $\Delta V_i$ is to raise $V_i$ above the nominal upper voltage limit, then the individual control actions $\Delta U_j$ which cause it to rise are reduced in proportion in order to keep $V_i$ within limits. Likewise, if $V_i$ is predicted to fall below the nominal low voltage, all $\Delta U_j$ which have the effect of lowering $V_i$ are reduced. Where $V_i$ is already violated, the controls actions are adjusted if necessary so as not to make the violation any worse.

E. The Final System

The overall scheme is illustrated in fig. 6

If the user has not set a limit to the number of controllers to be used, a limit $n_c$ is calculated based on the severity index. The rules of the dispatch system are then applied.

The selected controllers’ settings $\Delta U$ are found from defuzzification of the respective setting fuzzy sets where the crisp setting for the $j$th controller is

$$\Delta U_j = \frac{\sum_{i=1}^{n_c} \sum_{p=1}^{4} \alpha_{ij} V_{ij}}{\sum_{i=1}^{n_c} \sum_{p=1}^{4} \alpha_{ij}}$$

and $p$ is the rule number, $\alpha_{ij}$ is the implication of the rule executed for $i$th bus and $j$th controller, and $V_{ij}$ is the control move on the $j$th controller suggested by the $p$th rule to relieve a violation at the $i$th bus.

If any violations remain and the improvement in severity index from the last iteration is below a threshold, an extra control in addition to those used at previous iterations is allowed. If, however, the severity index has increased, the solution from the last iteration is retained and the expert system exited.

VI. RESULTS

The new fuzzy expert system (method NEW) was run to suggest control actions to relieve voltage violations caused by each of a full set of single and double circuit contingencies applied to three scenarios with progressively higher loadings, A, B and C, on a 20 machine, 100 busbar reduction of the U.K. National Grid [13] and by single line outages on a modified IEEE 57 bus test system [14]. 6 contingencies caused voltage violations on scenario A, 11 on scenario B and 19 on scenario C while 38 did so on IEEE-57. The results are presented in tables I and II with the number of controllers limit factor $\omega_v = 0.12$.

Comparisons are shown with the fuzzy expert system of [8] (FUZZY) and with that of [2] (TREE). ave $i$ is the average percentage improvement in severity index over all the contingencies which caused voltage violations. The average cost is determined according to a cost per MVAr for generation and a cost per unit MW loss for compensation. The average number of controllers used $n_c$, the number which are moved to their control limits $n_l$, and the number of contingencies for which converged solutions were found are shown. To give a
true comparison, the average times correspond to contingencies where all three methods reached converged solutions.

Table III shows the effect of changing the weighting factor $w_v$ used in determining the initial limit on the number of controllers used in the NEW method. The timings are averaged across all the contingencies.

VII. CONCLUSIONS

It can be seen from tables I and II that the new method achieves the overall lowest cost solution in a time comparable with the other two while using as few controllers as the TREE method and giving an improvement in security as good as the FUZZY method. The figure for the number of controllers which are moved to their operational limits in table II is slightly misleading as the new method uses up control margin in correcting violations caused by contingencies for which the other methods find no solution. A better reflection of the tendency to use up control margin is given in table I.

The FUZZY and TREE methods recommend fewer sets of actions for which the final load flow converges. While this tendency would probably be smaller were the sensitivity matrix re-calculated at each iteration, it reflects the fact that, for the FUZZY approach, a large number of actions makes the linear approximation less accurate, and that for the TREE approach control margin (in particular, reactive reserve) is used up.

Further work will focus on the utilisation of the new method to co-ordinate preventative and corrective actions and on the extension of the ideas to overload alleviation and loss reduction.

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References


Security-Constrained Reactive Dispatch Using Fuzzy Logic

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Abstract
The work described in this paper addresses the problem of dispatching reactive power to maintain voltage security on a large inter-connected power system. It makes novel use of fuzzy logic to model the operator’s decision-making process while using sensitivity analysis to derive the necessary movements in control settings within an expert system that seeks a low cost, low number-of-controllers solution. Results are presented for a 20 machine, 100 bus reduced model of the UK National Grid. Comparisons are shown with a linear programming approach.

