All-optical nonlinearities in semiconductor multi-quantum well waveguides for optical switching at 1.55mum

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ALL-OPTICAL NONLINEARITIES IN SEMICONDUCTOR MULTI-QUANTUM WELL WAVEGUIDES FOR OPTICAL SWITCHING AT 1.55μm.

Submitted by Ian Edward Day
for the degree of PhD
of the University of Bath
1995

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Abstract

A polarisation rotation gate is demonstrated in an GaAs/AlGaAs MQW waveguide at a wavelength of 1.55\(\mu\)m using the ultrafast Kerr nonlinearity. The two photon absorption (TPA) and polarisation rotation characteristics of two GaAs/AlGaAs MQW waveguides have been investigated at wavelengths either side of the half band gap wavelength. The Kerr coefficient was obtained from characterisation of nonlinear polarisation rotation. The optimum gating action was achieved at low coupled powers (<60W), operating 28nm below the half band gap wavelength. The effects of TPA limiting the polarisation rotation, were observed.

The carrier assisted nonlinearities bandfilling and plasma effect were modelled and used to design a MQW layer structure which would exhibit efficient nonlinear refraction. Such MQW waveguides were tested to evaluate their nonlinear refraction characteristics using self phase modulation techniques (SPM). Phase modulation in excess of \(\pi\) radians was demonstrated for a coupled pulse energy of 30pJ.

The design, construction and evaluation of two all-optical switches using the bandfilling nonlinearity and operating at \(\sim 1.55\)\(\mu\)m are described. A polarisation rotation gate is constructed using an InGaAsP/InGaAsP MQW waveguide as the nonlinear element. An integrated all-optical device, a nonlinear directional coupler is also demonstrated. All-optical switching of a 30ps optical pulse is demonstrated for a pulse energy of 10.5pJ in a polarisation rotation gate.

The recovery time of carrier assisted nonlinearities has been measured using SPM techniques. With an applied field of \(\sim 270kV/cm\) the recovery time of the nonlinearity was reduced to 18±3ps whilst maintaining 50% of the phase modulation observed with no bias applied. Carrier screening of applied fields has been observed in an InGaAsP electro-absorption modulator using high speed photo-current measurements. Measurements were carried out on similar devices, one with InGaAlAs barriers and one with a strained active layer. The strained layer device exhibited the fastest carrier decay time (<40ps).
"Of the writing of many books, there is no end, and much study wearies the body!"

Eccl. 12v12.
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**Journal Publications**

The following papers have been published in refereed journals arising from the work presented in this thesis.


Conference Publications

The following papers have been presented at national and international conferences arising from work presented in this thesis.


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Outline of Thesis

This thesis begins with an overview of semiconductor quantum wells in chapter 2. The properties of quantum wells are discussed in detail including the ability to tailor band gaps; increased density of states at the band edge; increased oscillator strength due to increased e-h wave function overlap and the splitting of the fourfold degenerate hole band into separate light and heavy holes at \( k=0 \). Optical nonlinearities of MQW material are overviewed in chapter 3. The classical theory of optical nonlinearities is discussed in detail, with reference to intensity dependent (Kerr type) nonlinear refraction and two photon absorption. Carrier assisted nonlinearities, which cannot be explained with classical theory, are also introduced. Bandfilling and the plasma effect are elaborated in detail. This discussion considers both the physical origin of the nonlinearity and defines quantitatively the effect of the nonlinearity on the refractive index of the material. Appropriate material systems in which to exploit these nonlinearities for all-optical switching at a wavelength of 1.55\( \mu \)m are established.

In chapter 4, the Kerr type nonlinearity in an MQW waveguide is exploited to form an all-optical switch. A novel polarisation rotation gate is constructed using a GaAs/AlGaAs MQW waveguide as the nonlinear element. Carrier assisted nonlinearities are considered in chapters 5, 6 and 7. The carrier assisted nonlinearities of MQW material are considered in order to design an InGaAsP/InGaAsP based MQW waveguide which will exhibit efficient nonlinear refraction at a wavelength of 1.55\( \mu \)m. The nonlinear refraction of such a waveguide (manufactured to the specifications arrived at by this design procedure) is measured. All-optical gates, a polarisation rotation gate and a nonlinear directional coupler, are subsequently constructed from the same MQW material designed in chapter 5. The design and performance of these gates is described in chapter 6.
In chapter 7, measurements are presented of the recovery time of carriers in InGaAsP/InGaAsP MQW structures under the application of a reverse bias applied perpendicularly to the plane of quantum wells. Theoretical modelling of the results is carried out to determine the dominant mechanism which limits carrier escape from the quantum wells. Lastly in chapter 8, the conclusions of this work are set out.
An introduction to all-optical switching is presented. The motivation for the work presented in this thesis is outlined and an overview is given of optical switching in semiconductor waveguides.
1. Introduction

The aim of the work described in this thesis was to study all-optical refractive nonlinearities around the wavelengths of 1.55µm in III-V semiconductor quantum wells, specifically GaAs/AlGaAs and InGaAsP/InP material systems. These nonlinearities will be exploited in waveguide structures to construct all-optical switching devices, using a knowledge of the nonlinearities to reduce the optical powers required to achieve switching of optical pulses to those which can be produced by diode laser sources. Switching of femtosecond pulses will be demonstrated in GaAs/AlGaAs waveguides by exploiting Kerr-type nonlinearities. All-optical switching of picosecond optical pulses will be demonstrated in InGaAsP/InP waveguides using carrier enhanced nonlinearities, predominantly bandfilling. The recovery time of carrier assisted nonlinearities will be studied in order to reduce this time below the carrier recombination time.

1.1 Review and Motivation

At present, long haul telecommunications data is carried in single mode optical fibre links. The use of optical fibre has revolutionised telecommunications, allowing transmission distances of several tens of kilometres between repeater stations. Currently, the operating wavelength of these fibre systems is 1.3µm, the dispersion minimum of the silica fibre [1]. The distance between repeaters (which detect the optical signal, and re-transmit the data back onto the fibre) is limited by loss in the silica fibre. Therefore, in order to increase the distance over which signals can be transmitted without the need for regeneration, attention has been focused at the 1.55µm wavelength band, corresponding to the loss minimum of the silica fibre [1]. Interest in this wavelength band has been further stimulated by the development of laser diode pumped Erbium doped Optical Fibre Amplifiers (EDFA) [2], giving ~30dB (typically) of gain without the need to remove the optical signal from fibre.
However, in the drive to transmit signals with ever increasing bandwidths over long distances, a serious problem exists with the transmission medium (silica fibre). The fibre core has a wavelength dependent refractive index, therefore different frequencies will propagate along the fibre at different velocities. This phenomenon is known as chromatic dispersion. Present optical fibre links use laser diodes to inject infra-red light into the fibres. Under high speed modulation, the output wavelength of a diode laser optical pulse changes throughout the evolution of the pulse (chirp) due to changes of carrier concentration in the laser [3]. The different frequency components of the pulse travel at unequal velocities along the silica fibre, interfering with adjacent pulses in a data stream and rendering the signal undetectable. For example, a 30ps transform limited pulse will have broadened to ~56ps after transmission along 30km of single mode optical fibre (dispersion parameter 15ps/nm.km). Considerable progress has been made in the reduction of chirp in directly modulated lasers, by using multi-section diode lasers [4], and, more recently, strained active layer lasers [5]. Dispersion shifted fibre, with the dispersion minima of the fibre shifted to 1.55μm, has been developed [6] to overcome this problem, but it will be many years before the fibre installed at present is replaced with this new fibre.

Considerable effort has been made to overcome the dispersion problems associated with the transmission of high bandwidth signals over long distances. Dispersion management techniques have been successfully employed using a negative chirp modulator [7]. Such a modulator pre-chirps the optical signal, allowing the dispersion in the optical fibre to correct this distortion as the pulse is transmitted. By using this technique, transmission of a 10Gbit/s signal over a distance of 100km has been demonstrated [8]. Other methods, employing dispersion compensation techniques such as phase conjugation and chirped fibre grating Bragg filters, have also been demonstrated. An optical phase conjugator is a device which takes an input signal centred on the optical carrier frequency and outputs its phase conjugate, that is, a signal with a spectrum which
has been inverted about the carrier frequency. This effect is obtained by a complex nonlinear process known as degenerate four-wave-mixing [9]. Thus, by placing a phase conjugation device half way along a length of dispersive fibre, the dispersion in the two halves of the fibre can be cancelled out [10]. Using chirped fibre grating Bragg filter involves the construction of a structure within an optical waveguide which has a large negative group velocity dispersion (GVD) per unit length. The structure takes the form of a Bragg reflector whose pitch varies linearly along its length [11]. Such a filter may be fabricated to have the same dispersion in a few centimetres as a long haul optical fibre link (tens of kilometres), but with the opposite sign. When inserted in-line, the filter will compensate for the dispersion of the fibre [12]. This technique can also be implemented using longer lengths of dispersion compensating fibre which has negative chromatic dispersion [13].

Perhaps the most significant development in overcoming dispersion effects in long distance optical fibre links is the use of solitons [14]. Optical solitons arise from a balance between the propagation effects of GVD and intensity dependent refractive index in the optical fibre [14]. The fundamental soliton, which propagates without change in its temporal or spectral form, can be considered as that propagation mode which continuously balances the oppositely induced phase shifts of negative GVD and self-phase modulation. Thus transmission at Gigabit per second data rates over thousands of kilometres is possible in future telecommunications systems [15,16].

A further limit to the bandwidth of current fibre network is that optical links exist only between exchanges. All optical data on the fibre must, on arrival at an exchange, be detected, converted to electrical signals and switched using electronics. The major part of the data will not drop down to the local network, but be re-converted to an optical signal and passed on to the next exchange until it reaches its destination. In order to simplify this process, it would be desirable to extract from the fibre network only that
data which is intended for a particular exchange, leaving the rest of the data as an optical signal in the fibre network. A device which would allow such extraction is an optical switch.

Therefore there exists a requirement in both present and future telecommunications systems for high speed, low chirp, optical modulators and switches to make greater use of the potential bandwidth offered by single mode optical fibres (~50,000GHz) [1].

1.2 Multi-Quantum Wells in Semiconductor Waveguides

Initially, this requirement for high bandwidth optical modulation and switching in optical fibre telecommunications systems stimulated much research into optical fibre switches utilising the nonlinear refraction exhibited by silica fibres (see for example [17-21]). These fibre-based devices demonstrated all-optical switching on femtosecond timescales as the nonlinear refraction is a result of nonlinear susceptibility induced by an applied optical field [22]. However, the nonlinear refractive index exhibited by these fibres is comparatively weak, requiring high optical powers (>100W) and/or several tens of metres of fibre to operate. Short (picosecond or femtosecond) optical pulses of an appropriate wavelength with such large optical powers cannot, at present, be generated by compact and efficient sources such as laser diodes. Mode-locked colour-centre lasers pumped by bulky YAG lasers which requiring kilowatts of electrical power must be used [23]. Such lasers are considered to be unsuitable for telecommunications applications due to their size and power requirements.

In recent years there has been much interest in all-optical nonlinearities in semiconductor multiple quantum well (MQW) structures since quantum confinement in semiconductor material was first demonstrated by Dingle et. al. [24]. With the advent of epitaxial growth techniques such as metal-organic vapour phase epitaxy (MOVPE), also called metal organic chemical vapour deposition (MOCVD) [25], and molecular beam epitaxy (MBE) [26] it was possible to grow single atomic layers. This technology
allowed the production of structures exhibiting quantum effects due to confinement caused by the composition of the materials grown. These structures have been greatly developed for the production of high efficiency laser diodes which utilise the two-dimensional carrier confinement [27].

A semiconductor quantum well structure is a multi-layer structure grown from materials chosen such that the potential structure formed exhibits quantum confinement of carriers [28]. The use of MQW structures in conjunction with optical waveguides formed by semiconductors with differing refractive indices has stimulated great interest in such MQW waveguide devices for opto-electronic applications. The confinement of carriers in MQW waveguides enhances the refractive and absorptive nonlinearities of the semiconductor material and has enabled the observation and exploitation of such effects [29]. Thus, the carrier confinement exhibited by MQW structures has not only lead to the development of high efficiency laser diodes, but has stimulated interest in semiconductor MQW waveguides for optical modulation and switching.

1.3 Recent Advances in Optical Modulation and Switching

Recently there has been a great deal of work published concerning electro-absorption modulators (see for example [30-34]). To date, it is these devices which have principally been investigated for use as external modulators for use in optical fibre communications systems. These devices, based on multiple quantum well (MQW) ridge waveguide structures, rely on an applied electric field to modulate the output light intensity. Electro-absorption modulators have a p-i-n layer structure. A p-i-n structure is used to allow the application of a bias voltage across the intrinsic region of the device. The quantum wells are grown into the intrinsic region.

The nonlinearity utilised by this device is the Quantum Confined Stark Effect (QCSE). Application of a bias voltage perpendicular to the quantum wells causes a shift in the wavelength of the absorption band edge of the quantum wells to longer
wavelengths. Therefore, considering light passing along the device at a fixed wavelength below the absorption band edge, application of bias voltage increases the absorption of the device to that light. In order to achieve a high switching rate the applied bias must be switched at RF frequencies. This use of an RF drive to the modulator limits the switching speed of such devices to ~50GHz due to capacitive effects in the waveguiding structure [35]. To achieve high modulation frequencies a near travelling wave electro-absorption modulator has been developed [35]. At high modulation frequencies, the RF drive to the device contacts can be considered to be a travelling wave along the device. By ensuring that the applied RF signal propagates in the same direction as the optical signal and at approximately the same velocity, high bandwidth modulation can be achieved. However, capacitive effects, limited by the physical dimensions of the device, restrict this type of modulator to switching frequencies of around 50GHz [36].

In order to achieve higher switching rates, other methods for optical switching must be sought which do not have the problems inherent in providing high speed electrical contacts to an optical device. One such method is all optical switching, i.e. using an optical signal either to modulate another signal (Cross Phase Modulation) or to modulate itself (Self Phase Modulation). The prime motivation for looking at all optical switching is that it offers the possibility of achieving greater switching rates than is fundamentally possible with optoelectronic modulators and switches.

There are two principle routes to producing all optical switching using all optical nonlinearities in multi-quantum well waveguide structures. The first method for producing all optical switching is to utilise carrier based nonlinearities. Carriers, whether generated by current injection or optical absorption, cause a reduction in the material refractive index. This change in refractive index will cause light passing along the guide to experience a phase shift relative to the case with no carriers present. This phase shift
can then be employed as the basis for an interferometric switching device. Possible devices which would use such a nonlinearity are directional couplers and Mach-Zehnder interferometers.

Such carrier based nonlinearities have great potential for communications applications because they are relatively efficient, requiring only a few watts of optical power to produce sufficient phase shift necessary for switching (i.e. \( \pi \) radians). These power levels are of the order of those capable of being produced by diode lasers, and are therefore more suitable for telecommunications applications due to their low power requirements and small physical dimensions. Carrier based nonlinearities have been recently demonstrated experimentally using carriers generated by optical absorption at wavelengths around 1.55\( \mu \)m (i.e. a wavelength suitable for optical fibre communications) [37-41]. The principle carrier based nonlinearity of interest for optical switching is phase space (or band) filling. This is the bleaching of absorption due to the presence carriers and hence an associated change in refractive index. Others include the plasma effect (the change in refractive index due to the presence of free carriers) and band gap renormalisation (the change in band gap due to carrier screening). Therefore it is these carrier induced nonlinearities which have been investigated (see chapters 6 and 7) in order to demonstrate all-optical switching at wavelengths around 1.55\( \mu \)m.

These carrier induced nonlinearities, generated by photon absorption, are limited in their recovery time due to the carrier recombination time which is of the order of nanoseconds. Thus the switching speed which can be achieved with such a recovery time is less than a Gigahertz. The limitation arises because in order to switch off a carrier based nonlinearity, carriers must be removed from the quantum well and the intrinsic region of the waveguide. It is this which has limited their use in optical switching. However, switching speeds can be increased by the application of a dc bias [42-45]. The dc bias has the effect of pulling carriers out of the quantum wells and sweeping them out of the intrinsic region of the waveguide. Theoretical modelling indicates that
thermionic emission is the dominant mechanism for carrier escape from shallow quantum wells [46]. Resonant tunnelling, when the n=1 sub band of one quantum well is aligned with the n=2 sub band of the adjacent quantum well, with a recovery time of 20ps has also been reported [47], as has thermal assisted tunnelling [46]. Carrier sweepout has been investigated in the MQW structures used for all-optical switching to determine the limiting mechanism for carrier escape from quantum wells (see chapter 7).

Very recently nonlinearities in active semiconductor waveguides have been shown to have great potential for all-optical switching, with optical demultiplexing being demonstrated at 40Gbit/s [48]. These nonlinearities have been observed experimentally [49] and some modelling of these nonlinearities has been carried out (see for example [50,51]) though the precise nature of the physics behind these nonlinearities is not yet fully understood. An investigation of these nonlinearities is beyond the scope of this work.

The problems associated with carrier based nonlinearities may be avoided by utilising a third order refractive nonlinearity. Because this nonlinearity does not require carriers to operate its speed is not limited by carriers. These nonlinearities are therefore of interest for their potential of terahertz switching rates [52-56]. However these nonlinearities are inefficient when compared with carrier induced nonlinearities, requiring peak optical powers of the order of a hundred Watts to operate.

In order to achieve the necessary power levels in the waveguide to achieve switching losses due to absorption of photons must be avoided. It is therefore necessary to work at wavelengths below half the band gap. At such wavelengths carriers cannot be generated by either linear or two photon absorption. The optical powers necessary for ultrafast nonlinearities are at present only capable of being produced by mainframe laser systems and are beyond the peak powers which laser diodes are presently capable of producing. Therefore these ultrafast nonlinearities are, at present, of limited use for telecommunications applications. However, these optical nonlinearities are attractive
for terahertz all-optical switching because there is minimal absorption of light, implying that devices employing the third order nonlinearity can have a large throughput. Future diode laser sources may make utilising third order nonlinearities a viable option for switching at telecommunications wavelengths. A novel all-optical gate is demonstrated in chapter 4 in the form of a birefringent polarisation rotation gate which exploits the asymmetric properties of quantum wells (see chapters 2 and 3).
1.4 References


Introduction to the Physics of Multi-quantum Well Structures.

The properties of quantum confined states in semiconductor material are discussed. These properties include the ability to tailor band gaps simply by altering the thickness of quantum well material; increased density of states at the band edge; increased oscillator strength due to increased e-h wave function overlap and the splitting of the fourfold degenerate hole band into separate light and heavy holes at \( k=0 \). The advantages of quantum confined semiconductor structures for opto-electronic interactions are introduced.
2.1 Introduction

In recent years the ability to grow very thin (nanometre) layers of semiconductor material has led to the production of structures where the physical dimensions of the structure are comparable with the de Broglie wavelength of the carriers. The quantisation of the energy levels in thin semiconductors has been of interest for some time [1]. With the advent of epitaxial growth techniques such as metal-organic vapour phase epitaxy (MOVPE), also called metal organic chemical vapour deposition (MOCVD) [2], and molecular beam epitaxy (MBE) [3] it was possible to grow single atomic layers. This technology allowed the production of structures exhibiting quantum effects due to confinement caused by the composition of the materials grown. As discussed in chapter 1, such structures have been of great interest for the development of high efficiency semiconductor lasers. The properties of quantum confined structures are also of interest for all-optical switching applications. Therefore, the properties of quantum confined structures are discussed in this chapter. These properties form the basis of the optical switches discussed in subsequent chapters. The quantum effects demonstrated by such a structure are a result of the carriers existing in a 2-dimensional system, the physics of which is very different from that of bulk material. A typical layer structure is shown in figure 2.1.1.

The confinement of carriers arises from the choice of materials used. In order to produce confinement, semiconductor material with different band gaps are grown. It is usual practice to use materials with the same lattice constant to produce a uniform crystal structure. A schematic drawing of the potential structure of the MQW material shown in figure 2.1.1 is shown in figure 2.1.2. Any carriers excited in the structure will be confined to the potential wells, and will exhibit quantum confinement effects providing the width of the potential well is sufficiently narrow.
The first optical confirmation of the quantisation of energy levels was made by Dingle et. al. in 1974 [4]. The advantages of quantum confinement are numerous and include: the ability to tailor band gaps simply by altering the thickness of quantum well material; increased density of states at the band edge; increased oscillator strength due to increased e-h wave function overlap. Quantisation also has the significant effect of splitting the fourfold degenerate hole band into separate light and heavy holes at k=0. For an introduction to quantum wells, see Weisbuch [5].
The high density of states close to the band edge of an MQW semiconductor is the major advantage of quantum well systems when looking for carrier based effects. A high carrier density can be achieved with far few carriers than would be possible in the bulk case because of the high density of states even at low energies and the small physical dimensions of the QW. It is this potential efficiency of carrier based nonlinearities that is the motivation for looking at MQW structures to produce efficient nonlinear optical switches.

The aim of this project is to exploit these quantum confinement effects for all-optical switching at a wavelength of 1.55\textmu m for telecommunications applications. Therefore, suitable materials for this application will be transparent to light with a wavelength of 1.55\textmu m. Two such material systems are GaAs/AlGaAs and InGaAsP/InP. The InP based material system is more flexible than the GaAs system as quaternary InGaAsP can be manufactured with a band edge between 0.75-1.35eV, depending on composition, while keeping the same lattice constant as InP [6]. This allows potential well height to be chosen independently of the desired band gap and well thickness.

In order to observe optical nonlinear effects in semiconductor quantum well structures, high optical intensities must be achieved in the quantum wells. Semiconductor optical waveguides allow the confinement of light in either one or two dimensions [7]. These structures use the refractive properties of semiconductors to confine and guide light. By placing the quantum wells in the guiding region of the waveguide, an intense optical field can be achieved for modest optical powers which can interact with the quantum well material. The theory of one dimensional slab waveguides is discussed in Appendix 3. This theory is developed to model two-dimensional confinement of light in waveguides and thus enable the design of ridge waveguides suitable for generating the high optical intensities required for observing nonlinearities in quantum wells.
2.2 Quantum Mechanics of Isolated Quantum Wells

The confinement of carriers in semiconductor heterostructures has been briefly outlined in the previous section (see Appendix 3 for more details). If the width of the potential well is made to be of a comparable wavelength to the de Broglie wavelength of the carriers (~5-10nm), quantum mechanical effects dominate the properties of carriers in the potential well. In particular, the allowed energy levels and the density of states are modified from those of bulk (3D) material.

A schematic drawing of a one-dimensional potential well is shown in figure 2.2.1. The well width is w, with a potential step V.

\[ -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + H \psi = E \psi \]  

(2.2.1)

where \( H=0 \) inside the well and \( H=V \) otherwise. The solution to this equation is of the form:

\[ \psi = A \exp(k_x x) \quad x \leq -w/2 \]  

(2.2.2)

\[ \psi = B \sin(k_x x + \delta) \quad -w/2 \leq x \leq w/2 \]  

(2.2.3)

\[ \psi = C \exp(-k_x x) \quad x \geq w/2 \]  

(2.2.4)
where

\[ k_b = \sqrt{\frac{2m^* (V - E)}{\hbar^2}} \quad k_{qw} = \sqrt{\frac{2m_{qw}^* E}{\hbar^2}} \]

and \( w \) is the quantum well thickness, \( E \) is the energy above bottom of the quantum well, \( m^* \) is the carrier effective mass, and \( k \) is the wave number. Different energy levels in the well (\( n=1,2,3... \)) will have different \( k \) values but will still satisfy equations (2.2.2)-(2.2.4). The subscripts \( qw \) and \( b \) refer to the quantum well and barrier, respectively, and \( A, B, C, \) and \( \delta \) are constants [9]. The boundary conditions for this potential structure are:

\[ \psi_n = \psi_{n+1} \quad \text{at} \quad x = \pm w/2 \quad (2.2.5) \]

\[ \frac{1}{m^*} \frac{\partial \psi_n}{\partial x} = \frac{1}{m_{n+1}^*} \frac{\partial \psi_{n+1}}{\partial x} \quad \text{at} \quad x = \pm w/2 \quad (2.2.6) \]

Using these boundary equations and given the potential well is symmetric (so bound states must have even or odd parity) the following eigenvalue equation can be derived:

\[ \tan \left( \frac{k_{qw} w}{2} \right) = \frac{k_b m_{qw}^*}{k_{qw} m_b^*} \quad (2.2.7) \]

Solutions of this equation can be calculated numerically. A sketch of the three lowest energy solutions for the energy levels in a finite potential well is shown in figure 2.2.2. It should be noted that this sketch assumes that the potential well is sufficiently deep to support three levels. Also shown is a typical potential structure for a GaAs/AlGaAs quantum well. Quantum well depths for electrons (\( \Delta E_c \)) and holes (\( \Delta E_v \)) are calculated from the bulk values of band gap, taking into account the 39%/61% split between conduction band and valence band depths in InP based structures and a 60%/40% split in GaAs based material [9].
Figure 2.2.2 Sketch of the allowed energy levels in a one dimensional potential well (a). Also shown is a typical potential structure of a GaAs/AlGaAs quantum well (b).

It should be noted that the lowest allowed energy level is not at the bottom of the potential well. This shift in the lowest allowed energy level in a quantum well results in a change in the energy of allowed transitions in the quantum well relative to the corresponding bulk material.

Note that the three dimensional dispersion relation (energy $E$ versus wave vector $k$) of a carrier in a quantum well is given by:

$$E = E_n + \frac{\hbar^2 k_y^2}{2m} + \frac{\hbar^2 k_z^2}{2m}$$  \hspace{1cm} (2.2.8)

where $E_n$ is the $n^{th}$ quantisation level in the $x$ direction, which can be found from the solution to the Schrödinger equation, and $k_y$ and $k_z$ are the de Broglie wave numbers in the $y$-$z$ plane of the well.

It should also be noted from equation 2.2.7 that the positions of the allowed energy levels are dependent on the effective mass of carriers and the potential barrier height. Therefore the position of the energy levels in the conduction band and the valence band are different because both the barrier heights $\Delta E_c, v$ and the effective masses are different.
Furthermore, the difference in the effective masses of the heave hole (hh) and light hole sub-bands (lh) gives rise to two different eigenvalue states. Therefore the degeneracy of the hh and lh sub-bands at k=0 in a bulk GaAs or InGaAsP semiconductor is lifted in the quantum well structure due to the quantum confinement. This is shown pictorially in figure 2.2.3.

[Diagram showing E-k diagrams of bulk and quantum well semiconductors for the n=1 states showing the splitting of the heavy hole (hh) and light hole (lh) sub-bands.]

Figure 2.2.3 Schematic E-k diagrams of bulk and quantum well semiconductors for the n=1 states showing the splitting of the heavy hole (hh) and light hole (lh) sub-bands.

For a system where carriers are free to move in only two dimensions, the density of states, g(E) (i.e. the number of carriers in the energy range E to E+dE) is very different from the $E^{1/2}$ behaviour exhibited in bulk (three dimensional) material. Consider a ring of states in 2D phase space of radius k and thickness dk.

$$g(E)dE = \frac{2\pi kdk}{\left(\frac{2\pi}{L}\right)^2}.2$$

(2.2.9)

Assuming a parabolic system:
\[ k = \sqrt{\frac{2m^* E}{\hbar^2}} \]
\[ dk = \frac{1}{2} \sqrt{\frac{2m^*}{\hbar^2 E}} dE \quad (2.2.10) \]

Therefore
\[ g(E) = \frac{m^*}{A \pi \hbar^2} \quad (2.2.11) \]

where \( A \) is the area occupied by the states. This analysis holds only for one band; however, it is easily extended to many bands with the result:
\[ \rho(E) = \frac{m^2}{\pi \hbar^2} \sum_n H(E - E_n) \quad (2.2.12) \]

where \( \rho(E) \) is the density of states per unit area and \( H \) denotes the step function. Note that this is now independent of energy for a given energy level \( E_n \).

The density of states associated with the allowed sub-bands of the quantum well is shown in figure 2.2.4 as a function of energy \( E \). The dashed curve is for the bulk (three dimensional) parabolic band where the density of states goes as \( E^{1/2} \). The solid line represents the density of states for the quantum well, which has a step like form. In the valence band, the same picture can be followed to draw the energy sub-bands of the holes in the valence band of quantum well. In this case the energy levels are measured from the top of the valence band.
The result of these quantum mechanical effects can be seen in the absorption spectra of quantum well material. The quantum confinement causes a shift in the absorption band edge of the material to shorter wavelength. This shift can be calculated from the above quantum mechanics and allows the tailoring of the absorption band edge by simply altering the thickness of the quantum well. An example of this is shown in figure 2.2.5 with a comparison of the absorption spectra of bulk GaAs and AlGaAs/GaAs MQW material (9.6nm wide wells, 9.8nm barriers) after [9] and [10]. Note also the change in the shape of the absorption spectra at energies above the band gap energy. For the bulk material, the curve is approximately parabolic, as would be expected for a $E^{1/2}$ density of states. However, for the quantum well material, the absorption is approximately constant until absorption into the higher order sub-band ($n=2$) occurs. The effect of the splitting of the hh and lh sub-bands can also be observed, with the two peaks in the absorption edge corresponding to e-hh and e-lh transitions.
Figure 2.2.5  Comparison of the absorption spectra of bulk GaAs and MQW GaAs/AlGaAs. After [9] and [10].
2.3 Selection Rules for Transitions in MQW Structures

In bulk III-V semiconductors the heavy hole (hh) and light hole (lh) sub-bands are degenerate at the centre of the energy band diagram (the $\Gamma$ point). In quantum wells, however, this degeneracy is lifted and the hh and lh sub bands are at different energies at the $\Gamma$ point. This is due simply to the differing effective masses of the lh and hh bands (see section 2.2).

Inter band absorption can involve transitions from various hole sub-bands to various electron sub-bands, but not all transitions are 'allowed'. The transitions must satisfy a number of selection rules which can be summarised as follows [11]:

1. The same rules for the changes in parity apply in quantum wells as for bulk material, i.e. for the electric dipole interaction the condition for a non-zero transition matrix element is that the change in azimuthal quantum number, $\Delta l$, must satisfy $\Delta l = \pm 1$.

2. Subject to (1), transitions are allowed between confined states with envelope wave functions of the same spatial symmetry (i.e. even to even or odd to odd transitions).

3. In the infinite well case, the envelope wave functions of states with different principal quantum numbers, n, are orthogonal. Hence inter band transitions can only occur for sub-bands with the same quantum number.

These selection rules are shown diagrammatically in figure 2.3.1 after [12]. The derivation of these rules makes some approximations which can become invalid in certain circumstances, allowing "forbidden" transitions. For example, rule 3 does not strictly apply for finite quantum wells. However, the selection rules do give an indication of which transitions are likely to be strongest. The combination of selection rules and the fact that lh and hh bands are not degenerate leads to a polarisation-dependent optical absorption in MQW material. This is not the case in bulk material.
Figure 2.3.1 Optical Transition Selection Rules in a quantum well after [12]. TE and TM refer to the polarisation of light incident on the waveguide. The numbers beside the arrows indicate the relative transition matrix element.
2.4 Electrically Biased MQW Structures

The effects induced by applying an electric field on the absorption of quantum wells has been extensively studied and well documented [13]. When the electric field is applied perpendicular to the plain of the quantum wells, then the induced effects are qualitatively very different from those in bulk material [14]. In bulk semiconductors the electric field induced effect on the absorption edge is dominated by the Franz-Keldysh effect [15]. This effect induces a general broadening in the absorption edge when a uniform DC electric field is applied to a semiconductor crystal. A tail is induced in the absorption edge at photon energies below the band gap, and oscillations are induced in the absorption spectrum.

In quantum well material, the applied field tilts the band structure as shown in figure 2.4.1.

![Schematic diagram of the tilt of the conduction and valence bands of a quantum well due to an applied electric field perpendicular to the plane of the well.](image)

The electron and hole wave functions move to opposite ends of the well to be at the bottom of the triangular potential induced by the external field, thus reducing the
oscillator strength. The potential barrier for electrons and holes is reduced, enabling the carriers to delocalise from the wells. This effect is more pronounced for electrons than holes in the heavy hole band due to the electrons' lighter effective mass. The energy eigenvalues are, not surprisingly, modified from the rectangular case. The result is a shift to lower energy of the e-hh transition. This is shown in figure 2.4.2, where measurements show the absorption spectra at wavelengths around the e-hh transition wavelength are plotted as a function of applied reverse bias (after [16]). Furthermore, the selection rules are broken for a triangular potential well and 'forbidden' transitions become allowed. The simplest way of describing this red shift is in terms of quantum mechanical tunnelling. Application of the perpendicular electric field leads to a less well localised wave function, which can tunnel horizontally out of the well before making the transition to the conduction band.

![Figure 2.4.2](image)

**Figure 2.4.2** Photocurrent spectra at wavelengths around the e-hh transition wavelength are plotted as a function of applied reverse bias for an InGaAsP/InP MQW structure (after [16]).
In the previous section, the quantum mechanics of an isolated quantum well were discussed. However, in many practical applications there is a requirement to place several quantum wells in close proximity in an SCH to increase the overlap of an optical mode with quantum well material. If the barriers between the quantum wells are sufficiently thin, the carrier wave functions of adjacent finite potential wells will have significant overlap. Carriers will then be free to travel through the structure. Such a structure is known as a superlattice. This is distinct from a multiple quantum well (MQW) structure, where the barriers between the quantum wells are sufficiently thick to prevent significant interaction between adjacent wells. In order to exploit the properties of quantum wells in a MQW structure, care must be taken to avoid the formation of a superlattice, thus losing the desired quantum confinement.

