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TENNIS STRING-BED RESPONSE MEASUREMENT

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ABSTRACT

With so many tennis string constructions and materials available, it is interesting to know how they affect the dynamical behaviour when they are woven to form the string-bed. This article investigates the usefulness of a high-speed camera in developing mechanical properties of tennis string-beds. Experimental findings of the ball-string-bed interaction are presented. The results show how recordings of the string-bed's displacement are processed to plot frequency for different materials. The change in frequency between two different gauges of the same material is also plotted. The results include a full response of the string-bed and a means by which damping characteristics can be determined.

KEYWORDS

Tennis, dynamic stiffness, damping, frequency

1. INTRODUCTION

In the current procedure of tennis racket stringing, players use the initial tension at which their rackets are strung as a reference to how it will perform during play. However, a newly tensioned string-bed is affected by several factors [4], not least time, giving rise to a degree of unpredictability in its dynamic response.

The focus of this study is to further knowledge of string-bed by showing a new experiment which enable the damping properties to be determined. Currently, the representation of a string-bed in models is a linear spring, which is supported by the string materials' highly elastic behaviour. However, being able to measure a string-bed's dynamic properties will help in understanding more about the vast range of string types available.

On the market today is a wide range of string materials and constructions. Players are guided by manufacturers' descriptions on string qualities such as spin and durability to help them choose which type best suit their requirements. However, precise interpretation of a string's playability is not possible because no single unit of measure is assigned to it. This makes it extremely difficult to compare playability across a range of strings offered by a manufacturer and, furthermore, across the manufacturers themselves. The initial tension at which at racquet is strung is used by tennis players as a point of reference to how it will perform dynamically during play, but performance may also be affected by the variation in string patterns so it would be useful to have an insight into the dynamical behaviour of the string-bed to assist with string settings and selection.

String performance is described in terms of playability and durability [1]. As a rule of thumb, a string classified as highly playable is so at the detriment of durability and *vice versa*. The balance between these two properties is generally linked to a player's ability where a highly proficient player selects a string primarily on playability forsaking durability and cost, whereas a recreational player may avoid expensive, highly-playable materials such as natural gut and select a durable synthetic alternative. To assist players in selecting a string based on these two properties, manufacturers often provide non-dimensional ratings, which although they give general guidance means it is still difficult to gauge how much more playable one string is over another.

2. STRING-BED RESEARCH

2.1. Introduction

This section gives a basic review of research on single strings and the woven string-bed in a tennis racquet. The single string tests research improves the understanding of why different materials are used in tennis. The string-bed research, investigates values for properties which could be useful when modelling, such as, string-bed stiffness. Also, the interaction between the string-bed and ball how they interact with the tennis ball, and to what extent they create injury.

2.2. String performance

2.2.1 Single string

A study into the performance of widely available materials tested ninety different strings in a laboratory [6]. In these tests, single lengths of string were tensioned in an apparatus which measured the force within the string by a load cell and applied traverse impacts by a pendulum. In one test, the string's ability to retain tension was measured after it had been both clamped and held for 1000 seconds and then subjected to ten impacts from the pendulum. The test used string materials of natural gut, nylon, polyester and Kevlar.

The test clearly shows that the performance of natural gut is the best of all the materials and Kevlar, despite retaining tension well after initial tensioning and clamping, suffers the highest loss in tension after the series of ten impacts from the pendulum. Nylon strings are manufactured in a range of constructions but it is concluded that they exhibit broadly similar performances and do not significantly overlap the performances of either natural gut or polyester. In addition to the measurement of tension loss, other performance indicators such as energy loss and dynamic stiffness are given which categorise string types thereby helping an informed decision to be made on which type of string to select.

The dynamic stiffness was determined by measuring the change in tension and the increase in length throughout the side impact from the hammer. The single string was 320 mm long and tensioned to 28 kg (61.7 lb). The dwell time of the hammer or the time in which the tension increased and lowered was 30 ms. If the increase in string length is DL and the increase in string tension DT, then string stiffness is DT/DL. Typical k results recorded for nylon, kevlar and natural gut were 200, 600 and 108 lb/in respectively.