Keywords: Expert systems, fuzzy logic, reactive dispatch.

1 Introduction
Control of voltage is one of the primary requirements of the management of a large electric power utility, both for quality of supply to consumers and security of the system, and is generally viewed as the need to maintain the voltage magnitudes at all the nodes in the system within pre-determined limits. This paper looks at a way of dispatching a best configuration of the available control devices to meet the specified voltage requirements after a fault or contingency.

When taking action to correct voltage violations, operators consider the sensitivity to a control move of the voltage to be corrected, a controller’s margin, and secondary effects where existing violations might be aggravated or new ones created. An attempt is made to minimise cost, fuel and the number of controls used. Since the co-ordination of these criteria is extremely complex, computer analysis tools have been under development since the early 1980s[1]. However, the definition of an analytical objective function that enables each of these aims to be achieved while guaranteeing a viable solution within a given timescale is non-trivial. Human operators, on the other hand, are extremely capable of reaching judgements on an ideal balance, hence more recent developments in the field have focused on the application of expert systems [2, 3]. This paper makes use of fuzzy logic to model the operator’s decision-making process while using sensitivity analysis [4] to derive the necessary movements in control settings.

2 Fuzzy Logic
Conventional logic assumes that a variable has one precise value i.e. it is crisp. Fuzzy logic attempts to model the vagueness of human reasoning by reflecting uncertainty about a variable’s value through the assignment of a set of values to the variable each of which has a degree of membership of the set which reflects the likelihood of the variable having that value [5]. A membership function defines the degree of membership over the range of possible values or universe of discourse, and such a function can be assigned for an adjective (known as a linguistic variable or a fuzzy set) that describes the set of values. It is this property that gives fuzzy logic its power to model qualitative reasoning.

The universe of discourse of a fuzzy set may be finite in which case the set can be expressed as a summation of singletons (a single point in the universe of discourse which has a non-zero degree of membership), or infinite (even if over a finite range). The support of a fuzzy set is the set of points in the universe of discourse with positive non-zero degrees of membership of the fuzzy set. A set with a finite support $y_1, y_2, \ldots, y_n$ is the union (denoted by +) of its constituent fuzzy singletons and is $A = \mu_1/y_1 + \ldots + \mu_n/y_n$ or $A = \sum_{i=1}^{n} \mu_i/y_i$ where $\mu$ is the degree of membership of $y$ in the fuzzy set $A$.

Logical operations such as NOT, AND and OR can be performed on fuzzy sets. Those for AND are of the form [6]

\[ A \text{ AND } B = \sum_{i=1}^{n} \mathcal{F}(\mu_A(x_i), \mu_B(x_i)) \]  (1)

where $\mu_A(x)$ is the degree of membership of $x$ in $A$. $A$ AND $B$ is equivalent to the standard set operation $A \cap B$.

2.1 Fuzzy expert systems
Choice of a so-called artificial intelligence technique is dependent on the kind of knowledge that is to be encapsulated. It has commonly been the experience of engineers that knowledge existing in the form of empirical data for classification problems is often best stored and applied using an artificial neural network while heuristic knowledge leading to classifications or control actions is often best captured by a series of 'IF...THEN' statements. In this latter category, fuzzy expert systems have found widespread use in recent years. Their advantages over conventional production-rule based expert systems have been characterised as including [7, 8]:

- fuzzy sets neatly symbolise natural language terms used by experts;
- since knowledge captured in 'IF...THEN' statements is often not naturally true or false, fuzzy sets afford representation of the knowledge in a smaller number of rules;
- fuzzy rules can be tuned on- or off-line;
- a smooth mapping can be obtained between input and output data.