Schrödinger’s equation for the infinite superlattice, from which the allowed energy states are determined, is the same as for the quantum well. However, the boundary conditions differ in that the allowed solutions are not confined to the potential wells, but are periodic throughout the lattice. A schematic diagram of a one-dimensional superlattice is shown in figure 2.5.1.
In order to solve equation (2.2.1) for periodic boundary conditions, the same approach is used as that used in the Krönig-Penny model of free electrons in metals [17]. Some modification is required to include different effective masses for the quantum well and barrier material. The solution to Schrödinger's equation for the potential structure shown in figure 2.5.1 yields the following dispersion relation [18]:

\[
\frac{k_b^2 m_b^2 - k_{qw}^2 m_{qw}^2}{2k_b k_{qw} m_{qw} m_b} \sin(k_{qw} d) \sinh(k_b g) + \cos(k_{qw} d) \cosh(k_b g) = \cos(kf) \tag{2.5.1}
\]

where

\[
k_b = \sqrt{\frac{2m_b(V - E)}{\hbar^2}} \quad k_{qw} = \sqrt{\frac{2m_{qw} E}{\hbar^2}}
\]

as before. This equation reduces to the standard Krönig-Penny dispersion relation when the carrier effective masses are equal, and for infinite barrier widths the equation reduces to the one dimensional potential well solution.

For allowed solutions, the left hand side of equation (2.5.1) must be in the range -1 to +1, as the right hand side of the equation contains a cosine which can only take values within this range. The parts of the solution which lie in the range -1 to +1 correspond to superlattice energy bands, i.e. if the wave function in the quantum well has these energy levels it will satisfy the periodic boundary conditions of a superlattice and couple to adjacent quantum wells. The allowed superlattice energy bands will be centred on the allowed energy levels for a single quantum well of the same dimensions as those in the superlattice. Providing the superlattice energy band width is narrow compared with the thermal broadening of the 1D energy levels (25meV), the quantum wells will remain effectively uncoupled.
2.6 Conclusions

The properties of quantum confined states in thin semiconductors have also been introduced. The quantum confinement of states causes these states to be shifted in energy, moving the e-hh and e-lh transitions to shorter wavelengths. Thus the effective band gap of the structure can be tuned by varying the thickness of the quantum well material. The degeneracy of the lh and hh sub-bands at k=0 is broken due to the difference in effective masses of the carriers in the sub-bands. The quantum confinement also introduces polarisation asymmetry in the absorption of the quantum wells. TM polarised light couples only to the e-lh transition. TE polarised light couples to both the e-hh and e-lh transitions, the relative oscillator strength for the e-hh transition being the greater of the two.

The density of states of the quantum confined system is independent of energy for a given sub-band giving a high density of states at the bottom of the quantum well. The properties of superlattices, where the wave functions of carriers in adjacent quantum wells are allowed to overlap, was also discussed.

The high density of states close to the band edge of an MQW semiconductor is the major advantage of quantum well systems when looking for carrier based effects. A high carrier density can be achieved with far fewer carriers than would be possible in the bulk case, because of the high density of states even at low energies and the small physical dimensions of the QW. It is this potential efficiency of carrier based nonlinearities that is the motivation for looking at MQW structures to produce an efficient nonlinear optical switches.
2.7 References


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Introduction to Optical Nonlinearities in Multi-Quantum Well Semiconductor Waveguides.

Optical nonlinearities in semiconductor waveguides are introduced. Particular reference is made to refractive nonlinearities in InP and GaAs based multi-quantum well material for all-optical switching at a wavelength of ~1550nm for telecommunications applications. A classical forced oscillator model is used to introduce the non-resonant, ultra fast, effect of nonlinear susceptibility which leads to a Kerr-type intensity dependent refractive nonlinearity and two photon absorption. This nonlinearity is best exploited at wavelengths around the half band gap wavelength. Resonant effects due to carrier induced nonlinearities, which are strongest at wavelengths close to the e-hh transition wavelength, are also introduced. The effects of bandfilling and the plasma effect are described quantitatively. Band gap renormalisation, excitonic effects and the quantum confined Stark effect are also discussed briefly, as are opto-thermal nonlinearities.
3.1 Introduction

Almost all materials exhibit some optical nonlinearity when they are excited by an intense optical beam. In most cases this nonlinearity is so small as to be virtually undetectable. However, the use of optical waveguides has enabled high intensity optical fields to be generated in semiconductor material using modest optical powers, capable of being produced by commercially available lasers. In semiconductors an enhancement of the nonlinearity occurs at wavelengths near resonance (the e-hh transition), enabling measurement and exploitation of nonlinearities. This enhancement is particularly significant in MQW semiconductor material. Therefore, it is the nonlinearities in semiconductor MQW material which are investigated in this project for the application of all-optical switching. In particular, nonlinearities in are sought which can be exploited (in appropriate material systems) to form all-optical switches at wavelengths close to 1.55\(\mu\)m for telecommunications applications.

Some of the nonlinear optical properties of semiconductor materials can be explained using the classical forced oscillator model. In this chapter the classical forced oscillator model will be described and applied to nonlinear third order susceptibility (section 3.2). This nonlinearity leads to a Kerr-type intensity dependent refractive index and two photon absorption (TPA) where an electron-hole pair is created by the absorption of two photons whose energy is less than the band gap energy. Both these effects are non-resonant i.e. the nonlinearity induced by the optical field remains only as long as the optical field is present. Therefore, the recovery time of the intensity dependent refractive index is of the order of femtoseconds. Such a nonlinearity is very attractive for high speed all-optical switching.

In order to achieve high optical fields in semiconductors, light at wavelengths longer than the band gap wavelength must be used, or the light will be absorbed and electron-hole pairs created. However, in trying to achieve an intense optical field in a semiconductor in order to exploit the nonlinear refraction, TPA, which is an intensity
dependent absorption, works to limit the optical intensity. Therefore in order to efficiently exploit nonlinear refraction without the limiting effects of TPA, it is necessary to work at wavelengths close to the half band gap wavelength where TPA small. An appropriate MQW material system with a half band gap wavelength around 1.55μm is GaAs/AlGaAs. Therefore, in section 3.2, the effects due to third order susceptibility are discussed with particular reference to the GaAs/AlGaAs material system.

The introduction of electron-hole pairs into semiconductor material leads, by optical absorption, to a perturbation of the refractive index due to several distinct physical mechanisms. Such effects are distinct from the Kerr-type nonlinearity in that the nonlinearity induced by the optical field remains for as long as the photo-generated carriers are present. This may be up to a few nanoseconds after the optical field has been removed. These carrier effects are of particular importance in quantum wells due to the high carrier concentrations which can be generated by a relatively small number of carriers. The high carrier concentrations are a direct result of the 2D confinement of carriers in quantum wells and the associated modifications in the density of states. While carrier induced nonlinearities are limited in their recovery time to the time to remove the carriers which induced the nonlinearity, they are particularly attractive for telecommunications applications because they are far more efficient than Kerr-type nonlinearities.

In contrast to the Kerr-type nonlinearity where it is necessary to operate at a large detuning from the absorption band edge in order to achieve high optical intensities, the carrier induced nonlinearities are particularly strong close to the absorption band edge. It is still necessary use wavelengths longer than the e-hh transition wavelength to avoid complete absorption of the optical signal. However, operating at a detuning from the band edge of a few tens of nanometers allows sufficient carrier generation to occur via band tail states without causing unacceptably high loss of the optical signal.
The appropriate MQW material system to observe these carrier induced nonlinearities at a wavelength of 1.55\textmu m is InGaAsP/InP. Using quaternary material it is possible to obtain a desired effective band edge independent of the quantum well width and potential well depth (see chapter 2). Therefore this material system offers great flexibility in the design of the quantum well for optimum nonlinear behaviour while operating at a fixed wavelength.

The dominant nonlinearities at detunings of a few nanometres from the absorption band edge are bandfilling (the bleaching of absorption due to phase space filling) and the plasma effect (nonlinearity due to the presence of free carriers). Therefore, these effects will be discussed qualitatively in section 3.3. Also discussed briefly are the nonlinearities due to the quantum confinement of states (excitonic effects). The effects of carrier screening of the built-in field of a p-i-n structure (quantum confined stark effect (QCSE) and band gap renormalisation) will also be briefly discussed. Lastly opto-thermal nonlinearities are described.
3.2 Classical Description of Optical Nonlinearities

When a dielectric medium is excited by an intense optical field, the electron orbital is elongated by the applied electric field. This elongation induces a small dipole component into the electron charge distribution. Thus the electron charge distribution is polarised. This polarisation interacts with the electro-magnetic field of the optical field thus altering the propagation of light in the dielectric medium.

Free carriers in any material (bulk or quantum confined) will interact with an incident optical field to produce a modification in the material dielectric constant. Indeed, in metals, consideration of free carriers as simple force oscillators leads to an excellent first approximation of the absorption properties of metals [1].

If a free electron is considered to be a classical particle, but with an effective mass \( m^* \), oscillating in a one dimensional harmonic potential, the equation of motion describing the polarisation (response) of the electron to the external field, \( \xi \), is given by the simple forced oscillator differential equation below [1]:

\[
\frac{d^2 y}{dt^2} + \gamma \frac{dy}{dt} + \omega_0^2 y = \frac{e \xi}{m^*} \quad (3.2.1)
\]

where \( y \) is the particle displacement, \( \gamma \) is the damping constant, and \( \omega_0 \) is the resonant frequency of the oscillation.

If it is assumed that displacement is proportional to the amplitude of the applied electric field and in the same direction, then it can be shown that:

\[
y = \frac{e \xi}{m^*} \left( \frac{1}{\omega_0^2 - \omega^2 + i \alpha \omega} \right) \quad (3.2.2)
\]

where \( \omega_0 \) is the resonant frequency, \( \omega \) is the frequency of the external field and \( e \) is the electronic charge. The standard definitions of polarisation \( P \) induced due to the application of an electric field are:

\[
\varepsilon \varepsilon_0 \xi = \varepsilon \varepsilon_0 \xi + P \quad (3.2.3)
\]
\[ P = ne \gamma \]  

(3.2.4)

where \( \varepsilon_r \) is the relative dielectric constant of the medium and \( n \) is the carrier concentration.

The electronic susceptibility \( \chi \) is defined as:

\[ \chi = \frac{P}{\varepsilon_0 \varepsilon_r} \]  

(3.2.5)

Therefore

\[ \chi = \frac{ne^2}{\varepsilon_0 m^*} \frac{1}{(\omega_0^2 - \omega^2 + i\gamma \omega)} \]  

(3.2.6)

The susceptibility has both real and complex parts i.e. \( \chi = \chi' + i\chi'' \), therefore rearranging equation (3.2.6) the susceptibility becomes:

\[ \chi = \frac{Ne^2/\varepsilon_0 m^* \omega_0^2}{(\omega_0^2 - \omega^2)^2 + (\gamma \omega)^2} \{ (\omega_0^2 - \omega^2) - i\gamma \omega \} \]  

(3.2.7)

The real part of \( \chi \) (i.e. \( \chi' \)) represents refractive index and the imaginary part, \( \chi'' \), represents the absorption. At the resonant frequency absorption is at a maximum, and the refractive index goes through zero, but the refractive index remains non zero at detunings from resonance where absorption is very small.

For any material the real (\( \chi' \)) and imaginary (\( \chi'' \)) parts of susceptibility are linked by the Kramers-Krönig transformation [2]:

\[ \chi'(\omega) = \frac{2}{\pi} P \int_{0}^{\infty} \frac{\Omega}{\Omega^2 - \omega^2} \chi''(\Omega) d\Omega \]  

(3.2.8)

\[ \chi''(\omega) = \frac{2}{\pi} P \int_{0}^{\infty} \frac{\omega}{\Omega^2 - \omega^2} \chi'(\Omega) d\Omega \]  

(3.2.9)

where \( P \) denotes the Cauchy principal value and \( \Omega \) is a dummy frequency variable. This transform implies that the refractive index can be calculated at any frequency from knowledge of the absorption at all frequencies. Furthermore, any change in absorption must result in a change in refractive index of the material and vice versa.
When absorption of a photon takes place (at resonance), electrons make transitions between the valence band and the conduction band quantum states across the semiconductor forbidden gap (an energy separation of $E_g$). The above classical model may be applied to a semiconductor by setting the resonant frequency, $\omega_o$, to $\omega_o = E_g / h$ where $h$ is Planck's constant. Thus the resonance at the band gap of the semiconductor acts in part to determine the resulting polarisation of an optical beam incident upon the semiconductor even for frequencies much smaller than $E_g / h$.

In equation (3.2.5), the polarisation scales linearly with the external applied field. When a semiconductor material is subjected to an intense optical field, the polarisation becomes a nonlinear function of the applied field. In this case, the linear theory can be extended to the nonlinear case by expressing the polarisation as a power series of the optical field [3]:

$$P = \sum_n \epsilon_n \chi^{(n)}(\omega) \xi^n$$  \hspace{1cm} (3.2.10)

where $\chi^{(n)}$ is the $n^{th}$ order susceptibility and each component of the electric field $\xi$ is characterised by its wave vector $k$ and frequency $\omega$. $\chi^{(n)}$ can be derived in the same manner as $\chi^{(1)}$ (equation 3.2.6) by adding higher order terms to the potential in equation 3.2.1.

The third order nonlinear susceptibility term, $\chi^{(3)}$, describes nonlinear effects such as intensity-dependent (Kerr-type) refractive index, two photon absorption (TPA), four-wave mixing which leads to optical phase conjugation and stimulated Raman/Brillouin scattering. The effects that are of particular interest for all-optical switching in semiconductor waveguides are the intensity dependent refractive index and TPA. The second order susceptibility $\chi^{(2)}$ describes nonlinear effects such as second harmonic generation, parametric amplification/oscillation and the Pockel's electro-optic effect [4]. The intensity dependent refractive index change can be written as:
\[ \mu = \mu_1 + \mu_2 I \]  \hspace{1cm} (3.2.11)

where \( \mu_1 \) is the linear refractive index, \( I \) is the optical intensity and \( \mu_2 \) is the nonlinear refractive index (Kerr) coefficient. The Kerr coefficient, \( \mu_2 \), is almost always referred to as \( n_2 \). However to avoid confusion with carrier concentrations it is referred to as \( \mu_2 \) here. \( \mu_2 \) can be expressed as [5]:

\[ \mu_2 = \frac{\chi^{(n)}}{2c\mu_2^2\epsilon_0} \]  \hspace{1cm} (3.2.12)

where \( c \) is the velocity of light. Calculations of the magnitude of \( \mu_2 \) have been carried out for various semiconductors [6]. An example is shown in figure 3.4.1 for GaAs.

![Graph showing the nonlinear refractive index of GaAs at wavelengths around half the band gap (1500nm) of the semiconductor.](image)

**Figure 3.4.1**  Nonlinear refractive index, \( \mu_2 \), of GaAs at wavelengths around half the band gap (1500nm) of the semiconductor. The vertical bars represent the experimental data and the continuous curve stands for the theoretical prediction based on a simple two-parabolic-band model. After A. Villeneuve *et al.* [6].

These calculations show that a large \( \mu_2 \) exists at wavelengths longer than the half band gap wavelength. In order to achieve high optical fields in semiconductors, light at
wavelengths longer than the band gap wavelength must be used, or the light will be absorbed and electron-hole pairs created. However, in trying to achieve an intense optical field in a semiconductor in order to exploit the nonlinear refraction, TPA, which is an intensity dependent absorption, works to limit the optical intensity. Therefore in order to efficiently exploit nonlinear refraction without the limiting effects of TPA, it is necessary to work at wavelengths close to the half band gap wavelength where TPA is small [6], allowing high optical intensities to be achieved in semiconductor waveguides, and hence nonlinear refraction can be observed.

The differential equation governing the rate of change of intensity, $I$, with distance, $z$, along a waveguide due to linear and two photon absorption is given by:

$$\frac{dI}{dz} = -\alpha I - \beta_{TPA} I^2$$  

(3.2.13)

where $\alpha$ is the linear absorption coefficient, $\beta_{TPA}$ is the two photon absorption coefficient and $I$ is the optical intensity. The solution to this equation is:

$$I = I_0 \left( \frac{\alpha + \beta_{TPA} I}{\alpha + \beta_{TPA} I_0} \right) e^{-\alpha L}$$  

(3.2.14)

where $I$ is the intensity at the output of the waveguide, $I_0$ is the coupled light intensity and $L$ is the device length. Hence the power loss in the waveguide is:

$$\Delta P = P_0 \left\{ 1 - \frac{\alpha e^{-\alpha L}}{\alpha + \left( \frac{P_0 \beta_{TPA}}{A_{eff}} \right) \left( 1 - e^{-\alpha L} \right)} \right\}$$  

(3.2.15)

where $A_{eff}$ is the effective device cross sectional area, and $P_0$ is the coupled optical power.

From equation (3.2.13) it is possible to show that the output power $P$, the input power $P_0$, the length of waveguide $L$, the effective length $L_{eff} = (1 - \exp(-\alpha L))/\alpha$, and the effective area of the waveguide mode $A_{eff}$, satisfy the equation:

$$\frac{P_0}{P} = \exp(\alpha L) + \exp(\alpha L) \frac{L_{eff}}{A_{eff}} \beta_{TPA} P_0$$  

(3.2.16)

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Therefore, from a plot of reciprocal transmission, the two photon absorption coefficient, $\beta_{\text{TPA}}$, can be obtained. For an operating wavelength of $1.55\mu$m, a suitable semiconductor with a band gap at half this wavelength is GaAs. Suitable tuning of the band edge is achieved using GaAs/AlGaAs MQW material. The use of quantum well material also enhances the third order nonlinear susceptibility of the material due to an enhanced oscillator strength [7]. The use of the nonlinearity in all-optical switching is described in chapter 4.
3.3 Carrier Assisted Nonlinearities in Semiconductor Quantum Wells

3.3.1 Introduction

In the above description of nonlinear optical effects, only the classical nonlinear effects have been considered. However, in semiconductors there are additional effects which occur due to real carrier generation. When photons with energies close to or above the band gap are incident on the semiconductor, then the photons will be absorbed and photo-generated carriers created. These carriers will modify the dielectric constant of the semiconductor and consequently alter the semiconductor refractive index. These carrier induced nonlinearities, which are not explained by the classical theory alone, are discussed in the following sections. In particular these effects are discussed with reference to the enhancement of nonlinear effects induced by quantum confinement of carriers in MQW structures. The scope of this discussion is limited to the principal carrier enhanced nonlinearities observed in InP based semiconductors at wavelengths close to the absorption band edge. It is these effects which will be exploited for all-optical switching devices in following chapters.

3.3.2 Plasma Effect

The refractive index change in a semiconductor due to the presence of free carriers can be calculated from the classical forced oscillator model described in section 3.2. Here the change in refraction considered is solely due to the interaction of a free carrier with the optical beam. This effect shows no resonance at wavelengths corresponding to the energy of the absorption band edge.

If we consider again the solution to the simple forced oscillator equation (equation 3.2.1) where the carrier is considered to be a simple particle forced to oscillate by the applied electric field of the optical signal, the solution is:

\[ y = \frac{e_0 \xi}{m^*} \left( \frac{1}{\omega_0^2 - \omega^2 + i \gamma \omega} \right) \]  

(3.3.1)
Again, the standard definitions of polarisation $P$ induced due to the application of an electric field are:

\[
e\varepsilon_0 \varepsilon_r \xi = \varepsilon_0 \xi + P
\]  

(3.3.2)

\[
P = n e \gamma
\]  

(3.3.3)

In the case of a free carrier, the induced polarisation is in the same direction as the applied electric field and the displacement. Therefore by consideration of the above equations we can write:

\[
\varepsilon_r = 1 + \frac{n e \gamma}{\varepsilon_0 \xi}
\]  

(3.3.4)

Hence

\[
\varepsilon_r = 1 + \frac{n e^2}{m e_0} \left( \frac{1}{\omega_0^2 - \omega^2 + i \omega \gamma} \right)
\]  

(3.3.5)

where $n$ is the charge density. For the case of free electrons, $\omega_0=0$ and $\gamma=0$; therefore :

\[
\varepsilon_r = (\mu - ik)^2 = 1 - \frac{n e^2}{m e_0} \omega^2
\]  

(3.3.6)

where $\mu$ is the refractive index of the material and $k$ is related to the absorption coefficient.

By differentiating this expression (3.3.6), the change in refractive index due to the presence of carriers is given by [8]:

\[
\Delta \mu = -\frac{\Delta n e^2}{2 \mu n^* e_0 \omega^2}
\]  

(3.3.7)

In order to include the contributions from both holes and electrons, it is convenient to use in equation (3.3.7) a reduced effective mass, $m^*$, defined in terms of the electron and hole effective masses by:

\[
\frac{1}{m^*} = \frac{1}{m_e} + \frac{1}{m_h}
\]  

(3.3.8)
It is assumed that the number of holes and electrons is equal in defining equation (3.3.8). This is a good approximation for the refractive index shift in a 2D system close to the band edge where photon energies are much greater than inter sub band resonance energies [9].

### 3.3.3 Bandfilling

The absorption of a semiconductor material is strongly dependent on the number of carriers already occupying states in the conduction and valence bands. These carriers block further transitions (by the Pauli exclusion principle) thus bleaching the absorption of the material. The bleaching of the absorption spectra necessarily induces a change in the refraction spectra via the Kramers-Kröning relations [2]. This change in refraction has a strong resonance at wavelengths around the e-hh transition wavelength. Thus bandfilling is an efficient route to achieving nonlinear refraction at wavelengths close to the absorption band edge. The magnitude of the change in absorption due to the presence of carriers in the quantum wells can be calculated as shown below from the shift in the quasi Fermi levels in the conduction and valence bands.

The change in absorption coefficient, \( \alpha \), as a result of carriers, accounting for both absorption and stimulated emission can be shown to be [10]:

\[
\alpha(\omega) = \alpha_0(\omega) \{f_c(\omega) - f_v(\omega)\}
\]  

(3.3.9)

where \( f_c \) and \( f_v \) are the quasi Fermi electron occupation probabilities in the conduction and valence bands respectively, and \( \alpha_0(\omega) \) is the initial absorption spectrum. Assuming an initially cold system (i.e. no thermally created electron hole pairs) with no dopants then \( f_c = 0 \) and \( f_v = 1 \). Therefore the change in absorption due to the presence of carriers is:

\[
\Delta\alpha(\omega) = \alpha_0(\omega) \{f_v(\omega) - f_c(\omega) - 1\}
\]  

(3.3.10)
In order to calculate the change in absorption it is therefore necessary to evaluate the Fermi functions, \( f_v \) and \( f_c \). The Fermi function is defined as:

\[
f_{c,v}(E) = \frac{1}{1 + \exp\left(\frac{E - F_{c,v}}{kT}\right)}
\]  

(3.3.11)

where \( F_{c,v} \) is the quasi Fermi level of conduction or valence band (i.e. the Fermi level calculated relative to the band edge). Thus, in order to calculate the change in absorption due to the presence of carriers, an expression must be found for the quasi Fermi levels in the conduction and valence bands. This can be done, remembering that:

\[
n_{2D} = \int_0^\infty g(E)f(E)dE
\]

(3.3.12)

where \( n_{2D} \) is the sheet carrier density in the quantum well and \( g(E) \) is the two dimensional density of states. In a 2D parabolic system, \( g(E) \) has an especially simple form (see chapter 2) which allows this integral to be calculated analytically. However, because we are considering band tail absorption, the bands cannot be considered to be truly parabolic. It is therefore necessary to include an exponentially decaying density of states for energies just below the band edge [11]. Considering just the first sub band in the conduction band, equation (3.3.12) should be modified to become:

\[
n_{2D} = \frac{m_i^*}{\pi \hbar^2} \int_0^{E_0} \frac{\exp\left(\frac{E}{E_0}\right)}{1 + \exp\left(\frac{E - F_c}{kT}\right)}dE + \frac{m_i^*}{\pi \hbar^2} \int_0^\infty \frac{dE}{1 + \exp\left(\frac{E - F_c}{kT}\right)}
\]

(3.3.13)

where \( E_0 \) is the decay constant of the exponential band tail. In principle, this can be solved numerically for \( F_c \). However, in practice, for calculating phase shift this process needs to be repeated many times and is therefore not a viable option due to limitations of computing power. However, the error in finding the quasi Fermi level using equation (3.3.14), below, is less than 10% at low \( (10^{16}\text{cm}^{-1}) \) carrier concentrations and negligible at the carrier concentrations necessary to produce \( \pi \) radians phase shift. In this case:
A similar expression holds for the valence band. Using the above expressions, the quasi Fermi levels, and hence the change in absorption coefficient, can be calculated around the band edge. This change in absorption is related to a change in refractive index via the Kramers-Kröning relation [12].

\[
F_e = kT \ln \left( \exp \left(\frac{n_2 \pi \hbar^2}{m^* k T}\right) - 1 \right)
\]  

(3.3.14)

\[
\Delta n(\omega) = \frac{2}{\pi} \int_0^n \frac{\Delta \alpha(\Omega)}{\Omega^2 - \omega^2} d\Omega
\]

(3.3.15)

This relationship links the refractive index at a particular wavelength with the entire absorption spectrum over all wavelengths. It can therefore be used to find the change in refractive index if the initial absorption spectrum is known. In practice, it is necessary to know the absorption spectrum only around the wavelength of interest.

It should be noted that the shift in the quasi Fermi levels is dependent on the carrier effective mass. The lighter the carrier effective mass, the further the corresponding quasi Fermi level shifts up the sub-band for an equal number of carriers. Thus it is the electrons which dominate the induced phase shift.

3.3.4 Band Gap Renormalisation

The introduction of many carriers into a quantum well will, in addition to reducing the availability of states, produce a screening of the Coulomb potential. It is a general principle of solid state physics that the band structure of a material is defined as the potential seen by a single particle. In the case of a large number of carriers present in a material, these carriers will cause a perturbation of the band structure effectively seen by the carriers. The perturbation of the single particle energies causes a reduction in the effective band gap. Exact determination of this renormalisation requires elaborate numerical calculation [13]. However, the change in band gap \( \Delta E_g \) can be modelled as having a one third power dependence on carrier concentration [13]:

\[
51
$\Delta E_g = -\left( \frac{e}{2\pi \varepsilon_0 \varepsilon_s} \right) \left( \frac{3}{\pi} \right)^{\frac{1}{3}} N^{\frac{1}{3}} \quad (3.3.16)$

where $N$ is the carrier concentration and $\varepsilon_s$ is the material static dielectric constant.

### 3.3.5 Excitonic Nonlinearities

In the previous sections, Coulombic interaction between photo-excited holes and electrons has not been considered. The holes and electrons were treated as independent particles. The nonlinearities associated with these carriers were assumed to be a linear superposition of the contributions of electrons and holes. However, the Coulombic interaction of photo-excited holes and electrons does give rise to nonlinear effects.

The electrons in the conduction band and holes in the valence band, as electrically charged particles, interact through the Coulomb potential. The attractive inter-band interaction causes a strong correlation between electrons and holes and can lead to the formation of bound states known as excitons. These excitons may be regarded in the same way as the classical description of a hydrogen atom (see for example [14]), being characterised by the Bohr radius defined as [14]:

$$a_{3D} = \frac{4\pi \varepsilon_0 h^2}{m_e e^2} \quad (3.3.17)$$

where $m_e$ is the reduced mass of the electron-hole pair. The binding energy, $b_{3D}$, of the ground state exciton is given by:

$$b_{3D} = -\frac{e^4 \mu_r}{8 \varepsilon_0 h^2} \quad (3.3.18)$$

As with the hydrogen atom, there is an infinite series of exciton energy levels. The energy level of the $i^{th}$ level is given by:

$$E_i^{3D} = E_g - \frac{b_{3D}}{i^2} \quad (3.3.19)$$

$52$
where $b_{3D_e}$ is the ground state binding energy of the exciton and $i$ is an integer ($i=1,2,3...$).

This exciton effect can be seen as a sharp peak in the absorption spectrum of a semiconductor at a wavelength just below the band gap wavelength, providing the measurement is carried out at low temperatures otherwise in bulk semiconductors at room temperature the excitonic peak is thermally broadened due to scattering with LO phonons. This effects mask the excitonic resonances in the absorption spectrum.

Increasing the carrier density causes the Coulomb interaction which formed the exciton to be screened. The exciton will therefore ionise. Photo-generated carriers cause screening of the Coulombic potential and result in a change in the absorption spectrum and thus also the refractive index. The nonlinear refraction coefficient for this effect has been calculated to be [4]:

$$\mu_2 = \frac{\alpha_{ex} \tau}{4\pi I_s (1 + \Delta^2)^2}$$

where $\Delta$ is the energy detuning from resonance, $I_s$ is the saturation intensity, $\alpha_{ex}$ is the exciton absorption coefficient and $\tau$ is the exciton lifetime. In quantum well material, the electrical confinement of carriers significantly increases the binding energy of excitons. Simple calculations for an infinite 1D potential well [15] show the 2D confined ground state exciton to have four times the energy and half the radius of a 3D exciton. In practice, finite potential barrier heights make this analysis an over-simplification of the picture. However this enhancement in exciton binding energy is clearly observed in the absorption spectra of MQW material, where exciton resonances are clearly observed, even at room temperature (see chapter 5).

In InP based MQW material, any calculation of the change in refraction around the absorption band edge due to photo-generated carriers should include the effects of
band gap renormalisation and exciton saturation. However, these effects tend to cancel each other in practice. It has been found that a band filling model is sufficient to account for experimental results of absorption saturation [16].

3.3.6 Quantum Confined Stark Effect

In bulk semiconductors, an electric field applied to the semiconductor induces an effect on the absorption edge of the material dominated by the Franz-Keldysh effect [17]. This effect induces a general broadening in the absorption edge when a uniform DC electric field (~$10^5$ V/cm) is applied to a semiconductor crystal. The absorption band tail is broadened at photon energies below the band gap and oscillations are induced in the absorption spectrum at photon energies above the absorption band edge. The Franz-Keldysh effect does not include any effect arising from coherence between electrons and holes (i.e. excitonic effects).

The corresponding effect in quantum wells, where the field is applied perpendicular to the quantum wells is known as the Quantum Confined Stark Effect (QCSE) [18]. QCSE is distinct from the other effects described so far in previous sections in that it is an electro-absorption effect. When an electric field is applied perpendicular to a quantum well stack, the potential wells are tilted and are no longer rectangular (see figure 2.6.1). The energy eigenvalues are modified from the rectangular case. The result is a shift to lower energy of the e-hh transition i.e "red-shifted". Furthermore, the selection rules which applied for a rectangular well are broken by the applied field and 'forbidden' transitions become allowed. The simplest way of describing this red shift is in terms of quantum mechanical tunnelling. Application of the perpendicular electric field leads to a less well localised wave function, which can tunnel out of the well before making the transition to the conduction band. Finding an analytical solution for this shift in band gap is a non-trivial problem. However, approximate solutions have been derived [19]:
\[ \Delta E_g = \frac{0.002}{\hbar^2} m^* e^2 \xi^2 L_z^4 \]  

(3.3.21)

where \( \xi \) is the applied electric field and \( L_z \) is the quantum well width. From this change in the band gap, the change in refraction can also be calculated. In the case of a passive phase modulator device, it is not the shift in band gap due to an applied field which is of interest, but the carrier screening of the built-in field of the p-i-n structure into which the quantum wells are grown. This carrier screening will lead to a change in absorption spectrum and hence a refractive index change.

### 3.3.7 Thermal Nonlinearities

All the previous sections have described the effects of photo-generated carriers on the refractive index of the semiconductor. It has been assumed that these carriers decay radiatively, so the change in refraction persists for the radiative recombination time of the material. However, it is possible for carriers to recombine by non-radiative routes. In radiative recombination, the energy of the photon absorbed by the semiconductor when the electron-hole pair was created is re-emitted as another photon as the electron and hole recombine. Therefore the semiconductor has no net energy gain. In non-radiative recombination, the photo-generated carriers give up their energy to the crystal lattice.

Non-radiative recombination can occur through traps and dislocations in the crystal structure [20], defects, surface recombination [21] and Auger recombination [22] which involves the emission of a third particle (phonon). As a result of these processes the temperature of the sample increases. This leads to a shift in the band gap due to expansion of the lattice.

The rise in lattice temperature induces a contribution to the nonlinear refraction of the semiconductor that is opposite to the contributions due to electronic effects. Under continuous illumination conditions, thermal effects in semiconductors usually dominate
over carrier induced nonlinearities. Therefore, care must be taken when investigating carrier induced nonlinearities to avoid joule heating of the waveguide by, for example, using a pulsed beam.

An increase in temperature results in a decrease in the band gap and a consequent shift of the absorption spectrum to longer wavelength (red shift). This change in absorption causes a change in the refractive index of the semiconductor. The sign of the index change is positive [23] and therefore the change competes with the negative contribution arising from carrier induced nonlinearities.
3.4 Conclusions

The classical forced oscillator model of optical nonlinearities has been discussed. The application of this model to nonlinear third order susceptibility was described. This nonlinearity lead to a Kerr-type intensity dependent refractive index and two photon absorption (TPA). In both of these effects the nonlinearity induced by the optical field remains only as long as the optical field is present. Therefore, the recovery time of the intensity dependent refractive index is of the order of femtoseconds. Such a nonlinearity is, therefore, very attractive for high speed all-optical switching. In trying to achieve an intense optical field in a semiconductor, in order to exploit the nonlinear Kerr-type refraction, TPA, which is an intensity dependent absorption, works to limit the optical intensity. Therefore, in order to efficiently exploit nonlinear refraction without the limiting effects of TPA, it is necessary to work at wavelengths close to the half band gap wavelength where TPA small. The appropriate MQW material system with a half band gap wavelength around 1.55µm is GaAs/AlGaAs. The Kerr-type nonlinear refraction forms the basis of the all-optical switch demonstrated in chapter 4, where efficient all-optical switching is demonstrated in a GaAs/AlGaAs MQW waveguide at a wavelength below the half band gap wavelength.

