PHD

The calibration of acoustic sources in underwater restricted environments

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Abstract

Underwater electroacoustic transducers are conventionally calibrated in laboratory tanks by using pulsed sinusoidal waves and time-gating the signals to obtain the direct path signal, before any reflections from the tank walls arrive. This enables the steady-state response of the transducer to be obtained, which, together with the voltage applied, can be used to calibrate the transducer and obtain the transmitting voltage response (TVR). However, for low frequencies and high $Q$ projectors, the time needed to reach a steady-state response can be greater than the time available before the first reflection arrives. For signals in this range, an alternative to expensive lake or sea trials is needed in the laboratory.

The reverberant field approach that has been developed relies on a continuous-wave noise signal being radiated into the tank, and enables all the tank modes to be excited over the frequency range used. The sound field is then sampled at a series of separations from the source, to produce an averaged pressure spectrum for each position. These are then used to plot a graph of pressure squared versus the reciprocal of separation squared, for each frequency within the frequency range. From the gradient and y-intercept of the graph, an estimate of the projector TVR is calculated at each frequency. Also an estimate of the TVR can be made from the spatially-averaged value of pressure squared over all the hydrophone locations.

An extensive programme of measurements in eight different tanks, of different size and construction, and with different sound sources, has been performed. The measurements were made over a frequency range of 10kHz to 100kHz. These show that the gradient method is in good agreement with a reference calibration to within 1(±1)dB for the majority of measurements, where the direct sound field is dominant. The spatially-averaged pressure squared method has produced results accurate to within 0.2(±2)dB, when the reverberant field is dominant.
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<td>A</td>
<td>amplitude of vibration</td>
</tr>
<tr>
<td>A</td>
<td>absorption of sound by the chamber</td>
</tr>
<tr>
<td>A</td>
<td>absorption of the surface area of the water</td>
</tr>
<tr>
<td>A</td>
<td>amplitude response in the directions ( \phi ) and ( \theta )</td>
</tr>
<tr>
<td>a</td>
<td>equilibrium radius of vibrating surface</td>
</tr>
<tr>
<td>a</td>
<td>diameter of piston face</td>
</tr>
<tr>
<td>( \hat{A} )</td>
<td>average amplitude response over the whole surface area</td>
</tr>
<tr>
<td>( \hat{a} )</td>
<td>average absorptivity of the surface of the chamber</td>
</tr>
<tr>
<td>( \hat{a}_a )</td>
<td>absorptivity of the air</td>
</tr>
<tr>
<td>( a_n )</td>
<td>absorptivity of region ( n )</td>
</tr>
<tr>
<td>( \hat{a}_{ak} )</td>
<td>average absorptivity of the tank</td>
</tr>
<tr>
<td>B</td>
<td>susceptance</td>
</tr>
<tr>
<td>C</td>
<td>electrical capacitance</td>
</tr>
<tr>
<td>C</td>
<td>electrical capacitance of the resonant system of a piezoelectric ceramic</td>
</tr>
<tr>
<td>C</td>
<td>( y )-intercept of pressure squared versus the reciprocal of separation squared graph (( y = mx + C ))</td>
</tr>
<tr>
<td>c</td>
<td>speed of sound, speed of sound in the fluid</td>
</tr>
<tr>
<td>( C_a )</td>
<td>acoustic capacitance</td>
</tr>
<tr>
<td>( C_m )</td>
<td>compliance</td>
</tr>
<tr>
<td>( C_0 )</td>
<td>electrical capacitance of piezoelectric ceramic (static capacitance)</td>
</tr>
<tr>
<td>D</td>
<td>electric displacement</td>
</tr>
<tr>
<td>d</td>
<td>piezoelectric constant ( d )</td>
</tr>
<tr>
<td>d</td>
<td>distance from the acoustic centre of a projector</td>
</tr>
<tr>
<td>d</td>
<td>separation of projector and hydrophone acoustic centres</td>
</tr>
<tr>
<td>d</td>
<td>logarithmic decrement (decay rate in Np / cycle)</td>
</tr>
<tr>
<td>( \frac{dE}{dt} )</td>
<td>rate at which energy falls on to a unit area of the boundary</td>
</tr>
<tr>
<td>( d_i )</td>
<td>difference between the ( i^{th} ) reverberant field calibration TVR (( G, Y ) or ( S )) and the ( i^{th} ) free-field calibration TVR</td>
</tr>
<tr>
<td>( d_{\text{min}} )</td>
<td>minimum distance between the sound source and microphone</td>
</tr>
</tbody>
</table>
needed for the measurement of the reverberant field

\[ \frac{dN}{df} \] number of normal modes per Hz

d_{PH} separation of acoustic centres of the projector and hydrophone
d\(V\) element of volume in the media
d\(0\) reference distance at which the transmitting pressure is specified in the definition of \(S_F\) (usually 1m)
d\(1\) separation of acoustic centres of the projector to the transducer and hydrophone; and the transducer to the hydrophone (reciprocity)
d' logarithmic decrement (decay rate in dB / cycle)
E electric field
E_{AS} energy that will strike \(\Delta S\) by direct transmission
F force
f frequency of vibration, acoustic frequency, frequency
f\(_c\) Schroeder frequency
f\(_c\)(air) Schroeder frequency for air (four modes per modal bandwidth), (also equals \(f_0\))
f\(_c\)(water) Schroeder frequency for water (four modes per modal bandwidth)
FFR\(_i\) \(i^{th}\) TVR of the free-field calibration
f\(_m\) maximum frequency
f\(_0\) resonant frequency, gives the frequency for which there are \(n\) modes in a bandwidth (for an enclosure with a reverberation time \(T_{60}\) and speed of sound \(c\) (Schroeder))
G conductance
G gradient of the logarithm to base ten of the sound intensity (pressure squared) versus time
G gradient derived reverberant field TVR estimate
g piezoelectric constant \(g\)
gmd mean difference derived from the gradient estimates
H hydrophone
I current
I acoustic intensity
I spatially averaged reverberant sound intensity
i subscript matrix notation, \(i = 1...6\)
i: \( \sqrt{-1} \)

\( I_d \): direct sound field intensity

\( I_h \): hydrophone input current

\( I_p \): projector input current

\( I_r \): reverberant sound field intensity

\( I_T \): transducer input current

\( J \): reciprocity parameter

\( j \): subscript matrix notation, \( j = 1...6 \)

\( j : \sqrt{-1} \)

\( k \): angular wavenumber

\( L \): electrical inductance

\( L \): electrical inductance of the resonant system of a piezoelectric ceramic

\( L \): inductance of a circuit

\( L_{eq} \): equivalent 'equal length' of enclosure, \( L_{eq} = V^{\frac{1}{3}} \)

\( L_x \): length of a rectangular enclosure

\( L_y \): width of a rectangular enclosure

\( L_z \): depth of a rectangular enclosure

\( L/\lambda \): number of wavelengths in the tank length or diameter

\( M \): free field voltage sensitivity

\( M \): hydrophone receive sensitivity

\( M \): receiving sensitivity

\( m \): mass

\( m \): subscript matrix notation, \( m = 1...3 \)

\( m \): gradient of pressure squared versus the reciprocal of separation squared graph \( (y = mx + C) \)

\( m_a \): acoustic mass

\( m_d \): mean difference between the reverberant field calibration and the free-field calibration

\( M_h \): free-field voltage sensitivity of the hydrophone

\( M_c \): free-field voltage sensitivity of calibrated hydrophone

\( M_s \): free-field voltage sensitivity of standard hydrophone

\( M_T \): free-field voltage sensitivity of the transducer

\( M_x \): free-field voltage sensitivity of uncalibrated hydrophone
N number of normal modes below frequency, f, in a chamber
N number of normal modes
n subscript matrix notation, n = 1...3
n modal frequencies
n number of frequency points in the calibration
P pressure amplitude
P projector
P combined pressure due to the direct and reverberant sound fields
P pressure
p acoustic pressure
p instantaneous pressure
Pa direct field pressure, at one metre from the projector, averaged over the whole surface area of the projector (i.e. averaged over 4πSr)
Pd effective pressure amplitude produced by the direct sound field (direct pressure)
Pd direct field pressure
Pei effective pressure amplitude of the 1th sound ray

Pp free-field sound pressure produced by the projector at the hydrophone or transducer
Pr spatially averaged effective pressure amplitude of the reverberant sound field (spatially averaged reverberant pressure)
Pr reverberant field pressure
PSA spatially averaged pressure amplitude
PT free-field sound pressure produced by the transducer at the hydrophone or projector
P0 direct field pressure, at one metre from the projector, in the ‘0’ mark direction
Q quality factor of resonator
Q quality factor
q charge
R electrical resistance
R resistance of circuit
r distance from the centre of the source
r radial distance from the centre of the piston
distance from dV to ΔS
radial distance from the effective centre of the source
separation of acoustic centres of projector and hydrophone
transducer separation
acoustic resistance (radiation resistance)
experimental ratio of direct field power to reverberant field power
experimental ratio of direct field pressure to reverberant field pressure
i\textsuperscript{th} TVR of the reverberant field calibration (G, Y or S)
mechanical (resistive) loss of the resonant system, of the piezoelectric ceramic, when resonating
distance from source of n\textsuperscript{th} maximum in axial pressure
acoustic radiation resistance of the transducer / water system
when resonating
mechanical resistance (Friction)
Rayleigh distance
theoretical ratio of direct field power to reverberant field power
theoretical ratio of direct field pressure to reverberant field pressure
ratio of the theoretical to experimental ratio of direct to reverberant field pressure at the minimum transducer separation
resistance of piezoelectric ceramic
ratio at the maximum transducer separation
ratio at the minimum transducer separation
strain
projector transmitting voltage response
sweep rate
transmitting sensitivity
surface area of the enclosure
spatially averaged pressure derived reverberant TVR estimate
standard deviation between the reverberant field calibration and the free-field calibration
surface area of the water in contact with the air
spatially averaged pressure
mean difference derived from the spatially averaged pressure estimates
\[ S_n \] surface area for a region \( n \) on the surface of the chamber

\[ S_P \] transmitting current response of the projector

\[ S_T \] transmitting current response of the transducer

\[ S_T \] total surface area of the chamber

\[ S_{ik} \] surface area of the water in contact with the tank

\[ S_x \] transmitting voltage response of the projector

\( T \) period of sinusoidal waveform

\( T \) stress

\( T \) transducer

\( T \) period of oscillation

\( t \) time

\( T_r \) reverberation time of the chamber

(intensity level drops 60dB over \( T_r \))

\[ TVR_{ya} \] y-intercept transmitting voltage response calculated from the average direct field pressure, \( P_a \)

\[ TVR_{yo} \] y-intercept transmitting voltage response corrected for the ‘0’ mark direction

\( T_{60} \) 60dB reverberation time

\( u \) particle velocity

\( U_0 \) speed amplitude of vibrating surface

\( V \) voltage

\( V \) electrical potential difference

\( V \) volume of fluid in the chamber

\( V \) volume

\( V \) volume of the chamber

\( V \) volume of the enclosure

\( v \) velocity

\( V_{PH} \) hydrophone output voltage due to input signal to projector

\( V_{PT} \) transducer output voltage due to input signal to projector

\( V_r \) open circuit output voltage of calibrated hydrophone

\( V_s \) open circuit output voltage of standard hydrophone

\( V_{TH} \) hydrophone output voltage due to input signal to transducer

\( V_{TP} \) projector output voltage due to input signal to transducer

\( V_x \) open circuit output voltage of uncalibrated hydrophone
\( V_x \)  
\text{driving voltage of the projector}

\( W \)  
\text{acoustic power}

\( W \)  
\text{acoustic power radiated by the source}

\( X \)  
\text{reactance}

\( x \)  
\text{displacement}

\( x \)  
\text{reciprocal of transducer separation squared \( x = 1/x^2 \)}

\( x \)  
\text{x-axis of pressure squared versus the reciprocal of separation squared graph \( y = mx + C \)}

\( x_v \)  
\text{volume displacement}

\( X-Y \)  
\text{indicates the horizontal plane of a Cartesian coordinate system}

\( X-Z \)  
\text{indicates the vertical plane of a Cartesian coordinate system}

\( Y \)  
\text{Admittance}

\( Y \)  
\text{y-intercept derived reverberant field TVR estimate}

\( y \)  
\text{pressure squared \( y = P^2 \)}

\( y \)  
\text{y-axis of pressure squared versus the reciprocal of separation squared graph \( y = mx + C \)}

\( ymd \)  
\text{mean difference derived from the y-intercept estimates}

\( Z \)  
\text{impedance}

\( Z_a \)  
\text{acoustic impedance}

\( Z_m \)  
\text{mechanical impedance}

\( Z_{PH} \)  
\text{transfer impedance when transmitting with the projector and receiving with the hydrophone}

\( \alpha \)  
\text{number of steady state cycles}

\( \Delta E \)  
\text{acoustic energy contributed to \( \Delta S \) by the entire shell}

\( \Delta f \)  
\text{frequency range of bandwidth}

\( \Delta f \)  
\text{modal bandwidth}

\( \Delta N \)  
\text{number of normal within the bandwidth \( \Delta f \), centred on \( f \)}

\( \Delta N/\Delta f \)  
\text{number of normal modes per unit frequency}

\( \Delta r \)  
\text{thickness of hemispherical shell}

\( \Delta R^* \)  
\text{error in the ratio}

\( \Delta S \)  
\text{element of a boundary}
\( \Delta S \) element of surface area
\( \Delta t \) time interval
\( \Delta x \) size of surface area element, \( \Delta S \), in \( X-Y \) plane
\( \Delta y \) size of surface area element, \( \Delta S \), in \( X-Z \) plane
\( \Delta \theta \) angle subtended by surface area element, \( \Delta S \), in \( X-Z \) plane
\( \Delta \phi \) angle subtended by surface area element, \( \Delta S \), in \( X-Y \) plane
\( \overline{\delta} \) average modal spacing
\( \varepsilon \) permittivity
\( \varepsilon \) acoustic energy density
\( \varepsilon_1 \) energy density of \( 1^{\text{th}} \) ray
\( \theta \) angle of inclination from the axis, angle made between \( r \) and the normal to \( \Delta S \), angle in the \( X-Z \) plane
\( \lambda \) wavelength
\( v \) auxiliary variable, inverse of the time taken for sound to travel \( L_{eq} \),
\( v = c/L_{eq} \)
\( \rho \) density of water
\( \rho_0 \) density of fluid or medium
\( \tau_c \) calibration time
\( \tau_E \) time constant that governs the growth and decay of acoustic energy in the chamber
\( \tau_r \) rise time of resonator
\( \phi \) phase angle
\( \phi \) angle in the \( X-Y \) plane
\( \omega \) angular frequency

'\( 0 \)' a 'zero' marked on a transducer indicating a reference direction for calibration
Abbreviations

SONAR   SOurn Navigation and Ranging
PZT     Lead Zirconate Titanate
NPL     National Physical Laboratory
ADP     Ammonium Dihydrogen Phosphate
TVR     Transmitting Voltage Response
ISO     International Standards Organisation
PWM     Plane Wave Model
ANSI    American National Standards Institute
RS      Receive Sensitivity
IEC     International Electrotechnical Commission
IEEE    parallel digital electronic data link (standard of the Institute of
        Electrical and Electronic Engineers, Inc. (U.S.))
FFT     Fast Fourier Transform
SIL     Sound Intensity Level
SPL     Sound Pressure Level
RS232   serial digital electronic data link
ADC     Analogue to Digital Converter
ASCII   American Standard Code for Information Interchange
ITC     manufacturer of transducers (International Transducer Corporation)
SA      Spectrum Analyser
OSC     Oscilloscope
BK      Brüel & Kjær
Sonardyne Sonardyne International Ltd.
ROV     Remotely Operated Vehicle
1.0. Introduction

Acoustics can be divided into a number of areas, depending on the medium the sound is travelling through and the frequency range used. One possible structure is to roughly divide acoustics up into airborne acoustics (environmental and architectural), ultrasonics (medical, engineering and materials), underwater acoustics (SONAR – military, fishing, geophysics and sea bed) and seismology (land and sea). The work in this thesis comes under the underwater umbrella, with an investigation of a new method of calibrating underwater transducers in restricted environments.

This chapter contains four sections, the first of which is a brief overview of the historical development of underwater acoustics and its applications. The second section is an introduction into the conventional method of underwater transducer calibration and why it is necessary. The third section introduces alternative methods of calibration and the fourth gives an overview of the thesis structure.

1.1. History and use of underwater sound

One of the earliest references to underwater sound was made by Leonardo da Vinci in 1490, where he notes that, if a ship is at rest, other ships can be heard if one end of a long tube is placed in the water and the other end in the ear. Probably the first quantitative measure of underwater sound was made in 1827 by Daniel Colladon, a Swiss physicist, and Charles Strum, a French mathematician, when they measured the velocity of sound in Lake Geneva, Switzerland. They did this by flashing a light and striking a bell underwater at the same time, and then timing the interval between the arrival of the light and the sound.

At the beginning of the twentieth century the first practical application in underwater sound was devised, when the submarine bell was used by ships for offshore navigation. A ship could determine its distance from a lightship, where a sound from the submarine bell and a blast from a foghorn were set off simultaneously, by timing the interval between the arrival of the two sounds (Urick, 1983). Underwater sound,
from these bells, could be detected at considerable distances by the use of a microphone mounted on the ships hull. The approximate direction of the sound could be determined by placing a microphone on opposite sides of the hull, and then sending the sound received by each device to the right and left ears of a listener (Kinsler, Frey, Coopen and Sanders, 1982).

In 1912 the steamship Titanic collided with an iceberg with the loss of hundreds of lives; this subsequently triggered the use of sound for sensing in the sea. L. A. Richardson filed a patent with the British Patent Office, five days after the collision of the Titanic, for echo ranging with airborne sound. A month later, on 10th May 1912, Richardson also filed another patent for the same method applied to underwater sound. R. A. Fessenden, who had been working on the same problem, filed for a U.S. patent on the 29th January 1913 and succeeded in detecting an iceberg at a distance of two miles on 27th April 1914. He used a new kind of moving-coil transducer for both submarine signalling (using Morse code) and echo location.

The outbreak of World War I created an urgent need for submarine detection, and stimulated a Russian engineer, Constantin Chilowsky, who worked with the French physicist Paul Langevin to develop an underwater source that transmitted sound across the Seine in Paris during the winter of 1915-1916. Chilowsky and Langevin used a condenser (electrostatic) projector and a carbon-button microphone placed at the focus of a concave mirror. A major step forward was made when Langevin turned to the piezoelectric effect and used a quartz-steel sandwich to replace the condenser projector as a sound source, and also employed one of the newly developed vacuum amplifiers. The high intensity of this sound generator enabled transmission of sound to a range of 8km, and, in 1918, produced the first detection of an echo from a submarine. Occasionally the echoes of submarines were received up to distances of 1,500 metres, but the extensive use of SONAR (SOund NAvi-gation and Ranging) for submarine detection had to wait until World War II (Clay and Medwin, 1977; Urick, 1983).

After the first world war, depth sounding by ships was developed and by 1925 the “fathometer” was devised by the Submarine Signal Company. In the United States, H. C. Hayes and others at the Naval Research Laboratory searched for practical
means of echo ranging on submarine targets. They solved the problem of finding a suitable sound projector for echo ranging by resorting to magnetostrictive projectors for generating the required acoustic power. However, piezoelectric materials were still widely used, and synthetic crystals of Rochelle salt and ammonium dihydrogen sulphate began to replace scarce natural quartz (Urick, 1983). Later on the piezoelectric materials lead titanate and lead zirconate titanate (PZT) began to be used for most transducers due to there ability to withstand very high potential differences and thus produce large sound pressures (Halmshaw, 1991).

Between the wars, SONAR improved by the use of advances in electronics, which enabled greater amplifying ranges to be used on signals as well as better processing and displaying sonar information. With these improvements in electronics and in acoustic signal processing, greater ranges could be achieved and smaller targets identified. Ultrasonic frequencies started to be used for listening and echo ranging, and enabled an increase in directionality to be obtained. Also during the war years, echo ranging of the sea depth was first used to find schools of fish.

By 1935, several accurate SONAR systems had been developed, and by the beginning of World War II, a large number of U.S., and other countries, ships were equipped for both underwater listening (passive) and echo ranging (active). Probably the most notable scientific accomplishment between the two world wars was an understanding of the propagation of sound in the sea. Thermal gradients in the sea refract the sound into its depths, and cause "shadow zones" where a target ship (submarine) can lie and not be detected. Thermal gradients also produce wave-guides within the sea, that enable very low frequency sound, attenuated very weakly, to travel around the world (Urick, 1983).

MacLean (1940) and Cook (1941) independently devised methods for calibrating electroacoustic transducers by using the reciprocity principle. This proved to be a breakthrough in the science of calibrating transducers, since only electrical measurements and a few easily determined constants are required to produce accurate calibrations (Bobber, 1970).
The sea is essentially impenetrable to electromagnetic radiation such as visible light, infrared, radio and radar. At present acoustic signals are the only feasible method of transmitting information by waves through water at distances beyond a few metres. Electroacoustic transducers are the most practical solution for sensing underwater sounds, and generally for producing well defined signals (Brüel and Kjær).

Sound propagation and measurement in water are used for a variety of applications. These include ship noise assessment, navigation of ships, location of fish and other targets, hydrographic surveys, civil engineering, oil and geophysical research, exploration of sediments and rocks in the sea, sea bed classification, the study of marine life and communication, whether it be speech or data transmission (telemetry) (Brüel and Kjær; Tucker and Gazey, 1966). Furthermore, ultrasound is used in several roles, with two of the most important being the medical field and the engineering field (non-destructive testing or evaluation).

1.2. Calibration of Transducers

Underwater transducers are usually electroacoustic transducers, where an input voltage is applied and an output pressure is produced. They also work in the reverse direction, where an input pressure produces an output voltage. The calibration of underwater transducers is needed, as in other areas of science, so that accurate and reproducible measurements can be made. The calibration of a transducer enables a measured output voltage to be converted to a specific incident pressure and vice versa. As the calibration of a transducer will change with frequency, sinusoidal waveforms are used to perform the calibrations at a series of frequencies.

The standard method for calibrating a transducer is to suspend two transducers in a body of water where one acts as a transmitter, a projector, and the other acts as a receiver, a hydrophone. Calibrations need to be carried out in free-field conditions, that is, where the signal received by the hydrophone comes directly from the projector and with no unwanted reflected signals from the boundaries of the body of water. It is the signal directly transmitted from the projector that is required for a calibration; the reflected signals are additional signals and the amplitude of these will have been reduced at the boundaries of the enclosure.
Free-field conditions are obtained by either using a very large expanse of water, or pulsing and time-gating the signals. A large expanse of water, such as a lake or the sea, means the acoustic wave has to traverse a large distance before it is reflected off a boundary and received back at the hydrophone. This large distance attenuates the signal, so that when it is received back, it is very small and can be ignored compared to the outgoing signal from the projector. Time-gating of a signal is used in small expanses of water such as a laboratory tank, where the attenuation of the reflected signals is very small for one reflection. The signal sent to the transmitter is a pulsed sinusoidal signal, where the amplitude envelope of the sinusoid is large for a very short time and small for a long time. This means the transmitter radiates a short "tone" burst, and is then idle for a long time before the signal is repeated. The long gap ensures that the multiple echoes from the tone burst have decayed before the next signal is transmitted. The received signal contains the signal direct from the projector and then a series of signals reflected off the walls of the tank. This signal is time-gated so that only the direct signal is captured and so a calibration can be made in free-field conditions.

The signals travelling between the transmitter and receiver are grouped into two categories as shown in Figure 1.1. First, there is the single direct path signal travelling in a straight line between the transducers, and second, the many reflected signals travelling by different paths. The reflected signals can bounce off the walls of the tank, at the water/air interface and off the transducers. The paths may include only one reflection, or many reflections, off one or more of the objects listed.
The direct path signal arrives first since it travels in a straight line and then the reflected signals arrive according to their path length. There is a time difference between the arrival of the direct path signal and the first reflected signal, which is called the 'free time'. The value of the free time depends on the volume of the water in the tank, the shape of the tank and the positions of the transducers in the tank.

There is a minimum time needed to perform a calibration on a direct path signal, called the calibration time. This is the time needed for the projector, or resonator, to reach a steady-state pressure output when excited by a voltage. This comprises of two parts, the rise time of the resonator, $\tau$, and a few cycles of the steady state signal in order to accurately measure the amplitude of the sinusoidal signal. The rise time can be defined as the product of the quality factor of the resonator, $Q$, and the period of the sinusoidal waveform, $T$ (Bobber, 1970: 169; Briel and Kjaer: 19; Kuntsal and Bunker, 1992). The calibration time is therefore $\tau_c = \tau_r + \alpha T = (Q + \alpha)T$, where $\alpha$ is the number of steady-state cycles of the signal required for the calibration. To perform a standard calibration in a tank, the free time must be equal to or larger than the calibration time. If it is not, then reflected signals will contaminate all, or part, of the steady-state section of the signal.
The free time in laboratory sized tanks is sufficient for most projectors, or acoustic sources, to be calibrated at most frequencies. However for a high $Q$ projector and low frequencies this is not the case and free-field conditions do not exist. As an example of typical $Q$ values for projectors, the values of the projectors used in this project ranged from 2.7 to 3.5 (calculated from the conductance curve of the projectors used in this project (Kuntsal and Bunker, 1992)).

1.3. Calibrations in non free-field conditions

There are two categories of calibrations that can be made in non free-field conditions, dependent on the amount of reflected signal present in the direct signal. The first case applies to time-gated signals, where the steady-state part of the signal is contaminated with one or two reflections. There are various methods of removing the reflected signals and, therefore, revealing the direct-path signal in order to perform a calibration. An overview of these methods will be given in the literature review (Chapter 2).

The second case is when a continuous-wave signal is produced by the projector and causes multiple reflections. A continuous-wave signal radiates out from the transmitter as a spherical wave front and is called the direct field. When the spherical wave front reaches the walls of the tank, it is partially reflected, and the many reflected waves interact to produce standing waves in the tank. The amplitude of the standing waves increases until the rate of absorption of the waves and the losses at the surfaces of the tank equal the rate of acoustic energy radiated into the tank. All the reflected waves, whether they form standing waves or not, are collectively called the reverberant sound field.

Calibrations are referred to as taking place in a restricted environment when they take place in an enclosure small enough to have significant sized reflected signals. A reverberant field is present, which can range from a few reflections to a large amplitude standing-wave field.

This project concerns a method of calibrating transducers in tanks where such a reverberant field is present, produced by a continuous wave signal sent to a projector.
The aim was to develop a method of calibration for low frequency signals, and high Q projectors, in tanks where the free time was insufficient for a standard calibration. For such low frequency signals, and high Q projectors, the direct part of a time-gated signal would be quickly swamped by many reflections. The methods used to recover the direct-path component from a signal contaminated with a few reflections would, therefore, not work. The method discussed in this thesis offers a way of calibrating this category of projector in a laboratory tank instead of the only other alternative, an expensive lake or sea trial.

Measurements were carried out in the laboratory tanks with projectors which had normal Q values, and at the standard frequencies used in these tanks. This then allowed conventional calibrations to be performed, so that the two methods could be compared. The accuracy of the new reverberant calibration method could then be established.

1.4. Structure of thesis

Before a more detailed explanation of calibration and the reverberant calibration method used in this thesis is given, a review of the background acoustic knowledge is necessary to fully understand the work. Chapter 2 contains a review of the background acoustics and a literature review on underwater calibration and reverberant fields in air acoustics. The underwater section contains information on calibrations in tanks with insufficient free times and calibrations in tanks with reverberant fields. Chapter 3 explains the theory of underwater free-field calibrations and the experimental equipment and methods employed in this project. It also contains the calibration and directional data measured for the transducers used in this work.

The reverberant field method used in this project requires the measurement of the reverberation times of the tanks used for a range of frequencies. This aspect of the project is therefore presented before the reverberant calibration method. Chapter 4 thus contains the theory of reverberation time decay and measurement, and also the experimental details of all the methods employed. This chapter also describes the different stages of the processing used on the experimental data for each method.
Chapter 5 contains the reverberation time experimental results and an analysis of these results for all the tanks measured.

The theory for the reverberant calibration method, and methods for analysing the acoustic fields, are presented in chapter 6 along with an explanation of the experimental equipment developed for these measurements. The procedure for carrying out the calibration measurements is explained, as is the procedure for processing the experimental data to produce a calibration. Measurements were taken in tanks at three sites: Bath University, the National Physical Laboratory (NPL) and Sonardyne International Ltd. A rig was developed at Bath and also used for the Sonardyne measurements. The measurements taken at NPL were performed on their equipment, which was altered to suit the calibration procedure.

The different stages of processing the experimental data are shown in chapter 7, along with details of the measurements taken in terms of the tanks and measurement parameters used. The reverberant calibration method produces three calibrations based on different procedures for analysing the data. The reverberant field calibration results for the three types of calibration are compared for the different tanks and measurement parameters. An overall measure of the variation between the calibrations and the reference free-field calibration is needed. This is achieved by calculating the mean difference between the two calibrations. The mean difference results are then shown for the different tanks and parameters selected, which gives a far more compact and overall picture of the results. The effect of transducer directionality on the results is investigated next, as is the effect of different sound field parameters. Finally, Chapter 8 discusses the results and makes conclusions about the application of this type of calibration procedure.
2.0. Background and Literature Review

The background knowledge for this thesis is presented in this chapter and covers the topics of underwater acoustic calibration and the basic physics of underwater transducers. This chapter also presents the literature review on underwater calibration and the acoustics of reverberant sound fields.

2.1. Background

This background section firstly covers electroacoustic transducers, which are used almost universally for underwater acoustic work, and details the main aspects of these transducers that need to be known for calibrations. Secondly it describes the acoustic theory underlying underwater calibrations. It includes sub-sections on piezoelectricity, equivalent circuits, transducer sensitivity and response, acoustic radiation from a sphere and the proximity criterion for transducers.

2.1.1. Piezoelectricity

An electroacoustic transducer converts electrical energy into acoustical energy and vice versa. Underwater transducers are electroacoustic and most usually have a piezoelectric active element, as opposed to a magnetostrictive one. The term piezoelectric is used for both single crystal materials and polarised polycrystalline ceramics. Single crystals like Rochelle salt, quartz, tourmaline, ammonium dihydrogen phosphate (ADP) and lithium sulphate display a piezoelectric effect due to the inherent asymmetry of the natural crystal structure. In polarised polycrystalline ceramics such as barium titanate (BaTiO₃) and lead zirconate titanate (PZT) the piezoelectric properties are produced by poling in the manufacturing process (Bobber, 1970).

The polycrystalline ceramics are manufactured using sintered ferroelectric ceramics, which are polarised by the application of a strong electric field (in excess of 1000 Vcm⁻¹), applied across the thickness of the sample above the Curie temperature. This
aligns the principal axis of the small crystals which then become ‘frozen’ as the material cools below the Curie temperature, and the plate behaves as if it is a disc of piezoelectric material (Halmshaw, 1991).

The piezoelectric effect is used to describe the effect where the surfaces of a crystal or ceramic become oppositely electrically charged when subjected to stress; the sign of the charge changes when the compression becomes tension. The converse effect, in which the crystal or ceramic expands along one axis and contracts along the other when subjected to an electric field, also occurs.

Piezoelectric ceramics can be easily manufactured and made into a variety of shapes as required. They tend to have a lower hardness than crystals, and have a lower resistance to wear, but can be driven at high voltages, and so generate a much greater acoustic output. However the ceramic materials can also suffer from ageing, where they become de-polarised over time. Most piezoelectric materials, crystal or ceramic, have a large acoustic impedance and so are not well matched to water.

All piezoelectric materials have certain properties, aside from stability, that affect their suitability as an electroacoustic element in a measurement transducer. These properties include the piezoelectric constants, dielectric constant, resistivity, and the fact that the crystals and ceramics are anisotropic. Piezoelectric constants are used to describe the relationship of an electrical parameter, such as charge density or electrical field, and a mechanical parameter, such as stress or strain. Since the material is anisotropic, the direction of the electrical and mechanical parameters is specified by crystallographic notation.

Tensor notation and matrix notation are used to describe crystal directions. In tensor notation subscripts 1, 2 and 3 are used to describe the mutually perpendicular directions, with combinations of two different numbers to describe shear motion. In matrix notation subscripts 1, 2 and 3 also describe the mutually perpendicular directions, but subscripts 4, 5 and 6 pertain to shear motions about the 1, 2 and 3 axes respectively. Figure 2.1 shows linear and shear motion, described in tensor and matrix notation.
Figure 2.1. Straight-line motion in axis 1 only: $T_{11}$ (tensor) or $T_1$ (matrix). Rotational or shear motion in the axes 2 and 3 around and perpendicular to axis 1: $T_{23}$ or $T_{32}$ (tensor) or $T_4$ (matrix).

The relationship between tensor and matrix notation is shown in Table 2.1.

<table>
<thead>
<tr>
<th>Tensor notation</th>
<th>11</th>
<th>22</th>
<th>33</th>
<th>23, 32</th>
<th>31, 13</th>
<th>12, 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix notation</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2.1. Relationship between tensor and matrix notation.

The piezoelectric constant $d$ is defined (Mason, 1964) by

$$d_{mj} = \frac{\partial D_m}{\partial T_j} = \frac{\partial S_j}{\partial E_m}$$  \hfill (2.1)
where the subscript \( m = 1..3 \), the subscript \( j = 1..6 \), \( D \) is the electric displacement, \( T \) is the stress, \( S \) is the strain and \( E \) is the electric field. The electric parameters \( D \) and \( E \) are first-rank tensors and are represented in matrix notation by the numbers 1 to 3. The mechanical parameters \( T \) and \( S \) are second-rank tensors and are represented in matrix notation by the numbers 1 to 6. The piezoelectric constants \( d \) and \( g \) are related (Mason, 1964) by

\[
d_{mi} = \varepsilon_{nm}^T g_{ni}
\]

where the subscripts \( m, n = 1..3 \), the subscript \( i = 1..6 \), \( T \) is stress and \( \varepsilon \) is the permittivity.

Manufacturers of piezoelectric ceramics quote the constants for the important crystal directions, and describe the simple case of a slab of piezoelectric material. The constant \( g \) is then defined, for a one dimensional case where the ceramic is infinite, as \( g = E/T \) (Vernitron). This definition is similar to the receive free-field voltage sensitivity \( M = V/p \), where \( V \) is the output voltage and \( p \) is the incidental pressure. Consequently, the \( g \) constant is related to hydrophone sensitivity and serves as the most useful criterion of a piezoelectric material for use in measurement hydrophones.

2.1.2. Equivalent Circuit of a Transducer

Piezoelectric and electrostrictive transducers can be considered to consist of a static capacitance, \( C_0 \), and a mechanical oscillator. The current flowing in the oscillator is independent of the current flowing in the static capacitance and this leads to the shunt equivalent circuit shown in Figure 2.2 (Tucker and Gazey, 1966). This circuit is only valid at frequencies near resonance; well away from resonance, the full transition line model needs to be used (Mason, 1964).
In the equivalent circuit $R_0$ is the resistance of the piezoelectric ceramic and $C_0$ is the capacitance of the piezoelectric ceramic. $L$ is the electrical inductance of the resonant system of the piezoelectric ceramic, $C$ is the electrical capacitance of the resonant system of the piezoelectric ceramic, $R_m$ represents the mechanical losses of the system when resonating and $R_r$ represents the radiation resistance of the transducer / water system when resonating.

When a reverberant field pressure impinges on the transducer, this will alter the radiation resistance term, $R_r$, representing the transducer / water system. Therefore, as the reverberant pressure field changes, $R_r$ will change producing a change in the admittance of the transducer. This will affect the matching of the transducer to the driving source.

The relationship between the impedances of the electrical, mechanical and acoustical parts of the transducer is complex but is not needed in detail here. It is explained in Bobber (1970) and Tucker and Gazey (1966). For current purposes, it is adequate to note that the equations that govern a resonating or network system are the same for electrical, mechanical and acoustical systems. Table 2.2 shows a list of electrical, mechanical and acoustical equivalents (Bobber, 1970; Blitz, 1964; Kinsler et al, 1982).
<table>
<thead>
<tr>
<th>Electrical</th>
<th>Mechanical</th>
<th>Acoustical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage, $V$</td>
<td>Force, $F$</td>
<td>Acoustic pressure, $p$</td>
</tr>
<tr>
<td>Charge, $q$</td>
<td>Displacement, $x$</td>
<td>Volume displacement, $x_v$</td>
</tr>
<tr>
<td>Current, $I$</td>
<td>Velocity, $v$</td>
<td>Particle velocity, $u$</td>
</tr>
<tr>
<td>Impedance, $Z$</td>
<td>Mechanical impedance, $Z_m$</td>
<td>Acoustical impedance, $z_a$</td>
</tr>
<tr>
<td>Resistance, $R$</td>
<td>Mechanical resistance, $R_m$</td>
<td>Acoustic resistance, $R_a$</td>
</tr>
<tr>
<td>Capacitance, $C$</td>
<td>Compliance, $C_m$</td>
<td>Acoustic compliance, $C_a$</td>
</tr>
<tr>
<td>Inductance, $L$</td>
<td>Mass, $m$</td>
<td>Inertance, $M$</td>
</tr>
</tbody>
</table>

Table 2.2. Electrical, mechanical and acoustical equivalents in resonant systems.

The circuit shown in Figure 2.1 can be represented as a complex impedance or admittance. The impedance, $Z = R + iX$, where $R$ is the resistance and $X$ the reactance. Similarly the admittance, $Y = G + iB$, where $G$ is the conductance and $B$ is the susceptance. The modulus of the impedance is given by $|Z| = \sqrt{R^2 + X^2}$ and the phase angle by $\phi = \arctan\left(\frac{X}{R}\right)$. The admittance can be expressed in terms of the resistance and the reactance. The admittance is the reciprocal of the impedance:

$$Y = \frac{1}{Z} = \frac{1}{R + iX}.$$  \hspace{1cm} (2.3)

Multiplying the top and bottom by the complex conjugate gives

$$Y = \frac{R}{R^2 + X^2} - \frac{iX}{R^2 + X^2}.$$  \hspace{1cm} (2.4)

Since $Y = G + iB$ then

$$G = \frac{R}{R^2 + X^2} = \frac{R}{Z^2}.$$  \hspace{1cm} (2.5)

and
If the conductance is wholly real then \( G = 1/R \), and if the susceptance is wholly imaginary then \( B = -1/X \). Similarly:

\[
R = \frac{-X}{R^2 + X^2} = \frac{-X}{Z^2}.
\]  

(2.6)

Consequently the conversion of series impedances to parallel impedances or admittances, or visa versa, can be easily calculated (Bobber, 1970).

### 2.1.3. Transducer sensitivity and response

The performance of a receiving transducer or hydrophone is described by its receiving sensitivity and that of a transmitting transducer or projector by its transmitting response. The sensitivity is defined by the ratio of output to input in the same way as a network gain is defined. As a hydrophone produces an output voltage for a given input pressure, and so the receiving sensitivity is defined as the output voltage / input pressure. Similarly a projector produces an output pressure for a given input voltage or current, and so the transmitting voltage response of a projector is defined as the output pressure / input voltage. Both sensitivity and response can relate pressure to either voltage or current. Usually either can be used for a projector, but only voltage is used for a hydrophone since the received current would be very small. Sensitivity and response are usually defined on a logarithmic scale, since the gain can vary with frequency over several orders of magnitude.

The hydrophone sensitivity is defined with reference to a free-field plane wave pressure (Bobber, 1970). Thus the hydrophone receive sensitivity (RS), \( M \), is defined as
\[ M = 20\log_{10} \left( \frac{V}{p} \right) \]  

(2.9)

where \( M \) has units of dB re 1V/\( \mu \)Pa, \( V \) is the open-circuit output voltage in volts and \( p \) is the free-field plane wave pressure in \( \mu \)Pa.

The projector response is defined with reference to a spherically divergent wave pressure (Bobber, 1970). Because the wave is spherically divergent the response needs to be defined with reference to a distance. This distance is one metre and if the wave is not spherically divergent at this point, because it is in the near field and not the far field (see section 2.1.5), then the pressure must be measured in the far field and corrected to the value it would be at one metre. This is achieved by assuming that this type of wave is inversely proportional to distance. Thus the projector transmitting voltage response (TVR), \( S \), is defined as

\[ S = 20\log_{10} \left( \frac{pd}{V} \right) \]  

(2.10)

where \( S \) has units of dB re 1\( \mu \)Pa/V at 1m, \( p \) is the free-field spherically divergent wave pressure in \( \mu \)Pa, \( d \) is the distance, in m, from the acoustic centre of the projector to the point in the far field (where the pressure needs to be known or measured) and \( V \) is the open circuit input voltage in V. The transmitting current response is defined in the same way, using input current instead of input voltage.

2.1.4. Radiation from a pulsating sphere

The simplest source for generating acoustic waves is a pulsating sphere whose radius varies sinusoidally with time. From symmetry, such a source will produce outgoing, harmonic, spherical waves into a medium that is infinite, homogeneous, and isotropic. The acoustic wave radiated by a pulsating sphere must be of the form

\[ p(r, t) = \frac{A}{r} \exp \left( j(\omega t - kr) \right) \]  

(2.11)
where $p$ is the complex acoustic pressure, $r$ is the distance from the centre of the source, $t$ is the time, $A$ is the complex amplitude of mass acceleration (of fluid displaced by the sphere) at the surface of the source, $\omega (=2\pi f)$ is the angular frequency and $k (=2\pi/\lambda)$ is the angular wavenumber of the oscillations of the sphere.

The radial component of the particle velocity must equal that of the surface of the sphere.

It can be shown (Kinsler et al, 1982: 164) that the acoustic intensity, in the long wavelength limit and for the radius of the source being, $a$, small compared to the wavelength so that $ka \ll 1$, is given by

$$ I = \frac{1}{2} \rho_0 c U_0^2 \left( \frac{a}{r} \right)^2 (ka)^2, \quad ka \ll 1. \quad (2.12) $$

In Equation 2.12, $I$ is the acoustic intensity, $\rho_0$ is the density of the fluid, $c$ is the speed of sound, $U_0$ is the speed amplitude of the vibrating surface of the sphere and $a$ is the equilibrium radius of this surface.

For a constant $U_0$, this intensity is proportional to the square of the frequency and dependent on the fourth power of the source radius. Thus small sources (with respect to wavelength) are inherently poor radiators of acoustic energy.

While pulsating spheres are difficult to construct and are of little practical importance, their theoretical importance is great since they serve as the prototype for an important class of sources, referred to as simple sources. A simple source is a closed surface, vibrating with arbitrary velocity distribution, but of such size that all dimensions are much smaller than the wavelength of the emitted sound (Kinsler et al, 1982: 163-164).

2.1.5. Proximity Criteria

A number of factors influence the separation of projectors and hydrophones. The minimum acceptable separation needs to satisfy several proximity criteria used in
 calibration measurements. The first of these is that there is no interference from reflections between the transducers. The next proximity criterion for a projector comes from the requirement implied in the definition of the transmitting current or voltage response that the pressure be that in a spherical diverging wave. If the pressure is not spherically divergent at one metre, then the pressure must be measured at a larger distance, where it is spherically divergent, and extrapolated back to one meter by assuming that the pressure is inversely proportional to the distance. The proximity criterion for a hydrophone comes from the definition of the free-field voltage sensitivity where the input free-field pressure is specified as that of a plane progressive wave. Plane waves can only be approximated from spherically diverging waves, when the radius of curvature of the spherical waves is very large or the segment is very small.

A spherical sound source produces a simple symmetrical pressure field, as described in section 2.1.4. For non-spherical sources the pressure field behaviour can be more complex. As an illustration the field of a plane circular piston can be considered. A plane circular piston produces an on-axis response given by (Kinsler et al, 1982: 176-177)

\[
P(r, \theta = 0) = 2\rho_0 c U_0 \left[ \frac{1}{2} kr \left[ \sqrt{1 + \left( \frac{a}{r} \right)^2} - 1 \right] \right]
\]

(2.13)

where \( P \) is the pressure amplitude, \( r \) is the radial distance from the centre of the piston source, \( \theta \) is the angle of inclination from the axis (perpendicular to the face of the piston), \( \rho_0 \) is the density of the medium, \( c \) is the speed of sound in the medium, \( U_0 \) is the speed amplitude of the surface of the piston, \( k \) is the wavenumber and \( a \) is the diameter of the piston face.

If \( r/a \gg 1 \) and \( r/a \gg ka \) then the pressure amplitude on the axis has the asymptotic form

\[
P_{ax}(r) = \frac{1}{2} \rho_0 c U_0 \frac{a}{r} \frac{ka}{r}.
\]

(2.14)
This reveals spherically divergent waves at distances satisfying \( r/a \gg 1 \) and \( r/a \gg ka \). Figure 2.3 shows the on-axis pressure (Equation 2.13) and its approximation (Equation 2.14); the approximation asymptotically approaches the true value as \( r \) increases. From the graph it can be seen there are two regions: the zone next to the face of the transducer, where the pressure varies rapidly, and the spherically-spreading zone far from the transducer, known as the near-field and far-field respectively.

![Graph showing pressure amplitude on the axis of a plane circular piston and its approximation](image)

Figure 2.3. Pressure amplitude on the axis of a plane circular piston (solid line, Equation 2.13) and the approximation which is asymptotic to it (dotted line, Equation 2.14), for \( \lambda = a/4 \). The Rayleigh distance is also shown (dashed line at \( r/a = 4\pi \)).

A distance is needed to distinguish the near-field or Fresnel zone, from the spherically-divergent far-field or Fraunhofer zone. Moving in towards the source from large \( r \), it can be shown that one encounters the first local maximum in axial pressure at \( r_1 = a^2/\lambda \). This value is sometimes used as a guide to distinguish these two zones; however the two curves are not close at this point and so the field is not spatially divergent. Consequently the transition distance is usually designated as the Rayleigh distance, \( r_{ray} = ka^2/\lambda = \pi a^2/\lambda \). The Rayleigh distance is marked on the
graph with a dashed line and it can be seen that at this distance the approximation is close to the on-axis pressure; hence the field is spatially divergent and the far field is taken to start here. The value of $r_{ay}$ is very small if the wavelength is large compared to the transducer diameter, and as such means there is effectively no near-field zone and only a far-field zone.

The proximity criteria for a single transducer is usually considered in terms of where the spherically-divergent far-field begins; the criteria apply also to the transducer as a hydrophone. From reciprocity theory, it is apparent that the proximity criteria for two transducers must be the same regardless of which is the projector and which the hydrophone (Kinsler et al, 1982; Bobber, 1970).

2.2. Literature Review

This literature review will cover four areas of interest for this thesis, the first two on underwater acoustics and second two on air acoustics. The air acoustics literature is used because there is very little work on reverberant sound fields in underwater tanks. There is plenty of work on reverberation in the ocean but very little on reverberant fields in rectangular enclosures such as laboratory tanks. However, airborne acoustics provides a wealth of information on reverberant fields in rectangular rooms.

The first section describes several methods of calibration based on the measurement of the direct path signal before the arrival of the first reflection. This is followed by a discussion of the limited literature available on calibrating underwater transducers in reverberant sound fields. The third section covers the literature on the measurement of the acoustic power radiated in reverberant fields. The fourth section investigates the nature of reverberant sound fields and how they decay. An understanding of the nature of these fields is required in order to understand and discuss the reverberant calibration technique developed in this thesis. The physics is the same for airborne and underwater acoustics, but with different values for speed of sound, material properties and absorptivities.
2.2.1. Calibration methods based on the measurement of the direct path signal before the arrival of the first reflection.

Underwater calibrations are usually performed using tone-burst signals sent to the projector and time-gating of the signals received by the hydrophone. This enables the steady-state part of the signal to be captured and the calibration performed. However, for low frequencies in laboratory sized tanks, the steady-state response of a projector is contaminated with reflections from the boundaries of the tank. This imposes a lower frequency limit on the calibrations that can take place in a given tank. The following two methods extend this frequency limit downwards and require only the measurement of the direct path signal, i.e. the hydrophone signal before the arrival of the first echo.

2.2.1.1. Predicting the steady state response of the projector from the uncontaminated initial transient signal.

Trivett and Robinson (1981) use a modified Prony method to extract the steady state amplitude from the transient part of a time-gated signal. The Prony (1795) method allows such an extrapolation of any signal that is described by a series of complex exponential expansions. The method can calculate approximate complex exponentials from the initial transient signal. These can then be used to predict the whole transient and the steady state amplitude (Xiaofeng and Wenjun, 1998) to a good degree of accuracy. Beatty, George and Robinson (1978) used this method to perform reciprocity calibration on time-gated signals, and extended the period of the lower frequency limit to eight times the observation period used in the calibration.

2.2.1.2. Achieving steady state response in the free time by using transient suppression.

The problem of not achieving a steady-state response from a projector, within the free time available in a tank, can be tackled in another way to that from the Prony extrapolation method. This is achieved using a transient suppression method as developed by Piquette (1992a, 1992b). A driving voltage waveform is applied to a transducer so that the usual turn-on and turn-off transients are suppressed. By using
circuit resonance (LCR) theory to deduce the appropriate driving voltage, the circuit can be operated in transient-suppressed mode. A steady-state signal is then available for calibration in tanks for which conventional time gating will not work.

2.2.2. Methods used for calibrating underwater transducers in reverberant sound fields.

These methods work by extracting the direct sound field from the combined direct and reverberant sound fields.

2.2.2.1. Determining the direct field by measuring the acoustic power.

The original concept for the core of the work presented in this thesis was devised by Hazelwood (1993, 1996) in which he determines the acoustic power radiated into a tank in three ways. He uses a continuous noise source, an ROV (Remotely Operated Vehicle), in a large tank to produce a reverberant field where the pressure is measured at a series of separations from the acoustic source. A graph of pressure squared versus the reciprocal of separation squared is plotted and the gradient and y-intercept calculated. From the gradient of the graph the acoustic power radiated into the tank is calculated and the y-intercept gives the spatially averaged reverberant field pressure squared. The reverberant field pressure and the absorption of the tank are then used to calculate the power radiated. A further set of measurements is taken away from the source and the tank walls, so that the reverberant field pressure is measured, and not the direct field or higher values of reverberant field near the tank walls. This reverberant field pressure is used with the absorption value as before, to calculate the power radiated. These three values of power can then be compared against the power that would be radiated in free-field conditions. This is calculated from the TVR of the source and the voltage applied to the source. Hazelwood reports a few test results to indicate the validity of the idea. The plan for this PhD was to test under what conditions this idea broke down and how accurate it was.

This basic idea was built upon and refined during this thesis in order to take accurate measurements. A rig was built to do this and measurements were taken in a range of tanks and a large number of positions within the tanks.
Hazelwood and Robinson (1998) carried out a series of measurements in test tanks to determine the accuracy of Hazelwood's method described above (Hazelwood, 1993; Hazelwood, 1996). They calculated the TVR of the projector from the gradient of the graph and found the result to be less than 1dB from the free-field TVR value. They also carried out a series of reverberant field measurements, at different positions in the tank, for a variety of projectors. The calculated reverberant TVR values were within 1dB of the free-field values.

2.2.2.2. Reverberant field measurements and the directionality of the projector.

Robinson (1999) gives an overview of various methods of calibrating transducers in continuous wave fields. He reports that the sensitivity of a transducer measured in a diffuse sound field is related to that from a free-field by the directivity factor of the transducer. This calibration is based on the spatially-averaged reverberant field pressure, as discussed in Bobber (1970).

2.2.2.3. Smoothing the reverberant field calibration in the frequency domain.

Robinson (1999) reports other methods of calibrating transducers in reverberant fields. One method involves obtaining the TVR calibration of a transducer as measured in a reverberant field, and then fitting a smoothing curve to the frequency response data. This attempts to smooth out the reverberant field perturbations and illustrate the underlying frequency response of the transducer. However, this response will contain a bias due to the reverberant field.

2.2.2.4. Averaging over position.

Since the reverberant field in an underwater tank is probably not diffuse, measurements of average pressure over a period of time will vary from place to place in the tank. One method of overcoming this variation with position is to measure the pressure over a period of time, at many positions within the tank, and then average them to give a mean time series. This could then be combined with the first method of averaging the calibration over frequency bins. However, this method will still contain a bias due to the reverberant field. As an example of averaging over
positions, Robinson (1999) presented the results of impedance loop averaging that I carried out as part of the initial work for this project. Some results of this work are presented in Appendix 1. This work involved measuring the impedance of a transducer at many positions within a small container of water. The impedance loop of a transducer is a clean loop when measured in free-field conditions, but when measured in a confined space the radiated waves reflect off the walls of the container and impinge on the transducer. The impinging waves alter the impedance of the transducer and cause the loop to contain large perturbations due to these reflections. However, if the many perturbed impedance loops are averaged to produce a single loop, it is relatively clear of perturbations and approximates well to the free-field loop.

