Scheme Design Considering Network Topology and Multi-attribute Decision-making for Under Frequency Load Shedding

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ABSTRACT--Power system emergency control is one key defense strategy in contingencies for protecting the system from cascading blackout. Under Frequency Load Shedding (UFLS) is one such strategy to ensure system stability by shedding load to retrieve balance between power supply and demand. Novel UFLS scheme design and a scheme optimization approach are proposed in this paper to find the optimal load-shedding schemes for different network partition resulted from contingencies. To obtain all possible UFLS schemes for a certain area, a candidate scheme set design algorithm is proposed based on value assigning of scheme parameters and then a relatively complete candidate scheme set is constructed. Considering network splitting caused by protection, several subsystems may exist from the reconstruction of independent network areas. The concept of homological area is defined and a graph-based method is used to analyze system topology change. Then, a multi-attribute decision-making (MADM) algorithm is introduced for order preference by similarity to an ideal solution (TOPSIS) for global optimizing candidate schemes. Optimal schemes for an area in both isolated area and homological area cases can be derived from all feasible UFLS schemes by MADM method. Simulation results demonstrate that the UFLS schemes can effectively restore system frequency in different network topologies.

Index Terms--Graph theory, Homological area, Desirable scheme screening, Multi-attribute decision-making, Under Frequency Load Shedding.

I. INTRODUCTION

Power systems have increased greatly in both size and complexity, which increase system vulnerability. Most of the time, systems are operated at the states close to their capacity limits [1], which could decrease system stability margin. The power system can normally operate in the case of small disturbances, such as the simultaneous failure of a double-circuit overhead line, etc. However, sudden loss of large generator or key tie-lines due to line tripping can produce severe generation-load imbalance. The frequency deviation caused by the imbalance may severely harm other generators, load-side equipment, and the system integrity [2].

In the case of a large generation loss, the scheduled power reserve may not be enough to restore system frequency. In such situation, the use of an Under Frequency Load Shedding (UFLS) scheme to curtail load to match generation is well known and established utility practice [3]. UFLS schemes are a last-resort tool to protect the system in case of a severe contingency, shedding an appropriate amount of load for quick recovery of system frequency to its rated value.

UFLS schemes can, according to [4], be categorized into three groups: traditional static UFLS schemes, semi-adaptive UFLS schemes and adaptive UFLS schemes. These schemes continuously measure frequency and optionally the rate of change of frequency (ROCOF) by means of type 81 relays [5]. Static UFLS scheme curtails a constant amount of load, based on experience, at predetermined frequency threshold with intentional time delay. To minimize over-shedding or under-shedding, several shedding steps are contained in static UFLS schemes. The semi-adaptive scheme provides a step forward. It measures ROCOF, $df/dt$, when a certain frequency threshold is reached. Based on the value of ROCOF, a different amount of load is shed. Measurement of ROCOF is evaluated only at the first frequency threshold. In the following steps, the load is shed in the traditional way. Unlike semi-adaptive UFLS scheme, adaptive UFLS scheme encompasses all intelligent algorithms, trying to figure out the most appropriate response of UFLS protection to a different set of possible system conditions [6]. Reference [7] makes use of the frequency derivative based on the System Frequency Response (SFR) model. Reference [8] uses both frequency and voltage variables to select an appropriate amount of load shedding based on two centralized adaptive load-shedding algorithms. Reference [9] proposes a global load shedding scheme integrated with the rate of frequency decline as an extra variable. Reference [10] describes an approach for obtaining a few-seconds-in-advance frequency prediction for WAMS-Based under frequency load shedding.

By analysing historic outages, it is found that systems are likely to be split into several subsystems accompanied by the activation of UFLS relays [11-12]. In such case, there may be one subsystem consisting of several network areas with their own UFLS schemes. These UFLS schemes may influence each other and then the frequency restoration of the whole.
subsystem if they are not properly coordinated. A new UFLS scheme is proposed for microgrids in [13]. Because the scheme is independent of microgrid parameters, it is not adaptive to parameter variations. A fast and optimal adaptive load shedding method for isolated power systems using Artificial Neural Networks (ANNs) is presented in [14], which performs effective load shedding in various loading scenarios. A novel UFLS adaptive scheme design method based on short-term frequency prediction is proposed in [15]. However, all aforementioned work is based on a single network area, which is regionalized based on administrative or geographical stations via network operation modes. Possible topological change of the system such as network splitting, caused by protection, is not considered. Thus, these schemes may perform well when the network is islanded but fail to restore system frequency when several areas are integrated to a subsystem and could even aggravate the consequence due to the violation of system limits. For example, on November 4, 2006, the Europe interconnected grid experienced a serious incident due to inappropriate operation of UFLS devices’ combination [16-17].

In this paper, we propose an improved UFLS scheme design method and the concept of homological area to design UFLS schemes for different system topologies. Based on the attributes of existing static UFLS schemes, a candidate scheme set design method is introduced, which could establish a complete set of candidate shedding schemes. Then, the recombination of electric partitions under various contingencies, especially in system splitting, is investigated. A novel method to find all homological areas of a system is then introduced by studying the bonds of system graphs derived from the dissected sketch map. A multi-attribute decision-making (MADM) algorithm based on TOPSIS (technique for order preference by similarity to an ideal solution) to select desirable schemes is also introduced. Simulation results show that the optimal UFLS scheme obtained by the proposed MADM method could restore the system frequency effectively in both isolated area and homological area cases. The main contributions of this paper are that it proposes: i) a new UFLS scheme design method to obtain a complete set of candidate schemes for network recovery; ii) the concept of homological area to obtain the optimal load shedding schemes for various network topologies; iii) a MADM algorithm based on TOPSIS to find the optimal load shedding schemes in various contingencies.

The remaining parts of this paper are organized as follows. Section II describes the attributes of existing UFLS schemes. In Section III, candidate set construction for shedding schemes is presented. In Section IV, the concept of the homological area for UFLS is described. The method to screen feasible schemes from candidate schemes is presented in Section V. In Section VI, the multi-attribute characteristics of UFLS schemes are discussed and a MADM algorithm based on TOPSIS is introduced. The proposed MADM algorithm is demonstrated on a practical system in Section VII. Section VIII concludes the paper.

II. ATTRIBUTES OF EXISTING UFLS SCHEMES

Traditional load shedding techniques are still widely used as the most effective UFLS methods because of their simplicity [18]. In these schemes, a fixed amount of load is shed to retrieve generation-load balance when system frequency falls below predetermined thresholds. At the same time, predefined time delay is determined for each shedding step in order to prevent tripping of UFLS relays during transient cases, such as system oscillation and voltage dip.