The resonant enhancement of optical nonlinearities due to photo-generated carriers is not accounted for by the classical forced oscillator model. The generation of real carriers in semiconductors induces several distinct carrier induced nonlinearities. Such effects are distinct from the Kerr-type nonlinearity in that the nonlinearity induced by the optical field remains for as long as the photo-generated carriers are present. These nonlinearities are of great interest for telecommunications applications as they require less power than Kerr-type nonlinearities. However, the recovery time of the nonlinearity is limited by the removal of the carriers and is therefore inherently slower than the recovery time of Kerr-type nonlinearity. These carrier effects are of particular importance in quantum wells due to the high carrier concentrations which can be
generated by a relatively small number of carriers. The high carrier concentrations are a direct result of the 2D confinement of carriers in quantum wells and the associated modifications in the density of states.

 Carrier-induced nonlinearities are particularly strong close to the absorption band edge. It is still necessary use wavelengths longer than the e-hh transition wavelength to avoid complete absorption of the optical signal. However, operating at a detuning from the band edge of a few tens of nanometers allows sufficient carrier generation to occur via band tail states without causing an unacceptably high loss of the optical signal. The appropriate MQW material system to observe these carrier induced nonlinearities at a wavelength of 1.55\( \mu \text{m} \) is InGaAsP/InP, which offers great flexibility in the design of the quantum well for optimum nonlinear behaviour while operating at a fixed wavelength.

 The dominant nonlinearities at detunings of a few nanometres from the absorption band edge are bandfilling (the bleaching of absorption due to phase space filling) and the plasma effect (nonlinearity due to the presence of free carriers). These effect of these nonlinearities in InGaAsP/InP MQW waveguides is investigated in chapter 5 in order to achieve efficient nonlinear refraction at a wavelength of 1.55\( \mu \text{m} \). This nonlinearity forms the basis of the all-optical switches demonstrated in chapter 6, where efficient all-optical switching is achieved using optical powers which can be obtained using amplified diode laser sources. These sources are suitable for telecommunications applications due to their efficient electrical power requirements and their compact size. The principal drawback to using carrier induced nonlinearities, i.e. their recovery time, is discussed in chapter 7. A route to decreasing the recombination time of the carriers and hence improve the recovery time of the nonlinearity, by the application of an electric field perpendicularly to the quantum wells, is investigated.
3.5 References


Polarisation Rotation Gating in AlGaAs/GaAs Multi-Quantum Well Waveguides.

A polarisation rotation gate is demonstrated in an GaAs/AlGaAs MQW waveguide at a wavelength of ~1.55 μm using the ultrafast Kerr nonlinearity. The nonlinear absorption and polarisation rotation characteristics of two GaAs/AlGaAs MQW waveguides have been investigated at wavelengths either side of the half band gap wavelength. From two photon absorption coefficient measurements and characterisation of nonlinear polarisation rotation the optical Kerr coefficient was obtained at a wavelength 80 nm shorter than the half band gap wavelength. Polarisation rotation gates using different nonlinear elements were constructed to operate at wavelengths 80 nm shorter than and 28 nm longer than the half band gap wavelength, in order to demonstrate the limiting effect of TPA. The most efficient gating action was achieved at low coupled powers (<60W), at a wavelength 28 nm below the half band gap wavelength. The effects of TPA, limiting the nonlinear polarisation rotation were clearly observed.
4.1 Introduction

Work on the development of integrated optical devices for nonlinear switching and processing has tended to focus on the performance of Mach Zehnder [1] and directional coupler [2] switches. Previous research into polarisation rotation gates [3] in birefringent optical fibres [4] have shown that optical switching can also be achieved using a waveguide which exhibits nonlinear birefringence.

The birefringent polarisation gate is a simple device which uses the optical Kerr effect to give amplitude modulation of an optical signal due to phase changes induced on that signal. Previous applications of the optical Kerr effect [3,5] have been demonstrated in nonlinear birefringent optical fibres. However, the optical Kerr effect is comparatively weak (with respect to MQW waveguides) in birefringent fibre [4], therefore gates constructed from such waveguides were far from compact with typical fibre lengths of 10-100m. Such devices also required high optical powers in excess of 100W to operate. Multi-quantum well (MQW) rib waveguides exhibit a nonlinear birefringence and show a much stronger optical Kerr effect. Therefore, a polarisation gate based on an MQW waveguide should also exhibit a nonlinear gating action and have more compact dimensions with more efficient power requirements than an optical fibre gate. Such a device would have applications where all-optical switching on femtosecond timescales is required, due to the femtosecond recovery time of the Kerr nonlinearity. The gates demonstrated here employ, at wavelengths near the half band gap, the ultrafast third-order refractive nonlinearity which has only recently been quantitatively measured [6] and used for switching in nonlinear directional couplers [2].

In this chapter, two nonlinear polarisation rotation gates were constructed, using two different GaAs/AlGaAs MQW waveguides, and operated at wavelengths 80nm above (a shorter wavelength) and 28nm below (a longer wavelength) the half band gap wavelength. These experiments demonstrate that nonlinear polarisation rotation gates can be constructed using compact GaAs/AlGaAs MQW waveguides with lengths of the
order of a few millimetres and operated using relatively low powers <100W. By operating these two gates above and below the half band gap wavelength, the limiting effect of TPA on intensity dependent nonlinearities such as nonlinear refraction (Kerr effect), is demonstrated.

In section 4.2, the principles of operation of a nonlinear polarisation rotation gate are introduced with particular reference to gates employing the Kerr effect in an MQW waveguide to give the required nonlinear birefringence. The basic theory of polarisation rotation gates is introduced. This theory is developed further in sections 4.3 and 4.4 to account for the linear and nonlinear losses which occur in MQW waveguides. In section 4.3, a polarisation rotation gate is constructed to operate at 1.512μm, 80nm above the half band gap wavelength of the MQW waveguide used. The intensity dependent transmission of the gate is measured in order to determine, by modelling of these results, the phase shift induced between TE and TM polarisations and hence deduce the Kerr coefficient.

In section 4.4 the polarisation rotation gate demonstrated in section 4.3 is compared with a similar gate operated at a wavelength 28nm below the half band gap wavelength. The limiting effects of TPA are clearly observed, with the gate operated below the half band gap showing more efficient gating than the gate operated above the half band gap. However, even for below half band gap operation, the effects of TPA limit the nonlinear gating action at high (~1kW) optical powers relinearising the transmission characteristic. A theoretical model of the polarisation rotation gate nonlinear transmission characteristics is also presented in section 4.4. This theory is fitted to the experimental data for the below half band gap gate. From this theory, the induced phase shift between TE and TM components is calculated. These experimental results show good agreement with this theory. The experiments show the transmitted power characteristic for low power (<100W) incident pulses is best suited for all-optical switching.
4.2 Operation of a Polarisation Rotation Gate

The operation of a birefringent waveguide polarisation gate is essentially the same as for a birefringent optical fibre polarisation rotation gate [3]. The gate relies on the use of a birefringent material to provide nonlinear polarisation rotation. A representation of a polarisation rotation gate using a MQW rib waveguide as the nonlinear birefringent element is shown in figure 4.2.1.

![Figure 4.2.1 Representation of a Waveguide Polarisation Rotation Gate](image)

Light launched into the birefringent waveguide at an angle off the principle axes of the waveguide (TE and TM) will exit the waveguide as elliptically polarised light. This is because components of the optical signal as resolved along the waveguide’s principal axes propagate down the waveguide with differing phase velocities. This elliptical light is then linearised using a quarter wave plate and blocked using a high extinction ratio polariser. The processes which cause the birefringence are the different effective refractive indexes for TE and TM polarised light in the waveguide and the asymmetry in absorption due to the quantum wells. The birefringence is perturbed
nonlinearly by an intense optical beam, therefore a change in the optical power entering
the waveguide will alter the phase angle between the output components of the signal.
The quarter wave plate is able to linearise only a given degree of ellipticity at certain
fixed angles, therefore a change in phase angle between components of the light incident
on the quarter wave plate will prevent the quarter wave plate from totally linearising the
light passing through it. A polariser is capable of blocking linear light; however, elliptical
light will be transmitted to some extent, depending on the degree of ellipticity. Hence
the power transmitted by the gate as a whole is power dependent. In a lossless birefringent
waveguide, this power dependent transmission can be shown to be [3]:

\[ P_t = P_c \sin^2(2\theta) \sin^2\left(\frac{\Delta\phi_B}{2}\right) \]  \hspace{1cm} (4.2.1)

where \( P_t \) is the power transmitted by a lossless ellipticity analyser, \( P_c \) is the power coupled
into a lossless waveguide and \( \Delta\phi_B \) is the phase difference between components of the
light on the birefringent axes relative to the phase difference at which the light is blocked.
\( \Delta\phi_B \) is given by:

\[ \Delta\phi_B = \frac{2}{3} \pi \mu_2 (P_c - P_{\text{lin}}) \cos 2\theta \frac{L_{\text{eff}}}{\lambda_p A_{\text{eff}}} \]  \hspace{1cm} (4.2.2)

where \( \mu_2 \) is the optical Kerr coefficient, \( L_{\text{eff}} \) the effective length of the fibre, \( \lambda_p \) the
wavelength of the light and \( A_{\text{eff}} \) the effective area of the waveguide mode of the incident
light which was launched linearly polarised at an angle \( \theta \) to the TM axis of the fibre.

Thus the power transmitted by the gate will follow a \( \sin^2 \) function of the phase shift
induced between TE and TM components in the waveguide. The maximum contrast
will be achieved when \( \theta = 22.5^\circ \), thus maximising the \( \sin^2 \) terms in equation (4.2.1) for
small \( \Delta\phi_B \). Such theory fits well the case of a lossless fibre polarisation rotation gate.

However, the operation of an MQW waveguide polarisation gate differs from that
of a lossless fibre gate in that there are two physical processes which give rise to
polarisation rotation. The first process by which polarisation rotation occurs is, as in
the case of the optical fibre gate, birefringence. TE and TM polarised components of an optical pulse polarised off the principle axes experience differing effective refractive indices in the waveguide. Thus TE and TM components move out of phase along the guide forming elliptically polarised light at the output of the waveguide. This asymmetry in nonlinear refraction arises from the asymmetry in $\chi^{(3)}$ which is itself a result of the splitting of the light and heavy hole sub-bands in quantum well material (see section 2.3).

The operation of the waveguide polarisation gate is complicated by the second polarisation rotation process, which is that the two photon absorption (TPA) coefficients for TE and TM polarisations are significantly different. In general there is also a difference in the linear absorption coefficients of semiconductor waveguides. If this were the case, then any change in the linear absorption coefficients would also lead to polarisation rotation. Also, the optimum incident angle would not be 22.5°. However, this is not so in the experiments here. This again arises from the splitting of the light and heavy hole sub-bands in MQW material. An intense optical pulse polarised off the principle axes will have its TE and TM components absorbed in differing proportions. The polarisation of the pulse emitted from the guide will therefore be effectively rotated with respect to the incident polarisation. As TPA is a nonlinear process, the rotation induced by TPA will itself be nonlinear with optical power. The modifications to the standard polarisation rotation gate equations (4.2.1 and 4.2.2) will be discussed in sections 4.3 and 4.4.
4.3 A Polarisation Rotation Gate used to Determine the Kerr Coefficient

4.3.1 Introduction

In this section, a polarisation rotation gate is constructed to operate at 1.512µm, 80nm above the half band gap wavelength of the GaAs/AlGaAs MQW waveguide used. The intensity dependent transmission of the gate is measured in order to determine, by modelling of these results, the phase shift induced between TE and TM polarisations and hence deduce the Kerr coefficient. The nonlinear absorption of the waveguide is measured for TE and TM polarised light at a wavelength of ~1.512µm. The asymmetry in nonlinear absorption at this wavelength due to the splitting of light and heavy hole sub-bands in the MQW material also contributes to the nonlinear polarisation rotation. A theoretical model describing the nonlinear birefringence and polarisation rotation in the MQW waveguide is constructed. From this model, the phase shift between TE and TM polarised light at the output of the waveguide due to nonlinear birefringence is calculated. Hence the optical Kerr coefficient $\mu_2$ is calculated for this material and wavelength.

4.3.2 Device Structure

The GaAs/AlGaAs single mode rib waveguide structure consisted of an undoped MQW region composed of 64 GaAs quantum wells of 4nm width separated by 63 Al$_{0.45}$Ga$_{0.55}$As barriers 4nm wide. The multi-quantum wells were grown over an n-doped Al$_{0.45}$Ga$_{0.55}$As buffer layer deposited on a (001) orientated n$^+$GaAs substrate. A 1µm thick p-doped Al$_{0.45}$Ga$_{0.55}$As (p~$5\times10^{16}$ cm$^{-3}$) layer was grown on the MQW layer and acted as the top cladding layer of the waveguide. This top layer was etched to form ribs of 0.55µm etch depth and 8µm width, running in the (110) crystal direction. The waveguide was manufactured by Bellcore Inc. Characterisation measurements were performed on a waveguide cleaved to a length of 1.5mm (except for the TPA
measurements which were carried out on a similar 7.9mm long waveguide). Room
temperature photoluminescence spectra of the two samples indicated the band gap of
the GaAs quantum wells to be at 795nm.

4.3.3 Nonlinear Absorption Measurements.

Figure 4.3.1 shows the experimental apparatus for the polarisation rotation gate
measurements. The experimental arrangement for measuring the linear and TPA
absorption coefficients is essentially the same as shown in figure 4.3.1 without the quarter
wave plate and polariser #2. A coupled cavity mode-locked KCl:Tl colour-center laser
[7] generated pulses of 90fs FWHM duration at a repetition rate of 82.3MHz and a
wavelength of 1.512μm. The exact pulse width was measured using a second harmonic
generation (SHG) autocorrelator. Longer pulses of ~30ps duration were also obtained
by blocking the external cavity of the colour centre laser. An attenuator wheel allowed
the intensity incident on the waveguide to be continuously varied. The polarisation of
the incident light was set using the half wave plate. The average incident optical power
was varied between 0.7mW and 10mW for both femtosecond and picosecond pulses.
The incident and transmitted power was measured over this power range for TE and TM
polarised light.

The linear absorption coefficient was calculated from the transmission of
picosecond pulses through the waveguide. The peak optical power was sufficiently small
that TPA was negligible. These results gave the linear absorption coefficient $\alpha \approx 4.5\text{cm}^{-1}$
for both TE and TM polarisations, indicating this loss is due to waveguide scatter. Also,
the effective length of the waveguide was 1.1mm, i.e. significantly shorter than actual
length of the device (1.5mm).
Figure 4.3.1 Experimental arrangement for the characterisation of nonlinear polarisation rotation.

The two photon absorption coefficient $\beta_{TPA}$ was determined by plotting the inverse transmission of femtosecond pulses as a function of coupled optical power using the following theory. The equation relating the optical intensity $I$ to the distance $z$ travelled in a medium with linear absorption coefficient $\alpha$ and TPA coefficient $\beta_{TPA}$ is:

$$\frac{dI}{dz} = -\alpha I - \beta_{TPA} I^2$$  \hspace{1cm} (4.3.1)

From this equation it can be shown that the output power $P$, the input power $P_o$, the length of waveguide $L$, the effective length $L_{eff} = (1 - e^{-\alpha L})/\alpha$, and the effective area of the waveguide mode $A_{eff}$, satisfy the equation:

$$\frac{P_o}{P} = e^{\alpha L} + \frac{L_{eff}}{A_{eff}} \beta_{TPA} P_o$$  \hspace{1cm} (4.3.2)

The gradient of the regression lines plotted in figures 4.3.2 and 4.3.3 are therefore proportional to the TPA coefficients.
Figure 4.3.2 Reciprocal transmission as a function of coupled pulse energy for TE polarised light.

Figure 4.3.3 Reciprocal transmission as a function of coupled pulse energy for TM polarised light.
For TE polarised light the value of $\beta_{\text{TPA}}$ obtained from figure 4.3.2 is 1.1cm/GW. For TM light the corresponding value obtained from figure 4.3.3 is 0.1cm/GW. From equation 4.3.2 it can be seen that $\alpha$ can also be extracted from the data in figure 4.3.3, giving $\alpha=4.0\text{cm}^{-1}$ for TE polarised light. This is in good agreement with the value extracted from the picosecond pulse measurements.

4.3.4 Measurement and Analysis of Nonlinear Polarisation Rotation.

Figure 4.3.1 shows the experimental arrangement. The polarisation of the incident light was set to be 20° to the TM axis by a high extinction ratio polariser (#1). Light transmitted through the waveguide was collimated and passed through a quarter-wave plate and a high extinction ratio polariser (#2). It is well known that suitable adjustment of the angle of the quarter-wave plate enables light of any degree of ellipticity to be linearised. This can then be either blocked or passed by the polariser. The final transmitted power was measured on the calibrated large area photodiode. The modal quality of the waveguiding was continuously monitored on the IR camera.

With an average incident power of 9.7mW for 90fs pulses, the transmitted light was linearised and blocked by the quarter wave plate and polariser at the output, hereafter referred to as the ellipticity analyser. The incident power was then stepped down to 0.7mW and the power transmitted by the ellipticity analyser recorded against incident power. The upper trace in figure 4.3.4 shows the ratio of measured power, $P_t$, over incident power, $P_o$, plotted against the shift in incident power from the power at which the transmitted light was linearised, $P_{\text{lin}}$. This demonstrates the power dependence of the ellipticity of the light, the so called ellipse rotation; the power ratio increases when the analyser no longer blocks the transmitted light.
In order to perform a quantitative analysis, the standard polarisation gating equation is used [3]:

\[ P_t = P_c \sin^2(2\theta) \sin\left(\frac{\Delta \phi_B}{2}\right) \]  

(4.3.3)

where \( P_t \) is the power transmitted by a lossless ellipticity analyser, \( P_c \) is the power coupled into a lossless waveguide and \( \Delta \phi_B \) is the phase difference between components of the light on the birefringent axes relative to the phase difference at which the light is blocked. \( \Delta \phi_B \) is given by:

\[ \Delta \phi_B = \frac{2}{3} \pi \mu_s (P_c - P_{lin}) \cos 2\theta \frac{L_{\text{eff}}}{\lambda P A_{\text{eff}}} \]  

(4.3.4)
where $\mu_2$ is the optical Kerr coefficient, $L_{\text{eff}}$ the effective length of the fibre, $\lambda_p$ the wavelength of the light and $A_{\text{eff}}$ the effective area of the waveguide mode of the incident light which was launched at an angle $\theta$ to the TM axis of the waveguide. This can be rewritten as:

$$\Delta \phi_B = K_p \cos 2\theta (P_c - P_{\text{lin}})$$

(4.3.5)

where

$$K_p = \frac{2}{3} \frac{\mu_2 L_{\text{eff}}}{\pi \lambda_p A_{\text{eff}}}$$

For the experimental case, we substitute,

$$P_m = MP_t$$

(4.3.6)

where $M$ is a factor to account for losses in the ellipticity analyser and loss at the uncoated output facet of the waveguide. In order to account for losses in the waveguide and coupling losses, the coupled power at the end of the waveguide can be written as:

$$P_c = R_o e^{-\alpha l} P_o = SP_o$$

(4.3.7)

so that the coupled power is reduced by factors to account both for coupling losses, $R_o$, and linear absorption loss within the guide. The issue of absorption losses will be discussed in the next section. Combining equations (4.3.4)-(4.3.7) for the experimental case gives:

$$\frac{P_m}{P_o} = \frac{1}{16} MS^2 K_p^2 \sin^2 4\theta (P_o - P_{\text{lin}})^2 + F$$

(4.3.8)

for the limit $\Delta \phi_B \to 0$, where $F$ represents a fraction of the incident power also being detected due to non-ideal analysers in the system. For the case of large $\Delta \phi_B$, equation (4.3.8) can be rearranged using equation (4.3.5) to give:

$$\Delta \phi_B = 2 \sin^{-1} \left( \left( \frac{P_m}{P_o} - F \right) \frac{1}{(MS \sin^2 2\theta)} \right)^{\frac{1}{2}}$$

(4.3.9)
Returning to the data, the upper trace in figure 4.3.5 shows the variation of the transmitted power ratio with the change in incident power. A parabolic fit to the data as suggested by equation (4.3.8) has been superimposed on the data points as a guide to the eye and holds only for small $\Delta \phi_B$.

Additionally, the lower trace in figure 4.3.5 shows values for induced $\Delta \phi_B$ against shift in incident power calculated using equation (4.3.9). This shows that a phase shift of $-\frac{\pi}{2}$ was obtained as the coupled power was stepped down. Using equations (4.3.5) and (4.3.7) we obtain:

$$\Delta \phi_B = K_p S \cos 2\theta (P_o - P_{o,lm}) \quad (4.3.10)$$

which can be fitted to the experimental data. Performing a linear regression on the data gives the straight line superimposed on the lower data points in figure 4.3.4. The fit correctly intercepts the $y$-axis at the origin and has a gradient of $6.7^\circ$/mW. Rearrangement of the coefficient in equation (4.3.10) allows evaluation of the effective nonlinear refractive index, the optical Kerr coefficient. Using values of $\lambda_p = 1.512 \mu$m, $A_{\text{eff}} = 3 \times 10^{-12} \text{m}^2$, $L_{\text{eff}} = 1.1 \text{mm}$, $\theta = 9^\circ$ and $S = 8.2\text{dB}$, consistent with calculated losses, gives $12 \times 10^{-18} \text{m}^2 \text{W}^{-1}$ for the nonlinear refractive index. This is in reasonable agreement with the value of $9 \pm 3 \times 10^{-18} \text{m}^2 \text{W}^{-1}$ measured by Tsang et al. on a GaAs/AlGaAs MQW waveguide in the mid-band gap region [6]. It should be noted that the value of $\theta$ used corresponds to the measured value at the output of the waveguide calculated from the linearisation position of the ellipticity analyser. This effective rotation is observed even in the absence of birefringent effects due to unequal absorption losses for TE and TM light modes.
4.3.5 Summary

A polarisation rotation gate was constructed to operate at 1.512μm, 80nm above the half band gap wavelength of the MQW waveguide used. The intensity dependent transmission of the gate was measured by linearising the output of the gate at high power, then reducing the incident power. From these measurements and the associated modelling, the Kerr (nonlinear refractive index) coefficient was found to be 12x10^{-18} m^2 W^{-1}, in reasonable agreement with the measured value.
4.4 Comparison of Polarisation Rotation Gates

4.4.1 Introduction

This section presents a detailed characterisation of the polarisation rotation switching action for two AlGaAs multi-quantum well waveguides. In section 4.3 a detailed characterisation of nonlinear polarisation rotation was carried out on a GaAs/AlGaAs MQW sample at a wavelength just above the half band gap wavelength. In this section a polarisation rotation gate is constructed using the same waveguide as that characterised in section 4.3. A further polarisation rotation gate is constructed using a different AlGaAs/AlGaAs MQW waveguide which is operated at a wavelength below the half band gap. The improved switching action of this gate demonstrates the limiting action of TPA in the previous gate. A theoretical model is also presented that shows good agreement with the experimental results. This model shows that the transmitted power characteristic for low power incident pulses operating at a wavelength beyond the half band gap is best suited for switching action.

4.4.2 Experimental Method

The waveguide used in the polarisation rotation gate working at a wavelength above the half band gap wavelength was a 7.9mm long waveguide of the same layer structure as the waveguide in section 4.3 (an undoped MQW region composed of 64 GaAs quantum wells of 4nm width separated by 63 Al\textsubscript{0.45}Ga\textsubscript{0.55}As barriers 4nm wide, photoluminescence peak 795nm). The operating wavelength was ~1.51\textmu m, a wavelength 80nm above the half band gap for e-hh transitions.

The AlGaAs single mode rib waveguide structure used in the below half band gap polarisation rotation gate consisted of an MQW region composed of 85 Al\textsubscript{0.10}Ga\textsubscript{0.90}As quantum wells of nominal width 7nm separated by 86 Al\textsubscript{0.26}Ga\textsubscript{0.74}As barriers of width 10nm. Experiments were performed in ribs of 2.5-4\textmu m width and 7.8mm length. Measurements of the linear absorption coefficient of the waveguides by the Fabry-Perot...
technique gave $\alpha \approx 1.4 \text{cm}^{-1}$ for both TE and TM polarisations for wavelengths in the $1.55\mu m$ region. The two photon absorption coefficients were measured to be $0.22 \text{cmGW}^{-1}$ for TE and $0.1 \text{cmGW}^{-1}$ for TM polarised light at $1.558\mu m$. By measurement of the photoluminescence peak of the quantum wells at low temperatures, the band gap of the sample at room temperature was predicted to be at $765\text{nm}$. Thus, the operating wavelength at $1.558\mu m$ was $28\text{nm}$ below the predicted two photon absorption band gap wavelength for e-hh transition as reflected by the magnitude of the two photon absorption coefficients. A summary of the waveguide properties and operating conditions is shown in Table 4.4.1.

<table>
<thead>
<tr>
<th>No. of Wells</th>
<th>PL Peak</th>
<th>Half Band Gap Wavelength</th>
<th>Operating Wavelength</th>
<th>Linear Loss</th>
<th>TPA coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>65, GaAs</td>
<td>795nm</td>
<td>1590nm</td>
<td>1510nm</td>
<td>4.5 cm$^{-1}$ (TE &amp; TM)</td>
<td>1.0 cm/GW (TE) 0.1 cm/GW (TM)</td>
</tr>
<tr>
<td>85, AlGaAs</td>
<td>765nm</td>
<td>1530nm</td>
<td>1558nm</td>
<td>1.4 cm$^{-1}$ (TE &amp; TM)</td>
<td>0.22 cm/GW (TE) 0.1 cm/GW (TM)</td>
</tr>
</tbody>
</table>

Table 4.4.1 A summary of the properties and operating conditions of the two waveguides from which the polarisation rotation gates were constructed.

Figure 4.3.1 shows the experimental arrangement used to observe polarisation rotation and gating. Two coupled cavity mode-locked colour centre lasers were used in the experiments. The above half band gap experiment was carried out using a KCl:TI coupled cavity mode-locked colour-centre laser which generated pulses of 90fs FWHM duration at a repetition rate of 82.3MHz and a wavelength of $1.512\mu m$. The exact pulse width was measured using an SHG autocorrelator. Longer pulses of $\approx 30\text{ps}$ duration were also obtained by blocking the external cavity of the colour-centre laser. The below half band gap experiment used a NaCl:OH coupled cavity mode-locked colour-centre laser [8]. This laser generated pulses of $300\text{fs}$ FWHM duration at a repetition rate of $81.9\text{MHz}$ and at a wavelength of $1.558\mu m$. Longer pulses, of $8\text{ps}$ duration, were obtained by blocking the external cavity of the colour centre laser. The pulse durations were
measured using an SHG autocorrelator. The polarisation of the incident light was set to be 22.5° to the TM axis using a high extinction ratio polariser (#1). Light transmitted through the waveguide was collimated and passed through a linearising quarter-wave plate and a second polariser (#2) to enable observation of polarisation rotation. The modal quality of the waveguiding was continuously monitored on the IR camera.

4.4.3 Results 1: Operation at Shorter Wavelength than the Half Band Gap Wavelength

These results were taken at a wavelength of 1.512μm, using the 64 well waveguide as the nonlinear element in the polarisation rotation gate. The operating wavelength was 80nm above the half band gap wavelength. With 10mW average power of 30ps FWHM pulses incident on the waveguide the transmitted light was linearised. The average incident power was kept constant and the pulse width was set to 90fs, thus changing only peak coupled power of the pulse train. The peak incident powers are recorded in Table 4.4.2.

<table>
<thead>
<tr>
<th>Pulse Width FWHM</th>
<th>Coupled Peak Power / W</th>
<th>$P_m/\mu W$</th>
<th>$P_m/\mu W$</th>
<th>$P_{\text{switch}}/\mu W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 ps</td>
<td>1</td>
<td>40.4</td>
<td>0.193</td>
<td>0.193</td>
</tr>
<tr>
<td>90fs</td>
<td>333</td>
<td>15.6</td>
<td>4.62</td>
<td>8.40</td>
</tr>
</tbody>
</table>

* Refers to setting of ellipticity analyser.

Table 4.4.2: Above half band gap polarisation gate results.

The powers transmitted by the ellipticity analyser were recorded when the polariser was both blocking and passing linearised light, for both peak powers used. Table 4.4.2 shows all power measurements from this procedure. The third column displays the power measured with the analyser set to transmit the linearised light. The reduction in
transmitted power, \( P_m \), for the femtosecond pulses arises from the nonlinear process of two photon absorption (TPA) present for the high peak intensities associated with the femtosecond pulse train. This shows that the maximum possible transmission of the gate is 15.6\( \mu \)W. The final column shows the powers measured with the ellipticity analyser set to block the transmitted picosecond pulses. For the femtosecond pulses, 8.4\( \mu \)W was passed by the gate giving a contrast ratio of 44:1 between the transmitted power in the phase shifted and blocked beams. This transmitted power is 54% of the maximum transmission for the femtosecond pulses. Rewriting equation (4.3.3) as

\[
P_t = P_{\text{max}} \sin^2 \left( \frac{\Delta \phi_{B\alpha}}{2} \right)
\]

we find that for the above result \( \Delta \phi_{B\alpha} = 95^\circ \) for \( P_t / P_{\text{max}} = 0.54 \). The phase shift between the two regimes was also measured using the ellipticity analyser. This gave \( \Delta \phi_{\alpha} = 63^\circ \).

The discrepancy between the measurements arises because TPA occurs predominantly for the TE mode and hence acts to rotate the elliptical light towards the TM axis, in addition to the effect of any induced birefringence. This modifies the passage of light through the analyser.

4.4.4 Results 2: Operation at Longer Wavelength than the Half Band Gap

**Wavelength: Low Power**

These measurements were carried out on the 85 well waveguide at a wavelength of 1.558\( \mu \)m, 28nm below the half band gap wavelength of the waveguide. Initial measurements of polarisation rotation below the half band gap were carried out using 8ps pulses with a maximum incident average power of 102mW on a 2.5\( \mu \)m width guide. The transmitted light was linearised and blocked at the output by the quarter wave plate and polariser with the incident light set to minimum power on the attenuator wheel. This gave a coupled peak power of 3W. Increasing the incident power by a factor of 20
increased the power transmitted through the gate by a factor of 344. This is a much greater contrast ratio than was achieved with the previous gate and demonstrates the improved switching action which can be achieved by operating at a wavelength below the half band gap. The full transmission characteristic is shown in figure 4.4.1 and demonstrates a strongly nonlinear characteristic for coupled powers as low as 60W.

Figure 4.6.1 Below half band gap polarisation rotation gate operated at low power (<60W coupled power).

4.4.5 Results 3: Operation at Longer Wavelength than the Half Band Gap

Wavelength: High Power

These measurements were carried out on the 85 well waveguide at a wavelength of 1.558μm, 28nm below the half band gap wavelength of the waveguide, as in the previous section. The powers used here are greatly in excess of those used in the previous section. With 300fs pulses incident on a 4μm width guide the transmitted light was linearised for a coupled peak power of 72W. The incident power was then increased by a factor of 33 to give a peak coupled power of 2.37kW corresponding to a maximum
incident average power of 146mW. The peak increase in transmitted power occurred for a coupled power 2.15kW and corresponded to the gate transmission rising by a factor of 163. Again this is far greater than the first gate demonstrated due to the lower TPA coefficient at the operating wavelength in this case. However, TPA is still having some effect as the transmission increase is substantially reduced from the low power results for the same waveguide and operating wavelength. The full transmission characteristic is shown in figure 4.4.2. It can be seen that the overall response is more linear than the low power case. This is discussed in more detail in the following section.

![Figure 4.4.2](image)

Figure 4.4.2 Below half band gap polarisation rotation gate operated at high power (>2kW coupled power).

4.4.6 Discussion

The transmitted power, $P_t$, assuming lossless conditions and monomode operation, is given by the standard polarisation gating equation [3],

$$P_t = P_e \sin^2(2\theta) \sin^2\left(\frac{\Delta \phi_B}{2}\right)$$  \hspace{1cm} (4.4.2)
where $P_c$ is the power coupled into a lossless waveguide and $\Delta \phi_b$ is the phase difference between components of the light on the birefringent axes relative to the phase difference at which the light is blocked. $\Delta \phi_b$ is given by:

$$\Delta \phi_b = K_p \cos 2\theta (\Delta P_c) \quad (4.4.3)$$

where $K_p$ is given by:

$$K_p = \frac{2}{3} \pi \mu_2 \frac{L_{\text{eff}}}{\lambda_p A_{\text{eff}}} \quad (4.4.4)$$

and $\mu_2$ is the optical Kerr coefficient, $L_{\text{eff}}$, the effective length of the waveguide, the wavelength of the light, $\lambda_p$, and the effective area of the waveguide, $A_{\text{eff}}$. The polarization of the incident light is at an angle $\theta$ to the TM axis of the waveguide.