2.2.2.5. Elimination of the reflected waves in the frequency domain.

Another method is reported by Robinson (1999) and described as “temporal smoothing” by Giangreco (1997). This method involves eliminating the effects of echoes by taking the Fourier transform of the frequency response data to calculate the “impulse” response of the transducer. The data is then windowed in the pseudo-time domain to eliminate the reflections that arrive after the direct path signal. The data is then transformed back to the frequency domain, where a smoothed version of the initial response is obtained.

2.2.2.6. Substitution calibration in the reverberant field.

McMahon and Hodson (1977) describe a computer-based broadband procedure to calibrate hydrophones in an open water acoustic tank. A reverberant sound field is generated in the water tank by driving a projector with a pseudo-Gaussian noise signal. This calibration is performed with the substitution calibration technique. This compares the output from an unknown hydrophone and a calibrated hydrophone. The unknown hydrophone is first placed in the reverberant field and the time signal digitally sampled. This is then transferred into the frequency domain by the use of a Fast Fourier Transform (FFT). This is repeated for 100 time signals, and the 100 spectra are averaged to produce a mean spectrum. The spectra are individually very noisy, and approximately show the underlying output of the
hydrophone. The averaged spectrum is not noisy, and clearly shows the underlying output. The unknown hydrophone is then removed from the reverberant field, and the known calibrated hydrophone placed at the same position in the field. The mean calibrated hydrophone output is then obtained as before. From a comparison of the two hydrophone outputs and the known hydrophone calibration, the unknown hydrophone calibration can be calculated. If the hydrophones are omnidirectional or have the same direction pattern (i.e. the same type of hydrophone), then this method leads to a good approximation of the calibration.

Robinson (1999) also mentions substitution calibration of a transducer in a reverberant field. This is obtained by comparing the averaged voltage spectra from the known and unknown transducers when placed at the same point in the reverberant field. The averaged spectra are easily obtained with modern analyser equipment. The unknown sensitivity is calculated from the ratio of the unknown and known transducer’s averaged voltage spectra, multiplied by the sensitivity of the known transducer. For this method to work, the pressure experienced by both hydrophones needs to be the same. A good indicator of this is the coherence function, which is defined as the square modulus of the cross-spectral density between the two signals, divided by the product of the power-spectral densities of each signal. Two signals are coherent if the coherence is unity; whereas if the coherence is less than one, the proportion indicates how suitable the reverberant field is for this type of measurement.

2.2.2.7. Substitution calibration using cross-correlation of the reverberant field.

Calibrations can be performed using the cross-correlation technique as reported by Robinson (1999) and Giangreco (1997). This method utilises broadband random noise to excite the reverberant field, and attempts to remove the effect of echoes by the use of a windowed cross-correlation function. The cross-correlation of the input signal and the output signal is equal to the sum of the cross-correlation of the input signal and the direct-path signal, and the cross-correlation of the input signal and the boundary echoes signal, both arriving at different times. If the peaks of the input and output cross-correlation function are narrow enough, a window can be applied which selects only the direct signal, and disregards the boundary echoes signal outside the
window. This technique can then be used to calibrate a hydrophone by the substitution method. A white noise signal is sent to a projector and a calibrated hydrophone placed in the reverberant field. The correlation function of the projector signal and the hydrophone signal is obtained and then windowed so that only the direct path signal is present. The calibrated hydrophone is removed and the uncalibrated hydrophone put in its place and the procedure repeated again. The cross-spectral density of the two windowed cross correlation functions is calculated by taking the Fourier transform. The uncalibrated sensitivity is then obtained calculating the ratio of the uncalibrated to the calibrated cross-spectral density and then multiplying by the calibrated sensitivity. For this technique to work, the windowing of the cross-correlation function needs to be successfully applied, and, for this to be the case, the function peak must be less than a certain width. Therefore an input signal with a narrow peak in its auto-correlation function is needed and this means it must have a large bandwidth. A Gaussian random noise signal has this property and should therefore enable the technique to be applied successfully. The coherence of the reverberant field also needs to be close to unity for this method to work.

2.2.2.8. Eliminating echoes in the cepstrum domain.

Robinson (1999) also reports on homomorphic signal processing or cepstral deconvolution. This method is similar to the cross-correlation technique, in that a form of deconvolution is used, but the windowing or filtering takes place in the complex cepstrum domain (Bogert, Healy and Tukey, 1963). This type of signal processing has been used to remove the effect of a transmission path, for example echoes, from signals in fields such as seismology (Ulrych, 1971) and audio signal processing (Oppenheim, Shafer and Stockham, 1968). The complex cepstrum is defined as the inverse Fourier transform of the complex natural logarithm of the complex spectrum, where the spectrum is the Fourier transform of the time domain signal (Randall, 1977). The independent variable of the cepstrum has been called the "quefrency" (Bogert et al, 1963), although it has dimensions of time and is similar to "τ" of the auto-correlation function. The spectrum can be expressed as $S(f) = A(f) \exp(i\phi(f))$, where the complex logarithm of the spectrum is given by:
\[ \ln(S(f)) = \ln[A(f)\exp(i\phi(f))] = \ln[A(f)] + i\phi(f). \]

The parts of the spectrum which vary only slowly, which is typical of a smooth transfer function, are gathered around the origin in the cepstrum domain; whereas the rapid spectral variations (e.g. interfering echoes) are not moved. Secondly, by taking logarithms, signals that are multiplied together in the frequency domain are summed together in the cepstrum domain. This therefore means that signals convolved in the time domain are summed in the cepstrum domain (Poche, 1977), (Le Gall and Gautard, 1998). These properties allow the direct path and reflected signals to be potentially separated in the cepstrum. This therefore means that just the direct path signal can be converted back to the time domain and a calibration performed (Oppenheim and Shafer, 1974). For signals that contain many reflections, it can be difficult to subtract the contribution of the echoes in the cepstrum with some prior knowledge of what they are.

2.2.2.9. Extracting the direct-path signal using time-delay spectrometry.

Robinson (1999) also reports on another method of calibration called time-delay spectrometry. This has been used in airborne acoustics (Heyser, 1967), ultrasonics (Ludwig and Brendel, 1988) and in underwater acoustics (Giangreco, 1997). In time-delay spectrometry, a projector is driven with a sinusoidal signal, \( V(t) \), where its frequency is swept over the frequency range of the calibration and is described by:

\[ V(t) = A\cos[2\pi(f_m - St)t]. \]

A is the signal amplitude, \( f_m \) is the maximum frequency and \( S \) is the sweep rate (=d\( f \)/dt). The signal received by the hydrophone consists of the direct-path signal and the reflected signals from the tank boundaries. The reflected signals arrive later than the direct-path signal and so the signals will have different frequencies. The reflected signals will have higher frequencies than the direct-path signal and can be removed with a narrow-band filter. The centre frequency of the filter must track the direct-path signal arriving at the hydrophone. This can be achieved by sweeping at the same rate as the drive signal, but offset by the time taken for the sound to travel the distance of the direct-path.
2.2.3. Sound power determination of reverberant sound fields in air.

Although little work has been reported on the use of reverberant techniques in underwater acoustics there is a considerable body of literature on the use of such techniques in airborne acoustics.

Tohyama, Imai and Tachibana (1989) state that the sound power radiated in a free-field can be estimated from sound power measurements in a reverberant field. They do, however, note that the output power radiated by a sound source is dependent on the acoustic pressure at the source location. Since the pressure in a room varies according to the modal structure of the room, the power output varies according to the sound sources position. The power output of a source in a free-field can be estimated by averaging the power output data over the room. In a high frequency band, average power measurements can be expected to represent the power output in the free-field, when the source is positioned randomly throughout the reverberant field. In a low frequency band the averaged power output can differ significantly from the power output in a free-field.

Agerkvist and Jacobsen (1993) use the traditional method of measuring the sound power radiated by a source, that is the spatially averaged mean square pressure method (ISO 3741, 1975), and apply it to low frequencies. This is quite convenient, and certainly cheaper in terms of facilities, than the alternative traditional method of placing a sound source in an anechoic room, which is effectively a free-field (ISO 3745, 1977). (This is also the case in underwater acoustics where anechoic tanks are expensive and the tiles do not absorb low frequencies very efficiently). The validity of the expression underlying the reverberation room method of measuring sound power is not obvious at low frequencies, since the expression is essentially based on statistical considerations. For the expression to be valid, the averaged reverberant field must produce an accurate power value, but at low frequencies there can be large errors in the averaged value. This is because the reverberant field varies so much because it does not have enough room modes to gives a statistically accurate result. At high frequencies there are plenty of room modes and the averaged value is accurate.
According to de Araujo and Yousri Gerges (1983), in general the sound power of a source as measured in a reverberation chamber is smaller than that measured in an anechoic chamber. However other work has shown that when the sound power is measured over all possible positions in a reverberation chamber, including positions near the chamber boundaries, the calculated average is equal to the free-field value (Maling, 1967; Yousri and Fahy, 1972; Yousri Gerges, 1979; Yousri and Fahy, 1974; Yousri Gerges, 1980). One possible reason for this difference is that only a few measurements are usually made in a reverberation chamber and that few are taken near the chamber boundaries.

Maa (1988) states that the acoustic impedance experienced by the source in the sound field produced in the chamber varies significantly from point to point, and this leads to the large variability of the power output of the source in a room (or tank). The average obtained over the whole room is approximately the same as the sound power produced in free space, provided that the source impedance is large compared to the field impedance and that the frequency is high enough so that the normal modes overlap. It was discovered that the power emission is a maximum on the boundary, and undulates about the free-field value as the source is moved towards the centre of the room. The average power emission is less than the free-field value if the average avoids the high power region near the boundaries, as is the case for most measurements.

Zeng, Maa and Crocker (1989) have calculated the sound power radiated by a monopole and dipole source in a reverberation chamber. This showed that the sound power emission of an ideal source is greater in reverberant than free-field conditions. In practice, measurements made at low frequencies show that the sound power measured in a reverberant chamber is less than that in an anechoic room, but at high frequencies, this difference disappears. This smaller sound power radiated in a reverberation chamber is shown to be due to the non-ideal internal resistance of the source. The internal resistance of an actual sound source can be calculated from the measured sound power emission ratio between reverberant and free-field conditions and the calculated sound power ratio.
2.2.4. Reverberant sound fields and their decay.

Kinsler et al (1982) talk in detail about room modes and their excitation and decay. The solution to the wave equation for a loss-less, rigid-walled rectangular cavity reveals a series of normal modes. These modes can be split up into three categories, as shown in Figure 2.4.

Figure 2.4. Three mode categories: (a) axial mode (parallel to two pairs of surfaces, and one axis), (b) tangential mode (parallel to one pair of surfaces) and (c) oblique mode (not parallel to any pair of surfaces).

The first is axial, where the propagation vector of the standing wave is parallel to one axis of the rectangular enclosure and therefore parallel to two pairs of surfaces. The second is tangential, where the propagation vector is parallel to one pair of surfaces. The third is oblique, where the propagation vector is not parallel to any surface. If the walls of the enclosure are not rigid, but lose acoustic energy, the normal modes will decay with time. This approximates to a real room since the walls of the room are not rigid and absorb acoustic energy.

In general, a reverberant room will respond strongly to any sound having frequencies near that of a mode of the enclosure. A simple measurement of the output from a
loudspeaker will, therefore, be dominated by the strongly responding modes of the room. Each standing wave has its own particular pattern of nodes and anti-nodes which, when combined with other modes, means that each room superimposes its characteristic field on the field of an acoustic source. The fluctuations in sound pressure, as a microphone is moved from point to point in the room or the frequency of the source is changed, mean that the true response of the source may be completely concealed. In air acoustics this problem can be overcome by taking measurements of the response of a loudspeaker in the open air or in an anechoic chamber. If the walls of an anechoic chamber are sufficiently absorbing, the reverberant sound field is so small that it is negligible compared to the direct field. (This is often the case in air acoustics but the tiles used in anechoic tanks are not very efficient absorbers and result in a significant reverberant field).

Each standing wave in a reverberant enclosure can be excited to its greatest extent by a sound source located in regions where the particular standing wave has a pressure anti-node. The pressure anti-nodes of all standing waves in a rectangular enclosure are maximised in the corners of a room. If a sound source is placed at the corner of a rectangular room, it will excite every mode of the room to its fullest extent. Likewise, a microphone placed in the corner of such a room will measure the maximum amplitude of all excited modes. When a sound source is placed at a pressure node the standing wave will only be weakly excited, or not at all.

If a continuous wave source is used in a room, the modes at that frequency will be excited. When the source is switched on, additional modes of the room will be excited at other frequencies, due to the transient response of the source and the room. All modes of the room are damped, and so these additional modes are only excited transiently and their amplitude will decay with time. The non-transient modes radiated by the source build up since they are continuously produced. These modes build up to a maximum value where the rate of energy radiated into the room equals the rate of energy absorbed by the room. The rate of decay of the transient waves varies with frequency (as does the growth rate of the non-transient waves radiated by the source). Since each mode has its own natural frequency, these modes will often interfere with each other and produce beats. The collection of all these decaying
modes is called the reverberant sound field. The theory for the growth and decay of sound in an enclosure is described in Chapter 6.

The rate of decay of sound in an enclosure will vary with frequency unless the rate of decay of each mode is the same. If modes vary in their rate of decay, which they do for most real situations, the gradient of the decay curve for each individual mode will be different. Therefore the reverberation time will change with frequency. In addition the gradient of the sound decay curve may change with time, representing different reverberation times for different sections of the graph. This is because the recorded decay curve is the amalgamation of the different decay curves for each frequency. So the gradient will depend on the initial amplitudes of the different modes and will change according to how the different modes die away, i.e. depend on how the absorption varies with frequency. Initially the rate of decay will be rapid, corresponding to a short reverberation time, and then will decrease with longer apparent reverberation times as the more weakly damped modes decay away. This is readily observable at low frequencies in rooms whose walls have distinctly different absorptivities. Formulae for calculating the reverberation time of an enclosure from its decay curve are derived in Chapter 4.

The natural modes of an enclosure can be plotted in vector frequency space, and in doing so the number of modes below a certain frequency can be calculated. The equation for this and the number of modes in a given frequency band are shown in section 6.1.4 in Equations 6.24 and 6.25 respectively. These equations show that the number of modes, and the number of modes per frequency band, increases with the frequency and volume of the enclosure. This means that the standing waves will overlap more at higher frequencies, so that as frequency increases the response of the room will become smoother. A standing wave has a particular direction and so as the number of modes increases with frequency, there are more standing waves with different directions and so the direction of the standing waves becomes more random. This is supported by the fact that equations for diffuse sound fields, such as Equation 4.9, agree with experimental results better as frequency increases.

The response of a room is observed to become less uniform as its symmetry is increased. This results in a number of modes with different harmonics in the x, y and
z directions having the same natural frequency. To increase the uniformity of the
distribution of normal modes, and so the reverberant field, Bolt (1946) has shown
that it is necessary to have the room dimensions in certain ratios. The acceptable
length ratios are of the form 1 : X : Y and constitute a region in XY space. The
approximate values that increase the uniformity of the normal modes are satisfied by
the joint conditions 2 < (X + Y) < 4 and (3/2)(X - 1) < (Y - 1) < 3(X - 1) (Kinsler et

Kinsler et al (1982) also note, in airborne acoustics, that normal modes that graze the
surface of a wall are only absorbed by half the amount that non-grazing normal
modes do. This is due to the average mean square pressure produced by these modes
at the walls being half that produced by them on other walls where they do not graze.
This shows that an absorbing surface is most effective in damping a normal mode
when it is located in a region of maximum mean square pressure. Oblique modes
spend the majority of their path grazing the walls of the room and so the mean square
pressure is half, however they are absorbed the most quickly since they spend there
time grazing the walls of the room. Tangential and axial modes spend far less of
their path near the walls and so are absorbed less quickly. Therefore the
reverberation times for tangential and axial modes are longer than for oblique modes.

In a rectangular room with all six sides having the same absorbing material the
reverberation times for the axial, tangential and oblique modes are 6:5:4 respectively.
In such a room the reverberation time measured for low frequencies may vary rapidly
with frequency as one then another mode is strongly absorbed. In this frequency
range, reverberation time only refers to a particular type of normal mode. In the mid-
frequency range, the decaying sound-curve is a fluctuating line where the initial part
is quickly decaying and represents the oblique modes being highly absorbed. The
middle part of the curve decays less rapidly and represents the tangential modes, and
the end part decays gradually, representing the axial modes. The time required for
the intensity level to drop by 60dB depends on the relative amounts of acoustic
energy in each of the three types of modes. At high frequencies, most of the acoustic
energy resides in the oblique modes and this means the first 20 to 30dB of the decay
curve is nearly a straight line. This first part can then be used to measure the
reverberation time of the majority of the acoustic energy present, the oblique modes. The rest of the curve is then the decay of the tangential and axial modes.

Tohyama and Yoshikawa (1981) derive theoretically an approximate formula of the space- and ensemble-averaged reverberation decay curve in a rectangular room. This approximate decay curve is written by the summation of the decay curve (over different starting conditions) in three (oblique, tangential and axial) types of diffuse fields. The formula contains not only the averaged sound absorption coefficient of the walls but the number of resonance wave modes excited in a rectangular reverberant room. The sound energy distribution rate, of the three types of diffuse fields in steady state conditions, is determined by the ratio of the number of wave modes within each type of diffuse field.

Cook, Waterhouse, Berendt, Edelman and Thompson, Jr., (1955) did pioneering work on determining the randomness of reverberant sound fields. The sound fields produced in reverberation chambers should ideally be completely random for the field to be useful in acoustical measurements. (The field needs to be random, i.e. diffuse, so that the reverberant field level, averaged over time, is the same throughout the chamber). Once a reverberant field has been established, it needs to be determined whether the field is random or not. This can be achieved by determining the cross-correlation coefficient for the sound pressure at two different points in the sound field. The measured variation of correlation coefficient with wave number and separation of the two points is useful in determining whether or not the sound field is random.

Jacobsen and Roisin (2000) says that measurements made in a reverberant room are often based on the assumption that the sound field is diffuse, so it is useful to validate that this is the case. To do this, definitions of a diffuse sound field are needed. The diffuse sound field is an idealised concept, and the sound field in a real room differs from this in fundamental ways. One way to test the diffuseness of a sound field is to compare the theoretical and measured spatial correlation functions. Since, theoretically, a diffuse sound field is completely random, the spatial (cross-) correlation function will be the correlation function of a random signal.
Another definition of a diffuse sound field that most acousticians would agree with, is that involving sound arriving from all directions. This leads to the concept of a sound field in an unbounded medium generated by distant, uncorrelated sources of random noise evenly distributed over all directions. Since the sources are uncorrelated there would be no interference (since superposition of incoherent waves) and the sound field would be completely homogeneous and isotropic. An approximation to this perfectly diffuse sound field would be a series of loudspeakers driven with uncorrelated noise in a large anechoic room. The sound field in a reverberant room driven with noise from one loudspeaker is different to this.

Kinsler et al (1982: 313) define a diffuse sound field as: the average energy density is the same throughout the volume of the enclosure, and all directions of propagation are equally probable.

Sepmeyer (1988) states that the low-frequency limit for the statistical behaviour of sound in rectangular rooms has been postulated by Schroeder (1962) to be when the average spacing of the modes of a room is less than one-third of their bandwidths. The frequency at which this occurs is called the Schroeder frequency, $f_c$, and is given by $f_c = 2 \times 10^3 (T_{60}/V)^{1/2}$.

Jacobsen and Roisin (2000) also state that a more realistic model of the sound field in a reverberant room above the Schroeder frequency is described by the sound field made up of plane waves with random phases arriving equally likely from all directions (usually called the plane wave model (PWM)). This model is for a pure tone and so the various plane waves interfere. This results in a sound field where the sound pressure level depends on position and the probability of the level being in a certain interval is the same at all positions. This model assumes infinitely many plane waves with completely random phases and so is also an idealised model. However, it gives a good approximation to the sound field in a reverberant room driven with a pure tone and with a frequency above the Schroeder frequency. The perfectly diffuse field described above can be obtained by averaging over an ensemble of fields with different source positions.
This second, more realistic, plane wave model can be extended by excitation with a band of noise. If a reverberant room is driven with a pure tone, the plane waves that make up the sound field have certain amplitudes and phases. If the exciting frequency is shifted slightly, the amplitude and phases of the plane waves will be changed, and so the entire interference pattern will be slightly changed. The longer the reverberation time of a room, the greater the range of sound field patterns there will be. This is because the old pattern remains for longer, and new patterns are produced as the frequencies of the exciting noise source change with time (Schroeder, 1962). Excitation of the room with a noise band is equivalent to averaging the sound field over the whole frequency band. This results in the sound field becoming more uniform. The effect of this spectral averaging depends not only on the bandwidth of the exciting signal, or the bandwidth of the analysis, but also on the damping of the room. Smaller damping leads to longer reverberation times and more effective averaging. This is because each new random excitation generates a new sound field pattern, and so the longer the reverberation time the more patterns are averaged together before an individual pattern has decayed away. There are many similarities between the sound field produced in a room driven with noise and a perfectly diffuse sound field, but there are also important differences. It can be concluded that diffuseness at low frequencies requires a large room and a long enough reverberation time so that the reverberant field has sufficient modal overlap.

According to Nelisse and Nicolas (1997) the correlation coefficient and the spatial uniformity can be used as an efficient method to characterise the diffuseness of a reverberant sound field. This can be given as a precise criterion in terms of the least permissible number of room modes to achieve an adequate diffusion. This is in good agreement with the Schroeder frequency limit for a diffuse field. More than one criterion is needed to correctly define a diffuse sound field. For example the uniformity of pressure in a room does not give sufficient information about the degree of diffuseness of the field.

A commonly accepted definition of a diffuse sound field is given as: An acoustic field is considered to be perfectly diffuse in a volume, \( V \), if the energy density is the same at all points in this volume, \( V \).
Rafaely (2000) notes that the diffuse sound field model is widely used in the analysis of sound in enclosures. The spatial and temporal correlation of sound in a diffuse field is useful in determining the diffuseness of reverberant room sound fields.

Sepmeyer (1988) says that measurements of space-averaged sound pressure squared are required to determine the sound power output of small sources in reverberant rooms. These measurements are contaminated by a direct field bias error. A standard (ANSI S1.21, 1972) is used to determine the sound power level of small sources in reverberant rooms. This standard states that a minimum distance between the sound source and microphone is needed for the measurement of the reverberant field to take place. This distance, $d_{\text{min}}$, is defined as $d_{\text{min}} = 0.08\left(\frac{V}{T_{60}}\right)^{1/2}$, where $V$ is the volume of the enclosure and $T_{60}$ is the 60dB reverberation time (time taken for the pressure level to drop by 60dB).

Kuttruff (1991) talks in detail about impulse response and reverberation time measurements. Measurement of the impulse response of a room leads to a complete description of the changes a sound signal undergoes when it travels from one point in a room to another. From system theory, all properties of a linear transmission system are contained in its impulse response. A room can be considered as an acoustical linear transmission system, and the impulse response to the room leads to the modal structure of the room.

The impulse response of a system is the output signal from the system after being excited by a very short impulse signal. Such a signal is a Dirac or delta impulse, which is a pulse of extremely short duration and unit area (Lynn, 1979). It has the property that its amplitude spectrum is unity for all frequencies. The impulse response spectrum, the spectrum of the delta impulse after it has passed through the linear transmitting system, describes how the amplitudes and phases of all the frequencies have changed by passing through the system. The transfer function of the system describes these changes in the frequency domain and is the Fourier transform of the impulse response. The output response of a system is described by the convolution of the impulse response and the input signal for the time domain. The output response of the system, in the frequency domain, is described by the
transfer function and input signal multiplied together. The impulse response or
transfer function described the changes made to a signal between two points in the
room and is only valid for these points. A different impulse response will be
obtained by a different set of points in the room. The transfer function describes the
modes of the room excited between these two points.

Impulse responses and correlation functions have important uses in room acoustics.
The cross-correlation of two signals from different positions in a room can be used to
determine the impulse response between these two points. Correlation functions can
be used to characterise a causal relationship between two different time functions
(cross-correlation) or the degree of randomness of one time function (auto-
correlation).

In air acoustics the reverberation time of a room is measured so that a sufficiently
large dynamic range of the decaying sound signal is recorded. The standard
equipment to do this consists of signal generator sending a signal to an amplifier, and
then to a loudspeaker in the room. A microphone in the room experiences a sound
pressure and then sends a signal to an amplifier, then to a filter and then on to a
logarithmic recorder. The exciting signal is applied for a short period of time so that
the reverberant pressure in the room has reached steady-state conditions. It is then
stopped and the logarithmic recorder plots the decaying sound field trace. The signal
from the generator is usually a sinusoidal signal, modulated so that the frequency
varies over 10Hz. (The signal is modulated over a frequency band so that many
modes decay together producing a reasonably smooth decay curve. Furthermore, the
frequency band over which the signal is modulated is narrow so that the change in
reverberation time with frequency can be found, and detail is not lost by using too
large a band). An alternative is a random noise generator followed by a filter, which
separates out the signal into frequency bands. Different equipment will need to be
used for reverberation decay measurements in underwater test tanks, due to the very
short reverberation times involved.

Because of the validity of the reciprocity principle, the locations of the sound source
and receiver can be interchanged without altering the results. This applies equally to
the reverberation time and impulse response measurements, which are very similar.
If the sound field is completely diffuse, the decay curves measured at different source and receiver locations should be the same.

The decay curve is subject to irregular level fluctuations, which are superimposed on the general fall in level and can be considered to be complicated beats or the result of incomplete cancellation. These random fluctuations about the true decay curve can be reduced or removed by averaging a large number of individual reverberation curves. These curves needed to be obtained by random noise excitation of the room. This time consuming procedure leads to the same result as another method called "the method of integrated responses", which was first proposed by Schroeder (1965). It is based on the ensemble average of all possible decay curves, for a certain place and bandwidth of exciting noise, and the corresponding impulse response. The individual decay curves of the ensemble average are produced by random starting conditions of the exciting signal. Each bandwidth of exciting noise is random and so the different starting conditions lead to different decay curves for a given position. The ensemble average gives the intrinsic decay curve for a given set of source and receiver positions. This ensemble averaged decay curve changes with position unless the sound field is completely diffuse.

Integrating the square impulse response over certain limits is equivalent to averaging over all possible decay curves, which are obtained at the same source and receiver positions with white noise excitation. The method of integrating impulse responses is more advantageous than other reverberation measurement methods since its result is not dependent on the random starting excitation of the sound source. The standard reverberation decay method produces traces which decay with strong fluctuations, whereas the integrated impulse response method produces a decay curve with none of these fluctuations.

Practically, the integrated impulse response method and averaging an ensemble of decay curves (very time consuming) can lead to large errors if the right integration time limits and time limits are not chosen. If the time limit is too long the integrated experimental noise builds up to a large level and limits the useful dynamic range of the decay curve. Conversely, if the time limit is too short then this will cause a downward bend on the decay curve. Therefore, to obtain an undistorted decay curve
with a large dynamic range, it seems that there is no other way to determine the integration time limits than to determine them by trial and error, in order to ensure an accurate result. This method is therefore not as advantageous as first thought.

Chu (1978) has made a comparison of two techniques to determine reverberation time using a digital acquisition system. The first technique was Schroeder's "integrated impulse method" and the second was an ensemble average of a large number of reverberant decays. It was found that the two methods are accurate and superior to a single reverberant decay curve and that the two methods agree well with each other, even for non-uniform decays.

Bartel and Yaniv (1982) made measurements in partially reverberant rooms to investigate the departure from linearity of sound decays. The measurements showed that 'smooth' decay curves, displaying a curvature characterised by a monotonically decreasing decay rate, could be obtained provided that the ensemble averages included decays recorded at several source and receiver locations.

Hirata (1982) states that if a room shape is regular the logarithmic sound decay curve is not a straight line, whereas an irregular room produces a straight line. When a room shape is not irregular enough for the sound field to be dominated by oblique waves, convex curvatures of logarithmic decay curves are observed in the middle and high frequency ranges. The curvature of the decay curve also depends on the location of any absorption material.

This completes the literature review, and the main points of the chapter will be briefly summarised. This chapter has described the background material needed for an understanding of this work, and covered the topics of piezoelectricity, transducer equivalent circuits, transducer sensitivity and response, acoustic radiation from a sphere and proximity criterion for transducers. It has also reviewed the literature on reverberant calibration in underwater acoustics and extracted, from the large amount of literature on airborne acoustics, some of the conclusions that are relevant about reverberant fields and their decays.
3.0. Reference Calibrations

The aim of this project was to investigate alternative approaches to the calibration of transducers. It was therefore necessary to have reference measurements obtained by standard calibration techniques for comparison. This chapter describes the theory for the standard calibration techniques used for underwater transducers, the methods and equipment used to carry out the calibration measurements and the transducer calibration results. It also includes some relevant background information on underwater transducers.

3.1. What is calibration and why is it needed?

Calibration is the process of relating a measurement or measurement device to a standard. A standard is based on a unit that is nationally or internationally used for comparison. This enables a quantity to be measured, and known to be the same wherever it is measured. This allows the exchange of information, since it is known that the quantity being discussed between two parties, is the same as the standard, and therefore the same quantity. Calibration is necessary, since no two devices can be made exactly the same, therefore a quantity measured by two different devices will produce slightly different results. The process of calibration means that the differences between the two devices can be compensated for, and a quantity measured by the two devices will produce the same result, within experimental errors.

The transducers used in this work needed to be calibrated with respect to a national standard so that the reverberant calibrations, produced by this project, could be compared to this standard (the free-field reference calibration).

3.2. Calibration Theory

The purpose of a calibration is to determine the performance of a device so that it may be used to make accurate and reproducible measurements. For underwater
situations the calibration of sound-radiating transducers (projectors or sound sources) and sound-receiving transducers (hydrophones) is required. There are two types of calibration that can be performed: primary and secondary. Primary methods involve determining the sensitivity from measurements of voltage, current, electrical and acoustical impedance. The secondary methods involve the use of a transducer that has been calibrated by a primary method, and is then used as a reference standard. Secondary methods require fewer measurements and normally introduce less additional error than in the primary standard. Secondary methods are therefore more generally used for calibrations, although the secondary method is never more accurate than the primary method (Bobber, 1970).

Transducer calibration usually implies the measurement of the free-field voltage sensitivity of the transducer. A free-field is needed so that no sound reflections arrive at the transducer and therefore cause errors in the calibration measurements. Such a field is achievable in a very large tank or the ocean (limited by ocean noise). However, very large tanks and ocean trials are very expensive; usually medium to small sized tanks are used. The size of the tanks causes restrictions on the type of measurements that can be made as discussed in sections 1.2 and 1.3.

3.2.1. Reciprocity Method

The reciprocity method is one of a family of primary calibration methods using the reciprocity principle. The reciprocity calibration used in underwater acoustics is called conventional reciprocity and is the most widely used method. Conventional reciprocity is more accurately known as three transducer spherical-wave reciprocity. Reciprocity depends on one electroacoustic transducer being reciprocal; that is, the ratio of its receiving sensitivity, \( M \), to its transmitting sensitivity, \( S \), is equal to a constant, \( J \), called the reciprocity parameter. This means that if a transducer is transmitting a pressure, \( P \), with a voltage, \( V \), then if the situation is reversed, then that pressure, \( P \), impinges on the transducer and generates a voltage that is related to \( V \), by a constant. The reciprocity parameter is dependent on the acoustic medium, frequency and boundary conditions, but is independent of transducer type or construction. To be reciprocal, a transducer must be linear, passive and reversible.
Conventional reciprocity calibration requires three transducers of which one serves only as a projector, $P$, one is a reciprocal transducer, $T$, which serves as both a projector and hydrophone, and the other, $H$, serves only as a hydrophone. Any of the transducers can be the one being calibrated, but the hydrophone free-field voltage sensitivity is considered here. All the measurements are made in the far-field so that only spherical waves impinge on the transducers. The measurements: (a), (b) and (c), shown in Table 3.1 are needed to carry out a reciprocity calibration. Measurement (d) is a reciprocity check of the reversible transducer $T$. The following derivation follows that in (Bobber, 1970).

<table>
<thead>
<tr>
<th>INPUT CURRENT</th>
<th>PROJECTOR</th>
<th>HYDROPHONE</th>
<th>OUTPUT VOLTAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $I_p$</td>
<td>$P$</td>
<td>$H$</td>
<td>$V_{ph}$</td>
</tr>
<tr>
<td>(b) $I_p$</td>
<td>$P$</td>
<td>$T$</td>
<td>$V_{pt}$</td>
</tr>
<tr>
<td>(c) $I_T$</td>
<td>$T$</td>
<td>$H$</td>
<td>$V_{th}$</td>
</tr>
<tr>
<td>(d) $I_T$</td>
<td>$T$</td>
<td>$P$</td>
<td>$V_{tp}$</td>
</tr>
</tbody>
</table>

Table 3.1. Measurements needed for a reciprocity calibration.

The free-field voltage sensitivity, $M_H$, of the hydrophone is obtained from the measurements as follows. The projector, $P$, is held at a fixed distance, $d_1$, from the transducer, $T$, and hydrophone, $H$. The free-field sound pressure, $P_p$, produced by $P$ at $H$ or $T$ is $I_P S_P d_0 / d_1$, where $S_P$ is the transmitting current response of $P$, and $d_0$ is the reference distance at which the transmitting pressure is specified in the definition of $S_P$. Therefore, from the free-field voltage sensitivity of the hydrophone, $M_H$, and the free-field voltage sensitivity of the transducer, $M_T$, the voltages generated by the hydrophone, $V_{ph}$, and the transducer, $V_{pt}$, are given by

$$V_{ph} = M_H P_p = \frac{M_H I_P S_P d_0}{d_1} \quad (3.1)$$

and

$$V_{pt} = M_T P_p = \frac{M_T I_P S_P d_0}{d_1}. \quad (3.2)$$
From Equations 3.1 and 3.2 the following equation can be formed:

\[
\frac{V_{PH}}{V_{PT}} = \frac{M_H}{M_T} \tag{3.3}
\]

If \( T \) is a reciprocal transducer then

\[
\frac{M_T}{S_T} = J \tag{3.4}
\]

is true; and if Equation 3.4 is substituted into Equation 3.3 then the voltage sensitivity of the hydrophone can be shown to be given by

\[
M_H = \frac{JS_T V_{PH}}{V_{PT}}. \tag{3.5}
\]

The sound field pressure, \( P_T \), produced by \( T \) at the position of \( H \), a distance \( d_1 \) from \( T \), is \( I_T S_T d_0 / d_1 \), where \( S_T \) is the transmitting current response of \( T \). Therefore from the free-field voltage sensitivity of the hydrophone, \( M_H \), the voltage generated by the hydrophone, \( V_{TH} \), is given by

\[
V_{TH} = M_H P_T = \frac{M_H I_T S_T d_0}{d_1} \tag{3.6}
\]

and if Equation 3.5 is rearranged and substituted into Equation 3.6, the expression for \( M_H \) can be found to be

\[
M_H = \left( \frac{V_{TH} V_{PH} d_1}{V_{PT} I_T d_0} \right)^{\frac{1}{2}} J. \tag{3.7}
\]

The reciprocity parameter, \( J \), is derived in the literature (MacLean, 1940; Foldy and Primakoff, 1945) as
\[ J = \frac{2d_0}{\rho f} \]  

where \( \rho \) is the density of the medium, \( f \) is the frequency and \( d_0 \) is the reference distance defined in the transmitting response for a projector \((d_0 = 1\text{m})\). The ratio \( d_1/d_0 \) does not normally appear in Equation 3.7 since either the voltages are adjusted to the value they would be at \( d_1 = 1\text{m} \) and \( d_1/d_0 = 1 \), or \((d_1/d_0)(2d_0/\rho f)\) is combined into a new definition of \( J \). Assuming either of these situations Equation 3.7 becomes

\[ M_H = \left( \frac{V_{PH}V_{TH}}{V_{PT}I_T} \right)^{\frac{1}{2}} . \]  

If the projector \( P \) is also a reciprocal transducer and the additional measurement (d) in Table 3.1 is performed, then measurements (b) and (d) constitute a reciprocity check. That is, both \( P \) and \( T \) are assumed reciprocal if

\[ V_{PR}/V_P = V_{TP}/V_T . \]

From measurements (a), (c) and (d),

\[ M_H = \left( \frac{V_{TH}V_{PH}}{V_{TP}I_P} \right)^{\frac{1}{2}} . \]  

The numerators of Equations 3.9 and 3.10 are identical and the denominators must, therefore, be equal. The addition of a fourth measurement to the necessary three provides both a reciprocity check and some redundancy that increases the reliability of the measurements (Bobber, 1970).

The hydrophone receive sensitivity, \( M_H \), can also be expressed as (Robinson, 1999; IEC565, 1978)

\[ M_H = \left( J \frac{d_{PH}d_{TH}}{d_{PT}} \frac{Z_{PH}Z_{TH}}{Z_{PT}} \right) \]  

where \( d_{PH} \) is the separation between \( P \) and \( H \) and \( Z_{PH} \) is the transfer impedance when transmitting with \( P \) and receiving with \( H \). The transfer impedance is the ratio of the
voltage across the terminals of the receiving device to the current driving the transmitting device. For a spherical wave field, the reciprocity parameter, $J$, is given by $J = 2\rho f$, assuming $d_0 = 1\text{m}$, and where $\rho$ is the water density and $f$ is the acoustic frequency. The transmitting response of any one of the devices may also be described with a similar equation (IEC565, 1978).

For each measurement of voltage or current the amplitude of the steady-state portion of the tone-burst signal is used. Since the transfer impedance is a ratio of electrical quantities, systematic errors in the measurement may be eliminated by using the same measurement channel (preamplifier, filter, digitiser) for each, and calibrated attenuators may be used to equalise the signals to minimise errors from non-linearities in the measurement chain. The measurements made at NPL used this system to obtain very accurate measurements and also used a current probe to measure the drive current. The use of a calibrated current transformer, and calibrated electrical attenuators, provide the traceability back to national standards of electrical measurement: the Ampere and the Ohm (Robinson, 1999).

3.2.2. Comparison Method

The comparison method is the main secondary calibration method, which involves subjecting the hydrophone to be calibrated and a calibrated reference hydrophone to the same free-field pressure, and then comparing the output voltages. The measurements effectively take place in a free-field because the projector usually emits a pulsed sound wave. This means that the signal is received at the hydrophone and has finished before the reflections off the walls of the tank have arrived. By the time the next pulse has arrived the reflected sound has decayed away. First the calibrated hydrophone is placed in the sound field and its open circuit output voltage, $V_r$, is recorded. Next the uncalibrated hydrophone is placed in the same position and its open circuit output voltage, $V_x$, is recorded. Since the calibrated hydrophones free-field voltage sensitivity, $M_r$, is known the free-field voltage sensitivity of the uncalibrated hydrophone, $M_x$, can be calculated as

$$M_x = \frac{M_r V_x}{V_r} \quad (3.12)$$
The sensitivity of a hydrophone describes the relationship between pressure experienced and voltage generated by the hydrophone. The sensitivity will in general be a function of frequency, and therefore, this process needs to be repeated over a range of frequencies.

The secondary calibration of projectors is more complicated since care has to be taken to prevent errors. Projector calibration requires good free-field conditions for an accurate calibration as the response of the transducer depends on the medium into which it radiates sound. This is because any reflected sound incident on the projector can alter the acoustic impedance it perceives and therefore affect the amount of energy radiated into the medium. Calibration of projectors by comparison to standard reference projectors is less common, because of the many errors that can be associated with this method. These are a consequence of projectors being larger than hydrophones, which makes them more prone to diffraction effects, and projectors having more spurious resonances and non-linearities.

Projectors can be calibrated by driving the projector in a free-field environment with a voltage, \( V_x \). A standard hydrophone with free-field voltage sensitivity, \( M_s \), is placed at a distance, \( d \), from the projector on the acoustic axis of the projector. The open circuit output voltage, \( V_s \), of the hydrophone is measured; the transmitting voltage response, \( S_x \), of the projector can then be calculated from

\[
S_x = \frac{V_x d}{M_s V_s}. \tag{3.13}
\]

3.3. Experimental Apparatus for Calibration

Some intermediate, secondary calibrations were carried out at Bath University. However, accurate reciprocity method calibrations were later carried out at the National Physical Laboratory (NPL). The apparatus for calibrations at Bath and NPL will now be described as well as the calibration procedure.
3.3.1. Reciprocity method calibrations

Three transducer spherical-wave reciprocity calibrations were carried out at the National Physical Laboratory, where there is a dedicated rig. A schematic diagram of this rig is shown in Figure 3.1. It shows a cylindrical wooded tank with three stepper motor stages on it. These are controlled by a computer via an IEEE link which also controls a vector analyser, a calibrated attenuator and a signal generator. The rig also consists of a power amplifier and a calibrated current probe.

The $P$, $T$ and $H$ transducers were placed in the tank so that their acoustic centres were at the same height. The computer adjusted the separation of the transducers via the stepper motor controller and rotated the transducers so that their ‘0’ marks faced each other for each of the measurement stages. The transducer acting as the projector was connected to the main output of the current probe, and the transducer acting as the hydrophone was connected to the input of the attenuator. For each measurement stage, the appropriate transducers were manually connected to the input and output.

The measurements were made by sending a tone-burst signal to the projector, for each of a series of frequencies. Each individual frequency measurement was controlled by the computer setting the signal generator to produce a tone burst of the required frequency, which was then sent to the transducer via a power amplifier and a calibrated current probe. The probe output was measured with the vector analyser so that the input current could be calculated using the probe’s calibration. The hydrophone output is passed via the calibrated attenuator on to the vector analyser. If the measured voltage level was too small or too large, the attenuator was adjusted under computer control, and the vector analyser then measured the voltage level again. This continued until an appropriate signal level was achieved. The voltage level before the attenuator was then calculated from the measured voltage and the calibration of the attenuator. The transducer calibrations were then calculated from the series of projector current and hydrophone voltage measurements.
Figure 3.1. Experimental rig, for reciprocity calibration measurements, at the National Physical Laboratory.
3.3.2. Comparison method calibrations

Comparison calibrations were made at Bath as a quick way of determining the calibration of a transducer before carrying out an accurate reciprocity calibration at NPL. The calibration of a projector was determined by using a calibrated hydrophone as a reference. A schematic diagram of the rig is shown in Figure 3.2. It consisted of a tank set up along with a synthesised function generator, which digitally produces accurate signals, an amplifier and a digital LeCroy oscilloscope. The tank arrangement consisted of an optical bench with a projector and hydrophone mounted on it. The two transducers were aligned so that their acoustic centres lay on an acoustic axis parallel with the optical bench and with their centres at the same height. They were also oriented so that the '0' mark on the transducers faced each other. The separation of the acoustic centres of the transducers was then measured. A tone-burst signal of a specific frequency was sent to the projector and oscilloscope, and the received signal was then amplified and recorded using the oscilloscope. The pulsed signal was arranged so that several cycles of the steady-state part of the signal were recorded with no reflections interfering. The oscilloscope averaged 100 samples of both the projector input voltage and hydrophone output voltage received via the hydrophone amplifier. This reduced any noise present on the signal so that a clear signal could be analysed. The time base of the oscilloscope was arranged so that a few cycles of each signal were displayed, and the instrument was programmed to calculate the peak-to-peak value of the signals. These peak-to-peak values were recorded for a range of frequencies. The secondary calibration was then calculated from the two sets of voltage readings and the calibration of the amplifier.

3.3.3. Amplifier calibrations

Amplifier calibrations were needed for both secondary calibrations of the transducers and for the reverberant calibrations discussed in chapter 6. This simply involved transmitting a continuous wave sine signal to an amplifier and measuring the input and output voltages. A schematic diagram of this experimental set up is shown in Figure 3.3. The input and output voltages were averaged 100 times by the oscilloscope and the peak-to-peak voltage measured. These measurements were made for a range of frequencies and the gain calibration calculated.
Figure 3.2. Experimental rig, for comparison method calibration measurements, at Bath University.
3.3.4. Transducer impedance

The impedance response of a transducer was measured at Bath and NPL with a Hewlett Packard '4192A LF Impedance Analyser'. The transducer was placed in a tank of water, making sure it was not near the sides of the tank or the surface of the water. The analyser was then used to measure the impedance at a range of frequencies and the data was downloaded to a computer via an IEEE link.

3.3.5. Directional measurements

The directional response of each transducer was determined by measurements made at NPL. The experimental set up for this is shown in Figure 3.4, which is similar to the rig for the reciprocity calibrations. The figure shows a cylindrical tank with stepper motor stages on top of the tank. The stepper motors were controlled by a stepper controller which, in turn, was controlled by a computer via an IEEE link. The computer also controlled a signal generator, attenuator and vector analyser via an IEEE link. The transducer, which had its directional response measured, was placed in one of the stepper stages and a hydrophone was placed in one of the free stages. Neither transducer needed to be calibrated.

The computer adjusted the signal generator to produce a tone-burst signal of a specific frequency, which was then passed on to the power amplifier, and from that on to the projector. The signal from the hydrophone was sent to a calibrated attenuator and then on to the vector analyser. The computer ordered the vector
Figure 3.4. Experimental rig, for the measurement of directional response, at the National Physical Laboratory.
analyser to sample the signal and if it was too large or too small it told the calibrated attenuator to alter the signal level until it was in an acceptable range. The computer then calculated the voltage level coming from the hydrophone from the measured signal level and the value of attenuation. The directional response of the transducer in question was made by rotating it, while measuring the output voltage from the hydrophone. It did not matter which transducer acted as the projector or hydrophone. The directional response could be measured for a range of individual frequencies.

3.4. Calibrated Transducers

In total eight transducers were used in reverberant field calibrations at three different locations. These were Bath University, the National Physical Laboratory and Sonardyne Ltd. The transducers consisted of four devices which were used as projectors and four devices as hydrophones. Table 3.2 shows the details of the transducers used: transducer code, manufacturer, transducer name, serial number, ownership, calibration date, physical description, transmit resonant frequency, transmitting voltage response (TVR) at transmit resonance and receive sensitivity (RS) at receive resonance. The difference between transmit and receive resonance, if indeed there was a receive resonance, was small for the transducers. The Q of the projector resonance curves, was calculated from the projectors conductance curves (Kuntsal and Bunker, 1992). The values obtained were P1 (Q=3.5), P2 (Q=2.8), P3 (Q=2.7) and T1 (Q=3.3). However, these values were derived from measurements made at 1kHz frequency intervals, and so more accurate values of Q could be derived if a smaller frequency interval was chosen.
### Table of Transducers (part 1)

<table>
<thead>
<tr>
<th>Code</th>
<th>Manufacturer</th>
<th>Transducer</th>
<th>Serial Number</th>
<th>Ownership</th>
<th>Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>ITC</td>
<td>ITC1001</td>
<td>N/K</td>
<td>NPL</td>
<td>N/K</td>
</tr>
<tr>
<td>P2</td>
<td>ITC</td>
<td>ITC1032</td>
<td>N/K</td>
<td>NPL</td>
<td>N/K</td>
</tr>
<tr>
<td>P3</td>
<td>ITC</td>
<td>ITC1032</td>
<td>925</td>
<td>Bath Uni</td>
<td>Feb 2000</td>
</tr>
<tr>
<td>T1</td>
<td>Graseby</td>
<td>BALL</td>
<td>N/A</td>
<td>Bath Uni</td>
<td>Feb 2000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Resonant Frequency / kHz</th>
<th>TVR / dB re 1μPa/V at 1m</th>
<th>RS / dB re 1V/μPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>spherical projector</td>
<td>18</td>
<td>150</td>
<td>-190°</td>
</tr>
<tr>
<td>P2</td>
<td>spherical projector</td>
<td>34</td>
<td>148</td>
<td>-193°</td>
</tr>
<tr>
<td>P3</td>
<td>spherical projector</td>
<td>34</td>
<td>147</td>
<td>-193</td>
</tr>
<tr>
<td>T1</td>
<td>spherical transducer</td>
<td>150</td>
<td>117 (at 50kHz)</td>
<td>-205 (at 50kHz)</td>
</tr>
</tbody>
</table>

Table 3.2. Table showing the code, manufacturer, transducer, serial number, ownership, calibration date, description, resonant frequency, transmitting voltage response (TVR) and receive sensitivity (RS) for the transducers used. (* = manufacturer’s data (nominal); + = value for P3.)

TVR and RS are at resonance for transmit and receive respectively, unless otherwise stated.
### Table of Transducers (part 2)

<table>
<thead>
<tr>
<th>Code</th>
<th>Manufacturer</th>
<th>Transducer</th>
<th>Serial Number</th>
<th>Ownership</th>
<th>Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>Bruel &amp; Kjaer</td>
<td>BK8103</td>
<td>1176386</td>
<td>Bath Uni</td>
<td>June 1999</td>
</tr>
<tr>
<td>H2</td>
<td>Reason</td>
<td>TC4034</td>
<td>319007</td>
<td>NPL</td>
<td>N/K</td>
</tr>
<tr>
<td>H3</td>
<td>Sonardyne</td>
<td>Sonardyne Ball</td>
<td>N/K</td>
<td>Sonardyne</td>
<td>N/A</td>
</tr>
<tr>
<td>H4</td>
<td>Bruel &amp; Kjaer</td>
<td>BK8103</td>
<td>1767557</td>
<td>Bath Uni</td>
<td>Feb 2000</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Code</th>
<th>Description</th>
<th>Resonant Frequency / kHz</th>
<th>TVR / dB re 1μPa/V at 1m</th>
<th>RS / dB Re 1V/μPa</th>
</tr>
</thead>
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<tr>
<td>H1</td>
<td>cylindrical hydrophone</td>
<td>120*</td>
<td>116 (at 50kHz)*</td>
<td>-213.6 (at 50kHz)</td>
</tr>
<tr>
<td></td>
<td>$\phi = 9.5$ mm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H2</td>
<td>cylindrical hydrophone</td>
<td>$\approx 330^*$</td>
<td>109 (at 50kHz)*</td>
<td>-218.7 (at 50kHz)</td>
</tr>
<tr>
<td></td>
<td>$\phi = 16$ mm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H3</td>
<td>spherical hydrophone</td>
<td>75</td>
<td>145</td>
<td>-203</td>
</tr>
<tr>
<td></td>
<td>$\phi = 22$ mm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H4</td>
<td>cylindrical hydrophone</td>
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<td>116 (at 50kHz)*</td>
<td>-212.4 (at 50kHz)</td>
</tr>
<tr>
<td></td>
<td>$\phi = 9.5$ mm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2. Table showing the code, manufacturer, transducer, serial number, ownership, calibration date, description, resonant frequency, transmitting voltage response (TVR) and receive sensitivity (RS) for the transducers used. (* = manufacturer’s data (nominal).)

TVR and RS are at resonance for transmit and receive respectively, unless otherwise stated.
3.5. Calibration Results

All eight transducers were calibrated for the reverberant field calibration measurements. All the transducers were calibrated at NPL using the reciprocity method except for the Sonardyne hydrophone (H3), which had a single figure nominal calibration using the comparison method. The transmitting response, receive sensitivity and directional plots for the transducers will now be shown.

3.5.1. Transmitting Responses

Four transducers were used as projectors: ITC1001 (P1), ITC1032 (P2 & P3) and BALL (T1). Figure 3.5 shows the transmitting voltage response of these four projectors.

![Graph showing transmitting voltage response against frequency for projectors: P1 (green), P2 (red), P3 (blue) and T1 (black).](image)

Figure 3.5. Transmitting voltage response against frequency for projectors: P1 (green), P2 (red), P3 (blue) and T1 (black).
3.5.2. Receive Sensitivities

Four transducers were used as hydrophones: BK8103 (H1 & H4), TC4034 (H2) and Sonardyne (H3). Figure 3.6 shows the receive sensitivity of hydrophones H1, H2 and H4, with a single representative value, over the frequency range, for hydrophone H3.

![Graph showing receive sensitivity against frequency for hydrophones H1 (red), H2 (green), H3 (single representative value over the frequency range) (black) and H4 (blue).]

Figure 3.6. Receive sensitivity against frequency for hydrophones: H1 (red), H2 (green), H3 (single representative value over the frequency range) (black) and H4 (blue).
3.5.3. Directional Response

Figure 3.7 shows the directional response of transducers H4, P3 and T1 in the X-Y plane (normal to the transducer symmetry axis) at 30kHz. Here zero degrees corresponds to the ‘0’ mark on the transducer, denoting the direction in which the transmitting voltage response and the receive sensitivity were measured.