For a traditional scheme, four key parameters should be determined: the number of load shedding steps, frequency threshold for each step, total amount of load to be shed, and time delays of UFLS relays. It is complicated to set the magnitude of load curtailment for each step. Traditionally, the total amount of load shed is set based on experience and then distributed to each shedding step with certain algorithms. There are several distribution methods roughly in three categories: equal step-size schemes, incremental step-size schemes and decremental step-size schemes [19].

- For equal step-size UFLS schemes, the amount of load curtailment for each step is \( \Delta P_{k} / N_F \), where \( \Delta P_{k} \) is the total amount of load to be shed and \( N_F \) is the number of load-shedding steps in the base round.
- For incremental step-size UFLS schemes, load shed for the next step is bigger than that for the current step, which is opposite to decremental step-size schemes.
- Decremental step-size UFLS schemes recover system frequency faster than incremental step-size schemes, while the frequency may rise much higher than the nominal value resulted from over-shedding.

III. CANDIDATE SET DESIGN FOR SHEDDING SCHEMES

Four parameters including the number of load shedding steps, frequency thresholds, load shed, and time delays are to be determined for traditional load shedding schemes. Once they are established, a load-shedding scheme is derived.

A. Determination of Parameters

The number of load-shedding steps contains two parts: base steps \( N_F \) and backup steps \( N_B \). From [20] it is seen that \( N_B \), frequency thresholds and the amount of load curtailment of backup rounds are all derived from the base round, and as a result only \( N_F \) is an independent variable.

Number of load-shedding steps: Determine the minimum /maximum allowable number of load shedding steps, denoted by \( N_{F\text{\text{min}}} \) and \( N_{F\text{\text{max}}} \) respectively. Then, with the step interval \( \Delta N \), a sequence of load-shedding steps in ascending order ranging from \( N_{F\text{\text{min}}} \) to \( N_{F\text{\text{max}}} \) are obtained. The set for \( N_F \) (denoted by \( \Phi_{N_F} \)) is calculated as follows:

\[
\begin{align*}
N_F(1) &= N_{F\text{\text{min}}} \\
N_F(i) &= N_F(i-1) + \Delta N; \quad N_{F\text{\text{min}}} \leq N_F(i) \leq N_{F\text{\text{max}}}
\end{align*}
\]

Frequency threshold: \( f_{s1} \) denotes the frequency threshold of the first step of the UFLS relay. Based on the minimum/maximum \( f_{s1} \) (denoted by \( f_{s1\text{\text{min}}} / f_{s1\text{\text{max}}} \)), the frequency threshold set of the first step (denoted by \( \Phi_{f_{s1}} \)) can be obtained with given frequency interval \( \Delta f_{s1} \), which is similar to that of \( \Phi_{N_F} \). The difference between two consecutive frequency thresholds is denoted by \( \Delta f \) and the
set for $\Delta_f$ (denoted by $\Phi_{\Delta_f}$) is obtained in the same way as for $\Phi_{N_f}$ after $\Delta_f^{\min}$ and $\Delta_f^{\max}$ are given.

**Time delay:** Similar to the number of load shedding steps, time delays for a UFLS scheme also contains two parts: base time delay $t_E$ and backup time delay $t_B$. However, minimum/maximum base time delay and time interval (denoted by $t_F^{\min}/t_F^{\max}$ and $\Delta t_E$ respectively) are usually different from those of backup time delay (denoted by $t_B^{\min}/t_B^{\max}$ and $\Delta t_B$ respectively). Thus, base and backup time delay sets (denoted by $\Phi_{t_E}$ and $\Phi_{t_B}$ respectively) should be computed separately in the same way as $\Phi_{N_f}$. Generally, time delays for all steps of the base round are the same and so are for the backup round.

**Load-shedding amount:** It is more flexible to design load-shedding size for each step, which affects the frequency restoration directly. The total amount of load shed should be estimated first. As long as maximum/minimum allowable total load-shed (denoted by $\Delta P_L^{\max}/\Delta P_L^{\min}$ respectively) are determined, the set for $\Delta P_L$ (denoted by $\Phi_{\Delta P_L}$) can be derived with the given interval $\delta_{\Delta P}$.

**B. Load Shedding Model**

Load-shed size for each step is another important factor influencing the system frequency restoration for a UFLS scheme. As discussed in Section II, there are mainly three types of UFLS schemes based on the load shed for each step. Equal, decremental and incremental load curtailments for each step are subject to equation (2), (3) and (4) respectively.

$$\sum_{i=1}^{N_f} \Delta P_i = \Delta P_L$$ \hspace{1cm} (2)

$$\sum_{i=1}^{N_f} \Delta P_i = \Delta P_L, i \leq j < i \leq N_f$$ \hspace{1cm} (3)

$$\sum_{i=1}^{N_f} \Delta P_i = \Delta P_L, i \leq j < i \leq N_f$$ \hspace{1cm} (4)

where $\Delta P_i$ is the load shed in the $i$th shedding step.

Let $\delta_{\Delta P} = \Delta P_{i+1} - \Delta P_i \hspace{1cm} (1 \leq i \leq N_f - 1)$, where $\delta_{\Delta P}$ denotes the difference between two consecutive load-shedding steps. Let $\delta_{\Delta P} = g(i)$, where $g(i)$ is a function of $i$, and the three types of load shedding schemes can be obtained when $g(i)$ is zero, incremental and decremental respectively. By introducing $\delta_{\Delta P}$, equations (2)-(4) can be integrated into a universal form:

$$\left\{ \begin{align*}
\Delta P_L &= N_f \cdot \Delta P_1 + \sum_{i=1}^{N_f-1} i \cdot \delta_{\Delta P} \\
\delta_{\Delta P} &= \Delta P_{i+1} - \Delta P_i = g(i)
\end{align*} \right.$$ \hspace{1cm} (5)

where $\Delta P_1$ indicates load shed for the first shedding step.

Once $\Delta P_L$ and $N_f$ are determined, for different $\Delta P_1$ and $g(i)$, $\Delta P_i \hspace{1cm} (1 < i \leq N_f)$ can be obtained through (5). The corresponding set of load curtailment \{ $\Delta P_1, \Delta P_2, \ldots$ \}, is denoted by $\Phi_{\Delta P}$. Various combinations of $\Delta P_L$ and $N_f$ for different UFLS schemes can be obtained from the two-fold Cartesian product [21] of $\Phi_{\Delta P}$ and $\Phi_{N_f}$, which is denoted by $\Phi_{\Delta P} \times \Phi_{N_f}$. Then, based on equation (5) and the $i$th element of $\Phi_{\Delta P} \times \Phi_{N_f}$, corresponding $\varphi_{\Delta P}(i)$, $(i=1,2,\ldots)$ is obtained, where $n$ is the number of elements of $\Phi_{\Delta P} \times \Phi_{N_f}$. At last, all possible load-shedding schemes are represented by a set $\Phi_{\Delta P} = \bigcup_{i=1}^{n}$.