The 'IF...THEN' statements in a fuzzy expert system are comprised of fuzzy antecedents and consequents such as $x$ IS low where low is a fuzzy set. The system can be thought of as working in 4 steps [6] (illustrated in figure 1):

1. Fuzzification. The degrees of membership of the crisp input variables of the corresponding input fuzzy sets are found.
2. Inference. The implication of the rule is derived from application of the degree of membership of the antecedent to that of the consequent.
3. Composition. All the membership functions assigned to each output variable are combined to form a single membership function for each one.

4. Defuzzification. Output fuzzy sets are converted to crisp values.

2.2 Cardinality

For a finite fuzzy set A, the cardinality |A| is defined as

\[ |A| = \sum \mu_A(x_i) \]  

where \( \mu_A(x_i) \) is the membership of the element \( x_i \) in the fuzzy set A.

There are a number of reported examples, e.g., that in [9], for the cardinalities of consequent clauses of a set of fuzzy rules being used in decision support in enabling the finding of precedence in a list of feasible decisions.

3 Sensitivity Analysis

Sensitivity analysis provides a linear approximation of the effects of control moves on variables to be controlled [4]. For balanced operation of a general power system where the active and reactive sub-systems have been decoupled

\[ G_Q(x_q, u_q, p) = 0 \]  

where \( G_Q \) is the set of reactive power injection equations. \( x_q \) is the vector of dependent variables consisting of a load bus voltage bus magnitudes \( V_i \) and \( N_y \) reactive injections \( Q_y \) at generator or compensation buses, \( u_q \) is the vector of independent variables consisting of \( N_g \) generator or static voltage compensator (SVC) voltage set-points \( V_y \), \( N_s \) shunt susceptances \( B \) and \( N_t \) transformer tap ratios \( t \), and \( p \) is the vector of transmission system parameters.

If small changes \( \Delta V_i \) and \( \Delta u_q \) are added to \( u_q \), increments in \( \Delta x_q \) will result. For balanced operation to continue

\[ G_Q(x_q + \Delta x_q, u_q + \Delta u_q, p) = 0 \]  

Taking the Taylor series expansion of (4) and neglecting higher order terms,

\[ G_{Qr}(x_q, u_q, p)\Delta x_q + G_{Qq}(x_q, u_q, p)\Delta u_q = 0 \]  

where the Jacobians \( G_{Qr} \) and \( G_{Qq} \) are

\[ G_{Qr} = \frac{\partial G_Q}{\partial x_q} \]  

\[ G_{Qq} = \frac{\partial G_Q}{\partial u_q} \]  

Equation (5) is rearranged to find the reactive sensitivity matrix \( S_Q \) relating \( \Delta x_q \) and \( \Delta u_q \) through

\[ \Delta x_q = S_Q\Delta u_q \]  

or

\[ \begin{bmatrix} \Delta V_i \\ \Delta Q_i \end{bmatrix} = \begin{bmatrix} S_{V_i} & S_{V_B} & S_{V_V} \\ S_{Q_i} & S_{Q_B} & S_{Q_V} \end{bmatrix} \begin{bmatrix} \Delta B \\ \Delta V_q \end{bmatrix} \]  

where

\[ S_Q = \frac{G_{Qq}(x_q, u_q, p)}{G_{Qq}(x_q, u_q, p)} \]  

4 A Fuzzy Expert System for Reactive Dispatch

A new fuzzy expert system has been developed which chooses controllers which correct violations at a low cost, allows a limit to be set on the number of controllers used and co-ordinates use of one controller for rectification of more than one violation where possible.