Figure 3.7. Directional response in the X-Y plane at 30kHz for hydrophone H4 (blue), projector P3 (red) and transducer T1 (green). Note the expanded dB scale.
Figure 3.8 shows the directional response of the transducers P1 and P2 in the X-Z plane at 20kHz. Here 90° denotes the '0' mark on the transducer, using the IEC565 (1978) convention. The cable is in the 180° direction.

Figure 3.8. Directional response in the X-Z plane at 20kHz for projectors P1 (blue) and P2 (red).

This chapter has described the reference calibrations which will be used for comparison purposes with the reverberant calibrations. Some of these calibrations will also require a knowledge of the reverberation time of the tank used. This will be considered in the next chapter.
4.0. Reverberation Time

The reverberant calibration method used in this work requires a knowledge of the reverberation time of the tank and so this needs to be measured. This chapter contains the theory of how sound decays in an enclosure and how to determine its reverberation time. The method used to measure the reverberation time, including the equipment used and the data processing is presented.

4.1. Introduction

The standard calibration methods, described in Chapter 3, rely on free-field techniques (employing the direct sound field only). This project concerned the calibration of transmitting transducers in non free-field environments in the presence of a reverberant field. The reverberant field calibration method enables the calibrations to be calculated in three different ways; two of these ways require the reverberation time and volume of the tank to be known. Reverberant calibrations were made in eight tanks and so reverberation time measurements were made in all of these tanks. This chapter describes the various methods that were used to measure the reverberation time.

As the reverberation time of the chamber will change with frequency it was necessary to know the reverberation time at a number of frequencies so that the TVR could be calculated at each frequency.

The reverberation time may also change with position in the chamber, but an average value could be calculated from measurements made at random positions within the chamber. The reverberant calibration method samples the sound field at many points, and so a representative average value of reverberation time was required.
In order to measure the reverberation time of a chamber, a reverberant field must be generated and its resultant decay recorded. From the decay of the field, the reverberation time can be determined, as described in this chapter. The different methods of exciting the reverberant field, recording and analysing the decaying signal will be discussed. Their influence on the accuracy, range of frequencies and frequency resolution of the reverberation time result, will be discussed in Chapter 5.

4.2. Theory of the growth and decay of sound in an enclosure

When a small sound source is operated continuously in an enclosure, there are two types of sound field present. The sound initially radiates from the source uniformly in all directions spreading with a spherical wavefront; this sound field is called the direct field. When the sound reaches the boundary of the chamber, some of the sound is reflected back into the chamber and some is absorbed by the walls of the enclosure. The reflected sound can bounce off the walls many times before it is absorbed. During this time, sound waves can interact with each other, producing standing waves within the tank. All of these reflected sound waves are called the reverberant field.

When a sound source is switched on in the enclosure, the sound initially radiates away from the source as a uniform spherical wavefront (direct field). Then as it encounters the walls of the tank, some sound is reflected and some is absorbed by, transmitted through and re-radiated by the walls. The re-radiated sound level is very small compared with the reflected sound level. With each successive encounter with a wall, the wave loses a proportion of its energy, but new waves are generated all the time from the continuous wave source, so the level of the reverberant field increases with each additional reflection, although the increases become progressively smaller. The reverberant field level approaches an asymptotic value for large times. The sound in an enclosure grows exponentially, with a growth constant related to the absorption at the walls. The absorption at the water surface is usually very small by comparison to the wall surface, and can be ignored for most tanks.
Once the sound in the enclosure has reached a steady-state value, the sound source is switched off. After the direct field reaches the boundaries of the enclosure, only the reverberant field is present. With each successive reflection, the sound waves lose a proportion of their energy, just as for the growth case, until the reverberant field is no longer present. The decay and growth of sound are both governed by the same exponential time constant since the same mechanism, tank wall absorption, is responsible for both. This argument follows that in room acoustics.

In Chapter 6 the growth of sound in an enclosure is derived and leads to the fundamental differential equation governing this growth (Equation 6.7). From this equation, the equation governing the decay of uniform diffuse sound is obtained by setting the acoustic power, $W$, to zero. If the source is turned off at time $t = 0$, the reverberant pressure decays exponentially (Kinsler et al, 1982) as

$$P_r^2(t) = P_r^2(0)e^{-t/\tau_E} \tag{4.1}$$

where $P_r = \left(\frac{1}{N} \sum_{i=1}^{N} P_{el}^2\right)^{1/2}$ is the spatially averaged effective pressure amplitude of the reverberant sound field ($P_r^2$ is proportional to the reverberant intensity), $P_{el}$ the effective pressure amplitude of the $i^{th}$ sound ray, $t$ is time and $\tau_E$ is the time constant that governs the growth and decay of acoustic energy in the chamber. The time constant, $\tau_E$, is given by

$$\tau_E = \frac{4V}{Ac} \tag{4.2}$$

where $V$ is the volume of fluid in the chamber, $c$ is the speed of sound in the fluid and $A$ is the absorption of the sound. The absorption is the equivalent area of free space needed for the observed loss of sound energy. Free space is an opening in the chamber where the fluids have the same acoustic impedance inside and outside, and no sound is reflected back into the chamber. In air acoustics this is an open window.
and in underwater acoustics it is an opening to a large reservoir of water. The absorption is defined as

\[ A = S_T \hat{\alpha} = \sum_n S_n a_n \]  

(4.3)

where \( S_T \) is the total surface area of the chamber, \( \hat{\alpha} \) is the average absorptivity of the surface of the chamber, \( S_n \) is the surface area for a region \( n \) on the surface of the room and \( a_n \) is the absorptivity of this region \( n \). The sum of all \( S_n \) equals \( S_T \). The spatially averaged reverberant sound intensity, \( I \), consequently decays as

\[ I(t) = I(0) e^{-\frac{A c}{4V} t}. \]  

(4.4)

Taking base ten logarithms of both sides of Equation 4.4 yields

\[ \log_{10} I(t) = \log_{10} I(0) - \frac{A c}{4V} t \log_{10}(e). \]  

(4.5)

The definition of reverberation time (Kinsler et al, 1982), used in airborne acoustics, is that the intensity level drops 60dB over the reverberation time, \( T_r \); that is one millionth of its original value i.e. \( I(T_r) = 10^{-6} I(0) \). The pressure level also drops 60dB over the reverberation time, but that is one thousandth of its original value i.e. \( P(T_r) = 10^{-3} P(0) \). The sound intensity level (SIL) and sound pressure level (SPL) are respectively given by

\[ SIL = 10 \log_{10} \left( \frac{I(T_r)}{I(0)} \right) \]  

(4.6)

and

\[ SPL = 20 \log_{10} \left( \frac{P(T_r)}{P(0)} \right). \]  

(4.7)
Substituting reverberation time into Equation 4.5 gives

\[
\log_{10}\{I(T_r)\} = \log_{10}\{I(0)\} - \frac{Ac}{4V} T_r \log_{10}(e),
\]

\[
\log_{10}\{10^{-6} I(0)\} = \log_{10}\{I(0)\} - \frac{Ac}{4V} T_r \log_{10}(e),
\]

\[
\log_{10}\{10^{-6}\} + \log_{10}\{I(0)\} = \log_{10} I(0) - \frac{Ac}{4V} T_r \log_{10}(e);
\]

this reduces to

\[
-6 = -\frac{Ac}{4V} T_r \log_{10}(e).
\]

Rearranging gives

\[
T_r = \left(\frac{6}{\log_{10}(e)}\right) \left(\frac{4V}{Ac}\right) = 13.82 \left(\frac{4V}{Ac}\right). \tag{4.8}
\]

Substituting the speed of sound for water, \(c = 1490\text{ms}^{-1}\), into Equation 4.8 gives

\[
T_r = 0.037 \frac{V}{A}. \tag{4.9}
\]

From Equation 4.5 it can be seen that if \(\log_{10} I(t)\) is plotted against \(t\) then the y-intercept is \(\log_{10} I(0)\) and the gradient, \(G\), is

\[
G = -\frac{Ac}{4V} \log_{10}(e). \tag{4.10}
\]
Rearranging the first part of Equation 4.8 and substituting it into Equation 4.10 and then rearranging gives

\[ T_r = \frac{-6}{G}. \]  

(4.11)

Since the trace is decaying the gradient is negative and the reverberation time is positive.

Sound can be absorbed or lost from a vessel in a number of ways. Sound is absorbed by water and the walls of the tank or transmitted through the walls or the top surface of the water into the external medium. At low frequencies the absorption of sound by water will be negligible compared with the losses at the walls of the chamber. At an interface sound is reflected and refracted. The proportions of each are governed by the reflection and transmission coefficients at the interface which intern is determined by the characteristic impedance of the media. A simple model for this behaviour was proposed in an earlier report and is presented in Appendix 2, but is not related to the calibration work. In a tank, the surface of the water loses a tiny proportion of the sound from the system, and reflects most of it back into the water. Sound travels more easily into the walls of the tank, since there is a better impedance match. In the walls, the sound can be absorbed far more rapidly than in the water, and being a solid can also support transverse waves. If the walls are made up of layers, the sound can be reflected, refracted and absorbed at each successive interface.

The reverberant field is made up of many reflected wave components, some of which may form standing waves in regular shaped chambers. These standing waves or modes can be axial (propagation vector parallel to one of the axes), tangential (propagation vector parallel to one pair of surfaces) or oblique (propagation vector can have components in all three orthogonal directions). The path length of the mode must be an integer number of wavelengths, whether that is in one (axial), two (tangential) or three (oblique) dimensions. The way the reverberant field decays will depend on the frequency of the modes. Since high frequencies are absorbed more
than low, the high frequency modes disappear from the reverberant field before the low frequencies. This means that there are effectively several time constants operating in the system. The decaying signal may not be linear on a logarithmic scale, but consist of several linear sections of different gradient merging into each other. The decaying signal is further complicated because axial, tangential and oblique modes decay at different rates and the proportion of these modes change with frequency.

In the ray model of acoustics, a sound field may be assumed to be diffuse after a large number of reflections. The definition of a diffuse sound field is that the energy density is the same throughout the volume of the enclosure, and all directions of propagation are equally probable. This model oversimplifies the behaviour of sound in an enclosure because it neglects the existence of normal modes. A diffuse sound field in an enclosure can be more generally defined as: the amplitude and phase of the pressure have random values with time and position within the enclosure.

A reverberant field is considered to be diffuse when the energy density is the same everywhere in the field, but the amplitude and phase of the pressure still vary with position. However the diffuse field will not extend to the whole of the sound field. This is because the sound field will be different close to the projector, where the direct field dominates, or close to absorbing surfaces, where the reverberant field is higher, than for the rest of the chamber.

4.3. Methods of exciting the reverberant field

The signal used to drive the transducer and excite the reverberant field influences the level and frequency content of the field. The field can be excited with signals varying from a short pulse to a continuous signal, and with signals ‘containing’ a variety of spectral content.

Long pulses enable the reverberant field to reach a higher steady-state level. This enables the decay to be followed over a larger range and therefore a better determination of the reverberation time to be made.
The frequency range of the pulse determines that of the reverberant field, which determines how many modes are excited. The rate of decay of individual modes can vary greatly. The total decay curve is the summation of all the decay curves excited. The decay curve will therefore not be linear on a logarithmic scale, since many decay constants govern the total system. The decay curve usually undulates about a mean straight line due to the different rates of decay. However, if there are major modes present, then the decay may change from one gradient to another, causing distinct curves in the path of the decay curve. The greater the frequency range of the exciting signal, the more modes are excited. Provided the decay time of the modes do not vary significantly over this range, the larger number of modes means the overall decay curve is an average of more curves and will result in a smoother decay.

Generally for a small frequency range the variation in the reverberation times due to changes in the vessel properties will be small and not significant. However over a larger frequency range the reverberation time can change significantly and so averaging over a large range can lead to bias and loss of accuracy.

In practice the signals that have been used to excite the reverberant field are pulsed signals. The time between successive pulses is large so that the signal decays away before the next pulse arrives. The pulses can be of different lengths and different spectral content; the types that have been used in the current work are as follows:

a. Tone Burst:  
   i. One Cycle;  
   ii. 10 Cycles;  
   iii. Long pulse (100s of cycles).

b. Noise Burst: 
   i. Short Pulse (~ 1ms);  
   ii. Long Pulse (100s of ms);  
   iii. Long Pulse (100s of ms) – with the use of filters on reception.

The use of filters enabled the reverberation time for a small frequency range to be determined.
For a one cycle tone-burst, the energy released into the tank is small, so only a small proportion of the decay curve can be observed. Its bandwidth will be large however, approximately the frequency of the tone used. For ten cycles, the reverberant level is higher and the decay curve can be followed for longer, and the bandwidth is approximately one tenth of the frequency of the tone used. For the long pulse the decay curve is long but the bandwidth is very narrow.

To obtain a smooth decay curve, a small to medium frequency range is required. Measurements have shown that the ten cycle signal gives a smooth curve without the frequency range being too narrow or wide as is the case for the other two. The ten cycle signal gives a sufficient decay curve under the conditions that have been measured. Another approach would be to keep the bandwidth constant by fixing the length of the pulse, but changing the frequency of the sine wave in it.

The other signal type used is a noise burst, where white noise over the frequency range of interest is gated into different length pulses. Using noise has the advantage of exciting many different modes randomly instead of coherently. The advantages and disadvantages of coherent versus incoherent signals will be described in the next section. The pulses of noise used contained more energy than the tone-bursts, since the bursts could be longer while retaining an appropriate bandwidth. It is therefore advantageous to use noise-bursts instead of tone-burst, since the decay curve is longer. If white noise is used, the bandwidth of the noise burst is large. This can be a problem, but may be overcome by the use of filters to select an appropriate bandwidth on transmission or reception. However, filters with a rapid roll-off are needed to ensure that the bandwidth is well defined, especially if the projector response varies rapidly with frequency.

4.4. Measurement of the reverberant decay

The experimental set up for the measurement of the reverberant decay consisted of two rigs, one for the tone-burst signals and one for the noise-burst signals. The two rigs were very similar with only minor differences between them. Figure 4.1 shows the rig for the tone-burst signals. It consisted of a synthesised function generator to
Figure 4.1. Experimental rig for measuring reverberation time using tone bursts.
produce the tone-burst, which was sent to the projector and also recorded by the LeCroy digital oscilloscope. The hydrophone output was amplified, recorded by the oscilloscope and transferred to a computer via an IEEE link. The projector and hydrophone were supported on movable optical benches so that the reverberation time could be measured for any location within the tank. The synthesised function generator could produce a pulse of sine wave cycles of varying frequencies and number of cycles.

Figure 4.2 shows the rig for the noise-burst signals. It was essentially the same as for the tone-burst rig except the noise-burst was generated differently. A pulse generator was used to gate the noise-burst pulses produced by the synthesised function generator; the noise had a white noise spectrum from D.C. to 10MHz. The signal from the pulse generator was connected to the amplitude modulation input of the synthesised function generator, which then acted as an envelope to the white noise signal.

There are different ways of recording the decaying pressure at a point in the field. Firstly, one can simply record the response to a single pulse. Secondly, for a coherent field produced by using a repeatable tone burst, the response to a number of pulses can be averaged coherently, giving an improvement in the signal to noise ratio. Finally, the magnitude of the signals may be averaged. This is most appropriate for noise bursts where successive pulses are uncorrelated.

When using noise to excite the reverberant field, the bandwidth is very large unless filters are used. This will result in the reverberation time being averaged over a range of frequencies. Narrower bandwidths, for producing reverberation time versus frequency graphs, can be obtained if short pulses of noise are used. This relies on capturing the entirety of the signal (hence the use of a short pulse) and then performing a FFT on the signal to convert it into the frequency domain. There the spectrum over the appropriate bandwidth is extracted and converted back into the time domain. This effectively results in the use of filters, but in the processing stage instead of the experimental stage. This method is only appropriate if the signal has not been averaged incoherently since the averaging process (usually the magnitude
Figure 4.2. Experimental rig for measuring reverberation time using noise bursts.

AM = Amplitude Modulation
of the signal) results in the loss of the phase information and a FFT cannot be performed.

There are, consequently three ways to use pulsed noise. Firstly the whole bandwidth of the noise burst can be used with incoherent averaging. Secondly, separate bandwidths (generated by processing) can be used with no averaging. Finally, separate bandwidths, generated on transmission, can be used with incoherent averaging. The third method has the advantages in terms of frequency resolution and signal level, but requires a long measurement process, especially if the field is measured at many positions within the tank.

When a short pulse signal is transmitted from a projector the received signal from the hydrophone can be considered to consist of three parts; initially a section of noise, then the direct arrival from the transmitter and last the reverberant part of the signal. A schematic diagram showing the transmitted and received signals is shown in Figure 4.3.

![Figure 4.3. Short pulse transmitted and received signals.](image-url)
For a long pulse signal the direct and reverberant parts of the signal arrive before the burst has finished. If the oscilloscope is triggered off the negative edge of the pulse envelope (the end of the pulse), and a delay added equivalent to the time needed for the sound to travel between the projector and the hydrophone, then the captured signal will only contain the reverberant part of the signal. A schematic diagram for this situation is shown in Figure 4.4. This method can also be used for a short pulse to give only the reverberant part of the signal, however it is advantageous to measure the background noise level for a single shot signal. This is because the background noise level can be used to reduce the noise floor of the trace and therefore increase the decay range of the signal.

![Diagram of transmitted and received signals](image)

4.5. Processing the decay curve to yield reverberation time

From the theory section (4.2) it can be seen that the reverberation time can be obtained from the gradient of a graph of the logarithm of the signal versus time, with the gradient being calculated using a least squares fit (linear regression).
Whether using single shot or multiple shot average the reverberant part of the signal must be extracted and processed appropriately to obtain the reverberation time. So if the signal captured contains more than the reverberant part of the field, preprocessing must occur to extract it from the larger signal.

The processes that have been developed to determine the reverberation time will now be described. Section A3.1, in Appendix 3, contains a list of all the major stages of the reverberation time processing.

### 4.5.1. Single shot record

First, the mean of the signal is calculated and any D.C. level removed. Figure 4.5 shows an example decay curve at this stage. The mean squared noise value at the beginning and end of the signal is then determined. The start of the direct arrival is found by finding the first point that the signal exceeds half of the maximum value in the record. The reverberant field part of the signal is then found by determining if there is a gap in the signal after the start of the direct field.

![Figure 4.5. Tone burst single shot decaying waveform.](image-url)
This is achieved by testing for a low-level signal, compared with the record maximum, that exceeds 300µs. If there is a gap, then the signal after this gap is defined as the reverberant part of the signal. If the gap is less than 300µs then the direct and reverberant parts of the signal are considered to merge into each other and the signal is defined as starting at the beginning of the direct field. A time less than this is not significant because fluctuations in the decaying signal, due to beat frequencies, are often of similar size.

At this stage the signal fluctuates significantly so to smooth out the variations a 500 or 40 point mean square average is performed over the signal, therefore reducing the number of points in a given time. The number of points over which the signal is averaged depends on the original number of points in the record, as described in section A3.1 of Appendix 3. The mean squared noise is then subtracted from the signal and the square root taken. Removing the background noise increases the dynamic range over which the signal decays. This will be explained in more detail in section 4.5.2 along with whether it is better to use beginning or end noise in section A3.1. The next stage is to take the logarithm of the decay curve which produces a fluctuating signal which follows a straight line graph with a negative slope until it reaches the noise floor where it approaches a plateau. An example of this graph is shown in Figure 4.6 with a decibel scale.

The decay part of the graph (that follows a straight line) is shown as a solid line while the plateau section is shown as a dashed line. If a mean squared (ms) average had not been performed then the fluctuations would be very large. If the calculated ms noise had not been removed from the ms averaged signal then the level of the plateau would have been increased by 10dB to 30dB. The removal of the noise can therefore effectively double the span of the straight-line part of the graph and makes the determination of its gradient more accurate.
In order to determine the gradient of the straight-line part of the logarithmic graph, this region has to be extracted. This is done by applying a two point average over the graph, to reduce the fluctuations even further, which produces a reasonably straight line with a negative slope (for short times) and a plateau (at longer times), as shown in Figure 4.7. Although this is not necessary for the example in this tank, in other tanks the decaying signal does not follow such a straight line, and the improved signal helps in marking the transition from decaying signal to plateau region. This figure also shows two circles marking the last 30% of the trace. The mean of this region is calculated and used to represent the level of the plateau.
Figure 4.7. Tone-burst single-shot averaged decay curve clearly showing that the signal decays to a noise floor.

Also on the figure are two dashed red lines corresponding to levels (11dB in this case) above and below the plateau. If the trace between the two circles keeps within the two lines then the trace is taken as having a plateau; if it does not then it is assumed that there is no plateau. The level of these lines needed either side of the plateau level varies with the individual trace with a plateau, but for most traces the level of the lines remains fixed for a particular tank. If there is a plateau then the end of the decaying region is taken to be where the decaying curve first crosses the level 10dB above the plateau. The slope of the straight-line section is determined from the logarithmic graph before the second average was applied (Figure 4.6). If there is no plateau then the decay region is taken to fill the whole of the graph, and the straight-line gradient is determined from the whole record. The extracted decay region is shown as the solid line in Figure 4.6. A least squares fit linear regression is then applied to the graph and the gradient determined, as shown in Figure 4.8. The solid straight line is the linear regression fit as shown in Figures 4.6 and 4.8. From this gradient the reverberation time is then easily calculated.
4.5.2. Multiple-shot averaged record

Hear the decay curve is recorded using a digital LeCroy oscilloscope and then its magnitude is calculated. The magnitude is sampled 50 times and the average calculated. This averaged signal is then converted into a logarithmic scale and the signal saved by the LeCroy, as shown in Figure 4.9. This figure shows a far less noisy signal than was the case for the single shot record. The reverberant field decay is taken to start at the maximum value, since the record has little fluctuations, and is shown by the solid line in the figure. The signal is processed from this stage onwards as for the single shot record except no noise is removed. The straight-line decay part of the signal is extracted and the gradient of this region and the reverberation time are then calculated. Figure 4.10 shows the signal at this end stage with considerably less fluctuations than for the non-averaged tone burst case, with the straight line representing the linear regression fit.
Figure 4.9. Averaged multiple-shot noise-burst signal on a logarithmic scale.

Figure 4.10. Linear regression applied to the straight-line region of the multiple-shot average waveform (with dots indicating the data points).
In this method, the mean square noise level prior to the signal was not removed as this did not reduce the noise floor. For this technique to work, it is important that there is no D.C. offset on the signal. If there is a D.C. offset, then the calculation of the mean square noise will be incorrect, since a bias will be introduced in to the calculation of the noise. Consequently, the noise floor will not be reduced when the calculated noise value is removed, as a new noise level (offset) will have been introduced. When the magnitude is taken before the noise level is calculated the D.C. bias is not known and so this method cannot work.

The background noise is removed to lower the noise floor on single shot records and therefore increase the dynamic range of the decay, which enables a greater accuracy in the determination of reverberation time. Although the background noise cannot be removed for multiple shot averaging, using this experimental set up, it is not necessary. This is because multiple shot averaging of incoherent signals results in a smoother decay curve than that for a single shot. Consequently the reverberation time can be accurately determined from the initial part of the decay curve where the noise floor is not significant.

4.5.3. Single shot record of a short noise pulse

The noise burst is processed in the same way as for the tone burst and generates a value for reverberation time. Figure 4.11 shows the decaying waveform and Figure 4.12 shows the processed straight-line decay curve with the linear regression fit line.
Figure 4.11. Single-shot noise-burst decaying waveform.

Figure 4.12. Linear regression applied to single-shot noise-burst waveform.
If the entirety of the pulse is captured then the signal can be converted into the frequency domain and split into different frequency bands. Each band is then converted back into the time domain and the reverberation time is calculated using the single shot procedure. Figure 4.13 shows an example decay curve for one of the bands converted back into the time domain.

Figure 4.13. Extracted frequency band decaying waveform from the single shot noise burst.

Figure 4.14 shows a straight-line decay curve for one of the frequency bands and the linear regression fit line. This procedure results in reverberation time values for each frequency band. The generated decay curves are not as clean as the original pulse but are still reasonable and give an adequate decay curve. The results are similar to the values for the tone burst signals but less accurate.
This chapter has discussed the measurement of reverberation time, the results for a number of tanks used in this work are presented in Chapter 5, along with an analysis of the data.
5.0. Reverberation Time in Different Tanks

This chapter describes the results of a programme to measure the reverberation time in eight tanks at the University of Bath, the National Physical Laboratory and Sonardyne. The chapter starts with a description of the tanks and then compares the results of the different measurement methods introduced in Chapter 4 for one of the tanks. The results for all eight tanks are then summarised and used to calculate the average absorptivity of the tanks. Finally the results are compared.

5.1. Description of tanks used for measurements.

Eight tanks were used for the measurements and the reverberation time was measured in all of these tanks by one or more methods. The tanks ranged in size from 0.16m³ to 120m³ and were either rectangular or cylindrical in shape. The tanks were made of various materials which included polypropylene, glass reinforced polymer, steel, concrete, glass and wood.

Table 5.1 shows the properties of the nine tanks: location, shape, construction, dimensions (depth of water), volume of water and nominal reverberation time at 30kHz (the reverberation time depends on frequency and depth of water). The values are quoted at this frequency as it is the resonance frequency of projector P3, which was used for the majority of measurements except those in tanks B4 (in which transducer T1 was used) and N1 (in which projector P2 was used). As can be seen the first letter of the tank code refers to the location (Bath, NPL or Sonardyne).
## Table of Tanks (Part 1)

<table>
<thead>
<tr>
<th>Code</th>
<th>Location</th>
<th>Shape</th>
<th>Construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Bath University</td>
<td>Rectangular</td>
<td>Polypropylene with encapsulated steel bands</td>
</tr>
<tr>
<td>B2</td>
<td>Bath University</td>
<td>Rectangular</td>
<td>Polypropylene inner skin with steel outer shell</td>
</tr>
<tr>
<td>B3</td>
<td>Bath University</td>
<td>Rectangular</td>
<td>Concrete tank sunk into the ground</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Dimensions (length x width x depth) / m</th>
<th>Volume of Water / m³</th>
<th>Nominal Reverberation Time / ms (at 30kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>2.72 x 1.51 x 1.32</td>
<td>5.42</td>
<td>79(±3)</td>
</tr>
<tr>
<td>B2</td>
<td>1.86 x 1.18 x 1.09</td>
<td>2.39</td>
<td>60(±2)</td>
</tr>
<tr>
<td>B3</td>
<td>3.06 x 1.52 x 1.68</td>
<td>7.81</td>
<td>48(±1)</td>
</tr>
</tbody>
</table>

Table 5.1. Table showing the code, location, shape, construction, dimensions (depth of water), volume of water and nominal reverberation time (at 30kHz) for the tanks used.
### Table of Tanks (Part 2)

<table>
<thead>
<tr>
<th>Code</th>
<th>Location</th>
<th>Shape</th>
<th>Construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>NPL</td>
<td>Rectangular</td>
<td>Polypropylene with encapsulated steel bands</td>
</tr>
<tr>
<td></td>
<td>(dismantled in 1999)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N2</td>
<td>NPL</td>
<td>Cylindrical</td>
<td>Wooden barrel with steel rings</td>
</tr>
<tr>
<td>N3</td>
<td>NPL</td>
<td>Rectangular with concave panels</td>
<td>Glass Reinforced Polymer panels bolted together</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Dimensions (length x width x depth or diameter x depth) / m</th>
<th>Volume of Water / m³</th>
<th>Nominal Reverberation Time / ms (at 30kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>2.0 x 1.5 x 1.4</td>
<td>4.2</td>
<td>60(±2)</td>
</tr>
<tr>
<td>N2</td>
<td>φ = 5.5, d = 5.0</td>
<td>120</td>
<td>68(±5)</td>
</tr>
<tr>
<td>N3</td>
<td>2.0 x 1.5 x 1.5</td>
<td>4.5</td>
<td>109(±5)</td>
</tr>
</tbody>
</table>

Table 5.1. Table showing the code, location, shape, construction, dimensions (depth of water), volume of water and nominal reverberation time (at 30kHz) for the tanks used.
### Table of Tanks (Part 3)

<table>
<thead>
<tr>
<th>Code</th>
<th>Location</th>
<th>Shape</th>
<th>Construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Sonardyne</td>
<td>Rectangular with panels</td>
<td>Steel panels bolted together, with inner plastic sheet lining</td>
</tr>
<tr>
<td>B4</td>
<td>Bath University</td>
<td>Rectangular</td>
<td>Glass plates joined with silicone sealant, with wooden brace top and bottom and a polystyrene cushion</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Dimensions (length x width x depth) / m</th>
<th>Volume of Water / m$^3$</th>
<th>Nominal Reverberation Time / ms (at 30kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>4.88 x 3.66 x 3.51</td>
<td>62.7</td>
<td>185(±7)</td>
</tr>
<tr>
<td>B4</td>
<td>0.988 x 0.488 x 0.336</td>
<td>0.162</td>
<td>182(±6)</td>
</tr>
</tbody>
</table>

Table 5.1. Table showing the code, location, shape, construction, dimensions (depth of water), volume of water and nominal reverberation time (at 30kHz) for the tanks used.
5.2. Effect of the reverberation time method on the results

Three main types of measurement method were used to obtain these results, with one category further subdivided. They were:

A. Tone-Burst Single-Shot
B. Noise-Burst Single-Shot
   (i). Whole Waveform
   (ii). Frequency Band Subsets
C. Noise-Burst Multiple-Shot Average

The following are detailed results for the concrete tank B3 at Bath. They show the breakdown of the results for the different reverberation time methods and how accurate and reproducible they were.

5.2.1. Method A: Tone-Burst Single-Shot

These results were generated by exciting the sound field with a tone burst consisting of ten cycles of the test frequency, at a repetition frequency of 10Hz. Using this pulse repetition frequency, or lower, left sufficient time for the sound field to decay to the background noise level. The sine wave frequencies used ranged from 10kHz up to 100kHz in steps of 10kHz. Reverberation time measurements were made for each frequency at five different observation positions.

Figure 5.1 shows a graph of reverberation time verses frequency for all five positions with error bars representing the standard error derived from the least squares fit of the decay curve. From the graph it can be seen that the results for the different locations are generally in agreement. The variations are generally to within one to two standard errors.
Figure 5.1. Reverberation time versus frequency for five different positions within tank B3, measured using method A.

Figure 5.2 shows a graph of reverberation time, averaged over all five positions, versus frequency with error bars representing the standard error derived from the distribution of values over the five positions. The graph shows a smoothly decreasing curve with frequency, apart from the first point. This would be expected since absorption in the tank walls increases with frequency. The standard error generally decreases with frequency, this is probably due to the increased modal density at higher frequencies. This means there is greater averaging for a given bandwidth and therefore a smoother decay curve. This leads to a more accurate determination of the decay gradient and so a smaller standard error.
5.2.2. Method B: Noise-Burst Single-Shot

These results were generated by exciting the sound field with a noise burst, of length 1.0ms, which was pulsed at a repetition frequency of 10Hz. The noise burst had a frequency range limited by the response of the transducer, which enabled frequencies over the range 1kHz to 100kHz to be transmitted. Reverberation time measurements were again made at five different positions. Each noise pulse was processed to give a reverberation time for the pulse as a whole (method B(i)). The pulse was also split up into separate frequency bands and the reverberation time for each of these bands calculated (method B(ii)). The centre frequencies of the bands were chosen to be the same as those of the corresponding tone bursts used in method A. This enabled comparison of the two methods.
Figure 5.3 shows a graph of reverberation time versus frequency for all five positions obtained using method B(ii). The error bars show the standard error and were derived for the least squares fit to the decay curve. The graph also shows that the results for different positions agree with each other, within one to two standard errors. The two methods give very similar results within the error limits, but method B does appear to be less accurate with slightly larger standard errors. This could be due to a dynamic range problem of the oscilloscope. When the signal is split into frequency bands the energy in each band will be less, making the dynamic range of the signal less than that of the whole. Also, the bands away from resonance have a far lower signal level than at resonance. These two effects mean that the voltage level may be smaller and the digitising error more significant. This means that the error in the results will be greater than those for method A, particularly at frequencies away from resonance. This is shown in Figure 5.3 to a small extent above resonance, where the transmitting response drops off slowly, and far more below resonance, where it drops off quickly.

![Graph of Reverberation Time versus Frequency](image)

Figure 5.3. Reverberation time versus frequency for five different positions within tank B3, measured using method B(ii).
Figure 5.4 shows a graph of reverberation time, averaged over all five positions, versus frequency with error bars representing the standard error derived from the distribution of values over the five positions. The graph shows an increase and then a decrease of reverberation time with increasing frequency. The decrease with frequency is fairly smooth, but the values around the peak undulate.

![Graph showing reverberation time versus frequency](image)

Figure 5.4. Reverberation time, averaged over all five positions, versus frequency for tank B3 measured using method B(ii).

Figure 5.5 shows a graph of reverberation time versus position, calculated using the whole of the noise burst and method B(i), for each of the five noise-bursts. The graph shows that the five pulses give the same result within the errors shown. These results use the whole of the noise burst without splitting into bands where the spectral content is dependent on the sensitivity of the transmitting transducer. The frequency range used was 1kHz to 100kHz with a peak in the resonance of the transducer (P3) at 30kHz. The calculated reverberation time is therefore a weighted average depending on the varying signal levels over the frequency range. These results are therefore weighted at 30kHz and are consistent with the individual frequency band results for 30kHz.
Figure 5.5. Reverberation time versus position for method B(i), using the whole noise pulse, in tank B3.

5.2.3. Method C: Noise-Burst Multiple-Shot Average

These results were generated by exciting the sound field with a noise burst of length 1.0ms, with a pulse repetition frequency of 10Hz. The acoustic noise burst had a frequency range limited by the response of the transducer and the bandwidth of the exciting noise burst signal. The frequency range was 1kHz to 100kHz, so the exciting field was therefore exactly the same as for method B. However the signal was recorded in a different way. Instead of a single shot capture of the signal, the signal was sampled and its magnitude taken fifty times. The mean of these fifty magnitudes was then calculated. The sampling of the acoustic signals and calculations were all performed by the LeCroy digital oscilloscope. These multiple shot average measurements were then made at five different positions.
Figure 5.6 shows a graph of reverberation time versus position for each of the five multiple shot averaged noise bursts. The error bars show standard error and were derived from the least squares fit of the decay curve. The graph shows that the five pulses give the same result within the estimated uncertainties.

The results for methods A and B(ii) are compared in Figure 5.7, which shows reverberation time versus frequency. The two curves are in good agreement with each other, with no differences of more than two combined standard errors. The standard error of method B(ii) is generally larger than that for method A, which explains the greater variation of the results for method B(ii). The average of these two curves is taken as the reverberation time response of the tank with a combined standard error obtained from the two errors.

There is a larger difference between the results for method A and B(ii) at 10kHz, than for the results at the other frequencies. This difference is due to the standard error in the B(ii) result at 10kHz being much larger than the other frequencies. This
is probably due to the original decaying noise waveform being split into different frequency bands, and the reverberation time then derived from each bands decay curve. The signal level from the projector (P3) is much lower at 10kHz than at resonance, or above resonance up to 100kHz, and consequently the dynamic range available to digitise the signal at 10kHz is much less than at resonance. Therefore the 10kHz decaying waveform has a much smaller dynamic range than the resonance waveform and so the uncertainty in the 10kHz waveform will be greater than at resonance. There is therefore a greater standard error in the 10kHz reverberation time result and so the mean value could be significantly different to the true value, and so significantly different from the method A result at 10kHz. The 10kHz reverberation time result for method A has the same dynamic range as for the other method A frequencies, since each one was captured individually at full dynamic range.

Figure 5.7. Reverberation time versus frequency for methods A (square) and methods B(ii) (triangle), for tank B3 (Bath Concrete).
The error in the 10kHz B(ii) result will also be greater because the modal density is lower at lower frequencies, and so less smoothing of the decay curve will occur leading to a larger error. These two sources of error probably explain the larger difference between the method A and B(ii) results at 10kHz compared to the other frequencies.

So in conclusion method A results agree with method B(ii) results within the expected uncertainties. Methods B(i) and C agree with each other to within experimental errors and give results for the whole signal that agree with those from methods A and B(ii) for the value of reverberation time near to the resonance of the projector at 30kHz.

5.3. Reverberation time results for all the tanks.

The following section contains the reverberation time results for all the tanks used. The tanks were of different size, shape and construction and at different sites. The reverberation time results depend on which methods were used at each site. The best results were obtained when both methods A and B(ii) were used so that an accurate description of the reverberation time response was found. In some cases this frequency response was obtained but only using one method, either A or B(ii). For most of the tanks an overall reverberation time was found, using method B(i) or C. This was just a confirmatory result for most tanks, but for two of the tanks these were the only results obtained and so the reverberation time frequency response is not known.
5.3.1. Type of measurement taken in each tank.

Table 5.2 shows the type of reverberation time measurement made for each tank and how the final result was obtained. When a measurement was made with method B(i) it was not always possible to carry on the frequency analysis by using method B(ii). To split the noise signal into frequency bands the whole of the signal was needed, but if a long pulse was used it was not always possible to record it all. A long pulse was used to obtain a higher reverberant field level in the tank. From Table 5.2 it can be seen that this was the case for two tanks: N1 and N3.

<table>
<thead>
<tr>
<th>Tank</th>
<th>Method A</th>
<th>Method B(I)</th>
<th>Method B(ii)</th>
<th>Method C</th>
<th>Final Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>Mean (A+B(ii))</td>
</tr>
<tr>
<td>B2</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>Mean (A+B(ii))</td>
</tr>
<tr>
<td>B3</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>Mean (A+B(ii))</td>
</tr>
<tr>
<td>B4</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>YES</td>
<td>A</td>
</tr>
<tr>
<td>N1</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>B(i)</td>
</tr>
<tr>
<td>N2</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>B(ii)</td>
</tr>
<tr>
<td>N3</td>
<td>NO</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>B(i)</td>
</tr>
<tr>
<td>S1</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>Mean (A+B(ii))</td>
</tr>
</tbody>
</table>

Table 5.2. Table indicating which types of reverberation time measurement were taken in each tank. Method: A (Tone-Burst), B (Noise-Burst) – (i) Whole Signal – (ii) Separate Frequency Bands, C (Averaged Noise-Burst). For tank description see Table 5.1. Final result indicates how the end result was derived.
5.3.2. Reverberation time results for all tanks.

A graphical representation of reverberation time versus frequency for six tanks and a single value for two tanks, centred around the projector resonance of 30kHz, is shown in Figure 5.8. How the results were obtained is shown in Table 5.2.

Figure 5.8. Reverberation time versus frequency for six tanks and representative values for two others at 30kHz. Black circle – Bath Plastic (B1); blue – Bath Metal (B2); red - Bath Concrete (B3); cyan – Bath Glass (B4); black triangle – NPL Plastic (N1); green circle – NPL Wood (N2); black square – NPL Glass Reinforced Polymer (N3) and green triangle – Sonardyne Metal (S1).

Figure 5.8 indicates that the reverberation time response of tanks B1, B2, B3 and N2 do not vary much with frequency, but generally decrease, and are similar to each other with a value of approximately 50ms. The result for tank N1 lies within this range and these results can be collectively called the bottom group. There are two discrepancies for these results: the 10kHz value for tank N2, which is far higher but also has a large error bar, and the 20kHz value for tank B1 which is also far higher but with the same size error bar as the rest of tank B1. The 10kHz value for tank N2
could be a true value since it does fit in with the tanks curve, which generally decreases in reverberation time with increasing frequency. However, since the standard error is large it could be a false value, which could be due to the low signal level at 10kHz when using the method B(ii) (as described for figure 5.7, in section 5.2.3). The 20kHz value for tank B1 may be the true value since the standard error is no larger than the rest of the tank values. The value is derived from the methods A and B(ii), but any increase in error from method B(ii) would reveal itself in the standard error, which it does not. Also the 10kHz value would be affected far more than the 20kHz value, but in fact the 10kHz value fits in with the rest of the lower frequency values. Maybe the tank absorption at this frequency range is less than at the other frequencies. The approximate average standard error for tank B1 is 3ms, 2ms for tank B2, 1ms for tank B3 and 6ms for tank N2, with the 10kHz points generally having a larger uncertainty than the average.

Tank N3 has a value of 109(±5)ms which places it between the top and bottom group. The top group comprises of two tanks: S1 and B4. These results vary with frequency more, with those in tank S1 decreasing with frequency and those in tank B4 increasing with frequency. The values in tank S1 range from 208ms to 114ms, with varying error bars having an approximate value of 10ms. Those in tank B4 range from 142ms to 183ms, also with varying error bars having an approximate value of 6ms.

Generally the absorption of a material to sound increases as frequency increases, and so the absorption of a tank will increase with increasing frequency. Therefore the reverberation time will decrease with increasing frequency, since the sound decays quicker at higher frequencies. However, from Figure 5.8, it can be seen that the reverberation time of tank B4, constructed of glass, increases with frequency. This is opposite of what has just been argued and the reverberation time responses of the other tanks. The increasing reverberation time of the glass tank indicates that the absorption of the glass decreases with frequency. There is evidence for this in the texts of other authors where the attenuation of sound in glass is generally shown to decrease with frequency (Turner and Pretlove, 1991; Woods, 1972; Krautkramer and Krautkramer, 1983; Kaye and Laby, 1986; Tennent, 1990). However, this is only an
indication of this behaviour over a part of the attenuation spectrum for glass, and at a far lower frequency than that used in the tank measurements.

Selected reverberation time results are shown in Table 5.3. It shows reverberation time with standard error against frequency values of 30kHz, 50kHz and 70kHz; for each of the tanks.

<table>
<thead>
<tr>
<th>Tank \ Frequency</th>
<th>30kHz</th>
<th>50kHz</th>
<th>70kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bath Plastic (B1)</td>
<td>79(±3)</td>
<td>55(±3)</td>
<td>61(±2)</td>
</tr>
<tr>
<td>Bath Metal (B2)</td>
<td>60(±2)</td>
<td>55(±3)</td>
<td>49(±2)</td>
</tr>
<tr>
<td>Bath Concrete (B3)</td>
<td>48(±1)</td>
<td>44(±1)</td>
<td>38(±1)</td>
</tr>
<tr>
<td>Bath Glass (B4)</td>
<td>182(±6)</td>
<td>161(±6)</td>
<td>183(±1)</td>
</tr>
<tr>
<td>NPL Plastic (N1)</td>
<td>60(±2)</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>NPL Wood (N2)</td>
<td>68(±5)</td>
<td>50(±5)</td>
<td>37(±2)</td>
</tr>
<tr>
<td>NPL GRP (N3)</td>
<td>109(±5)</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Sonardyne Metal (S1)</td>
<td>185(±7)</td>
<td>148(±5)</td>
<td>142(±6)</td>
</tr>
</tbody>
</table>

Table 5.3. Reverberation time with standard error, in milli-seconds, against frequency and tank.

5.3.3. Calculation of average absorptivity for all tanks.

The reverberation time of the tank can be related to the average absorptivity of the tank walls using theory from section 4.2. Substituting Equation 4.10 into Equation 4.16 and rearranging gives the reverberation time, \( T_r \), as

\[
T_r = \frac{0.037V}{A} = \frac{0.037V}{\sum_n S_n a_n} = \frac{0.037V}{S_T \hat{a}}
\]  

(5.1)

where \( V \) is the volume of the water, \( A \) is the absorption of sound at the water boundary, \( S_n \) is the surface area for a region \( n \) on the boundary of the water, \( a_n \) is the absorptivity of this region \( n \), \( S_T \) is the total surface area of the water and \( \hat{a} \) is the average absorptivity at the water boundary. The sum of all \( S_n \) equals \( S_T \).
Therefore to obtain the average absorptivity of the tank, and not the average absorptivity of the whole surface in contact with the water, the absorptivity of the water to air interface needs to be known. If it can be assumed that most of the sound incident on the water to air interface will be reflected then the absorptivity of this interface will be assumed to be zero. Since the reflection coefficient of the water to tank interface is considerably lower than that for the water to air interface this assumption is valid provided great accuracy is not required. With a knowledge of the surface area of the water in contact with the tank and the air, the average absorptivity of the tank walls can be calculated. The absorption of the surface area of the water, $A$, can now be expressed as

$$A = S_{tk} \hat{\alpha}_{tk} + S_a \hat{\alpha}_a$$  \hspace{1cm} (5.2)

where $S_{tk}$ is the surface area of the water in contact with the tank, $\hat{\alpha}_{tk}$ is the average absorptivity of the tank / water interface, $S_a$ is the surface area of the water in contact with the air and $\hat{\alpha}_a$ is the absorptivity of the air / water interface. Since the absorptivity of water to air interface, $\hat{\alpha}_a$, is assumed to be zero the second part of Equation 5.2 reduces to zero. Equation 5.2 is then substituted into Equation 5.1 and rearranged to give

$$\hat{\alpha}_{tk} = \frac{0.037V}{S_{tk} T_r}.$$  \hspace{1cm} (5.3)

Equation 5.3 is now used to calculate the average absorptivity of the different tanks. This gives a measure of the intrinsic absorption of the material the tank is made from and is therefore useful in designing tanks of specific reverberation times or estimating reverberation times of existing tanks.

The tank walls and water within the tank comprise a tank system. The absorption within the tank system consists of two parts, the loss of sound from the system and the attenuation of sound in the system. Sound is lost from the system in three ways, from the water surface to the air, from the tank walls to the air and from the base of
the tank to the floor. The attenuation of sound in the system occurs in the water and in the tank walls. Appendix 2 describes the results of a simple thought experiment on this system with the losses and attenuation of sound in it; a brief summary will be given here. The proportion of sound that travels from one medium to another is governed by the reflection coefficient of the boundary. This can be calculated from the characteristic acoustic impedance of the materials involved. From this it can be seen that sound easily travels through a water to tank wall interface and so both the water and wall have high sound levels within them. The attenuation of sound in water, at the low frequencies being measured of 10kHz to 100kHz, is very small. However the attenuation of sound in the walls of the tank is significant and so virtually all the sound is attenuated in the walls of the tank. Sound can also leave the tank system in the three ways mentioned above. However from the reflection coefficients it can be seen that, for most tanks, the amount that leaves is very small compared to the amount of sound that can enter the walls of the tank. Since the attenuation of sound in the walls is considerable the absorption of sound in the system is mainly due to this mechanism and not by sound leaving the system. The thought experiment indicates that when sound leaves the system it primarily does so through the floor, then the water to air interface and lastly through the wall to air interface. This thought experiment suggests that for most tanks the absorptivity of the tank walls is the dominant mechanism for absorption in the tank. The exception to this is when there is a large amount of coupling of the tank to the ground, such as if a tank is sunk into the floor.

The calculated average absorptivity (at 30kHz) for the eight tanks, along with the tank material and coupling to the ground, is shown in Table 5.4.

The average absorptivities of the tanks in Table 5.4 do not take into account the thickness of the walls of the tank. This will give a misleading interpretation of the absorptivity of a material, as will the different couplings to the ground. The absorption of the tank walls depends on the attenuation coefficients of the materials used but these will vary with frequency and the type of sound wave present.
<table>
<thead>
<tr>
<th>Tank Code</th>
<th>Dimensions (length x width x depth or diameter x depth) / m</th>
<th>Volume of Water, $V / m^3$</th>
<th>Surface Area of Water in Contact with Tank, $S_A / m^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>2.72 x 1.51 x 1.32</td>
<td>5.42(±0.03)</td>
<td>15.27(±0.07)</td>
</tr>
<tr>
<td>B2</td>
<td>1.86 x 1.18 x 1.09</td>
<td>2.39(±0.02)</td>
<td>8.82(±0.05)</td>
</tr>
<tr>
<td>B3</td>
<td>3.06 x 1.52 x 1.68</td>
<td>7.81(±0.04)</td>
<td>20.04(±0.07)</td>
</tr>
<tr>
<td>B4</td>
<td>0.988 x 0.488 x 0.336</td>
<td>0.1620(±0.0003)</td>
<td>1.474(±0.002)</td>
</tr>
<tr>
<td>N1</td>
<td>2.0 x 1.5 x 1.4</td>
<td>4.2(±0.2)</td>
<td>12.8(±0.6)</td>
</tr>
<tr>
<td>N2</td>
<td>$\phi = 5.5, d = 5.0$</td>
<td>119(±2)</td>
<td>110(±1)</td>
</tr>
<tr>
<td>N3</td>
<td>2.0 x 1.5 x 1.5</td>
<td>4.5(±0.2)</td>
<td>13.5(±0.6)</td>
</tr>
<tr>
<td>S1</td>
<td>4.88 x 3.66 x 3.51</td>
<td>62.7(±0.1)</td>
<td>77.8(±0.1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tank Code</th>
<th>Reverberation Time, $T_r / ms$ (at 30kHz)</th>
<th>Average Absorptivity of tank, $\hat{\alpha}_t$ (at 30kHz)</th>
<th>Tank Material</th>
<th>Coupling of Tank to Ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>79(±3)</td>
<td>0.166(±0.006)</td>
<td>Polypropylene</td>
<td>Sits on wooden beams</td>
</tr>
<tr>
<td>B2</td>
<td>60(±2)</td>
<td>0.167(±0.006)</td>
<td>Polypropylene</td>
<td>Sits in steel tank on ground</td>
</tr>
<tr>
<td>B3</td>
<td>48(±1)</td>
<td>0.301(±0.007)</td>
<td>Concrete</td>
<td>Sunk into ground</td>
</tr>
<tr>
<td>B4</td>
<td>182(±6)</td>
<td>0.0223(±0.0007)</td>
<td>Glass</td>
<td>Sits on polystyrene cushion on desk</td>
</tr>
<tr>
<td>N1</td>
<td>60(±2)</td>
<td>0.20(±0.02)</td>
<td>Polypropylene</td>
<td>Sits on ground</td>
</tr>
<tr>
<td>N2</td>
<td>68(±5)</td>
<td>0.59(±0.04)</td>
<td>Wood</td>
<td>Sits on ground</td>
</tr>
<tr>
<td>N3</td>
<td>109(±5)</td>
<td>0.113(±0.009)</td>
<td>Glass Reinforced Polymer</td>
<td>Sits on ground</td>
</tr>
<tr>
<td>S1</td>
<td>185(±7)</td>
<td>0.161(±0.006)</td>
<td>Steel</td>
<td>Sits on ground</td>
</tr>
</tbody>
</table>

Table 5.4. Shows the following details needed to calculate the average absorptivity (at 30kHz) of the tank and the context for the absorptivity value: tank code, dimensions (depth of water), volume of water, area of water in contact with tank, reverberation time (30kHz), tank material and coupling of tank to the ground.
To analyse this data with the limited information about how significant coupling to the ground is it will be assumed that the coupling is small and can be ignored since the proportion of the area in contact with the ground is small. However, for the concrete tank (B3) this is not the case since it is sunk into the ground with coupling on five sides. This tank has the second largest absorbitivity of the group, at 30%, where such a large figure is probably due to the large coupling. The values for the rest of the group are due mainly to the attenuation of sound in the walls of the tank. The largest absorptivity of the group is the wooden tank (N2), which has a value of 59%. The three polypropylene tanks B1(17%), B2(17%) and N1(20%) have similar values, however so does steel tank S1 at 16%. The glass reinforced polymer tank N3 has a value of 11%, which represents its composition with a value between the other plastic tanks and the glass tank B4, with a value of 2.2%.

The discussion of the attenuation of sound in these materials is complicated by the different attenuation coefficients of compressional and shear waves. This is further complicated by the proportion of each wave in the wall, which is due to the angle of incidence of the ray in the water to the wall. This leads to the consideration of the relative proportion and amplitude of the axial, tangential and oblique modes, with frequency. Such a discussion is outside the scope of this project.

The results presented in this chapter show that it is possible to make accurate measurements of the reverberation time in acoustic tanks. In the subsequent chapters those results will be applied to the calibration of transducers by reverberant techniques.
6.0. Calibrations in the Presence of a Reverberant Field

This chapter describes a method of calibrating a transmitting transducer (projector) in the presence of a reverberant field. The theory for this method is described along with methods for analysing the acoustic fields. The experimental apparatus and procedures used are explained, and finally the processing of the measurements to produce a calibration is described.

6.1. Theory

In Chapter 3 (reference calibrations) it was seen that the calibration of high Q projectors at low frequencies is not possible, with free-field calibration techniques, in tanks below a certain size. This is because the steady state response of the projector has not been reached before the arrival of the first reflection. However, non free-field techniques do exist, as discussed in the literature review, Chapter 2. These are based on predicting the steady state behaviour of the transducer from signals which have been received before the first few reflections. Calibrations can then be performed since the free-field steady state response has been determined. These methods extend the lower frequency range of the tank, but there still exists a limit below which calibrations can not be performed.