In the presence of linear and two photon absorption equation (4.4.2) will be modified in two ways. Firstly, the transmitted power will be proportional not to the coupled power but to the power transmitted by the lossy guide. Taking into account the linear loss coefficient, $\alpha$, and two photon absorption coefficient, $\beta_{\text{TPA}}$, the power transmitted by a waveguide of length $L$ is,

$$P_{\text{out}}(L, \alpha, \beta_{\text{TPA}}) = \frac{P_c}{(1 + C) \exp(\alpha L) - C} \quad (4.4.5)$$

where $C$ is defined as $C = \beta_{\text{TPA}} P_c / \alpha A_{\text{eff}}$. Secondly, equation (4.4.4) will be modified by the inclusion of $L_{\text{eff}, \beta_{\text{TPA}}}$ the effective length in the presence of two photon absorption. This can be calculated using the standard definition and equation (4.4.5) to be,

$$L_{\text{eff}, \beta_{\text{TPA}}} = \frac{A_{\text{eff}}}{\beta_{\text{TPA}} P_c} \{ \ln[(1 + C) \exp(\alpha L) - C] - \alpha L \} \quad (4.4.6)$$

The modified polarisation gating equation has been used to model the experimental data are displayed as the solid lines in figures 4.4.1 and 4.4.2. For both cases an optical Kerr coefficient of $6 \times 10^{-14} \text{cm}^2/\text{W}$ was used in agreement with previous measurements [1,2]. Analysis of the power transmitted on the principal axes of the waveguides showed that, due to the astigmatism of the focus into the waveguide modes, there can be different
mode coupling losses for the TE and TM polarisation modes. The asymmetric coupling losses at the waveguide input is equivalent to having an effective input angle, $\theta_{\text{eff}}$. The effective mode area $A_{\text{eff}}$ was trivially assumed to the product of the active area depth and the rib width. The coupling was optimised for TE polarised light. The coupling coefficient for the TM mode used in the model was $\sim 85\%$ of the coupling coefficient for TE polarised light. The output powers given by the polarisation gating equation have been scaled to overlay the experimental results. This adjustment is necessary until the theory is extended to account for the averaged switching over the ultrafast pulse shapes.

For the low power results a good fit was found using $\theta_{\text{eff}} = 16^\circ$. The fit then yields a value of $0.22\pi$ for $\Delta \phi_B$ for the maximum coupled power. This relatively small phase shift allows a high change in transmission for the gate between the "on" and "off" states. For the high power results the best fit was found for $\theta_{\text{eff}} = \theta = 22.5^\circ$ and yields a value of $0.95\pi$ for $\Delta \phi_B$ for the highest value of coupled power. Clearly, the value of $\Delta \phi_B$ approximates $\pi$ for the highest coupled powers and may actually have passed $\pi$ for the final four points although this is not predicted by the theory. It is also clear by inspection that the total phase shift and total transmitted power are much lower than might be expected from extrapolation of the low power results. This can be explained by examination of equations (4.4.5) and (4.4.6). Equation (4.4.5) shows that two photon absorption rapidly acts to reduce the power transmitted by the guide. Most crucially, the action of the two photon absorption is to reduce $L_{\text{eff}}$ as shown by examination of equation (4.4.6) so that the net phase shift does not scale linearly with coupled power. The combined effect of these two factors is to tend to relinearise the transmission function as shown by figure 4.4.2.
4.4.7 Summary

The performance of a GaAs based MQW waveguide polarisation rotation gate has been evaluated for operation at a wavelength 80nm above and 28nm below the half band gap wavelength. For above half band gap operation (i.e. an operating wavelength shorter than the half band gap wavelength) a contrast ratio of 44:1 was achieved between the transmitted power in the phase shifted and blocked beams. This transmitted power was 54% of the maximum transmission for the femtosecond pulses. The induced phase shift between TE and TM components was measured to be $\Delta \phi_p = 63^\circ$, with some additional polarisation rotation caused by asymmetry in TPA coefficients giving an effective $\Delta \phi_p = 95^\circ$.

The below half band gap experiment (operating at a wavelength longer than the half band gap wavelength) was carried out using an AlGaAs/AlGaAs MQW waveguide. Using low power picosecond pulses, an increase in the coupled power by a factor of 20 (from 3W to 60W peak power) gave an increase in the gate transmission by a factor of 344. The induced phase shift, $\Delta \phi_p$, was calculated to be 40° ($0.22\pi$ radians). This was repeated using femtosecond pulses (300fs) to achieve higher peak powers. Increasing the coupled power from 72W to 2.37kW (a factor of 33) gave an increase in the gate transmission by a factor of 163. For the peak coupled power, $\Delta \phi_p$ was calculated to be 171° ($0.95\pi$ radians).

The highest contrast ratio was achieved using below half band gap operation, at low powers (<60W) where the effects of TPA in limiting polarisation gating are minimised. By operating at high powers or above the half band gap the effect of TPA is to limit the increase in gate transmission and linearise the gate transmission function.
4.5 Conclusions

Two nonlinear polarisation rotation gates were constructed, using two different GaAs/AlGaAs MQW waveguides, to operate at wavelengths 80nm above and 28nm below the half band gap wavelength. These experiments demonstrated that nonlinear polarisation rotation gates can be constructed using compact GaAs/AlGaAs MQW waveguides with lengths of the order of a few millimetres and operated using relatively low powers <100W. By operating these two gates above and below the half band gap wavelength, the limiting effect of TPA on intensity dependent nonlinearities such as nonlinear refraction (Kerr effect), was demonstrated. This represents a significant improvement over fibre polarisation gates in that the semiconductor waveguide device is two orders of magnitude more compact and operates at lower powers.

The waveguide operated at a wavelength above the half band gap was a GaAs/AlGaAs single mode rib waveguide structure with an undoped MQW region composed of 64 GaAs quantum wells of 4nm width separated by 63 Al$_{0.45}$Ga$_{0.55}$As barriers 4nm wide. The two photon absorption coefficients for TE and TM polarised light at a wavelength of 1.512µm were measured to be 1.1GW/cm$^2$ and 0.1GW/cm$^2$ respectively. The intensity dependent transmission of the gate was measured in order to determine, by modelling of these results, the phase shift induced between TE and TM polarisations and hence deduce the Kerr coefficient. The induced birefringence between TE and TM polarised light was found to be 6.7°/mW. Hence the optical Kerr coefficient was calculated to be $12\times10^{18}$m$^2$W$^{-1}$. This is in reasonable agreement with the published value of $9\pm3\times10^{18}$m$^2$W$^{-1}$ in the mid-band gap region.

The polarisation rotation gate demonstrated operated 80nm above the half band gap was compared with a similar gate operated at a wavelength 28nm below the half band gap wavelength. The AlGaAs structure used in the below half band gap polarisation rotation gate consisted of an MQW region composed of 85 Al$_{0.16}$Ga$_{0.84}$As quantum wells of nominal width 7nm separated by 86 Al$_{0.26}$Ga$_{0.74}$As barriers of width 10nm. The TPA
coefficients measured at a wavelength of 1.558\,\mu m were 0.22\,cm/GW for TE polarised light and 0.1\,cm/GW for TM polarised light. The limiting effects of TPA were clearly observed, with the gate operated below the half band gap showing more efficient gating than the gate operated above the half band gap, with contrast ratios of 344:1 and 44:1 respectively. However, even for below half band gap operation, the effects of TPA limit the nonlinear gating action at high (~2\,kW) optical powers relinearising the transmission characteristic and limiting the increase in transmission to a factor of 134. A theoretical model of the polarisation rotation gate nonlinear transmission characteristics was also presented. This theory was fitted to the experimental data for the below half band gap gate. From this theory, the induced phase shift between TE and TM components was calculated to be 40° for a coupled peak power of 60\,W and 171° for a coupled power of 2.37\,kW. The experiments show the transmitted power characteristic for low power (<100\,W) incident pulses and operation below the half band gap wavelength is best suited for all-optical switching. This gate is, to the author's knowledge, the first demonstration of a polarisation rotation gate using an AlGaAs MQW waveguide as the nonlinear element.
4.6 References


Design and Evaluation of a Phase Modulator for Efficient Self Phase Modulation Using Carrier Enhanced Nonlinearities.

The carrier assisted nonlinearities of bandfilling and the plasma effect were modelled theoretically to determine nonlinear refraction in a semiconductor waveguide for light at a wavelength of 1.55 µm and a detuning of a few tens of meV from the n=1 e-hh transition where absorption losses are low. This model was used to design a MQW layer structure for a semiconductor waveguide to exhibit efficient nonlinear refraction and hence efficient nonlinear phase modulation. The fabricated waveguides were tested to evaluate their nonlinear refraction characteristics using self-phase modulation (SPM) techniques. Phase modulation in excess of π radians was demonstrated in the optimised MQW waveguide for a coupled pulse energy of 30 pJ.
5.1 Introduction

In the previous chapter, the use of Kerr-type nonlinearities as the basis for an all-optical nonlinear switch was demonstrated. The optical power required for the operation of that switch was of the order of a hundred watts. This requirement for such large optical powers makes such a device unattractive for telecommunications applications, as these power levels are not at present available from compact sources such as laser diodes and Erbium doped fibre amplifiers. Therefore, more efficient nonlinearities must be utilised if all-optical switching is to be achieved with a diode laser source.

Carrier-assisted optical nonlinearities are far more efficient at inducing refractive nonlinearity at wavelengths close to the e-hh transition. Therefore it is these nonlinearities which are investigated in this chapter to design, fabricate and test a semiconductor waveguide which would exhibit sufficient nonlinear refraction to allow all-optical switching at the pulse powers and energies which can be produced by amplified diode laser sources at a wavelength of ~1.55μm.

In section 5.2, the carrier-assisted nonlinearities of bandfilling and the plasma effect are modelled theoretically to determine nonlinear refraction in a semiconductor waveguide for light at wavelengths in the band tail of the e-hh transition (a few tens of milli-electron Volts from the e-hh transition). Wavelengths in the band tail are chosen to allow some carrier generation to occur, and hence induce nonlinearity, without totally absorbing the signal, thus allowing self switching of a single optical pulse. An alternative approach would be to use two beams of different wavelengths, one with a wavelength shorter than the band gap wavelength which would be totally absorbed, the other at a longer wavelength where no absorption occurred but which would still experience the refraction change induced by the short wavelength beam. This approach is not used here due to difficulties in implementing it experimentally. This model is used to design a
MQW layer structure for a semiconductor waveguide which will exhibit efficient nonlinear refraction. For comparison, a similar waveguide with a bulk active layer is also designed.

The fabricated waveguides are tested to evaluate their nonlinear refraction characteristics using self phase modulation (SPM) techniques. These are described in section 5.3 Absorption spectroscopy is also carried out on the fabricated wafers to determine their absorption at wavelengths close to 1.55μm. Phase modulation in excess of π radians is demonstrated in the optimised MQW waveguide for a coupled pulse energy of 30pJ.
5.2 Waveguide Design

5.2.1 Introduction

A knowledge of the properties of quantum wells and of carrier enhanced nonlinearities described previously is used here to design a waveguide which will exhibit an efficient carrier-enhanced refractive nonlinearity at a wavelength of 1.55μm. The waveguide considered here is an InGaAsP/InP based MQW ridge waveguide. An InP based system is used as it allows the tailoring of the effective band gap of the quantum wells independently of the quantum well width through the variation in the arsenic alloy fraction of the quantum well material [1].

The structure considered here is a multi quantum well structure with InGaAsP quantum wells and InGaAsP barriers (InGaAsP barriers having a different composition to the quantum well material in order to provide 2-dimensional carrier confinement). Optical confinement is achieved by using InP cladding regions.

The following sections analyse the design parameters of quantum well width, barrier width and the number of quantum wells. Each of these parameters is optimised for achieving an efficient refractive nonlinearity with the restrictions that the waveguide length will be 1mm, the absorption of the waveguide shall not be more than 3dB over the length of the waveguide and the operating wavelength shall be ~1.55μm. The absorption and operating wavelength requirements were chosen in order to be compatible with the requirements of an all-optical telecommunications system. The length requirement was chosen to avoid excessive scatter loss and assist in the fabrication of the device.

The final waveguide design is presented, including the ridge width and depth to ensure single mode operation of the waveguide at a wavelength of ~1.55μm. For this design, the theoretical nonlinear refraction is calculated as a function of coupled pulse energy.
5.2.2 Quantum Well Width

In designing the optimum quantum well width a trade-off is involved. A wide quantum well increases the overlap integral (confinement factor) between the optical mode and the quantum wells in the waveguide intrinsic region. This in turn increases the phase modulation produced by the device (see appendix 3) as a greater proportion of the optical mode overlaps with the quantum wells where the change in refractive index occurs. However, increasing the quantum well width beyond a certain limit will produce no additional increase in phase modulation [2] (effectively losing the 2D confinement of the carriers in the quantum wells).

Whilst, in theory, using quaternary material for the quantum well means barrier width and effective band gap can be chosen independently, in practice this is not the case. Growers of epitaxial structures tend to grow only specific compositions of quaternary material (specified by Q, the material band gap wavelength in microns). At BT Labs, the standard compositions grown are Q=1.0, 1.1, 1.2, 1.3, 1.55. Therefore it is these material compositions of InGaAsP which must be used in the design of the quantum well.

The operating wavelength is chosen to meet the absorption requirements outlined in section 5.2.1., and is set to give the maximum allowed absorption. From the absorption data shown in section 5.2.5 the required waveguide absorption (3dB in a 1mm long device, \(\alpha=4.5\text{cm}^{-1}\)) will be obtained at a detuning from the e-hh transition energy of approximately 50meV for TE polarised light. Thus for an operating wavelength of \(\sim1.55\mu\text{m}\), the required wavelength for the e-hh transition is \(\sim1.46\mu\text{m}\).

In order to achieve the widest possible well, the composition Q=1.55 should be used. Given this composition of well material, the barrier composition can be chosen from the remaining standard compositions. In order to achieve the greatest possible overlap integral, an InGaAsP composition with Q=1.0 would be used. However, in order to reduce the potential barrier for carriers in the wells a composition with Q=1.2 was
chosen for the barrier quaternary material. This was to reduce the thermionic emission
time constant for carrier escape from the quantum wells. Shallow potential barriers are
important for minimising the recovery time of the carrier enhanced nonlinearity. For
these compositions of quaternary material, a well width of 7.5nm should give an e-hh
transition wavelength of 1.46\mu m. This was calculated from the quantum mechanics of
a finite 1D potential well (see chapter 2).

5.2.3 Barrier Width

Selecting the optimum barrier width for a 7.5nm quantum well involves another
trade-off. Thin barriers increase the overlap integral. However, if the barriers are too
thin then the wave functions of carriers in the quantum wells can overlap, forming a
superlattice. This would also reduce the 2D confinement of the carriers, therefore
reducing phase modulation efficiency. To find the thinnest barriers allowed, the
following approach is used. The allowed energy levels in the quantum well were
calculated from the solution to Schrödinger’s equation in a one dimensional quantum
well of finite depth (see chapter 2). The allowed energy bands for a superlattice with
the dimensions of the quantum well width and a chosen value of barrier width are
calculated using the standard Kronig-Penny analysis [3], modified to account for the
differing carrier effective masses in the quantum wells and barriers [4].

The solution for the allowed energy bands of the superlattice will give bands centred
on the 1D allowed energy levels. Providing the superlattice energy band width is narrow
compared with the thermal broadening of the 1D energy levels (25 meV), the quantum
wells will remain effectively uncoupled.
Figure 5.2.1  Schematic of a one dimensional superlattice.

The solution to Schrödinger’s equation for the potential structure shown in figure 5.2.1 yields the following dispersion relation [4]:

\[ \left( \frac{k_b^2 m^*_q - k_{qw}^2 m_b^*}{2k_b k_{qw} m_q w m_b} \right) \sin(k_{qw}d) \sinh(k_b \gamma) + \cos(k_{qw}d) \cosh(k_b \gamma) = \cos(kf) \]  

(5.2.1)

where

\[ k_b = \sqrt{\frac{2m^*_q (V - E)}{\hbar^2}} \]

\[ k_{qw} = \sqrt{\frac{2m^*_q E}{\hbar^2}} \]

This equation reduces to the standard Kronig-Penny dispersion relation when the carrier effective masses are equal, and for infinite barrier widths equation (5.2.1) reduces to the one dimensional potential well solution:

\[ \tan \left( \frac{k_{qw} w}{2} \right) = \frac{k_b m^*_q}{k_{qw} m_b} \]  

(5.2.2)

where \( w \) is the quantum well thickness, \( m^* \) is the carrier effective mass and \( k \) is the wave number. The subscripts \( qw \) and \( b \) refer to the quantum well and barrier respectively (see chapter 2).
For allowed solutions the left hand side of equation (5.2.1) must be in the range -1 to +1 as the right hand side of the equation contains a cosine which can only take values within this range. The parts of the solution which lie in the range -1 to +1 correspond to superlattice energy bands, i.e. if the wave function in the quantum well has these energy levels it will satisfy the periodic boundary conditions of a superlattice and couple to adjacent quantum wells.

For a quantum well (InGaAsP Q=1.55) width of 7.5nm, a barrier (InGaAsP Q=1.2) width of 12nm gives an allowed energy band width of ~3meV. (The effective masses used in these calculations are shown in appendix 1. A 39%/61% conduction band offset is assumed [5]). This is sufficiently small compared to thermal broadening of the quantum well allowed energy level at room temperature (25meV) to allow the quantum wells to remain uncoupled.

5.2.4 Number of Quantum Wells

Calculation of the optimum number of quantum wells in a layer structure comprising 7.5nm thick InGaAsP (Q=1.55) quantum wells and 12nm thick InGaAsP (Q=1.2) barriers involves complex calculations which are described in the following sections. The following analysis attempts to calculate the nonlinear refraction efficiency of a waveguide containing the above quantum wells and barriers. The number of quantum wells is varied and the optimum quantum well number found for a fixed carrier concentration.

Essentially, this process seeks to find the optimum balance between a large number of quantum wells (and hence a large $\Gamma$) and a small number of quantum wells where the carrier concentration is high and therefore the nonlinear refraction is also high.
5.2.5 Modelling of Absorption Edges

In order to carry out modelling of the nonlinear refraction induced by the presence of photogenerated carriers in a MQW waveguide, it is necessary to have a knowledge of the absorption spectrum of the quantum well material. The following figures 5.2.2 and 5.2.3 show data supplied by M. Fisher (B.T. Labs) in order to facilitate these calculations. The data shows the measured linear absorption coefficient of the barrier and well material, $A_t$. In order to extract the linear absorption coefficient of the well material, $A_t$, must be scaled to account for the width of the barrier material in the sample.

![Graph showing linear absorption coefficient as a function of wavelength for InGaAs/InP MQW material.](image)

Figure 5.2.2a Linear absorption coefficient as a function of wavelength for InGaAs/InP MQW material.
The decay of absorption becomes exponential at a detuning of approximately 10meV, with a decay constant of 7.05meV (calculated from a linear regression). These values are consistent with those measured by A. Von Lehmen \textit{et al.} [6]. The $E_0$ parameter reflects the energy associated with microfields in the crystal due to optical phonons or structural defects [7]. The average electric microfield intensity produced by these perturbations results in tunnelling of electrons into lower energy states, thus extending the band tail. Therefore $E_0$ is a sensitive indicator of damage-related effects.
Figure 5.2.3 shows the InGaAs/InP material absorbence spectrum in more detail, the points are the measured data, the curve shown is the fit which is used in the phase shift calculations. In the phase shift calculations, the absorption is scaled to take account of the absorption of the well material alone. Above the band edge, the absorption is assumed to be constant at 8750 cm\(^{-1}\) i.e. assuming a 2-dimensional density of states and only one allowed sub-band in the conduction band well. At detunings from the band edge of less than 10 meV, a cosine function is fitted to the data as the gradient of the cosine matches the plateau at the band edge, and is a reasonable fit at the transition to exponential decay. In this region the linear absorption coefficient of the well material alone, \(\alpha\), is:

\[
\alpha = 8750 \cos \left( \frac{\Delta E}{11.8} \right) \text{ cm}^{-1} \tag{5.2.3}
\]
where $\Delta E$ is the detuning from the band edge in meV. For detunings in excess of 10meV the fit takes the following form:

$$\alpha = 24779 \exp\left( \frac{\Delta E}{7.05} \right) \text{ cm}^{-1}$$  \hspace{1cm} (5.2.4)

### 5.2.6 Calculation of Phase Shift

The theory used here to calculate the phase shift along the guide calculates the number of carriers generated due to the absorption of a short (30ps) optical pulse at a wavelength of 1.55\(\mu\)m incident upon the waveguide. This calculation is carried out iteratively to account for bleaching of the absorption of the latter parts of the optical pulse due to carriers generated by the initial parts of the optical pulse.

The total number of carriers generated is used to calculate the phase shift due to the plasma effect. The change in refractive index, caused by the change in absorption coefficient due to bandfilling, is calculated by numerically solving the Kramers-Kröng relation.

The theory assumes that no relaxation (either spontaneous or stimulated recombination) of carriers occurs during the optical pulse, i.e. a short pulse width (30ps), and that all carriers have relaxed before another optical pulse is incident on the waveguide, i.e. a low pulse repetition frequency. The theory is also limited to low initial values of linear absorption coefficient, where the number of photo-generated carriers does not cause $\alpha$ to fall close to zero. If this were the case then carriers, generated by TPA, which scatter from the barriers into the quantum wells and would cause $\alpha$ to fall below zero. Any accurate calculation of the change in absorption coefficient would then require a rate equation model, which has not been considered.

The two absorption processes considered here are linear and two photon absorption. The differential equation governing the rate of change of intensity with distance along a waveguide is given by:
where $\alpha$ is the linear absorption coefficient, $\beta_{TPA}$ is the two photon absorption coefficient, $I$ is the optical intensity and $z$ is the distance along the waveguide. This equation can be solved (see section 3.2) to show that the number of photons absorbed in the waveguide is given by:

$$N = \frac{P_0 t_p \lambda}{hc} \left( 1 - \frac{\alpha e^{-\alpha L}}{\alpha + \left( \frac{P_0 \beta_{TPA}}{\alpha} \right) (1 - e^{-\alpha L})} \right)$$  \hspace{1cm} (5.2.6)$$

where $P_0$ is the coupled optical power, $t_p$ is the optical pulse width, $\lambda$ is the free space wavelength, $h$ is Planck's constant, $c$ is the speed of light and $L$ is the device length. The number of photo-generated carriers can then be calculated remembering that, for every two photons lost to TPA only one pair of carriers is generated. From this expression the loss in the waveguide due to linear and two photon absorption can be calculated.

The photo-generated carriers are thought to produce phase shift by two dominant mechanisms; the plasma effect and the shift of the absorption band edge due to bandfilling. The Quantum Confined Stark Effect (QCSE) will give a small amount of phase shift due to carriers screening the built-in field of the p-i-n structure. The maximum phase shift due to QCSE can be estimated to be one radian [8] but is not expected to have that large a magnitude here. The carrier density generated in this case will be small compared to the transparency carrier density, due to a low absorption coefficient. The built-in field will not therefore be greatly screened. The change in refractive index due to the plasma effect (i.e. the presence of free carriers) is:

$$\Delta \mu = \frac{-e^2 \Delta N}{2m^* \varepsilon_0 \mu_0^2}$$  \hspace{1cm} (5.2.7)$$
where $\mu$ is the refractive index, $N$ is the number of free carriers, $m^*$ is the carrier effective mass and $\omega$ is the incident angular frequency (see chapter 3). The shift of the absorption spectra due to the filling of the valence and hole sub-bands can be shown to be:

$$\Delta\alpha(E) = \alpha_0(E) \{ f_v(E) - f_c(E) - 1 \}$$  \hspace{1cm} (5.2.8)$$

where $\alpha_0(E)$ is the unshifted absorption spectra, $f_v$ is the valence band electron Fermi function and $f_c$ is the conduction band electron Fermi function (see chapter 3).

In order to evaluate this change in absorption, the quasi Fermi levels in the valence and conduction bands must be calculated for a given carrier density, as discussed in chapter 3.

Assuming the photo-generated carriers are evenly distributed in the quantum wells, the phase shift due to the shift of the absorption band edge can be found by evaluating the standard Kramers-Kröning relationship linking absorption and refractive index [3]:

$$\Delta\mu(E) = \frac{c}{2\pi^2} P \int_0^\infty \frac{\Delta\alpha(\xi)}{\xi^2 - E^2} \, d\xi$$  \hspace{1cm} (5.2.9)$$

where $E$ is the energy corresponding to the operating wavelength and $P$ denotes the Cauchy principle value. In order to evaluate this expression numerically for a given change in absorption spectra, care is necessary when dealing with the singularity at the operating energy. Most workers in the field tend to use an approach similar to that described by Hutchings [9]. If

$$I = P \int_0^\infty \frac{f(x)}{x - d} \, dx$$  \hspace{1cm} (5.2.10)$$

(where $P$ denotes the Cauchy principle value), then

$$I = \lim_{\delta \to 0} \int_0^{d-\delta} \frac{f(x) - f(2d-x)}{x - d} \, dx + \int_{2d}^\infty \frac{f(x)}{x - d} \, dx$$  \hspace{1cm} (5.2.11)$$

The standard Kramers-Kröning Integral, expressed in terms of wavelength is:
Equation 5.2.12 can be rearranged in terms suitable for use in equation 5.2.11 simply by factorising the denominator. The integration is then carried out numerically using a closed Simpson’s rule algorithm. However, this method is very time consuming. A far more elegant approach is used here, utilising Fourier transforms. A more formal and general definition of the Kramers-Kronig relations may be found in [10]. If

\[ \alpha(\xi) = \alpha'(\xi) + j\alpha''(\xi) \]  

then

\[ \alpha'(E) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\alpha'(\xi)}{\xi - E} d\xi \]  

\[ \alpha''(E) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\alpha''(\xi)}{\xi - E} d\xi \]  

These equations may be solved by using the following relations [11]:

\[ KK\{\alpha'(E)\} = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\alpha'(\xi)}{\xi - E} d\xi \]  

\[ KK\{\alpha''(E)\} = F\{-j.sgn(t)F^{-1}\{\alpha''(E)\}\} \]  

where KK denotes the Kramers-Krönig transform, F the Fourier transform and F⁻¹ the inverse Fourier transform. Also \( \alpha(t) \) is the measured time response of the system and:

\[ F\{\alpha(t)\} = \alpha'(E) + j\alpha''(E) \]  

This analysis applies for any causal system which satisfies equation 5.2.13. In the case of absorption and refraction (the two parts of the complex dielectric constant of the material) a constant of \( \lambda/4\pi \) must also be taken into account [12].

Therefore, in order to obtain the shift in refraction spectrum from a change in absorption spectrum, the following procedure was used.

1. Take a change in absorption spectrum with points equally spaced in energy.
2. Multiply by \( \lambda/4\pi \).
3. Take the Fourier transform of the result.

4. Multiply the first half of the points by \(-j\), the rest by \(+j\).

5. Take the Inverse Fourier transform of the result to obtain the change in refraction spectrum.

The availability of efficient and robust FFT algorithms for modern PCs makes this procedure far more attractive than numerical integration techniques, and is at least two orders of magnitude faster to calculate.

Figures 5.2.4 and 5.2.5 show a typical result of these calculations for a quantum well carrier concentration of \(1.5 \times 10^{23} \text{cm}^{-3}\) in 7.5nm wide quantum wells. 5.2.4a shows the occupation probability as a function of energy for electrons (solid line) and holes (dashed line). 5.2.4b shows the resulting bleaching of absorption (dashed) and the initial absorption spectrum (solid). 5.2.4c shows the change in absorption spectrum.
Figure 5.2.4 Solution of bandfilling equations for a well carrier concentration of $1.5 \times 10^{23} \text{cm}^{-3}$ in 7.5nm wide quantum wells. (a) shows the occupation probability as a function of energy for electrons and holes. (b) shows the resulting bleaching of absorption and the initial absorption spectrum. (c) shows the change in absorption spectrum, and (d) the integrand of the Kramers-Kronig relation at a wavelength of 1.55μm (Equation 5.2.9).

Figure 5.2.5 Change in refraction as calculated from the Kramers-Kronig relations (equation 5.2.9) for a carrier concentration of $1.5 \times 10^{23} \text{cm}^{-3}$ in 7.5nm wide quantum wells, with an initial absorption spectrum as shown in figure 5.2.3.
Figure 5.2.4d is the integrand of the Kramers-Kronig relation (Equation 5.2.7) at an energy corresponding to a wavelength of 1.55μm. Note the singularity at the wavelength of interest. The resulting change in refraction, as calculated by the Fourier transform method is shown in figure 5.2.5.

Having found the change in refractive index due to the plasma effect and bandfilling, the phase shift along the waveguide can be found from:

$$\Delta \phi = \frac{2\pi}{\lambda} \Delta n L \Gamma$$  \hspace{1cm} (5.2.19)

where $\Gamma$ is the waveguide confinement factor and $\Delta \phi$ is the phase shift. Note that $L$ is used instead of the effective waveguide length $L_{\text{eff}}$ as the carriers are assumed to be evenly distributed along the length of the waveguide.

### 5.2.7 Calculation of Phase Shift to optimise number of Quantum Wells

In order to optimise the number of quantum wells for maximum phase shift efficiency, the theory described in section 5.2.6 is used to calculate the phase shift resulting from a 30ps gaussian pulse with peak coupled powers of 0.4W and 1.0W for quantum well numbers of between 1 and 20. The waveguide absorption ($\Gamma \alpha$) is kept constant at 4.5cm$^{-1}$ by varying the operating wavelength from 1.505μm for a single quantum well, to 1.556μm for 20 quantum wells. This is shown in figure 5.2.6. The overlap integral calculated for 7.5nm wide InGaAsP (Q=1.55) quantum wells, 12nm wide InGaAsP (Q=1.2) barriers and InP cladding such that the total intrinsic region width is 0.4μm as shown in figure 5.2.7.
Figure 5.2.6 Required operating wavelength to keep $\Gamma \alpha$ constant at 4.5 cm$^{-1}$.

Figure 5.2.7 Overlap integral calculated for TE polarised light in a structure with 7.5 nm wide InGaAsP ($Q=1.55$) quantum wells, 12 nm wide InGaAsP ($Q=1.2$) barriers and InP cladding such that the total intrinsic region width is 0.4 $\mu$m.
Figures 5.2.8 to 5.2.10 show the predicted carrier concentration per quantum well (figure 5.2.8), refractive index drop per quantum well (figure 5.2.9) and total phase shift (figure 5.2.10) as a function of the number of quantum wells for coupled peak powers of 0.4W and 1.0W. As would be anticipated, the well carrier concentration drops as the number of quantum wells is increased as the same number of carriers generated by a constant waveguide absorption are distributed between a greater number of quantum wells (figure 5.2.8). The resulting change in refractive index of the well material mirrors this drop in carrier concentration (figure 5.2.9). However, the resulting phase shift increases with the number of quantum wells, as the increase in waveguide confinement, $\Gamma$, dominates over the drop in well material refractive index in determining the phase shift (figure 5.2.10). The increase in phase shift is observed to reach a plateau at 12 quantum wells.

![Graph showing well carrier concentration as a function of quantum well number under excite by a 30ps gaussian optical pulse with coupled powers of 0.4W (squares) and 1.0W (circles).]
Figure 5.2.9 Well material refractive index change as a function of quantum well number under excitation by a 30ps gaussian optical pulse with coupled powers of 0.4W (squares) and 1.0W (circles).

Figure 5.2.10 Phase shift as a function of quantum well number under excitation by a 30ps gaussian optical pulse with coupled powers of 0.4W (squares) and 1.0W (circles).
Therefore the optimum number of quantum wells to use in a phase modulator device is 12. The optimum layer structure for a quantum well phase modulator is 12 InGaAsP (Q=1.55) quantum wells 7.5nm wide separated by 12nm InGaAsP (Q=1.2) barriers. The guiding layer is 0.4μm wide to reduce scatter loss by free carrier absorption in the doped p and n regions of the p-i-n structure. The performance of this structure is investigated more fully in section 5.2.9.

5.2.8 Lateral Waveguide Design

In order to make use of the nonlinear properties engineered in the MQW layer structure described in the previous sections, light must be confined in the MQW region, and guided along the device. It has been assumed up until now that the MQW layer structure would be operated with light confined single mode under a 3μm wide ridge. In this section the calculations carried out to ensure single mode operation of the waveguide are described i.e. the calculation of the required etch depth. A schematic diagram of the sections of the ridge waveguide used in the calculation is shown in figure 5.2.11.

![Schematic diagram of a ridge waveguide with the dimensions used in the calculations for the etch depth.](image)

The effective index method for calculating waveguide properties is described in appendix 3. It is applied here to a five layer problem, to find the effective refractive
index of the regions either side of the ridge and of the section under the ridge. The waveguide is then treated as a three layer slab, in order to determine the propagation constants and ensure single mode guiding in the lateral plane. The effective refractive index of the guiding (i) region is taken to be the weighted average of the refractive index of the material in that region. The calculations show that an etch depth of 0.9μm will give a $V_p$, the normalised propagation constant, of $0.62\pi$, well within the limit for single mode operation.

In order to compare the performance of the MQW layer structure, a waveguide with an active region of bulk InGaAsP was also designed. This waveguide was as similar as possible to the MQW device design, except that the intrinsic region consisted of InGaAsP (Q=1.45-nominally). This band edge was chosen to keep the waveguide absorption approximately the same as for the MQW device accounting for the differences in the overlap integral. The etch depth required for single mode operation was 1.0μm. These etch depth details are required before the layer structures can be grown as the wafers will be subsequently wet etched. The etch depth will be determined by the inclusion of an etch stop layer in the wafer structure. The final layer structure designs are shown in figures 5.2.12 and 5.2.13.
Figure 5.2.12 Final MQW layer structure design.

Figure 5.2.13 Final Bulk layer structure design.
5.2.9 Calculation of the Phase Shift for the Proposed Devices

The phase shift, change in refractive index, change in absorption coefficient, carrier concentration and waveguide loss have been calculated as a function of coupled power from the theory of section 5.2.6. The calculations have been carried out for the MQW and bulk devices described in the previous section. The results are shown in figures 5.2.14 (MQW) and 5.2.15 (Bulk).