4.1 The rule base

The decisions taken by operators to restore busbar voltages to within limits can be modelled by the following four rules, applied for each available controller and each violation:

1. IF voltage IS low AND sensitivity IS positive AND control margin IS enough to raise AND cost IS low THEN setting IS \( \Delta U_j \) max

2. IF voltage IS high AND sensitivity IS positive AND control margin IS enough to lower AND cost IS low THEN setting IS \( \Delta U_j \) min

3. IF voltage IS low AND sensitivity IS negative AND control margin IS enough to lower AND cost IS low THEN setting IS \( \Delta U_j \) min

4. IF voltage IS high AND sensitivity IS negative AND control margin IS enough to raise AND cost IS low THEN setting IS \( \Delta U_j \) max

The variables voltage, sensitivity and control margin are crisp variables, voltage is the voltage at a load bus \( V_i \), control margin is the margin available at a candidate controller compared with the output required to correct \( V_i \), and sensitivity is the sensitivity \( S_j \) of the load bus voltage to a change in the candidate controller setting. The variables high and low relating to voltage, positive and negative relating to sensitivity, enough to raise and enough to lower relating to margin and cost are fuzzy sets and are shown in figure 2. The fuzzy set setting is the suggested controller adjustment which must be defuzzified to give a single crisp value.

The control margin values to be fuzzified for the \( j \)th controller are \( U_j^{\min} = U_j^p \) and \( U_j^{\max} = U_j^p \) where \( U_j^p \) is the current control setting and \( U_j^{\min} \) and \( U_j^{\max} \) are the minimum and maximum control settings. The values \( \Delta U_j^{\max} \) and \( \Delta U_j^{\min} \) of setting are the margin limited changes in control setting where

\[ \Delta U_j^{\max} = \min(M_j, U_j^{\max} - U_j^p) \]  

\[ \Delta U_j^{\min} = \max(M_j, U_j^{\min} - U_j^p) \]  

\( \Delta U_j^{\max} \) is found from

\[ \Delta U_j = \frac{\Delta V_j}{S_{ij}} \]
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The deadbands in figure 2 are defined with $m$ in (5) as the switched susceptance changes [10].

$A_{vi}$ is the deviation outside the set limits of the voltage at the $i$th load bus.

The sensitivity matrices are composed together into one finite fuzzy setting for each controller with different control settings suggested by the expert system where

$$\Delta V_{i} = \frac{\sum_{i=1}^{L} \sum_{j=1}^{N} \mu_{vi}(\Delta U_{pj}) \Delta U_{pj}}{\sum_{i=1}^{L} \sum_{j=1}^{N} \mu_{vi}(\Delta U_{pj})}$$

(14)

and $\mu$ is the rule number, $\Delta U_{pj}$ is the implication of the rule executed for $i$th bus and $j$th controller, and $v_{vi}$ is the control move on the $j$th controller suggested by the $i$th rule to relieve a violation at the $i$th bus.

$\eta = \sum_{i=1}^{L} \left( \frac{\Delta V_{i}}{\Delta V^{max}} \right)^{n}$

(15)

and $\Delta V_{i}$ is the deviation of the voltage magnitude at bus $i$ from nominal, $\Delta V^{max}$ is the maximum permissible deviation and $n$ is some chosen integer.

Though there is some risk of stopping at a local minimum, if the severity index at the end of an iteration is higher (i.e., the system is less secure) than at the end of the previous iteration, then the previous set of control actions is chosen as the final one and the expert system exited.

4.3 Termination conditions

Where removal of all the violations is not possible, a final power system state can result which is less secure than the initial one. To overcome this, the severity index $\eta$ of the contingency under consideration is determined [12] both before application of any control actions and after each iteration of the expert system where

$$\eta = \sum_{i=1}^{L} \left( \frac{\Delta V_{i}}{\Delta V^{max}} \right)^{n}$$

and $\Delta V_{i}$ is the deviation of the voltage magnitude at bus $i$ from nominal, $\Delta V^{max}$ is the maximum permissible deviation and $n$ is some chosen integer.

4.4 Limiting the number of controllers

Application of the rules and defuzzification of setting would result in non-zero control actions for every controller that would have any effect on the relief of any load bus voltage violation. However, the operator is likely to want to restrict the number of control moves to limit the number of communications needed and prevent any possible transient problems caused by the largely unpredictable interaction of the changes over the immediate post-action period. Since the limit on the number of moves may be too low to enable the removal of all the violations, only the initial number of controller actions is restricted and freedom is given for additional controllers to be selected by further iterations of the expert system where necessary to remove remaining violations. As an alternative to the user setting the limit, a function

$$n_{c} = f(\omega_{c}, N, \eta)$$

(16)

may be defined where $n_{c}$ is the initial maximum number of controllers, $N$ is the total number of controllers, $\eta$ is the contingency severity index and $\omega_{c}$ is some weighting. In this way, the initial limit on the number of controllers is appropriate to the severity of the system insecurity.