The technique described in this chapter allows projectors to be calibrated when they are running in continuous mode and not a pulsed mode. The continuous waves radiating from the transducer produce a spherically divergent direct sound field and a standing wave reverberant field. The technique aims to extract the direct field from the total field so that a calibration can be performed as if this is the free-field response of the projector without any reflections present. This technique can be used to perform calibrations at lower frequencies than the limit imposed by the free-field techniques or the predicting steady state behaviour techniques. In practice there are accuracy problems with this technique, particularly at low frequencies.
6.1.1. A simple model for the growth of sound in an enclosure

If a sound source is operated continuously in an enclosure, absorption in the medium and at the surrounding surfaces prevents the acoustic pressure amplitude from becoming infinitely large. In underwater acoustics at low to medium frequencies the absorption of sound in the medium is negligible so that both the rate at which the amplitude increases and its ultimate value are controlled by surface absorption only. If the total sound absorption is large, the pressure amplitude quickly reaches an ultimate value only slightly in excess of that produced by the direct wave alone. By contrast, if the absorption is small, considerable time will elapse before the ultimate, significantly higher amplitude is attained.

When a sound source is started in an enclosure that does not have large absorption the reflections at the boundaries produce a sound energy distribution that becomes more and more uniform with increasing time. If the absorption is very low after a large number of reflections the energy distribution will approach complete uniformity, except close to the source or to the absorbing surfaces.

The relationship between energy density and the energy flux across the boundaries of the enclosure is now derived, and follows that in Kinsler et al (1982). In Figure 6.1a, $\Delta S$ is an element of a boundary and $dV$ an element of volume in the medium at a distance $r$ from $\Delta S$, where $r$ makes an angle $\theta$ with the normal to $\Delta S$. Let the acoustic energy density $\varepsilon$ be uniform throughout the region so that the acoustic energy present in $dV$ is $\varepsilon dV$. The amount of this energy that will strike $\Delta S$ by direct transmission, $E_{\Delta S}$, is $\varepsilon dV$ attenuated by $4\pi r^2$, multiplied by the projection of $\Delta S$ on the sphere of a radius $r$ centred on $dV$ and is therefore given by

$$E_{\Delta S} = \frac{\varepsilon dV}{4\pi r^2} \cos(\theta) \Delta S.$$  

(6.1)
Figure 6.1. Volume and surface elements used for deriving growth of sound in an enclosure.

Now let $dV$ be part of a hemispherical shell of thickness $\Delta r$ and radius $r$ centred on $\Delta S$ as shown in Figure 6.1b. The acoustic energy $\Delta E$ contributed to $\Delta S$ by this entire shell can then be obtained by assuming that energy arrives from any direction with equal probability. Integrating over the hemisphere with $dV = 2\pi r \sin(\theta) r \Delta r d\theta$ yields

$$\Delta E = \frac{\varepsilon \Delta S \Delta r}{2} \int_0^{\pi/2} \sin(\theta) \cos(\theta) d\theta = \frac{\varepsilon \Delta S \Delta r}{4}.$$  
(6.2)

This energy arrives during a time interval $\Delta t = \Delta r/c$, so that Equation 6.2 can be rewritten as $\Delta E/\Delta t = \varepsilon c \Delta S/4$. Thus, the rate $dE/dt$ at which energy falls on a unit area of the boundary is

$$\frac{dE}{dt} = \frac{\varepsilon c}{4}.$$  
(6.3)

If it is assumed that, at any point within the enclosure, energy is arriving and departing along individual ray paths and that the rays have random phases at the point, then the energy density $\varepsilon$ is the sum over all rays of the energy densities $\varepsilon_i$ of the individual rays. Now, if the $i^{th}$ ray has effective pressure amplitude $P_{ei}$ we have $\varepsilon_i = P_{ei}^2 / (\rho_0 c^2)$ and thus
where

\[ P_r = \left( \sum_i P_{ei}^2 \right)^{1/2} \]  \hspace{1cm} (6.5)

is the spatially averaged effective pressure amplitude of the reverberant sound field. Note that the energy density \( \varepsilon \) calculated is that for a point, since it is the sum of the energy density rays \( \varepsilon_i \) at that point. However, since the energy density is uniform throughout the region, this is the same as the energy density of the whole volume. The energy density is not always uniform in a real system and so an average energy density is used in calculations.

If the sound absorption of the enclosure is \( A \), then the rate at which energy is being absorbed by all surfaces is

\[ \frac{Ac}{4} \varepsilon \] \hspace{1cm} (6.6)

from Equation 6.3. \( A \) has units of square meters.

This rate at which sound energy is absorbed by the surfaces, plus the rate \( Vd\varepsilon dt \) at which it increases in the medium throughout the interior of the enclosure, must equal the rate \( W \) at which it is being produced. The fundamental differential equation governing the growth of sound energy in an enclosure is therefore

\[ V \frac{d\varepsilon}{dt} + \frac{Ac}{4} \varepsilon = W. \] \hspace{1cm} (6.7)

If the sound source started at \( t = 0 \), the solution of this differential equation and use of Equation 6.4 gives
where

\[ P_r^2 = \frac{4W\rho_0 c}{A} \left( 1 - e^{-t/\tau_E} \right) \quad (6.8) \]

is the time constant governing the growth of the acoustical energy in the enclosure.

If \( A \) is small and \( \tau_E \) is large, a relatively long time will be required for the effective pressure amplitude \( P_r \) and energy density \( \epsilon \) to approach their ultimate values of

\[ P_r^2(\infty) = \frac{4W\rho_0 c}{A} \quad (6.10) \]

and

\[ \epsilon(\infty) = \frac{4W}{Ac}. \quad (6.11) \]

Equation 6.10 shows that a small value for the absorption in an enclosure produces a large value for the ultimate pressure amplitude.

This model is based on assuming that the acoustic energy has a diffuse distribution, so the equations derived do have limitations. Equation 6.7 is only valid after sufficient time has passed for each initial ray to have been reflected of the boundaries of the enclosure several times. This is to allow the sound field to become diffuse to a reasonable degree. Equation 6.10 indicates that the final pressure amplitude is independent of the volume and shape of the enclosure, is the same at all points in the enclosure, and depends only on the source strength and the room absorption \( A \). This is not true for enclosures having well defined sound focusing properties, irregular shaped enclosures having recesses and coupled enclosures. Also an enclosure with part of the surface area having significantly different absorption will cause the sound field to be non diffuse and so the equations will no longer be valid (Kinsler et al, 1982).
However for the more specific case of underwater test tanks the shapes are regular and do not focus sound significantly. The walls of the tank are usually made of the same material and so will have the same absorption, but the water / air interface will have a significantly different absorption. However the proportion of the total surface area of the water that is the water / air interface is small to medium. This could cause the reverberant field to not be diffuse or partially diffuse if the absorption between the two regions is significantly different and the water / air surface area proportion is large enough. However, measurements in underwater test tanks generally indicate that this is not too large a problem since the reverberant sound field is usually partially diffuse, apart from near the transmitter and the tank walls.

6.1.2. Direct and reverberant sound fields

When sound is continuously radiated into an enclosure two sound fields are produced. The first is the direct sound field, which is the direct arrival from the source. The second is the reverberant sound field, which is produced from the reflections off the surfaces of the enclosure. The effective pressure amplitude, $P_d$, produced by the direct sound field, which is assumed to be radiated uniformly (spherically) in all directions, is given by (Kinsler et al, 1982)

$$P_d^2 = \frac{\rho_0 c W}{4\pi r^2}$$  \hspace{1cm} (6.12)

where $r$ is the radial distance from the effective centre of the sound source, $W$ is acoustic power radiated by the source, $\rho_0$ is the volume density of the fluid and $c$ is the speed of sound in the fluid. The spatially averaged effective pressure amplitude, $P_r$, of the reverberant sound field is obtained from Equation 6.10 and is given by (Kinsler et al, 1982)

$$P_r^2 = \frac{4\rho_0 c W}{A}$$  \hspace{1cm} (6.13)
where $A$ is the total sound absorption of the enclosure. Combining the Equations 6.12 and 6.13 for the direct and reverberant sound fields, $P^2 = P_d^2 + P_r^2$, gives the total mean square pressure at a point in the sound field as

$$P^2 = \rho_0 c W \left( \frac{1}{4\pi r^2} + \frac{4}{A} \right). \quad (6.14)$$

To determine the relative levels of the two sound fields the ratio of their intensities can be calculated. The ratio of the reverberant sound field intensity, $I_r$, to the direct sound field intensity, $I_d$, is given by

$$\frac{I_r}{I_d} = \frac{4\pi r^2}{(A/4)} = \frac{16\pi r^2}{A}. \quad (6.15)$$

Equation 6.15 shows that for locations very close to the source, $4\pi r^2 << (A/4)$, the direct field dominates and the reverberant field has very little effect. This means that the shape, size and absorption of the enclosure has negligible effect. The reverberant sound field will dominate at distances where $4\pi r^2 >> (A/4)$ and the sound pressure level will be reduced by 3dB for each doubling of the total sound absorption of the enclosure (Kinsler et al, 1982).

6.1.3. Calibration in the presence of a reverberant field

In this section it will be shown that the direct sound field can be extracted from the total sound field and so facilitate calibration. Firstly the equation for the total sound field will be rewritten in terms of reverberation time instead of absorption. This is done since reverberation time is a directly measurable quantity where as absorption is calculated via reverberation time.

Recalling Equation 4.8, from the chapter on reverberation time, and rearranging gives
\[ A = \frac{24V}{T_c c \log_{10}(e)} \]  

(6.16)

and substituting Equation 6.16 into Equation 6.14 gives

\[ p^2 = p_0cW \left( \frac{1}{4\pi r^2} + \frac{cT_c \log_{10}(e)}{6V} \right) \]  

(6.17)

where \( V \) is the volume of water and \( T_r \) is the reverberation time.

Now consider a projector radiating sound into a tank and a hydrophone placed at a distance \( r \) from this source. If measurements of pressure are made at different separations, \( r \), then a graph of pressure squared against the reciprocal of separation squared can be plotted. From Equation 6.17 it can be seen that the gradient, \( m \), and the y-intercept, \( C \), of this graph are defined as

\[ m = \frac{\rho_0cW}{4\pi} \]  

(6.18)

and

\[ C = P_r^2 = \frac{\rho_0c^2 T_r W \log_{10}(e)}{6V}. \]  

(6.19)

In Equation 6.19 the y-intercept, \( C \), is the spatially averaged reverberant sound pressure amplitude squared. Equation 6.18 can be rearranged to give the acoustic power radiated into the tank as

\[ W = \frac{4\pi m}{\rho_0c}. \]  

(6.20)

Equation 6.19 can also be rearranged for the acoustic power radiated into the tank and gives
The acoustic power is therefore easily calculated using the gradient (using equation 6.20) or using the y-intercept and a measurement of the reverberation time in the tank (from equation 6.21).

The y-intercept, \( C \), is the spatially averaged reverberant field pressure amplitude squared. This can alternatively be obtained by spatially averaging the squared pressure in the sound field, at locations where the reverberant field dominates the direct field.

From Equations 6.4 and 6.5 it was seen that the energy density at a point was the sum of the individual ray energy densities. The pressure at a point is therefore calculated from the square root of the sum of the individual ray pressures squared and not the sum of the ray pressures. The sound ray pressures are not coherent and so the energy is summed and not the pressure. To calculate a spatially averaged pressure it is therefore necessary to average energy densities and so it is derived from the mean of pressure squared. The spatially averaged pressure amplitude, \( P_{SA} \), is given by

\[
P_{SA} = \left( \frac{1}{N} \sum_{n} P_{n}^2 \right)^{\frac{1}{2}}
\]  

(6.22)

where \( P_n \) is the amplitude of the pressure in the sound field at position \( n \). If this is then squared then the result is effectively the spatially averaged reverberant field pressure amplitude squared if the direct field is small compared to the reverberant field. So the acoustic power radiated into the tank, \( W \), can be obtained by substituting the y-intercept, \( C \), with the spatially averaged pressure amplitude, \( P_{SA} \), in Equation 6.21 to give

\[
W = \frac{6VP_{SA}^2}{\rho_0 c^2 T_r \log_{10}(e)}, \quad P_d << P_r
\]  

(6.23)
where $P_d$ is the direct field pressure and $P_r$ is the reverberant field pressure. For the direct field pressure to be very small compared to the reverberant field pressure the absorption of the tank needs to be small. Therefore the wall material of the tank needs to be unabsorbent or the tank needs to be small. This means that the reverberation time needs to be large relative to the volume of water (i.e. compared to the size or transit time across the tank).

Recalling Equation 6.12, for the direct sound field pressure, and using the value of acoustic power given in Equations 6.20, 6.21 or 6.23, the direct field sound pressure, at a distance $r$ from the projector, can be calculated. The transmitting voltage response of the projector can then be calculated from this direct field pressure, the voltage applied across the projector and the use of Equation 2.10 (equation for TVR of projector).

These three ways of calculating the transmitting voltage response of a projector in a reverberant sound field depend on the field being diffuse. A diffuse field is one where the acoustic energy density is the same everywhere. This means that the energy distribution can be assumed to be completely uniform and have random directions of flow. In terms of the pressure in the reverberant field this means that the amplitude and phase are completely random in time and space and independent of each other.

However a truly reverberant field is almost impossible to achieve in underwater acoustics, due to the size and absorption of the tanks and the frequencies involved. In practice these equations are still useful for a partially diffuse field. The results from them have differing degrees of error depending on the situation, which will be seen in the results chapter (Chapter 7).

6.1.4. Modal density

A reverberant field is constructed of many different standing waves. The number of excited modes in a given frequency band gives an indication of how diffuse the field is. One way to determine this is to calculate the number of possible modes below a
given frequency. The number, \( N \), of normal modes below frequency, \( f \), in a chamber can be calculated (Kinsler et al, 1982) to be

\[
N = \frac{4\pi V}{3c^3} f^3
\]  

(6.24)

where \( V \) is the volume of the chamber and \( c \) is the speed of sound in the chamber. Differentiating Equation 6.24 with respect to frequency yields the number of normal modes, \( \Delta N \), having frequencies in a band of width, \( \Delta f \), centred on \( f \) as

\[
\frac{\Delta N}{\Delta f} \approx \frac{4\pi V}{c^3} f^2.
\]

(6.25)

Equation 6.24 demonstrates that the number of normal modes increases rapidly with increase of frequency and Equation 6.25 shows that the number of normal modes per unit frequency also increases rapidly with frequency.

6.1.5. Calculation of average projector directionality and the effect on the y-intercept reverberant calibration result

The transmitting voltage response of a transducer can vary with direction over the whole surface area. The y-intercept TVR is based on the square of the spatially averaged reverberant field pressure, \( C \), as described in Equation 6.21. This reverberant field pressure is a result of radiation from the whole surface of the projector, and results in an average TVR estimate that is not necessarily the same as that in the ‘0’ mark direction of the projector. Only if the amplitude response in the ‘0’ mark direction is the same as the average amplitude response will the y-intercept TVR give the same answer. In order to enable comparisons the average amplitude response of the projector needs to be calculated, using the directivity patterns, and compared to the ‘0’ mark direction response.

Assume that the amplitude response of the projector has the value of 1 at the ‘0’ mark position and varies for other directions. Summing the amplitude over the whole surface area and dividing by the surface area leads to the average amplitude
response compared to the ‘0’ mark position. Firstly the surface needs to be divided into small elements and the area of each calculated so that the weighting is assigned to the correct proportion of the surface area. Consider a sphere of radius $r$, where the horizontal plane is designated by X-Z and the vertical axis is designated Y. A ‘0’ mark is defined so that the angle $\phi$, in the X-Z plane, is zero and the angle $\theta$, in the X-Y plane, is also zero. A small surface area element, $\Delta S$, makes angles $\phi$ and $\theta$ with the ‘0’ mark direction in the X-Z and X-Y planes respectively. The surface element has length, $\Delta x$, and height, $\Delta y$, which subtends the small angles $\Delta \phi$ and $\Delta \theta$ as shown in Figure 6.2.

Figure 6.2. Diagram to explain the calculation of the average amplitude response of a projector. A sphere of radius $r$ with $\phi$ angles in the X-Z plane and $\theta$ angles to the X-Y plane. The radius of the circle centred on the Y-axis and containing the surface element, $\Delta S$, is $r \cos(\theta)$. The ‘0’ mark is at $\theta$ and $\phi$ equals zero radians.
From Figure 6.2 it can be that the height of $Ay$ is $r\Delta\theta$ and that the length of $Ax$ is $r\cos(\theta)\Delta\phi$, where the angles are in radians. The radius of the circle centred on the Y-axis and containing $Ax$, and not the origin, is $r\cos(\theta)$. The area of the sphere is calculated by first summing the small surface areas, $\Delta S = \Delta x \Delta y$, for a narrow strip from $\theta = -\pi/2$ to $\theta = \pi/2$. Then a series of these strips is summed together from $\phi = 0$ to $\phi = 2\pi$, to give the total surface area of the sphere. The average amplitude response over the whole surface area, $\tilde{A}$, is

$$
\tilde{A} = \frac{1}{4\pi^2} \int_{\phi=0}^{\phi=2\pi} \int_{\theta=-\pi/2}^{\theta=\pi/2} A(\theta,\phi) r d\theta r\cos(\theta) d\phi,
$$

(6.26)

which reduces to

$$
\tilde{A} = \frac{1}{4\pi} \int_{\phi=0}^{\phi=2\pi} \int_{\theta=-\pi/2}^{\theta=\pi/2} A(\theta,\phi)\cos(\theta) d\theta d\phi,
$$

(6.27)

where $A$ is the amplitude response in the direction $\theta$ and $\phi$.

If the response of the projector is not known in all directions then an approximation to the average amplitude response can be made. This is found by measuring the directional response of the transducer the X-Y and X-Z planes. The value of the amplitude response for a particular direction, and therefore angles $\theta$ and $\phi$, is assumed to be given by the amplitude in the X-Y plane multiplied by the amplitude in the X-Z plane.

Now the average amplitude response is known relative to the ‘0’ mark direction, the effect on the y-intercept TVR can be calculated. The direct field pressure, at one metre from the projector in the ‘0’ mark direction, is defined as $P_0$. The direct field pressure, at one metre, averaged over $4\pi$ Sr is $P_a$, which is derived from the reverberant field. Recalling Equation
2.10, the y-intercept transmitting voltage response calculated from this averaged direct field, $TVR_{ya}$, is defined as

$$TVR_{ya} = 20 \log_{10} \left( \frac{P_a r}{V} \right), \quad (6.28)$$

where $P_a$ is the direct field pressure averaged over the whole surface area of the projector, $V$ is the potential difference across the projector and $r$ is the separation of the transducers. This is not the correct calibration if $P_a$ does not equal $P_0$, since the TVR is defined for the direct field pressure in the '0' mark direction. The correct y-intercept transmitting voltage response calibration, $TVR_{y0}$, is defined as

$$TVR_{y0} = 20 \log_{10} \left( \frac{P_0 r}{V} \right), \quad (6.29)$$

where $P_0$ is the direct field pressure in the '0' mark direction. Substituting in the definition for $P_a$ above leads to

$$TVR_{y0} = 20 \log_{10} \left( \frac{P_a r}{AV} \right) = 20 \log_{10} \left( \frac{P_a r}{V} \right) - 20 \log_{10} (\hat{A}), \quad (6.30)$$

which along with Equation 6.28 leads to

$$TVR_{y0} = TVR_{ya} - 20 \log_{10} (\hat{A}). \quad (6.31)$$

To calculate the correct value of the y-intercept TVR a series of $\hat{A}$ values will be needed for each frequency point in the calibration since the average directionality response changes with frequency.
6.1.6. Schroeder frequency

Schroeder (1962) postulated that sound in a rectangular room behaves in a statistical manner above a certain frequency limit. This limit is the low-frequency limit where the average spacing of the room mode frequencies is less than one third of their bandwidths. According to Nelisse and Nicolas (1997) the Schroeder frequency limit gives a good indication that the sound field is diffuse. This is so since it is in good agreement with two other methods of measuring sound field diffuseness, the correlation coefficient and the spatial uniformity.

Sepmeyer (1988) states that the Schroeder frequency, $f_c$, is given by

$$f_c = 2 \times 10^3 \left( \frac{T_{60}}{V} \right)^{\frac{1}{2}},$$

(6.32)

where $T_{60}$ is the 60dB reverberation time and $V$ is the volume of the enclosure. This equation is valid for air acoustic rooms. However, this relationship can be derived from more elemental acoustical and electrical circuit concepts. This will now be derived and follows that in Sepmeyer (1988). Consider the situation where there are $n$ modal frequencies within a modal bandwidth $\Delta f$. This situation is equivalent to $n$ marks, modal frequencies, on a scale and the spaces between the marks represent the average mode spacing $\bar{S}$. As with any scale the number of marks is one greater than the number of spaces. The average frequency spacing between modes is

$$\bar{S} = \frac{\Delta f}{(n-1)},$$

(6.33)

where $n$ is the number of modes within the modal bandwidth $\Delta f$. Therefore Schroeder’s criterion requires four modes per modal bandwidth.

From the mathematics of a single degree of freedom resonator, such as a LCR series circuit, it is known that

$$Q = \frac{f_0}{\Delta f},$$

(6.34)
where $Q$ is the quality factor, $f_0$ is the resonant frequency and $\Delta f$ is the separation in frequency of the half power (-3dB) points. From Pain (1999:43-47), the logarithmic decrement of a resonant circuit is equal to $RT/2L$, where $T$ is the period of the oscillation, $R$ the resistance and $L$ the inductance of the circuit. The decay rate, $d$, of the voltage or current is

$$d = \frac{RT}{2L} = \frac{R\pi}{2\pi f_0 L} = \frac{\pi}{Q} \text{ Np / cycle} \quad (6.35)$$

or

$$d' = \frac{8.68\pi}{Q} = \frac{27.3}{Q} \text{ dB / cycle} \quad \text{or} \quad \frac{27.3f_0}{Q} \text{ dB / s.} \quad (6.36a, 6.36b)$$

Since $T_{60}$ is the time required for the mode to decay 60dB, then

$$T_{60} = \frac{60Q}{27.3f_0} = 2.2Q/f_0. \quad (6.37)$$

Rearranging Equation 6.37 and substituting it into Equation 6.34 and then also rearrange gives

$$\Delta f = 2.2/T_{60}. \quad (6.38)$$

Now, the average mode spacing for rectangular enclosures needs to be known. This was presented by Maa (1939), where the number of modes per Hz is given by

$$\frac{\partial N}{\partial f} = \frac{4\pi Vf^2}{c^3} + \frac{\pi Sf}{2c^2} + \frac{(L_x + L_y + L_z)}{2c} \quad (6.39)$$

where $N$ is the number of modes, $f$ is frequency, $c$ is the speed of sound, $V$ and $S$ are the volume and surface area of the enclosure and $L_x, L_y$ and $L_z$ are the lengths of the $x, y$ and $z$ sides of the rectangular enclosure. Note that the first term of Equation 6.39 is the approximation to the number of modes per Hz, as quoted in Equation 6.25.
Let $L_{eq}$ be the equivalent length of a cube of volume, $V$, with the same volume as the enclosure. Let $V^{1/3} = L_{eq}$ and $S = 6.2L_{eq}^2$; then $L_x + L_y + L_z \approx 3L_{eq}$ is true.

For convenience, set $c/L_{eq} = \nu$. The number of modes per Hz then becomes

$$\frac{\partial N}{\partial f} = \frac{4\pi}{\nu} \left[ \left( \frac{f}{V} \right)^2 + \frac{0.775f}{\nu} + \frac{3}{8\pi} \right] = \frac{1}{\bar{\delta}}. \quad (6.40)$$

Equation 6.40 is then solved for $f$, determining the frequency for a given $\bar{\delta}$, the average frequency spacing between modes. Equations 6.33 and 6.38 are then substituted into Equation 6.40, since the reverberation time relates to the bandwidth which along with the number of modes in a bandwidth gives $\bar{\delta}$ which leads to

$$f_0 = \left( \frac{T_{60}}{V} \right)^{1/2} \left( \frac{(n-1)c^3}{8.8\pi} \right)^{1/2} - \frac{3.1c}{8V^{1/3}}. \quad (6.41)$$

Equation 6.41 gives the frequency for which there are $n$ modes in a bandwidth, for an enclosure with a reverberation time $T_{60}$. Applying Schroeder's criterion of four modes per modal bandwidth, $n = 4$, and the speed of sound in air, $c = 343\text{ms}^{-1}$, then the Schroeder frequency in air becomes

$$f_{c(\text{air})} = f_{04} = 2.1 \times 10^4 \left( \frac{T_{60}}{V} \right)^{1/2} - 133V^{-1/3}, \quad (6.42)$$

(Sepmeyer, 1988). Note that the first term of Equation 6.42 is the same as Equation 6.32 except that the constant is shown to greater accuracy. The Schroeder frequency in water, $c = 1490\text{ms}^{-1}$, now becomes

$$f_{c(\text{water})} = 1.89 \times 10^4 \left( \frac{T_{60}}{V} \right)^{1/2} - \frac{577}{V^{1/3}}. \quad (6.43)$$
At this frequency there are four modes per modal bandwidth, which means that above this frequency the reverberant acoustic field is diffuse and behaves in a statistical manner.

6.1.7. Ratio of experimental direct and reverberant sound fields

The ratio of the direct to reverberant sound field indicates the relative levels of the fields and is useful in determining under what sound field conditions the gradient, y-intercept and spatially averaged pressure results are accurate. The pressure or power ratios can be calculated and indicates the relative pressure levels (sound field levels) or power levels (energy in the fields) respectively. The other issue to be considered is where in the acoustic field to measure this ratio. First of all the measurements need to be made in the far field, of the direct field, and not the near field to obtain a spherically divergent wave. So the minimum distance from the source is the near field / far field boundary and the maximum distance will be determined by the tank size. Within this range the direct field decreases in amplitude with distance from the source, and the reverberant field undulates throughout the region with a constant average pressure if the field is diffuse. It was decided to take the minimum and maximum transducer separations, in the reverberant calibration measurements, as the positions to sample the ratio of the fields. These points were used because the direct field was dominant for the minimum position and the reverberant field was dominant for the maximum position. These positions thus indicated the ratio at the two extremes of the fields combinations. The level of the reverberant field depends on the power radiated and the absorption of the tank. The direct / reverberant field ratio therefore primarily depends on the distance from the source and the absorption of the tank. The spatial fluctuations of the reverberant field will also affect the ratio, but the cancel out if spatially averaged in a diffuse field.

This ratio will be calculated for pressure and power at the maximum and minimum transducer separations used in the reverberant field calibrations. The direct and reverberant field pressure at these positions needs to be measured to calculate these ratios. However, this data already exists in the reverberant calibration data and is extracted as follows.
From the graph of pressure squared versus the reciprocal of separation squared the gradient and y-intercept can be obtained through a least squares fit.

So \( y = P^2 \), \( x = 1/r^2 \) and the equation of fit is \( y = mx + C \), where \( P \) is pressure, \( r \) is transducer separation, \( m \) is the gradient and \( C \) the y-intercept of the best fit line. The value of \( y \) can be split up into the direct and reverberant pressure squared, \( y = P_d^2 + P_r^2 \) where \( d \) represents the direct field and \( r \) represents the reverberant field. The direct field pressure squared can thus be represented as

\[
P_d^2 = y - P_r^2 = y - C = mx = m/r^2, \tag{6.44}
\]

where the y-intercept, \( C \), and \( x \) are defined above. The experimental ratio of direct field power to reverberant field power is calculated by

\[
R_{EdrPower} = \frac{P_d^2}{P_r^2} = \frac{mx}{C} = \frac{m}{r^2C}. \tag{6.45}
\]

The experimental ratio of direct field pressure to reverberant field pressure is calculated by

\[
R_{EdrPressure} = \frac{P_d}{P_r} = \sqrt{\frac{m}{r^2C}} = \sqrt{R_{EdrPower}}. \tag{6.46}
\]

6.1.8. Ratio of theoretical direct and reverberant sound fields

The theoretical ratio of direct to reverberant sound field can also be calculated by knowing the distance from the source and the absorption of the tank or more practically the reverberation time and volume of water in the tank. For direct comparison the same separations were used as for the experimental ratios. So for each tank the same maximum and minimum transducer separations were used for the experimental and theoretical ratios.
To calculate these theoretical ratios it is necessary to recall Equations 6.12, 6.13 and 6.16, the equations for the direct field pressure squared, reverberant field pressure squared and the absorption of the tank. Substituting Equation 6.16 into Equation 6.13, produces the reverberant field pressure squared, $P_r^2$, as

$$P_r^2 = \frac{\rho_0 c^2 W T_r \log_{10}(e)}{6V}, \quad (6.47)$$

where $\rho_0$ is the volume density of water, $c$ is the speed of sound in water, $W$ is the acoustic power radiated into the water, $T_r$ is the reverberation time of the tank and $V$ is the volume of water. The theoretical ratio of the direct field power to the reverberant field power is

$$R_{TdrPower} = \frac{P_d^2}{P_r^2} = \frac{3V}{2\pi r^2 c T_r \log_{10}(e)}. \quad (6.48)$$

The theoretical ratio of the direct field pressure to the reverberant field pressure is

$$R_{TdrPressure} = \frac{P_d}{P_r} = \sqrt[3]{\frac{3V}{2\pi r^2 c T_r \log_{10}(e)}} = \sqrt{R_{TdrPower}}. \quad (6.49)$$

6.2. Experimental procedure

This section consists of a description of the experimental rig that was built to take reverberant field calibration measurements. It also describes the procedure for carrying out the measurements and the limitations and problems that were encountered.

6.2.1. Experimental rig

From the theory it was seen that in order to perform these reverberant calibration measurements it was necessary to measure the pressure for a variety of separations
between the projector and hydrophone. The measurements were made in a straight line and the voltage across the projector was also measured.

At Bath an automated rig was created to enable many measurements to be carried out in different tanks and positions within tanks. This required the use of stepper motors to control the position of the transducers, which enabled far greater accuracy to be obtained than would have been possible by hand and removed the chance of mistakes when carrying out the measurements. A LeCroy digital oscilloscope was used to measure the voltages applied to and received by the transducers. This also enabled a computer to be used to record waveforms captured by the LeCroy, via an IEEE link. One of two different stepper motor controlled translation stages was used to position the hydrophone depending on which tank measurements were taken in. The computer was also used to control the stepper motors using a RS232 link for a one dimensional translation stage and an IEEE link for a two dimensional translation stage. A diagram of the experimental rig created is shown in Figure 6.3, for the one dimensional case. This rig was also used at Sonardyne, but without the use of the stepper motor, to take reverberant calibration measurements.

The rig consisted of an optical bench and stepper motor controlled translation stage parallel with each other, attached to the top of the water tank. The optical bench supported the projector that could be moved up and down its length. The translation stage supported the hydrophone, which could also be moved up and down the length of the stage and was controlled by the computer. The projector and hydrophone were aligned so that their acoustic centres lay on the same acoustic axis, which was parallel to the optical bench and stepper motor.

The projector signal was produced by a Brüel and Kjær noise generator, type 1405, which was set to produce 100kHz white noise (D.C. to 100kHz (-3dB)). This signal was amplified by a Brüel and Kjær power amplifier, type 2713, which was set to a gain that would produce a suitably large sound field, and then onto the projector and LeCroy oscilloscope to be recorded. The power amplifier was adjusted so that the field was large enough to produce an output from the hydrophone that was far greater than the background electronic noise level, so that the background noise could be ignored.
Figure 6.3. Experimental rig, for reverberant calibration measurements, at the University of Bath and Sonardyne.
The amplified continuous white noise signal generated a reverberant field in the tank. A white noise signal was used to produce a more diffuse field than would be generated with a broadband pulse. The random nature in frequency, amplitude and phase meant random modes were excited at random and varied randomly in amplitude. This meant dominant modes were less likely to build up; this made the field more diffuse than would normally be the case for an underwater tank.

The sound field in the tank was measured using a hydrophone whose electrical output was amplified by a calibrated Brookdeal precision A.C. amplifier, type 9452. The signal was then sent to the LeCroy oscilloscope to be captured. However, before it was recorded the signal level was sampled to check that it was not too large or small for the dynamic range of the oscilloscope's analogue to digital converter (ADC). If the signal level was not suitable then the oscilloscope scale setting was changed to ensure that the signal optimally fitted the dynamic range of the ADC before the signal was recorded. This was achieved by writing a QuickBASIC program to control the oscilloscope via an IEEE link. This QuickBASIC program also controlled the stepper motor to position the hydrophone in the tank. The LeCroy used 8 bit digitisation of the voltage signal and could sample up to 50,000 points in a record.

6.2.2. Measurement procedure

The first task that needed to be done when performing a reverberant calibration run was to place the transducers in the appropriate holders so that their direction for calibration, the ‘0’ marks, were facing each other. It was also necessary to ensure that the acoustic centres of the two transducers were at the same height in the water. The separation of the acoustic centres of the transducers and the number of positions at which hydrophone measurements were to be made were then input into the computer program. The program then calculated the required separations. If the positions were equally spaced then the data points in the graph of pressure squared versus the reciprocal of separation squared would not be equally spaced out along the horizontal axis but most of the points would be towards the zero end of the axis. A good fit will not then be achieved since there will not be many points representing one end of the linear regression fit line. To overcome this problem it was decided to
have the points equally spaced on the graph which required that the separations were $1/r^2$ distributed, with most points close to the projector i.e. in the direct field. However this solution appears to mean that the sound field is not equally sampled because the direct field is sampled more than the reverberant field, since it is dominant in this region. This is not a problem because the reverberant field is still being sampled adequately since, assuming the field is reasonably diffuse, it is similar everywhere. Also the direct field changes quickly near to the projector but not far away. A possible compromise between these two is to sample the field at $1/r$ separations. The computer program was written so that it could arrange the hydrophone positions in these three types of ways, with the actual positions being calculated from the initial separation value, the length of the sample traverse and the number of measurement positions.

A set or run of reverberant field calibration measurements was made by recording the signal from the hydrophone for a number of separations between the projector and receiver. This involved the program controlling the oscilloscope recording the signal going to the projector and then the signal from the hydrophone via the amplifier. It then moved the hydrophone a pre-calculated distance and recorded the new signal from the hydrophone. The amplified hydrophone signal was then recorded for each of the calculated separations.

The voltage-time records were transferred from the oscilloscope to the computer, via the IEEE link, using the LeCroy's own data format. The QuickBasic program converted this format into an ASCII file type, which can be read by most software.

6.3. Processing of the measured data

The stages necessary to calculate the calibration of the projector using the reverberant calibration method will now be described. Section A3.2, in Appendix 3, contains a list of all the major stages of the reverberant calibration processing. A MATLAB program was written to carry out the processing of the data. MATLAB is a versatile mathematical package based on matrices. First an average spectrum of both the projector and hydrophone signals was needed.
6.3.1. Generating an averaged spectrum

Since a random, white noise signal was being transmitted from the transducer, any short sample of the received signal would not give a true representation of the overall signal spectrum. In fact, due to the random nature of the source, the spectral levels would be subject to statistical fluctuations. Consequently it was necessary to average a number of spectra in the frequency domain in order to obtain a true representation of the signal spectrum. This was easily achieved at NPL with a vector analyser, however this facility was not available at Bath, so an oscilloscope was used to capture a long time signal. A trace of 50,000 points was recorded and split into 50 sections of 1,000 points each. A window function was then applied to the individual sections, to ensure that they started and finished at zero and had a gradient of zero at either end. The window function used was a one eighth cosine bell curve, which consists of the first and last eighth of the section being respectively multiplied by a cosine taper consisting of one half of a cosine wave. A 1024 point Fast Fourier Transform (FFT) was then performed on each windowed section to produce the spectrum of the signal. The spectra were then averaged to produce a mean spectrum. Averaging 50 times reduces the statistical fluctuations by a factor of approximately seven, so that is to about 14% of the original level.

6.3.2. Calculation of projector transmitting voltage response from the reverberant field measurements

To calculate the transmitting voltage response of the projector from the reverberant field measurements a series of calibration data files needed to be loaded. These were the calibration for the amplifier used between the hydrophone and oscilloscope, the hydrophone calibration data and the reverberation time frequency response of the tank. If the accuracy of the final result was to be determined then the free-field projector calibration also needed to be known. These files were required to have the same frequency range and frequency step size so that the same frequency points could be evaluated.
The averaged voltage spectra recorded by the oscilloscope, for each of the hydrophone positions, was converted into a pressure spectrum using Equation 1.9, the amplifier calibration and the calibration of the hydrophone.

At this stage the pressure-frequency matrices for the receiver and the voltage-frequency matrix for the transmitter were averaged over 2kHz bands to reduce the fluctuations in the frequency responses. For the spectra calculated from the oscilloscope this meant the number of points in the spectra was reduced by a factor of 4.

For each frequency band matrices of pressure squared and the reciprocal of separation squared were then calculated. A least squares fit (linear regression) was then performed between the pressure squared matrix and the reciprocal of separation squared matrix. This produced values for the gradient, error in the gradient, y-intercept, error in the y-intercept and the correlation coefficient of the fit.

For each frequency band the acoustic power radiated into the tank, and its error, was then calculated three ways. Firstly it was calculated using Equation 6.20 and the gradient of the graph. Secondly it was calculated using Equation 6.21, from the y-intercept, the reverberation time and the volume of water in the tank. Finally, it was calculated using Equation 6.23, using the spatially averaged pressure squared from the hydrophone positions, the reverberation time and the volume of water. The spatially averaged pressure squared was calculated using Equation 6.22. The pressure at one metre from the centre of the projector was then calculated using Equation 6.12 and each of these acoustic powers. Assuming the transmitter is isotropic the transmitting voltage response of the projector, and its error, was then calculated using Equation 1.10, with the three pressure estimates and the voltage applied across the projector.

If the directional response of the projector was not uniform then the calculation of pressure from the acoustic powers could be wrong. The acoustic powers calculated from Equations 6.21 and 6.23 give the power averaged of all directions of radiation from the transmitter. The calculated pressure is therefore the averaged over $4\pi$ Sr at one metre. This may not be the direct field pressure in the '0' mark direction and
therefore not the TVR for the calibration direction, but a mean TVR averaged over all directions. However, if the projector is isotropic then these two TVR will be a valid. The power calculated from Equation 6.20 will be the power radiated in the ‘0’ mark direction, since it is an estimate of the direct field in the calibration direction. It is therefore a valid calculation of the pressure and so also the TVR of the projector, even if the transmitter is anisotropic.

Determining the TVR from the spatially averaged pressure is the same principle as used in air acoustics to determine calibrations. The difference is that in air acoustics the reverberant field is sampled randomly with position and measurements are not made near the transmitter (near field) or close to the walls of the room. The field is spatially sampled randomly since the reverberant field is usually completely diffuse so that it does not matter where the field is sampled. Since the reverberant field is not completely diffuse, in underwater tanks, the calibration obtained will fluctuate about the true curve. Also the calibration may have a bias on it if the direct field is not small compared to the reverberant.

6.4. Rig for measurements at the National Physical Laboratory

The rig for taking reverberant calibration measurements at the National Physical Laboratory is shown in Figure 6.4. It consisted of a noise generator, producing 100kHz white noise (D.C. to 100kHz (-3dB)), which sent a signal to a power amplifier. The power amplifier sent its output to the projector and its monitor voltage output to the vector analyser. The monitor voltage output was approximately one tenth of the output to the projector. The monitor voltage was used to avoid overloading of the vector analyser. In order to know the exact voltage applied to the projector the monitor voltage output was calibrated. The output from the hydrophone was sent to an amplifier and then to the vector analyser. The exact gain of the amplifier needed to be known so that the output voltage from the hydrophone could be calculated.
Figure 6.4. Experimental rig, for reverberant calibration measurements, at the National Physical Laboratory.
The projector and hydrophone were mounted on stepper motor controlled translation stages so that they can be easily and accurately moved. The acoustic centres of the transducers were at the same height in the water and the '0' marks of the transducers, faced each other. A computer was used to control the position of the translation stages and to remotely operate the vector analyser via an IEEE link, or both of these pieces of equipment could have been used by hand. The stepper motor controller displayed the separation of the acoustic centres of the two transducers.

Measurements were made of the hydrophone output for different separations of the transducers and also the voltage applied across the projector. The vector analyser sampled 50 time segments, carried out an FFT on each of them and then calculated the mean of these spectra. This was the signal that was recorded. The vector analyser therefore performed the first stage of the analysis, that was implemented by the MATLAB program, at Bath. The rest of the analysis was the same as for the Bath measurements, where matrices were needed for the transducer separation, amplifier gain and hydrophone calibration.

In Chapter 7 the different stages of processing the experimental data are shown along with the range of measurements taken. Then the reverberant calibration results are presented in conventional spectral form and then in an analytical way to compare them against each other in different tanks, and using different projectors. The affect at directionality of the projectors is investigated and acoustic field data is presented and analysed. Finally the directionally compensated reverberant calibration results are compared against the acoustic field parameters determined.
7.0. Results for Calibrations in the Presence of a Reverberant Field

This chapter describes the results of the series of measurements to calibrate projectors in a reverberant sound field using the eight tanks described in section 5.1. The circumstances of the reverberant calibration measurements taken in the tanks are described in Table 7.1. The first section of the chapter describes example results at each stage of the reverberant calibration calculation process. The different stages of this process were described in Chapter 6. The chapter then proceeds to give a sample of the large number of results obtained in conventional spectral form and then in a systematic way in order to compare results obtained for the different tanks used with various projectors. The affect of directionality of the projectors on the results is investigated and a compensation attempted. Next a few parameters concerning the acoustic fields the measurements were taken in are presented and analysed. Finally the directionally compensated reverberant calibration results are compared against the parameters for the acoustic fields.

7.1. Example reverberant calibration results for each stage of the calculation process.

In Chapter 6 it was seen that the sound field at a position was sampled either using the LeCroy oscilloscope or a vector analyser. Most of the example results in this section are taken from measurements and calculations for run one in tank B3 (using the oscilloscope), but some are for run one in tank N2 (using the vector analyser).

The oscilloscope records 50,000 points and a typical trace is shown in Figure 7.1, which shows recorded voltage versus time. The time between samples is 2µs, therefore the total record length is 100ms. This trace shows white noise with a frequency band between D.C. and 100kHz(-3dB point). Similar traces were obtained for all the amplified hydrophone outputs and the voltage applied to the projector.
<table>
<thead>
<tr>
<th>Projector</th>
<th>Hydrophone</th>
<th>Gain of Amplifier / dB</th>
<th>Bandwidth of Amplifier</th>
<th>Signal Capture Device</th>
<th>Tank</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1, P2</td>
<td>H2</td>
<td>N/A</td>
<td>N/A</td>
<td>SA</td>
<td>N1</td>
<td>NPL</td>
</tr>
<tr>
<td>P3</td>
<td>H1</td>
<td>40</td>
<td>1kHz – 100kHz</td>
<td>OSC</td>
<td>B1</td>
<td>Bath</td>
</tr>
<tr>
<td>P3</td>
<td>H1</td>
<td>40</td>
<td>1kHz – 100kHz</td>
<td>OSC</td>
<td>B2</td>
<td>Bath</td>
</tr>
<tr>
<td>P3</td>
<td>H1</td>
<td>40</td>
<td>1kHz – 100kHz</td>
<td>OSC</td>
<td>B3</td>
<td>Bath</td>
</tr>
<tr>
<td>P3</td>
<td>H3</td>
<td>39.5</td>
<td>Broad Band</td>
<td>OSC</td>
<td>S1</td>
<td>Sonardyne</td>
</tr>
<tr>
<td>T1</td>
<td>H4</td>
<td>50</td>
<td>1kHz – 100kHz</td>
<td>OSC</td>
<td>B4</td>
<td>Bath</td>
</tr>
<tr>
<td>P3</td>
<td>H4</td>
<td>28.9</td>
<td>100Hz – 1MHz</td>
<td>SA</td>
<td>N2</td>
<td>NPL</td>
</tr>
<tr>
<td>P1, P3</td>
<td>H4</td>
<td>28.9</td>
<td>100Hz – 1MHz</td>
<td>SA</td>
<td>N3</td>
<td>NPL</td>
</tr>
</tbody>
</table>

Table 7.1. Table shows projector, hydrophone, gain of amplifier, bandwidth of amplifier, signal capture device, tank and location of tank for the reverberant field calibration measurements. SA = Spectrum Analyser, OSC = LeCroy Oscilloscope, NPL = National Physical Laboratory, Bath = Bath University.
The 50,000 point trace was then split up into 50 traces of 1,000 points each. A window function was then applied to the individual sections, to ensure that they started and finished at zero and had a gradient of zero at either end. The window function used was a one eighth cosine bell curve, which consists of the first and last eighth of the section being respectively multiplied by the last and first half of a cosine function period, which had one added to it and was then divided by two. A 1024 point Fast Fourier Transform (FFT) was then performed on each windowed section to produce the spectrum of the signal.

![Figure 7.1](image)

Figure 7.1. A 50,000 point voltage versus time trace recorded by the digital oscilloscope in tank B3 for measurement run one, position one.

Figure 7.2 shows an example of one of these spectra, which shows voltage versus frequency. The trace contains 512 points with a frequency interval between points of 488Hz.

This spectrum contains many random fluctuations but the true response of the projector is apparent. However the response can be made much clearer by averaging the 50 noisy spectra to reduce the fluctuations and produce a cleaner mean spectrum.
Figure 7.2. Voltage versus frequency spectrum of section one with a frequency interval of 488Hz, for measurement using the oscilloscope, in position one, run one, tank B3.

Figure 7.3. Averaged voltage versus frequency spectrum with a frequency interval of 488Hz, for measurement using the oscilloscope, in position one, run one, in tank B3.
Figure 7.3 shows this averaged spectrum of voltage versus frequency, where the fluctuations are reduced by a factor of $\sqrt{50} \approx 7$ if the reverberant field is uncorrelated between sections. This spectrum shows a frequency response with more limited fluctuations.

The reason the fluctuations in the time signal are reduced by averaging will now be explained. The random signal sent to the projector produces a direct field which is the random excitation of the projectors TVR. This direct field then goes on to produce a reverberant field, which is randomly excited with the TVR weighting. Each section recorded, therefore contains the randomly excited direct and reverberant field. When the sections are averaged together the direct field signal reveals the underlying TVR signal. This is because each random excitation is modified by the projectors TVR, and so generates a signal that is randomly above or below the TVR level. When the signal is averaged the random excitations progressively cancel each other out, and so the amount the signal is above or below the TVR is reduced.

This is analogous to a signal contaminated with noise, where the original signal is revealed by averaging many times. The original signal is present in every sample (coherent) and the noise is different in every sample (random), and so the averaging process reinforces the coherent signal and cancels out the random signal. The reverberant field fluctuations are also randomly excited, and so by averaging, the reverberant field is also revealed. The averaging process therefore progressively reveals the direct and reverberant field. However, since the reverberant field is effectively random for a diffuse field, this cancels itself out to a large extent leaving the underlying direct field.

The statistics for the reduction in the random fluctuations of the signal level are the same as that for reducing the uncertainty in an observation by averaging many times. The standard error in the mean of $n$ observations is $1/\sqrt{n}$ times the standard error in a single observation (Squires, 1985). Therefore the random fluctuations in the averaged spectrum are $1/\sqrt{50}$ of the random fluctuations in the spectrum of one section. This is only true if the reverberant field in each section is uncorrelated, i.e. not coherent. If the reverberant field is correlated to some extent between sections
(persistence of reverberant signal), then the signal is not random and the reverberant signals in each section will not tend to cancel each other out. If the reverberant signal between sections is partially correlated then some reduction in the fluctuations will occur, however if the reverberant field is totally correlated between sections then there will be no reductions in the reverberant field fluctuations. Whether the reverberant field is correlated will be discussed in a few pages.

The vector analyser performed the same role as the oscilloscope and the first stage of the MATLAB analysis program. It sampled fifty time segments, carried out an FFT on each segment and took the mean of these spectra. The spectrum contains 4096 points, with a frequency interval between points of 62.5Hz. The end result is essentially the same, except that the oscilloscope samples all the time segments at one time. Whereas the vector analyser samples each time segment, performs an FFT then samples the next and so on. There is a time gap between each sample of approximately half as second. Figure 7.4 shows an example averaged voltage versus frequency spectrum, captured by the vector analyser.

![Figure 7.4. Averaged voltage versus frequency spectrum with a frequency interval of 62.5Hz from the vector analyser for measurement in position one, run one, in tank N2.](image)
This spectrum has fluctuations on a much finer frequency interval than the averaged oscilloscope spectrum shown in figure 7.3. The oscilloscope spectrum contains 205 points between 0kHz and 100kHz with a frequency interval of 488Hz, whereas the vector analyser spectrum contains 1601 points between 0kHz and 100kHz with a frequency interval of 62.5Hz. The vector analyser spectrum appears to have more rapid fluctuations since its frequency interval is far smaller than that of the oscilloscope spectrum and therefore the higher rate fluctuations have not been removed by averaging over a larger sample bandwidth. Figure 7.5 shows the result of averaging eight of the vector analyser frequency bins together to produce a spectrum that contains 201 points between 0kHz and 100kHz with a frequency interval of 500Hz. The spectra obtained by the two approaches can now be compared since they have almost identical frequency range, number of points and frequency interval.

![Graph](image_url)

Figure 7.5. Averaged voltage versus frequency spectrum with a frequency interval of 500Hz from the vector analyser for measurement in position one, run one, in tank N2.

These two spectra are shown in figures 7.3 and 7.5 and reveal that the oscilloscope spectrum is more noisy than the vector analyser spectrum. This is representative of
all the measurements taken irrespective of tank. A spectrum with large fluctuations within it indicates that the reverberant field variations are significant compared to the level of the direct field. The difference in noise on the spectra may be due to the reverberant field being correlated on a short time scale. The oscilloscope signal length is 100ms whereas the time between samples on the vector analyser is approximately 500ms. The analyser took approximately 30 seconds to sample all fifty traces. If the sound field is correlated over a short time (100ms) and not over a longer time scale (30s) then the vector analyser signal will achieve a better estimate of the true spectrum. For comparison the reverberation time for these tanks ranges from 48ms up to 186ms (remember the reverberation time is defined to -60dB). A given standing wave reverberant field pattern will remain the same until it decays away. Therefore, within a period of time of the order of the reverberation time, the field pattern will change gradually as each new random 'field' decays away. It will take a period of time as long as the reverberation time for the reverberant field pattern to change completely. Since the field, for the oscilloscope, was very similar for a large part of this time the reduction in noise was not $\sqrt{50}$, but less, since the fields would have been partially correlated. The fields for the vector analyser measurements were probably not correlated and so the reduction in the fluctuations was probably $\sqrt{50}$.

The hydrophone pressure spectra were then calculated using the sensitivity of the transducer and the gain of the amplifier. Next the pressure and voltage spectra were averaged over a bandwidth of 2kHz to reduce the noise further. This was applied to both the oscilloscope calculated spectra and the original vector analyser spectra, which were averaged over 4 points and 32 points respectively.

The next step to obtain the reverberant calibration, as described in Chapter 6, was to square each individual pressure representing a frequency in the pressure-frequency matrices. The reciprocal of the separation-squared matrix was also calculated. These matrices then allow the graph of pressure squared versus the reciprocal of separation squared to be plotted. This graph is plotted for each 2kHz frequency bin. Figure 7.6 shows this graph for one of the 46 frequency bins for run one in the tank B3. The points represent the pressure data and the line the least squares fit to this
data. The original 512 points of the spectra, from the oscilloscope calculations, were reduced to 184 points to reduce the bandwidth to the range 10kHz to 100kHz. The averaging over four frequency bins then reduced the number of points to 46.

![Graph showing pressure squared versus the reciprocal of separation squared squared for frequency bin 1 of 46, in run one tank B3. The points represent the pressure data and the line the least squares fit to this data.](image)

Figure 7.6. Pressure squared versus the reciprocal of separation squared graph for frequency bin 1 of 46, in run one tank B3. The points represent the pressure data and the line the least squares fit to this data.

![Graph showing pressure squared versus the reciprocal of separation squared squared for all 46 frequency bins. The blue lines represent the pressure data and the red lines the least squares fit to this data. This graph illustrates the range of data points and best fit lines.](image)

Figure 7.7 shows the pressure squared versus the reciprocal of separation squared graph for all 46 frequency bins. The blue lines represent the pressure data and the red lines the least squares fit to this data. This graph illustrates the range of data points and best fit lines.
Figure 7.7. Pressure squared versus the reciprocal of separation squared graph for all 46 frequency bins, in run one tank B3. The blue lines represent the pressure data and the red lines the least squares fit to this data.

The gradients of all these best fit lines are plotted against frequency bin in Figure 7.8. This gradient is proportional to the power radiated by the projector and so will show the features of the TVR of the projector. This is the case as Figure 7.8 shows the peak in the projector's TVR. The graph also shows fluctuations in the gradient which are due to the fluctuating nature of the data points in the pressure squared versus the reciprocal of separation squared graph. This, in turn is due to the reverberant nature of the field.
Figure 7.8. Gradient of the pressure squared versus the reciprocal of separation squared plot, versus frequency for run one of tank B3.