**C. Candidate Set Construction**

After determining the following parameter sets: $\Phi_{f_{ji}}, \Phi_{M_f}, \Phi_{t_E}$, $\Phi_{t_B}$ and $\Phi_{\Delta P}$, all possible UFLS schemes, also called candidate schemes, can be obtained as follows:

1) Construct Cartesian product of $\Phi_{f_{ji}}, \Phi_{M_f}, \Phi_{t_E}$ and $\Phi_{\Delta P}$, i.e. $\Phi_{SF} = \Phi_{f_{ji}} \times \Phi_{M_f} \times \Phi_{t_E} \times \Phi_{\Delta P}$. Then candidate UFLS schemes for the base round can be obtained.

2) Derive frequency thresholds and the load to be shed for the backup round from $\Phi_{SF}$, denoted by $\Phi_{SB}$.

3) Construct Cartesian product of $\Phi_{SB}$ and $\Phi_{t_E}$, and then determine UFLS schemes for the backup round, denoted by $\Phi_{SB} = \Phi_{SB} \times \Phi_{t_E}$.

4) It can be seen from step 2) that for every element of $\Phi_{SF}$, there is a corresponding element in $\Phi_{SB}$. Thus the number of elements of $\Phi_{SF}$ is equal to that of $\Phi_{SB}$. Another feature for them is that each element of $\Phi_{SF}$ is related to one element of $\Phi_{SB}$ exactly. Based on the correlation between them, the complete candidate scheme set (denoted by $\Phi_{C}$) can be established by combining the pair elements from $\Phi_{SF}$ and $\Phi_{SB}$.

**IV. HOMOLOGICAL AREAS IN AN UFLS SCHEME**

Graph theory is utilized to define the homological area and analyse homological areas for UFLS in this section.

**A. Definition**

Generally, a specific UFLS scheme is designed for a single network area, called an area in Fig.1. However, when the UFLS relays are activated, a multi-area subsystem may consist of several areas due to network splitting. Then UFLS schemes for each area should be activated. The incoordination among these schemes may fail to force the subsystem to enter a new equilibrium state. Therefore, simulations and optimizations should be conducted for all candidate UFLS schemes in both their own UFLS regions and the whole multi-area subsystem for coordination.

Only the areas connected to the main power grid via weak tie lines are easily islanded in a contingency. The number of tie lines connecting an area to the other parts of the power system is defined as connection degree. In Fig.1, the connection degree of Area 1 is 1 and that of Area 4 is 8. To simplify analysis, there are two constraints on areas division:

1) Only when the connection degree of an area or a
multi-area subsystem is less than 3, it can be split from the original system. For example, an area with two connection degree is split from the system, which means its one tie line is in maintenance and the other tie line fails.

2) The original power system is split into only two parts. The constraints are based on the assumption that only the action of UFLS relays is considered and the active splitting behaviour of system operation is ignored.

![Diagram of the study system](image)

**Fig. 1** Configuration of the study system

Only the cases subject to the two constraints are considered for the following two reasons:

1) Implementation of UFLS is less significant if system operators are involved into system splitting. The functionality of UFLS will be affected once the operator interferes the cases when several tie lines (more than two) are tripped. Thus, the active behaviour of the operator will not be taken into consideration.

2) By introducing candidate sets of UFLS schemes and UFLS homological areas explained below, the situation that the grid is split into more than 2 parts can be analysed in the similar way as that of cases with system split into two parts.

An area catering to the above two constraints can be split from the system and is called an isolated area. An isolated area has no connection with other parts of the original system and is the smallest unit in areas division. An area is also the smallest unit in UFLS implementation. The areas with background colour in Fig.1 are isolated areas. Multi-area subsystems that satisfy the above two constraints are defined as UFLS homological areas, referred as a homological area for simplicity. In Fig.1, there are eight homological areas and only two homological areas are presented as examples. Areas 14 and 15 can form a homological area. However, areas 11 and 12 cannot form a homological area, because if these two areas are removed from the interconnected system, the grid will be split into 3 parts. In addition, areas 1, 2, 3, 4, 5, 6 and 7 cannot form a homological area either as the connection degree of the new multi-area subsystem is larger than 3.

When UFLS relays are activated after system disturbances, an area may be disconnected from the main grid, forming an isolated area or still in a homological area. The frequency characteristics of a UFLS scheme in an isolated area will be very different from that in a homological area. Therefore, it is necessary to optimize the candidate schemes in these cases.

B. Graph Bond

Different area splitting means different grid faults. Thus, to design effective UFLS schemes which can restore system frequency in various contingencies, all isolated and homological areas resulted from different faults should be found first. Then simulate and screen candidate schemes in both isolated and homological areas to select optimal ones.

Fig.2 is a single-line graph of Fig.1, where vertices represent areas and edges tie lines. The graph is denoted by $G = (V, E)$, where $V$ is the vertex set and $E$ is the edge set. A bond is a minimal nonempty edge cut which does not have any other cut-set as a subset [22]. Suppose $B = E[X, Y]$ is a bond, $X$ and $Y$ are two sets of randomly selected vertices of a graph $G$, $B$ has the following attributes:

1) $G$ will be partitioned into two disjoint sub-graphs with the removal of $B$;
2) If not all edges belong to $B$ are deleted, $G$ will still be a connected graph.

![Diagram of single-line connected graph](image)

**Fig. 2** Single-line connected graph of Fig.1

For example, $B = \{ (v_4, v_5), (v_5, v_8) \}$ is a bond of $G$, where $(v_4, v_5)$ denotes an edge whose ends are an unordered pair of vertices: $v_4$ and $v_5$.

The following step is to assign weights that represent the number of tie lines between two vertices to each edge of $G$. The weight of a bond is defined as the sum of weight of each edge in the bond, which is denoted by $w$. If $w$ of $B$ is less than 3, the bond can be deleted to render $G$ disconnected, which is like area splitting in the real power system.

By removing $B$ from $G$ (denoted by $G - B$ in mathematics, where "−" means removing a sub-graph from the original graph), the remaining parts represent two homological areas or an isolated area and a homological area. Therefore, homological area analysis of a grid is to find out all bonds, whose $w$ is less 3, in the connected graph.