The controllers are ranked in order of suitability according to the cardinality of each controller's fuzzy setting which is found using equation (2).

4.5 Prevention of new violations

The effects of the set of chosen defuzzified controller settings are tested using the sensitivity matrix on all the load bus voltage magnitudes to ensure that the combined actions do not aggravate existing violations or create new ones.

Some control actions $\Delta U_{ij}$ will raise $V_{i}$, while others will lower it. If the net effect $\Delta V_{i}$ is to raise $V_{i}$ above the nominal upper voltage limit, then the individual control actions $\Delta U_{ij}$ which cause it to rise are reduced in proportion. Likewise, if $V_{i}$ is predicted to fall below the nominal low voltage, all $\Delta U_{ij}$ which have the effect of lowering $V_{i}$ are reduced. Where $V_{i}$ is already violated, the control actions are adjusted if necessary so as not to make the violation any worse.
4.6 The final system

The overall scheme is illustrated in Figure 3.

![Figure 3: Outline of the new fuzzy expert system](image)

If the user has not set a limit on the number of controllers to be used, a limit \( n_c \) is calculated based on the severity index. The rules of the dispatch system are then applied. If any violations remain and the improvement in severity index from the last iteration is below a threshold, an extra control in addition to those used at previous iterations is allowed. If, however, the severity index has deteriorated, the solution from the last iteration is retained and the expert system exited.

5 Results

The new fuzzy expert system (method NEW) was run to suggest control actions to relieve voltage violations caused by each of a full set of single and double circuit contingencies applied to three scenarios with progressively higher loadings, A, B and C, on a 20 machine, 100 busbar reduction of the U.K. National Grid. 6 contingencies caused voltage violations on scenario A, 10 on scenario B and 20 on scenario C.

<table>
<thead>
<tr>
<th>Case</th>
<th>ave %</th>
<th>( f(x) )</th>
<th>( n_c )</th>
<th>( n_u )</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEW</td>
<td>A</td>
<td>24.060</td>
<td>0.086</td>
<td>6</td>
<td>1.000</td>
</tr>
<tr>
<td>LP</td>
<td>A</td>
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<td>0.939</td>
<td>6</td>
<td>1.333</td>
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<td>22.310</td>
<td>0.077</td>
<td>10</td>
<td>1.300</td>
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<tr>
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<td>0.955</td>
<td>10</td>
<td>1.000</td>
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<td>0.111</td>
<td>18</td>
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<td>C</td>
<td>13.697</td>
<td>0.959</td>
<td>17</td>
<td>0.765</td>
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</tbody>
</table>

Table 1: Results for 20 machine 100 bus reduced U.K. system

The results are presented in Table 1 along with results obtained for the same scenarios using a linear programming based reactive dispatch routine similar to that in [1] (method LP). The average execution time of each dispatch is shown along with the average improvement in severity index \( f(x) \) obtained by the actions suggested, the average number of controls used \( n_c \), the number of converged solutions \( n_u \), and the average cost.

6 Conclusions

Table 1 shows that the new fuzzy expert system achieves results comparable with those obtained by a linear programming approach but in a fraction of the time for a reasonably sized system. This promises that the method will prove practicable on a realistic system both for on-line use and planning, and for co-ordination of preventative and corrective actions.

The fuzzy expert system allows easier adjustment of the objectives by the user than an equivalent linear programming based routine. A limit on the number of controllers used can be easily set and the deadbands of the linguistic variables changed intuitively to reflect different operational priorities. Such ease of adjustment and the iterative solution method makes the method more likely to reach a good solution.

References