From the best fit lines in Figure 7.7 the y-intercept can be found for each frequency bin. This graph of y-intercept, or spatially averaged reverberant field pressure squared, versus frequency is plotted in Figure 7.9. It also shows the peak in the projector TVR since the y-intercept is proportional to the power radiated. The shape of the curve will be different to Figure 7.8 since the y-intercept is also proportional to the reverberation time frequency response of the tank. The graph fluctuates due to the reverberant nature of the field.
Another way to obtain the reverberant field calibration is to calculate the spatially averaged reverberant field pressure from the hydrophone pressure spectra and not the y-intercept of the graph. The measured pressure data is shown in Figure 7.10 in a three dimensional mesh plot of hydrophone pressure reading versus frequency and distance from the acoustic source. The plot clearly shows the pressure decaying with distance and the resonance peak of the projector.

This plot also shows the two acoustic fields, the direct field and reverberant field. The direct field is characterised by the pressure decreasing with distance from the source. The reverberant field is characterised by the pressure values for each frequency keeping a constant average value over distance. The pressure values will fluctuate around the average value. However in this plot only the furthest two points from the source appear to be part of the reverberant field and this result would be better illustrated with more points further from the source. Therefore to determine the spatially averaged reverberant field pressure the reverberant field needs to be
large compared to the direct field. If this was the case then averaging the hydrophone pressure spectra would reveal the spatially averaged reverberant field pressure. However, this would not be the case for the data shown in Figure 7.10 as this average would contain a sizeable bias due to the relative size of the direct field.

Figure 7.10. Pressure versus frequency and distance from the acoustic source, for run one of tank B3. The colour represents the pressure value, with red being high pressure and blue low.

If the reverberant field was not large compared to the direct field then the averaged pressure squared value will not be close to the averaged reverberant field pressure squared value. Figure 7.11 shows a graph of spatially averaged pressure versus frequency for the data shown in Figure 7.10. It shows the peak in the projector TVR since both the direct field pressure and the spatially averaged reverberant field pressure are proportional to the TVR. If the reverberant field is large compared to the direct field then the shape of the curve will be similar to that in Figure 7.9 since both are based on estimates of the reverberant field. The fluctuations in the graph are again due to the random nature of the reverberant field.
From the graphs in Figures 7.8, 7.9 and 7.11 the acoustic power radiated into the tank can be calculated as described in chapter 6. However, depending on the nature of the acoustic field, the gradient and y-intercept graphs can become negative. This is due to the direct field being inaccurately determined. The gradient and y-intercept depend on the direct field being accurately measured. When the reverberant field is large compared with the direct field then the direct field cannot be determined with any accuracy. When the graph of pressure squared versus the reciprocal of separation squared is plotted, the best-fit line is not well determined and the gradient can become negative. This is because fluctuations in the reverberant field completely swamp the small changes in the direct field, leading to random values in the gradient, which can become negative. The y-intercept can occasionally become negative if the fluctuations in the reverberant field are large and this subsequently causes a large error in the gradient.

If the reverberant field were diffuse it would contain large fluctuations in pressure but these would not be correlated. A non-diffuse field would be correlated and so the averaging of the pressure spectra would not reduce the fluctuations as much as a
diffuse field would. This means greater averaging of the pressure spectra reduces the fluctuations in the reverberant calibration for a diffuse reverberant field, but has a limited effect in a non-diffuse field.

The transmitting voltage response of the projector was calculated three ways; by using the gradient, y-intercept and spatially averaged pressure. Figure 7.12 shows these three estimates of TVR versus frequency.

![Transmitting Voltage Response](image)

Figure 7.12. Transmitting voltage response versus frequency for run one of tank B3. Blue – gradient; green - y-intercept, red - spatially averaged pressure and black line – reference calibration.

The graph shows that the gradient calibration values (blue points) are a reasonable match to the reference calibration of the projector (black line). This would be expected since the gradient versus frequency graph is smooth and has no negative points. This therefore indicates that the reverberant field is not large since the direct field could be extracted. The y-intercept values (green points) in Figure 7.12 are not such a good fit as the gradient values. This is due to the y-intercept versus frequency graph having larger fluctuations. Another indication of this is that one point is
negative. The y-intercept values are not as accurately determined as the gradient values. Finally the spatially averaged pressure values (red points) in Figure 7.11 form a smooth curve, as for the gradient values, but have a large bias on them. The spatially averaged pressure versus frequency graph is also smooth. This bias then indicates that the direct field is large compared to the reverberant field, which confirms what the gradient results indicated. This would be expected for tank B3, the concrete tank sunk into the ground at Bath. The absorption of the tank is high, from Table 5.4, and therefore the reverberant field is small.

7.2. Reverberant field calibration results

Measurements were taken in eight tanks using four projectors for various types of reverberant calibration measurement. A calibration measurement was a run, which consisted of the hydrophone being moved to several separations and the pressure sampled. A group or type of measurements was a series of runs where an aspect of the experimental set up was altered. The groups used included changing the start position of a run in a tank for a series of parallel runs. For example a run would consist of a series of measurements along an axis parallel to one side of the tank. The rest of the group would involve runs parallel to the first run, where the start positions for each run would move normal to this axis (parallel to the other side of the tank). Other groups included changing the orientation of a run in a tank (changing the angle between the axis of the run and the side of the tank) and the distance over which a run was carried out. There are too many results to show each TVR versus frequency graph so sample of the calibration results are shown in this section. A more analytical approach to analysing the results is discussed in section 7.3 and a detailed overview of all the results is given in section 7.4, using this method. A breakdown of all the groups and runs taken in the tanks is shown in Table 7.2. The figure references are for the results displayed using the method described in section 7.3.
Table of Reverberant Calibration Measurements by Group (part 1)

<table>
<thead>
<tr>
<th>Tank</th>
<th>Group Code</th>
<th>Group Code Numbers</th>
<th>Number of Calibrations</th>
<th>Figure</th>
<th>Group number on graph</th>
<th>Parameter Varied</th>
<th>Projector</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>pl</td>
<td>1 - 5</td>
<td>5</td>
<td>7.17</td>
<td>1</td>
<td>For each run, the projector was moved to a new position across the width of the tank (hydrophone scan along the length of the tank)</td>
<td>P3</td>
</tr>
<tr>
<td></td>
<td>po</td>
<td>1 - 3</td>
<td>3</td>
<td></td>
<td>2</td>
<td>Orientation of the projector (fixed position) was changed for each run (hydrophone scan along radials from the projector position)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>pw</td>
<td>1 - 4</td>
<td>4</td>
<td></td>
<td>3</td>
<td>For each run, the projector was moved to a new position along the length of the tank (hydrophone scan across the width of the tank)</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>md</td>
<td>1 - 5</td>
<td>5</td>
<td>7.18</td>
<td>1</td>
<td>For each run, the projector was moved to a new position down the depth of the tank (hydrophone scan along the length of the tank)</td>
<td>P3</td>
</tr>
<tr>
<td></td>
<td>mf</td>
<td>1 - 10</td>
<td>10</td>
<td></td>
<td>2</td>
<td>Distanced traverse by the projector was changed for each run</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2. Table displaying reverberant calibration measurements categorised by tank, group code, group code numbers, number of calibrations, Figure showing the mean difference results, group number in this Figure, parameter varied in the group measurements and projector used.
Table of Reverberant Calibration Measurements by Group (part 2)

<table>
<thead>
<tr>
<th>Tank</th>
<th>Group Code</th>
<th>Group Code Numbers</th>
<th>Number of Calibrations</th>
<th>Figure</th>
<th>Group number on graph</th>
<th>Parameter Varied</th>
<th>Projector</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2</td>
<td>ml</td>
<td>1 - 7</td>
<td>7</td>
<td>7.18</td>
<td>3</td>
<td>For each run, the projector was moved to a new position across the width of the tank (hydrophone scan along the length of the tank)</td>
<td>P3</td>
</tr>
<tr>
<td></td>
<td>mo</td>
<td>1 - 11</td>
<td>11</td>
<td></td>
<td>4</td>
<td><strong>Orientation</strong> of the projector (fixed position) was changed for each run (hydrophone scan along radials from the projector position)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>mt</td>
<td>1 - 5</td>
<td>5</td>
<td></td>
<td>5</td>
<td>Water temperature was measured for each run</td>
<td></td>
</tr>
<tr>
<td></td>
<td>mw</td>
<td>1 - 7</td>
<td>7</td>
<td></td>
<td>6</td>
<td>For each run, the projector was moved to a new position along the length of the tank (hydrophone scan across the width of the tank)</td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>cl</td>
<td>1 - 6</td>
<td>6</td>
<td>7.19</td>
<td>1</td>
<td>For each run, the projector was moved to a new position across the width of the tank (hydrophone scan along the length of the tank)</td>
<td>P3</td>
</tr>
</tbody>
</table>

Table 7.2. Table displaying reverberant calibration measurements categorised by tank, group code, group code numbers, number of calibrations, Figure showing the mean difference results, group number in this Figure, parameter varied in the group measurements and projector used.
### Table of Reverberant Calibration Measurements by Group (part 3)

<table>
<thead>
<tr>
<th>Tank</th>
<th>Group Code</th>
<th>Group Code Numbers</th>
<th>Number of Calibrations</th>
<th>Figure</th>
<th>Group number on graph</th>
<th>Parameter Varied</th>
<th>Projector</th>
</tr>
</thead>
<tbody>
<tr>
<td>B4</td>
<td>gla</td>
<td>1 – 12</td>
<td>12</td>
<td>7.20</td>
<td>1</td>
<td>For each run, the alignment of the projector with the hydrophone scan axis was changed (tangential distance between projector centre and axis)</td>
<td>T1</td>
</tr>
<tr>
<td></td>
<td>gld</td>
<td>1 – 16</td>
<td>16</td>
<td></td>
<td>2</td>
<td>For each run, the projector was moved to a new position down the depth of the tank (hydrophone scan along the length of the tank)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>g1l</td>
<td>1 – 12</td>
<td>12</td>
<td></td>
<td>3</td>
<td>For each run, the projector was moved to a new position across the width of the tank (hydrophone scan along the length of the tank)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>g1n</td>
<td>1 – 7</td>
<td>7</td>
<td></td>
<td>4</td>
<td>Number of hydrophone positions in a scan</td>
<td></td>
</tr>
<tr>
<td></td>
<td>g1o</td>
<td>1 – 18</td>
<td>18</td>
<td></td>
<td>5</td>
<td>Orientation of the projector (fixed position) was changed for each run (hydrophone scan along radials from the projector position)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>g1p</td>
<td>1 – 6</td>
<td>6</td>
<td></td>
<td>6</td>
<td>Various distributions of hydrophone positions in a scan</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2. Table displaying reverberant calibration measurements categorised by tank, group code, group code numbers, number of calibrations, Figure showing the mean difference results, group number in this Figure, parameter varied in the group measurements and projector used.
### Table of Reverberant Calibration Measurements by Group (part 4)

<table>
<thead>
<tr>
<th>Tank</th>
<th>Group Code</th>
<th>Group Code Numbers</th>
<th>Number of Calibrations</th>
<th>Figure</th>
<th>Group number on graph</th>
<th>Parameter Varying</th>
<th>Projector</th>
</tr>
</thead>
<tbody>
<tr>
<td>B4</td>
<td>g1r</td>
<td>1 – 13</td>
<td>13</td>
<td>7.20</td>
<td>7</td>
<td>Repeat run of the scan, no parameters changed</td>
<td>T1</td>
</tr>
<tr>
<td></td>
<td>g1s</td>
<td>1 – 6</td>
<td>6</td>
<td></td>
<td></td>
<td>Initial separation of transducers was changed for each run</td>
<td></td>
</tr>
<tr>
<td></td>
<td>g1t</td>
<td>1 – 9</td>
<td>9</td>
<td></td>
<td></td>
<td>Distance traversed by the hydrophone (in a scan) was changed for each run</td>
<td></td>
</tr>
<tr>
<td>N1</td>
<td>dp</td>
<td>1</td>
<td>1</td>
<td>7.21</td>
<td>1</td>
<td>None</td>
<td>P1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td>None</td>
<td>P2</td>
</tr>
<tr>
<td>N2</td>
<td>npl</td>
<td>1 – 2</td>
<td>2</td>
<td>7.22</td>
<td>1</td>
<td>Repeat run of the scan, no parameters changed</td>
<td>P3</td>
</tr>
<tr>
<td>N3</td>
<td>npl</td>
<td>3 – 5</td>
<td>3</td>
<td>7.23</td>
<td>1</td>
<td>Repeat run of the scan, no parameters changed</td>
<td>P3</td>
</tr>
<tr>
<td></td>
<td>npl</td>
<td>6 – 9</td>
<td>4</td>
<td></td>
<td>2</td>
<td>Repeat run of the scan, no parameters changed</td>
<td>P1</td>
</tr>
<tr>
<td>S1</td>
<td>sm</td>
<td>3</td>
<td>1</td>
<td>7.24</td>
<td>1</td>
<td>None</td>
<td>P3</td>
</tr>
</tbody>
</table>

Table 7.2. Table displaying reverberant calibration measurements categorised by tank, group code, group code numbers, number of calibrations, Figure showing the mean difference results, group number in this Figure, parameter varied in the group measurements and projector used.
Figure 7.13 shows the transmitting voltage response of projector P3 against frequency for tank B1, group 'pl' for all five runs. The five parallel runs, with the scan axis at different distances from the tank wall, are all plotted on this graph for the gradient (G), y-intercept (Y) and spatially averaged pressure (S) derived results. The graph shows that the G, Y and S results do not vary much with each run. The gradient calibration results fit well with the reference calibration apart from some deviation at 70kHz to 85kHz. The y-intercept results fit well up to 60kHz and after that become a lot lower than the reference calibration.

These deviations are probably due to the gradient and y-intercept not being properly determined, which is probably due to the fluctuating nature of the reverberant field and the signal having a low value compared to the large amplitude signals near resonance. The spatially averaged results follow the correct shape of the reference calibration but are biased above this. This is due to the direct field being significant compared to the reverberant field. The graph shows that the three methods of
producing the TVR give reproducible results within the group. The gradient and spatially averaged pressure results vary over a range of approximately 2dB and the y-intercept results over a range of about 3dB below 60kHz and 5dB above.

Figure 7.14 shows the transmitting voltage response of projector P3 against frequency for tank B1, all three groups and all of the runs in the groups. The graph contains twelve runs each for the gradient, y-intercept and spatially averaged pressure. These are broken up into the three groups as follows: 'pl' which contains five runs, 'po' which contains three and 'pw' which contains four. The $G$, $Y$ and $S$ results are similar over the three groups and therefore with Figure 7.13. This shows that reverberant calibration measurements give similar results when performed along the length, across the width and over scan orientations in tank B1. The $G$ and $S$ results vary over a range of 3dB and the y-intercept over a range of 5dB below 60kHz and greater above. The results vary over a slightly larger range than for the previous graph.

Figure 7.14. Transmitting voltage response versus frequency for tank B1, source P3, all three groups and all runs in the groups. Blue – gradient; green - y-intercept, red - spatially averaged pressure and black line – reference calibration.
Figure 7.15 shows the averaged transmitting voltage response versus frequency for source P3 derived from the gradient, y-intercept and spatially averaged pressure for tank B1. The TVR has been averaged over each group so that there are three traces for each of the \( G \), \( Y \) and \( S \) results. This shows the results in Figure 7.14 in a less cluttered way and that they are consistent with each other. Since any individual reverberant calibration has error due to the random excitement of the reverberant field, averaging several runs produces a more accurate representation of the true calibration. The variation in the results is reduced with the gradient varying over 1.5dB, spatially averaged pressure varying over 1dB and y-intercept varying over 2dB below 60kHz.

![Figure 7.15](image-url)  
Figure 7.15. Transmitting voltage response versus frequency averaged over a group for tank B1, source P3, with one trace for each of the three groups for each colour. Blue – gradient; green - y-intercept, red - spatially averaged pressure and black line – reference calibration.
Figure 7.16 shows the transmitting voltage response versus frequency, derived from the gradient, for projector P3 in five of the tanks. The tanks used were the three large tanks at Bath (B1, B2 & B3) and two large tanks at NPL (N2 & N3). Each individual run for all of the measurement groups are shown in the graph. The black points (tanks B1, B2 and B3) form the majority of the points in a band generally around the black line (reference calibration), with the blue points (tank N3) at the top of this band and the red points (tank N2) at the bottom of this band. There is a far greater variation in the results between the tanks than for the single tank, B1, with most of the results agreeing within 7dB.

Figure 7.16. Transmitting voltage response versus frequency calculated from gradient for five tanks (B1, B2, B3, N2 & N3), source P3, all groups in the tanks and all runs in the groups. Black points - B1, B2 and B3; red points – N2; blue points N3 and black line – reference calibration.
Figure 7.17 shows the averaged transmitting voltage response versus frequency, derived from the gradient, for projector P3 in the five tanks used in Figure 7.16 (B1, B2, B3, N2 & N3). The TVR has been averaged over each group to give one trace for each group. The variation in the averaged calibration results has been reduced, with the majority agreeing to within 5dB. The graph shows the traces for the groups and only one trace for some tanks. The traces for tanks N2 (red) and N3 (blue) are distinctly different from the collection of traces representing tanks B1 (black), B2 (black) and B3 (green). The traces for tanks B1 and B2 form a central band generally around the reference calibration (black line), and the trace for tank B3 is within this band. The trace for tank N2 is below the central band representing the large Bath tanks and the trace for tank N3 is above this band.

![Graph showing transmitting voltage response versus frequency averaged over a group and calculated from gradient for five tanks (B1, B2, B3, N2 & N3), source P3, with one trace for all of the groups in the tanks. Black – B1 and B2; green – B3; red – N2; blue – N3 and black line – reference calibration.](image-url)
Figure 7.18 shows the averaged transmitting voltage response versus frequency, derived from the y-intercept, for projector P3 in the same five tanks used before (B1, B2, B3, N2 & N3). The TVR from the y-intercept results has been averaged over each group to give one trace for each group. The graph shows a range of spread of 5dB to 10dB with the exception of the trace for tank N2 (red), which is significantly above the rest. The next highest traces are tanks B3 (green) then N3 (blue) which are slightly higher than the groups representing tanks B1 (black) and B2 (black) for most of the frequency range. The results are similar but again the values above 60kHz are a lot lower than the reference calibration (black line).

Figure 7.18. Transmitting voltage response versus frequency averaged over a group and calculated from y-intercept for five tanks (B1, B2, B3, N2 & N3), source P3, with one trace for all of the groups in the tanks. Black – B1 and B2; green – B3; red – N2; blue – N3 and black line – reference calibration.
Figure 7.19 shows the averaged transmitting voltage response versus frequency, derived from the spatially averaged pressure, for projector P3 in the same five tanks used before (B1, B2, B3, N2 & N3). The TVR from the spatially averaged results has been averaged over each group to give one trace for each group in the five tanks. The spread of the spatially averaged pressure results is 10dB to 15dB and this is because this method only gives an accurate calibration if the direct field is small compared to the reverberant field. The tanks used had a wide variety of absorptions so that the direct field was large in some tanks and small in others compared to the reverberant field. This is why the calibrations range from agreeing with the reference calibration (black line) to a large bias above it. The highest trace is that for tank N2 (red), the most anechoic, then tanks B3 (green) and N3 (blue) are next at about the same level with tanks B1 (black) and B2 (black) below these.

Figure 7.19. Transmitting voltage response versus frequency averaged over a group and calculated from spatially averaged pressure for five tanks (B1, B2, B3, N2 & N3), source P3, with one trace for all of the groups in the tanks. Black – B1 and B2; green – B3; red – N2; blue – N3 and black line – reference calibration.
Figure 7.20 shows the averaged transmitting voltage response versus frequency for source T1 derived from the gradient, y-intercept and spatially averaged pressure for the small glass tank B4. The TVR has been averaged over each group so that there are nine traces for each of the G, Y and S results. The spread of the Y and S results ranges from 3dB to 8dB depending on frequency. The Y and S reverberant calibrations agree reasonably well with the calibrations lying above and below the reference calibration over different frequency regions. The gradient reverberant calibration results are almost universally above the reference calibration and by a large amount for some groups. However, these gradient results are fairly meaningless for this tank; the explanation for this is given in the next two paragraphs.

![Transmitting Voltage Response vs Frequency](image)

Figure 7.20. Transmitting voltage response versus frequency averaged over a group for tank B4, source T1, with one trace for each of the nine groups for each colour. Blue – gradient; green - y-intercept; red - spatially averaged pressure and black line - reference calibration.

The absorption in the glass tank (B4) is very low, as shown by the large reverberation time for a small volume of water. This means the reverberant field is large as shown by these results. The gradient of tank B4 pressure squared versus the
reciprocal of separation squared graphs varies between positive and negative. The graphs are fairly evenly balanced with slightly more positive than negative points. This happens because the fluctuations in the reverberant field are swamping the direct field to such an extent that the direct field is not being determined accurately. This problem arises when the fluctuations of the reverberant field are so large that they are far larger than the change in $y$ due to the direct field. When this happens the gradient can become negative and so the power becomes negative which is meaningless since a positive power is being radiated.

Since this would produce a negative TVR it was decided to take the modulus of the gradient and calculate the power and TVR from that. However the negative gradient values that are now positive are arbitrarily related to the reference gradient and often gave a large positive value since the value was originally a large negative value. This leads to TVR values well above the reference value and so the gradient values are meaningless, and are only included to illustrate this point.

This is only a problem if the fluctuations in the reverberant field are large compared to the change in the direct field. The large fluctuations cause a far smaller error in the $y$-intercept since there are a fairly equal number of positive and negative gradients and so the $y$-intercept is still well calculated. This is so since the $y$-intercept is large for this graph, due to the reverberant field being large, and so will not go negative. This does lead to a larger uncertainty for the $y$-intercept but this is not a problem unless the value for this is small. So providing the average reverberant field pressure is not small there is not a problem. Since the size of the reverberant field fluctuations is dependent on the level of the average field, a large fluctuation is more likely to occur with a large level and that decreases the chance of the $y$-intercept becoming negative. Only measurements taken in the tank B4 have caused negative gradients, but very occasionally negative $y$-intercepts have occurred for tank B4 and some of the other tanks. With reverberant field level being so large in this tank it means that the $S$ values agree well with the $Y$ values and the reference calibration.
Figure 7.21 shows the transmitting voltage response against frequency for projector P2, tank N1 and the single run in the group ‘dp2’. The gradient results follow the reference calibration curve but are offset by 1dB to 2dB below the curve. The y-intercept results are a bad fit where the points are approximately 5dB below the reference calibration and then drop even further at the resonance of the projector. However the spatially averaged pressure results are a good fit with the points biased by 1dB above the reference calibration and follow the curve well. This indicates that the reverberant field is significantly larger than the direct field since the $S$ results agree well. However the accuracy of the gradient results indicates that the direct field is well determined and so is not that small, which is confirmed by the small bias on the $S$ results.

![Transmitting voltage response versus frequency for tank N1, source P2 and the single run in the group ‘dp2’. Blue – gradient; green - y-intercept; red - spatially averaged pressure and black line – reference calibration.](image)

Figure 7.21. Transmitting voltage response versus frequency for tank N1, source P2 and the single run in the group ‘dp2’. Blue – gradient; green - y-intercept; red - spatially averaged pressure and black line – reference calibration.
Figure 7.22 shows the averaged transmitting voltage response versus frequency for source P1 derived from the gradient, y-intercept and spatially averaged pressure for tanks N1 and N3. The TVR has been averaged over each group, where there is one group for each tank so that there are two traces for each of the G, Y and S results. The tank N1 group contained one run and the tank N3 group contained four runs.

The gradient results fit well with the reference calibration and are generally spread over 2dB but increases to 4dB at the lower frequencies. The y-intercept results spread over 2dB to 4dB and are biased approximately 2dB below the reference calibration. The spatially averaged results show the least spread, which are less than 2dB but have a bias above the reference calibration of 2dB to 3dB. This indicates that the direct field was significant compared to the reverberant field in both tanks. The graph shows similar results in the two tanks, particularly for the S results.
Measurements were taken in tank S1 with source P3, but due to damage to the hydrophone and amplifier during the experiment replacement equipment had to be used for which only a single frequency calibration was available. Because of this the reverberant calibrations do not fit the reference calibration well over the whole frequency range. For this reason the calibrations are not presented, but are shown in section 7.4 where the average of the whole frequency range is used.

This discussion has indicated what was happening and why the three different methods gave varying results. A more rigorous way of analysing the results was required to determine more precisely what was happening, and to quantify the difference between the results.

7.3. Determining the difference in the reverberant field calibrations and the reference calibrations.

The previous section gave a qualitative discussion of how the reverberant calibrations varied with frequency, for changing tank, projector and the three different methods. This gave an overview of the situation but a more rigorous quantitative approach to the analysis was needed. This was achieved by calculating the difference between the free-field calibration and the reverberant calibration for each frequency point and then calculating the mean. This is, therefore, the mean difference between the two calibrations in dB. This reduces the two calibrations to a single number so that a quantitative difference is known although the knowledge of how the calibrations change relative to each other, with frequency, is no longer known. This value is referred to as the mean difference, which is expressed in dB.

In order to calculate the mean difference and the standard deviation between the reverberant field calibration and the free-field calibration, the difference $d_i$ between the two needs to be known and is calculated as

$$d_i = RFR_i - FFR_i,$$  \hspace{1cm} (7.1)

where $RFR$ is the TVR of the reverberant field calibration ($G$, $Y$ or $S$), $FFR$ is the TVR of the free-field calibration. The mean difference, $md$, is defined as
\[ md = \frac{1}{n} \sum_{i=1}^{n} d_i , \quad (7.2) \]

where \( n \) is the number of frequency points in the difference vector, \( d \). The standard deviation, \( s \), of the difference vector, \( d \), is calculated by

\[ s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (d_i - md)} , \quad (7.3) \]

where \( md \) is the mean difference and \( n \) is the number of frequency points.

If the reverberant field is correlated, then the standard deviation is the standard error of the mean difference value. If the reverberant field is not correlated, then the standard error is the standard deviation divided by the square root of the number of frequency points, \( n \). In practice, the field is probably partially correlated, and the standard error value is somewhere between these two values. How correlated the field is, depends on the number of modes excited in the tank and the reverberation time of the tank, for a given exciting random source. The shorter the reverberation time the smaller the correlation, as discussed in section 7.1. The voltage-time trace captured on the oscilloscope is 100ms long and the reverberation times for the various tanks range from approximately 40ms to 200ms. This means the traces will be partially correlated to differing degrees so the error in the mean difference value will be taken as the standard deviation. This assumes a correlated reverberant field and will overestimate the true error.

Every experimental run produces three reverberant field calibrations, which are derived from the gradient, y-intercept and spatially averaged pressure. A mean difference and standard deviation value is calculated for each of these calibrations. The mean difference value gives an indication of how close a fit the overall reverberant field calibration is to the free-field (reference) calibration. It indicates the overall difference between the two, whereas the standard deviation indicates the degree of deviation between the two.
This approach to analysing the data leads to a small error in the mean difference value because averaging dBs causes a small bias in the result. However, with the difference between the two TVR being relatively small, this is only a very small error. The averaging of each calibration can be carried out in the linear domain, converted to dBs, and then the difference found, but this leads to a false value for standard deviation. This is because the variation of the sample is in the linear domain, whereas it needs to be known in the logarithmic domain. Averaging in the dB scale may lead to a small error but it is small compared to the standard deviation.

7.4. Mean difference reverberant field calibration results

There are too many results to show here in detail, so examples and overviews for each tank will be shown. Measurements were taken for different positions and projector orientations within the tanks in order to see how these changes would alter the accuracy of the results. The results are displayed as mean difference values, as described in section 7.3, against runs, groups, or tanks. The group measurements taken are described in Table 7.2 where the scan location is varied and include measurements where, for each run, the projector was moved to a new position along the length, across the width, and down the depth of the tank. Measurements were also taken where, for each run, the projector orientation was changed, for a fixed position, and the hydrophone scan taken along radials from the projector position. The hydrophone always traverses along radials from the projector, with the '0' marks of the transducers facing each other. Other types of group were also used, and the number of measurement positions in each group varied from three to eighteen.
Figure 7.23 shows mean difference versus position of the projector, P3, across the width of the tank B1, for the group 'pl' runs 1 to 5. Each measurement run was along the length of the tank, with the start position of the run being moved to a new position across the width of the tank, for each run. The figure shows mean difference points derived from spatially averaged pressure (SAP), gradient and y-intercept of the pressure squared versus the reciprocal of separation graph. From now on these three types of mean difference result will be referred to as spatial mean difference ($smd$), gradient mean difference ($gmd$) and y-intercept mean difference ($yld$) respectively.

Figure 7.23. Mean difference versus position of the projector, P3, for each run, across the width of the tank B1, for group 'pl' runs 1 to 5. Projector traverses along the length of the tank and the 0mm position is approximately half way across the width of the tank. Mean difference derived from: spatially averaged pressure (red), gradient (blue) and y-intercept (green). The error bars represent standard deviation.
The graph shows that the $gmd$ results agree with each other to within the estimated uncertainty (standard deviation), the $ymd$ results agree with each other and the $smd$ results agree with each other. Furthermore, the results in each of these three groups agree with each other to a far greater extent than the error bars would indicate, therefore suggesting that the reverberant field is not very correlated in this case. The graph indicates distinct differences between the $gmd$, $ymd$ and $smd$ results. The gradient results are close to zero indicating that the two calibrations do not differ significantly. However the y-intercept results are significantly below the gradient results. The spatially averaged results are above the gradient results suggesting a significant direct field compared to the reverberant. As would be expected this agrees with the calibrations shown in Figure 7.13 for the same group. The low values for the $ymd$ results are due to the low TVR values for the y-intercept results above 60kHz.

Figure 7.24 shows the mean difference results plotted against group for tank B1 and projector P3. The $gmd$, $ymd$ and $smd$ results are plotted for a particular group at the same x-axis position on the graph. Group one, ‘pl’, represents the projector moved across the width of the tank, group two, ‘po’, represents the projector, and therefore also the hydrophone scan, orientation varied within the tank and group three, ‘pw’, represents the projector moved along the length of the tank. Group ‘pl’ is labelled this way to represent the direction of traverse, i.e. along the length of the tank, even though the projector moves across the width of the tank throughout the group. For group ‘po’ the zero degrees direction is when the hydrophone traverses along the length of the tank. Any angle of traverse is measured from this line along the centre of the tank. Groups one, two and three show the five, three and four runs respectively.
Figure 7.24. Mean difference versus group number for measurements made in tank B1, with projector P3. Each group number shows the several mean difference results for spatially averaged pressure (red), gradient (blue) and y-intercept (green). Group number one represents the projector (and therefore also the hydrophone scan axis) moved across the width of the tank, two represents the changing orientation of the projector (and hydrophone scan axis) and three represents the projector (and hydrophone scan axis) moved along the length of the tank.

All three groups consistently show the gmd results close to zero, the smd results above the gmd results and the ymd results below the gmd results. The gradient results range from 0dB to 2dB, the y-intercept results range from -2.5dB to -5.5dB and the SAP results range from 3dB to just over 4dB.

Figure 7.25 shows mean difference versus group for measurements made in tank B1, with projector P3. The groups are as described for Figure 7.24 but with a single value representing the gradient, y-intercept and SAP results. The mean difference value for a group is the average of all the calibration runs in that group, i.e. for group 1: ‘pl’ – average of five runs. Each calibration is converted into linear form, the
mean calculated of all the calibrations, this is then converted back into the dB scale and then the mean difference is calculated. This is done for all three calibration types for each group.

Figure 7.25. Mean difference versus group number for measurements made in tank B1, with projector P3. Each group number shows the mean difference result, for the average of the calibrations in that group, for spatially averaged pressure (red), gradient (blue) and y-intercept (green). The error bars represent standard deviation. Group number one represents the projector (and hydrophone scan axis) moved across the width of the tank, two represents the varied orientations of the projector (and hydrophone scan axis) and three represents the projector (and hydrophone scan axis) moved along the length of the tank.

The $g_{md}$ results are almost identical and agree with each other to well within the standard deviations, this is the same for the $y_{md}$ and $s_{md}$ results. The $s_{md}$ results are distributed about 3.5dB, the $g_{md}$ results about 1dB and the $y_{md}$ results about $-3.5$dB. As expected this agrees with the conclusions from Figure 7.24.
Figure 7.26 shows the mean difference results versus group for tank B2, with projector P3. The individual gradient, y-intercept and SAP results agree with each other to within standard deviation limits. The $g_{md}$ and $s_{md}$ results are very similar and are distributed about approximately $1$dB. The $y_{md}$ results are distributed about approximately $-5$dB, again because the calibration values above $60$kHz are low. The $g_{md}$ and $s_{md}$ results being similar indicate that the direct field is smaller than the reverberant field but large enough to be measured accurately.

Figure 7.26. Mean difference versus group number for measurements made in tank B2, with projector P3. Each group number shows the mean difference result, for the average of the calibrations in that group, for spatially averaged pressure (red), gradient (blue) and y-intercept (green). The error bars represent standard deviation. The group numbers represent the aspect of this scan that is changed: 1 - projector location down depth, 2 - hydrophone distance traversed, 3 - projector location across width, 4 - projector and hydrophone scan axis orientation, 5 - temperature of water and 6 - projector location along length.
Figure 7.27 shows the mean difference values for group ‘cl’, tank B3 and projector P3. The gradient value is approximately 1dB the, y-intercept value is -2dB and the SAP value is above 6dB. This indicates that the direct field is large by comparison to the reverberant field, which is what would be expected for a tank with high absorptivity walls as shown in Table 5.4.

![Graph showing mean difference versus group number for measurements made in tank B3, with projector P3. Each group number shows the mean difference result, for the average of the calibrations in that group, for spatially averaged pressure (red), gradient (blue) and y-intercept (green). The error bars represent standard deviation. Group number one represents the projector (and hydrophone scan axis) moved across the width of the tank.](image-url)
Figure 7.28 shows the mean difference versus group number for the measurements made in tank B4 using transducer T1. The y-intercept and SAP results agree with themselves, and each other, within error limits. However, the gradient results agree with themselves, for the most part, but do not agree with the ymd and smd results, except for two groups. The gradient results are not well determined since they range from 1dB to almost 11dB, with most distributed about 7dB.

Figure 7.28. Mean difference versus group number for measurements made in tank B4, with transducer T1. Each group number shows the mean difference result, for the average of the calibrations in that group, for spatially averaged pressure (red), gradient (blue) and y-intercept (green). The error bars represent standard deviation. The group numbers represent the aspect of the scan that is changed: 1 – projector/hydrophone alignment, 2 – projector location down depth, 3 – projector location across width, 4 – number of hydrophone positions, 5 – projector and hydrophone scan axis orientation, 6 – hydrophone distribution of points, 7 – repeat run, 8 – initial separation of transducers and 9 – hydrophone distance traversed.
This is because the gradient is not determined accurately, with there being large numbers of negative gradient values. This is the case since the reverberant field is so large that the direct field cannot be ascertained from the pressure squared versus the reciprocal of separation squared graph. However groups five and six give more sensible results, perhaps due to the direct field not being too small in these cases or maybe due to the general distribution of points because the gradient cannot be determined.

The $y_{md}$ and $s_{md}$ results range from 2dB to $-0.5$dB and agree with each other very well. The two results agree with each other and give sensible results since they are both determined from the reverberant field level which has been shown to be very large compared to the direct field, therefore indicating that these should provide accurate results. The $y_{md}$ results are not low, as was the case for tanks B1 to B3, because the reverberant field level is large. The reverberant field level would be expected to be large compared to the direct since the absorptivity of the glass walls is very low, as shown in Table 5.4.
Figure 7.29 shows the mean difference versus group for the measurements taken in tank N1 with projector P1 and P2. Group one represents a single run with projector P1 and group two represents a single run with projector P2. The gradient results are at $-1\text{dB}$ and $-2\text{dB}$ and agree with each other to within errors. The SAP results do not agree with each other within errors and are at just below $1\text{dB}$ and just above $2\text{dB}$. They indicate that the direct field is significant compared to the reverberant field. The $ymd$ values are at $-3.5\text{dB}$ and at $-6.5\text{dB}$ and agree with each other within errors. The $y$-intercept values are below zero because the calibrations were below the free-field calibration. The group two results was even lower than the group one results because the calibration for projector P2 was very low above $30\text{kHz}$, the same effect as for projector P3 in tanks B1 to B3.

![Figure 7.29](image.png)

Figure 7.29. Mean difference versus group number for measurements made in tank N1, with projectors P1 and P2. Each group number shows the mean difference result, for the average of the calibrations in that group, for spatially averaged pressure (red), gradient (blue) and $y$-intercept (green). The error bars represent standard deviation. Group number one represents the single calibration for projector P1 and two represents the single calibration for projector P2.
Figure 7.30 shows the mean difference versus group for the measurements made in tank N2 with projector P3. The gradient is a reasonable value at close to $-1$ dB and the y-intercept value is near 2 dB. The SAP value is very high at 12 dB and is due to a large direct field level compared to the reverberant field level. Since the y-intercept value is derived from the reverberant field and not the combined field it gives a reasonable result. It would be expected that the reverberant field is very low since the absorptivity of the wood is very high and the size of the tank is large, as shown in Table 5.4.

![Graph showing mean difference versus group number](image)

**Figure 7.30.** Mean difference versus group number for measurements made in tank N2, with projector P3. Each group number shows the mean difference result, for the average of the calibrations in that group, for spatially averaged pressure (red), gradient (blue) and y-intercept (green). The error bars represent standard deviation. Group number one represents an average of two calibrations for projector P3.
Figure 7.31 shows the mean difference versus group for tank N3 with projectors P1 and P3. Group one represents measurements made with projector P3 and group two represents those made with projector P1. The gradient results are at 0.5dB and 3dB and do not agree with each other given the standard deviations. The y-intercept results agree with each other given the errors and are below zero with values of -2dB and -3dB, which is due to the calibrations being lower than free-field calibration. The SAP values are at 3dB and 6dB and do not agree with each other given the errors concerned. This indicates that the direct field is significant compared with the reverberant field.

![Graph showing mean difference versus group number for measurements made in tank N3](image)

Figure 7.31. Mean difference versus group number for measurements made in tank N3, with projectors P1 and P3. Each group number shows the mean difference result, for the average of the calibrations in that group, for spatially averaged pressure (red), gradient (blue) and y-intercept (green). The error bars represent standard deviation. Group number one represents the average calibration for projector P3 and two represents the average calibration for projector P1.

Figure 7.32 shows the mean difference versus group for the measurements taken in tank S1 with projector P3. The single group represents one run taken in the tank at
Sonardyne. The results for this tank can only be taken as a guide since single value calibrations were only available for the hydrophone and hydrophone amplifier. Normally there is a series of calibration-frequency values but one value represents the whole frequency band. This could lead to large discrepancies in the calculated reverberant projector calibration if the two calibrations varied significantly over the frequency band. The original hydrophone and amplifier were damaged during the experiment and replacements were borrowed that only had single frequency guidance calibrations. The gradient and y-intercept values were approximately -0.5dB and 0.5dB respectively. These results are a good fit normally and very good considering the hydrophone and amplifier calibrations. The SAP result is 8dB which is very high and indicates that the direct field was large compared to the reverberant field, which was to be expected given the size of the tank.

Figure 7.32. Mean difference versus group number for measurements made in tank SI, with projector P3. Each group number shows the mean difference result, for the average of the calibrations in that group, for spatially averaged pressure (red), gradient (blue) and y-intercept (green). The error bars represent standard deviation. Group number one represents the single calibration for projector P3.
7.5. Comparison of the reverberant field calibrations for each tank

In this section the results for all 25 groups are collated together and compared. Figure 7.33 shows the mean difference results versus group for all the groups and is a compilation of Figures 7.25 to 7.32. The key to the group-numbers is displayed in Table 7.3 which shows group-number against tank, projector, group and changing variable in each group.

![Figure 7.33](image_url)

Figure 7.33. Mean difference versus group number for measurements made in all eight tanks, with different projectors. Each group number shows the mean difference result, for the average of the calibrations in that group, for spatially averaged pressure (red), gradient (blue) and y-intercept (green). The error bars represent standard deviation. This figure is a compilation of figures 7.25 to 7.32. Table 7.3 is the key for this graph and describes the relationship between group number, tank, projector, group and variable changed.

From the graph groups 1 to 10, the three large Bath tanks, show that the gradient results give values close to zero indicating a good match between reverberant and free-field (reference) calibration. For these groups the y-intercept values are low due to the low calibration values above 60kHz, but given the size of the error bars they
are not that bad a fit. Tank B2, groups 4 to 9, show that the SAP results are close to zero indicating a significant reverberant field. Tank B1, group 1 to 3, and tank B3, group 10, show that the SAP results are significantly above zero, particularly for tank B3. This indicates that the direct field is significant for tank B1 and large for tank B3. Tank B4, groups 11 to 19, show that the y-intercept and SAP results both agree and are at zero indicating a good agreement between the reverberant and free-field calibration. This shows that the direct field is very small compared to the reverberant field. The gradient results are very high and as explained earlier are not a viable method for the size of the direct field.

Tank N1, groups 20 and 21, show reasonable agreement for the gradient and y-intercept results, with the y-intercept results being low but having a larger error. Projector P1 results are a better fit than projector P2 and the SAP results are high, particularly for P1 and indicates a significant direct field. Tank N2, group 22, has the gradient slightly below zero and the y-intercept above zero, but has a very large value for the SAP. This indicates a large direct field compared to the reverberant. Tank N3, groups 23 and 24, show the gradient results above zero and the y-intercept below with projector P1 results close or at zero and projector P3 results showing a worse fit. The SAP results are high, particularly for P3, indicating a significant direct field. Finally tank S1, group 25, shows good agreement for the gradient and y-intercept results with them both being very close to zero. The SAP result is very high indicating a large direct field.
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<td></td>
<td>g1r</td>
<td>Repeat Run</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td>g1s</td>
<td>Initial separation of transducers</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td>g1t</td>
<td>Distance traversed by hyd</td>
</tr>
<tr>
<td>20</td>
<td>N1</td>
<td>P1</td>
<td>dp1</td>
<td>None</td>
</tr>
<tr>
<td>21</td>
<td>P2</td>
<td></td>
<td>dp2</td>
<td>None</td>
</tr>
<tr>
<td>22</td>
<td>N2</td>
<td>P3</td>
<td>np1l</td>
<td>Repeat Run</td>
</tr>
<tr>
<td>23</td>
<td>N3</td>
<td>P3</td>
<td>np13</td>
<td>Repeat Run</td>
</tr>
<tr>
<td>24</td>
<td>P1</td>
<td></td>
<td>np16</td>
<td>Repeat Run</td>
</tr>
<tr>
<td>25</td>
<td>S1</td>
<td>P3</td>
<td>sm</td>
<td>None</td>
</tr>
</tbody>
</table>

Table 7.3. Table displaying the key to Figure 7.33 and showing group-number against tank, projector, group and variable changing during the group.
The results will now be plotted against tank and projector, which illustrates the explanation of Figure 7.33 in a less cluttered way. All the calibrations for a particular tank and projector combination have been averaged together to produce a value for each of the three methods. The calibrations were first converted into linear form, then all the calibrations averaged together, then converted back into dB’s and then the mean difference calculated. This was done for all the calibrations for each group and tank-projector combination. The results are shown in Figure 7.34 and show mean difference versus each tank-projector combination for the gradient, y-intercept and spatially averaged pressure method.

Figure 7.34. Mean difference versus tank-projector number for the results of measurements made in all eight tanks, with different projectors. Each tank-projector number shows the mean difference result, for the average of the calibrations in that tank for a particular projector, for spatially averaged pressure (red), gradient (blue) and y-intercept (green). The error bars represent standard deviation. Due to the number of groups for each tank and projector, this means that the results for tanks B3, N1, N2, N3 and S1 remain as shown in the figures above. However the results for tanks B1, B2 and B4 are the average of all the calibrations for each individual tank. The key for the tank-projector number is shown in Table 7.4.
It must be remembered that each projector was not calibrated over the same frequency range due to the resonant frequency of the projector. This detail is shown in the key for Figure 7.34, which is Table 7.4 and shows tank-projector number against tank, projector, frequency range and frequency interval. The frequency range shows the start and stop frequency for the measurements and the frequency interval is the step between frequency points on the calibration curves.

<table>
<thead>
<tr>
<th>Tank-Projector Number</th>
<th>Tank</th>
<th>Projector</th>
<th>Frequency Range / kHz</th>
<th>Frequency Interval / kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B1</td>
<td>P3</td>
<td>10 - 100</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>B2</td>
<td>P3</td>
<td>10 - 100</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>B3</td>
<td>P3</td>
<td>10 - 100</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>B4</td>
<td>T1</td>
<td>10 - 100</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>N1</td>
<td>P1</td>
<td>10 - 30</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>N1</td>
<td>P2</td>
<td>10 - 40</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>N2</td>
<td>P3</td>
<td>10 - 100</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>N3</td>
<td>P3</td>
<td>10 - 100</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>N3</td>
<td>P1</td>
<td>10 - 30</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>S1</td>
<td>P3</td>
<td>10 - 100</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 7.4. Table showing tank-projector number against tank, projector, frequency range over which measurements were made and frequency interval between the measured points for figure 7.34.

7.6. The effect of projector directionality on the y-intercept and spatially averaged pressure calibrations.

The TVR of a projector is not uniform with direction and the calibration of the transducer is defined for a particular direction. The TVR values calculated from the gradient are derived from the measurement of the direct field in the '0' mark direction of the projector. The y-intercept value of TVR is based on the reverberant field pressure, which is due to the combined effect of varying direct field pressure with direction and the interaction with the boundary of the tank. The correct value of y-intercept TVR is only obtained if the reverberant field pressure is due to the direct
field pressure in the '0' mark direction. This is true since it is the '0' mark direction for which the calibration is valid. The actual reverberant field pressure is caused by the summation of direct field pressure over the whole surface area of the projector. The average amplitude response of the projector needs to be known relative to the '0' mark direction. This means the amplitude response needs to be averaged over $4\pi \text{sr}$. This is a long and laborious task but an approximation to it can be easily calculated. The X-Y and X-Z amplitude response of the projector needs to be measured with the values at the '0' mark position set to unity. These responses can then be approximately averaged over the whole surface area so that the averaged response is known relative to the '0' mark direction, as shown in section 6.15.

From the equation to calculate the acoustic power radiated by the projector from the y-intercept, Equation 6.21, it is seen that the y-intercept value, $C$, is the square of the reverberant field pressure, $P_r$. The reverberant field pressure measured is that due to the averaged response over the whole surface area of the projector. What needs to be known is the reverberant pressure were the response of the transducer is the same in all directions as that at the '0' mark direction. This can be calculated if the average response of the transducer is known relative to the '0' mark direction. The '0' mark reverberant pressure is the average reverberant pressure (measured) divided by the average response of the projector over the whole surface area. The y-intercept value of the TVR can be compensated for the directionality response by subtracting the average response in dBs. The calculation of the approximate average response over the whole surface area and the compensation of y-intercept TVR is described in section 6.1.5.

A series of amplitude directionality measurements were made at NPL on the four projectors, at sample frequencies. These included X-Z measurements made by NPL on their projectors P1 and P2, and just X-Y measurements on Bath's projectors P3 and T1, also made at NPL. Table 7.5 shows the average amplitude response and average amplitude response in dBs against projector, X-Y frequency and X-Z frequency. Each X-Y and X-Z response is calculated assuming a unity response for the variation in the other plane and enables the effect of just the one plane to be ascertained. No measurements were made in both planes for the same projector, but
Table of Average Projector Directionality Response (Part 1)

<table>
<thead>
<tr>
<th>Projector</th>
<th>X-Y Frequency / kHz</th>
<th>X-Z Frequency / kHz</th>
<th>Average Amplitude Response</th>
<th>Average Amplitude Response / dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITC1001 (P1)</td>
<td>Unity Response</td>
<td>20</td>
<td>0.92</td>
<td>-0.70</td>
</tr>
<tr>
<td>ITC1001 (P1)</td>
<td>Unity Response</td>
<td>30</td>
<td>0.87</td>
<td>-1.20</td>
</tr>
<tr>
<td>ITC1032 (P2)</td>
<td>Unity Response</td>
<td>20</td>
<td>0.91</td>
<td>-0.85</td>
</tr>
<tr>
<td>ITC1032 (P2)</td>
<td>Unity Response</td>
<td>40</td>
<td>0.93</td>
<td>-0.67</td>
</tr>
<tr>
<td>ITC1032 (P3)</td>
<td>10</td>
<td>Unity Response</td>
<td>0.99</td>
<td>-0.10</td>
</tr>
<tr>
<td>ITC1032 (P3)</td>
<td>30</td>
<td>Unity Response</td>
<td>0.98</td>
<td>-0.17</td>
</tr>
<tr>
<td>ITC1032 (P3)</td>
<td>50</td>
<td>Unity Response</td>
<td>1.01</td>
<td>0.12</td>
</tr>
<tr>
<td>ITC1032 (P3)</td>
<td>70</td>
<td>Unity Response</td>
<td>1.43</td>
<td>3.08</td>
</tr>
<tr>
<td>ITC1032 (P3)</td>
<td>100</td>
<td>Unity Response</td>
<td>0.86</td>
<td>-1.28</td>
</tr>
<tr>
<td>BALL (T1)</td>
<td>30</td>
<td>Unity Response</td>
<td>1.13</td>
<td>1.03</td>
</tr>
<tr>
<td>BALL (T1)</td>
<td>50</td>
<td>Unity Response</td>
<td>0.94</td>
<td>-0.57</td>
</tr>
<tr>
<td>BALL (T1)</td>
<td>100</td>
<td>Unity Response</td>
<td>1.10</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 7.5. Table displaying projector, X-Y frequency, X-Z frequency, average amplitude response over surface area and average amplitude response over surface area in dB's. Unity response means the amplitude is one for all angles in the X-Y or X-Z plane.
Table of Average Projector Directionality Response (Part 2)

<table>
<thead>
<tr>
<th>Projector</th>
<th>X-Y Frequency / kHz</th>
<th>X-Z Frequency / kHz</th>
<th>Average Amplitude Response</th>
<th>Average Amplitude Response / dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITC1032 (XY=P3, XZ=P2)</td>
<td>10</td>
<td>20</td>
<td>0.90</td>
<td>-0.96</td>
</tr>
<tr>
<td>ITC1032 (XY=P3, XZ=P2)</td>
<td>30</td>
<td>20</td>
<td>0.88</td>
<td>-1.15</td>
</tr>
<tr>
<td>ITC1032 (XY=P3, XZ=P2)</td>
<td>av 10 &amp; 30</td>
<td>20</td>
<td>0.91</td>
<td>-0.87</td>
</tr>
<tr>
<td>ITC1032 (XY=P3, XZ=P2)</td>
<td>30</td>
<td>40</td>
<td>0.90</td>
<td>-0.94</td>
</tr>
<tr>
<td>ITC1032 (XY=P3, XZ=P2)</td>
<td>50</td>
<td>40</td>
<td>0.90</td>
<td>-0.88</td>
</tr>
<tr>
<td>ITC1032 (XY=P3, XZ=P2)</td>
<td>av 30 &amp; 50</td>
<td>40</td>
<td>0.90</td>
<td>-0.93</td>
</tr>
<tr>
<td>ITC1032 (XY=P3, XZ=P2)</td>
<td>30</td>
<td>av 20 &amp; 40</td>
<td>0.89</td>
<td>-1.06</td>
</tr>
</tbody>
</table>

Table 7.5. Table displaying projector, X-Y frequency, X-Z frequency, average amplitude response over surface area and average amplitude response over surface area in dBs. The same type of projector was used for all measurements in part 2, but projector P3 was used for plane X-Y and projector P2 for plane X-Z. Av represents that measurements for these two frequencies were averaged together.
measurements were made of two different ITC1032 projectors, one in the X-Y plane (P3) and one in the X-Z plane (P2). Since the response difference between projectors of the same type will be small this gives a good indication of the overall average response of either projector ITC1032. This table also shows results where two X-Y or X-Z measurements were averaged together to obtain an approximate response for the frequency half way between the two.