The MQW device results are calculated for a 1mm long waveguide with 12, 7.5nm wide InGaAsP quantum wells (Q=1.55), 12nm wide InGaAsP (Q=1.2) barriers and InP cladding, with a heterostructure width of 0.40μm. The linear absorption coefficient is taken to be 9.5cm$^{-1}$ (4.5cm$^{-1}$ linear absorption and 5cm$^{-1}$ scatter) at an operating wavelength of 1.55μm. The TPA coefficient is taken to be 50cm/GW [8]. The carrier effective masses in the quantum wells are taken to be those of bulk InGaAsP at a band gap of 1.46μm (see appendix 1). The optical pulse width is 30ps. The absorption band edge is that described in section 5.2.5. The effective band gap of the waveguide was estimated using the 1D quantum well model and found to be 1.46μm.

![Image](image-url)

**Figure 5.2.14** Theoretical calculation of the bleaching of absorption and carrier concentration as functions of coupled pulse energy for the MQW device.
Figure 5.2.14 shows the increase in carrier concentration with increasing pulse energy coupled into the device. The associated bleaching of linear absorption at 1.55\(\mu\)m (the signal wavelength) due to phase space filling is also plotted. The carrier concentration remains well below transparency carrier density (~\(2 \times 10^{18} \text{cm}^{-3}\)) for all the pulse energies considered here, therefore the approximations of a low carrier density should remain valid in this case.

![Figure 5.2.14](image)

Figure 5.2.15 Theoretical calculation of waveguide absorption as a function of coupled pulse energy for the MQW device.

Figure 5.2.15 shows the loss of optical power in the waveguide due to linear and two photon absorption as a function of pulse energy. Initially, the loss drops, as would be expected from a falling linear absorption coefficient. However, the loss due to two photon absorption increases with increasing pulse energy, counterbalancing the increase in transmission due to linear absorption bleaching. Thus the overall waveguide loss remains approximately constant for the pulse energies under consideration here.

![Figure 5.2.15](image)
Figure 5.2.16 shows the induced phase shift due to the photogenerated carriers as a function of pulse energy. Both nonlinear mechanisms (bandfilling and the plasma effect) give phase shifts which are roughly linear with a slight curvature, as would be expected with a low level of photo-generated carriers. Bandfilling is the dominant mechanism for inducing phase shift in this case, giving rise to a phase shift an order of magnitude greater than the phase shift due to the plasma effect. For a coupled pulse energy of 30pJ, the total phase shift is in excess of $\pi$ radians, which is the minimum necessary for complete all-optical switching.

The calculations for the bulk device were carried out with the same parameters as for the MQW device as far as was possible. The main difference was that the calculation of the Fermi functions and occupation probabilities was carried out using a three dimensional density of states.
Figure 5.2.17  Theoretical calculation of the bleaching of absorption and carrier concentration as functions of coupled pulse energy for the bulk device.

Figure 5.2.17 shows the increase in carrier concentration with increasing pulse energy coupled into the device. Due to the larger volume of the bulk device, the carrier concentration is approximately a third of that of the MQW device. The associated bleaching of linear absorption at 1.55\textmu m (the signal wavelength) due to phase space filling is also plotted. Again due to the lower carrier density there is less bleaching of absorption than for the MQW device. From these calculations it would be expected that the phase modulation performance would be significantly poorer than that of the MQW device. However, the overlap between the optical mode and the guiding region is much greater in this case. It is this which is expected to compensate for the lower carrier concentration, and hence a less efficient nonlinear refraction.
Figure 5.2.18 Theoretical calculation of waveguide absorption as a function of coupled pulse energy for the bulk device.

Figure 5.2.18 shows the loss of optical power in the waveguide due to linear and two photon absorption as a function of pulse energy. In this case the bleaching of linear absorption by phase space filling is more than compensated for by the loss due to two photon absorption. Thus the overall waveguide loss increases approximately linearly for the pulse energies under consideration here. Again, this is a result of the lower carrier concentration in the bulk device than the MQW device.
Figure 5.2.19  Theoretical calculation of phase shift as a function of coupled power due to bandfilling and the plasma effect for the bulk device.

Figure 5.2.19 shows induced phase shift due to the photogenerated carriers as a function of pulse energy. Again, both nonlinear mechanisms (bandfilling and the plasma effect) give phase shifts which are roughly linear with a slight curvature, as would be expected with a low level of photo-generated carriers. The change in refractive index is far smaller than for the quantum well material due to the smaller carrier concentration in the bulk device. Bandfilling is again the dominant mechanism for inducing phase shift in this case. However, the proportion of the phase shift due to the plasma effect is greater than for the MQW device. As was expected, although the change in refractive index is far smaller than for the MQW device, the resulting phase shift is of the same order of magnitude as for the MQW device. This is due to the greater overlap between the optical mode and the guiding region. For a coupled pulse energy of 30pJ the total phase shift is just less than π radians.
5.3 Waveguide Testing

5.3.1 Introduction

The wafer designs described in section 5.2 were manufactured at BT Labs. to the specifications given in the previous section. Wafer AT2043 was the bulk wafer, AT2047 the quantum well wafer. Absorption measurements were carried out on the wafers prior to any further processing to determine the absorption band edge and verify the quality of the growth. This is described in section 5.3.2. 3μm ridges were etched on both wafers, and the wafers cleaved to a length of 2mm to form devices suitable for testing of the phase modulation characteristics of the bulk and MQW layer structures. (Whilst the designs were for 1mm long waveguides these devices were damaged and 2mm long waveguides had to be used). The phase modulation characteristics of the two waveguides were determined by the technique of self phase modulation (SPM), where the spectrum of light passing through the waveguide is measured. The phase modulation is then extracted by modelling the SPM spectra.

5.3.2 Wafer Absorption Measurements

The absorption spectra of the wafers AT 2043 (bulk) and AT 2047 (MQW) were measured in the wavelength region 1.2μm-1.7μm, around the absorption band edge of the samples. A knowledge of the band tail absorption spectra is essential for choosing the correct operating wavelength for observation of SPM. Operating too close to the band edge will result in poor transmission through the guide. Whilst this absorption will result in photo-generated carriers, and hence induce nonlinearity, the losses will be unnecessarily high. However, operating too far from the band edge there is little photogeneration of carriers. The measurements were carried out by measuring the transmission of infra-red light through suitably prepared sections of wafer using a Fourier transform spectrometer.
Sections of the two wafers (AT2043 and AT2047) were cleaved into 1cm squares for the purposes of the measurement. From each wafer, four such sections were used. Two of each set of four sections were wet etched to remove the ternary capping layer from the samples. The other samples were dry etched, removing a thickness of 300µm, to remove all the active layers from the InP substrate. These samples provided the reference spectra from which the true absorption spectra of the active layers was deduced. One of each of the pairs of samples thus prepared were then anti-reflection coated. The absorption measurements were then carried out. The uncoated samples were mounted such that the light was incident on the samples at 72°, the Brewster angle for the material. In this way Fabry-Perot effects were eliminated from the absorption spectra. For both the quantum well and bulk samples, spectra from the test and reference samples were measured, and the absorption spectra calculated by subtracting the two spectra so obtained. This was repeated for the coated wafers, mounted at normal incidence. In principle, these two measurements should provide data on absorption for both TE and TM polarisations. In the case of light incident at the Brewster angle, there are components of the incident light on both TE and TM axes, whereas for normal incidence light only couples to the TE polarisation. However, even for light incident at the Brewster angle, light is mostly on TE polarisation in the samples due to the high refractive index of InP. Therefore it was not possible to extract data for both TE and TM polarised light.

The results of the measurements are shown in figure 5.3.1 and 5.3.2. Figure 5.3.1 shows the direct output of the spectrometer. The vertical scale, marked absorbence is a logarithmic scale directly related to the light absorbed by the sample. The absorbence of the bulk sample is much higher than that of the MQW sample because the thickness of the bulk active region is much greater than the total thickness of the quantum wells (0.4µm as opposed to 90nm). The inset shows an expanded wavelength scale around the band edge. The absorbence falls below zero at wavelengths longer than 1.48µm. This would indicate gain rather than absorption in this region. However, as this is not
physically possible, it suggests that the absorption of the reference sample was, for some reason, higher than of the test sample. This may be some artifact of the dry etching used to remove the active layers in the reference sample. The linear absorption coefficient, $\alpha$, was then calculated using the following formula (after first shifting the absorbence zero level):

$$\alpha = \frac{1}{L_z} \ln(10^a)$$

(5.3.1)

where $L_z$ is the total width of the absorbing material, and $a$ is the absorbence. The absolute values of $\alpha$ are set from a knowledge of the waveguide absorption at 1.531\(\mu\)m, as measured on devices fabricated from these wafers.

![Absorbance spectra for bulk (AT 2043) and MQW (AT 2047) wafers. Inset shows expanded wavelength scale.](image)

Figure 5.3.1  Absorbence spectra for bulk (AT 2043) and MQW (AT 2047) wafers. Inset shows expanded wavelength scale.

It can be seen from figure 5.3.2 that the absorption spectra are greatly different for the bulk and quantum well cases. In the MQW case, there is a sharp exciton peak at 1.46\(\mu\)m, corresponding to the e-hh transition, also a more rounded exciton peak around
1.4μm corresponding to the e-h transition. The separation of these peaks is that which would be expected from the splitting of the light and heavy hole sub-bands in a 7.5nm wide quantum well. Note there are no transitions to higher levels in the well as the quantum wells have been designed such that there is no n=2 level in the conduction sub-band. The absorption is then flat out to 1.3μm, due to the two dimensional density of states. The sharp rise in absorption around 1.2μm is probably due to the Q=1.2 InGaAsP barriers. The bulk spectrum shows a more rounded absorption edge with less absorption overall than for the quantum well sample. The absorption rises moving into the band as expected for a three dimensional density of states.

The absorption spectra show that at 1.55μm, the linear absorption of both bulk and quantum well samples is of the order of 1-10 cm⁻¹, ideal for use in SPM experiments. Further, they also demonstrate that the model used in the design of the waveguides is reasonably accurate.

![Figure 5.3.2 Absorption spectra (TE polarised light) for bulk (AT 2043) and MQW (AT 2047) wafers. Inset shows expanded wavelength scale.](image-url)
5.3.3 Self Phase Modulation Experimental Method

The quantum well waveguide used was the p-i-n, SCH-MQW ridge waveguide described in section 5.2. The loss of the waveguide has been measured to be 10cm\(^{-1}\) at 1.55\(\mu\)m for TE polarised light. Photocurrent measurements were used to determine the fraction of the absorption of the waveguide which generates carriers. The previous absorption measurements do not give this information. Hence the linear absorption coefficient, \(\alpha\), was 5cm\(^{-1}\), with the remaining loss due to scatter. The device length was 2mm.

The bulk ridge device used was fabricated to be as similar to the quantum well ridge device as possible, to allow comparison between bulk and quantum well devices. The only differences are that the ridge etch depth is 1.0\(\mu\)m, and the intrinsic guiding layer consists only of InGaAsP (Q=1.45). However waveguide losses are higher than in the MQW device, with a loss of 15cm\(^{-1}\) for TE polarised light. TM polarised light is not guided by this structure.

The experimental apparatus is shown in figure 5.3.3. A NaCl:OH Colour Centre Laser is used to generate approximately 10ps wide pulses (measured on an SHG autocorrelator) at a repetition rate of 82MHz, with peak powers of up to 500W. The tuning range of the laser was \~\text{1.5-1.6}\mu.m. The actual operating wavelength was set to be 50meV beyond the absorption band edge; a wavelength of 1.55\(\mu\)m for the MQW ridge device and 1.542\(\mu\)m for the bulk device.
Figure 5.3.3 Experimental apparatus for self phase modulation measurements.

The infra-red light was coupled into the buried heterostructure waveguide using a lensed fibre, with an attenuator wheel at the input of the fibre to control the power incident on the waveguide. The incident polarisation was set using a fibre polariser. For both ridge devices, bulk optics were used to launch light into the waveguides. The output of the waveguide was monitored continuously on a camera. The SPM spectra were measured using a scanning Fabry-Perot interferometer, and recorded using a storage oscilloscope. Care was taken during the measurements to mark centre frequency on the recorded traces. Care was also taken to ensure no slab mode from the waveguides was analysed by the Fabry-Perot interferometer, by the use of slits and pinholes.

SPM spectra were recorded as a function of incident power for pulse energies up to approximately 1nJ. The devices were unbiased during the measurement, but terminated into 50Ω to prevent charge screening as much as possible. All measurements were carried out using TE polarised light.
5.3.4 Self Phase Modulation Modelling

The phase modulation information was extracted from the SPM spectra by modelling the spectra of a chirped pulse. If an optical pulse propagating is in a medium of length $L$, which has a time varying refractive index, $\Delta \mu(t)$, then the pulse will undergo a phase shift, $\Delta \phi(t)$, given by:

$$\Delta \phi(t) = \frac{2\pi L}{\lambda} \Delta \mu(t) \quad (5.3.2)$$

This phase shift is the self phase modulation which manifests itself as a shift in the spectrum of the optical pulse. That a phase shifted optical pulse should experience a change in its optical spectrum can readily be understood by remembering that, by definition, frequency is rate of change of phase. Therefore, the shift in the angular frequency, $\Delta \omega$, and the frequency spectrum, $\Delta \nu$, are given by:

$$\Delta \omega(t) = -\frac{\partial \phi(t)}{\partial t}$$
$$\Delta \nu = -\frac{1}{2\pi} \frac{\partial \phi(t)}{\partial t} \quad (5.3.3)$$

This time varying frequency shift (chirp) will modify the frequency spectrum of the optical pulse, which can be measured and the time varying phase shift associated with it can be extracted by modelling the spectrum of chirped optical pulse. The resulting shape of the chirped pulse frequency spectrum depends on both the magnitude and speed of recovery of the phase shift. Therefore, the magnitude and recovery time of the phase shift can be extracted using this technique.

The modelling was carried out in the following way. Firstly, the carrier temporal profile was generated using the simple rate equations to model the absorption of an asymmetric sech$^2$ pulse due to linear and two photon absorption. The pulse asymmetry was calculated from SPM spectra of a length of monomode fibre which exhibits an ultra fast Kerr-type nonlinear refractive index. This data showed that a CW component was present in the output of the colour-centre laser in addition to the 8ps mode locked pulses. A CW background was therefore included in the modelling of the incident pulse.
Carriers created due to linear absorption are excited into the well. Carriers created by TPA are excited into the barriers. It was assumed that the principal source of nonlinearity was due to carriers in the wells, therefore it was the well temporal carrier profile which was calculated. Time constants were included for carriers to scatter into the wells from the barriers, and for carriers to escape from the wells.

The diagram below schematically describes the physical processes which were modelled in order to obtain a well carrier population temporal distribution for use with the SPM modelling.

![Diagram of carrier dynamics between a quantum well and a barrier.](image)

Figure 5.3.4 Model of carrier dynamics between a quantum well and a barrier.

The rate equations used in the modelling were:

\[
\frac{d(N_w)}{dt} = \frac{d(N_{Lin})}{dt} + \frac{N_b - N_w}{\tau_{bw} \tau_{we}} \tag{5.3.4}
\]

\[
\frac{d(N_b)}{dt} = \frac{d(N_{TPA})}{dt} - \frac{N_b - N_w}{\tau_{be} \tau_{we}} \tag{5.3.5}
\]

Where \(N\) is the number of carriers and the subscripts \(w\) and \(b\) refer to wells and barriers respectively (note that all material in the guiding layer which is not well material is referred to as 'barrier' material). The subscripts \(Lin\) and \(TPA\) refer to carrier generation.
processes of linear absorption and Two Photon Absorption respectively. The values of \( \alpha \) and \( \beta_{TPA} \) (linear and two photon absorption coefficients) are scaled to account for the relative volumes of material in the barriers and wells. The \( \tau \)'s refer to time constants for processes by which carriers enter/leave the well/barrier states. The \( \tau_{sc} \) is the scattering time from barrier states to well states. The \( \tau_{ee} \) is the time for carriers to leave the wells. It is a lumped term including recombination and sweep out and it is intended to represent any physical process by which carriers will leave the well and therefore no longer contribute to the bandfilling phase shift. The \( \tau_{be} \) is a similar term for the barrier states.

This carrier profile was then used to generate a time-varying phase change profile by assuming a linear relationship between carrier concentration and phase. The resulting time-varying phase shift is superimposed on the sech\(^2\) pulse, which has had its temporal profile modified due to two photon absorption. The spectrum associated with such a pulse is generated using an FFT. This spectrum was then fitted to the experimental data, using the parameters of phase shift and decay constants, accounting for pulse asymmetry and two photon absorption. For the bulk device, the rate equation model is greatly simplified as there were no barriers in the material. This method of extracting phase modulation information is accurate to within 20% providing that the phase modulation is in excess of \( \pi \) radians. It is also strongly dependent on an accurate knowledge of the pulse shape and spectrum of the incident optical pulse.

5.3.5 Results and Discussion

Figures 5.3.5a and 5.3.6a show typical SPM spectra measured for the MQW ridge device and the bulk ridge device respectively. The spectra generated by the analysis are shown in figures 5.3.5b and 5.3.6b. They show very good agreement between the model and experiment. By fitting the adjustable parameters in the model to the measured data the phase modulation as a function of coupled pulse energy could be extracted. This is shown in figures 5.3.7 and 5.3.9 for the MQW and bulk devices respectively.
Figure 5.3.5a  Spectral plot of output of waveguide, showing chirp due to self phase modulation. MQW ridge device, coupled pulse energy 97.6pJ (10.75W). -0.12THz to 0.26THz.

Figure 5.3.5b  Modelling of self phase modulated frequency spectra. Well escape time constant, 10000ps, barrier to well scattering time 4.4ps, phase modulation -3.9\pi rad.
Figure 5.3.6a  Spectral plot of output of waveguide, showing chirp due to self phase modulation. Unbiased bulk ridge device, coupled pulse energy 85.4pJ (9.9W). -0.3THz to 0.48THz.

Figure 5.3.6b  Modelling of self phase modulated frequency spectra. Carrier escape time constant, 10000ps, TPA scattering time 0.2ps, phase modulation -2.4\pi rad.
Figure 5.3.7  Self Phase Modulation vs. Coupled pulse energy for the MQW ridge device.

Figure 5.3.8  Well capture time required to fit to data for the MQW ridge device.
The results for the MQW device appear to show some saturation of the nonlinearity for coupled pulse energies above 40pJ. In order to obtain a good fit to experimental data, the barrier-to-well scattering time is reduced from 4.5ps to less than 1ps for coupled pulse energies below 10pJ, as shown in figure 5.3.8. Physically this would suggest that the well states are sufficiently filled for coupled pulse energies above 40pJ to impede carrier capture into the wells. This would support the view that the nonlinearity was being saturated due to the occupation of well states.

Figure 5.3.9 shows the measured phase modulation for the bulk device. In this case the phase modulation performance is less efficient than for the MQW sample, though the higher waveguide losses in the bulk device make direct comparison difficult.

In both the bulk and MQW ridge devices, the nonlinearity can be seen to be saturating, though the precise form of this saturation differs between the two devices.
The blocking of states in the MQW device (possible evidence for which is shown in figure 5.3.8) is a process could account for the differences in saturation behaviour between figures 5.3.7 and 5.3.9.

Comparison of these results with the theory of section 5.2 shows reasonable agreement between theory and experiment. The phase shifts predicted by the theory are close to those achieved in practice. However, the precise form of the phase shift at low pulse energies (<20pJ) for the MQW device is not predicted by the theory. It should be noted that the error in fitting phase shifts to measured data for phase shifts below \( \pi \) radians is large. This may account for the shape of the figure 5.3.7.
5.4 Conclusions

The carrier assisted nonlinearities bandfilling and the plasma effect were modelled theoretically to determine nonlinear refraction in a semiconductor waveguide for light at a wavelength corresponding to a detuning of 50meV from the n=1 e- hh transition. This model was used to design a MQW layer structure for a semiconductor waveguide which will exhibit efficient nonlinear refraction. For comparison, a similar waveguide with a bulk active layer is also designed. The optimum layer structure given the requirements of low loss and a 1mm length is 12 InGaAsP (Q=1.55) quantum wells 7.5nm wide separated by 12nm wide InGaAsP (Q=1.2) barriers. The guiding layer is 0.4μm wide. A bulk layer structure was also designed for comparison with the MQW waveguide.

Absorption spectroscopy was carried out on the fabricated wafers. The results show a good quality of growth. The n=1 e- hh transition is at a wavelength of 1.466μm, very close to its design specification. The bulk wafer had a band edge of 1.45μm. The fabricated waveguides were tested to evaluate their nonlinear refraction characteristics using self phase modulation (SPM) techniques. The chirped spectra of light exiting the waveguide due to nonlinear refraction in the waveguide were measured. Modelling of these spectra taking account of the carrier generation processes involved and nonlinear absorption allowed the phase modulation information to be extracted. Phase modulation in excess of π radians was demonstrated in the optimised MQW waveguide for a coupled pulse energy of 30pJ. These pulse energies are compatible with those which can be produced by laser diodes emitting picosecond pulses. Therefore this carrier assisted nonlinearity is an attractive route for producing all-optical switches for telecommunications applications.

Saturation of the refractive nonlinearity was observed for coupled pulse energies above 40pJ. This was attributed to a filling of the available states in the conduction sub-band. The bulk waveguide also exhibited nonlinear refraction. The nonlinear refraction was less efficient than for the MQW waveguide. This was predicted by the theory. However, direct comparison between the two samples was difficult due to different waveguide losses.
5.5 References


In this chapter, the design, construction and evaluation of two nonlinear all-optical switches operating at a wavelength of ~1.55μm are described. A polarisation rotation gate is constructed using an InGaAsP/InGaAsP MQW waveguide as the nonlinear element. An integrated all-optical device is also demonstrated in the form of a nonlinear directional coupler. Both devices use the bandfilling nonlinearity to induce nonlinear refraction as a route to achieving all-optical switching. All-optical switching of a 30ps optical pulse is demonstrated for a pulse energy of 10.5pJ in a polarisation rotation gate.
6.1 Introduction

In chapter 4, an optical switch was demonstrated in an AlGaAs MQW waveguide utilising the non-resonant 3rd order Kerr-type nonlinearity. In order to reduce the power necessary to achieve switching towards that available from diode lasers, a carrier enhanced refractive nonlinearity in an InGaAsP/InGaAsP MQW waveguide has been considered here. This nonlinearity has been shown to give an efficient refractive nonlinearity (see chapter 5). For photon energies close to the band gap energy, electron-hole pairs are generated by absorption. The resultant nonlinear refractive index change arises from a variety of mechanisms, predominantly bandfilling and the plasma effect. The recovery times of these nonlinearities are limited by the carrier escape time from the quantum wells (≤50ps [1,2]). In quantum well material, this change in refractive index is greater for TE polarised light than for TM polarised light because of the difference in effective band edges for these two polarisations. This anisotropy results in a nonlinear birefringence which can be utilised to form the basis of a nonlinear switch by employing a polarisation rotation gate [3].

The efficient phase modulation exhibited by the bandfilling nonlinearity in InGaAsP/InGaAsP MQW material can also be exploited to form an integrated nonlinear all-optical switch. Unlike the polarisation rotation gate, this device requires no optical components in addition to the MQW waveguide [4]. In its simplest form a nonlinear directional coupler is two parallel ridge waveguides whose spacing is sufficiently close to allow the interaction of optical modes supported by the twin guides. Light coupled into one waveguide will couple to the other guide as the light propagates along the waveguide. A change in the refractive index of the waveguides will perturb the optical modes in the waveguide and therefore modify the coupling between the guides. In this case, bandfilling provides the necessary nonlinear refraction in order to demonstrate diode pumped all-optical switching at ~1.55μm. Such a device has been demonstrated in GaAs MQW material operating at a wavelength of 855nm [5] with a recovery time of 130ps [6].
6.2 Polarisation Rotation Gate

6.2.1 Introduction

The polarisation rotation gate was introduced in chapter 4 where a polarisation rotation gate was constructed using a GaAs/AlGaAs MQW waveguide as the nonlinear element. The nonlinearity exploited in that case was the optical Kerr effect at a wavelength corresponding to half the band gap of the material. The work presented here is a similar device, however the nonlinear element is an InGaAsP/InGaAsP MQW waveguide with the operating wavelength in the absorption band tail of the material. The design of this particular waveguide (for optimised phase shift efficiency) is described in chapter 5. The nonlinearity exploited in this case in bandfilling. Bandfilling provides the birefringence necessary for the operation of a polarisation rotation gate due to the splitting of the light hole and heavy hole valence bands in MQW material. This is described more fully in section 6.2.2.

In order to demonstrate nonlinear optical switching, a source of relatively high power picosecond optical pulses was required. The previous polarisation rotation gate experiments were carried out using a mode-locked colour-centre laser. Such a laser provides transform limited picosecond and femtosecond pulses with peak pulse energies of up to the order of 100µJ. The SPM measurements show that the pulse energies necessary to achieve phase shifts of approximately π radians are of the order of 100pJ. These powers are beyond the range a laser diode can produce at a wavelength of ~1.55µm. However, the advent of erbium doped fibre amplifiers with small signal gains of up to 40dB [7] allows such powers to be achieved without the need for the complexity of a colour centre laser. The construction of a Ar:Ion pumped, erbium doped fibre amplifier (EDFA) is described. This amplifier was used in conjunction with a Q-switched three contact distributed feedback (DFB) laser [8] to provide 30ps optical pulses with peak pulse energies of up to 250pJ. This source was used to probe the nonlinear polarisation rotation in the InGaAsP/InGaAsP MQW waveguide.
6.2.2 Theory of Operation

The theory of operation of a nonlinear polarisation rotation gate was discussed extensively in chapter 4. The basic requirement for a nonlinear polarisation rotation gate is to have a nonlinear birefringent element. The bandfilling nonlinearity investigated in chapter 5 provides just such a birefringent nonlinearity in MQW material. Bandfilling is a refractive nonlinearity in which photo-generated carriers modify the absorption spectrum by blocking the available states in the valence and conduction bands. This bleaching of the absorption spectrum must also induce a refractive nonlinearity via the Kramers-Kröning relations. The magnitude of the refraction is a function in wavelength of detuning from the absorption band edge (where the bleaching of absorption is greatest). This is shown pictorially in figure 6.2.1 using the equations derived in chapter 5.

![Figure 6.2.1](image)

**Figure 6.2.1** Refractive index shift as a function of wavelength due to bleaching of the absorption spectrum. The effective band edge of the device is 1.46µm for the e-hh transition and TE polarised light.
In multi-quantum well material, the effective band edges seen by TE and TM polarised light are at different wavelengths due to the splitting of the light hole and heavy hole valence sub-bands (as discussed in section 2.4). A schematic diagram of this is shown in figure 6.2.2. Calculations for the InGaAsP/InGaAsP MQW structure under consideration here show the difference in effective band edges between TE and TM polarised light to be ~60nm. It is this difference in the effective band edges that leads to the required nonlinear birefringence. For TM polarised light, the effective operating wavelength is a few tens of nanometers further from the e-hh transition than for TE polarised light as TM polarised light couples only to the light hole valence sub band. Figure 6.2.1 shows that this greater detuning from the band edge leads to a significant nonlinear birefringence as nonlinear refraction is a function of the effective detuning from the e-hh transition.

![Schematic representation of the light hole and heavy hole valence sub-band splitting and the resulting different effective band edges for TE and TM polarised light.](image)

Calculations of the phase shift as a function of power at the appropriate effective detunings from the band edge for TE and TM polarised light have been carried out. These calculations are similar to those described in chapter 5 for the optimisation of phase modulation efficiency. It is assumed that all the carriers are generated by the
absorption of TE polarised light. For the carrier population thus created, the nonlinear refraction and hence phase modulation are calculated for TE polarised light (figure 6.2.3a). The effective detuning of TM polarised light from the e-hh transition wavelength (the TE band edge) is then calculated. The phase shift at this detuning is then calculated for TM polarised light, accounting for the difference in quantum well overlap for TM polarised light. The results of those calculations are shown in figure 6.2.3b. The parameters used in the calculations are those of the MQW structure used in the experiment (see section 6.3).

It can be seen from figure 6.2.3 that the power necessary to achieve a difference in phase shift of $\pi$ radians between the TE (figure 6.2.3a) and TM (figure 6.2.3b) polarisations (approximately the power required for a complete switch) is 500mW (15pJ pulse energy). In practice the situation is more complicated, as the asymmetry in waveguide loss and coupling efficiency between TE and TM polarised light must also be considered. However these calculations do show that it should be possible to observe all-optical switching at powers that can be achieved using amplified diode lasers.

![Figure 6.2.3a](image_url)

**Figure 6.2.3a** Phase shift as a function of coupled power for a 30ps optical pulse. The e-hh transition wavelength is 1.46\(\mu\)m with an operating wavelength 1.55\(\mu\)m and TE polarised light.
Figure 6.2.3b  Phase shift as a function of coupled power for a 30ps optical pulse. The e-hh transition wavelength is 1.46μm with an operating wavelength 1.55μm and TM polarised light.

6.2.3 Experimental Method

The waveguide used was a 2 mm long p-i-n MQW ridge waveguide, whose layer structure is described in chapter 5. The erbium doped fibre amplifier is shown schematically in figure 6.2.4. This amplifier was constructed from a length of erbium doped fibre using bulk optics and a 50:50 fibre coupler. The erbium doped fibre consisted of a 3.2μm core doped to a concentration of 1.5±0.5×10^{18} cm^{-3}, with a 90μm diameter cladding (refractive index step 12×10^{-3}) with an overall diameter of 250μm (with acrylate coating). The fibre was supplied by BNR Europe Ltd. The length of erbium doped fibre was chosen to be as long as possible, while not allowing the amplifier to lase from scattering in the fibre. For this fibre that length was 18m. The pump source was an Ar:Ion laser (Spectra Physics 2030, 20W all-line) operating in single line mode at a wavelength of 528.4nm. The internal aperture of the Ar:Ion laser was restricted to give
an output power of 200mW. The pump was coupled into the erbium fibre via a 50:50 fibre coupler (50:50 coupling for wavelengths between 1.52-1.57μm). The fibre coupler was fusion spliced onto the erbium fibre. The other end of the erbium fibre was placed in an angled capillary tube filled with index matching gel in order to prevent the amplifier lasing from facet reflections. Light from the diode laser was coupled into the erbium doped fibre via a quarter wave plate and a polarising beam splitter. The isolator in front of the diode laser was to prevent back-amplified spontaneous emission from the erbium fibre being incident on the laser facet, and possibly causing the amplifier to lase or to destroy the diode laser.

The quarter wave plate is used to make light entering the erbium fibre circularly polarised. The light passes down the fibre to the 50:50 coupler, where it is partially reflected by the mirror, then returns for a second pass along the amplifier. The phase change induced by the reflection at the mirror causes the light passing through the quarter wave plate to be polarised orthogonally to the incident polarisation. Therefore, the polarising beam splitter routes the output of the amplifier towards the output isolator. The double pass configuration allows the maximum amplification possible for this combination of fibre and pump source.
The experimental arrangement is shown in figure 6.2.5. The laser diode shown was a three contact InGaAsP DFB semiconductor laser emitting at a wavelength of 1.536μm. This wavelength is close to the peak gain of the erbium doped fibre amplifier (EDFA) and corresponds to a detuning of ~50meV from the band edge of the waveguide. The laser was driven under Q-switched operation [8] in order to produce 30ps optical pulses (measured using an autocorrelator) with peak powers of 100mW, at a repetition rate of 1.78MHz. The optical pulses were amplified using the EDFA to give peak pulse energies of 190pJ and peak powers of 6.3W. The low repetition rate was used to prevent the amplifier becoming saturated. The output of the amplifier was constantly monitored using a photodiode (18ps rise time) and a 25ps rise time sampling oscilloscope to ensure that the laser pulses remained of high quality. The output facet of the waveguide was monitored using the IR camera to ensure monomode operation of the waveguide. The
output of the gate was filtered using monochromator (set to a resolution of 1nm) in order to remove amplified spontaneous emission generated by the amplifier from the optical signal.

Figure 6.2.5 Experimental Apparatus

The polarisation gate was constructed as described in chapter 4. The waveguide is birefringent and therefore light entering the waveguide polarised at a angle off the principle axes leaves the waveguide elliptically polarised. The elliptical light was then linearised using the quarter wave plate and blocked using the polariser. As a result of the anisotropy of the carrier enhanced nonlinearity, the waveguide also exhibited a nonlinear birefringence. Thus, as the optical power was increased, the polarisation state of the light leaving the waveguide was changed and so the quarter wave plate no longer completely linearised the polarisation. This resulted in a transmission of the gate which was dependent on input power.

The gate transmission was first minimised for low power CW light by appropriate manipulation of the quarter wave plate and polariser for an incident polarisation of 22.5\(^\circ\) off the TE polarisation of the waveguide. The signal from the Q-switched diode was
chopped before the amplifier and was detected using a lock-in amplifier and photodiode at the output of the monochromator. The coupling loss to the waveguide was 10dB (determined from Fabry-Perot loss and transmission measurements) and therefore the peak pulse energy in the waveguide was 19pJ (630mW). By varying the incident pulse energy the nonlinear gate transmission was measured.