C. Homological Area Analysis for UFLS

1) Searching for Bond

In order to analyse homological areas of a grid, all edge cuts of the grid’s graph should be found out first. Then select bonds from edge cuts satisfying the weight constraints and homological areas can be derived from these bonds. All edge cuts of graph $G$ can be obtained by following steps [23]:

1) Find a spanning tree of graph $G$ randomly in Fig.3.
2) Determine fundamental bonds. Table I shows the fundamental bonds of the spanning tree in Fig.3.
3) Find out all symmetric difference (denoted by $B_p, B_{p+1} \ldots \sim$ ) of each two fundamental bonds (denoted by $B_1, B_2 \ldots \sim$ ). The symmetric difference of
$B_i$ and $B_j$ is the set of elements which are in either of
the sets but not in their intersection, calculated by
$$B_i \oplus B_j = (B_i \cup B_j) \cap \overline{(B_i \cap B_j)}$$  \hspace{1cm} (6)

![Fig.3 A spanning tree of graph G](image)

<table>
<thead>
<tr>
<th><strong>TABLE I</strong></th>
<th><strong>FUNDAMENTAL BONDS OF THE SPANNING TREE</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds</td>
<td>Edges</td>
</tr>
<tr>
<td>$B_1$</td>
<td>$(v_1, v_4)$</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$(v_2, v_4)$</td>
</tr>
<tr>
<td>$B_3$</td>
<td>$(v_3, v_4)$</td>
</tr>
<tr>
<td>$B_4$</td>
<td>$(v_4, v_7)$</td>
</tr>
<tr>
<td>$B_5$</td>
<td>$(v_5, v_7)$</td>
</tr>
<tr>
<td>$B_6$</td>
<td>$(v_{6}, v_{7})$, $(v_8, v_9)$</td>
</tr>
<tr>
<td>$B_7$</td>
<td>$(v_{6}, v_{7})$, $(v_8, v_9)$</td>
</tr>
<tr>
<td>$B_8$</td>
<td>$(v_8, v_9)$</td>
</tr>
</tbody>
</table>

There might be more than one spanning trees. However, there is only one bond result. No matter which spanning tree is selected in step 1), the same bond results will be found by following steps 2) and 3).

The fundamental bonds and all symmetric difference of any two bonds together form the cut space (in mathematics, the set of all cut sets of an undirected graph is called the cut space) of graph $G$, which contains all other bonds that are not included in the fundamental bond set. To form a homological area set based on the cut space of $G$, two selecting steps are executed: 1) to select bonds from the cut space; 2) to select bond whose weight is less than 3. In order to reduce computational burden, the selecting process is usually reversed, i.e. selecting edge cuts whose weight are less than 3 from cut space first and then finding bonds from edge cuts.

Let $C = \{C_1, C_2, \ldots, C_i \leq k\}$, $C_i$ the cut set of the cut space of $G$, where $w_i$ is the weight of $C_i$. Then find out bonds from $C$. For $\forall C_i \subset C$, let $e(v_i, v_j)$ be an edge of $C_i$. Then, traverse each edge of the disconnected components of $G-C_i$ from $v_i$ and $v_j$ respectively. The sets of vertices corresponding to the two walks are denoted by $V_i$ and $V_j$ respectively. Apparently, the endpoints of any edge in $C_i$ belong to two disjoint components of $G-C_i$. If $V_i \cup V_j \neq \emptyset$, $G-C_i$ has exactly two parts and $C_i$ is a bond. Otherwise, $C_i$ is not a bond.

Take the edge cut $C = \{v_4,v_5\}, (v_5,v_8)$ as an example. $V_4 = \{v_4,v_5,v_1,v_2,v_7,v_8\}$ and $V_8 = \{v_8,v_9,v_{13},v_{12},v_{11},v_{10},v_{17},v_{16},v_{15},v_{14}\}$ are two vertex sets with start vertex $v_4$ and $v_8$, respectively. At the same time, $V_4 \cup V_8 = \emptyset$. Thus $C = \{v_4, v_5\}, (v_5, v_8)$ is a bond.

(2) Searching for Isolated Areas and Homological Areas

In above selecting process, there are two complementary vertex sets after removing $C$, denoted by $V_s$ and $\overline{V_s}$. $V_s$ denote the vertex set with fewer vertices. If there is only one vertex in $V_s$, it represents an isolated area; otherwise, it represents a UFLS homological area with several areas. All isolated and homological areas can be obtained by the aforementioned approach. In one system area, the amount of load curtailment is decided by the percentage of its active power. An area with heavy load tends to have low system frequency due to that the percentage of active power shortage could be large. Therefore, the area needs to be equipped with UFLS devices. An area with light load tends to have high system frequency when the adjacent area with heavy load is disconnected from it. An area containing more than half of the grid’s area is regarded as the main system of the power grid. In this case, it usually has a small percentage of active power shortage and thus is not treated as a homological area.

Table II presents all homological areas and isolated areas for the system in Fig.1, where $IA$ represents an isolated area and $HA$ a homological area. The connection degree means the tie lines number between the area set and the remaining parts. There are 7 isolated areas and 8 homological areas.

<table>
<thead>
<tr>
<th><strong>TABLE II</strong></th>
<th><strong>HOMOLOGICAL AREAS AND ISOLATED AREAS OF GRAPH G</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Area set</td>
<td>Connection degree</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>6,7</td>
<td>2</td>
</tr>
</tbody>
</table>

(3) UFLS Schemes for Homological Areas

After finding all homological areas, scheme selection from candidate scheme sets of member areas forms all candidate schemes for the homological area. By rearranging shedding steps of these schemes in descending frequency thresholds, UFLS schemes for the homological area are obtained.

Take the homological area with area 6 and 7 in Fig.3 for an example. The scheme set for area 6 is assumed to be $\{sch_b\}$ and that for area 7 $\{sch_7_1, sch_7_2\}$. If area 6 has low system frequency, the operators can only choose the UFLS scheme $\{sch_b\}$. By contrast, if area 7 has low frequency, the operators can choose either one scheme from the two UFLS candidate scheme set $\{sch_7_1, sch_7_2\}$. Then, for the homological area consisting of both area 6 and area 7, the global UFLS scheme is formed by one UFLS scheme of area 6 and one UFLS scheme of area 7. It is because that the candidate schemes for the homological area are formed by the candidate scheme sets of member areas. The scheme set for the homological area is $\{(sch_b, sch_7_1), (sch_b, sch_7_2)\}$.