From the results it can be seen that the y-intercept TVR values will increase and decrease depending on the frequency in question. Projectors P1 and P2 show an increase in TVR but this is only shown for the low frequencies for a small range. This improves the fit of the y-intercept TVR to the free-field TVR although it does not completely explain the difference between the two calibrations, so some other effect may be altering the response. The results for projectors P3 are over a far greater range and increase for the low frequencies then decrease for the medium frequencies and increase again for the high frequencies. This improves the fit below 60kHz, but makes the fit even worse above this frequency where the TVR goes even lower, but does make it better at 100kHz. Projector T1 is also over a large frequency range and changes from decreasing, to increasing to decreasing the y-intercept TVR. This helps improve the fit changing the y-intercept TVR in the correct direction as frequency changes. However one or more other effects is causing the changing fit with frequency, since this has only partially solved the problem.

This effect will also alter the SAP results since they are based on the reverberant field pressure, but they are also based on the direct field so the degree of compensation depends on the relative size of the two fields.

A compensating value for the mean difference results has only been approximately calculated. To calculate an accurate average response for the projector, the directionality needs to be known at close frequency intervals over the whole 3D-space.
7.7. Analysis of tank sound fields

It would be advantageous to know if the reverberant calibration method, based on the $p^2$ versus $1/r^2$ graph, and the spatially averaged pressure method will work in a given tank. These methods will only work in certain types of sound fields and so if the tank sound field type can be determined then it will be known if the method is suitable for that tank. The reverberant calibration method requires that the direct field is measurable in the combined field and that the reverberant field is diffuse and that its fluctuations are not too large. Also the spatially averaged pressure method requires that the direct field is very small compared to the reverberant field and that the reverberant field is diffuse. To more rigorously determine the sound field type for which these methods are suitable, the sound fields of the tanks needed to be analysed.

Several theoretical and experimental acoustic field parameters were used to analyse the sound fields, and indicate how diffuse the fields were and the ratio of direct to reverberant fields. The theoretical parameters considered were the ratio of tank length to wavelength, the number of tank modes at a specific frequency, the ratio of the number of tank modes within a frequency band to the width of that frequency band, the Schroeder frequency and the ratio of the direct field pressure to the reverberant field pressure.

The experimental results calculated were the ratio of the direct and reverberant field pressure for the minimum and maximum transducer separations used. Also calculated was the direct and reverberant field sound power measured for the maximum and minimum transducer separations used. For completeness the ratio of the theoretical and experimental sound pressure for the minimum transducer separation was also calculated. The theory for all but the simplest of these parameters is shown in sections 6.14 and 6.16 to 6.18.
7.7.1. Ratio of tank length or diameter to wavelength, $L/\lambda$

The ratio of tank length or diameter to wavelength demonstrates the number of pressure anti-nodes in that dimension of the tank and therefore indicates the uniformity of the pressure field at that frequency. The sound field uniformity depends on the distribution of the pressure anti-nodes over the whole tank and is therefore an amalgamation of all the field patterns over the frequency range in question. Although this ratio only gives an indication of the pressure field uniformity at the frequency in question, it is still useful as it indicates the number of anti-nodes for one dimension. The actual number of anti-nodes will be greater due to the combination of three dimensions and so the if the number of anti-nodes is greater than, say ten, the true sound field will probably be sufficiently uniform for the methods above. Ten is chosen as a reasonable number that will give sufficient anti-nodes for the field to be uniform. According to Nelisse and Nicolas (1997), spatial uniformity is one indicator of sound field diffuseness, which itself is needed for spatial averaging to be successful.

<p>| Number of wavelengths in tank length or diameter, $L/\lambda$ |
|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|</p>
<table>
<thead>
<tr>
<th>f / kHz</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>S1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>18</td>
<td>12</td>
<td>21</td>
<td>6.6</td>
<td>13</td>
<td>37</td>
<td>13</td>
<td>33</td>
</tr>
<tr>
<td>30</td>
<td>55</td>
<td>37</td>
<td>62</td>
<td>20</td>
<td>40</td>
<td>111</td>
<td>40</td>
<td>98</td>
</tr>
<tr>
<td>60</td>
<td>110</td>
<td>75</td>
<td>123</td>
<td>40</td>
<td>81</td>
<td>221</td>
<td>81</td>
<td>197</td>
</tr>
<tr>
<td>100</td>
<td>183</td>
<td>125</td>
<td>205</td>
<td>66</td>
<td>134</td>
<td>369</td>
<td>134</td>
<td>328</td>
</tr>
</tbody>
</table>

Table 7.6. Ratio of length or diameter to wavelength, where each row has the same wavelength. All the tanks are rectangular except tank N2 which is cylindrical.

Table 7.6 shows that the number of pressure anti-nodes along the length of the tanks. As can be seen all the tanks have a ratio greater than ten for the four frequencies selected, except tank B4 for the lowest frequency of 10kHz. This nominally means that all the tanks except for tank B4 at 10kHz have a sufficiently uniform sound field. The higher the frequency the more anti-nodes and therefore the more uniform the field. Tank B4 at 10kHz will still have a reasonable number of anti-nodes throughout the tank and so will still probably be sufficiently uniform.
7.7.2. Number of tank modes, \( N \), below frequency, \( f \)

The number of tank modes below a given frequency is also useful to indicate if the reverberant field will be diffuse, since the higher the number of modes the greater the modal density. This was calculated for all eight tanks at four representative frequencies using Equation 6.24, as shown in Table 7.7.

<table>
<thead>
<tr>
<th>( f / \text{kHz} )</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>S1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>6.9x10³</td>
<td>3.0x10³</td>
<td>9.9x10³</td>
<td>2.1x10²</td>
<td>5.3x10³</td>
<td>1.5x10⁵</td>
<td>5.7x10³</td>
<td>7.9x10⁴</td>
</tr>
<tr>
<td>30</td>
<td>1.9x10⁵</td>
<td>8.2x10⁴</td>
<td>2.7x10⁵</td>
<td>5.5x10³</td>
<td>1.4x10⁵</td>
<td>4.1x10⁶</td>
<td>1.5x10⁵</td>
<td>2.1x10⁶</td>
</tr>
<tr>
<td>60</td>
<td>1.5x10⁶</td>
<td>6.5x10⁵</td>
<td>2.1x10⁶</td>
<td>4.4x10⁴</td>
<td>1.2x10⁶</td>
<td>3.3x10⁷</td>
<td>1.2x10⁶</td>
<td>1.7x10⁷</td>
</tr>
<tr>
<td>100</td>
<td>6.9x10⁶</td>
<td>3.0x10⁶</td>
<td>9.9x10⁶</td>
<td>2.1x10⁵</td>
<td>5.3x10⁶</td>
<td>1.5x10⁸</td>
<td>5.7x10⁶</td>
<td>7.9x10⁷</td>
</tr>
</tbody>
</table>

Table 7.7. The number of tank modes, \( N \), below the frequency, \( f \), for the eight tanks.

Table 7.7 shows that the number of modes below the indicated frequency is in the thousands even at 10kHz, apart from tank B4. This indicates that the reverberant field is complicated and probably diffuse at the higher frequencies, even for tank B4. Another useful criterion is the number of modes per Hz, as shown in the next section.

7.7.3. Ratio of the number of tank modes, within a given bandwidth, to that bandwidth at a specified frequency, \( \Delta N/\Delta f \)

This rate of change of the number of tank modes with respect to frequency gives the number of modes per Hz. It is therefore a useful indication of the modal density at a particular frequency. This was calculated for all eight tanks at four representative frequencies using Equation 6.25, as shown in Table 7.8.
Table 7.8 shows that the number of modes per Hz is approximately unity for most of the tanks at 10kHz and a lot lower for tank B4 and higher for the large tanks. However, by 30kHz the number of modes per Hz is approximately ten or over for all of the tanks except B4, the small tank. This is probably sufficient a sufficient modal density to allow the reverberant field to be diffuse. However, tank B4 still has only 6 modes per Hz at 100kHz and may not be diffuse.

7.7.4. The Schroeder frequency of the tanks, $f_c$

The Schroeder frequency is the frequency where the reverberant sound field is postulated as behaving in a statistical manner. At and above this frequency the average spacing of the resonant frequencies is less than one-third of their bandwidths. According to Nelisse and Nicolas (1997) the Schroeder frequency limit gives a good indication that the sound field is diffuse. This is the case since it is in good agreement with two other methods of measuring sound field diffuseness, the correlation coefficient and the spatial uniformity. This has been calculated for all eight tanks at four representative frequencies using Equation 6.41, with the shown in Table 7.9.
Table 7.9. The Schroeder frequency, $f_c$, of the eight tanks for the different reverberation times at the frequency $f$.

Table 7.9 shows the Schroeder frequency, indicating theoretically that the reverberant field is diffuse above the frequencies indicated for each tank and nominal frequency, $f$. The frequency $f$, is used to determine if the field is diffuse at that frequency, since the reverberation time changes with frequency. The reverberation time for that frequency is used to calculate the Schroeder frequency and therefore indicates if the field is diffuse at that frequency.

The results indicate that the reverberant field is theoretically diffuse for all the tanks, except B4, over the whole frequency range of the reverberant calibration, 10kHz to 100kHz. Even the field in tank B4 is diffuse from 20kHz onwards, which is most of the frequency range.

7.7.5. Ratio of the experimental direct field pressure to the reverberant field pressure at the minimum transducer separation.

The ratio of the experimentally measured direct field pressure relative to the reverberant field pressure enables the relative levels of the two fields to be ascertained. Measurements at the minimum and maximum separation of the transducers indicates whether the reverberant calibration method is suitable over this range. This also indicates if the spatially averaged pressure method is valid for these two distances. These measurements values were all obtained from analysis of the original oscilloscope or vector analyser data for the reverberant calibration measurements. The pressure spectra enabled the pressures at specific frequencies to be found, and so the ratio at a particular frequency to be calculated. The
experimental pressure ratio at the minimum and maximum positions were calculated using Equation 6.46, with the minimum results shown in Table 7.10.

<table>
<thead>
<tr>
<th>Ratio of experimental direct to reverberant field pressure at minimum transducer separation, $R_{EdrPressureMin}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f / \text{kHz}$</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uncertainty in the ratio of experimental direct to reverberant field pressure at minimum transducer separation, $\Delta R_{EdrPressureMin}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f / \text{kHz}$</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

Table 7.10. Ratio of experimental direct field pressure to the reverberant field pressure when measured at the minimum transducer separation used.

This ratio indicates that the direct field is larger than the reverberant field when the value is greater than one. Table 7.10 shows that generally the ratio increases with frequency, with only tanks B3 and N3 showing some deviation from this. The results show that the direct field is larger than the reverberant field for the large Bath tanks (B1, B2 and B3), except for the 10kHz value for tank B2. The reverberant field is significant compared to the direct field for these three tanks, with the ratios ranging from 0.7 to 4.0. The reverberant field is larger than the direct for tank B4, the glass tank, with ratio being 0.2 and 0.4. The three NPL tanks and the Sonardyne tanks have a larger direct field, but behave similarly to the large Bath tanks with ratios ranging from 1.2 to 7. These results are to be expected since the receiver was near the source.

The uncertainty in the ratio was derived using Equation 6.46 and the uncertainties in the input variables. The input variables were the gradient and y-intercept of the least
squares fit of the pressure squared versus the reciprocal of separation squared graph and so the uncertainty is the standard error derived from the fit.

7.7.6. Ratio of the experimental direct field pressure to the reverberant field pressure at the maximum transducer separation.

These experimental results were obtained as for those presented in section 7.7.5, using Equation 6.46 to calculate the ratios, except that the data was taken for the maximum transducer separation used.

<table>
<thead>
<tr>
<th>Ratio of experimental direct to reverberant field pressure at maximum transducer separation, $R_{EdrPressureMax}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$ / kHz</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uncertainty in the ratio of experimental direct to reverberant field pressure at maximum transducer separation, $\Delta R_{EdrPressureMax}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$ / kHz</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

Table 7.11. Ratio of experimental direct field pressure to the reverberant field pressure when measured at the maximum transducer separation used.

The ratios indicate that the reverberant field is the dominant field for all the tanks, with the exception of tank N1 where it is equal to the direct field for frequencies 60kHz and 100kHz. However for these two values the standard error is very large, masking the true value. The ratios generally range from 0.1 to 0.5, with even the values for tank B4 not going much below 0.1. These results are within expected values since the receiver was a long distance from the source.
7.7.7. Ratio of the experimental direct field power to the reverberant field power at the minimum transducer separation.

The results here are the same as for section 7.7.5 with a minimum transducer separation, except that acoustic power is used instead of pressure. The experimental power minimum and maximum position ratios were calculated using Equation 6.45. It is the power ratios that determine the gradient and y-intercept values in the pressure squared versus the reciprocal of separation squared graphs.

<table>
<thead>
<tr>
<th>Ratio of experimental direct to reverberant field power at minimum transducer separation, $R_{EdrPowerMin}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f / \text{kHz} )</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uncertainty in the ratio of experimental direct to reverberant field power at minimum transducer separation, $\Delta R_{EdrPowerMin}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f / \text{kHz} )</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

Table 7.12 shows the experimental power minimum position ratio for all eight tanks with the ratios increasing with frequency as for the pressure values. The minimum position power ratio indicates that the direct field power is greater than the reverberant field power for all tanks except tank B4 and the 10kHz value for tank B2. They range from approximately unity to 20 for most of these values. However, very large values were measured for tanks N1 and N2, with values of 60 and 30 respectively, for frequencies 60kHz and 100kHz. The direct field power was small compared to the reverberant field power for tank B4, with values around 0.05 and 0.3.
7.7.8. Ratio of the experimental direct field power to the reverberant field power at the maximum transducer separation.

These results are the same as section 7.7.7, using Equation 6.45 to calculate the ratios, except that the maximum transducer separation is used.

<table>
<thead>
<tr>
<th>f / kHz</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>S1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.04</td>
<td>0.02</td>
<td>0.13</td>
<td>0.01</td>
<td>0.05</td>
<td>0.12</td>
<td>0.18</td>
<td>0.026</td>
</tr>
<tr>
<td>30</td>
<td>0.10</td>
<td>0.10</td>
<td>0.28</td>
<td>0.01</td>
<td>0.19</td>
<td>0.20</td>
<td>0.09</td>
<td>0.05</td>
</tr>
<tr>
<td>60</td>
<td>0.23</td>
<td>0.07</td>
<td>0.12</td>
<td>0.03</td>
<td>2</td>
<td>0.3</td>
<td>0.15</td>
<td>0.07</td>
</tr>
<tr>
<td>100</td>
<td>0.27</td>
<td>0.3</td>
<td>0.6</td>
<td>0.03</td>
<td>2</td>
<td>0.3</td>
<td>0.12</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 7.13. Ratio of experimental direct to reverberant field power at maximum transducer separation, $R_{EdrPowerMax}$

<table>
<thead>
<tr>
<th>f / kHz</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>S1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.003</td>
</tr>
<tr>
<td>30</td>
<td>0.02</td>
<td>0.02</td>
<td>0.07</td>
<td>0.01</td>
<td>0.03</td>
<td>0.04</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>60</td>
<td>0.09</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>10</td>
<td>0.1</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>100</td>
<td>0.08</td>
<td>0.1</td>
<td>0.2</td>
<td>0.01</td>
<td>10</td>
<td>0.4</td>
<td>0.07</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 7.13 shows the experimental power ratio for the maximum position where generally the ratio increases with frequency. These ratios show that the reverberant field power is far greater than the direct field power, except for tank N1 (60kHz and 100kHz), as was the case for the pressure maximum ratio.

7.7.9. Ratio of the theoretical direct field pressure to the reverberant field pressure at the minimum transducer separation.

The results in this section are similar to those in section 7.7.5 except the pressures at the minimum experimental separation have been theoretically calculated using the theory of acoustic fields and the reverberation time of the tank at the frequency in
question. This ratio was calculated using Equation 6.49, and gives an indication of how well the theory agrees with experiment. The degree of theoretical and experimental agreement is shown in the next section (7.7.10).

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>S1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.36</td>
<td>1.0</td>
<td>2.37</td>
<td>0.208</td>
<td>1.52</td>
<td>4.4</td>
<td>2.3</td>
<td>3.0</td>
</tr>
<tr>
<td>30</td>
<td>1.39</td>
<td>1.04</td>
<td>2.25</td>
<td>0.185</td>
<td>1.52</td>
<td>5.3</td>
<td>2.3</td>
<td>3.2</td>
</tr>
<tr>
<td>60</td>
<td>1.8</td>
<td>1.24</td>
<td>2.43</td>
<td>0.191</td>
<td>1.52</td>
<td>7.3</td>
<td>2.3</td>
<td>3.6</td>
</tr>
<tr>
<td>100</td>
<td>1.86</td>
<td>1.32</td>
<td>2.80</td>
<td>0.184</td>
<td>1.52</td>
<td>6.5</td>
<td>2.3</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Uncertainty in the ratio of theoretical direct to reverberant field pressure at minimum transducer separation, \( \Delta R_{tdrPressureMin} \)

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>S1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.06</td>
<td>0.1</td>
<td>0.08</td>
<td>0.009</td>
<td>0.07</td>
<td>0.9</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>30</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.005</td>
<td>0.07</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>60</td>
<td>0.1</td>
<td>0.04</td>
<td>0.04</td>
<td>0.003</td>
<td>0.07</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>100</td>
<td>0.06</td>
<td>0.03</td>
<td>0.07</td>
<td>0.003</td>
<td>0.07</td>
<td>0.5</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 7.14. Ratio of theoretical direct field pressure to the reverberant field pressure when measured at the minimum transducer separation used.

Table 7.14 shows that theoretical pressure ratio at the minimum position generally increases with frequency, just as for the experimental case. The cause of the ratio sometimes decreasing with frequency is due to the reverberation time occasionally decreasing with frequency. From Equation 6.49 it can be seen that the only change in the ratio with frequency, will be caused by change in the reverberation time with frequency. The change in the value of the speed of sound with frequency, over this range, is negligible. The theoretical ratios indicate that the direct field pressure is greater than the reverberant pressure for all the tanks except tank B4, the glass tanks, where the reverse is true. The uncertainty in the ratio was derived using Equation 6.49 and the uncertainties in the input variables. The input variable with easily the largest uncertainty is the reverberation time, where the uncertainty is the standard error derived from the least squares fit of the decay curve.
7.7.10. Comparison of the theoretical and experimental ratios of the direct field pressure to the reverberant field pressure.

In order to compare the results in sections 7.7.5 and 7.7.9 the ratio of the theoretical to experimental ratios of the direct field pressure to reverberant field pressure were calculated (for the minimum transducer separation). This ratio demonstrates the difference between the theoretical pressure minimum position ratio in section 7.7.9 and the experimental equivalent pressure minimum position ratio in section 7.7.5. This therefore demonstrates the how good the agreement is between theory and experiment. This was simply calculated by dividing the one result by the other.

<table>
<thead>
<tr>
<th>f / kHz</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>N1</th>
<th>N2</th>
<th>N3</th>
<th>S1</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.15</td>
<td>1.45</td>
<td>1.17</td>
<td>0.93</td>
<td>1.26</td>
<td>1.34</td>
<td>0.48</td>
<td>1.63</td>
</tr>
<tr>
<td>30</td>
<td>0.73</td>
<td>0.57</td>
<td>0.76</td>
<td>0.74</td>
<td>0.64</td>
<td>1.27</td>
<td>0.66</td>
<td>1.22</td>
</tr>
<tr>
<td>60</td>
<td>0.64</td>
<td>0.81</td>
<td>1.27</td>
<td>0.36</td>
<td>0.20</td>
<td>1.45</td>
<td>0.51</td>
<td>1.18</td>
</tr>
<tr>
<td>100</td>
<td>0.60</td>
<td>0.39</td>
<td>0.66</td>
<td>0.36</td>
<td>0.20</td>
<td>1.20</td>
<td>0.59</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Table 7.15. Ratio of the theoretical to experimental, ratio of direct field pressure to the reverberant field pressure when measured at the minimum transducer separation used.

Table 7.15 shows the ratio of the theoretical to experimental pressure ratio for the minimum position. It shows values between 0.2 and 1.45 indicating broad agreement between theory and experiment, with the fluctuations about unity being due to the theory not accounting for the fluctuations in the reverberant field. There also appears to be a slight trend where at low frequency the ratio is higher than one and at higher frequencies the ratio is less than one.
7.8. Comparison of reverberant calibration results with tank sound field properties

A comparison needs to be made between the reverberant calibration results and the tank sound field properties so that it can be determined under what conditions the reverberant calibration methods are viable. Before this can be done the effect of projector directionality on the reverberant calibration mean difference results needs to be calculated. The mean difference results shown in Figure 7.34, obtained by averaging all the calibrations over a given tank-projector combination, are the results before any directionality effect is taken into account. A restatement of these results is shown for comparison in Table 7.16, under the columns for original mean difference results.

The average amplitude response of each projector, for different planes or combination of planes through the transducer, at various frequencies was shown in Table 7.5. It was decided to use the single plane results, and not the combined X-Y and X-Z plane results, because the single plane results covered the frequency range required for each projector; whereas the combined plane results only covered a smaller range for projectors such as P3. Also it was not known how transferable the responses were between two transducers of the same type. The average over the whole frequency range in question was needed so that the effect of the projector directionality could be applied to the y-intercept and spatially averaged pressure mean difference results. This average was calculated by taking the mean of the average amplitude responses known over the frequency range. These results are shown in Table 7.16, under the column "directivity / dB".

The effect of average directionality over frequency on the mean difference results is shown in Table 7.16, under the columns for directionality effect on mean difference results. This is calculated using Equation 6.31, as shown in section 6.15, and is only applied to the y-intercept and spatially averaged pressure results. Table 7.16 also shows the uncertainty (standard deviation) in the results, where the original mean difference results uncertainties are again shown in Figure 7.34. The directionality result is taken as having no error so the effect of directionality of the mean difference results is nil, and so the uncertainties in these new results is the same as for the old.
### Table of Mean Difference Results showing the Affect of Projector Directionality

<table>
<thead>
<tr>
<th>Tank-Projector Number</th>
<th>Tank</th>
<th>Projector</th>
<th>Original Mean Difference Results</th>
<th>Directionality / dB</th>
<th>Directionally Compensated Mean Difference Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>G / dB</td>
<td>Y / dB</td>
<td>S / dB</td>
</tr>
<tr>
<td>1</td>
<td>B1</td>
<td>P3</td>
<td>0.9</td>
<td>-3</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>B2</td>
<td>P3</td>
<td>0.8</td>
<td>-5</td>
<td>0.7</td>
</tr>
<tr>
<td>3</td>
<td>B3</td>
<td>P3</td>
<td>0.8</td>
<td>-2</td>
<td>6.3</td>
</tr>
<tr>
<td>4</td>
<td>B4</td>
<td>T1</td>
<td>6</td>
<td>0.4</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>N1</td>
<td>P1</td>
<td>-1</td>
<td>-4</td>
<td>2.3</td>
</tr>
<tr>
<td>6</td>
<td>N1</td>
<td>P2</td>
<td>-2</td>
<td>-7</td>
<td>0.8</td>
</tr>
<tr>
<td>7</td>
<td>N2</td>
<td>P3</td>
<td>-1.2</td>
<td>1.6</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>N3</td>
<td>P3</td>
<td>3.0</td>
<td>-3</td>
<td>6.0</td>
</tr>
<tr>
<td>9</td>
<td>N3</td>
<td>P1</td>
<td>0.5</td>
<td>-1.8</td>
<td>3.0</td>
</tr>
<tr>
<td>10</td>
<td>S1</td>
<td>P3</td>
<td>-0.7</td>
<td>0.4</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uncertainty (standard deviation) in the Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>G / dB</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

Table 7.16. Table displaying tank-projector number, tank, projector, (original) mean difference results averaged over all the TVR measurements for a tank-projector combination, directionality effect of the projector, and the mean difference results recalculated for the directionality effect on the $Y$ and $S$ results. $G$ represents the gradient, $Y$ represents the y-intercept and $S$ represents the spatially averaged pressure results. The uncertainty in the mean difference results is the standard deviation.
In Table 7.16 the gradient results are just shown for comparison and are not affected by the directionality of the transducer and so the same results appear in the original and directionally affected mean difference results. For the y-intercept results, tank B1 and B2 measurements are made slightly worst by compensating for the directionality, and there is no affect on tank B3 result. Tanks B4, N1 and N2 results are slightly improved, whereas for tank N3 the P3 results remains unchanged and the P1 result is improved. The tank S1 result is made slightly worse, but given the single calibration values for these measurements the result is not that accurate. For the spatially averaged pressure results tanks B1, B2, B3 and B4 results are improved slightly, but tank N1 results are made worse. Tank N2 result is very slightly improved and tank N3, projector P3, is slightly improved. Tank N3, projector P1, is made worse and tank S1 is made slightly better.

Most of the gradient result errors are 1dB or less, with two being 2dB. This means the gradient results could all be very close to zero, except tank B4, tank N1 (projector P2), tank N2 and tank N3 (projector P3). Of these only tank B4 and tank N3 (projector P3) is significantly greater than zero. The y-intercept errors are significantly larger than the gradient errors and range from 0.5 to 4dB. The y-intercept results could therefore be far closer to zero, with tanks B2, N1 (both projectors) and tank N3 (both projectors) not very close to zero. Of these only tank B2 and tank N1 (projector P2) are significantly far from zero. However, for the spatially averaged results only tanks B2, B4 and N1 (projector P2) could be close to zero, all the rest are significantly greater than zero.

The overall affect of the directionality is small compared to the size of the Y and S results. The error in the Y results stems from the directionality effect and the low TVR values at some frequencies when the output power is very low. This second effect seems to have a far greater affect than the influence of directionality. The errors in the spatially averaged values are due to the directionality effect and the bias due to the direct field. Again the directionality effect seems small compared with the other error, the bias. Given that the directionality compensation made as many y-intercept results worse as it improved, then the accuracy of the compensation is probably questionable. However, the spatially averaged pressure results are mostly improved. The y-intercept values seemed to be more vulnerable to errors, than the
spatially averaged pressure results, probably due to the greater inaccuracies in
determining the gradient (the intrinsic uncertainty in linear regression fit causing the
uncertainty in the y-intercept value). The errors in the directionality value are due to
insufficient frequency and spatial resolution in the measurement of the projectors.
Directionality plots at far closer frequency intervals are required as well as smaller
angular spacing and measurements over the whole of the projector sphere, not just in
two planes. It was impractical to improve on these measurements given the time
available at the end of the project and the manual facilities (no automated calibration
rig) and lack of X-Z facilities at Bath.

To compare these transducer directionality effects on the mean difference results,
details of the acoustic fields in the tanks need to be known. It was decided to look at
how diffuse the field would be theoretically, and experimentally and theoretically
determine the balance of the direct and reverberant fields. If the reverberant field is
diffuse then taking the mean of the reverberant field pressure at a number of
positions will produce an accurate value of the spatially averaged field pressure.
However, if the field is not diffuse then this will probably produce a bias since the
field sampled will probably not be representative of the average. For a non-diffuse
field the whole field would need to be sampled to obtain an accurate result. This
method therefore requires a fairly diffuse field since only part of the field is sampled.

The ratio of the direct to reverberant field is needed for comparison since it has been
shown that this affects the accuracy of the results. This is to be expected since the
relative levels of the direct and reverberant fields affect the accuracy to which the
gradient and y-intercept of the pressure squared versus the reciprocal of separation
squared graph can be determined. If the reverberant field is too large or fluctuates
too much then the direct field will be swamped by it. In the graph the gradient will
not be able to be extracted from combined field signal. The inaccuracy of
determining the gradient can cause large errors in the value of the y-intercept
calculated. This ratio also indicates if the spatially averaged pressure results are
accurate since it needs the direct field to be small.

So the gradient and y-intercept method require that the reverberant field is not too
large so that the direct field can be accurately determined, and that the reverberant
field is diffuse. The spatially averaged pressure requires that the direct field is very small, so that its contribution may be ignored, and that the reverberant field is also diffuse.

Table 7.17 shows a summary of the important reverberant field results from section 7.7. It shows the number of modes per Hz and the Schroeder frequency for the tank in question. A single value is given which is representative of the properties over the whole frequency range in question. This was calculated by taking the mean over the four frequency values shown in section 7.7. This representative value can then be used for comparison against the mean difference values in Table 7.16. The power ratio of direct to reverberant fields is shown, also averaged over the four frequencies to give a representative value. The power ratio is used since it represents the energy in the fields and is used in the equations to calculated all three reverberant calibration estimates. The experimental and theoretical ratios are shown for the minimum and maximum positions. The minimum and maximum position ratios indicate where the methods are suitable.

Table 7.17 shows that theoretically the field will be diffuse for all the tanks apart from B4. The number of modes per Hz is in the tens or higher apart from tank B4. The Schroeder frequency is well below 10kHz, the lower frequency range of reverberant calibration measurements, for all the tanks apart from tank B4 and so indicates that the field will be diffuse. Even in tank B4 the Schroeder frequency is 18kHz indicating that the majority of the field, in the tank, is diffuse. The uncertainties in the Schroeder frequency (based on standard error from reverberation time) are very small and do not alter the conclusions of where the field is diffuse.

The power ratio data indicates that at the minimum position, close to the projector, the direct field is dominant for all the tanks apart from B4 where the reverberant field is dominant. However, for the maximum position, far from the projector, the reverberant field is dominant for all the tanks except tank N1 where it is not possible to determine which is dominant due to the size of the uncertainty (based on the standard error from the gradient and y-intercept). The theoretical results agree with the trend of the experimental results but do vary significantly from them in places due to the fluctuating nature of the reverberant field. The theoretical results do not
### Table of Tank Reverberant Field Properties

<table>
<thead>
<tr>
<th>Tank</th>
<th>Tank-Projector Numbers</th>
<th>Number of modes per Hz, $\Delta N/\Delta f / \text{Hz}^2$</th>
<th>Schroeder Frequency, $f_s / \text{Hz}$</th>
<th>Experimental Power Ratio at Minimum Distance, $R_{EWR}^{min}$</th>
<th>Experimental Power Ratio at Maximum Distance, $R_{EWR}^{max}$</th>
<th>Theoretical Power Ratio at Minimum Distance, $R_{TWR}^{min}$</th>
<th>Theoretical Power Ratio at Maximum Distance, $R_{TWR}^{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>1</td>
<td>75.2</td>
<td>1,689</td>
<td>6</td>
<td>0.16</td>
<td>2.64</td>
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<td>2,291</td>
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<td>1.35</td>
<td>0.040</td>
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<td>0.28</td>
<td>6.1</td>
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<tr>
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#### Uncertainty in the Results

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<tr>
<th>Tank</th>
<th>Tank-Projector Numbers</th>
<th>Uncertainty in $R_{EWR}^{min}$</th>
<th>Uncertainty in $R_{EWR}^{max}$</th>
<th>Uncertainty in $R_{TWR}^{min}$</th>
<th>Uncertainty in $R_{TWR}^{max}$</th>
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</thead>
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<tr>
<td>B1</td>
<td>1</td>
<td>0.3</td>
<td>5</td>
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<td>0.03</td>
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<tr>
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</tr>
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<td>B3</td>
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<td>1</td>
<td>3</td>
<td>1</td>
<td>0.007</td>
</tr>
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<td>3</td>
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<td>0.09</td>
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</table>

Table 7.17. Table displaying tank against tank-projector number, Number of modes per Hz, Schroeder frequency, and the experimental and theoretical direct to reverberant field power ratios at the minimum and maximum positions. The uncertainty in the results is the standard error.
take into account the modal nature of the reverberant field, just the absorption of the tank, hence the discrepancy in the results. The theoretical results also indicate that the direct field is dominant for the minimum position, except tank B4 where the reverberant field is dominant. The theoretical maximum position also says that the reverberant field is dominant for all the tanks.

The errors in experimental power ratios are small, with the exception of tank N1, and so not significantly alter the results, only slightly reduce the size of the ratios bring them into closer agreement with the theoretical values. The larger errors for the experimental results are due to the fluctuations in the reverberant field. As mentioned the exception to this is the tank N1 which has a very large error compared to the ratio. This is due to the errors in determining the gradient and y-intercept results, which intern are probably due to very large fluctuations in the reverberant field and therefore due to strong resonance's and beat frequencies. The errors in the theoretical power ratios are very small and are therefore not significant. The errors are due to uncertainties in the reverberation time and volume of the tank.

The comparison between the directionally compensated mean difference results and the tank sound field properties is now made. The important results for this comparison are shown in Table 7.18, which shows the directionally compensated mean difference results against the Schroeder frequency and experimental and theoretical power ratios at the minimum positions. The Schroeder frequency indicates if the reverberant field is diffuse and the power ratios indicate if the direct or reverberant field dominates. The minimum positions are shown for the ratios since most of the results are disproportionately taken closer to the projector and the reverberant field usually dominates far from the projector. The uncertainty in the $G$, $Y$ and $S$ results is the standard deviation and the uncertainty in the Schroeder frequency and theoretical power ratio is the standard error derived from the least squares fit of the decay curve (reverberation time). The uncertainty in the experimental power ratio is the standard error derived from the least squares fit of the pressure squared versus the reciprocal of separation squared graph.

The Schroeder frequency indicates that the fields are diffuse for all the tanks except B4, and it is diffuse for the majority of that. The reverberant field need to be at least
Comparison Table of Directionally Compensated Mean Difference Results and Tank Field Properties

<table>
<thead>
<tr>
<th>Tank-Projector Number</th>
<th>Tank</th>
<th>Projector</th>
<th>Directionally Compensated Mean Difference Results</th>
<th>Schroeder Frequency, $f_c$ / Hz</th>
<th>Experimental Power Ratio at Minimum Distance, $R_{EWdWrmin}$</th>
<th>Theoretical Power Ratio at Minimum Distance, $R_{TWdWrmin}$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$G$ / dB</td>
<td>$Y$ / dB</td>
<td>$S$ / dB</td>
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</tr>
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<tr>
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<td>P1</td>
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<td>1,907</td>
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<tr>
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<td>P2</td>
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**Uncertainty in the Results**

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<th></th>
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<th></th>
<th>$G$ / dB</th>
<th>$Y$ / dB</th>
<th>$S$ / dB</th>
<th>$f_c$ / Hz</th>
<th>$R_{EWdWrmin}$</th>
<th>$R_{TWdWrmin}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>N1</td>
<td>P2</td>
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<td>4</td>
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<td>5</td>
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<td>P3</td>
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<td>1</td>
<td>6</td>
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<tr>
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<td>P3</td>
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<td>3</td>
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<td>N3</td>
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<td>0.5</td>
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<td>3</td>
<td>1</td>
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</tr>
<tr>
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<td>S1</td>
<td>P3</td>
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<td>2</td>
<td>2</td>
<td>3</td>
<td>8</td>
<td>2</td>
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</table>

Table 7.18. Table displaying tank-projector number, tank, projector, directionality compensated mean difference results, Schroeder frequency, experimental and theoretical ratio for the direct to reverberant power fields at the minimum position. $G$ represents the gradient, $Y$ represents the y-intercept and $S$ represents the spatially averaged pressure results. The uncertainty in the mean difference results is the standard deviation and the uncertainty in the Schroeder frequency and experimental and theoretical power ratios is the standard error.
partially diffuse for these methods to work since only part of the field is sampled. If the reverberant field is not diffuse then to obtain an accurate estimate of the average reverberant field level the whole field needs to be sampled. For a completely diffuse field any small region will give an accurate estimate of the average reverberant field. Therefore to obtain an accurate estimate of the average reverberant field from the measurements taken (medium size proportion of the field) then the reverberant field needs to be at least partially diffuse.

The gradient results for the large Bath tanks are close to zero and are therefore in good agreement with reference calibration of the projector and indicates that the direct field is dominant. The power ratios show that the direct field is dominant for these tanks, and is therefore in good agreement with the gradient results. If the gradient results are well determined, due to the direct field being dominant, then the y-intercept results would expect to be reasonably well determined. However, they are not due to the low level of the signal at higher frequencies and the greater error associated with the y-intercept. The y-intercept is more susceptible to errors because points far from the origin can disproportionately affect the y-intercept. The SAP results are high due to the dominant direct field, with the size of the acoustic field power ratio seeming to be related to the size of the SAP value. Tank B2 has both the lowest ratio and SAP value of the three. Next the tank B1 has a medium ratio and SAP value and tank B3 has the highest ratio and SAP result.

The power ratios show that the reverberant field is dominant for the tank B4, as expected from the low absorption due to material and size. The gradient result for tank B4 is very high and is due to the direct field not being determined at all. The gradient could not be measured due to the size and fluctuations of the reverberant field which caused the gradient to be both positive and negative in equal proportion. As explained earlier the negative gradients are turned into positive values and lead to fairly meaningless results if significant gradient points are negative. The results for the other tanks are valid since only one negative frequency point was found in any of the other results. The y-intercept and SAP tank B4 results are a good fit, as expected since the reverberant field completely dominates.
For tank N1, projector P1, the gradient result is not too bad a fit and the SAP result is high indicating a large direct field. This is agreed with by the ratios which are both high, but the experimental ratio is very high, far higher than the theoretical, and therefore suggests that a large reverberant field pressure resonance (standing wave) occurred at this position in the tank. The y-intercept result is low, due to the y-intercept being inaccurately determined. The tank N1, projector P2, result also has similar results but with less well determined gradient and y-intercept but a better SAP value.

Tank N2 has a reasonably well determined gradient and y-intercept result but a very poor SAP result. This is to be expected since the ratio, and small absorption, indicates that the direct field is dominant. The more sensible y-intercept result indicates that the fluctuations on the pressure squared versus the reciprocal of separation squared graph are fairly small, as possibly expected if the direct field is very large.

Tank N3, projector P3, has a high SAP result and a corresponding high direct field, but unexpectedly also has a high gradient result. The y-intercept result is low but since the gradient was not well determined then the y-intercept will not be either, perhaps because the fluctuates were too great, or in reality the reverberant field is not diffuse. However, tank N3 projector P1, has the gradient and y-intercept results close to zero and with the SAP result high, which is what would be expected from a dominant direct field.

Finally tank S1 has a very high ratio and therefore direct field, and is in agreement with the gradient and y-intercept results being close to zero and the SAP result being high.

Now that the results have been presented and discussed at some length an overview and discussion will be presented in Chapter 8.
8.0. Discussion

This chapter comprises four sections with discussions on the literature review, reverberation time, the reverberant calibration method and results and further work.

8.1. The literature review

The direct field signal from a projector is required to perform a calibration since only the direct signal and no reflected signals are required. Traditionally there are two ways of achieving this, these are to use a large body of water for the calibration, such as a lake or the sea, or to pulse and time gate the signals. Where there is insufficient free-time within the body of water two more approaches are also possible. The first of these is to obtain the direct path signal with only a few reflections present, using either the Prony approach to extrapolate the steady state signal or the transient suppression method to produce a steady state signal. The second category is where the number of reflections is greater than a few, and then a continuous wave signal needs to be used to produce a reverberant standing wave pressure field. There are many methods of extracting the direct path signal from the combined reverberant and direct fields, including the methods presented in this thesis.

The reverberant field level changes with position throughout the volume of the tank. The pressure experienced by the projector alters its damping term, which therefore alters its response, and so therefore the power radiated into the tank changes with position. If the source impedance is large compared to the sound field impedance, the source impedance change is very small with changing pressure and so the power output change is also very small.

If the low frequency reverberant field power is averaged over most of the tank, it is found that the average power is lower than the free-field value. However this does not usually include positions near the boundary of the tank. The power at the boundaries is higher than elsewhere, and if the power is average over the whole of the tank, including near the walls, then the average power equals the free-field value.
This difference between regions is due to insufficient modal overlap, since the equations for the averaged reverberant field pressure are based on statistical considerations. For high frequencies there is sufficient modal overlap and no discrepancy between the two regions.

In conclusion there are two aspects of acoustic power measurements that need to be considered. The first is that the pressure measurements need to be averaged over the whole tank, including the boundary regions at low frequencies. The second is that the power determined by measuring the spatially averaged pressure will vary with projector position in the tank, unless the impedance of the projector is large compared to the impedance of the reverberant field. These aspects need to be considered, to see if they have a significant effect, when making reverberant field calibrations in a tank.

In order to obtain an accurate y-intercept and SAP calibration, the reverberant field needs to be diffuse and relatively smooth. Such a reverberant field enables a more accurate determination of the acoustic power radiated into the tank and therefore a more accurate calibration of the projector from the y-intercept and SAP methods. There are a number of criterion that can be used to indicate, and measurements that can be taken to determine, whether this is the case.

The number of tank modes in a given frequency band gives an indication of how diffuse or smooth the reverberant field will be. The approximate number of modes in a given frequency band is proportional to the volume of water in the tank multiplied by the centre frequency of the band squared, multiplied by the frequency bandwidth and divided by the speed of sound in water cubed. Therefore the larger the volume of water, centre frequency of the band and bandwidth, the larger the number of modes, and the more diffuse and smooth the reverberant field will be.

One method of measuring whether a sound field is diffuse is to determine the cross-correlation coefficient for the sound pressure at two different points in the sound field, obtained at the same time. This indicates whether the field is correlated between these two points and therefore if the field is random and therefore diffuse.
Another method for determining if a sound field is diffuse is to calculate the Schroeder frequency. The low-frequency limit for statistical behaviour of sound in rectangular enclosures, has been postulated by Schroeder, to be when the average spacing of the modes of a tank is less than one-third of their bandwidths. The frequency at which this occurs is called the Schroeder frequency, and is proportional to the square root of the following fraction: the 60dB reverberation time divided by the volume of water.

One definition of a diffuse sound field is that sound is equally likely to travel in any direction, with the energy density being the same throughout the field. This leads to the concept of a sound field in an unbounded medium generated by distant, uncorrelated sources of random noise evenly distributed over all directions. Since the sources are uncorrelated there would be no interference and the sound field would be completely homogeneous and isotropic. This would be an idealised perfectly diffuse sound field.

A more realistic model of a diffuse reverberant sound field above the Schroeder frequency is a sound field made up of plane waves with random phases arriving from all directions. This model is for a pure tone and so the various plane waves interfere. The sound pressure level in such a field depends on position, with the probability of the level being in a certain interval being the same at all positions. This model assumes an infinite number of plane waves with completely random phases and so is also an idealised model. This gives a good approximation to the sound field in a reverberant tank driven with a pure tone and with the frequency above the Schroeder frequency.

This second more realistic plane wave model can be extended by excitation with a band of noise. When the reverberant tank is driven with a pure tone, the plane waves that comprise the field have certain amplitudes and phases. If the exciting frequency is changed slightly, the amplitudes and phases of the plane waves will be changed and so the entire interference pattern will be slightly changed. Excitation by white noise generates random amplitudes and phases of the driving signal leading to new interference patterns over time. However the old interference patterns are still present and the length of time they persist depends on the absorption of the room, the
reverberation time. The longer the reverberation time the longer it takes for an old pattern to decay away. The auto-correlation coefficient of the pressure at a point in a reverberant sound field is related to the reverberation time, since the signals separated by a time greater than the reverberation time will not be correlated. Exciting a tank over a band of noise is equivalent to averaging the sound field over the whole frequency band and results in the sound field becoming more uniform.

The decay of sound in an enclosure depends on the relative proportions of axial, tangential and oblique modes. In a rectangular enclosure the reverberation times of the three different types of mode are longest for axial modes then tangential and least for oblique. The gradient of the decay curve changes over time as the different modes decay away at their different rates. At high frequencies the initial part of the decay curve is dominated by oblique modes since these modes are very abundant at these frequencies.

The decay curve is subject to irregular level fluctuations, which are superimposed on the general fall in level. The fluctuations about the general decay level can be reduced by averaging over a large number on individual decay curves, which have been obtained with random noise excitation of the tank. This is very time consuming and leads to the same result as Schroeder’s “method of integrated impulse responses”. It is based on the ensemble average of all possible decay curves, for a certain place and bandwidth of exciting noise, and the corresponding impulse response. The individual decay curves of the ensemble average are produced by random starting conditions of the exciting signal. Each bandwidth of exciting noise is random and so the different starting conditions lead to different decay curves for a given position. The ensemble average gives the intrinsic decay curve for a given set of source and receiver positions. The ensemble average decay curve changes with position unless the sound field is completely diffuse.

The individual decay curves have large fluctuations superimposed on the general decay curve. Averaging many decay curves, or using “the integrated impulse response method”, produces an intrinsic smooth decay curve. This intrinsic decay curve does change with different source and receiver positions.
8.2. Reverberation Time

Accurate reverberation time measurements were needed so the calibrations based on the y-intercept and spatially averaged pressure results could be made. The tank reverberation time responses were determined using two methods and compared and averaged. The first method was a tone-burst single shot, where a ten cycle tone burst signal was pulsed at a repetition frequency of 10Hz. Each decay curve was then analysed to calculate its reverberation time. The tone-burst signal frequency ranged from 10kHz to 100kHz in steps of 10kHz. The second method was a noise burst single shot, where a noise burst of length 1.0ms was pulsed at 10Hz. The noise burst contained frequencies from 1kHz to 100kHz. The recorded pulse was transferred to the frequency domain and then split up into various frequency bands and then each band converted back into the time domain, where the reverberation time was determined. The frequency bands had a width of 10kHz and progressed from one band to the next over the range from 10kHz to 100kHz. Both methods had decay curves over the same approximate frequency ranges. Measurements were made at five different positions within a tank for each method. The spatial average for each method was then calculated. The spatial average for the two methods were similar for each tank, and the two results were then averaged together to produce a mean value.

From the literature review it can be seen that either method will excite one possible decay curve from the set of all possible decay curves, between two points, which could produce an intrinsic ensemble average decay curve. The decay curve excited depends on the starting conditions. Averaging the decay curves over several positions will give a better estimate of the global tank decay curve. The tone-burst signals will produce similar starting conditions which will give rise to a small bias in the curve, although averaging over several positions will give a reasonable representative value of the tank decay curve. The noise-burst signals have random starting conditions and so averaging over several positions should give rise to a more accurate result. The tone-burst results are still valid since they give similar results to the noise-burst signals, and the two results agree with each other within their standard errors.
The random errors in the reverberation time results range from 0.1 dB (1% error) to 2.3 dB (30% error), but the vast majority of the results are within 0.2 dB (2% error) and 1.0 dB (12% error). If the tank N2 results are ignored, the upper limit goes down to 0.7 dB (8% error) with only the very low frequency points of tanks S1 and B2 above this. These results indicate the extreme limits of the reverberation time values versus frequency, with the majority of results less than the upper limit.

These reverberation time uncertainties will cause errors in the reverberant field calibration results. They will affect both the y-intercept and spatially averaged pressure results to the same degree, altering the calibrations to the same extent as the reverberation time error. How significant these errors are will be discussed in section 8.3.

### 8.3. Reverberant calibration method and results

The reverberant calibration method is based on measuring the sound field spectrum excited by random noise produced from the test projector. The spectrum captured is very noisy since it is produced by noise and is a combination of the direct and reverberant fields. However, one of the key features of this method is that the direct field spectrum can be obtained to a reasonable degree of accuracy by averaging many sampled spectra together. This is the same situation as for a signal contaminated by noise which can be extracted from the noise by averaging over many pulses. The direct field spectrum, in each individually sampled signal, is the result of the projector response to the random excitation. When these random direct field spectra are averaged, they produce a good approximation of the response of the projector. However, the captured signals also contain the reverberant field. Since the reverberant field spectrum is also random, since the exciting signal is random, the true reverberant field spectrum is revealed in the same way as the direct field. The averaged spectrum shows the direct field and the time averaged reverberant field. The increase in the signal to noise ratio with increasing number of samples is the same for both the direct and reverberant fields.

The spectra obtained from the vector analyser had a better signal to noise ratio than those calculated from the oscilloscope traces because of the time the sample was
captured over. The 50 vector analyser spectra were captured over 30s whereas the 50 oscilloscope traces were all recorded over 100ms. The signal to noise ratio difference is a result of the reverberation time of the tank. As time passes a new random signal is emitted from the projector and sets up a new reverberant field pattern. However, the reverberant field pattern persists in the tank until the sound has been completely absorbed. This is very different to the direct field where the signal is constantly changing, and has no persistence and is not correlated. The reverberant field could be said to persist in the tank, constantly decreasing in amplitude, until such a value where it is so small it is of no consequence. This will be taken for arguments sake as -60dB, so that each new field persists for as long as the reverberation time at that frequency. So the reverberant field sampled at an interval less than the reverberation time can be considered to be partially correlated. Therefore spectra that were captured during this reverberation time will be partially correlated and so the reduction in the noise level will be less than the square root of the number of samples. The reverberation time of the eight tanks used in these experiments ranged from 48ms to 186ms, so the oscilloscope data could be considered to be partially correlated and the vector analyser data to not be correlated. The oscilloscope data will therefore have a lower signal to noise ratio than the vector analyser data and, therefore, result in a less accurate estimate of the projector calibration.

For each frequency band the graph of pressure squared versus the reciprocal of separation squared was plotted and the gradient and y-intercept calculated using a least squares fit routine. From the gradient the acoustic power was calculated and from that the calibration of the projector. From the y-intercept and the reverberation time of the tank the acoustic power was calculated and again the projector calibration found. The third method was to use the spatially averaged pressure from the pressure spectra readings and the reverberation time to calculate the projector calibration. This also gave an estimate of acoustic power similar to the y-intercept, since they are both estimates of the reverberant field pressure squared.

The y-intercept and spatially averaged pressure estimates of the projector calibration require the reverberation time of the tank. Therefore the error in the reverberation time will contribute to the overall error in the estimates of these projector
calibrations. In section 8.2 it was quoted that the reverberation time random error ranged up to approximately 1dB for the vast majority of the results. This error is not insignificant and its contribution to the overall calibration error varies from being small to medium depending on the tanks and estimate methods used.

For comparison the reverberation time error and the reverberant calibration error are now given. The reverberation time standard error (varies with frequency), for the tanks are: B1 (0.3dB to 0.7dB), B2 (0.2dB to 1.1dB), B3 (0.2dB to 0.5dB), B4 (0.1dB to 0.6dB), N1 (0.4dB), N2 (0.3dB to 2.2dB), N3 (0.4dB) and S1 (0.2dB to 1.3dB). The reverberant calibration errors, vary over the different tanks, and are given for the three different types of reverberant calibration result. The figures are taken from Table 7.8, where the standard deviation in the mean difference results are: G (0.3dB to 2dB), Y (0.5dB to 4dB) and S (0.3dB to 2dB).

The relative levels of the direct and reverberant fields will affect the accuracy of the gradient, y-intercept and spatially averaged pressure estimates of the projector calibration. The degree of fluctuations in the reverberant field will also affect the accuracy of the gradient and y-intercept results, but not the spatially averaged pressure results provided an evenly distributed sample of the field is used. The gradient and y-intercept results require that the direct field be measurable in the combined field. This usually requires the direct field to be larger than the reverberant field for a good proportion of the scan. However if the reverberant field is sufficiently uniform then the direct field could be a small proportion of the reverberant field. Conversely if the fluctuations were very large then the direct field would have to be very large compared to the reverberant field. The fluctuations alter the best fit line in the pressure squared versus the reciprocal of separation squared graph. The larger the fluctuations the greater the error in the gradient and y-intercept.

The spatially averaged pressure measurement requires that the reverberant field is much greater than the direct field. The ratio of the direct field amplitude to the reverberant field amplitude enables the systematic error in the spatially averaged pressure value due to the direct field to be estimated. If the reverberant field is sampled evenly throughout the tank, including near the tank walls then the
fluctuations will cause a very small error since the field is evenly sampled spatially. If the field is not well sampled then a bias will occur in the result. Also with fewer samples a greater error will be produced by a field with large fluctuations, since one position could disproportionately alter the result due to a very high or low field level. With few sample positions any one position represents a large proportion of the tank area, therefore leading to errors. Hence the pressure squared versus the reciprocal of separation squared graph (power extraction graph) method requires a large direct to reverberant field ratio, and the spatially averaged pressure method requires a small ratio.

The reverberant field calibration measurements were taken in several groups for each tank. Several runs were made for each group, and the different groups included variations in the locations and orientation of the scan within the tank. It was found that measurements within a particular group did not vary much, giving fairly consistent gradient, y-intercept and spatially averaged pressure results. Results of many different groups for a particular tank and projector varied to a greater extent, generally from 1dB to 2dB. When averaged the results were fairly consistent for each $G$, $Y$ and $S$ result. The results did vary significantly from tank to tank and using different projectors.