6.2.4 Results and Discussion

The gate transmission is shown in figures 6.2.6 and 6.2.7. Increasing the incident pulse energy causes bandfilling due to photo-generated carriers, resulting in a nonlinear refractive index which has polarisation asymmetry at 1532nm. As discussed earlier, this polarisation asymmetry arises from the valence sub-band splitting in MQW material. This nonlinear birefringence causes polarisation rotation to occur. This can be seen in figures 6.2.6 and 6.2.7 where power is transferred from the 'passing' state to the 'blocking' state as the incident pulse energy is increased. Hence the gate transmission (figure 6.2.7) exhibits nonlinearity with power. Switching occurs for a coupled power of 350mW, a pulse energy of 10.5pJ. The low pulse powers necessary for this all-optical switch are considerably less than those required for switching using the nonresonant 3rd order Kerr nonlinearity.
Figure 6.2.6  Polarisation gate transmission as a function of pulse energy.

Figure 6.2.7  Normalised gate transmission as a function of coupled power.
6.3 Nonlinear Directional Couplers

6.3.1 Introduction

The nonlinear directional coupler (NLDC) is a simple integrated optical device, consisting of two parallel ridge waveguides of a fixed separation. A schematic diagram of such a structure is shown in figure 6.3.1. In this section, the theory of linear and nonlinear directional couplers is discussed. The theory of linear directional couplers is used in conjunction with the effective index method (see appendix 3) in order to design a linear directional coupler. Such a coupler, fabricated from the same wafer as that described in chapter 5, will exhibit the same refractive nonlinearities as those measured in the single ridge waveguides. It is these refractive nonlinearities which cause the linear coupler to exhibit nonlinear behaviour in the coupling ratio between the outputs of the two arms. As the power coupled into the device increases, the ratio of output intensities switches from ~1:100 at low power to ~100:1 in an "ideal" case. In practice complete switching is not possible due to waveguide losses and pulse break-up.

Figure 6.3.1 Schematic diagram of a twin ridge waveguide directional coupler.
The nonlinear behaviour of the NLDC can be modelled using coupled nonlinear differential equations which describe the propagation of the optical modes supported by the coupled ridge structure. The effects of linear and nonlinear (two photon absorption) loss are included following the work of Stegman et al. [9] and Delong et al. [10]. These equations are then solved numerically using known material parameters in order to model the nonlinear coupling behaviour.

The linear coupler was fabricated using the optimised phase shift wafer design (see chapter 5). The nonlinear coupling behaviour of this device was investigated by measuring the transmission of 30ps optical pulses with peak pulse energies of up to 500pJ as a function of power. These measurements demonstrate all-optical switching of a 30ps pulse in an integrated semiconductor waveguide switch.

6.3.2 Theory and Design of Linear Directional Couplers

Consider the twin ridge waveguide structure shown in figure 6.3.1. In the case where the separation between the guides is large, the structure will support an optical mode confined to the region below each of the two ridges and there will be effectively no interaction between the two modes. In this case the electric fields of the optical modes can be described in the following way (see for example [11,12]):

\[ E_{1z} = a_1(z)\phi_1(x,y) \]  
\[ E_{2z} = a_2(z)\phi_2(x,y) \]

where 
\[ a_i(z) = a_i(0)e^{-j\mu_i z} \]
and the \( e^{j\omega t} \) term has been omitted for simplicity. \( a_i(z) \) is the amplitude of the electric field in the optical modes. The subscripts 1 and 2 refer the optical modes confined by ridges 1 and 2 respectively. \( \beta \) is the single waveguide propagation constant defined by 
\[ \mu_{\text{eff}} = \beta/k_0 \]  
where \( \mu_{\text{eff}} \) is the effective refractive index seen by the propagating optical mode, and \( k_0 = 2\pi/\lambda \) where \( \lambda \) is the free space wavelength.
If an interaction between the two modes is allowed (i.e. reducing $D$) then considering a small distance $\Delta z$ along the guide:

$$\Delta a_1(z) = -j\beta_1 a_1(z) \Delta z + \kappa_{12} a_2(z) \Delta z$$  \hspace{1cm} (6.3.3)

Where $\kappa_{12}$ is the coupling factor between guide 1 and guide 2. In the limit as $\Delta z$ tends to zero:

$$\frac{da_1}{dz} = -j\beta_1 a_1 + \kappa_{12} a_2$$  \hspace{1cm} (6.3.4)

similarly for waveguide 2:

$$\frac{da_2}{dz} = -j\beta_2 a_2 + \kappa_{21} a_1$$  \hspace{1cm} (6.3.5)

Equations (6.3.4) and (6.3.5) form the coupled mode equations which describe the propagation of an optical field along the waveguides. The optical power in each waveguide is defined as $P_{1,2} = |a_{1,2}|^2$. In the specific case of a directional coupler, light is coupled into only one arm. Therefore $a_1$ is the applied field, $a_2$ is the induced field, $a_1(0)=1$, $a_2(0)=0$ (in normalised units) and $L$ is the interaction length. The coupled mode equations can be solved (see Appendix 4) to show that the power in each arm as a function of interaction length is:

$$P_1(L) = |a_1(L)|^2 = \cos^2(YL) + \frac{(\Delta \beta)^2}{Y^2} \sin^2(YL)$$  \hspace{1cm} (6.3.6)

$$P_2(L) = |a_2(L)|^2 = -\frac{\kappa^2}{\kappa^2 + (\Delta \beta)^2} \sin^2(YL)$$  \hspace{1cm} (6.3.7)

where

$$\Delta \beta = \frac{\beta_1 - \beta_2}{2} \quad Y^2 = \kappa^2 + (\Delta \beta)^2$$

and $\kappa_{12} = \kappa_{21} = -j\kappa$. In the case where guides 1 and 2 are identical, the equations simplify further to give the following:

$$P_1(L) = \cos^2(\kappa L)$$  \hspace{1cm} (6.3.8)
It can be seen from these equations that in the case of two identical parallel ridge waveguides with light injected into one arm of the coupler the power in each guide sinusoidally oscillates as a function of coupling length. If the guides are non-identical then the maximum fraction of the injected light which may be cross-coupled is $\frac{\kappa^2}{(\kappa^2 + \Delta\beta^2)}$.

A mathematically equivalent way of looking at the problem of two coupled waveguides, but one which is conceptually more easily understood, is to consider the twin ridge structure as one waveguide which is capable of supporting two "supermodes" [13] as shown in figure 6.3.2.

![Diagram of twin ridge waveguide directional coupler](image)

**Figure 6.3.2** Schematic diagram of a twin ridge waveguide directional coupler showing the two supermodes supported by such a structure.

The modes are created initially in phase with each other at $z=0$ when light is injected into one arm of the coupler. The two modes then propagate along the coupler with
different phase velocities due to the different effective indices. At any point along the
guide, the power in each arm is given by the linear superposition of the complex
amplitudes of the two modes. It can readily be seen that if the two modes have a relative
phase delay of $\pi$ radians between them, then all the power initially in arm 1 will have
been transferred to arm 2.

In order to design a directional coupler an explicit relationship is required for the
coupling coefficient $\kappa$, in terms of the effective refractive indices of the ridge ($\mu_i$) and
spacer regions ($\mu_b$). For ridges of width $w$, and centre to centre separation $D$, the coupling
coefficient, $\kappa$, is given by [13]:

$$\kappa = \frac{2k_0(\mu_i^2 - \mu_b^2)}{\sqrt{b(\mu_i^2 - \mu_b^2) + \mu_b^2}} \frac{b(1-b)}{2+bV} e^{-\sqrt{b}V\left(\frac{\mu-r}{D}\right)}$$  \hspace{1cm} (6.3.10)

where

$$V_p = k_0d \sqrt{\mu_i^2 - \mu_b^2}$$

$$b = \frac{\beta^2 - \mu_b^2k_0^2}{(\mu_i^2 - \mu_b^2)k_0^2}$$

$V_p$ is the normalised propagation constant and $b$ is the guide refractive index. $V_p$ was
calculated from the effective index method applied to a single ridge waveguide (see
chapter 5). The relationship between $V_p$ and $b$ has been calculated for single mode
optical waveguides [14]. From the results of this analysis $b$, was found to be to be $-0.55$
(for $V_p=2.45$). From equations (6.3.8) and (6.3.9) the minimum length required for
complete cross-coupling between the arms of the coupler is given by:

$$L_c = \frac{\pi}{2\kappa}$$  \hspace{1cm} (6.3.11)

From these equations, the coupling length as a function of ridge separation (between the
edges of the ridges) was calculated. The results are shown in figure 6.3.3.
Figure 6.3.3 Coupling length for complete cross-coupling at a wavelength of 1.55μm as a function of ridge separation, calculated for two, 3μm ridges etched to a depth of 0.9μm. The material parameters are those of the optimised phase shift wafer design (see chapter 5).

From these calculations the design for a 1mm coupling length directional coupler using the optimised phase shift wafer design was realised. It should be noted that the tolerance on ridge separation is quite critical in this regime. The precision with which the ridges would have to be etched in order to achieve exactly a 1mm coupling length are beyond that which can be achieved in practice. It should also be noted that all these calculations use the effective index method to calculate the waveguide propagation constants. This is a method which is only accurate to within a few percent. Therefore, to ensure the fabrication of a directional coupler which would cross-couple with an interaction length of 1mm, several ridge widths and ridge separations were tried. These are listed in the following table.
Table 6.3.1 Ridge widths and separations of the four, twin ridge waveguide directional couplers fabricated.

<table>
<thead>
<tr>
<th>Device Number</th>
<th>Ridge Width* / μm</th>
<th>Ridge Separation / μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>33401</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>33402</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>33403</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>33404</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

* Nominal ridge widths.

All the devices were fabricated using wafer AT2047. The etch depth was 0.9μm (set by the etch stop layer in the wafer).

6.3.3 Theory of Nonlinear Directional Couplers

In the case of the nonlinear directional coupler under consideration here, the refractive index of the waveguide is dependent on the pulse energy of the incident optical pulse. The coupled differential equations used previously to describe the propagation of light along a linear directional coupler can, with appropriate modification, also be used to describe the nonlinear directional coupler. This is done by including a term to account for the change in the propagation constant, $\Delta \beta$, due to the change in refractive index ($\Delta \mu = \Delta \beta/k_0$). In the case of carrier assisted nonlinearities, this change in refractive index is a function of pulse energy. It has been shown in section 5.3 that this refractive index shift is approximately linear with pulse energy for pulse energies $< 50 \text{pJ}$. Therefore the change in refractive index can be written simply as $\Delta \mu = \mu_2 E_p / A_{\text{ext}}$, where $\mu_2$ is a nonlinear refractive index coefficient determined from the experimental data in section...
5.3 (the nonlinearity being a carrier assisted nonlinearity here), $E_p$ is the pulse energy, $t_p$ is the pulse width and $A_{\text{eff}}$ is the effective mode area. The coupling equations then become:

$$-j \frac{d}{dz} a_1(z) = \kappa a_2(z) + \Delta \beta \frac{E_p}{t_p} k_o a_1(z)$$ (6.3.12)

$$-j \frac{d}{dz} a_2(z) = \kappa a_1(z) + \Delta \beta \frac{E_p}{t_p} k_o a_2(z)$$ (6.3.13)

As in any nonlinear process, the presence of waveguide absorption imposes a serious limitation on the efficiency of any nonlinear switching process [15]. Therefore, in modelling the nonlinear directional coupler, linear and two photon absorption effects must be taken into account. The appropriate modifications to equations (6.3.12) and (6.3.13) are shown below [9,10]:

$$-j \frac{d}{dz} a_1(z) = \kappa a_2(z) + \Delta \beta \frac{E_p}{t_p} k_o a_1(z) + j \alpha a_1(z) + j \frac{\beta_{\text{TPA}} E_p}{2 A_{\text{eff}} t_p} a_1(z)$$ (6.3.14)

$$-j \frac{d}{dz} a_2(z) = \kappa a_1(z) + \Delta \beta \frac{E_p}{t_p} k_o a_2(z) + j \alpha a_2(z) + j \frac{\beta_{\text{TPA}} E_p}{2 A_{\text{eff}} t_p} a_2(z)$$ (6.3.15)

where $\alpha$ is the linear waveguide loss, $\beta_{\text{TPA}}$ is the two photon absorption coefficient and $A_{\text{eff}}$ is the optical mode effective area. Solving these equations analytically is not a trivial matter. However, numerical solutions can be obtained relatively easily using software such as MATLAB. MATLAB employs 4th/5th order Runga-Kutta algorithms with automatic step control to obtain numerically stable solutions. By analysing these equations, some basic properties of nonlinear directional couplers can be established.

The power required to achieve a 50:50 coupling ratio of an infinitely long directional coupler is the critical power $P_c$, defined as:

$$P_c = \frac{A_{\text{eff}} \lambda}{\mu_2 L_c}$$ (6.3.16)

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At low powers (well below $P_c$) the power in each arm oscillates as a function of length. This is shown pictorially in figure 6.3.4, where the incident power has been set to be $0.5P_c$.

At the critical power, the maximum cross coupling that can be achieved is 50:50, irrespective of the length of the device. This is shown in figure 6.3.5. For powers beyond $P_c$ complete cross coupling is impossible as a function of length.

In practice, couplers are usually designed to be one coupling length long, hence at low power all the power incident in the bar arm emerges at the cross output facet. Equations (6.3.14) and (6.3.15) can be also be solved for a fixed length device as a function of power. For the same parameters as those used in the previous results, with the device length set to the coupling length, figure 6.3.6 shows the power response of a lossless NLDC.

![Figure 6.3.4](image)

**Figure 6.3.4** Power in the arms of an NLDC as a functions of interaction length. The incident power is set to $0.5P_c$ (the critical power) for this device.
Figure 6.3.5  Power in the arms of an NLDC as a function of interaction length. The incident power is set to $P_c$ (the critical power) for this device.

Figure 6.3.6a Transmission characteristic of a lossless NLDC. The device length is one beat length ($L_c$).
Unlike interferometric devices, the nonlinear directional coupler does not transfer power back to the bar arm once switching has been achieved. It is this property of NLDCs that makes them particularly attractive for switching applications.

However, in practice no semiconductor waveguide is lossless. In the case of a resonant nonlinearity, operating in the absorption band tail, there is both linear and two photon absorption. It is these effects which limit the maximum possible switching fraction. The parameters necessary to predict the performance of the nonlinear directional couplers fabricated from the same wafer as the ridge waveguides described in chapter 5 can be extracted from the phase modulation measurements described there. Figure 6.3.7 shows the results of this calculation. The device length is 1mm. The coupling length is set to 1.18mm; this best fits the experimental data described in the following sections for device 33404. The linear loss is 15cm$^{-1}$, $\beta_{\text{TPA}}=80$cm/GW and $A_{\text{eff}}$ is assumed to be 1$\mu$m$^2$. 

Figure 6.3.6b Normalised transmission of a lossless NLDC as a functions of incident power. The Device length is one beat length ($L_c$).
Figure 6.3.7a Theoretical transmission of a 1mm long nonlinear directional coupler including linear absorption, waveguide scatter and two photon absorption. Parameters are set to model the behaviour of device 33404 (see table 6.3.1).

Figure 6.3.7b Normalised transmission characteristic.
It can be seen from these calculations that we would expect to see 80% cross-coupling for a coupled pulse energy of 100pJ, with switching (i.e. a coupling ratio of 50:50) occurring at ~60pJ coupled pulse energy. This is within the range of powers available from an erbium amplified Q-switched three contact DFB laser.

6.3.4 Experimental Method

The nonlinear directional couplers were fabricated to the designs specified in the previous section. Each of the four devices was waveguide tested and found to be single moded for operation at a wavelength of 1.536μm (the gain peak of the erbium doped fibre amplifier). The experimental apparatus used to investigate the nonlinear behaviour of the directional couplers is shown in figure 6.3.8. The three contact laser diode was a bulk Distributed Feed Back (DFB) three section laser supplied by BNR Europe. The laser was 400μm long, split 100μm:200μm:100μm. The laser was driven under double pulsed operation [8] to give 30 ps pulses with peak pulse energies of ~4pJ at a repetition rate of 4MHz. This was the maximum repetition rate possible without causing the erbium doped fibre amplifier to saturate and limit the peak amplified pulse energy. The bulk isolator placed immediately after the three contact laser was used to prevent back amplified spontaneous emission from the amplifier reaching the laser facet. This was to protect the three contact laser should the erbium amplifier have lased off the three contact laser facet.
The optical power incident on the directional coupler device under investigation was controlled at the input to the amplifier by means of a half wave plate and polariser. The polariser was set to maximise transmission through the amplifier system. The half wave plate was then used to rotate the polarisation of the light incident on the polariser and hence continuously control the power entering the amplifier. Controlling the optical power in this way ensured that the coupling efficiency into the directional coupler was constant throughout the experiment.

The erbium doped fibre amplifier used is a commercial diode pumped erbium amplifier, supplied by BT & D Technologies, having a peak small signal gain of 38dB and a saturation output power (CW) of 14.6 dBm. This amplifier is far more compact than the Ar:Ion pumped amplifier used in section 6.2, due largely to the compact nature of the pump lasers. This system is therefore truly an all-diode pumped system. The configuration of this amplifier has been modified from that of a standard commercial
erbium amplifier where the erbium doped fibre is pumped from both ends with an isolator in the middle. In this case, both pumps propagate down the fibre in the same direction as the amplified signal. The second pump laser is coupled in to the erbium fibre half way along the length of the fibre. This is to prevent the amplified three contact laser pulse from damaging the pump lasers.

The output of the amplifier was connected to the lensed fibre via connectorised fiber to minimise losses and achieve the maximum possible power delivered to the waveguide. The fibre isolator was used to allow optical connections (via FC/PC connectors) be to broken/made while the amplifier is running. The output of the isolator was passed through a tuneable 1nm bandpass filter to filter out amplified spontaneous emission and excess 1480nm light from the amplifier pump lasers. A fibre polarisation control was used to set the incident polarisation to be linear. Light was coupled in and out of the waveguide using lensed fibres. The output of the waveguide was measured using an optical power meter.

The experiment was carried out in the following way. While monitoring the Q-switched pulse output from the amplifier using an 8ps rise-time photodiode and a 25ps rise-time S-4 head on a sampling oscilloscope, the drive conditions for the three contact laser were optimised for maximum power output. The output lensed fibre was aligned by forward biasing the waveguide to 40mA and optimising the coupling of light from the device. Using this method it was possible to discriminate between the outputs from the two waveguides. The input coupling was then aligned by monitoring the high speed photocurrent using low power pulses. Low power pulses were used to avoid saturation problems caused by charge build-up in the device. The coupling was optimised and the polarisation was set to TE by maximising this photocurrent. By looking at the photocurrent it was possible to discriminate between coupling into the two arms of the coupler.
The power transmitted by the waveguide was then measured as a function of pulse energy for incident pulse energies of up to ~500pJ. Light was coupled into the other input arm and the measurement repeated. This ensured that the output coupling remained constant throughout the measurement. These measurements were carried out on all four directional coupler devices. For the device which exhibited the strongest nonlinear coupling properties (33404), the measurements were repeated as a function of forward bias up to 5mA and reverse bias up to 0.75V.

6.3.5 Results and Discussion

The nonlinear directional coupling properties were measured for each of the four couplers (devices 33401-33404). The linear (low power) coupling ratios of each of the devices at a wavelength of 1.536μm are tabulated below.

<table>
<thead>
<tr>
<th>Device Number</th>
<th>Etch Widths/μm (Ridge/Spacer/Ridge)</th>
<th>Bar Coupling Ratio</th>
<th>Cross Coupling Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>33401</td>
<td>3/3/3</td>
<td>0.35</td>
<td>0.65</td>
</tr>
<tr>
<td>33402</td>
<td>2/2/2</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>33403</td>
<td>2.5/2.5/2.5</td>
<td>0.08</td>
<td>0.92</td>
</tr>
<tr>
<td>33404</td>
<td>3/2/3</td>
<td>0.05</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 6.3.2 Linear directional coupling properties of the MQW twin ridge waveguide directional couplers.

From low power CW transmission measurements on a single ridge waveguide etched at the same time as the nonlinear directional couplers, the linear loss of the device was measured to be $\alpha = 15\text{cm}^{-1}$. The coupling loss was measured to be 7dB when coupling using a lensed fibre. This figure was calculated from the relative collection efficiencies.
of bulk and lensed fibres measured with the device under 40mA forward bias, and from comparison with previous measurements on 3\textmu m ridge waveguides etched from the same wafer (devices 32037 and 32039).

The device which exhibited the most efficient nonlinear coupling behaviour was device 33404. Due to the limitations on pulse energies which were available for the experiment it was not possible to demonstrate a complete switch in all of the four devices. Therefore, the results presented here are the nonlinear coupling properties of device 33404 only. These results are shown in figures 6.3.9-6.3.11. Figure 6.3.9 shows nonlinear transmission measurements with the device unbiased, but terminated into a constant voltage supply. Figure 6.3.10 shows the results of the same measurements with the device under 3mA forward biased. Figure 6.3.11 shows the results for 0.75V reverse bias.

The results shown in figure 6.3.9 demonstrate clearly that nonlinear coupling behaviour is occurring. Figures 6.3.9a and 6.3.9b show that power is being transferred from the cross arm to the bar arm as the incident pulse energy is increased beyond 200pJ. However, for pulse energies of \geq 500pJ the transmitted powers in both arms do not increase with increasing power, nor does the coupling ratio change.
Figure 6.3.9a  Bar arm transmission as a function of incident pulse energy. Unbiased device.

Figure 6.3.9b  Cross Arm transmission as a function of incident pulse energy. Unbiased device.
Figure 6.3.9c  Normalised Transmission as a function of incident pulse energy. Unbiased device.

Figure 6.3.9d  Reciprocal transmission as a function of coupled pulse energy, showing the effects of two photon absorption. Unbiased device.
The observed behaviour is not entirely what is predicted by the modelling of this device (section 6.3.2). That the transmitted powers should reach a plateau is predicted from the two photon absorption properties of the waveguide at this wavelength. This is confirmed by the inverse transmission plot 6.3.9d, showing a 50% increase in absorption due to two photon absorption. However, the normalised transmission is not expected to level off. The model assumes that the change in effective refractive index of the waveguide is linear with power. However, from the SPM measurements carried out (chapter 5) this is not the case. The change in refraction does saturate with power as the available states for carriers become occupied, blocking further carrier generation. It is therefore believed that it is the saturation of the nonlinearity which is causing the switching ratio to saturate with increasing pulse energy.

The results with forward and reverse bias show that the directional coupler is very sensitive to the biasing conditions. It is only under the bias conditions shown that a complete switching action is seen. For forward biases either side of this operating point, the switching is less efficient. Under forward bias, some of the available states are filled by the injected current. The associated change in the refractive index of the waveguide tunes the linear transmission characteristic of the directional coupler to a more favourable operating point (i.e. towards complete cross coupling at low powers). However, if too many states are filled, the optical absorption is bleached and the nonlinearity quenched. The injection of carriers also increases the free carrier scatter loss of the waveguide. This reduces the efficiency of switching (see section 6.3.2). Therefore the optimum operating point under forward bias is a trade off between tuning the operating point, state filling and waveguide loss. In this case the optimum operating point is a forward bias of 3mA. It can be seen that some modest improvement in switching can be achieved over the unbiased case.
Figure 6.3.10 Normalised Transmission as a function of incident pulse energy. 3mA forward bias applied to device 33404.

Under the application of reverse bias, the quantum well absorption is increased. Here, the trade off is between efficient carrier generation (and hence efficient nonlinearity) and overall waveguide loss. In this case, the optimum operating point is 0.5V reverse bias. The efficiency of the nonlinear coupler is not strongly affected by the application of a reverse bias up to a bias of 0.5V, beyond which the nonlinear coupling behaviour is severely impaired. This suggests that the operating wavelength of the nonlinear directional couplers is not very critical when the efficiency of carrier generation is considered.
Figure 6.3.11 Normalised Transmission as a function of incident pulse energy. 0.5V reverse bias applied to device 33404.
6.4 Conclusions

The efficient all-optical bandfilling refractive nonlinearity has been exploited to form the basis of two nonlinear all-optical switches. These switches were the nonlinear polarisation rotation gate and the nonlinear directional coupler. Both switches used an InGaAaP/InGaAsP MQW waveguide as the nonlinear element. In both configurations, diode pumped all-optical switching of a 30ps optical pulse was demonstrated. The coupled pulse energy required was 10.5pJ for the polarisation rotation gate. This is, to the author’s knowledge, the first demonstration of a polarisation rotation gate operated at a wavelength of ~1.55μm with an InGaAsP MQW waveguide as the nonlinear element. The nonlinear directional coupler required ~100pJ coupled pulse energy in order to demonstrate switching under various forward and reverse bias conditions.

The increased power requirements for NLDC are accounted for partly by the greater linear loss of the device (15cm\(^{-1}\) as opposed to 10cm\(^{-1}\) for the polarisation rotation gate waveguide). Also, the directional coupler configuration is inherently a more inefficient switch than a polarisation rotation gate, requiring twice the phase shift in order to produce a switch. Due to the increased power required for the nonlinear directional coupler, two photon absorption was also a limiting factor in achieving efficient nonlinear switching. However the main limitation to switching was believed to be saturation of the bandfilling nonlinearity on the basis of the measurements presented here and in chapter 5.

In summary, an efficient all-optical switching requiring coupled pulse energies as low as 10.5pJ (~330mW peak power) has been demonstrated. This is the first time that all-optical switching has been demonstrated in a semiconductor waveguide using a diode laser pump source.
6.5 References


Carrier screening of applied fields has been observed in an InGaAsP electroabsorption modulator using high speed photo-current measurements. Two similar structures, one with InGaAlAs barriers and one with a strained active layer showed reduced susceptibility to carrier screening when compared with a standard quaternary layer structure device. The strained layer device exhibited the fastest carrier decay time (<40ps). Self phase modulation measurements have been carried out on a buried heterostructure waveguide with similar layer structure to the optimised phase shift device. With an applied field of ~270kV/cm the recovery time of the nonlinearity was reduced to 18±3ps whilst maintaining 50% of the phase modulation observed with no bias applied.
7.1 Introduction

Carrier enhanced nonlinearities in InP based material, operating at detunings from the absorption band edge of around 50meV, rely on photogenerated carriers to induce the nonlinear behaviour, and are far more efficient than the Kerr-type nonlinearity (see chapter 4). It has been demonstrated that a phase modulation of \( \pi \) radians can be achieved for coupled optical pulse energies as low as 30pJ, operating in a regime where waveguide absorption losses are of the order of 3dB/mm. Whist carrier enhanced nonlinearities are more efficient than ultrafast nonlinearities, they are inherently slower, as in order to switch off the nonlinearity, the carriers must be allowed to recombine or be removed from the active region. Carrier recombination times in InP material are of the order nanoseconds. Therefore, in order to achieve switching rates above 1GHz, carriers must be removed in some other fashion.

The method of carrier sweep-out demonstrated here is to apply a field perpendicular to the quantum well stack [1,2]. This is achieved by applying a reverse bias to electrical contacts placed on the top and bottom of the waveguide device. The waveguides used are p-i-n structures designed to allow any applied voltage to be dropped across the intrinsic region of the waveguide which contains the quantum well stack. The applied reverse field tilts the potential structure of the quantum wells, allowing carriers to escape more rapidly from the potential confines of the quantum wells due predominantly to thermionic emission. Once the carriers have escaped from the quantum wells, the applied bias also sweeps carriers out of the intrinsic region of the waveguide.

In order to measure the escape times of the carriers from the quantum wells, two methods have been employed. The first method, described in section 7.2, is to use high speed photo-current measurements. Carriers are created in a waveguide using a 30ps optical pulse. The waveguides are connected to an external electrical circuit, allowing the photo-current to be time resolved using a sampling oscilloscope. Providing the carriers do not become trapped in other parts of the circuit, the measured response times
will be limited by carrier escape times from the quantum wells. This method is a simple method of measuring carrier lifetimes on timescales from nanoseconds down to picoseconds. However, this method is limited in that the temporal resolution is limited to around ~40ps by the external electrical circuit, and the waveguide capacitance. In order to achieve a resolution of ~40ps, the waveguides must be mounted on low capacitance sub-mounts. Such mounts require technology not available to Bath University, therefore commercial high speed waveguide modulator devices were used in this experiment.

The technique described in section 7.3 is the self phase modulation (SPM) technique first described in chapter 5 and here used to measure the recovery time of the nonlinearity. Using SPM, greater temporal resolution can be achieved than with the high speed photo-voltage measurements, being limited only by the optical pulse width to ~8ps in this case. This method measures the phase modulation induced by the presence of carriers on an optical pulse travelling along the waveguide. From the spectra of the pulse exiting the waveguide, both the magnitude and speed of recovery of the phase modulation and hence carrier lifetime on the quantum wells can be established. This technique has previously been used to determine phase modulation information only. Here it is extended to extract temporal information also. The measured carrier lifetimes are compared with theoretical lifetimes of carriers in quantum wells which are subject to a reverse bias.
7.2 High Speed Photo-current Measurements

7.2.1 Introduction

The self phase modulation measurements detailed in chapter 5 require an optical pulse with a transform limited frequency spectrum. This is readily achieved with a mode-locked colour-centre laser. However, the frequency spectrum of a Q-switched pulse from a three contact laser diode is far from transform limited, and in some cases can show a double moded spectrum [3]. Therefore it is not possible to use the optical pulses from a Q-switched laser diode to carry out self phase modulation measurements.

A simple technique for measuring carrier lifetimes in semiconductor waveguides is to use the waveguides as photodiodes, providing suitable electrical connections can be made to the waveguide. The 30ps optical pulse from a Q-switched laser diode can be used to excite carriers in the waveguide. The photo-current from these carriers can then be measured on a sampling oscilloscope. Temporal resolutions of the order of 40ps can be achieved.

Current pulses from three MQW structures have been time-resolved after excitation of the structures by ultrafast laser pulses. Three devices designed as 1550nm QCSE modulators were used. These devices were manufactured by BNR Harlow. The QCSE modulators are E349 which has a similar structure to the design detailed in chapter 5, A2174 which has GaInAlAs barriers and E1149 which has a strained active layer. The dependence of the current pulse decay time on pulse energy and reverse bias across the devices has been measured. Typical raw data is presented which shows that exponential decay times can be obtained from the data. This is followed by plots showing the dependence of the measured decay times on pulse energy and reverse bias.

Simple theoretical analysis of the data is presented for device E349. Also, for this device the response time of the apparatus is deconvolved from the raw data. The current pulse decay time is compared with theoretical carrier escape times from the device active
layer for device E349. In calculating the carrier escape time, thermionic emission and the transit time across the intrinsic region of the device for carriers moving at the saturated drift velocity are considered in detail.

### 7.2.2 Devices

Detailed layer structures of all the BNR modulators are shown in Appendix 2. Details of the E349 structure are used below to predict carrier escape times from the device. The active region of the device consists of 14 InGaAsP (\(Q=1.58\)) quantum wells of 9nm width and 13 InGaAsP (\(Q=1.18\)) barriers of 10nm width clad with more barrier material to give a total guiding layer width of 0.826 \(\mu\text{m}\). It is assumed that all reverse bias applied to the device is dropped across the intrinsic region of the waveguide.

For the purpose of detailed modelling of carrier escape times it has been calculated that the well material (\(Q=1.58\)) has a band gap of 784.8meV with an electron effective mass of 0.044\(m_e\) and light and heavy hole masses of 0.056\(m_e\) and 0.43\(m_e\), respectively. The barrier material is calculated to have a band gap of 1050meV. The electron effective mass in the barrier material is assumed to be 0.063\(m_e\), the heavy and light holes 0.090\(m_e\) and 0.47\(m_e\), respectively. These figures are calculated from the data in Appendix 1. It is assumed that there is a 39:61 split in the band gap discontinuity when the carriers are confined to the wells. The effective electron and hole well depths have been calculated using a 1D finite potential well which gave an effective band edge for the structure of 1.50\(\mu\text{m}\) for emission of light. For this sample, the measured photo-current peak for absorption was at 1.489\(\mu\text{m}\), which is found to be in reasonable agreement with the simple model. This analysis gives an effective conduction band barrier height of 72meV and a valence band step for the heavy holes of 152meV.
The structure of sample A2174 is shown in Appendix 2.2. Five 5nm GaInAs quantum wells are confined with 7.5nm GaInAlAs (Q=1.15) barriers. The aluminium doping of the barriers is intended to reduce the valence band discontinuities in the quantum well layer. The total thickness of the intrinsic layer is 0.2525 µm.

The structure and composition of E1149 is shown in Appendix 2.3. Three 8nm GaInAsP (Q=1.59) quantum wells are separated by 8nm barriers of GaInAsP (Q=1.347). This is a strained layer device with the wells under a compression of 1% and the barriers correspondingly strained with 1% tension. The nominal intrinsic region thickness is 0.528µm.

Devices E349, A2174 and E1149 were all mounted on high bandwidth mounts as the devices were all designed as high speed modulators. The bandwidth of the mounts were in excess of 10GHz, with the device capacitances limiting the overall bandwidth.

7.2.3 Experimental Method

The experimental apparatus is shown in figure 7.3.1. Ultrashort laser pulses were generated from a 3-contact DFB diode laser using the double pulse Q-switching technique [3]. This gave laser pulses with a width of 30ps at 1536nm which were then amplified using an Ar:ion-pumped Er3+ fibre amplifier. This amplifier was described in detail in chapter 6. The amplifier output was coupled into a fibre and into the device using a fibre lens via a polarisation controller. Coupling into the device was optimised by maximising the collection of spontaneous emission from the device when driven under forward bias with a current of 20mA. A split off part of the amplifier output was used to monitor pulse quality. The light transmitted through the sample was not monitored.

The photo-current from the sample was time-resolved using a Tektronix 7603 oscilloscope (S4 head: 25ps rise time and 7S11/7T11 sampling units) acting as a 50Ω termination to the output from the device. The oscilloscope was triggered electrically
from the power supply of the diode laser. Bias was applied to the device using an 8ps
rise time bias tee and all connections were made using SMA semiflexible cable (rise
time 25ps).