Once candidate scheme sets for all homological areas are obtained, the following work is to find the globally optimal schemes for each area of the homological area.
V. SCREENING FOR FEASIBLE UFLS SCHEME

In this section, the approach to select feasible schemes from candidate schemes is shown.

Before the screening process for feasible schemes, simulation for every candidate scheme should be conducted first to get several indicators which reflect the performance of the scheme. The indicators are obtained from the frequency-time curves of the power system after the candidate UFLS schemes are implemented. The six indicators to evaluate the scheme performance are shown as follows.

1) Minimum frequency: \( f_{\text{min}} \rightarrow \max \);
2) Fall time: \( t_{\text{dec}} \rightarrow \min \);
3) Time of regulation: \( t_{\text{reg}} \rightarrow \min \);
4) Frequency deviation: \( \Delta f \rightarrow \min \);
5) Overshoot: \( \delta \rightarrow \min \);
6) Total load curtailment: \( \Delta P \rightarrow \min \).

There are certain constraints for the minimum, maximum and steady state frequencies after UFLS schemes are implemented. If the constraints are not satisfied, the severe system condition will not be eased, and even deteriorates further. The constraints for feasible UFLS schemes can be expressed as follows.

\[
\begin{align*}
    f_{\text{min,s}} & \leq f_{\text{min}} \\
    f_{\text{ss}} & \leq f_{\text{c}} \\
    f_{\text{max,s}} & = f_{\text{max}}
\end{align*}
\]

where, \( f_{\text{min,s}} \) and \( f_{\text{max,s}} \) are the minimum and maximum allowable frequencies respectively for the area in UFLS implementation; \( f_{\text{ss}} \) is the minimum allowable steady-state frequency for new generation-load balance.

To get valid UFLS schemes, a variety of system operation modes and multi grid faults should be taken into account, such as the parameter change of system devices, topology change caused by grid faults. Actually, the algorithm above is an exhaustion method. Thus, the schemes should be simulated under different conditions and a large number of groups of indicators can be obtained for each scheme correspondingly. Only when the multi-groups of indicators get under every possible condition satisfy (7), the scheme will be a feasible one. Suppose the candidate scheme set \( \Phi_k \) for area \( k \) is established by using the methodology shown in Section III and every scheme of \( \Phi_k \) is simulated in the area. Only when all groups of indicators of scheme \( i_k \), one element of \( \Phi_k \), satisfy (7), \( i_k \) is a feasible scheme. Otherwise, \( i_k \) is an infeasible scheme and should be removed from \( \Phi_k \). After all infeasible schemes are removed, the left schemes consist of a feasible set, denoted by \( \Phi^f_k \).

VI. OPTIMAL UFLS SCHEMES ON MULTI-ATTRIBUTE DECISION-MAKING

After the feasible scheme set is got in Section V, an optimization method could be conducted to obtain the optimal UFLS scheme. The multi-attribute characteristics of UFLS schemes are demonstrated in Part A of Section VI. A novel multi-attribute decision-making algorithm based on TOPSIS is presented in Part B. It should be noted that all schemes processes are offline. In reality, the candidate schemes for an area may require several hundreds of or more optimization processes, which take up a large volume of computation. In practical application, according to the certain network configurations, many perceived network topologies will be analysed to find out the optimal topology solutions offline. Thus, network operators can make decisions according to the preset scheme in response to real-time network operation situations. By existing SCADA system without changing any communication devices or adding new devices, UFLS relays can receive control information and then act according to the scheme from operators.

A. Multi-attribute Characteristics of UFLS Schemes

As shown in Section V, six indicators will be obtained after a specific UFLS scheme is simulated. \( f_{\text{min}} \) and \( t_{\text{dec}} \) reflect the performance of candidate schemes during the transient UFLS procedure. Generators and loads sensitive to the system frequency are apt to be influenced by frequency oscillation [26], and long-time operation under low frequency will do great harm to the system devices. An even worse case is the generator is tripped from the grid after its under frequency protection is activated. It will increase the imbalance between generation and load and the low system frequency will be lower. The power system may even collapse due to the vicious circle. Thus, \( f_{\text{min}} \) for an ideal UFLS scheme should be the maximum and \( t_{\text{dec}} \) minimum.

\( t_{\text{reg}}, \Delta f \) and \( \delta \) reflect the coordination of UFLS scheme between system inertia and speed regulator. They should be the minimum for the optimal scheme. \( \Delta P \) reflects the economics of the schemes, which should be the minimum when restoring the system frequency fast to the rated value.

The six indicators, which are also called attributes of the scheme, reflect the frequency restoring ability of UFLS schemes [25]. To evaluate the performance of a feasible scheme, all the attributes should be taken into account comprehensively. Apparently, for an optimal scheme, all the attributes should be better than that of any other schemes. However, it can be easily known from the basic characteristics of UFLS schemes that some attributes are destined to be in contradiction. For example, to enhance \( f_{\text{min}} \) and reduce \( t_{\text{dec}} \), \( \Delta P \) should be increased, which means \( \Delta P \) can’t be the minimum and \( \delta \) may also be larger. Thus, the UFLS scheme whose attributes reach their optimal value simultaneously actually doesn’t exit.

Based on the characteristics of UFLS schemes, the relatively optimal scheme can be obtained by adopting MADM methods. MADM is also called multi-objective decision making for finite schemes. The typical characteristics for MADM problems are as follows.

1) There are candidate schemes;
2) There are multiple attributes;
3) The dimensions for each attribute are different;
4) There is certain weight for every attribute.

There happens to be the same characteristics for UFLS schemes, and thus it is an MADM problem to find the optimal UFLS scheme is, which can be solved by MADM
methods.

When employing MADM theory, there should be only one group of attributes for each scheme. However, as demonstrated in Section V, there are several groups of attributes for each scheme. To solve the problem, the integrated attributes for each scheme are adopted through integrating several groups of attributes into one group by the certain algorithm. For the paper is concentrated on MADM, the attributes in this paper are all integrated attributes.

B. Multi-attribute Decision-making Based on TOPSIS

TOPSIS is an ordinal preference method to get the optimal scheme most similar to the ideal one, which is proposed by C.L. Hwang and K. Yoon [26]. TOPSIS is also one of the most effective methods to solve MADM problems. The core idea of TOPSIS is to choose the scheme, which is the most similar to the ideal scheme and furthest from the negative ideal scheme, as the optimal one. During the decision-making process, the scheme closer to the ideal scheme and also closer to the negative ideal scheme than any other one may also exist, which will hinder the optimal UFLS scheme selection. Therefore, the relative distance of one scheme to the ideal scheme is adopted by TOPSIS to conduct scheme evaluation. It should be noted that the following UFLS schemes are described by integrated attributes, which is different from that in Section II to V, where the schemes are described by their actual attributes.