The gradient results required that the direct field could be measured in the combined field. To do this the gradient had to be accurately determined from the pressure squared versus the reciprocal of separation squared graph. Likewise the y-intercept results needed to be accurately determined from this graph. To do this the direct field needed to be greater, or a significant proportion, of the reverberant field. Also the degree of fluctuations on the reverberant field affected the accuracy to which the gradient and y-intercept were determined. The y-intercept was more susceptible to error since points far from the origin disproportionately affect the value of the y-intercept, whereas each point was equally weighted for the gradient value. The spatially averaged pressure result required only the reverberant field and so the presence of the direct field caused a bias in the results. The SAP results required the direct field to be small compared to the reverberant field so that its value was not increased.
The y-intercept and SAP were both measuring the reverberant field and were both susceptible to the same uncertainties. These were that the reverberant field be diffuse and that the projector directionality be uniform. The reverberant field needs to be diffuse since only part of the field is sampled and so the energy density needs to be fairly uniform throughout the field. If it is not then the sampled spatially averaged pressure will not be the true reverberant field pressure for the tank. If the field is not diffuse then the whole tank needs to be sampled. The reverberant field level is determined by the absorption of the tank and the direct sound field radiated from the projector. However, the direct field level can vary with orientation due to projector directivity. The reverberant field is due to the sound radiated from the whole surface area of the projector and is, therefore, due to the average response over this area. If the projector response is not uniform with direction then a bias will occur in the y-intercept and SAP results. This can be compensated for, by measuring the directional response over all orientations and then taking the mean, where the reference calibration direction pressure amplitude is defined as unity. This value in dBs can then be subtracted off the measured value to give the compensation for TVR. This should compensate for the directionality effect for both results, however the two results are susceptible to errors of their own. Before this is discussed, the absorption of the tank needs to be mentioned. This is responsible for the level of the reverberant field along with the direct field power radiated. The absorption can be expressed in terms of the reverberation time of the tank and the volume of water in the tank. The error in the reverberation time can significantly affect the Y and S results if it is not accurately determined.

The other y-intercept error occurs when the TVR is low over a range of frequencies, which causes the ymd result to go low. This was observed when the TVR of the projector in question (most notably P3) rolled off (decreased) away from resonance. Consequently the power radiated at these frequencies was very low thus resulting in small signals compared to signals around the projector resonance. This in turn means that these low amplitude signals are only digitised by a small number of bits and therefore possibly cause a significant error in the measurement of the signal. As mentioned above the points far from the origin on the pressure squared versus the reciprocal of separation graph, have a disproportionate weight in determining the y-intercept value. This is because although points close to and far from the origin can
move the best fit line by the same amount, the points far from the origin have a
magnifying lever effect on the y-intercept. The points close to the origin affect the
y-intercept value to a far lesser extent. As far as the gradient is concerned, all the
points have equal weighting. If the digitising error is significant it could therefore
cause an error in the y-intercept. The reason the y-intercept goes low and not high is
because the y-axis is pressure squared. Assuming there are equal positive and
negative digitising errors in the pressure, then when squared, the positive errors
become larger than the negative errors. Since the positive errors are larger than the
negative errors, around the best-fit line, they therefore cause the y-intercept to go
lower.

This argument could also be applied to large amplitude fluctuations since they will
also cause the y-intercept to become low. However, this could not be applied to the
low power signals radiated when the TVR is low compared to the resonance of the
projector. In that situation both the direct and reverberant fields would be
intrinsically low, so the reverberant field would not be large compared to the direct
field. However, if the tank were reverberant or the reverberant field were not diffuse
then the fluctuations could be large.

Another possible cause of a low TVR value over a certain frequency range is the
directionality of the projector. If large diffraction effects occur over certain
frequencies then the directionality of the projector will vary wildly with angle. This
could occur at integer number of wavelengths around the circumference of the
projector. This means that the amplitude of the signal in the '0' mark direction may
be very different that the average over the whole surface area of the projector for a
particular frequency or range of frequencies. This could explain low y-intercept
TVR results over part of the frequency range, but it should also affect the SAP
results in the same way at the same frequencies if the reverberant field is dominant.

The other error for the SAP results is the bias caused by measuring the direct and
reverberant fields together and not just the reverberant field. For the transducers
considered here both the low y-intercept values and the SAP direct field bias values
were more significant effects than that due to the measured values of non-uniformity
in the directionality of the projectors.
The results for each tank-projector combination revealed general trends that agree with the discussion above. The gradient results were generally in best agreement with the reference calibration and the y-intercept results were low and the spatially averaged pressure results were high. These generally indicated that the direct field was dominant and therefore measured well, depending on diffuseness, producing the more accurate gradient results as well as the bias in the SAP results. The y-intercept results were probably low due to the low power at certain frequencies, due to projector roll-off.

The large Bath tanks produced good fits for the $G$ results, low fits for the $Y$ and high for the $S$ apart from tank B2 which was a good fit. This indicates a dominant direct field for tanks B1 and B3, (this was particularly true for B3, which was to be expected given the high coupling to the ground of the sunken concrete tank), and a far less dominant direct field in tank B2. The low $Y$ values probably reflected the roll-off for projector P3 over a large frequency range. The result for the small glass tank, B4, showed the dominant nature of the reverberant field, with the $G$ results being high, but meaningless, and the $Y$ and $S$ results producing a good fit. The NPL tank, N1, showed a low bias for the $G$ results and a reasonably high bias for the $S$ results and low values for the $Y$ results. The fluctuations in the results between the projectors indicated significant fluctuations in the reverberant field. The degree of discrepancy between projectors P1 and P2 in their $Y$ and $S$ results was not sufficiently accounted for by the projector directionality (as included in this work). This extra difference may be explained, along with the $G$ results, by the high level of fluctuations. The large discrepancy between the $Y$ results may additionally be explained by the large roll-off for the projector P2 response as opposed to P1, because the P2 measurements were made over a far larger frequency range than P1. The tank N2 results showed that the $G$ and $Y$ results were a better fit, but the $S$ results were very high, indicating a dominant direct field as expected since the absorption of the wooden tank is low. The $Y$ result was probably not low since the fluctuations were small because the reverberant field was small. For the tank N3 and projector P3 the $G$ result was high, possibly indicating large fluctuations or a non-diffuse field. The $S$ result was high indicating a large direct field and the $Y$ result was low, indicating low power output at the higher frequencies, possibly due to projector roll-off, and therefore digitising errors. The low $Y$ result could instead be due to large
variations in the directionality of the projector over this frequency range. The projector P1 results for tank N3, show that $G$ is a very good fit and $Y$ is slightly low and $S$ is slightly high. This indicates that the direct field is dominant, but not to the degree for the P3 measurements. The difference between the two could be due to the different frequency range or the different directionality of the projectors. The $Y$ results for projector P1 is not as low as for P3, probably since there is not a significant roll-off due to the frequency range being small. The tank S1 show that the $G$ and $Y$ results are a good match and that the $S$ value is very high. This indicates that the reverberant field is very high, but has small fluctuations explaining the good $Y$ result.

To interpret these results, the sound field the measurements were taken in needs to be considered so that any relationship between sound field type (or condition) and the calibration results can be determined. This should enable the best sound field conditions for each type of measurement to be determined. First of all, the reverberant field needs to be diffuse for all three estimates of TVR, since the whole field is not sampled. This was considered by calculating the number of wavelengths per dimension of the tank, the number of modes up to a specific frequency, the number of modes per Hz at a specific frequency and the Schroeder frequency of each tank. The Schroeder frequency is the theoretical frequency above which the reverberant field is random or diffuse. These results indicated that the fields in the tanks would be diffuse at all but the lowest frequencies, with more of the frequency range being non-diffuse for the field in tank B4. The Schroeder frequency indicated that all the tanks would be diffuse in the working range of 10kHz to 100kHz, apart from tank B4 below 20kHz. These tanks would therefore be suitable for measurements in terms of being diffuse. However if time sampled segments were taken at intervals of less than the reverberation time of the tank then the signals would be partially correlated, therefore not averaging out the fluctuations as much and so increasing the uncertainty in the final calibration result.

The $G$ and $Y$ calibration results were determined from the pressure squared versus the reciprocal of separation squared graph, where the accuracy of the determination of the gradient and y-intercept depended on the relative levels of the direct and reverberant fields as well as the degree of fluctuations. The direct field needs to be
measurable in the combined field so that the gradient and y-intercept can be
determined. This means the direct field has to be dominant or a significant
proportion of the reverberant field and the reverberant field fluctuations must not
swamp the direct field changes. However, for the situation where the direct field is
completely swamped by the reverberant field, the gradient cannot be measured but
the y-intercept can be measured providing the fluctuations average out, since the
gradient is effectively zero. The $S$ result requires that the reverberant field is
dominant to such a degree that the direct field is very small proportion of the
reverberant so that the error in $S$ is very small.

This was investigated by calculating the ratio of direct to reverberant field pressure
and power at the minimum and maximum transducer separations. These ratios were
calculated from the gradient and y-intercept values at the positions indicated. These
ratios were also calculated theoretically by using the reverberation times of the tanks
and the separation distances. The most useful ratios were the power ratios since that
is what is used in the least squares fit graph.

The results showed that there were significant variations between the experimental
and theoretical results, where the experimental ratio gave higher and lower values
that the theoretical. The differences were probably due to the fluctuations in the
experimental reverberant field; the theoretical results only modelled the average
level.

The results indicate that for the minimum position near the projector, the direct field
was dominant for all the tanks, within experimental uncertainty, except for tank B4
where the reverberant field was dominant. For the maximum position far from the
projector the reverberant field was dominant for all the tanks.

The absorption of the tank is proportional to the ratio of direct to reverberant field
power, and absorption is proportional to the surface area in contact with the water
and the absorptivity of the material. All the tanks except B4 have a medium to large
surface area in contact with the water and a medium to high absorptivity and so the
absorption of all these tanks is medium to high, enough to produce a dominant (or
certainly measurable) direct field. The glass tank, B4, has a small surface area and a
small absorptivity, which means its absorption is small and so the ratio is small and
the reverberant field is dominant (measured direct field was very small). (The
absorptivity and surface area data for the tanks is in Table 5.4, and the relevant
equations for this calculation are Equation 4.9, 5.2 and 6.48).

The uncertainty in the $G$, $Y$ and $S$ results is the standard deviation (reverberant field partially correlated). The uncertainty in the experimental and the theoretical direct to reverberant field pressure and power ratios is the standard error (from least squares fit of pressure squared versus the reciprocal of separation squared graph and the reverberation time decay graph).

From the literature review it was seen that to produce an accurate average reverberant field power the field had to be sampled at many positions throughout the field, including the high power region near the boundaries. For this result to be accurate the frequency had to be high with modes overlapping, that is above the Schroeder frequency with the field being diffuse. The source impedance also needed to be large compared to the sound field impedance for the average power to equal the free-field value. The output power of the source also varies with position and is highest near the boundaries of the enclosure and undulates about the free-field value as the source is moved to the centre of the enclosure. The average power output produces the free-field value when averaged throughout the tank, including near the boundaries. Both the source and receiver therefore need to be averaged throughout the tank, including near the walls of the tank.

Now an overview is given of the directionally compensated reverberant calibration results, compared against the acoustic field data. Generally the $G$ results were good when the direct field was dominant, this was the case for all the tanks except tank B4. For tank B4 the $G$ result was meaningless since the gradient value was negative, because the reverberant field was completely dominant. The $Y$ results were often low when there were large fluctuations compared with the direct field signal. The $Y$ results could also be low if there was a large roll-off with frequency possibly resulting in digitisation errors, or if the directionality of the projector varied significantly with angle for certain frequencies or frequency ranges. Even if there is not a large roll of with frequency or large changes in directionality the $Y$ results are
still more prone to error than the \( G \), due to the fluctuating nature of the reverberant field. The \( Y \) results appeared to be most accurate when the direct field was dominant with small fluctuations and no roll-off with frequency for the projector. The \( Y \) results also seemed to be accurate when the reverberant field was dominant (\( Y \) effectively measures \( S \)) and (then the gradient is zero and has no lever effect on the graph and therefore cannot cause greater error in the \( Y \) results or bias it negatively with roll-off). The \( S \) result was only accurate when the reverberant field was dominant, which was the case for the tank B4. Within these general trends some of the individual tank-projector combinations varied considerably. However these were generally the NPL results where limited runs were taken and so would be more susceptible to statistical fluctuations. The fluctuations within a run would be considerable but over many runs averaging reduced the spread and reduced the uncertainty of the results, as shown for the Bath tanks.

The \( G \) and \( Y \) results were susceptible to fluctuations because they rely on the direct field being extracted from the graph. The \( Y \) and \( S \) results will be inaccurate if the compensation for the directivity of the projector is wrong. This could have occurred since the directivity was not averaged over the whole of the surface area of the transducer. However, the directivity measurements taken only produced small compensations indicating that the uncertainty due to directivity is relatively small. The main source of errors for the \( Y \) results are the intrinsic increased susceptibility to error due to fluctuations, and the negative bias in the results due to large fluctuations when the direct field was small and the gradient was not zero. The main source of error in the \( S \) results was the presence of a direct field.

Although the \( G \) results rely on the direct field they are affected by the perturbations of the reverberant field and if the fluctuations do not cancel out an error occurs in the result. So the \( G, Y \) and \( S \) results are all susceptible to the reverberant field not being diffuse. For accurate measurements the projector impedance must be large compared with the sound field impedance and the reverberant field must be diffuse, above the Schroeder frequency. Both of these requirements were generally met, however, the average reverberant field power was only accurate if the field was sampled throughout the tank, including near the tank walls. This was not the case since measurements were made near the centre of the tanks, away from the walls. This
could lead to a lower power and therefore lower estimated TVR, depending on the
degree of the effect. This would only apply to the Y and S results and could possibly
explain some of the lower Y results. The S results would therefore be biased
positively (direct field) and negatively (not sampled close to wall). Furthermore the
power output of the projector depended on position within the tank, generally due to
the modal structure, and specifically higher near the walls and undulating when
approaching the centre of the tank. Since only one projector position was used for a
run this could cause a large error in the estimate of the free-field (direct field) power.
This could account for some of the variation in the Y and S (less so) results.

8.4. Further work

Now suggestions for further work will be made that concern solving the problems
associated with this reverberant calibration method.

The measurements made with the oscilloscope could be improved by sampling the
50 time segments, for a position, separately and over a larger time. If the time
between each segment is greater than the reverberation time of the tank then the 50
segments will not be correlated and so a more accurate representation of the average
field can be obtained. This is the same way as the vector analyser takes the
measurements and will improve the uncertainty in the results.

It would be useful to confirm by experimentation that the reverberant fields are
diffuse. To do this the cross-correlation coefficient needs to be determined for
several pairs of points in each tank. If the field is correlated then the reverberant
field is not diffuse and will therefore cause errors in the estimate of the TVR for all
three methods.

The error in the y-intercept results needs to be investigated to see how large an effect
the fluctuations have on the result, and to verify the suspected cause of the negative
bias in the results. That is the possible digitisation error (due to the roll-off of the
projector), the large fluctuations of the reverberant field compared to the direct field
or the possible rapid changes in directivity of the projector with frequency. Also see
if a low value occurs if a narrow frequency range is selected, and so a very small roll-off of the transducer is present.

The output pressure amplitude of each transducer needs to be measured over the whole surface area and then the mean calculated. This can then be compared to the average calculated from the X-Y and X-Z axes to see how different the results are. If there is a significant difference in the results then these new values can be applied to the mean difference results to produce accurate directionally compensated values. The average directionality could also be measured at closer frequency intervals since the directionality can change dramatically with frequency. However, this would all be very time consuming and require the development of a rig.

Since the reverberant field power increases at the boundaries of the tank (below the Schroeder frequency), for an accurate determination of the $Y$ and $S$ results the field needs to be sampled near the walls of the tank as well as the rest of the field. Therefore reverberant calibration measurements needs to be made throughout the whole of the tank, including a representative sample near the tank walls if any of the calibration frequencies are below the Schroeder frequency. The degree of increase of reverberant field power near the walls could be investigated.

One possible method of ensuring that both the source and receiver are moved over the whole of the tank area is to perform this calibration method whilst moving the projector as well as the hydrophone. This is done by increasing the separation of the transducers as required by the reciprocal separation squared distribution, but also randomly move the projector around the tank for each new measurement. This would ensure that both the projector and hydrophone have random positions throughout the field, including near the tank walls, whilst keeping the required separation. It may be necessary to rotate the transducers during this run to ensure that their calibrated directions are always facing each other. This could be performed on a rig with appropriate stages and controls.

Finally the conclusions to this work will be presented in Chapter 9.
9.0. Conclusion

The conclusions to this research on the reverberant calibration method based on the pressure squared versus the reciprocal of separation squared graph, along with spatially averaged pressure, will now be presented.

The \( G \) and \( Y \) estimates of the projector calibration are based on, respectively, the gradient and y-intercept of the above mentioned graph. The \( S \) estimates are based on the measurements taken for the \( G \) and \( Y \) results, which have been averaged to give the spatially averaged reverberant field pressure.

The uncertainty in the \( G \), \( Y \) and \( S \) results is the standard deviation (reverberant field partially correlated). The uncertainty in the experimental and the theoretical direct to reverberant field pressure and power ratios is the standard error (from least squares fit of pressure squared versus the reciprocal of separation squared graph and the reverberation time decay graph).

The results taken in the large Bath tanks show that the gradient method is accurate to better than 2dB with an uncertainty of about 1dB. However, most of the results show that \( G \) is accurate to within 1dB with an uncertainty of less than 1dB. The degree of variability depended on the level of tank fluctuations. Consistent and accurate results could be obtained by averaging the TVR from several measurement runs, which then yielded results of accuracy of less than 1dB. The y-intercept results were far less accurate with results from the three tanks ranging from \(-2\)dB to \(-6\)dB with uncertainties of \(2\)dB to \(3\)dB. The spatially averaged pressure results ranged from accurate to inaccurate with results ranging from \(0.3\)dB to \(5.9\)dB, with uncertainties of less than 1dB. These results are based on many measurement runs with the results showing a good degree of consistency for the \( G \) and \( S \) results with uncertainties of less than 1dB, but the \( Y \) results are less consistent with uncertainties of up to 3dB.

The results for tank B4 are again based on many calibration runs and show a good degree of consistency. The \( G \) results are meaningless due to the gradient values
being equally positive and negative, due to the direct field not being measurable because it was completely dominated by the reverberant field. The Y and S results are both very accurate with results of $-0.07\,\text{dB}$ and $0.2\,\text{dB}$ respectively, but with errors of $2\,\text{dB}$. This larger error than for the first three tanks is due to the results varying more because the reverberant field was so dominant. The fluctuations were larger but cancelled out over the many runs.

The tanks at NPL had very few measurements taken in them due to commercial time constrains. This is reflected in the results with far larger deviations from the true TVR calibration. The gradient result accuracies range from $-2\,\text{dB}$ to $+3\,\text{dB}$ with uncertainties of less than $2\,\text{dB}$ and less than $1\,\text{dB}$ for most of the results. The y-intercept result accuracies range from $-6\,\text{dB}$ to $1.2\,\text{dB}$ with uncertainties of $4\,\text{dB}$ or less. The SAP result accuracies range from $1.6\,\text{dB}$ to $11\,\text{dB}$, with uncertainties of $1\,\text{dB}$ or less, which reflect the accuracy of the results and the wide range of tank direct field levels relative to the reverberant field. This, in turn, reflects the wide range of absorptions of the tanks.

Only one run was made in the tank S1, similar to some of the NPL tanks, however they are accurate results despite the lack of detailed calibration curves. The errors for this tank would be considerably smaller if accurate calibration curves were available, since the results would follow the true calibration better.

The acoustic field data shows that theoretically all the tanks fields were diffuse, with the exception of the reverberant field in tank B4 for very low frequencies which could still be considered to be diffuse, over most of the frequency range. The data also showed that the direct field was dominant compared to the reverberant field near the projector, for all the tanks except tank B4. In tank B4 the reverberant field was dominant compared to the direct field close to the projector and far from it. For all the other tanks the reverberant field was dominant far from the projector. Since most of the measurements were biased towards the projector the direct field was dominant for most of the measurements for all of the tanks except tank B4, where the reverberant was dominant.
The overall results therefore show that, as measured, the gradient results are accurate to within 3dB, with an uncertainty of 1dB or less, when the direct field is dominant or a reasonable proportion of the reverberant field. The error for the tank S1 result has been ignored considering the lack of accurate calibration curves. However, for most tanks with a non-dominant reverberant field the gradient results were accurate to within 1dB and with errors of less than 1dB. Under the present arrangement the y-intercept results are not accurate with results ranging from -6dB to 1.2dB with uncertainties of 4dB or less for a dominant or non-small direct field. The spatially averaged pressure results ranged from 0.3dB to 11dB with an uncertainty of 2dB or less, for the non-dominant reverberant field. This of course would be expected if the reverberant field were not dominant. For the dominant reverberant field the gradient results are to be ignored, and the y-intercept and SAP results produce accurate values of less than 0.2dB, but with uncertainties of 2dB.

More generally the gradient method produces accurate results when the direct field is measurable in the combined field, that is the direct field being dominant to a significant proportion of the reverberant field. However, the method is useless where the reverberant field is dominant, this being the case since the method relies on the direct field which cannot be measured when the reverberant field is dominant. The y-intercept would be expected to be accurate if the direct field were dominant, but is very susceptible to reverberant field fluctuations causing considerable errors. Large negative y-intercept biases can be caused by large reverberant field fluctuations, possible by digitisation errors due to the TVR roll-off with frequency, or possibly by large changes in projector directivity with frequency. Another explanation for the low Y values is that the reverberant field is not sampled evenly throughout the tank, specifically not near the tank walls, which could lead to low power measurements of the reverberant field (if the field is not diffuse). However, when the reverberant field is dominant, the Y results are accurate due to there being a zero gradient and therefore the fluctuations cannot cause a bias in the negative direction, and so the fluctuations cancel out leaving an accurate result. The spatially averaged pressure method produces an accurate result when the reverberant field is dominant, and not when there is a significant direct field component present. The S method may also be susceptible to its value being pushed low if the reverberant field is not sampled near the walls of the tank (if the field is not diffuse). However, this effect will be
swamped if the direct field is large. Another effect to alter the $Y$ and $S$ results is if the projector directionality is not uniform, if it is not then the reverberant field measured may under or over estimate the TVR for the calibration direction.

The position of the projector in the tank will also affect the output power radiated into the reverberant field. This depends on position within the tank due to its modes, where the highest power output is near the walls and the power undulates as the projector moves towards the centre of the tank. The reverberant field impinging on the transducer will alter its impedance and therefore alter its power output. This effect will therefore alter the results for all three methods and could explain some of the variation in the results. Finally the reverberant field needs to be diffuse otherwise all three methods will be less accurate, even the gradient method since the fluctuations will not cancel out properly.

In conclusion the reverberant field needs to be diffuse for these three methods to work. Further more the gradient method works when the direct field is dominant or a significant proportion of the reverberant field, but is useless if the reverberant field is dominant. The y-intercept is only accurate when the direct field is dominant with small reverberant field fluctuations or, is far more reliable when the reverberant field is dominant. The spatially averaged pressure method is only accurate when the reverberant field is dominant.

One of these methods may be useful in carrying out a calibration in a tank if the acoustic properties of the tank are known. The two important questions that must be answered are “will the tank field be diffuse?” and “what is the direct to reverberant field ratio?”. These two properties can be established by a combination of theory and experiment. The Schroeder frequency can be calculated from the volume of the tank and, therefore, the frequency above which the tank field is diffuse can be determined. Secondly the reverberation time of the tank needs to be measured and from that, and the volume of the tank, the direct to reverberant field ratio can be calculated. Alternatively these two questions can be answered with two experimental measurements. First, test if the reverberant field is diffuse with the cross-correlation technique, and second, measure the direct to reverberant field ratio using the method set out in this thesis (plot graph of pressure squared versus the reciprocal of
separation squared). These two experimental measurements are far more time consuming that the above theory and experiment test, but do offer definitive answers.

If the tank is diffuse, the gradient method is suitable for fields where the direct field is dominant or a significant proportion of the reverberant field. Otherwise, the spatially averaged pressure method is suitable if the reverberant field is diffuse. However, for the SAP method the directionality of the projector in question needs to be taken into account. Also for the SAP method, the TVR may be low if it does not include regions near the walls of the tank. For both methods the position of the projector in the tank may affect the power output and therefore the TVR. To achieve more accurate results it may be necessary the average the results from many projector positions, including near the walls of the tank. These last three points may also improve the y-intercept result along with ensuring small averaged reverberant field fluctuations compared to the direct field, when it is measurable. If it is not measurable, a zero gradient is present and then the fluctuations do not cause a problem and cancel out to produce an accurate result.

The acoustic field needs to be measured over a time greater than the reverberation time of the tank. Ideally the individual time segments of the measured field, that produce the 50 spectra, need to be spread out with gaps between each segment of greater than the reverberation time of the tank. This is required to ensure that each sampled time segment is not correlated to any other, and the full benefit of averaging the spectra can be achieved.

The accuracy of the directivity compensation is not known since only a small sample of the pressure amplitude over the whole surface area of the projector was measured. The two orthogonal planes of measurement probably give a good indication of the directivity since the results did not show any significant unexpected variations and would probably give a representative average directivity. However, large changes in directivity for regions of the surface area cannot be discounted and so the whole surface area needs to be measured and the average directivity determined. These measurements also need to be taken in small frequency intervals so that any rapid changes in directivity with frequency can be found. This is possible since standing waves will be set up around the circumference of the projector with integer number
of wavelengths. High resolution spatial and frequency measurements like this would fully compensate for the directionality effect on the y-intercept and spatially averaged pressure results.

The three reverberant calibration methods investigated in this project are most useful at low frequencies and for projectors with high $Q$ values. It would be useful to know if this method will work accurately at frequencies below those that can be used for free-field calibrations. The most important criterion needed to enable these calibrations to take place is that the reverberant field is diffuse. This is necessary for the gradient, y-intercept and spatially averaged pressure measurements so that they can be accurately determined. A diffuse field is needed for the $G$ method so that the fluctuations balance out and give an accurate gradient, and so that the $Y$ and $S$ methods do not need to sample the whole reverberant field to give an accurate value of the spatially averaged reverberant field pressure. The frequency above which a tank can generally be considered to be diffuse is given by the Schroeder frequency. Theoretically above this frequency these three methods may give useful guide calibrations depending on the direct to reverberant field ratio in the tank. The $Y$ and $S$ results in tank B4 give adequate calibrations over the frequency range 10kHz to 100kHz. The Schroeder frequency of the tank, for its reverberation time at 10kHz, is 16.7kHz and therefore indicates that the method gives sufficiently accurate results slightly below the Schroeder frequency limit. The Schroeder frequency for the large Bath tanks ranges from about 1kHz to 2.5kHz and for the large wooden tank at NPL is about 400Hz. These results therefore indicate that reverberant calibrations could be carried out in these tanks down to the limits indicated. However, the frequency limit does depend on the reverberation time of the tank at the frequency in question. It therefore appears likely that the lower frequency calibration limit of underwater tanks can be extended downwards in frequency using these methods, albeit with larger uncertainties in the calibrations.

The accuracy of these three methods is put into content if they are compared to that of conventional free-field calibrations, which generally have an uncertainty of approximately 0.1dB (uncertainty often quoted in NPL calibrations) for an accurate calibration facility. Calibrations in a laboratory tank are usually accurate to 0.5dB or less, but can be more, depending on the experimental equipment. The reverberant
calibration methods are therefore not as accurate as conventional free-field calibrations in standard laboratory tanks, but for conditions where the uncertainty is near 1dB, it is an adequate calibration for many purposes or at least a good guide to the projector's calibration. Depending on whether the direct or reverberant field is dominant (or a similar level) the gradient or spatially averaged pressure method can be used to give a reasonable indication of the projector's calibration in a laboratory tank. If this level of calibration is all that is required, then these methods offer a cheap alternative to an expensive lake or sea trial for low frequency and high $Q$ projector calibration.


Vermiton Ltd. (no date). *Piezoelectric ceramics data bulletin 66011/F*.


A0. Appendices

A1.0. Impedance Loop Averaging 246

A2.0. Sections from the Transfer Report 251

A3.0. Processing Experimental Data 264

A4.0. Addendum to the Thesis 268

A5.0. Published Papers 288
Measurements of the impedance loop of transducer (T1) were taken in three vessels in the first year of the MPhil/PhD. These results are presented here as interesting findings and because they were not presented in an earlier report. These vessels were a cylindrical glass (Pyrex) beaker, a rectangular Perspex (Lucite) beaker and the concrete tank sunk into the ground at Bath University, tank B3. The impedance loops for the two beakers are severely distorted due to the large reflections from the beaker walls, however, by averaging over many loops the impedance (or admittance) loop of the transducer can be determined with only small distortions.

The cylindrical Pyrex beaker (2000ml) had an internal diameter of 12.5cm and contained water up to a height of 10cm, and therefore contained approximately 1.1x10^{-3} m^3 of water. The rectangular Perspex beaker contained the same volume of water, with internal base dimensions of 18.5cm and 10cm and therefore water to a height of 6cm. The rectangular concrete tank (B3) had dimensions of 3.06m x 1.52m x 1.68m (water height) and therefore contained 7.81m^3 of water.

A Hewlett Packard HP4192A LF Impedance Analyser (5Hz – 13MHz) was used to measure the admittance of the Graseby transducer (T1) in the three vessels mentioned above. The impedance was measured at discrete frequencies from 31kHz to 300kHz in steps of 1kHz. The analyser then produced this admittance run data in a text file as three columns of frequency (kHz), conductance (S) and susceptance (S). The transducer was placed in the water contained in the vessel and an impedance run made. The position of the transducer was then randomly changed and an impedance run was then made again. This process was repeated 50 times so than 50 impedance loop runs were obtained for 50 independent random positions within the vessel. This procedure was carried out for the Pyrex and Perspex beakers and the concrete tank (B3).

Theoretically the admittance loop of a electroacoustic transducer is a smooth loop and can be calculated from its equivalent circuit (for more information on impedance
or admittance loops see Tucker and Gazey (1966)). From section 2.1.2 (Equivalent circuit of a transducer) it was stated that the radiation resistance term of the equivalent circuit of a transducer represents the transducer / water system. When a sound wave impinges on the transducer it alters its radiation resistance term and causes a small secondary loop on the main admittance loop. When the admittance loop measurements were taken, the application of a potential difference across the transducer caused sound waves of that frequency to be radiated. These waves were then reflected off the beaker or tank walls and then impinged on the transducer causing secondary resonance loops on the main loop. The many reflected waves, the reverberant field, caused large numbers of secondary resonances on the main loop and completely distorted its shape. Figure A1.1 shows the 50 loops measured for the Pyrex beaker, plotted on top of each other. The 50 admittance loops were averaged together to form one new loop. This was done by calculating the average for the conductance at each frequency and then the susceptance. Figure A1.2 shows the averaged admittance loop for the Pyrex beaker. The figure shows that the distortions (secondary resonances) have been significantly reduced.

Figure A1.3 shows the 50 admittance loops (red) and the average admittance loop (blue) for the Pyrex beaker. Figures A1.4 and A1.5 show the 50 loops (red) and the average loop (blue) for the Perspex beaker and concrete tank (B3) respectively. Figure A1.6 shows the average admittance loop for the Pyrex beaker (red), Perspex beaker (blue) and the concrete tank (B3) (green).

The size of the distortions (secondary resonances) depends on the level of the reverberant field. The larger the field (less absorption) the greater the distortion. Consequently the Pyrex beaker has the largest distortions (low absorption - low absorptivity and low surface area), then the Perspex beaker (medium adsorption - high absorptivity, but low surface area) and least the concrete tank (high absorption - high absorptivity (coupled to ground) and high surface area). Figure A1.5 mainly shows undisturbed loops, but a few loops are smaller and represent a few loops taken near the tank walls.
Figure A1.1. Cylindrical Pyrex beaker 50 admittance loop (AL) plots.

Figure A1.2. Cylindrical Pyrex beaker average of 50 admittance loop plots.
Figure A1.3. Cylindrical Pyrex beaker all 50 AL plots (red) and average of all 50 AL (blue).

Figure A1.4. Rectangular Perspex beaker all 50 AL (red) and average of all 50 AL (blue).
Figure A1.5. Concrete tank (B3) all 50 AL (red) and average of all 50 AL (blue).

Figure A1.6. Average of all 50 AL for cylindrical Pyrex beaker (red), rectangular Perspex beaker (blue) and concrete tank (B3) (green).
A2.0. Sections from the Transfer Report

Appendix 2 contains relevant sections from my transfer report, the details of which are:


Sections of chapters and whole chapters are shown, which is either referred to in the main text of the thesis or is work of interest but was not pursued for the PhD. The contents of this appendix are as follows:

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   3.4. Thought Experiment 252

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      8.1.1. Water/Air system 255
      8.1.2. Wall/Water/Air system 257
      8.1.3. Energy coupling to the floor 259
   8.2. Results 260
   8.3. Discussion 260

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3.0 Theory

Each section of this chapter shows the theory that is used in the following chapters, so to see why and in what context the theory is used the relevant chapters will need to be referred to.

3.4 Thought Experiment

At the interface between water and air, some sound will be transmitted and some will be reflected (Kinsler and Frey, 1982); as shown in Figure 3.2.

![Figure 3.2: Interface between water and air.](image)

The intensity reflection coefficient is given below:

\[
R = \frac{I_r}{I_i} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2
\]  

(3.19),

and the intensity transmission coefficient is given below:

\[
T = \frac{I_t}{I_i} = \frac{4Z_1Z_2}{(Z_1 + Z_2)^2}
\]  

(3.20),

where \(R\) is the intensity reflection coefficient, \(T\) is the intensity transmission coefficient, \(I_i\) is the incident intensity, \(I_r\) is the reflected intensity, \(I_t\) is the transmitted intensity.
intensity, $Z_i$ is the impedance of medium 1 and $Z_2$ is the impedance of medium 2. The impedance's of medium 1 and 2 are defined as shown:

$$Z_1 = \rho_{0_1} c_1 \quad (3.21a),$$

$$Z_2 = \rho_{0_2} c_2 \quad (3.21b),$$

where $\rho_{0_1}$ is the volume density of the fluid medium 1, $c_1$ is the speed of sound in the fluid medium 1, $\rho_{0_2}$ is the volume density of the fluid medium 2 and $c_2$ is the speed of sound in the fluid medium 2. The attenuation of sound in fluid is given below:

$$I(t) = I(0)e^{-\alpha t} \quad (3.22),$$

where $t$ is time, $I$ is intensity, $c$ is the speed of sound in the fluid and $\alpha$ is attenuation in the fluid per unit distance.
8.0 Thought Experiments

8.1 Models

The objective of the thought experiments is to give some insight into the mechanisms governing the absorption of sound energy in the tank, which then controls the value of the reverberation time. For instance is the sound energy absorbed in the water or walls of the tank, or is it lost from the system through the waters surface, walls or floor. When this is known, and from the relative strength of the results, it might be possible to alter tanks so that they are more desirable for reverberant calibrations.

This first came about from the large difference in reverberation time between the cylindrical glass and rectangular beakers. What causes the difference, what mechanisms dominated the system. For the same fluid and volume of fluid the reverberation time is only controlled by the absorption of sound in the chamber. This is the case for the two beakers, the only difference is the shape of the vessel, thickness of walls and the wall material.

Two ways are thought of so that the shape of the vessel can affect absorption. The first is that for a fixed volume of water the shape of the vessel can alter the surface area of the vessel; this will only be a small effect. The second is that the shape of the vessel can focus the sound so that it can be absorbed more by an region. For example if the sound is focused next to the walls it can then be absorbed by the walls. This is not reasonable since rectangular or cylindrical beakers do not focus sound near the walls. Another example will be if the sound is focused in a region of the water, this seems reasonable given the shape of the cylindrical beaker. If the sound is absorbed non-linearly with pressure level then this might be the case.

The sections 8.1.1 to 8.1.3 show the different mechanisms that are thought to possibly control the absorption of sound energy in the tank or loss of sound energy from the tank.
8.1.1 Water/Air system

This model involves the loss of sound energy from the body of water and absorption of sound by the water. Imagine a water/air system, as shown in Figure 8.1, with the water held in position by an infinitely thin boundary that does not interfere with the transmission or reflection of sound. Imagine a sound is generated in the water and undergoes propagation in one dimensional, reflecting of the water/air interface at normal incidence.

![Figure 8.1: Theoretical set up for water/air system.](image)

At the interface between water and air, some sound will be transmitted and some will be reflected. The property governing this effect is the intensity reflection coefficient which is explained in the section 3.4. The equation governing the intensity reflection coefficient is given by:

\[
R = \frac{I_r}{I_i} = \left( \frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2
\]

(8.1).

So after reflection at one boundary the intensity is \(I_r = RI_i\). The number of reflections a sound ray undergoes after a time, \(t\), is \(ct/d\). Therefore the sound intensity after a time \(t\) is shown as:

\[
I(t) = I(0)R^{ct/d}
\]

(8.2),

where \(t\) is time, \(c\) is the speed of sound in the water, \(d\) is the width of the water column, \(R\) is the reflection coefficient of the water/air interface and \(I\) is the intensity.
of the sound. Attenuation also occurs in the water and attenuation in fluids is explained in section 3.4. The equation governing attenuation in fluids is given as:

\[ I(t) = I(0)e^{-\alpha t} \]  

(8.3).

Combining Equations 8.2 and 8.3 gives:

\[ I(t) = I(0)R^{ct/d}e^{-\alpha t} \]  

(8.4).

To calculate the reverberation time, substitute in the reverberation time, \( t_r \), into Equation 8.4 which then gives:

\[ I(t_r) = 10^{-6}I(0) = I(0)R^{ct_r/d}e^{-\alpha t_r} \]

\[ 10^{-6} = R^{ct_r/d}e^{-\alpha t_r} \]  

(8.5).

If base ten logs are taken of Equation 8.5 then the following equation is formed:

\[ \log_{10}(10^{-6}) = \frac{ct_r}{d}\log_{10}(R) - \alpha t_r \log_{10}(e), \]

\[ -6 = t_r \left\{ \frac{c}{d}\log_{10}(R) - \alpha \log_{10}(e) \right\} \]  

(8.6).

The term in the curly brackets of Equation 8.6 is the gradient of the intensity verses time graph. By rearranging Equation 8.6 the reverberation time is found to be:

\[ t_r = \frac{-6}{\left\{ \frac{1}{d}\log_{10}(R) - \alpha \log_{10}(e) \right\}} \]  

(8.7).

If the values of the speed of sound, \( c \) (1490ms\(^{-1}\)), width of water column, \( d \) (0.1m), intensity reflection coefficient of the water/air interface, \( R \) (0.998), and sound
attenuation coefficient in water, \( \alpha \) (2.6 X 10^{-4} Nm^{-1}s^{-1}), are fed into Equation 8.7 then the reverberation time is found to be 230ms. However 99.4\% of this reverberation time was due to the reflection term, while 0.6\% was due to the attenuation term.

8.1.2 Wall/Water/Air system

This model involves the loss of sound energy from the water/wall system and absorption by this system. The system shown in Figure 8.2 is a real system with the water held in position by a beaker, where \( d_3 = d_1 + 2d_2 \).

![Figure 8.2: Set up for wall/water/air system.](image)

The assumptions are that the system has one dimensional propagation of sound and that sound travels freely between the water/wall interface. This last assumption is made to make the calculation easy; the reflection and transmission coefficients at the water/wall boundary are ignored. Since this is the case the reflection equation is easily calculated using the wall/air intensity reflection coefficient and the distance \( d_3 \), as shown:

\[
I(t) = I(0)e^{-\alpha d_3}
\] (8.8).

However attenuation has to considered not only in water but in the wall as well. The combined attenuation effect is given below:

\[
I(t) = I(0)e^{-\alpha_1(\alpha_1 + \alpha_2) d_3}
\] (8.9),
where $\alpha_1$ is the attenuation coefficient in water, $\alpha_2$ is the attenuation coefficient in the wall, $\beta_1 = d_1/d_3$, is the fraction of the water path length over the total path length and $\beta_2 = 2d_2/d_3$, is the fraction of the wall path length over the total path length. Only longitudinal waves can exist in a fluid, but longitudinal and shear waves can exist in a solid. The reverberation time for this model is calculated using the reflection Equation 8.8 and the attenuation Equation 8.9 and is shown below:

$\frac{-6}{\left\{ c \left[ \frac{1}{d_3} \log_{10}(R) - (\alpha_1 \beta_1 + \alpha_2 \beta_2) \log_{10}(e) \right] \right\}}$ \hspace{1cm} (8.10).

This is first calculated for longitudinal waves in the wall, which in this case is Perspex. Using the values of speed of sound in water, $c$ ($1490\text{ms}^{-1}$), width of water/wall column, $d_3$ (0.11m), intensity reflection coefficient for wall/air interface, $R$ (0.99948), sound attenuation coefficient in water, $\alpha$ ($2.6 \times 10^{-4} \text{Npm}^{-1}$), water path fraction, $\beta_1$ ($100\text{mm}/110\text{mm} = 0.9091$), longitudinal sound attenuation in Perspex, $\alpha_{2L}$ (0.5755) and wall path fraction, $\beta_2$ ($2.5\text{mm}/110\text{mm} = 0.0909$) the reverberation time is calculated to be 150ms. However 15.3% of the reverberation time was due to the reflection term, 0.4% was due to the attenuation in water term and 84.4% was due to the longitudinal wave Perspex attenuation term.

This is recalculated using the shear wave attenuation coefficient in Perspex instead of the longitudinal one. The shear wave attenuation coefficient for Perspex, $\alpha_{2S}$, is found to be eight times that for the longitudinal attenuation coefficient for Perspex, $\alpha_{2L}$ (Read and Dean, 1978). Therefore the shear wave attenuation coefficient is $8 \times 0.5755 = 4.604$ and the reverberation time calculated to be 20ms. However 2.2% of the reverberation time is due to the reflection term, 0.06% due to the water attenuation term and 97.7% due to the shear wave Perspex attenuation term.
8.1.3 Energy coupling to the floor

This model involves the loss of sound energy from the beaker to the deck it is resting on. This is calculated as the energy transmitted into the deck. The reverberation time is calculated in the same way as reflection at an interface. The situation is shown in Figure 8.3.

The bottom of the beaker is either in contact with the floor or the air gaps between the beaker and floor. There is therefore an air/floor ratio. The reverberation time is calculated from the reflection coefficients of the beaker/air and beaker/floor boundary. The reverberation time due just for the beaker/floor coupling can be calculated if it is assumed there is no loss due to reflection from water/air or wall/air boundaries around the sides or top of the beaker; no loss due to attenuation in the beaker walls or water.

The proportion of the beaker area in contact with the floor, and not the air, is named $\gamma$. If $\gamma$ is assumed to be unity (100% contact), the reverberation time due to contact with the floor is found to be 720\,$\mu$s. The Perspex/concrete (beaker bottom wall/floor) intensity reflection coefficient is found to be 0.2654. The reverberation time measured experimentally for the Perspex beaker is 4\,ms. This can then be used to backtrack and calculate the proportion of beaker in contact with the floor, $\gamma$ is 29\%.
8.2 Results

Table 8.1 shows the loss mechanisms and their calculated reverberation times.

<table>
<thead>
<tr>
<th>Loss Mechanism</th>
<th>Reverberation Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflection Perspex/concrete (100% contact)</td>
<td>720μs</td>
</tr>
<tr>
<td>Reflection Perspex/concrete (29% contact)</td>
<td>4ms</td>
</tr>
<tr>
<td>Shear wave attenuation in Perspex</td>
<td>22ms</td>
</tr>
<tr>
<td>Compressional wave attenuation in Perspex</td>
<td>180ms</td>
</tr>
<tr>
<td>Reflection water/air</td>
<td>230ms</td>
</tr>
<tr>
<td>Reflection Perspex/air</td>
<td>980ms</td>
</tr>
<tr>
<td>Reflection glass/air</td>
<td>4.0s</td>
</tr>
<tr>
<td>Attenuation in water</td>
<td>36s</td>
</tr>
<tr>
<td>Measured value for Perspex beaker</td>
<td>4ms</td>
</tr>
<tr>
<td>Measured value for glass beaker</td>
<td>20ms</td>
</tr>
</tbody>
</table>

Table 8.1: Loss mechanisms and calculated reverberation times.

All of the reverberation time values in Table 8.1 are calculated for just the mechanism named in the context of the beaker set up.

8.3 Discussion

The volume of water in the models is the same as the volume of water in the two beakers. The thickness of the walls in the model is the same as the thickness of the walls in the Perspex beaker. It must be remembered that all the mechanisms above are idealised theoretical loss mechanisms and have many simplifying assumptions, not least being that everything is calculated in one dimension. Therefore the results only give a guide as to the relative magnitude of the effect.

From the results it seems that energy loss through the floor is a dominant mechanism as is shear wave attenuation in the Perspex walls. The measured Perspex beaker result of 4ms agrees with floor contact proportion of 29%. This seems like a
reasonable contact area although it could have a large error in the result. The shear wave attenuation in the Perspex walls has a value of 22ms which would indicate that the floor contact proportion is more important. This is not necessarily the case as these are only very rough guide results. The cylindrical glass measured result is 20ms which does not agree well with the floor contact proportion result. Also attenuation in glass will be a lot lower than in Perspex. However as has been said these are only rough guides and these results agree with each other given there probable errors. Another possibility to explain that the Perspex result is a lot lower than the glass result is that:

a) the Perspex beaker has a lot thicker walls than the glass beaker; as well as the attenuation in Perspex being higher.

b) the thought experiment was only thought of in terms of one dimension, but there will be a lot of waves at oblique incidence to the walls. Shear waves are set up far more in the walls by oblique incidence than normal incidence. Since there are far more shear waves and shear waves are attenuated far more than longitudinal waves, energy will be absorbed in the Perspex walls far more and therefore the reverberation time will be lower. It is thought that shear waves will not be easy to set up in glass and as such this effect will not have much of an effect for the glass beaker. This could explain why the Perspex beaker has a lot lower reverberation time that the glass beaker (even though they both rest on the ground and therefore have the same area in contact).

c) Because the shear waves are set up more at oblique incidence, a lot are generated in the walls, but the oblique incident waves in the walls cannot produce shear waves in the water and as such are reflected back into the walls. A lot of energy is trapped in the walls and because shear waves are highly attenuated most of the energy is absorbed in the walls, the reverberation time will therefore be very low.

d) Compressional waves could also be trapped in the walls due to the reflection coefficients. It the wall/air coefficient is high (which it is), the energy cannot escape that way. Also if the water/wall reflection coefficient is near 0.5 more energy is retained in the wall, and as the wall attenuates more than water does the energy is
absorbed. Also since the energy has been absorbed there is no longer equilibrium between the wall and water and energy goes into the wall to restore equilibrium. As the energy is absorbed again and again in the walls, the reverberation time will be short.

Attenuation of Compressional waves in Perspex is very low compared to shear waves, but is enormous compared to attenuation in water. Attenuation of sound in water is calculated to be 36s for a beaker 10cm across (only one dimensional). In terms of relative strength of effect water attenuation is non existent. Reflection at the water/air boundary is very high but not as high as the wall/air boundaries. However if sound can get into the walls the reflection coefficient for Perspex/concrete (wall/floor) is rather low and so energy is easily lost to the floor.
References


A3.0. Processing Experimental Data

Appendix 3 describes the various stages involved in processing experimentally measured data in order to determine various parameters. Section A3.1 enables the reverberation time of an enclosure to be determined from the measured decaying signal. Section A3.2 enables the reverberant field calibration to be calculated from the measurements made at various positions in the combined direct and reverberant sound fields. MATLAB was used to perform the calculations in both sections.

A3.1. Procedure for processing measured decaying sound field data to determine the reverberation time of the enclosure.

The list below describes the stages involved in processing a decaying signal in order to determine the reverberation time and its standard error. Stage (d) determines if there is a gap in the signal and therefore if the direct field component of the signal is separate from the reverberant part. If the direct field part is separate it removes it. At stage (e) a choice has to be made over whether or not to remove noise and how. If noise is removed it can be root mean squared noise or mean squared noise. Mathematically mean squared noise is the correct one to choose. It would be expected that there is not much difference between noise calculated at the beginning of the record (before the first arrival) and the end of the signal (signal decayed away to background level). This is the case as there is no statistical difference between the two. However due to the sample size of the noise sections the difference in value between the two can be significant. It has been found that using the end noise is preferable since this is the value of the noise where the signal is extracted from said noise. Using the end noise results in a large improvement of the dynamic range of the decay, whereas the beginning noise often only results in a slight improvement. The number of points the record is averaged over depends on the initial number of points recorded.

The decaying signals were recorded on a LeCroy oscilloscope at Bath and Sonardyne and a spectrum analyser at NPL. The LeCroy oscilloscope recorded 50,000 points
and is averaged over 500 points to give the end number of points as 100. The spectrum analyser recorded 4096 points and was averaged over 40 points to give the end number of points as 102. This resulted in the signals, recorded on both instruments, having a similar number of points after averaging.

The 'single shot record' and 'single shot record of a short noise pulse' both removed noise using mean square end noise. The 'multiple shot average' did not remove noise.

List of processing scheme stages:

a. Recorded waveform
b. D.C. offset removed
c. Beginning noise removed
d. Is there a gap of 300μs or greater in the signal?
   Yes: Remove direct field components
   No: No action
e. Chose noise removal type:
   0: \( N \) point rms of signal (no removal of noise)
   1: \( N \) point rms of signal and subtract rms beginning of signal noise
   2: \( N \) point rms of signal and subtract rms end of signal noise
   3: \( N \) point ms of signal, subtract ms beginning of signal noise and square root signal
   4: \( N \) point ms of signal, subtract ms end of signal noise and square root signal

\[ \text{rms} = \text{root mean square} \]
\[ \text{ms} = \text{mean square} \]

LeCroy oscilloscope: Records: 50000 points
\( N = 500 \) points
After average: 100 points
Spectrum Analyser: Records: 4096 points

\[ N = 40 \text{ points} \]

After average: 102 points

f. Convert to decibel scale: \[ 20 \log_{10} \text{ of signal} \]

g. Two point average of signal to reduce fluctuations

h. Is there a plateau?

\[ pl = \text{level of last } x \text{ proportion of record.} \]

\[ y = \text{proportion of plateau level, } pl \]

Does the last x proportion of record lie within bounds of \( pl - y \) and \( pl + y \) (i.e. is there a plateau)? (See Figure 4.6.)

Yes: Determine time, \( t_{cut} \), where signal at \( pl + 10 \text{dB} \). Then extract the signal from end of stage (f) between the times 0 and \( t_{cut} \)

No: Extract the whole of the signal from the end of stage (f)

i. Perform linear regression on extracted (straight-line decaying) signal

j. Calculate reverberation time and standard error from the gradient and error in the gradient of the linear regression

**A3.2. Procedure for processing the reverberant calibration data to determine the calibration of the projector.**

The processing of the reverberant calibration data has two stages, the averaging of spectra and the calculation of the transmitting voltage response of the projector.

List of stages for obtaining an averaged spectrum from the time-voltage signal from the LeCroy oscilloscope:

a. split time-voltage signal up into 50 sections

b. perform an FFT on each section

c. average the 50 spectra to produce an averaged spectrum
List of stages for obtaining the transmitting voltage response of the projector:

d. divide voltage spectra from hydrophone by the gain of the amplifier to produce
   the voltage spectra output from the hydrophone.
e. calculate the pressure spectra for the hydrophone positions by using Equation 1.9
   and the voltage spectra output from the hydrophone
f. generate the pressure squared matrix
g. generate the reciprocal of separation squared matrix from the separations of the
   transducers used in the measurements.
h. perform a least squares fit (linear regression) on the pressure squared matrix and
   the reciprocal of separation squared matrix
i. the acoustic power radiated into the tank is calculated using Equation 6.20 and the
   gradient of the graph
j. the acoustic power radiated into the tank is calculated using Equation 6.21, the
   y-intercept, the reverberation time frequency response and the volume of water
   in the tank
k. the acoustic power radiated into the tank is calculated using Equation 6.23, the
   spatially averaged pressure, the reverberation time frequency response and the
   volume of water in the tank
l. the pressure at one metre from the projector is calculated using Equation 6.12 and
   the three acoustic powers
m. the transmitting voltage response of the projector is calculated using Equation
   1.10, the three pressure responses at one metre and the voltage applied across
   the projector
A4.0. Addendum to the Thesis

This appendix is an addendum to the thesis, which is required as part of the minor corrections to the thesis, after the viva voce examination. The work required comes under three categories: (1) characterising the tanks direct to reverberant field range in terms of a distance, (2) an alternative method of determining the reverberation time of the tank using the data measured for the reverberant calibrations, and (3) writing a guide to the reverberant calibration technique developed in this thesis, for a novice to this area of work.

A4.1. Characterising the tank

The tank can be characterised by the spread of the direct and reverberant field. This can be achieved by determining the distance from the source where the direct and reverberant field have equal intensity or pressure. This distance can be called the characteristic tank length, and can be calculated by measuring the sound field or the reverberation time of the tank. The reverberant calibration technique yielded data about the acoustic field that can be used to calculate this length. Another measure of direct to reverberant field spread is the minimum distance between transducers that is used when measuring reverberant fields in the ANSI S1.21 (1972) standard.