The photo-voltage signal was maximised using the polarisation controller. This
corresponds to polarising the incident light on the TE axis of the sample. The erbium
amplifier currently gives a strong fluorescence background to the amplified pulse.
Increasing the pump fluence to the erbium amplifier increases the background level of
the fluorescence but had no observable effect on photo-voltage profile. It thus appears
that this background level of excitation has little effect on the carrier dynamics. The
maximum pulse energy incident on the device was 80pJ, corresponding to a peak power
of ~2.7W, and could be smoothly reduced to <1pJ as required. The mode coupling loss
from fibre lens to device was estimated to be 5dB for devices E349, A2147 and E1149.

Figure 7.2.1 Experimental set up for high speed photo-current measurements.
The photo-voltage was measured as a function of applied reverse bias for various pulse energies. The reverse bias was varied from 0V to 5.5V (3.7V for E349) and was limited in magnitude by the power dissipation in the matching resistor on the sample mount. Time-resolved photo-voltage was measured on the oscilloscope and the output from the oscilloscope was digitised using an 8-bit Thurlby Digital Storage Adaptor (DSA). An average over 9 traces was performed on the DSA before digitisation. The digitised traces were converted to ASCII files and then analysed in MATLAB.

7.2.4 Results and Discussion 1: E349

Typical photo-voltage data from the QCSE modulators is shown in figures 7.2.4 and 7.2.5 taken in this case for device E349. Figure 7.2.4 shows that for high pulse energies (here 53.5pJ) and low reverse biases (here 0.5V) the signal is long lived. Figure 7.2.5 shows a quickly decaying voltage trace taken at lowest power, 0.83pJ, and at a higher reverse bias, 3.7V. In order to extract the carrier lifetimes, the background level from the raw data is subtracted and the data is then displayed on a logarithmic scale so that the decay constant can be found. All the lifetimes reported here correspond to the fastest decay time which is observed immediately after the signal peak. Figure 7.2.5 shows the typical decay shape for low powers and high reverse biases. Clearly, one decay time is present before the signal level becomes dominated by noise. It has been assumed that the longer decay constants which are sometimes observed at later time delays arise due to carrier movement in the chip and mount outside of the intrinsic region.
Figure 7.2.4 Typical Photo-voltage data, using device E349. The incident pulse energy is 53.5pJ with 0V applied bias. The dotted line shows the fit to the data.

Figure 7.2.5 Typical Photo-voltage data, using device E349. The incident pulse energy is 53.5pJ with -3.7V applied bias. The dotted line shows the fit to the data.
Decay times were measured several times at each power and reverse bias setting used. The data have had the calculated response time of the device mount deconvolved from the experimentally measured times. The estimated rise time of the circuit is 47ps for device E349. This is a convolution of the rise times of the device and its mount (30ps), the semiflexible cable (25ps), the fast bias tee (8ps) and the S4 scope head (25ps). Of these rise times, the band-width of the device mount is the least well known and has been estimated from the work of Fells et al. [4].

Typical errors on the data can be deduced from the scatter of the displayed lifetimes. Possible sources of systematic error are the drifting of laser pulse energy or profile during a data run or gradual loss of optical coupling to the device. These errors have been minimised by constantly monitoring the laser pulse and by returning at the end of a data run to the experimental condition used at the start.

Figures 7.2.6 and 7.2.7 show the power dependence of the carrier decay constant against reverse bias for device E349 (figure 7.2.6, 0.83pJ incident; figure 7.2.7, 53pJ incident). There is a general trend that, at every power, the decay time is reduced for higher biases. Figure 7.2.6 shows the effect of pulse energy on the carrier decay constant for maximum applied bias. This plot is typical of the data taken, showing that, with increasing pulse energies, the carriers become more long lived. The decay constant reaches a plateau (figure 7.2.8) at 59ps after deconvolution of the calculated response time of the device so it appears not to be due to the resolution limit of the experimental apparatus. The deconvolution of the carrier decay time constant, \( \tau_d \), from the rise time of the electrical circuit, \( \tau_r \), is calculated from:

\[
\tau_d = \tau_m - \tau_r
\]  

(7.2.1)

where \( \tau_m \) is the measured decay time constant.
Figure 7.2.6  Carrier decay time constant as a function of bias for device E349. The incident pulse energy is 0.83pJ. The circles show measured points. The line is drawn as a guide to the eye.

Figure 7.2.7  Carrier decay time constant as a function of bias for device E349. The incident pulse energy is 71pJ. The circles show measured points. The line is drawn as a guide to the eye.
In order to make carrier assisted all-optical nonlinearities a viable route to all-optical switching for telecommunications applications, the escape time of carriers from the active region of a device must be reduced from the carrier recombination time (typically of the order of nanoseconds). To this end simple modelling of the carrier sweep-out times from quantum wells under a perpendicularly applied reverse bias has been performed. This analysis is intended to show the dominant mechanism by which carriers exit the quantum wells and follows the work of Fox et al. [5] which builds upon previous theoretical results [6-8]. Thermionic emission and tunnelling from the quantum wells have been considered. The time taken by carriers to cross the active region of the waveguide has been included [9].

The carrier escape times from quantum wells due to thermionic emission are calculated from the following equations [5]:

Figure 7.2.8 Carrier decay time constant as a function of pulse energy for device E349. A reverse bias of 3.7V was applied to the waveguide. The line is a regression line.
where \( k_b \) is the Boltzmann constant, \( T \) is absolute temperature, \( m_i \) is the effective mass of the relevant carrier in the well, \( L_w \) is the quantum well width and \( H_i(F) \) is the effective barrier height as a function of field, \( F \), defined as follows:

\[
H_i(F) = Q_i \Delta E_g - E_i^n - \frac{|e| FL_w}{2}
\]  

(7.2.3)

\( Q_i \) is the ratio of band gap discontinuities (such that \( Q_e + Q_h = 1 \)). \( \Delta E_g \) is the difference in band gaps between well and barrier material and \( E_i^n \) is the \( n^{th} \) sub-band energy.

Figure 7.2.9  Thermionic emission time constant as a function of reverse bias as calculated for a 7.5nm \( Q=1.55 \) well with \( Q=1.2 \) barriers.

As an example, these are evaluated for the parameters of the buried heterostructure device used in the SPM experiment (InGaAsP \( Q=1.55 \) wells, 7.5nm wide, InGaAsP \( Q=1.2 \) barriers) for the \( n=1 \) sub-band (no higher sub-bands exist in the conduction band well). The results are shown in figure 7.2.9 for electrons (solid line) and holes (dashed...
The band-gap offsets for InP based material give holes the greater barrier height to escape over. This, and the greater effective mass of heavy holes (with respect to conduction band electrons) give holes a significantly longer escape time constant (~8ps for a reverse bias of 5V) than for electrons (~0.15ps at 5V).

The time taken for a carrier to tunnel between one well and the adjacent well, when under reverse bias, is given by [5]:

$$\frac{1}{\tau_i} = \frac{\hbar \pi}{2L_o^2m_i} \exp\left(\frac{2L_o\sqrt{2m_iH'_i(F)}}{\hbar}\right)$$  \hspace{1cm} (7.2.4)

where $L_o$ is the barrier width, $m_i$ is the effective mass of the relevant carrier in the barrier material and $H'_i(F)$ is the effective barrier height for tunnelling defined as:

$$H'_i(F) = Q_i\Delta E_g - E_i^e - \frac{|e|F(L_w + L_o)}{2}$$  \hspace{1cm} (7.2.5)

Again, these are evaluated for the buried heterostructure device. For electrons, the barrier height is virtually eliminated by the application of a 3V reverse bias. However, the barrier for holes remains high. Given this and the greater effective mass of heavy holes and the exponential dependence of them both on tunnelling time, it is not surprising that the tunnelling rates for holes are several orders of magnitude higher than those for electrons (figure 7.2.10). Given that holes effectively cannot tunnel out of their wells for the fields applied in these experiments, it seems likely that thermionic emission is the dominant process for carrier escape in this particular structure.
Figure 7.2.10  Tunnelling time constants as a function of reverse bias as calculated for a 7.5nm Q=1.55 well, with 12nm Q=1.2 barriers.

Having escaped from the quantum well region, the carriers must be swept out of the intrinsic region of the device. However, the field dependent mobility of the heavy holes in InGaAsP is not well known. In order to estimate this contribution to the decay time of the photo-voltage, the transit time from quantum wells out of the intrinsic region at the hole saturated drift velocity has been added to the calculated escape time. This delays carrier escape by ~4ps. The drift velocity is assumed to saturate at reverse biases of ≥2V at the value measured by Hill et al. [9]. The drift time has not been modelled for reverse biases between 0V and 2V.

Figure 7.2.11 shows a direct comparison between experimental sweep out rates and theoretical thermionic emission rates, including an offset for hole transport across the active region of the waveguide. This offset assumes that the holes are travelling at the saturation drift velocity. At low applied biases (<2V), the holes are predicted to escape from the wells in ~100ps. However, the applied field is not sufficient to sweep the carriers out of the intrinsic region of the waveguide. Therefore the carriers fall back...
into the wells and the decay constant measures the carrier drift time out of the intrinsic region. For biases above 2V, the carriers are expected to travel at the saturated drift velocity [9]. In this region there is good agreement between the theoretical thermionic emission time constant for holes and the measured decay time, indicating that thermionic emission of holes is limiting the carrier sweep out.

No modelling has yet been done to explain the increase of the carrier lifetime with pulse energy. It is thought that this occurs due to field screening by the optically generated carriers.

![Graph showing theoretical calculation of thermionic emission times for holes from a single quantum well of device E349, including an offset of 8ps to account for carriers traversing the active region of the waveguide, assuming that the carriers are travelling at the saturation drift velocity (triangles). The measured decay constants for 0.83pJ incident pulse energy (circles) are included for comparison.](image)

Figure 7.2.11 Theoretical calculation of thermionic emission times for holes from a single quantum well of device E349, including an offset of 8ps to account for carriers traversing the active region of the waveguide, assuming that the carriers are travelling at the saturation drift velocity (triangles). The measured decay constants for 0.83pJ incident pulse energy (circles) are included for comparison.
7.2.5 Results and Discussion 2 : A2174

Photo-voltage pulses were measured after photoexcitation of sample A2174. Unlike voltage pulses measured for sample E349, the pulses decayed rapidly over ~0.5ns even when the device was unbiased and strongly pumped. At low pulse energies, ~1pJ, deconvolved decay times of ~50ps were measured from the unbiased device. Increasing the incident pulse energy on the unbiased device to a maximum of 81pJ caused a rise in the decay time constant to 175ps. This is a much weaker dependence on the incident pulse energy than that shown by device E349. When reverse bias was applied, decay times fell, reaching a minimum for reverse biases greater than 1.5V. Figure 7.2.12 shows the bias dependence of the decay time constant at two pulse energies. This data would indicate that there is some increase in decay constant with increased pulse energy, similar to the behaviour of device E349. However, figure 7.2.13 shows that there is no apparent power dependence of the decay time measured for a reverse bias of 2V. It may be that though some screening of the applied field is occurring, it is much smaller than that observed in device E349. This is due to the lower valence band quantum well depth seen by holes in this structure. In GaInAs/InGaAlAs quantum well material, the band offsets are such that the holes see a shallower potential step than in InGaAsP barrier quantum well material [10]. Therefore, the holes are able to escape from the quantum wells more readily. Hence the saturation effects due to charge screening are greatly reduced when compared to device E349. In these figures the displayed time constant is not deconvolved as the true response time of the mount appears to be faster than that quoted [4].
Figure 7.2.12 Carrier decay time constant as a function of bias for device A2174. Incident pulse energy 1.1pJ (squares) and 50pJ (circles).

Figure 7.2.13 Carrier decay time constant as a function of pulse energy for device A2174. Reverse bias applied to the waveguide is 2.0V.
7.2.6 Results and Discussion 3: E1149

The strained layer sample again showed rapid decay of the photo-voltage pulses in comparison to sample E349. This is shown in figure 7.2.14. For an incident pulse energy of 4pJ, the decay time was as fast as the calculated resolution of the system for all biases. The bias dependence for two higher incident pulse energies also shown. Note that this dependence is being displayed for reverse biases up to only 1.5V with the decay time constant falling rapidly over the first 0.5V of bias. The measurements show a marked reduction in carrier decay time compared with device E349. In this case, the reduction in carrier decay time is due to the reduced barrier height seen by the hole carriers in the valence sub-band quantum well due to the strain in the material [11].

Again, some screening of the applied field with increasing incident pulse energies can be observed due to photogenerated carriers. Of the three devices (E349, A2174 and E1149) this device shows the fastest carrier decay time, and the least susceptibility to charge screening of the applied field.

![Graph](image)

Figure 7.2.14 Carrier decay time constant as a function of bias for device E1149. The incident pulse energies are 4pJ (triangles), 25pJ (squares) and 55pJ (circles).
7.2.7 Summary

Photo-voltage decay times have been measured. It was possible to extract exponential decay times from the experimental results. The decay times measured decrease under the application of reverse bias to the device. Carrier induced screening of the applied field was clearly observed in device E349 (of similar structure to the design in chapter 5) when excitation is by energetic pulses. Theoretical modelling indicates that thermionic emission is the dominant escape mechanism from the quantum wells, but escape times from the active region are limited at low reverse biases by the drift time out of the intrinsic region. At low powers and high reverse biases the measured decay times become comparable to the resolution of the present experiment.

The strong bias dependence of the fall in the decay time for samples A2174 (InGaAlAs barriers) and E1149 (strained active layer) must partly arise because these samples have narrower intrinsic regions (70% and 37% narrower respectively) so that greater electric fields are being applied to the quantum wells at any value of reverse bias than for device E349. In the case of device A2174, the aluminium doping will cause the band offsets to shift such that electrons see the deepest potential step between quantum well and barrier states. Thermionic emission times are exponentially dependent on the potential barrier height, therefore it is likely that the hole escape time from the quantum well is greatly reduced from that of E349. In the case of the strained layer device (E1149) the effect of the strain is to perturb the hole bands such that the effective mass of the holes is reduced. This will also lead to a reduction in the thermionic emission times and hence the observed reduction in decay times compared with those of E349. The strained layer device (E1149) exhibited the fastest carrier decay times and carrier induced screening. Any improvement in carrier assisted nonlinearity recovery time over and above that demonstrated here will be most readily facilitated by utilising strained layer material.
7.3 Self Phase Modulation

7.3.1 Introduction

The measurement of the Self Phase Modulation (SPM) of light in a semiconductor waveguide can provide useful information on the magnitude of the intensity dependent nonlinear phase modulation in the waveguide (see chapter 5). The bias dependent SPM measurement can also be used to extract carrier escape times from the guiding region of the waveguide. SPM measurements on an MQW buried heterostructure laser structure are presented. The MQW layer structure of this device is similar to that of the device designed in chapter 5. Results are presented both with and without a reverse bias applied to the waveguide.

7.3.2 Experimental Method

The waveguide was a buried heterostructure MQW graded index separate confinement heterostructure device. The waveguide region was 0.17μm wide in the vertical direction and contained 9 InGaAsP (Q=1.55) of width 7.5nm wells separated by 6nm wide InGaAsP (Q=1.2) barriers. The waveguide length was 750μm. The device was contacted to allow a bias to be applied to the device. The facets were antireflection coated for a wavelength of 1.55μm. The absorption band edge of the quantum wells in the waveguide was 1.485μm for TE polarised light. From low power transmission measurements, the linear loss of the waveguide was estimated to be α=40cm⁻¹, though this was believed to be primarily due to scatter losses from photo-current measurements and absorption measurements on similar wafers (chapter 5).

The experimental apparatus is shown in figure 7.3.1. A NaCl:OH colour-centre laser was used to generate 8ps pulses (measured on an SHG autocorrelator) at a repetition rate of 82MHz, with peak powers of up to 500W. The tuning range of the laser was approximately 1.5-1.6μm. The actual operating wavelength was set to be 50meV beyond the absorption band edge of the sample. This corresponds to the same operating point
as used in the measurements in chapter 5. Therefore, the buried heterostructure device with a band edge of 1.485\,\mu m was measured at a wavelength of 1.58\,\mu m. The infrared light was coupled into the waveguides using a lensed fibre, with an attenuator wheel at the input of the fibre to control the power incident on the waveguide. The output of the waveguide was monitored continuously on a camera. The SPM spectra were measured using a scanning Fabry-Perot interferometer.

The SPM spectra were measured as a function of incident power for peak powers between 3.5\,W and 57.0\,W. For a peak incident power of 57.0\,W, SPM spectra for reverse biases up to 6.2\,V (34\,\text{MV/m} approximately) were measured. The transmission of the guide was measured for TE polarised light as a function of coupled power to obtain an estimate of $\beta_{TPA}$, the two photon absorption coefficient. Such a measurement can only yield an approximate value due to bleaching of the linear absorption as a result of bandfilling.
7.2.3 SPM Modelling

The SPM spectra shown in figures 7.3.2 and 7.3.3 are typical of the spectra taken in the experiments. They contain information about the magnitude of the self phase modulation experienced by light passing through the waveguide due to nonlinearities in the waveguide. As frequency is by definition, the rate of change of phase, these spectra also contain information on the rate of change of the self phase modulation. Physically, in the region of operation, any observed nonlinearity will be largely due to photogenerated carriers, and such nonlinearity will 'turn on' as the carriers are generated due to photoabsorption. Furthermore, the nonlinearity will 'turn off' as carriers leave the active region of the waveguide. Therefore, the SPM spectra can yield information on the carrier sweep out times from quantum wells.

The phase modulation and carrier sweep out time information were extracted from the SPM spectra by modelling the spectra of a chirped pulse. A sech\(^2\) pulse was convolved with an exponential decay function in order to produce a time varying refractive index shift. The resulting time varying phase shift was superimposed on the sech\(^2\) pulse. The spectrum associated with such a pulse was generated using a Fourier transform. This spectrum was then fitted to the experimental data, using the adjustable parameters of phase shift and decay constant, accounting for pulse asymmetry and two photon absorption. It is assumed that any nonlinearity is not saturated by the coupled pulse energies used. A more complete account of the theory is given in chapter 5. Typical results of the modelling carried out are shown in figures 7.3.2b and 7.3.3b.
Figure 7.3.2a Spectral plot of output of waveguide, showing chirp due to self phase modulation. The coupled pulse energy was 225pJ. The free spectral range was 400GHz. The device was unbiased.

Figure 7.3.2b Theoretical modelling of a SPM spectra. The well escape time constant is 1000ps, the barrier to well scattering time is 4ps and the phase modulation is \(-5.3\pi\) radians. To be compared with the figure 7.3.2a.
Figure 7.3.3a  Spectral plot of output of waveguide, showing chirp due to self phase modulation. The coupled pulse energy was 225 pJ. The applied reverse bias was 4.0V. The free spectral range was 400GHz.

Figure 7.3.3b  Theoretical modelling of a SPM spectra. The well escape time constant is 18ps, the barrier to well scattering time is 4ps and the phase modulation is -2.3\pi radians. To be compared with figure 7.3.3a.
7.3.4 Results and Discussion

The results of the measurements and analysis carried out are shown on figures 7.3.4-7.3.7. The plot of reciprocal transmission in figure 7.3.4 shows a 50% change in transmission for a coupled power of 22W, yielding $\beta_{\text{TPA}}=10\text{cm/GW}$. This value is low when compared with previous measurements which show $\beta_{\text{TPA}}=50\text{cm/GW}$ close to the absorption band edge [1]. This discrepancy is believed to be due to the bleaching of linear absorption. The phase shift as a function of coupled pulse energy with no bias applied to the device (figure 7.3.5) shows a roughly linear dependence, with $\pi$ radians phase shift for a coupled pulse energy of 55pJ. However, some saturation of the nonlinearity is observed for higher pulse energies.

![Graph showing reciprocal transmission as a function of coupled power](image)

**Figure 7.3.4** Reciprocal transmission as a function of coupled power for the buried heterostructure device (29311). The line shown is a regression line.
The bias dependent results at fixed power show the carrier decay constant rapidly decreasing with reverse bias (figure 7.3.6). At low reverse bias (0V-2V) there is a large uncertainty in fitting the decay constant to the measured data, hence the large error bars. The decay constant reaches a plateau beyond 4V reverse to a value of 18±3 ps. The reduction in phase shift due to the application of reverse bias is also shown (figure 7.3.7). Again, at low reverse bias there is an uncertainty in fitting the phase shift and decay constant. The reduction in phase shift due to the application of a reverse bias of 5V is approximately 50%. Note that the reduction in phase shift does not follow linearly with the decrease in recovery time. The recovery time decreased by nearly two orders of magnitude, for a reduction in phase shift of only ~50%.
Figure 7.3.6 Carrier decay time as a function of reverse bias across sample. The coupled pulse energy is 225pJ. The error bars show the uncertainty in fitting to the measured data.

Figure 7.3.7 Peak phase shift as a function of applied reverse bias. The coupled pulse energy is 225pJ. The error bars show the uncertainty in fitting to the measured data.
Simple modelling of the carrier sweep-out times from quantum wells under a perpendicularly applied reverse bias was again carried out. The analysis is intended to show the dominant mechanism by which carriers exit the quantum wells. The relevant papers from which these equations are taken are referenced at the end of this chapter [5-9].

Figure 7.3.8 shows a direct comparison between experimental sweep out rates and theoretical sweep out rates. The theoretical sweepout rates comprise of the thermionic emission time constant and the time for carriers to traverse the active region of the waveguide, assuming that the carriers are travelling at their saturation drift velocity. It would appear that thermionic emission is not the dominant process for all applied biases, as the experimental sweep-out times are nonlinear with applied bias. Nevertheless from the relative magnitudes of the sweep-out times, it would appear that at high applied bias thermionic emission does contribute significantly to carrier escape from quantum wells.

The apparent dependence of the recovery time on the hole lifetime is somewhat surprising since the refractive nonlinearity due to bandfilling is dominated by the presence of electrons. The thermionic emission time for electrons calculated from equation 7.2.1 is of the order of 0.2ps and the electron drift time is calculated to be 0.7ps from data in reference [9]. Thus the experimental refractive lifetime cannot be explained by this simple calculation of the electron dynamics but fits well with a similar calculation for the hole dynamics (see figure 7.3.8). Therefore it appears that charge screening effects, due to presence of holes, limit the movement of electrons and thus the speed of the refractive nonlinearity. The presence of holes limits the travel of electrons until such times as the holes can be swept out of the active region.

The discrepancy between the thermionic emission time constants (~8ps) and the measured carrier decay time (18±3ps) is not fully accounted for by carrier drift across the active region of the waveguide. It is thought that screening of the applied field due to the presence of carriers may be limiting the minimum carrier decay time observed.
While the carrier generation is not sufficient to induce carrier screening at zero bias, the increased absorption due to applied reverse bias leads to a greater carrier concentration. Such screening of the applied bias has been observed as a function of carrier concentration in a similar MQW waveguide device; this is discussed in section 7.2.

![Graph showing Well Carrier Escape Time vs. Applied Field](image)

**Figure 7.2.8** Direct comparison of experimental carrier decay times and theoretical thermionic emission times (line). The error bars show the uncertainty in fitting to the data.
7.4 Conclusions

Photo-current measurements were carried out on three MQW electro-absorption modulators; a quaternary device (E349) similar to the waveguide described in chapter 5, an InGaAlAs barriers device (A2174) and a strained active layer device (E1149). Carrier induced screening of the applied field was clearly observed in sample E349. The minimum photo-current decay time increased as a function of pulse energy. Theoretical modelling indicated that thermionic emission was the dominant escape mechanism from the quantum wells but escape times from the active region were limited at low reverse biases by the drift time out of the intrinsic region.

The InGaAlAs barriers device (A2174) and the strained active layer device (E1149) both exhibited significantly faster carrier decay times at all biases than the quaternary device. In the case of the InGaAlAs barriers device, the shift of the band offsets due to the aluminium doping greatly reduces the valence band potential well depth, thus significantly reducing the thermionic emission time constant. The effect of strain in device E1149 was to reduce both the effective mass of the holes and the hole potential well. Thus the strained layer device exhibited the fastest carrier decay times (<40ps), though the measurement of these times was limited by the external circuit used. In order to achieve a greater temporal resolution, self phase modulation techniques were used.

Self phase modulation measurements were carried out on a buried heterostructure waveguide with a similar layer structure to the optimised phase shift layer structure with a reverse bias applied perpendicular to the quantum well stack. Analysis of the SPM spectra allowed extraction of both phase modulation and nonlinearity recovery time information. A phase modulation of $4\pi$ radians was achieved for a coupled pulse energy of 200pJ with no reverse bias applied to the waveguide. This was considerably less efficient than the device described in chapter 5 due to high waveguide scatter losses. With an applied field of ~270kV/cm the recovery time of the nonlinearity was 18±3ps. The phase modulation associated with this recovery time was ~50% of the phase
modulation achieved with no bias applied to the waveguide. Modelling of carrier sweep out times from quantum wells under the application of a reverse bias indicated that the limiting factor in the recovery time of the nonlinearity is thermionic emission of holes.

The refractive nonlinearity due to bandfilling is dominated by electrons due to their low effective mass. Thus it would be expected that the recovery time of the nonlinearity fit well with the electron lifetime. However, the calculations indicate that it is the hole lifetime which dominates the recovery time of the nonlinearity. Therefore it appears that charge screening effects, due to the presence of holes, limit the movement of electrons and thus the speed of the refractive nonlinearity. The measured recovery time of the nonlinearity is slower than the predicted thermionic emission time. The discrepancy is not accounted for by hole drift across the active region. It is believed that carrier screening of the applied field is responsible for limiting the measured nonlinearity recovery time constant to 18±3ps, as was observed for the electro-absorption modulator E349. However, this recovery time is, to the authors knowledge, the fastest reported recovery time constant of the bandfilling nonlinearity measured in an InGaAsP MQW material system. From the photo-current measurement it would appear that the strained active layer material system would allow this recovery time to be reduced even further.
7.5 References


Conclusions and Outlook

Conclusions to the work in this thesis are presented and possible avenues of further work are discussed.
8. Conclusions

8.1 Conclusions

The aims of the work described here were to study all-optical refractive nonlinearities around wavelengths of 1.55\mu m in III-V semiconductor quantum wells, specifically GaAs/AlGaAs and InGaAsP/InP material systems. These nonlinearities were exploited in waveguide structures, and all-optical switching devices were constructed. Using a knowledge of the nonlinearities, the optical powers required to achieve switching of optical pulses were optimised to those which can be produced by diode laser sources in InGaAsP/InP MQW material systems. Switching of femtosecond pulses was also demonstrated in GaAs/AlGaAs waveguides by exploiting Kerr-type nonlinearities. All-optical switching of picosecond optical pulses was demonstrated in InGaAsP/InP waveguides using carrier enhanced nonlinearities, predominantly bandfilling. The recovery time of carrier assisted nonlinearities was investigated in order to reduce this time below the carrier recombination time.

Two nonlinear polarisation rotation gates were constructed, using two different GaAs/AlGaAs MQW waveguides, to operate at wavelengths 80nm above and 28nm below the half band gap wavelength. These experiments demonstrated that nonlinear polarisation rotation gates can be constructed using compact GaAs/AlGaAs MQW waveguides with lengths of the order of a few millimetres and operated using relatively low peak powers (<100W). This represents a significant improvement over fibre polarisation gates in that the semiconductor waveguide device is two orders of magnitude more compact and operates at lower powers. By operating these two gates above and below the half band gap wavelength, the limiting effect of TPA on intensity dependent nonlinearities such as nonlinear refraction (Kerr effect), was demonstrated.

The waveguide operated at a wavelength above the half band gap was a GaAs/AlGaAs MQW single mode rib waveguide structure with an undoped MQW region composed of 64 GaAs quantum wells of 4nm width separated by 63 Al_{0.45}Ga_{0.55}As barriers
4nm wide. The intensity dependent transmission of the gate was measured in order to determine, by modelling of these results, the phase shift induced between TE and TM polarisations and hence deduce the Kerr coefficient. The optical Kerr coefficient was calculated to be $12 \times 10^{-18} \text{m}^2 \text{W}^{-1}$. This is in reasonable agreement with the published value of $9 \pm 3 \times 10^{-18} \text{m}^2 \text{W}^{-1}$ in the mid-band gap region.

The polarisation rotation gate operated 80nm below the half band gap was compared with a similar gate operated at a wavelength 28nm above the half band gap wavelength. The AlGaAs structure used in the below half band gap polarisation rotation gate consisted of an MQW region composed of 85 Al$_{0.16}$Ga$_{0.84}$As quantum wells of nominal width 7nm separated by 86 Al$_{0.24}$Ga$_{0.76}$As barriers of width 10nm. The limiting effects of TPA were clearly observed, with the gate operated below the half band gap showing more efficient gating than the device operated above the half band gap, with contrast ratios of 344:1 and 44:1 respectively. However, even for below half band gap operation, the effects of TPA limit the nonlinear gating action at high (~2kW) optical powers relinearising the transmission characteristic. The experiments show the transmitted power characteristic for low power (<100W) incident pulses and operation below the half band gap wavelength is best suited for all-optical switching. This gate is, to the author's knowledge, the first demonstration of a polarisation rotation gate using an AlGaAs MQW waveguide as the nonlinear element.

In order to reduce the peak powers necessary for all-optical switching, carrier assisted nonlinearities operating at wavelengths close to the absorption band edge have also been investigated. The carrier assisted nonlinearities bandfilling and the plasma effect were modelled theoretically to determine nonlinear refraction in a semiconductor waveguide for light at a wavelength corresponding to a detuning of 50meV from the n=1 e-hh transition. This model was used to design a MQW layer structure for a semiconductor waveguide which would exhibit efficient nonlinear refraction. For
comparison, a similar waveguide with a bulk active layer was also designed. The optimum layer structure was 12 InGaAsP (Q=1.55) quantum wells 7.5nm wide separated by 12nm wide InGaAsP (Q=1.2) barriers. The guiding layer was 0.4μm wide.

The fabricated waveguides were tested to evaluate their nonlinear refraction characteristics using self phase modulation (SPM) techniques. Modelling of these spectra taking account of the carrier generation processes involved and nonlinear absorption allowed the phase modulation information to be extracted. Phase modulation in excess of π radians (sufficient for an all-optical switch) was demonstrated in the optimised MQW waveguide for a coupled pulse energy of 30pJ, a peak power of ~3.75W. Saturation of the refractive nonlinearity was observed for coupled pulse energies above 40pJ. This was attributed to a filling of the available states in the conduction sub-band. The bulk waveguide also exhibited nonlinear refraction. The nonlinear refraction was less efficient than for the MQW waveguide. This was predicted by the theory. However, direct comparison between the two samples was difficult due to different waveguide losses. The peak powers and pulse energies required are compatible with those which can be produced by diode lasers. Therefore this carrier assisted nonlinearity is an attractive route for producing all-optical switches for telecommunications applications.

The efficient all-optical bandfilling refractive nonlinearity was exploited to form the basis of two nonlinear all-optical switches. These switches were a nonlinear polarisation rotation gate and a nonlinear directional coupler. Both switches used an InGaAsP/InGaAsP MQW waveguide as the nonlinear element. In both configurations, diode pumped all-optical switching of a 30ps optical pulse was demonstrated. The coupled pulse energy required was 10.5pJ for the polarisation rotation gate. This is, to the author’s knowledge, the first demonstration of a polarisation rotation gate operated at ~1.55μm with an InGaAsP MQW waveguide as the nonlinear element. The nonlinear directional coupler required ~100pJ coupled pulse energy in order to demonstrate switching under various forward and reverse bias conditions. The increased power
requirements for NLDC were accounted for partly by the greater linear loss of the device. Also, the directional coupler configuration is inherently a more inefficient switch than a polarisation rotation gate, requiring approximately twice the phase shift in order to produce a switch. The main limitation to switching was saturation of the bandfilling nonlinearity, however this could be overcome by using a longer waveguide. Nevertheless, an integrated all-optical nonlinear switch capable of switching 30ps optical pulses has been demonstrated using a diode pumped source.

The diode pumped all-optical switches show great potential for application in telecommunications, however the recovery time of these devices is limited by the presence of carriers. The removal of carriers from the MQW structures using a reverse bias in order to improve the recovery time of bandfilling nonlinearity was investigated using photocurrent and SPM techniques.

Photo-current measurements were carried out on three MQW electro-absorption modulators; an InGaAsP device similar to the devices used in chapter 5, an InGaAlAs barriers device and a strained layer device. Carrier induced screening of the applied field was clearly observed in the InGaAsP device. Theoretical modelling indicated that thermionic emission was the dominant escape mechanism from the quantum wells. Both the InGaAlAs barriers device and the strained layer device showed improved carrier escape times and reduced susceptibility to carrier induced screening effects. This was due to the shift the hole sub bands relative to those in the InGaAsP device. The strained layer device exhibited the fastest carrier decay times and was least affected by carrier induced screening. Any improvement in carrier assisted nonlinearity recovery time over and above that demonstrated in the InGaAsP/InGaAsP buried heterostructure device would be most readily facilitated by utilising strained layer material.