Suppose there are \( m \) schemes to be evaluated and exist \( n \) evaluation indicators for the feasible UFLS schemes, which are denoted by \( M = \{1, 2, 3, \ldots \} \) and \( N = \{1, 2, 3, \ldots \} \) respectively. The feasible scheme set is denoted by \( \Phi^r = \{A_1, A_2, A_3, \ldots \} \). Attribute set of the schemes is denoted by \( G = \{X_1, X_2, X_3, \ldots \} \). The attributes of UFLS schemes are mainly categorized into two types: cost type, which is expected to be the minimum, and profit type, which are expected to be the maximum. Subscript sets of cost type and profit type attributes are denoted by \( J \) and \( J' \) respectively. Thus, only \( f_{\min} \) is in profit type and the other indicators are all in cost type.

The decision matrix for the MADM problem is in (8).

\[
D = \begin{bmatrix}
X_1 & X_2 & \cdots & \cdots \\
A_1 & x_{11} & x_{12} & \cdots & \cdots \\
A_2 & x_{21} & x_{22} & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & x_{m1} & x_{m2} & \cdots & \cdots
\end{bmatrix}
\]

(8)

where, \( x_{ij} \) is the \( j \)th attribute of the scheme \( A_i \).

The steps to solve the MADM problem by using the TOPSIS method are shown as follows:

1) Construct standard decision matrix \( R = (r_{ij})_{m \times n} \).

\[
\begin{align*}
R_i &= \frac{x_{ij} - x_{ij}^{\min}}{x_{ij}^{\max} - x_{ij}^{\min}}, i \in M; j \in J \\
R_j &= \frac{x_{ij}^{\max} - x_{ij}}{x_{ij}^{\max} - x_{ij}^{\min}}, i \in M; j \in J'
\end{align*}
\]

(9)

where, \( x_{ij}^{\min} = \min_{i \in M} \{x_{ij}\} \) and \( x_{ij}^{\max} = \max_{i \in M} \{x_{ij}\} \). Through standardization of \( R \), the metrics for each attribute of the schemes will be the same.

2) Determine weight vector \( w \) of the attributes.

\[
w = \{w_1, w_2, \ldots, \}
\]

(10)

The methods to assess the weight of scheme attributes are mainly divided into two types [27]: subjective and objective weighting methods. Based on the objectives and requirements of UFLS, the weights of scheme attributes are calculated with the entropy-based method [28], which considers subjective weights. The entropy of each attribute is calculated by using (11).

\[
p_{ij} = \frac{x_{ij}}{\sum_{k=1}^{m} x_{kj}}, i \in M; j \in N
\]

\[
E_j = -k \sum_{j=1}^{m} p_{ij} \ln p_{ij}, j \in N
\]

\[
k = \frac{1}{\ln m}
\]

(11)

where, \( E_j \) is the entropy of \( X_j \) and \( E_j \in [0, 1] \).

Based on mathematical theory, \( \lim_{p_{ij} \to 0} p_{ij} \ln p_{ij} = 0 \). So when \( p_{ij} = 0 \), \( p_{ij} \ln p_{ij} \) will be always zero. Then, weights of scheme attributes without subjective preferences are calculated through (12).

\[
d_i = 1 - E_j
\]

\[
w_j = \frac{d_j}{\sum_{j=1}^{N} d_j}, j \in N
\]

(12)

where, \( d_j \) is information deviation of \( X_j \) and \( w_j \) is the weight of scheme attribute without subjective preferences. At last, \( w \) is revised with the subjective weight vector \( \lambda = \{\lambda_1, \lambda_2, \lambda_3, \ldots \} \), which is predetermined by the decision maker. The revising method is expressed as follows.

\[
w_j^* = \hat{\lambda}_j w_j, j \in N
\]

(13)

Through revision, more accurate weight vector \( w^* = \{w_1^*, w_2^*, w_3^*, \ldots \} \) can be obtained.

3) Construct weighted standard decision matrix \( V = (v_{ij})_{m \times n} \).

\[
v_{ij} = w_j^* x_{ij}, i \in M, j \in N, w_j^* \in w^*, r_{ij} \in R
\]

(14)

4) Determine the ideal solution \( A^+ \) and negative ideal solution \( A^- \).
where, \( v_j^+ \) and \( v_j^- \) are the \( j \)-th attribute of \( A^+ \) and \( A^- \) respectively.

\[
\begin{align*}
   \begin{cases}
   v_j^+ &= \max_i v_j^i, j \in J^+ \\
   v_j^- &= \min_i v_j^i, j \in J^- \\
   v_j^- &= \max_i v_j^i, j \in J^+ \\
   v_j^+ &= \min_i v_j^i, j \in J^- 
   \end{cases}
\end{align*}
\]

(16)

5) Calculate the distance between the feasible scheme and the ideal scheme, which is denoted by \( S_i^+ \), and that between the feasible and the negative ideal scheme, which is denoted by \( S_i^- \), respectively. The distance in this paper is Euclidean distance by (17).

\[
\begin{align*}
   S_i^+ &= \sqrt{\sum_{j=1}^{n}(v_j^+ - v_j^i)^2} \\
   S_i^- &= \sqrt{\sum_{j=1}^{n}(v_j^- - v_j^i)^2}
\end{align*}
\]

(17)

where, \( S_i^+ \) is the distance between scheme \( i \) and the ideal scheme; \( S_i^- \) is the distance between scheme \( i \) and the negative ideal scheme.

\[
\begin{align*}
   S^+ &= \{ S_1^+, S_2^+, S_3^+, \ldots \} \\
   S^- &= \{ S_1^-, S_2^-, S_3^-, \ldots \}
\end{align*}
\]

(18)

6) Calculate the relative proximity between each scheme and the ideal scheme, which is denoted by \( C_i \) and calculated by (19).

\[
C_i = \frac{S_i^+}{S_i^+ + S_i^-}, 0 < C_i < 1; i \in M
\]

(19)

where, \( C_i = \{ C_1, C_2, C_3, \ldots \} \). The performance of the feasible schemes is in positive correlation with \( C_i \), which means the frequency restoration ability of the UFLS scheme with larger \( C_i \) is better than that with lower \( C_i \). So when \( C_i \) in \( C_i \) is arranged in descending order, the precedence which reflects the performance of the UFLS schemes is also obtained. Thus, when the feasible UFLS scheme set is assessed with the TOPSIS algorithm, the relatively optimal scheme for each area can be obtained no matter it is isolated or belongs to a homological area.