A4.1.1. Calculating the characteristic tank length from the acoustic field data measured for the reverberant calibration technique

The data collected for the reverberant calibration technique sampled the acoustic sound field at points varying from close to the source to those further away. This usually ranged from points where the direct field was dominant (close to the source) to points where the reverberant field was dominant (further from the source). It is this region that is needed so that the transition from the direct to reverberant field is measured so that the point where the two fields are equal (distance from the source equals the characteristic tank length) is known.
This is calculated by first determining the direct field squared pressure \( (P_d^2) \) and then the spatially averaged reverberant field square pressure \( (P_r^2) \), from the graph of pressure squared \( (P^2) \) versus the reciprocal of separation squared \( (1/r^2) \). From the graph \( P_d^2 = P^2 - C \), where \( C \) is the y-intercept and is equal to \( P_r^2 \). The gradient of the graph is therefore

\[
m = \frac{P_d^2}{\left( \frac{1}{r^2} \right)} = P_d^2 r^2.
\]  

(EA4.1)

Equating the direct and reverberant field square pressure to each other \( (P_d^2 = P_r^2) \) and then substituting in a rearranged Equation (EA4.1) and the y-intercept \( (C = P_r^2) \) leads to

\[
\frac{m}{r^2} = C.
\]  

(EA4.2)

In Equation EA4.2, \( r \) is the distance at which the direct and reverberant field sound intensities are equal. This is the experimental characteristic tank length, \( L_{ce} \), and by rearranging Equation EA4.2 is given by

\[
L_{ce} = r = \sqrt{\frac{m}{C}},
\]  

(EA4.3)

where \( m \) is the gradient and \( C \) is the y-intercept of the graph of pressure squared versus the reciprocal of separation squared.
A4.1.2. Calculating the characteristic tank length from the reverberation time and the volume of water of the tank

The characteristic tank length can also be found by using the theoretical equations for the direct and reverberant fields. Recalling Equation 6.12, the direct field squared pressure, $P_d^2$, is given by

$$P_d^2 = \frac{\rho_0 c W}{4\pi r^2},$$  \hspace{1cm} (6.12)

where $\rho_0$ is the volume density of the fluid, $c$ is the speed of sound in the fluid, $W$ is the acoustic power radiated by the source and $r$ is the radial distance from the effective centre of the source. Recalling Equation 6.13, the spatially averaged effective squared pressure amplitude of the reverberant field, $P_r^2$, is given by

$$P_r^2 = \frac{4\rho_0 c W}{A},$$  \hspace{1cm} (6.13)

where $A$ is the total sound absorption of the enclosure. Equating the direct and reverberant field square pressure to each other ($P_d^2 = P_r^2$) and then substituting in Equations 6.12 and 6.13 gives

$$\frac{\rho_0 c W}{4\pi r^2} = \frac{4\rho_0 c W}{A},$$  \hspace{1cm} (A4.4)

In Equation A4.4, $r$ is the distance at which the direct and reverberant field sound intensities are equal. This is the theoretical characteristic tank length, $L_{ct}$, and by rearranging Equation A4.4 is given by

$$L_{ct} = r = \sqrt{\frac{A}{16\pi}}.$$  \hspace{1cm} (A4.5)
Recalling Equation 6.16, the total sound absorption of the enclosure, A, is given by

\[ A = \frac{24V}{c \log_{10}(e) T_r}, \]  

(6.16)

where \( V \) is the volume of water in the tank, \( c \) is the speed of sound in the fluid and \( T_r \) is the reverberation time of the tank. Substituting Equation 6.16 into Equation A4.5, give the theoretical characteristic tank length, in terms of the reverberation time of the tank, as

\[ L_{ct} = \sqrt{\frac{3V}{2\pi c \log_{10}(e) T_r}}. \]  

(A4.6)

A4.1.3. The minimum transducer separation used in the ANSI S1.21 (1972) standard for reverberant field measurements

The ANSI S1.21 (1972) standard states that a minimum distance between the sound source and microphone is needed for the measurement of the reverberant field to take place. This distance, \( d_{\text{min}} \), is defined as

\[ d_{\text{min}} = 0.08 \frac{V}{T_r}, \]  

(A4.7)

where \( V \) is the volume of the enclosure and \( T_r \) is the 60dB reverberation time (i.e. the time taken for the sound level to decay by 60dB). The 60dB reverberation time is the standard reverberation time. Equation A4.7 can be used to characterise the tanks used in this work, by using the volume of water in the tank and the tank reverberation time.
A4.1.4. The Results of tank characterisation

The experimental characteristic tank length has been calculated for all the eight tanks used in this project. This has been achieved by using Equation A4.3, along with the gradient and y-intercept calculated for the reverberant calibration measurements. The theoretical characteristic tank length has also been calculated for all eight tanks by using Equation A4.6, along with the volume of water in the tanks and the measured reverberation time of the tanks. Finally the $d_{\text{min}}$ length, from the ANSI standard, has been calculated using Equation A4.7 and also the volume of water and the reverberation time of the tanks. Figures A4.1 to A4.5 show these three parameters, against frequency, for the tanks B1, B3, B4, N2 and S1.

![Figure A4.1. Theoretical characteristic tank length, $L_{\text{ct}}$, (blue), experimental characteristic tank length, $L_{\text{ce}}$, (red) and $d_{\text{min}}$ (green) versus frequency for tank B1, group pl, runs 1-5.](image-url)
Figure A4.2. Theoretical characteristic tank length, $L_{ct}$, (blue), experimental characteristic tank length, $L_{ce}$, (red) and $d_{\text{min}}$ (green) versus frequency for tank B3, group cl, runs 1-6.

Figure A4.3. Theoretical characteristic tank length, $L_{ct}$, (blue), experimental characteristic tank length, $L_{ce}$, (red) and $d_{\text{min}}$ (green) versus frequency for tank B4, group gl1, runs 1-a (not c, b greatly skews mean for Figure A4.8).
Figure A4.4. Theoretical characteristic tank length, $L_{ct}$, (blue), experimental characteristic tank length, $L_{ce}$, (red) and $d_{min}$ (green) versus frequency for tank N2, group npl, runs 1-2.

Figure A4.5. Theoretical characteristic tank length, $L_{ct}$, (blue), experimental characteristic tank length, $L_{ce}$, (red) and $d_{min}$ (green) versus frequency for tank S1, group sm, run 3.

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A4.1.5. Proportion of the tank

The proportion the tank characteristic length and $d_{\text{min}}$ is of the tank length gives an indication of the distribution of the direct and reverberant field in a tank. The tank equivalent length, $L_{\text{eq}}$, is defined as

$$L_{\text{eq}} = \sqrt[3]{V},$$  \hspace{1cm} (A4.8)

where $V$ is the volume of the water in the tank. This is a more useful parameter to compare the characteristic length against since it is directly related to the volume of the water and not one side of a rectangular parallelepiped. The lengths of the different sides of the tank are not equal and so any one side is not representative of the volume.

To give a proportion of the tank length for these three parameters a single representative value for them is needed. This is obtained by taking the mean of the individual value over the frequency range, thus giving a value for the tank. Using the tank equivalent length the ratios $L_{\text{ce}}/L_{\text{eq}}$, $L_{\text{ef}}/L_{\text{eq}}$ and $d_{\text{min}}/L_{\text{eq}}$ can be calculated for the eight tanks and so show the proportion of the tank length that the characteristic length and $d_{\text{min}}$ cover.

Table A4.1 shows the three mean lengths and the three proportions calculated for all eight tanks.
### Table of Tank Characterisation Parameters

<table>
<thead>
<tr>
<th>Tank</th>
<th>Group and Run</th>
<th>$L_{ce}$ / mm</th>
<th>$L_{ct}$ / mm</th>
<th>$d_{min}$ / mm</th>
<th>V / m$^3$</th>
<th>$L_{eq}$ / m</th>
<th>$L_{ce} / L_{eq}$</th>
<th>$L_{ct} / L_{eq}$</th>
<th>$d_{min} / L_{eq}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>pl: 1-5</td>
<td>486</td>
<td>253</td>
<td>744</td>
<td>5.42</td>
<td>1.76</td>
<td>0.277</td>
<td>0.144</td>
<td>0.424</td>
</tr>
<tr>
<td>B2</td>
<td>ml: 1-7</td>
<td>452</td>
<td>189</td>
<td>555</td>
<td>2.39</td>
<td>1.34</td>
<td>0.338</td>
<td>0.141</td>
<td>0.415</td>
</tr>
<tr>
<td>B3</td>
<td>cl: 1-6</td>
<td>553</td>
<td>375</td>
<td>1110</td>
<td>7.81</td>
<td>1.98</td>
<td>0.279</td>
<td>0.189</td>
<td>0.557</td>
</tr>
<tr>
<td>B4</td>
<td>g1l: 1-c (1-12)</td>
<td>-</td>
<td>26.4</td>
<td>77.7</td>
<td>0.162</td>
<td>0.545</td>
<td>-</td>
<td>0.0484</td>
<td>0.143</td>
</tr>
<tr>
<td>N1</td>
<td>dp: 1</td>
<td>352</td>
<td>228</td>
<td>671</td>
<td>4.20</td>
<td>1.61</td>
<td>0.218</td>
<td>0.141</td>
<td>0.416</td>
</tr>
<tr>
<td>N2</td>
<td>npl: 1-2</td>
<td>1020</td>
<td>1390</td>
<td>4100</td>
<td>119</td>
<td>4.92</td>
<td>0.207</td>
<td>0.283</td>
<td>0.833</td>
</tr>
<tr>
<td>N3</td>
<td>npl: 3-5</td>
<td>367</td>
<td>175</td>
<td>514</td>
<td>4.50</td>
<td>1.65</td>
<td>0.222</td>
<td>0.106</td>
<td>0.312</td>
</tr>
<tr>
<td>S1</td>
<td>sm: 3</td>
<td>491</td>
<td>547</td>
<td>1610</td>
<td>62.7</td>
<td>3.97</td>
<td>0.124</td>
<td>0.138</td>
<td>0.406</td>
</tr>
</tbody>
</table>

Table A4.1. Table showing tank, reverberant calibration group and run, experimental characteristic tank length ($L_{ce}$), theoretical characteristic tank length ($L_{ct}$), ANSI minimum transducer separation for measurements of the reverberant field ($d_{min}$), volume of water in the tank (V), equivalent tank length ($L_{eq}$), ratio of $L_{ce}$ to $L_{eq}$, ratio of $L_{ct}$ to $L_{eq}$ and ratio of $d_{min}$ to $L_{eq}$, for all eight tanks (B1, B2, B3, B4, N1, N2, N3 and S1). There are no values for $L_{ce}$ and $L_{ct}/L_{eq}$ for tank B4, since these parameters are based on the gradient of the pressure squared versus the reciprocal of separation squared graph. The gradient values for tank B4 are meaningless since the reverberant field completely dominates the direct field, and so no values are shown for these two parameters.
A4.2. Determining the reverberation time of the tank using data measured for the reverberant calibrations

The reverberation time of the tank can be determined from the data measured for the reverberant calibrations. This can be achieved using the gradient, volume of water and $P_r^2$, or alternatively the projector TVR, voltage applied to the projector, volume of water and $P_r^2$.

A4.2.1. Determining the reverberation time using the gradient and $P_r^2$.

Recalling Equation 6.18, the gradient, $m$, of the pressure squared versus the reciprocal of separation squared graph, is

$$m = \frac{\rho_0 c W}{4\pi}, \quad (6.18)$$

where the $\rho_0$ is the volume density of the fluid, $c$ is the speed of sound in the fluid and $W$ is the acoustic power radiated by the source. Recalling Equation 6.19, the spatially averaged effective squared pressure amplitude of the reverberant field, $P_r^2$, or the y-intercept, $C$, of the pressure squared versus the reciprocal of separation squared graph, is

$$C = P_r^2 = \frac{\rho_0 c^2 T_r W \log_{10}(e)}{6V}, \quad (6.19)$$

where $T_r$ is the reverberation time of the tank and $V$ is the volume of water in the tank. Dividing Equation 6.18 by Equation 6.19 gives

$$\frac{m}{C} = \frac{m}{P_r^2} = \frac{3V}{2\pi c \log_{10}(e) T_r}. \quad (A4.9)$$
By rearranging Equation A4.9, the reverberation time is given by

\[ T_r = \frac{3VC}{2\pi c \log_{10}(e)m} = \frac{3VP_r^2}{2\pi c \log_{10}(e)m}. \quad (A4.10) \]

Thus the reverberation time can be calculated from the volume of water, y-intercept and the gradient of the pressure squared versus the reciprocal of separation squared graph. This equation is based on the spatially averaged pressure squared, \( P_r^2 \), which is why the y-intercept can be used. But the reverberation time can therefore be calculated from the average of pressure squared measurements where the reverberant field is dominant. Therefore Equation A4.10 can be re-written as

\[ T_r = \frac{3VP_r^2}{2\pi c \log_{10}(e)m}, \quad P_d \ll P_r. \quad (A4.11) \]

**A4.2.2. Determining the reverberation time using the projector TVR and \( P_r^2 \).**

The reverberation time of the tank can also be found from the projector TVR, voltage applied to the projector, volume of water in the tank and the spatially averaged squared pressure, \( P_r^2 \). This can be calculated by recalling Equation 2.10, for the projector TVR and rearranging it to obtain the direct field pressure. Using the projector TVR and voltage applied to the projector, the direct field pressure at one metre can be calculated. Recalling Equation 6.12, for the direct field pressure, the acoustic power radiated can be calculated. Recalling Equation 6.21 or Equation 6.23, the acoustic power and the y-intercept or spatially averaged squared pressure can be used to calculate the reverberation time. The final equation to calculate the reverberation time in this way is

\[ T_r = \frac{3V_w P_r^2}{2\pi c \log_{10}(e)V_p^2 \cdot 10^{[(TVR/10)-12]}}, \quad (A4.12) \]
where $V_w$ is the volume of water in the tank, $V_p$ is the voltage applied to the projector and TVR is the transmitting voltage response of the projector. $P_r^2$ can be the y-intercept, $C$, or the spatially averaged squared pressure sampled where the reverberant field is dominant, $P_{SA}^2$ ($P_d << P_r$).

A4.2.3. Reverberation time results determined from the measurements made for reverberant calibrations.

The reverberation time has been calculated for all eight tanks using the $m/P_r^2$ method and the TVR and $P_r^2$ method. For each of these methods the reverberation time was calculated using the y-intercept, $C$, the spatially averaged squared pressure, $P_{SA}^2$, (for all the pressure values) and $P_{SA}^2$ calculated for the region where the reverberant field is dominant. This last value is calculated by squaring each pressure and taking the averaged of the two positions furthest from the projector (generally $P_d << P_r$).

Figures A4.6 to A4.10 show reverberation time against frequency, for the measured reverberation time results, and for the two methods above for the $C$ and $P_{SA}^2$ ($P_d << P_r$) results, for the tanks B1, B2, B3, B4, N2 and S1. The $P_{SA}^2$ values calculated for the whole of the pressure field give high reverberation time results compared to the measured reverberation time. These results have been left off the graphs, but are included in a table of results (Table A4.2).
Figure A4.6. Reverberation time versus frequency, for measured (blue), derived from $m/C$ (solid green), derived from $m/P^2_{SA}$ where $P_d << P_r$ (dotted green), derived from TVR and C (solid red) and derived from TVR and $P^2_{SA}$ where $P_d << P_r$ (dotted red), for tank B1, group pl and runs 1-5.

Figure A4.7. Reverberation time versus frequency, for measured (blue), derived from $m/C$ (solid green), derived from $m/P^2_{SA}$ where $P_d << P_r$ (dotted green), derived from TVR and C (solid red) and derived from TVR and $P^2_{SA}$ where $P_d << P_r$ (dotted red), for tank B3, group cl and runs 1-6.
Figure A4.8. Reverberation time versus frequency, for measured (blue), derived from $m/C$ (solid green), derived from $m/P_{sa}^2$ where $P_d << P_r$ (dotted green), derived from TVR and C (solid red) and derived from TVR and $P_{sa}^2$ where $P_d << P_r$ (dotted red), for tank B4, group g11 and runs 1-a (not c, b greatly skews mean).

Figure A4.9. Reverberation time versus frequency, for measured (blue), derived from $m/C$ (solid green), derived from $m/P_{sa}^2$ where $P_d << P_r$ (dotted green), derived from TVR and C (solid red) and derived from TVR and $P_{sa}^2$ where $P_d << P_r$ (dotted red), for tank N2, group npl and runs 1-2.
Figure A4.10. Reverberation time versus frequency, for measured (blue), derived from $m/C$ (solid green), derived from $m/P_{d_s}^2$ where $P_d << P_r$ (dotted green), derived from TVR and C (solid red) and derived from TVR and $P_{d_s}^2$ where $P_d << P_r$ (dotted red), for tank S1, group sm and run 3.

The mean values of the varying reverberation time results with frequency, are calculated to give representative values for each method and form of $P_r^2$. Table A4.2 shows representative measured reverberation time results for the eight tanks. It also shows the six representative results for the two methods and the three different forms of $P_r^2$, for the eight tanks.
Table of Measured and Derived Tank Reverberation Times

<table>
<thead>
<tr>
<th>Tank</th>
<th>Group and Run</th>
<th>Tr / ms</th>
<th>Trgy / ms</th>
<th>Trgs / m</th>
<th>Trgr / ms</th>
<th>TrTVRyint / ms</th>
<th>TrTVRSA P / ms</th>
<th>TrTVRSA Pr / ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>pl: 1-5</td>
<td>66.5</td>
<td>41.6</td>
<td>121</td>
<td>49.8</td>
<td>48.3</td>
<td>151</td>
<td>59.3</td>
</tr>
<tr>
<td>B2</td>
<td>ml: 1-7</td>
<td>50.2</td>
<td>20.0</td>
<td>53.8</td>
<td>21.9</td>
<td>21.3</td>
<td>63.1</td>
<td>24.3</td>
</tr>
<tr>
<td>B3</td>
<td>cl: 1-6</td>
<td>41.7</td>
<td>25.9</td>
<td>150</td>
<td>41.9</td>
<td>29.5</td>
<td>178</td>
<td>49.2</td>
</tr>
<tr>
<td>B4</td>
<td>g1l: 1-c (1-12)</td>
<td>172</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>230</td>
<td>245</td>
<td>232</td>
</tr>
<tr>
<td>N1</td>
<td>dp: 1</td>
<td>59.6</td>
<td>45.8</td>
<td>132</td>
<td>57.3</td>
<td>31.5</td>
<td>101</td>
<td>42.0</td>
</tr>
<tr>
<td>N2</td>
<td>npl: 1-2</td>
<td>47.8</td>
<td>94.7</td>
<td>953</td>
<td>70.8</td>
<td>70.9</td>
<td>722</td>
<td>53.5</td>
</tr>
<tr>
<td>N3</td>
<td>npl: 3-5</td>
<td>109</td>
<td>33.1</td>
<td>218</td>
<td>40.8</td>
<td>64.0</td>
<td>438</td>
<td>80.5</td>
</tr>
<tr>
<td>S1</td>
<td>sm: 3</td>
<td>158</td>
<td>220</td>
<td>1150</td>
<td>250</td>
<td>194</td>
<td>1080</td>
<td>225</td>
</tr>
</tbody>
</table>

Table A4.2. Table showing tank, reverberant calibration group and run, and then reverberation time for: measured (Tr), m/C derived (Trgy), m/ \( P_{SA}^2 \) derived (Trgs), \( m/ P_{SA}^2 \) for \( P_d \ll P_r \) derived (Trgr), TVR and y-intercept derived (TrTVRyint), TVR and \( P_{SA}^2 \) (TrTVRSA P) and TVR and \( P_{SA}^2 \) for \( P_d \ll P_r \) (TrTVRSA Pr). There are no values for Trgy, Trgs and Trgr for tank B4, since these parameters are based on the gradient of the pressure squared versus the reciprocal of separation squared graph. The gradient values for tank B4 are meaningless since the reverberant field completely dominates the direct field, and so no values are shown for these three parameters.
The results, for the $m/P_r^2$ and TVR & $P_r^2$ methods, for both the $P_r^2$ forms (y-intercept, and spatially averaged squared pressure where the reverberant field is dominant) show that they are adequate estimates of the measured reverberation time. These are only moderately accurate since these estimated reverberation times are in good agreement with the measured reverberation times for part of the frequency range, but are a bad fit for other parts of the range. The spatially averaged squared pressure results for the whole acoustic field are high compared to the measured results. This is because the spatially averaged square pressure results for the whole field includes the direct field, which is significant, and therefore causes a bias in $P_{SA}^2$. This therefore causes the reverberation time results based on this value to be high compared to the measured results.

A4.3. Guidance notes to a novice user on how to calibrate projectors in a reverberant sound field using the technique developed in this thesis.

Presented here are guidelines on how to perform the reverberant calibration technique, developed in this thesis, for use by novice users. In order to calibrate a projector, in a laboratory sized tank, a diffuse reverberant field is required. This is produced with a continuous wave noise signal sent to the projector. The reverberant field needs to be diffuse, and a guide to see if the field will be diffuse is to calculate the Schroeder frequency for the tank. The Schroeder frequency, $f_c$, is given by

$$f_c = 2 \times 10^3 \sqrt{\frac{T_r}{V}}$$

where $T_r$ is the reverberation time of the tank and $V$ is the volume of water in the tank. The field will generally be diffuse if the exciting noise signals used is above the Schroeder frequency. Measuring the reverberation time of a tank accurately is not a quick or easy procedure, and so it will probably be faster to try this technique out on a calibrated projector, and if the reverberant calibration gives a close match to the reference calibration then the field is probably diffuse.

The equipment needed to carry out the reverberant calibration is a projector, a calibrated hydrophone, a power amplifier, a signal generator to produce a noise signal, a vector analyser or an oscilloscope to capture the signal, and some way to accurately move the hydrophone away from the projector in a straight line.
This can be achieved using a computer controlled stepper motor or manually using an optical bench arrangement (slow and laborious). A calibrated amplifier for the signal from the hydrophone may be needed if the signal is not large enough. Alternatively a more powerful power amplifier, between the signal generator and projector, could be used, particularly if the intrinsic noise from the hydrophone is large compared with the wanted acoustic signal.

A continuous wave white noise signal is generated by the signal generator, and then sent to the power amplifier, and then on to the projector. The signal sent to the projector needs to be recorded by a vector analyser or an oscilloscope. The signal from the hydrophone is sent to a vector analyser or an oscilloscope to be recorded. If the signal is not large enough a calibrated amplifier will need to be put between the two. The projector and hydrophone need to be aligned so that they are in the same orientation and that their calibrating ‘0’ mark directions are facing each other. The initial separation of the transducers needs to be close, so that the hydrophone is in the direct field, but also far enough away so that it is in the far field. The hydrophone is moved away from the projector so that it moves from the direct field to the reverberant field. This is done in a series of steps, and at each position the hydrophone signal needs to be recorded, and the transducer separation noted. Also, the signal sent to the projector needs to be recorded once.

The signal recorded from the hydrophone, or applied to the projector, needs to be an averaged voltage spectrum. This is where the vector analyser records a series of time trace samples and performs a Fast Fourier Transform (FFT) on them all. A series of voltage spectra now exist, and the mean of these spectrum is taken to produce an averaged spectrum. This is done for every hydrophone position and once to the signal applied to the projector. This is necessary to reveal the underlying signal in the recorded noise trace. This process can also be performed on an oscilloscope and the FFT and averaging performed on a computer. In this project 50 traces were recorded for each hydrophone position, and the averaged spectrum calculated. Thus, an averaged voltage spectrum exist for each hydrophone position and one for the signal applied to the projector.
The projector TVR can now be calculated. The averaged hydrophone voltage spectra are corrected to their original value, if an amplifier was used on the hydrophone, by using the calibration of the amplifier. The voltage spectra are converted into pressure spectra using the calibration of the hydrophone.

The number of frequency points in the pressure spectra and projector voltage can be reduced by averaging several adjacent points. This will improve the fit of the graph that is plotted next. The reduced number of frequency points means that each new point represents a larger frequency band.

The pressure spectra and transducer separation data is then used to plot points on a graph of pressure squared versus the reciprocal of separation squared. This is done for each frequency band in the spectra. Therefore one plot is produced for each frequency band, with one pressure value from each spectra plotted against the appropriate separation. One plot is produced for each frequency band. A linear regression fit is performed for each plot. Therefore a vector is produced for gradient and y-intercept, against frequency. A vector can also be produced of spatially averaged squared pressure, against frequency. This is produced by taking the mean of squared pressure, over all the hydrophone positions, for each frequency band.

From the gradient, y-intercept and spatially averaged squared pressure vectors, an estimate of the acoustic power radiated into the tank can be made. From these three estimates of power, three estimates of direct field pressure can be made. From these three direct field pressures and the projector voltage spectra, three estimates of the transmitting voltage response (TVR) of the projector can be made.

If the projector is not omnidirectional, then this will need to be taken into account when calculating the direct field pressure from the acoustic power, for the y-intercept and spatially averaged squared pressure.

Further details of how to take the measurements, to calculate the projector TVR and to compensate for the projector directionality, can be found in chapter 6.
The gradient derived TVR is found to be most accurate when the direct field is dominant or a good proportion of the reverberant field. Where the reverberant field is dominant the spatially averaged squared pressure results are most accurate. The y-intercept results are less accurate than the other two, except for the following conditions. The gradient results are meaningless when the reverberant field is dominant, and the spatially averaged squared pressure results have a large positive bias on them when the direct field is dominant or a significant proportion of the reverberant field.
A5.0. Published Papers

Work carried out during this project has been presented in two published papers, one at a conference in London and the other at a conference in Copenhagen. The two papers are reproduced over the following pages and the details of the papers and conferences are shown below:

A5.1. Projector sensitivity measurements in reverberant fields


Conference held by the Institute of Acoustics (I.O.A.), on Underwater Acoustic Calibration and measurement, at the National Physical Laboratory, Teddington, Middlesex, UK, on 20th to 21st July 1998.

Everitt and Humphrey (1998) reported initial results using a variation on this technique. A continuous white noise source was used to produce a reverberant field in a tank of water and the pressure was measured at a series of transducer separations. The graph of pressure squared versus the reciprocal of separation squared was plotted and the gradient determined using a least squares fit linear regression. The power radiated into the tank was then calculated from the gradient and then this used to calculate the pressure at one metre from the projector in a free-field. The TVR of the projector was then calculated using this pressure and the voltage applied to the projector. Measurements were taken using two projectors in four tanks and the results presented. The TVR calculated from the gradient and the free-field TVR were similar, but the difference varied from 0.3dB to 3.1dB, for a 1kHz bandwidth, depending on the tank and projector.
A5.2. Transducer transmitting sensitivity measurements in restricted environments


Conference held jointly by the Ultrasonics International 99 and 1999 World Congress on Ultrasonics at the Technical University of Denmark, Copenhagen, Denmark, on 29th June to 1st July 1999.

Everitt and Humphrey (2000) reported extended results based on the gradient method used in the paper above (Everitt and Humphrey, 1998). Measurements were taken in three tanks for a variety of different traverses. Traverses were made parallel to the walls of the tank for a series of locations at different distances from the walls and at different depths, and also for a series of inclined traverses. Individual calibration measurements show good agreement between gradient TVR and free-field TVR with differences of no more than 1dB. Results averaged over a series of measurement runs also show accuracy to within 1dB. There appears to be no correlation between the accuracy of this technique and the position in the tank provided the measurements are not made close to the tank walls. The results indicate that the construction of the tank has a small effect on the accuracy of the technique, with variations between the tanks typically of less than 1dB.
PROJECTOR SENSITIVITY MEASUREMENTS IN REVERBERANT FIELDS

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ABSTRACT

The measurement of transducer transmitting sensitivity is normally performed under free-field conditions. In laboratory tanks this is often achieved by using a time gate to isolate the direct signal from reflected signals. However, for high Q, low frequency projectors, the free-field time available in most laboratory tanks may be too short. Therefore measurements have to be made in the presence of a reverberant field. Here one technique of extracting the direct field is investigated by plotting the variation of pressure squared against the reciprocal of distance squared. The gradient of this graph is proportional to the acoustic power radiated into the tank and from this the direct field pressure and projector sensitivity can be calculated.

1. INTRODUCTION

Projector sensitivity calibration requires the measurement of transmitted pressure for a known voltage applied to the transducer. The transmitted pressure is usually determined using a calibrated hydrophone and its sensitivity curve. Once the pressure is known at an arbitrary range the pressure at one metre can be calculated and hence the projector sensitivity. This type of measurement is normally performed under free-field conditions (where no reflections are present). In laboratory tanks a time gate can be used to isolate the direct signal from the reflections so that the measurements can effectively be made under free-field conditions. The time available between the arrival of the direct path signal and first reflection at the hydrophone is called the 'free time'. The pulsed signal must have finished, or reached a steady state, in this time interval if the result is not to be contaminated by reflected signals. However, high Q low frequency projectors require free-times greater than those available in most laboratory tanks. Measurements can be made in very large tanks to overcome this problem but these are prohibitively expensive. Alternatively 'sea trials' can be undertaken but these are also very expensive, so a solution is needed using laboratory sized tanks. It is, therefore, interesting to consider the possibility of making measurements under reverberant conditions, similar to those used in airborne acoustics.

When free times are needed which are greater than those available in the laboratory tank there is no advantage in using pulsed signals [1] and measurements might as well be made with continuous wave signals. The use of continuous waves means that there is a build up of reflected waves giving rise to a reverberant field. When pressure measurements are taken in the presence of the reverberant field the resultant field is a superposition of the direct and reverberant fields. Simple measurements of the total field will give a false estimate of the direct field pressure and, therefore, sensitivity. The direct field can, however, be extracted from these measurements by measuring the acoustic power in the tank. In this paper we describe a series of preliminary measurements to investigate this technique.
PROJECTOR SENSITIVITY MEASUREMENTS IN REVERBERANT FIELDS

2. THEORY

The projector sensitivity is defined by:

\[ S_p = 20 \log_{10} \left( \frac{p}{V_p} \right) \text{ dB re } 1 \mu Pa/V \text{ at } 1 \text{ m}, \]  

(1)

where \( S_p \) is projector sensitivity, \( p \) is the pressure (in \( \mu Pa \)) generated by the projector at a distance \( r \) and \( V_p \) is the potential applied across the projector [2,3].

The hydrophone sensitivity is defined by:

\[ S_h = 20 \log_{10} \left( \frac{V_h}{p} \right) \text{ dB re } 1 \text{ V/\( \mu Pa \)}, \]  

(2)

where \( S_h \) is hydrophone sensitivity, \( V_h \) is voltage received from the hydrophone and \( p \) is the pressure (in \( \mu Pa \)) incident on the hydrophone [2,3].

It is known that the pressure in an enclosure is due to two fields, the direct (initial) field and the reverberant (reflective) field [4]. If the direct sound field is assumed to be radiated uniformly in all directions then the direct sound pressure \( P_d \) at a distance \( r \) from the sound source is given by

\[ P_d^2 = \frac{Q}{4\pi r^2} \]  

(3)

where \( Q \) is acoustic power radiated by the source, \( \rho_0 \) is the volume density of the fluid and \( c \) is the speed of sound in the fluid [4]. The equilibrium value of the spatially averaged reverberant sound pressure \( P_r \), is given by

\[ P_r^2 = \frac{4\rho_0 c Q}{A} \]  

(4)

where \( A \) is the total sound absorption of the chamber [4]. Combining the equations (3) and (4) for the direct and reverberant sound fields (incoherently), gives the total pressure at a point in the sound field as

\[ P^2 = \rho_0 c Q \left( \frac{1}{4\pi r^2} + \frac{4}{A} \right) \]  

(5)

Now consider a projector radiating sound in to a tank and a hydrophone placed at a distance \( r \) from the this source. If measurements of pressure are made at different separations \( r \), then a graph of pressure squared against the reciprocal of separation squared can be plotted [5] & [6]. From equation (5) it can be seen that the gradient, \( m \), and y-intercept, \( C \), of this graph are defined as follows:

\[ m = \frac{\rho_0 c Q}{4\pi} \]  

(6)
PROJECTOR SENSITIVITY MEASUREMENTS IN REVERBERANT FIELDS

\[ C = P_r^2 = \frac{4\rho_0 cQ}{A}. \]  

From equation (6) it can be seen that the acoustic power radiated into the tank can be found from the gradient of the graph. From this value of power and equation (3) the pressure at a separation \( r \) can be calculated. This is the pressure due to the direct field only and can be used for sensitivity calibration purposes.

This method of extracting the power and, therefore, the direct field is only accurate if the reverberant field is diffuse. If the reverberant field is not diffuse the residuals of the gradient are not evenly distributed about the true gradient and can result in an error in the value of power. However a diffuse field distributes the residuals well, virtually cancelling out the reverberant field error in the gradient (direct field). To obtain a diffuse field many reflections are needed [4] and therefore the tank needs a long reverberation time. The spatially averaged reverberant field pressure, \( P_r \), will therefore be large for a diffuse field but will ensure an accurate determination of the gradient. In practice the reverberant field is made more diffuse by driving the projector with a noise source which excites the modes of the tank fairly evenly.

The reverberation time of the tank, \( T_r \), is given by

\[ T_r = \frac{0.164V}{A}, \]  

where \( V \) is the volume of the water in the tank and \( A \) is the total absorption of the tank and is given by

\[ A = \sum_i S_i a_i, \]  

where \( S_i \) is the \( i \)th surface area and \( a_i \) is the \( i \)th absorptivity of the boundary of the tank [4]. To obtain a long reverberation time and hence a diffuse field it is therefore necessary to have a large volume of water and a small absorptivity at the boundaries of the tank.

3. EXPERIMENTAL SET UP AND PROCESSING OF DATA

The experimental system, as shown in figure 1, consists of a 100 kHz white noise continuous wave signal used to drive the projector producing a diffuse reverberant field in the tank. The signal sent to the projector is also sent to a digital oscilloscope where it is recorded for use in calculating the projector sensitivity. The hydrophone measures the pressure at its position in the tank, and its output is amplified before being recorded using the digital oscilloscope. This amplification ensures an adequate signal to noise ratio for the received signals at the oscilloscope.
The received hydrophone signals were recorded and saved to disk for a series of projector/hydrophone separations. Each record contained 50,000 points, sampled at 4 µs, giving a total record length of 200 ms. This sampling time gives a Nyquist frequency of 125 kHz which is appropriate for a maximum frequency of 100 kHz in the transmitted signal.

In order to process the data each record was divided into 50 sections of 1000 points each, to give a section such as that shown in Figure 2. A Fast Fourier Transform was then carried out on each section to give 50 noise spectra. The mean of these spectra was then taken to give the mean voltage spectrum of the received hydrophone signal. Averaging the noise spectra in this way helps to reduce the variance of the spectra. The same process was used to analyse the voltage applied to the projector. The hydrophone voltage spectra were converted to pressure spectra using the hydrophone calibration curve which gave the sensitivity as a function of frequency.
PROJECTOR SENSITIVITY MEASUREMENTS IN REVERBERANT FIELDS

The power transmitted in any frequency band can be determined from the graph of pressure squared, for that frequency band, against the reciprocal of separation squared. An example of this for one frequency band is shown in Figure 3. The gradient of the graph is proportional to the acoustic power radiated into the tank in that frequency band. From the power radiated in each frequency band the pressure at one metre is calculated using the formula for the direct field, assuming uniform radiation. This, together with the spectrum for the projector drive voltage, enables the transmission sensitivity of the projector to be calculated. This is then compared with the free-field sensitivity of the projector which was determined previously by the National Physical Laboratory.

The sensitivity determined in this way is still influenced significantly by the reverberant field structure. This effect can be reduced by averaging the power data over a wider frequency range to obtain a smoother sensitivity curve. This was achieved by averaging the estimates of radiated power. An alternative approach would be to average the power spectra before taking the gradient of the pressure squared graph.

4. RESULTS

Measurements were made in four different tanks using two different projectors, an ITC1001 and an ITC1032. Tank 1 is made of polypropylene, 2.0 by 1.5 by 1.4 m in size, held together with steel body bands encapsulated in polypropylene. Tank 2 is a concrete tank, sunk into the ground, approximately 3.06 by 1.52 by 1.68 m in size. Tank 3 is a metal tank with a 9 mm thick polypropylene inner liner, 1.86 by 1.18 by 1.09 m in size. Finally tank 4 is made of 9 mm thick polypropylene, 2.72 by 1.51 by 1.32 m in size, held together with steel body bands encapsulated in polypropylene. The transmission sensitivity results determined from the acoustic power in the four tanks are shown in Figures 4 to 9. In all of the graphs the data points (x) denote the transmission sensitivity determined from acoustic power, whereas the solid line (—) denotes the reference data determined from free field measurements.

Figure 4 shows the transmission sensitivity obtained using individual bands of 244 Hz. As can be seen, the data has a significant scatter but is distributed around the reference curve. However, when the data is averaged, in power, over a greater bandwidth the amount of scatter is significantly reduced with the points following the
PROJECTOR SENSITIVITY MEASUREMENTS IN REVERBERANT FIELDS

genereal shape of the curve. This is shown in Figure 5, which shows the same results as Figure 4, except that the transmission sensitivity is averaged over four bins (to give a bandwidth of 977 Hz).

The results for projector ITC1001 taken in all four tanks, using an average over approximately 1 kHz, are shown in Figures 5 to 8. The graphs appear to show that the transmission sensitivity results are below the reference sensitivity curve for tanks 1 & 2. The points fit quite well with the curve for tank 3 and the points are above the curve for tank 4. Figure 9 shows similar results obtained for the ITC1032 projector in tank 3; here the results and reference curve agree well, even better than for the ITC1001 projector in tank 3.

Figure 6: Transmission sensitivity (dB re 1 μPa/V at 1 m) of the ITC1001 projector for a bandwidth of 1 kHz in tank 1.

Figure 7: Transmission sensitivity (dB re 1 μPa/V at 1 m) of the ITC1001 projector for a bandwidth of 977 Hz in tank 2.

Figure 8: Transmission sensitivity (dB re 1 μPa/V at 1 m) of the ITC1001 projector for a bandwidth of 977 Hz in tank 4.

Figure 9: Transmission sensitivity (dB re 1 μPa/V at 1 m) of the ITC1032 projector for a bandwidth of 977 Hz in tank 3.
PROJECTOR SENSITIVITY MEASUREMENTS IN REVERBERANT FIELDS

The differences between the reverberant field and free-field calibrations have been investigated as follows. Firstly the difference between the two calibrations was calculated on a point to point basis and then averaged over the whole frequency range. This gives the average difference, or systematic shift, between the two curves in dB. The results of this analysis are shown in Figures 10 and 11 for the two projectors and four tanks using bandwidths of approximately 250 Hz and 1 kHz respectively. (The bandwidths are actually 250 Hz and 1 kHz for tank 1, and 244 Hz and 977 Hz for tanks 2 to 4.)

<table>
<thead>
<tr>
<th>Projector</th>
<th>Tank 1</th>
<th>Tank 2</th>
<th>Tank 3</th>
<th>Tank 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITC 1001</td>
<td>-2.1</td>
<td>-2.6</td>
<td>-2.7</td>
<td>1.3</td>
</tr>
<tr>
<td>ITC 1032</td>
<td>-2.3</td>
<td>-3.3</td>
<td>-0.7</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

Figure 10 : Mean difference (in dB) between the reverberant field and free-field calibrations for a bandwidth of approximately 250 Hz for the four tanks and two projectors.

<table>
<thead>
<tr>
<th>Projector</th>
<th>Tank 1</th>
<th>Tank 2</th>
<th>Tank 3</th>
<th>Tank 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITC 1001</td>
<td>-1.5</td>
<td>-2.1</td>
<td>-1.1</td>
<td>2.1</td>
</tr>
<tr>
<td>ITC 1032</td>
<td>-2.0</td>
<td>-3.1</td>
<td>0.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure 11 : Mean difference (in dB) between the reverberant field and free-field calibrations for a bandwidth of approximately 1 kHz for the four tanks and two projectors.

5. DISCUSSION

The results presented in Figures 4 to 8 indicate that the trend of the reverberant field calibrations generally follow that expected from the reference free-field calibrations. Clearly there is a significant scatter in the 250 Hz bandwidth results, but those for a 1 kHz bandwidth have a much smaller scatter with a range of about ± 2 dB. On some of the graphs there is potential evidence of a periodicity in the fluctuations (especially Figures 7 and 9) which may be related to the modal structure within the tanks. The results shown do indicate mean shifts or offsets from the reference results. The graphs, and Figure 11, indicate that these are largest for tanks 1 and 2, while they are of the opposite sign for tank 4. It is not understood why this is so and further work needs to be done to understand this.

As was mentioned earlier, in order to obtain an accurate result for the transmission sensitivity from power measurements an accurate determination of the gradient of the pressure squared against the reciprocal of separation squared graph is needed. To obtain an accurate gradient requires a diffuse, reverberant field so that the residuals of the gradient cancel each other out evenly, leaving just the original gradient due to the direct field.

The projectors ITC1001 and ITC1032 are fairly omnidirectional, however the ITC1001 varies by approximately 0.5 dB with direction and the ITC1032 varies by approximately 1 dB with direction. This will produce a slightly less diffuse reverberant field and therefore introduce an error in the gradients of the graphs.

It is interesting to consider the relative suitability of the tanks used for this type of measurement. Tank 1 has a volume of 4.2 m³, tank 2 a volume of 7.82 m³, tank 3 a volume of 2.39 m³ and tank 4 a volume of 5.41 m³. Given that the larger the volume of water the greater the reverberation time of the tank and so the more diffuse the reverberant field, it would be expected that the amount of scatter on the reverberation calibration results would be less for larger tanks. Conversely for a highly absorbent tank it would be expected that the amount of scatter on the graphs would be large. To test this the reverberant field level in the tanks was measured, a higher
PROJECTOR SENSITIVITY MEASUREMENTS IN REVERBERANT FIELDS

reverberation level indicating a longer reverberation time. Tank 3 appears to have a reverberant field level approximately 20% higher than tank 2, and tank 4 has a field level approximately 15% higher than tank 3. It would be expected that tank 2 has a lower reverberant field level, despite its larger volume, due to the higher absorption of its concrete walls which are coupled into the earth. Also tank 4 would be expected have a higher reverberant field level than tank 3 as a result of its larger volume. Therefore, the amount of scatter on the sensitivity graphs would be expected to be greatest for tank 2 and least for tank 4. However the scatter of the results on the sensitivity graphs do not show any particular trend. This may be due to the limited range of reverberation times of the tanks tested so far. The reverberant results, averaged over approximately 1 kHz, are in better agreement with the free field results than with those averaged over 250 Hz. This is assumed to be due to the effect of standing waves in the tank cancelling each other out when averages are taken over a wider bandwidth.

6. CONCLUSIONS

The results seem to indicate that this method has significant promise but is not yet a very accurate method of determining projector sensitivity. Using a more diffuse field seems to improve the accuracy as does averaging over a wider bandwidth which tends to cancel out the effects of the standing wave modes of the tank. It is intended to pursue this technique with a detailed study to investigate the influences of the tank size, shape and construction, as well as processing techniques. The potential advantages of making multiple scans and using longer data sequences will also be investigated. Overall the aim is to determine the potential of this technique and how it is influenced by the modal structure of the tank.

7. ACKNOWLEDGEMENTS

The authors would like to thank Stephen Robinson and the National Physical Laboratory for the loan of the projectors, supply of reference calibration data, assistance with one set of tank measurements and advice received. We would further like to thank Richard Hazelwood for discussions and advice. The support of EPSRC in the form of a studentship (SJE) is gratefully acknowledged.

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Transducer transmitting sensitivity measurements in restricted environments

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Abstract

Transducer transmitting sensitivity is normally measured under free-field conditions. In laboratory tanks, this is often achieved by using time gating to isolate the direct signal from the reflected signals. However for high-Q, low-frequency projectors, the free-time availability in laboratory tanks may be too short. A measurement technique has been developed to perform these calibration measurements in the presence of a reverberant field. The measurements have been performed at ultrasonic frequencies so that the free-field sensitivity can be determined in order to assess the accuracy of this new technique. The technique involves extracting the direct field from the combination of the direct and reverberant fields by plotting the variation of pressure squared against the reciprocal of separation squared. The gradient of this graph is proportional to the acoustic power radiated into the tank, and from this, the direct field pressure and projector sensitivity can be calculated. This paper investigates this technique and looks at the influences of tank size and construction, as well as the effect of averaging over different bandwidths. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Calibration, Reverberant, Tank, Transducer

1. Introduction

To calibrate a transmitting transducer (projector), the direct field pressure at a known distance from the transducer must be measured for a known applied voltage in order to calculate the transmitting sensitivity. In laboratory tanks, the pressure is usually measured with a calibrated hydrophone by reference to its sensitivity curve. The direct field of the sound source is obtained by time-gating the signal sent to the projector so that the hydrophone receives the direct signal in the absence of reflections. The time available to receive this signal prior to the arrival of tank wall reflections is called the ‘free time’ and is dependent on the position of the transducers in the tank and the tank’s size and shape. In this time, the transducer output needs to reach its steady-state level, and the time needed for this depends on the frequency of the signal and Q of the projector. In most laboratory tanks, projectors cannot be accurately calibrated, but for low-frequency and high-Q transmitters, there is insufficient free time. This can be overcome by using large tanks, lakes or sea trials, but these are expensive, so a solution using laboratory-sized tanks would be advantageous.

One possible solution is to utilize the methods used in room acoustics where a continuous wave signal is employed. This results in the build-up of reflected waves giving rise to a reverberant field. The resultant field is a superposition of the direct and reverberant fields. Simple measurements of the total field will give a false estimate of the direct field pressure and, therefore, sensitivity. The direct field can, however, be extracted from these measurements by measuring the acoustic power in the tank.

2. Theory

The projector sensitivity is defined by:

\[ S_p = 20 \log_{10} \left( \frac{p}{V_p} \right) \text{ dB rel 1 \mu Pa/V at 1 m}, \] (1)

where \( S_p \) is the projector sensitivity, \( p \) is the pressure (in \( \mu \)Pa) generated by the projector at a distance, \( r \), and \( V_p \) is the potential applied across the projector [1,2].
The hydrophone sensitivity is defined by:
\[
S_h = 20 \log_{10}\left(\frac{V_h}{p}\right) \text{ dB re } 1 \text{ V} \mu\text{Pa},
\]  
(2)

where \(S_h\) is the hydrophone sensitivity, \(V_h\) is the voltage received from the hydrophone, and \(p\) is the pressure (in \(\mu\text{Pa}\)) incident on the hydrophone [1,2].

It is known that the pressure in an enclosure is due to two fields, the direct (initial) field and the reverberant (reflective) field [3]. If the direct sound field is assumed to be radiated uniformly in all directions, then the direct sound pressure, \(P_d\), at a distance, \(r\), from the sound source is given by
\[
P_d = \frac{\rho_0 c Q}{4\pi r^2},
\]  
(3)

where \(Q\) is the acoustic power radiated by the source, \(\rho_0\) is the volume density of the fluid, and \(c\) is the speed of sound in the fluid [3]. The equilibrium value of the spatially averaged reverberant sound pressure, \(P_r\), is given by
\[
P_r = \frac{4\rho_0 c Q}{A},
\]  
(4)

where \(A\) is the total sound absorption of the chamber [3]. Combining Eqs. (3) and (4) for the direct and reverberant sound fields (incoherently) gives the total pressure at a point in the sound field as
\[
P^2 = \rho_0 c Q \left(\frac{1}{4\pi r^2} + \frac{4}{A}\right).
\]  
(5)

Now consider a projector radiating sound into a tank and a hydrophone placed at a distance, \(r\), from this source. If measurements of pressure are made at different separations, \(r\), then a graph of pressure squared against the reciprocal of separation squared can be plotted [4,5]. From Eq. (5), it can be seen that the gradient, \(m\), of this graph is given by:
\[
m = \frac{\rho_0 c Q}{4\pi}.
\]  
(6)

Thus, the acoustic power radiated into the tank can be found from the gradient of the graph. From this value of power and Eq. (3), the direct pressure and sensitivity can be calculated.

This method of extracting the power and, therefore, the direct field is only accurate if the reverberant field is diffuse. If the field is not diffuse, the residuals of the gradient are not evenly distributed about the true gradient, and this can result in an error in the value of power. To obtain a diffuse field, many reflections are needed [3], and therefore, the tank needs a long reverberation time. In practice, the reverberant field is made more diffuse by driving the projector with a noise source that excites the modes of the tank fairly evenly.

3. Experimental set-up and data processing

The experimental system, shown in Fig. 1, consisted of a noise generator supplying a continuous 100 kHz white-noise signal to drive the projector and produce a reverberant field in the tank. The projector signal was recorded by a digital oscilloscope for use in calculating the projector sensitivity. The hydrophone output was amplified and also recorded by the digital oscilloscope for a number of transducer separations. Each trace contained 50 000 points, sampled at 2 μs, giving a recording length of 100 ms; this was divided into 50 sections of 1000 samples each, and a Fast Fourier Transform was then performed on each section. These 50 spectra were averaged to give a mean spectrum where the intrinsic noise of the signal had been reduced so that the spectrum of the projector could be seen. The same process was used to analyse the voltage applied to the projector. The hydrophone voltage spectrum was converted to a pressure spectrum using the hydrophone calibration.

For any frequency band, the power transmitted was determined from the graph of pressure squared against the reciprocal of separation squared [6]. The gradient of the graph was proportional to the acoustic power radiated into the tank. From this, the pressure at one metre was calculated using Eq. (3) for the direct field, assuming uniform radiation. This, together with the spectrum for the projector drive voltage, enabled the transmission sensitivity of the projector to be calculated. This will be referred to as the reverberant field sensitivity.
4. Results and discussion

A series of reverberant field sensitivity calibrations have been performed and compared with a free-field sensitivity calibration, which was performed using a hydrophone calibrated at the National Physical Laboratory. An example of the comparison of these two sensitivities is shown in Fig. 2. As can be seen, the general trend of the reverberant field sensitivity follows that of the free-field sensitivity, with random fluctuations of no more than 2 dB. These are expected for this technique but may be greater if the reverberant field is not diffuse, as described in the theory section.

A quantitative way of analysing the data is needed. One approach is to calculate the difference between the reverberant field sensitivity and the free-field sensitivity for each frequency point and then compute the mean and standard error of this difference over the whole frequency range. Table 1 shows these values for the results shown in Figs. 2–5. If the root mean square average of the reverberant sensitivity curve is taken over four frequency bands, to give a resolution of 2 kHz, the curve is smoothed as shown in Fig. 3. This reduces the fluctuations by a factor of two but does not reduce the mean difference (Table 1).

Three different tanks were used for this study, as described in Table 2. In each tank, measurements were made to see how the accuracy of the method varied with position in the tank. One set of measurements was

Table 1
Mean and standard error of difference between reverberant and free-field sensitivities, calculated over the frequency range 10–100 kHz

<table>
<thead>
<tr>
<th>Fig. 2</th>
<th>Fig. 3</th>
<th>Fig. 4</th>
<th>Fig. 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean difference (dB)</td>
<td>0.5</td>
<td>0.7</td>
<td>0.34</td>
</tr>
<tr>
<td>Standard error (dB)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.05</td>
</tr>
</tbody>
</table>
made with the traverse direction parallel to the length of the tank, centred along the length and half way across the width. Other measurements were then made along parallel traverses displaced across the width of the tank. The average of five such measurements for tank 1 (Fig. 4) shows a better agreement than that in Fig. 2. Many measurements were taken in the three tanks for different positions of length, width, depth and orientation. It was found that changing the tank or position within the tank varied the mean difference, although the differences were not normally significant, provided the measurements were not carried out close to the tank walls. The global average of all the reverberant field sensitivity measurements is shown in Fig. 5. Table 1 shows that there is no increase in accuracy over that for averaging in one tank.

Although there are small changes in the mean difference for different positions, analyses for systematic variations in the results do not generally show any correlation with position. Fig. 6 shows the mean difference and standard error for different locations of traverses in tank 1, which shows no significant correlation. This is not unexpected, as the reverberant field should have the same characteristics throughout the tank except near the walls.

5. Conclusions

A method of calibrating projectors in small tanks using reverberant techniques has been demonstrated and shown to produce average results accurate to within 1 dB. Individual measurements over a frequency band of 2 kHz show random uncertainties of about 1 dB. There appears to be no correlation between the accuracy of this technique and the position in the tank provided the measurements are not made close to the tank walls. These results indicate that the construction of the tank has a small effect on the accuracy of the technique, with variations between tanks typically of less than 1 dB. Further measurements need to be made in tanks with a greater range of reverberation times so that the effects of anechoic tanks and highly reverberant tanks can be seen. The technique is appropriate to high-Q sources and has potential for lower frequency projectors and other sources in restricted environments.

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