The photocurrent measurements were limited in their temporal resolution to ~40ps due to the external electrical circuit. Therefore, to achieve greater temporal resolution, self phase modulation techniques were employed. SPM measurements were carried
out on a buried heterostructure waveguide with similar layer structure to the optimised phase shift layer structure. A reverse bias was applied perpendicular to the quantum well stack to sweep carriers out of the quantum wells, thus improving the recovery time of carrier induced nonlinearities. Analysis of these spectra allowed extraction of both phase modulation and nonlinearity recovery time information. With an applied field of ~270kV/cm the recovery time of the nonlinearity was reduced to 18±3ps. The phase modulation associated with this recovery time was ~50% of the phase modulation achieved with no bias applied to the waveguide. Modelling indicated that the limiting factor in the recovery time of the nonlinearity was thermionic emission of holes. The apparent dependence of the recovery time on the hole lifetime is not what would be expected since the refractive nonlinearity due to bandfilling is dominated by the presence of electrons rather than holes. Therefore it appears that charge screening effects, due to the presence of holes, limit the movement of electrons and thus the speed of the refractive nonlinearity. The measured recovery time of the nonlinearity is slower than the predicted thermionic emission time. The discrepancy is not accounted for by hole drift across the active region. It is believed that carrier screening of the applied field is responsible for limiting the measured nonlinearity recovery time.

The measured nonlinearity recovery time constant of 18±3ps is, to the author's knowledge, the fastest recovery time constant to be reported for carrier assisted nonlinearities in semiconductor waveguides. Previous results have been limited to ~50ps. This recovery time constant, and the efficient switching demonstrated in both the polarisation rotation gate and the directional coupler make such devices an attractive route to optical switching. Whilst the recovery time is not as fast as the current fastest electro-absorption modulators, the photocurrent measurements show that there is considerable scope for improving the recovery time of the bandfilling nonlinearity by using strained layer material. Also, the nonlinear gates do not require the use of high bandwidth electrical circuitry, requiring only a dc bias to be applied to the waveguide.
Another route to all-optical switching is using nonlinearities induced when waveguides are driven forward bias as opposed to reverse bias. These devices suffer from a high scatter loss due to free carrier scattering and the physics of nonlinearity is not yet fully understood. Therefore in these respects, the reverse biased gates do have some advantages. However, a high bit rate demultiplexer (40Gbit/s) has been demonstrated using a waveguide under forward bias therefore such nonlinearities are also worthy of further investigation.

Thus, all-optical refractive nonlinearities were studied at wavelengths around 1.55μm in III-V (GaAs/AlGaAs and InGaAsP/InP) semiconductor quantum wells. These nonlinearities were exploited in waveguide structures, and all-optical switching devices were constructed. With an understanding of these nonlinearities, the optical powers required to achieve switching of optical pulses were optimised to those which can be produced by diode laser sources in InGaAsP/InP MQW material systems. The recovery time of carrier assisted nonlinearities was reduced by two orders of magnitude by applying a reverse bias to the waveguide. A recovery time constant of 18±3ps was demonstrated. Switching of femtosecond pulses was also demonstrated in a novel polarisation rotation gate using GaAs/AlGaAs waveguides as the nonlinear element by exploiting Kerr-type nonlinearities.

8.2 Outlook

All-optical switching of a 30ps optical pulse at a wavelength of 1.55μm has been demonstrated with diode pumped sources by employing carrier assisted nonlinearities in InGaAsP MQW waveguides. The recovery time of this nonlinearity has been shown to be 18±3ps with a reverse bias applied perpendicular to the quantum well stack. Thus such devices have great potential for applications in telecommunications. Future
research in all-optical switching will seek to reduce the power requirements and recovery
times still further. This work has shown several avenues which may be pursued to
achieve these goals.

The escape times of holes from quantum wells by thermionic emission have been
shown to limit the nonlinearity recovery time. Photocurrent measurements have shown
that the use of strained layer material can significantly reduce these escape times. Thus
this material system has the potential for demonstrating switches with improved recovery
times. By improving carrier escape times, the effects of carrier induced screening of
applied bias should also be reduced. The power requirements of such optical switches
are also of interest. The switches demonstrated here show that the power requirements
of optical switches are strongly dependent on the geometry of the switch when exploiting
the same nonlinearity. Investigation and optimisation of other switching configurations,
e.g. asymmetric directional couplers, should yield more efficient switches. Recent work
on nonlinearities in forward biased MQW waveguides has also shown some promising
results (e.g. demultiplexing of a 40Gbit/s signal). There is a great deal of scope for
determining the mechanisms of such nonlinearities and thus optimising such nonlinear
switches.
APPENDIX 1 - Material Constants

A1.1 InGaAs Data

The following data have been used for calculations concerning InGaAs lattice matched to InP (as supplied by Dr. M. Fisher, B.T. Labs.):

Conduction Band effective mass:

\[
\frac{m_c}{m_0} = 0.041 \quad (A1.1)
\]

Light Hole effective mass:

\[
\frac{m_{l_2}}{m_0} = 0.051 \quad (A1.2)
\]

Heavy Hole effective mass:

\[
\frac{m_{h_1}}{m_0} = 0.56 \quad (A1.3)
\]

The band gap of InGaAs is 0.75eV

A1.2 InGaAsP Data

The following data has been used for calculations concerning In\(_{1-x}\)Ga\(_x\)As\(_y\)P\(_{1-y}\) lattice matched to InP:

Ga content \(x\):

\[
x = \frac{0.1894y}{0.4184 - 0.013y} \quad (A1.4)
\]
Energy Gap:

\[ E_s = 1.35 - 0.738y + 0.138y^2 \quad (eV) \quad (A1.5) \]

Electron effective mass:

\[ \frac{m_e}{m_0} = 0.08 - 0.039y \quad (A1.6) \]

Light hole effective mass:

\[ \frac{m_{lh}}{m_0} = 0.12 - 0.069y \quad (A1.7) \]

Heavy hole effective mass:

\[ \frac{m_{hh}}{m_0} = (1 - y)\{0.79x + 0.45(1 - x)\} + y\{0.45x + 0.4(1 - x)\} \quad (A1.8) \]

Refractive index:

\[ \mu^2 = (1 - y)\{8.4x + 9.6(1 - x)\} + y\{13.1x + 12.2(1 - x)\} \quad (A1.9) \]

Valence band spin-orbit splitting:

\[ \Delta = 0.11 + 0.31y - 0.9y^2 \quad (eV) \quad (A1.10) \]
APPENDIX 2 - BNR Modulator Layer Structures

The following layer structures were provided by BNR Europe Ltd. All specifications are nominal.

A2.1 E349 Layer Structure

<table>
<thead>
<tr>
<th>Nominal Specification</th>
<th>Doping</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Layer Thickness (µm)</strong></td>
<td><strong>Material</strong></td>
</tr>
<tr>
<td>300</td>
<td>InP substrate</td>
</tr>
<tr>
<td>0.8</td>
<td>InP buffer layer</td>
</tr>
<tr>
<td>0.04</td>
<td>GaInAsP (λ_pl=1.18µm) lower wave guide</td>
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<tr>
<td>0.010</td>
<td>GaInAsP (λ_pl=1.18µm)</td>
</tr>
<tr>
<td>0.0090</td>
<td>GaInAsP (λ_pl=1.58µm)</td>
</tr>
<tr>
<td><strong>MQWs Cycle x14</strong></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>GaInAsP (λ_pl=1.18µm) upper wave guide</td>
</tr>
<tr>
<td>0.12</td>
<td>InP spacer layer</td>
</tr>
<tr>
<td>0.020</td>
<td>GaInAsP (λ_pl=1.18µm) etch stop</td>
</tr>
<tr>
<td>0.35*</td>
<td>InP</td>
</tr>
<tr>
<td>1.5</td>
<td>InP</td>
</tr>
<tr>
<td>0.2</td>
<td>InGaAs contact layer</td>
</tr>
<tr>
<td>0.17</td>
<td>InP anti-passivation layer</td>
</tr>
</tbody>
</table>

* no allowance made for diffusion of zinc during overgrowth
## Nominal Specification

<table>
<thead>
<tr>
<th>Layer Thickness (μm)</th>
<th>Material</th>
<th>Type</th>
<th>Value (cm⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>InP substrate</td>
<td>S</td>
<td>5x10¹⁷</td>
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<tr>
<td>1.0</td>
<td>InP buffer layer</td>
<td>Se</td>
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<tr>
<td>0.09</td>
<td>GaInAaAs (λ_pl=1.15μm) lower wave guide</td>
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<tr>
<td>0.005</td>
<td>GaInAlAs (λ_pl=1.15μm)</td>
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<tr>
<td>0.0075</td>
<td>GaInAsP (λ_pl=1.58μm)</td>
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### MQWs Cycle x5

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<th>Value (cm⁻³)</th>
</tr>
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<tbody>
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<td>0.1</td>
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<td>2.5</td>
<td>InP</td>
<td>Zn</td>
<td>1x10¹⁸</td>
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<tr>
<td>0.4</td>
<td>InGaAs contact layer</td>
<td>Zn</td>
<td>&gt;10¹⁹</td>
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<tr>
<td>0.02</td>
<td>InP anti-passivation layer</td>
<td>Zn</td>
<td>2x10¹⁸</td>
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</table>
### Nominal Specification

<table>
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<th>Layer Thickness (μm)</th>
<th>Material</th>
<th>Doping</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>InP substrate</td>
<td>substrate uniformity control wafer placed upstream</td>
</tr>
<tr>
<td>0.2</td>
<td>InP buffer layer</td>
<td>Si 5x10&lt;sup&gt;17&lt;/sup&gt;</td>
</tr>
<tr>
<td>0.05</td>
<td>GaInAsP (λ&lt;sub&gt;PL&lt;/sub&gt;=1.03μm) doped lower wave guide interface layer</td>
<td>Si 5x10&lt;sup&gt;17&lt;/sup&gt;</td>
</tr>
<tr>
<td>0.05</td>
<td>GaInAsP (λ&lt;sub&gt;PL&lt;/sub&gt;=1.17μm) doped lower wave guide</td>
<td>Si 5x10&lt;sup&gt;17&lt;/sup&gt;</td>
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<tr>
<td>0.06</td>
<td>GaInAsP (λ&lt;sub&gt;PL&lt;/sub&gt;=1.30μm) undoped lower wave guide</td>
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</tr>
<tr>
<td>0.004</td>
<td>GaInAsP barrier (λ&lt;sub&gt;PL&lt;/sub&gt;=1.347μm) for 162Å in InP 1% tension</td>
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</tr>
<tr>
<td>0.008</td>
<td>GaInAsP well (λ&lt;sub&gt;PL&lt;/sub&gt;=1.59μm) 1% compression</td>
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</tr>
<tr>
<td>0.004</td>
<td>GaInAsP barrier (λ&lt;sub&gt;PL&lt;/sub&gt;=1.347μm) for 162Å in InP 1% tension</td>
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</table>

### MQWs Cycle x3

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</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>GaInAsP (λ&lt;sub&gt;PL&lt;/sub&gt;=1.30μm) anti-saturation layer</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>GaInAsP (λ&lt;sub&gt;PL&lt;/sub&gt;=1.17μm) upper wave guide</td>
<td>None</td>
</tr>
<tr>
<td>0.12</td>
<td>GaInAsP (λ&lt;sub&gt;PL&lt;/sub&gt;=1.03μm) upper wave guide interface layer</td>
<td>None</td>
</tr>
<tr>
<td>0.2</td>
<td>InP Zn delay layer</td>
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</table>
APPENDIX 3 - Semiconductor Optical Waveguides

A3.1 Introduction

In this appendix, the basic properties of optical confinement in semiconductor slab waveguides are discussed. The theory of one-dimensional waveguides is presented. This theory is extended to two dimensional waveguides in order to design single mode ridge waveguides. Finally, the use of separate confinement heterostructures (SCH) to allow electrical confinement of carriers in the same region as a two-dimensionally confined optical mode is discussed. SCH waveguides of this type were used in previous chapters to demonstrate optical nonlinearities in GaAs/AlGaAs and InGaAsP/InP based MQW semiconductor material.

A3.2 One-Dimensional Waveguide Theory

Maxwell’s equations in a homogeneous, isotropic and source free medium can be written as [1]:

\[ \nabla \cdot \mathbf{D} = 0 \quad (A3.1) \]
\[ \nabla \cdot \mathbf{B} = 0 \quad (A3.2) \]
\[ \nabla \times \mathbf{E} = \mu_p \frac{\partial \mathbf{H}}{\partial t} \quad (A3.3) \]
\[ \nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} \quad (A3.4) \]

where \( \mathbf{D} \) is the electric displacement \( (\mathbf{D} = \varepsilon \mathbf{E}) \), \( \mathbf{B} \) is the magnetic induction \( (\mathbf{B} = \mu_p \mathbf{H}) \), \( \mathbf{E} \) and \( \mathbf{H} \) are the electric and magnetic fields respectively and \( \varepsilon \) and \( \mu_p \) are the permittivity and permeability of the medium. In a non-magnetic medium \( \mu_p = \mu_o \) where \( \mu_o \) is the permeability of free space. In a lossless medium, \( \varepsilon = \varepsilon_r \mu_o \) where \( \mu \) is a real number corresponding to the refractive index of the medium (throughout the thesis, \( \mu \) is used to represent refractive index to avoid confusion with carrier density, \( n \)).

The \( \mathbf{E} \) and \( \mathbf{H} \) fields of a monochromatic wave travelling in the z-direction can be written as:
\[ E = E(x,y)e^{j(\omega t - \beta z)} \]  
\[ H = H(x,y)e^{j(\omega t - \beta z)} \]

where \( \beta \) is the normalised propagation constant in the z-direction and \( \omega \) is the angular frequency. Substituting these equations into Maxwell's equations (A3.1-A3.4) the wave equation is found to be:

\[ \nabla^2 E(x,y) + \omega^2 \mu_p \varepsilon E(x,y) = 0 \]  

A one dimensional slab waveguide is shown in figure A3.1. It consists of a dielectric layer of thickness \( d \) and refractive index \( \mu_I \), sandwiched between two semi-infinite layers of refractive index \( \mu_I \) and \( \mu_{II} \). Layers I and III are known as the top and bottom cladding regions respectively, and layer II as the guiding region. Optical confinement is only in the x (transverse) direction.

Two self consistent solutions of the wave equation (A3.7) in the one-dimensional slab waveguide exist where \( \partial / \partial y = 0 \). The transverse electric (TE) field with zero component in the z-direction is the first solution; its non-zero components are \( E_y, H_x \) and \( H_z \). The
second solution is referred to as the transverse magnetic (TM) field with non-zero components $H_y$, $E_z$ and $E_z$ [2]. Therefore for the TE waveguide mode, equation (A3.7) reduces to:

$$\frac{\partial^2 E_y}{\partial x^2} + (\mu_0^2 k_0^2 - \beta^2) E_y = 0$$

(A3.8)

Similarly for the TM waveguide mode:

$$\mu_i^2 \frac{\partial}{\partial x} \left[ \frac{1}{\mu_i^2} \frac{\partial H_y}{\partial x} \right] + (\mu_i^2 k_0^2 - \beta^2) H_y = 0$$

(A3.9)

where $k_0^2 = \mu_0 \varepsilon_0 \omega^2$ and $\mu_i$ is the refractive index of the $i^{th}$ layer. If the refractive index is uniform within each layer then the above two equations can be written as:

$$\frac{\partial^2 \psi_y}{\partial x^2} + (\mu_i^2 k_0^2 - \beta^2) = 0$$

(A3.10)

where $\psi_y = E_y$ or $H_y$. The dispersion relations, which relate all the parameters necessary and sufficient to uniquely define the waveguiding properties of the slab layer structure, can be obtained from equation (A3.10) by applying the boundary conditions for guided modes in a slab waveguide. The boundary conditions are $\psi_y \to 0$ as $x \to \pm \infty$, $E_z$ and $\partial E_z/\partial x$ (for the TE mode) and $H_y$ and $\mu_i^{-1} \partial H_y/\partial x$ (for the TM mode) are continuous at each interface. The dispersion relations for the TE and TM modes are:

$$k_n = \tan^{-1} \left[ \tan^{-1} \left( \frac{\eta_n k_0}{\mu_n^2 k_0} \right) + m \pi \right]$$

(A3.11)

where

$$k_i = k_0 (\mu_{\text{eff}} - \mu_i^2)^{1/2}$$

(A3.12)

$$k_{ii} = k_0 (\mu_{\text{eff}} - \mu_{ii}^2)^{1/2}$$

(A3.13)

$$k_{iii} = k_0 (\mu_{\text{eff}} - \mu_{iii}^2)^{1/2}$$

(A3.14)
with $m=0,1,2,3...$ and $\mu_{\text{eff}} = \beta / k_o$ is the effective mode refractive index. The term $\eta_i$ accounts for the different boundary conditions for TE and TM modes such that $\eta_i = 1$ for TE modes and $\eta_i = \mu_i^2$ for TM-modes. Thus the guides' mode solutions can be obtained by solving equation (A3.10) for discrete modes. Setting $m=0$ gives the propagation constant ($\beta = k_o \mu_{\text{eff}}$) of the fundamental mode. The propagation constant $\beta$ of all the guided modes must lie in the range $\mu_{\text{eff}} k_o < \beta < \mu_i k_o$ from consideration of physically allowed solutions to equations (A3.8) and (A3.9). The solutions to equation (A3.10) within each layer are:

Layer I ($x>d$)

$$\psi_y = A e^{-k_i(x-d)} \quad \beta > \mu_i k_o \quad (A3.15)$$

Layer II ($0 \leq x \leq d$)

$$\psi_y = B \cos(k_i x) + C \sin(k_i x) \quad \beta < \mu_i k_o \quad (A3.16)$$

Layer III ($x<0$)

$$\psi_y = D e^{k_{\text{III}}x} \quad \beta > \mu_{\text{III}} k_o \quad (A3.17)$$

where A, B, C and D are arbitrary constants. Thus the solution to the wave equation for guided modes in a one dimensional slab waveguide is, as would be expected, oscillatory in the guiding region and exponentially decaying outside of that region. Figure A3.2 shows a sketch of the three lowest order TE modes of a slab waveguide.

![Figure A3.2 Sketch of the three lowest order TE modes of a one dimensional slab waveguide.](image-url)

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It is convenient to define normalised parameters in order to describe waveguide propagation modes. The normalised propagation constant, \( b \), is defined as:

\[
b = \frac{\beta^2 - \mu_t^2 k_0^2}{(\mu_n^2 - \mu_t^2) k_0^2} \quad 0 \leq b \leq 1 \tag{A3.18}
\]

The normalised thickness parameter, \( V_p \), is defined as:

\[
V_p = k_0 d \sqrt{\mu_n^2 - \mu_t^2} \tag{A3.19}
\]

The asymmetry parameter, \( a \), is defined as:

\[
a = -\frac{\mu_t^2 - \mu_m^2}{\mu_n^2 - \mu_t^2} \tag{A3.20}
\]

Using equations (A3.15-A3.17) and the boundary conditions stated previously for TE modes, it can be shown that:

\[
\tan(V_p \sqrt{1-b}) = \sqrt{1-b} \left[ \frac{\sqrt{b} + \sqrt{b + a}}{(1-b) - \sqrt{b}(b + a)} \right] \tag{A3.21}
\]

In order for the slab waveguide to guide light, the propagation constant must lie in the range \( \mu_t \mu_n k_0 < \beta < \mu_n k_0 \), i.e. \( 0 \leq b \leq 1 \). If \( \beta = \mu_n k_0 \) for a particular wavelength, light will no longer be confined by the waveguide, but will refract into the substrate. If this occurs, the waveguide cannot propagate light at this frequency. This is known as the cut off condition. For the case of a symmetric waveguide \( (\mu_t = \mu_m \); therefore \( a = 0 \) at cut off \( (b = 0) \) it can be seen from equation (A3.21) that \( V_p = \pi \). For single moded operation of the waveguide \( V_p \leq \pi \). From these equations can be calculated the thickness of the guiding region necessary to ensure only single mode propagation is allowed at a given wavelength and for a given material system.

The fraction, \( \Gamma \), of the light in a guided optical mode which is actually confined within the guiding layer (the optical confinement factor) can be approximated by the following formula derived by Botez [3]:

\[\text{223}\]
Typically for a weakly guiding semiconductor structure where the difference between the refractive indices of the guiding and cladding layers is \( \approx 0.1 \), a guiding layer \( \approx 0.2-0.4 \)\( \mu m \) thick would give single mode operation with \( \Gamma \) of in the order of \( \approx 0.7-0.8 \).

This approach to solving Maxwell’s equations in a three slab waveguide can be extended to waveguides with a greater number of slabs [4]. For the four layer slab waveguide shown in figure A3.3 the dispersion relation is given by [5]:

\[
k_{\|}d_{\|} = \tan^{-1} \left[ \frac{\eta_1 k_1}{\eta_{III} k_{III}} \right] + \tan^{-1} \left[ \frac{\eta_{III} k_{III} (\eta_{IV} k_{IV}) e^{k_{IV}d_{IV}} - (\eta_{III} k_{III} - \eta_{IV} k_{IV}) e^{-k_{IV}d_{IV}}}{(\eta_{III} k_{III} + \eta_{IV} k_{IV}) e^{k_{IV}d_{IV}} + (\eta_{III} k_{III} - \eta_{IV} k_{IV}) e^{-k_{IV}d_{IV}}} \right] + m\pi
\]

where

\[
k_{IV} = k_0 (\mu_{eff}^2 - \mu_{IV}^2)^{\frac{1}{2}}
\]

Figure A3.3 A schematic drawing of a four layer slab waveguide

**A3.3 Two-Dimensional Confinement in Semiconductor Waveguides**

The previous section considered a waveguide with one dimensional confinement. However, in order to maintain high optical intensities over long distances, it is necessary to introduce confinement in both \( x \) and \( y \) directions. This lateral confinement can be achieved by means of lateral variations in index. Some examples of two dimensional waveguides are shown in figure A3.4. In general, Maxwell’s equations cannot be solved
analytically for the two dimensional guide; therefore, approximate methods such as the effective index method [6] or numerical techniques such as those using finite element analysis [7] must be used.

While numerical techniques allow very accurate determination of the waveguide propagation constants, they require complex programming and considerable computation. By contrast, the effective index method requires very little computation, and will usually give results of greater accuracy than the accuracy of the measured material parameters. The effective index method only becomes unacceptably inaccurate where the thickness of the slab layers is of the order of nanometres (i.e. very much smaller than the wavelengths of light confined in these structures, which are typically 1-2μm).

![Waveguide Diagram](image)

Figure A3.4 Two common two-dimensional confinement waveguide geometries. (a) shows a ridge waveguide where a central ridge is etched in the top cladding layer. (b) shows a buried heterostructure configuration where the high index material is etched away, then surrounded with low index material.

**A3.4 Effective Index Method of 2D Waveguide Design**

The effective index method is an approximate method which converts the analysis of a two-dimensional waveguide into two one dimensional problems. This is illustrated in figure A3.5. The ridge waveguide cross-section is divided into three regions. Each region represents a slab waveguide. The effective refractive indices for each region are then found using the model described in section A3.2. These effective indices are then
used in an equivalent symmetric three layer slab guide with thickness \( w \) to find the effective index of the original ridge waveguide. In this way the lateral confinement of light under the ridge can be determined. Confinement in the vertical direction can be calculated again using the slab model described in section A3.2.

![Diagram](image)

**Figure A3.5** Illustration of the effective index method applied to a ridge waveguide. The guide (a) is considered as three slab waveguide regions; a central slab, B, clad with two identical slabs, A. For each of these four slab regions effective indices \( \mu_A \) and \( \mu_B \) can be calculated. These effective indices are used as the indices for a slab orientated vertically (c) to determine the lateral mode confinement.

### A3.5 Electrical Confinement and Separate Confinement Heterostructures

In the previous sections, the confinement of light in semiconductor waveguide structures has been considered. Whilst the ability to confine light in two dimensions and thus form optical waveguides is of great interest for many applications in nonlinear optics, it is the ability of these structures to provide electrical confinement as well (with suitable choice of semiconductor material) which has caused the rapid growth of the opto-electronics industry in recent years. Figure A3.6 shows a simplified potential structure for a simple three layer semiconductor structure of the kind discussed in the previous sections, in this case a GaAs/AlGaAs heterostructure.
The difference in the band gaps of these semiconductors \( (E_g) \) causes the potential wells for electrons and holes shown in figure A3.6. It is indeed fortunate for the optoelectronics industry that the refractive indices of these semiconductors are greater for the narrow band gap semiconductor (GaAs) than for the wider band gap semiconductor (AlGaAs). Without the confinement of both carriers and optical fields in the same region, the semiconductor laser would not be possible.

However, it is possible to further refine the simple three layer structure of figure A3.6 to include separate confinement of electrical and optical fields, and form a separate confinement heterostructure (SCH) using quantum wells. The use of quantum wells allows high carrier concentrations to be achieved using a relatively small number of carriers compared to that which would be required in a simple three layer (bulk) heterostructure. This is of great importance for in the exploitation of all-optical and opto-electronic nonlinearities in semiconductors. However, with the physical dimensions of semiconductor quantum wells (typically a few nanometres wide) there is insufficient high refractive index material to confine an optical mode effectively to...
the region of a quantum well (or several quantum wells). The solution to this problem is to place the quantum wells within the guiding region of an ordinary three layer slab waveguide. This is shown pictorially in figure A3.7.

Figure A3.7a A schematic drawing of a separate confinement heterostructure taking an InGaAsP/InGaAsP/InP SCH-MQW structure as an example. The layer structure is shown on the left, with the associated refractive index profile on the right. The approximate position of the guided optical mode is shown by the dashed line.

Figure A3.7b The potential profile of the separate confinement heterostructure for an InGaAsP/InGaAsP/InP structure. The confinement of electrons (solid circles) and holes (open circles) is also shown. Not to scale.
Figure A3.7a shows the refractive index profile and layer structure of a SCH. The optical mode is confined to the guiding layer of the waveguide by the heterostructure formed by the two lower index materials. By placing the quantum wells in the guiding layer, substantial overlap of the optical mode and the quantum wells is achieved. Figure A3.7b shows the corresponding potential structure of the SCH. The carriers are confined to the quantum wells by the potential structure. The overall efficiency of any interaction of the optical mode with the quantum wells is dependent on the fraction of the confined optical mode which overlaps with the quantum wells.

In a planar slab waveguide, light confined within the slab of greater refractive index is not totally confined within the guiding layer. There is some overlap of the optical mode with the cladding layers because of the finite refractive index step between the guiding and cladding layers. This effect becomes significant where the physical dimensions of the guiding slab are of the same order of magnitude as the wavelength of the light within the slab. In quantum well material, where the dimensions of the quantum well are much less than the wavelength of the infra-red light of interest (1.5μm) this effect is very pronounced. The optical confinement factor, Γ, is defined as the ratio of the optical mode which overlaps with the quantum wells. Mathematically, this can be described as:

\[ \Gamma = \frac{\int_{q_w} |E(x)|^2 \, dx}{\int_{-\infty}^{\infty} |E(x)|^2 \, dx} \]  

(A3.25)

Evaluating this integral for the structure shown in figure A3.7 requires substantial numerical calculations to solve the wave equation self consistently for all the interfaces of the SCH structure [8]. A typical value of \( \Gamma \) for quantum well material with 5-10nm wide wells and barriers is 1% overlap per well.
A3.6 References


A4.1 Theory of Linear Directional Couplers

Consider the twin ridge waveguide structure shown in figure 6.3.1. In the case where the separation between the guides is large, the structure will support an optical mode confined to the region below each of the two ridges and there will be effectively no interaction between the two modes. In this case the electric fields of the optical modes can be described in the following way [see for example 1,2]:

\[
E_{1z} = a_1(z) \phi_1(x, y) \quad (A4.1)
\]

\[
E_{2z} = a_2(z) \phi_2(x, y) \quad (A4.2)
\]

where

\[
a_i(z) = a_i(0) e^{-j \beta_i z}
\]

and the \(e^{j \omega t}\) term has been omitted for simplicity. \(a_i(z)\) is the amplitude of the electric field of the \(i\)th optical mode. The subscripts 1 and 2 refer the optical modes confined by ridges 1 and 2 respectively. \(\beta\) is the single waveguide propagation constant defined by \(\mu_{\text{eff}} = \beta/k_0\) where \(\mu_{\text{eff}}\) is the effective refractive index seen by the propagating optical mode, and \(k_0 = 2\pi/\lambda\) where \(\lambda\) is the free space wavelength.

If interaction between the two modes is allowed (i.e. reducing \(D\)) then considering a small distance \(\Delta z\) along the guide:

\[
\Delta a_i(z) = -j \beta_i a_i(z) \Delta z + \kappa_{12} a_2(z) \Delta z \quad (A4.3)
\]

Where \(\kappa_{12}\) is the coupling factor between guide 1 and guide 2. In the limit as \(\Delta z\) tends to zero:

\[
\frac{da_1}{dz} = -j \beta_1 a_1 + \kappa_{12} a_2 \quad (A4.4)
\]

similarly for waveguide 2:

\[
\frac{da_2}{dz} = -j \beta_2 a_2 + \kappa_{21} a_1 \quad (A4.5)
\]
Equations (A4.4) and (A4.5) form the coupled mode equations which describe the propagation of an optical field along the waveguides. Optical power in each waveguide is defined as \( P_{1,2} = |a_{1,2}|^2 \). For power conservation, \( P_T = |a_1|^2 + |a_2|^2 \) is a constant, therefore:

\[
\frac{dP_T}{dz} = \frac{d}{dz} (|a_1|^2 + |a_2|^2) = 0 \quad (A4.6)
\]

Now if the equations (A4.4) and (A4.1) are considered it can be noted that:

\[
\frac{d|a_1|^2}{dz} = \kappa_{12}^* a_1 a_2^* + \kappa_{12} a_1^* a_2 \quad (A4.7)
\]

Similarly for guide 2:

\[
\frac{d|a_2|^2}{dz} = \kappa_{21}^* a_1 a_2^* + \kappa_{21} a_1^* a_2 \quad (A4.8)
\]

Now if equations (A4.7) and (A4.8) are added, and remembering the result of (A4.6) it can be seen that for arbitrary \( a_1 \) and \( a_2 \):

\[
\kappa_{12}^* + \kappa_{21} = 0 \quad (A4.9)
\]

For most optical components \( \kappa_{12} \sim \kappa_{21} \), therefore it will be assumed that \( \kappa_{12} = \kappa_{21} \).

Therefore:

\[
\kappa_{12}^* + \kappa_{21} = 0 \quad (A4.10)
\]

So it can be concluded that the coupling coefficient, \( \kappa \), is purely imaginary and therefore we can write \( \kappa = -j\kappa \). Therefore it may be shown that the coupled wave equations become:

\[
\frac{d a_1}{dz} = -j\beta_1 a_1 - j\kappa a_2 \quad (A4.11)
\]

\[
\frac{d a_2}{dz} = -j\beta_2 a_2 - j\kappa a_1 \quad (A4.12)
\]

In order to solve these equations Laplace transforms are used. These are defined by:
\[ \bar{a}(s) = \int_0^\infty a(z)e^{-sz}dz \quad \int_0^\infty \frac{da}{dz} e^{-sz}dz = s\bar{a} - a(0) \quad (A4.13) \]

If it is assumed that \( \kappa \) is not a function of \( z \), then after some algebraic manipulation it can be shown that:

\[ \bar{a}_1(s) = \frac{a_1(0)(s + j\beta_1) - j\kappa a_2(0)}{(s + j\beta_o)^2 + Y^2} \quad (A4.14) \]

\[ \bar{a}_2(s) = \frac{a_2(0)(s + j\beta_2) - j\kappa a_1(0)}{(s + j\beta_o)^2 + Y^2} \quad (A4.15) \]

where the following definitions have been made:

\[ \frac{\beta_1 + \beta_2}{2} = \beta_o \quad \Delta\beta = \frac{\beta_1 - \beta_2}{2} \quad Y^2 = \kappa^2 + (\Delta\beta)^2 \]

A mathematics data book may be consulted to find the solutions of these Laplace transforms to yield the following solutions to the coupled wave equations (A4.11) and (A4.12) for the amplitudes of the electric fields in guides 1 and 2:

\[ a_1(z) = e^{-j\beta_o z} a_1(0) \left( \cos(Yz) - \frac{j\Delta\beta}{Y} \sin(Yz) \right) - \frac{j\kappa}{Y} e^{-j\beta_o z} a_2(0) \sin(Yz) \quad (A4.16) \]

\[ a_2(z) = e^{-j\beta_o z} a_2(0) \left( \cos(Yz) + \frac{j\Delta\beta}{Y} \sin(Yz) \right) - \frac{j\kappa}{Y} e^{-j\beta_o z} a_1(0) \sin(Yz) \quad (A4.17) \]

In the specific case of a directional coupler, light is coupled into only one arm. Therefore \( a_1 \) is the applied field, \( a_2 \) is the induced field, \( a_1(0)=1, a_2(0)=0 \) (in normalised units) and \( L \) is the interaction length. Then the power in each arm as a function of interaction length becomes:

\[ P_1(L) = |a_1(L)|^2 = \cos^2(YL) + \frac{(\Delta\beta)^2}{Y^2} \sin^2(YL) \quad (A4.18) \]

\[ P_2(L) = |a_2(L)|^2 = \frac{\kappa^2}{\kappa^2 + (\Delta\beta)^2} \sin^2(YL) \quad (A4.19) \]
In the case where guides 1 and 2 are identical, the equations simplify further to give the following:

\[ P_1(L) = \cos^2(\kappa L) \]  \hspace{1cm} (A4.20)

\[ P_2(L) = \sin^2(\kappa L) \]  \hspace{1cm} (A4.21)

It can be seen from these equations that in the case of two identical parallel ridge waveguides with light injected into one arm of the coupler the power in each guide sinusoidally oscillates as a function of coupling length. If the guides are non-identical then the maximum fraction of the injected light which may be cross-coupled is \( \kappa^2/(\kappa^2 + \Delta \beta^2) \).

A4.2 References

"Now all has been heard; here is the conclusion of the matter: Fear God and keep His commandments, for this is the whole of man."

Ecclesiastes 12v13