VII. DEMONSTRATION EXAMPLES

A. Classical Model of Generating Unit and Load

![Fig.4 Block diagram of a generating unit with governor](image)

The classical model of a generating unit with governor [29] is used to analyse system responses of different UFLS schemes and the block diagram is shown in Fig.4.

In Fig.4, \( M = 2H \), \( H \) is the inertia of the system; \( D \) is the damping of the generator; \( R \) is the adjustment coefficient of the governor; \( T_0 \) is the inertia time constant of the governor; \( \Delta \omega_r \) is the deviation of the rotor speed.

B. Scheme Design

A large power system is shown in Fig.5, and the area partition for the system is also presented. It is seen that the system is rather large, which includes 15 areas. The whole system is in a radial pattern and the connection between some areas is relatively weak. So when the tie-lines for those areas fail and are removed, the main grid is easily to be split.

![Fig.5 Area partition of a large power system in China](image)

(1) Homological Area Analysis

Based on the system shown in Fig.5, the grid partition map can be obtained in Fig.6. According to the operation principle when network partition happens, the areas in Fig.5 are partitioned based on the voltages of the tie-lines. If there exist 220kV tie-lines between one area and the main grid, then the tie-lines will be reserved in Fig.6. A special case is that if one area is only connected to the main grid via 110kV tie-lines, the 110kV tie-lines will have to be reserved in Fig.6. A tie-line between two areas is reserved means that the tie-line is regarded as a connection line in the corresponding partition map between those two areas. A connection line in partition map means a connection degree. Only when the connection degree is less than 3 for an area or a multi-area subsystem, the area can be split from the original system.

![Fig.6 Partition map of the power system](image)

<table>
<thead>
<tr>
<th>Table III</th>
<th>BASIC PARAMETERS OF EACH AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>Active power output of generators</td>
</tr>
<tr>
<td></td>
<td>( P_i ) (MW)</td>
</tr>
<tr>
<td>1</td>
<td>62</td>
</tr>
<tr>
<td>2</td>
<td>226</td>
</tr>
<tr>
<td>3</td>
<td>78</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
</tr>
</tbody>
</table>
The basic parameters of each area are shown in Table III, including the active power output of the generators, load, active power shortage (load minus active power output), active power shortage level and inertia of the area (the resistance of the area to any change in its state). Based on the homological area analysis method in Section IV, all the isolated and homological areas for Fig.6 are obtained in Table IV. As for the shadow parts in Table IV, they may form a homological area which consists of area 10, 11, 12, 14 and 15. The total active power shortage level for the homological area is positive, which means UFLS is necessary. However, the homological area is almost half the size of the grid and the safety control strategy for the system is enough to recover itself to the normal operation state without activating UFLS relays. Thus the large homological area is not taken into account during UFLS analysis.

### Table IV

**Homological and Isolated Areas of the Grid**

<table>
<thead>
<tr>
<th>Isolated areas set</th>
<th>Active power shortage level (%)</th>
<th>Homological area</th>
<th>Area set</th>
<th>Active power shortage level (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I₁</td>
<td>88</td>
<td>H₁</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>I₂</td>
<td>37.9</td>
<td>H₂</td>
<td>12,13</td>
<td>11.4</td>
</tr>
<tr>
<td>I₃</td>
<td>73.3</td>
<td>H₃</td>
<td>12,13,14</td>
<td>11.4</td>
</tr>
<tr>
<td>I₄</td>
<td>10.6</td>
<td>H₄</td>
<td>12,3,4,6,7</td>
<td>4.5</td>
</tr>
<tr>
<td>K</td>
<td>3.1</td>
<td>H₅</td>
<td>10,11,12</td>
<td>13,14,15</td>
</tr>
</tbody>
</table>

### (2) UFLS Schemes Design

It can be seen from Table IV that area 12 is an isolated area and also belongs to the homological areas H₁, H₂, H₃ and H₄. Then the UFLS schemes for area 12 should be optimized in both cases. For the sake of simplicity, suppose the UFLS schemes for areas 10, 11, 13, 14 and 15, which are related to area 12, are all the same, shown in Table V.

### Table V

**UFLS Scheme for Area 10, 11, 13, 14 and 15**

<table>
<thead>
<tr>
<th>Frequency threshold (Hz)</th>
<th>Step size ΔP (%)</th>
<th>Time delay (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base round</td>
<td>49</td>
<td>0.2</td>
</tr>
<tr>
<td>48.5</td>
<td>8</td>
<td>0.2</td>
</tr>
<tr>
<td>47.5</td>
<td>8</td>
<td>0.2</td>
</tr>
<tr>
<td>49</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>Backup round</td>
<td>48.5</td>
<td>5</td>
</tr>
</tbody>
</table>

C. MADM Algorithm Based on TOPSIS

(1) Candidate Scheme Set

The candidate UFLS scheme set for area 12 can be constructed by using the method proposed in Section III. The parameter set for candidate set construction can be obtained from Table V, and \( \Phi_{\text{set}} = \{40\%\}, \Phi_{\text{step}} = \{4\}, \Phi_{\text{delay}} = \{2\} \). The candidate UFLS schemes are designed with equal, decremental and incremental step sizes respectively. For incremental step-size schemes, the load curtailment for the first step is 10\%ΔP₂. For decremental step-size schemes, the load curtailment for the first step is 30\%ΔP₂. Then based on equation (5), the load shed for each step of the three kinds of schemes is shown in (20).

\[
\Phi_{\text{th}} = \{8\%,8\%,8\%,8\%,8\%\}, (4\%,6\%,8\%,10\%,12\%), (12\%,10\%,8\%,6\%,4\%)
\]

Other parameters for scheme design are as follows:

\[
\Phi_{\text{w}} = \{49\}, \Phi_{\text{wb}} = \{0.25,0.5\}, \Phi_{\text{w}} = \{2\}, \Phi_{\text{w}} = \{12\}
\]

Then by using the method in Section III, the candidate scheme set for area 12, denoted by \( \Phi_{\text{12}} = \{\phi_1,\phi_2,\cdots,\phi_n\} \), can be constructed and is shown in Table VI.

<table>
<thead>
<tr>
<th>Table VI</th>
<th>CANDIDATE UFLS SCHEMES FOR AREA 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candidate scheme ( \phi_i )</td>
<td>Candidate scheme ( \phi_i )</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>Load shed (%)</td>
</tr>
<tr>
<td>49</td>
<td>8</td>
</tr>
<tr>
<td>48.75</td>
<td>8</td>
</tr>
<tr>
<td>48.5</td>
<td>8</td>
</tr>
<tr>
<td>48.25</td>
<td>10</td>
</tr>
<tr>
<td>48</td>
<td>12</td>
</tr>
<tr>
<td>49</td>
<td>3.8</td>
</tr>
<tr>
<td>48.75</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Then the following work is to conduct simulations with the six schemes in area 12 in both isolated and homological conditions. After UFLS simulation in I₅, H₁, H₂, H₃ and H₄ respectively, thirty groups of scheme attributes are obtained from the frequency-time curves. The model shown in Fig.5 is adopted for simulation and the total active power shortage level is set as 20%.

According to the basic operation rules, the constraints in (7) are set. Then all feasible schemes are determined.

(2) Optimization of UFLS Schemes Based on TOPSIS

Before optimizing the six UFLS schemes for area 12, the parameters required by TOPSIS are shown as follows.

\[
m = 6, M = \{1,2,6\} \quad n = 6, N = \{1,2,6\}
\]

\[
\Phi^i = \{\phi_1,\phi_2,\phi_3,\phi_4,\phi_5,\phi_6\}
\]

\[
G = \{f_{\text{min}}, f_{\text{sum}}^1, f_{\text{sum}}^2, \Delta P, \delta, \Delta P\}
\]

\[
J = \{1\}, J' = \{2,3,4,5,6\}
\]

Based on the decision-making process by using TOPSIS method presented in Section VI, all the parameters are calculated. At last, relative proximity is obtained according to
(19) as \( C_s = \{0.6740,0.6018,0.6010,0.3863,0.5080,0.5350\} \).

 Obviously, the scheme \( \varphi_1 \) is closest to the ideal solution for its value of relative proximity is the largest. Thus scheme \( \varphi_1 \) is the optimal scheme for area 12 after considering homological areas from possible network partition.

(3) UFLS Simulation

To validate the optimization results got based on TOPSIS, the simulation results with the six schemes are presented.

Fig.7 Frequency curves for the six schemes when area 12 is isolated

When area 12 is an isolated area \( (I_o) \), the simulation results are shown in Fig.7. It can be seen from Fig.7 that only the backup round of scheme 4 function at around 12s when the frequency is lower than 49 Hz. The system frequency returns to its rated value at about 17s for scheme 4, which is the longest in the six schemes. Thus scheme 4 is not an optimal scheme. For scheme 5, the frequency overshoot is about 0.3 Hz, which means over shedding happens and should be avoided. For scheme 6, the restored system frequency is 49.8 Hz and the scheme is not the optimal one, either. It is a little difficult to evaluate the other three schemes. Though the steady frequency of scheme 3 is closest to the rated value, the overshoot is larger than the other two schemes. For scheme 1 and 2, their steady frequency and overshoot are almost the same, but the minimum frequency for scheme 2 (around 48.42 Hz) is lower than that of scheme 1. So the optimal UFLS scheme for area 12 is scheme 1.

Fig.8 Frequency curves for the six schemes when area 12 is in \( H_1 \)

When area 12 is in the homologous area \( H_1 \), the simulation results are shown in Fig.8. It can be seen from Fig.8 that the simulation results for the six schemes in \( H_1 \) performs better than that in \( I_o \), as shown in Fig.7. In Fig.8, the performance of scheme 5 is worst for its recovery time is longest. In addition, scheme 5 may restore the system frequency when area 12 is isolated, shown in Fig.7, but fails when area 12 is in the homologous area \( H_1 \). For scheme 2, 3 and 4, the steady frequency is lower than that of scheme 1 and 6. So scheme 1 and 6 are relatively better schemes. Though the overshoot of scheme 1 is larger than that of scheme 6, the steady frequency of scheme 1 is higher than that of scheme 6. So it is hard to determine which scheme is better just from the frequency curves in Fig.8. By adopting TOPSIS method, scheme 1 is found to be the optimal scheme.

When area 12 is in the homological area \( H_2 \), the simulation results are shown in Fig.9. As shown in Fig.9, the performance for the six schemes is almost the same with the size of the homological area becomes larger. The performance of scheme 4 and 5 is a little worse than that of the other four schemes. The steady frequency of scheme 4 is the lowest (about 49.9 Hz) and the maximum frequency is lower than 50 Hz. The maximum frequency of scheme 5 is almost 50.1 Hz, which is the highest of the four schemes and indicates over shedding. The overshoot of scheme 1 and 6 is larger than that of scheme 2 and 3, but the steady frequency of scheme 2 and 3 is higher than that of scheme 1 and 6. So only by TOPSIS method, the optimal scheme is obtained.

Fig.9 Frequency curves for the six schemes when area 12 is in \( H_2 \)

When area 12 is in the homologous area \( H_2 \), the simulation results are shown in Fig.10. Fig.10 is almost the same with Fig.9, which means the load shedding effects with the six schemes in \( H_2 \) and \( H_3 \) are similar. The steady frequencies in Fig.10 are rather close to each other, and only the overshoot of the schemes is different. Only by using TOPSIS method, it can be found out which scheme is relatively better.

Fig.10 Frequency curves for the six schemes when area 12 is in \( H_3 \)

When area 12 is in the homologous area \( H_3 \), the simulation results are shown in Fig.11. For \( H_3 \) is the largest homological area, the performance of the six schemes are almost the same, as shown in Fig.11. It will be rather difficult or impossible to decide which scheme is the optimal one from Fig.11 directly. But when adopting TOPSIS method, the decision-making process will be easier and the optimal scheme is available for various system topologies.
Through conducting simulation with the six schemes for area 12 in $I_1, H_1, H_2, H_3$ and $H_4$ respectively, the optimal scheme ($\theta$) obtained by using TOPSIS method performs the best during system frequency restoration in different cases. The decision-making process is also more precise and easy.

VIII. CONCLUSIONS
This paper proposes a new design approach for load shedding. Besides, it has taken the influence of grid topology change into consideration in network contingencies by defining the concept of homological area. To design the desirable UFLS schemes, a novel MADM algorithm based on TOPSIS is also proposed. From the demonstration examples, it can be seen that by the proposed exhaustive scheme design approach all feasible UFLS schemes for an area are obtained. By the MADM algorithm based on TOPSIS, optimal schemes for an area both in isolated and homological cases are achieved. The simulation results show that a scheme, which performs well in its own area, may fail to restore system frequency in the corresponding homological area. The simulation also shows that more UFLS schemes are designed and the optimal schemes able to restore the system frequency in various network topologies can be selected by the MADM method. The UFLS schemes proposed in this paper have already been applied to the actual power systems in Sinkiang and Shanghai of China.

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REFERENCES

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