This corpus of work may be furthered by showing how the results of these single-string tests relate to the performance of a fully-strung racquet.

2.2.2 String-bed

Whilst the single string tests give empirical data on different materials, it is necessary to understand what their parameters are when woven into the string-bed. A parameter of particular importance to researchers which create models of the ball-string-bed interaction is the stiffness. The stiffness of the string-bed was measured for three different contact areas which straddle the diameter of a tennis ball [9]. Figure 1 shows the string-bed stiffness against string-bed deflection after figure 5 [9]. The applied force was distributed over a circular area and repeated for diameters of 35mm, 55mm and 65mm, on the string-bed of a Spalding Heat 90 racquet. The plots show the stiffness increasing with deflection from 38 kN/m to 62 kN/m approximately implying that in a model a non-linear spring should be used.

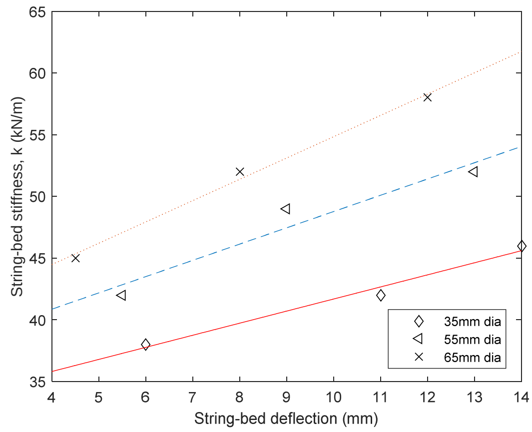

Figure 1 Experimental data for string-bed stiffness, determined by applying a distributed force perpendicular to the string plane [9]

In another study [3], the string-bed stiffness was determined by bracing the racquet head and applying the force over an area of 12 cm^2. It explains how the effect k (spring constant) of the strings is a function of several factors which include, the tension at which the racquet is strung, string gauge, type of string and size of the racquet head. The experiments measured k as a function of the applied force for several racquets and found is ranged from 2 to 3.5 kN/m.

In a study which compared spring damper models of an impact between a ball and string-bed to experiments, the set-up shown in figure was used to determine one of the parameters, maximum string-bed deflection [9]. Slender aluminium bars were attached to the string-bed and passed through a frame so that they moved perpendicularly to the string-bed, whose displacement could then be

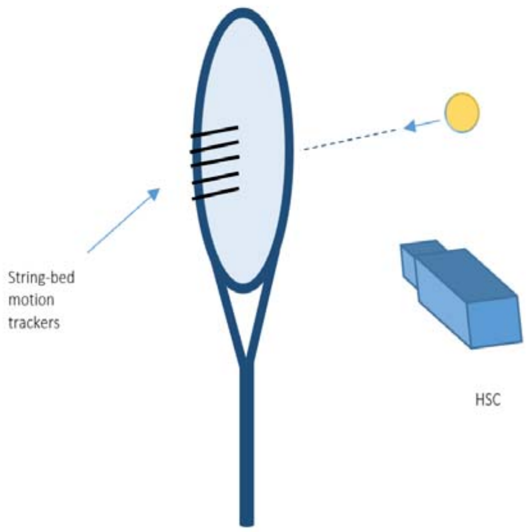

Figure 2 Measurement of maximum string-bed displacement using aluminium trackers

tracked and recorded by a HSC. Although this contact method might influence a free vibrational response, good agreement between the model and experiment was found for the CoR and maximum string-bed deflection, which were also shown to be dependent on the value of string-bed stiffness.

Experimental work investigated the string-bed's performance in a test that measured the incident and rebound velocities of a steel boule [6]. The 760 g boule did not deform during the impact like a tennis ball so any losses are attributed to the string-bed only. The results show rebound velocities to be 95% +/- 2% of the incident velocities suggesting that energy loss due to friction between the strings is negligible. The results are only for impacts perpendicular to the string-bed resembling a ground-stroke which is hit flat, rather than an angled one which imparts spin onto the ball and induces more lateral movement of the cross-strings. Interestingly, the results represent both new and old strings showing that the age of a string does not affect playability in terms of energy being returned to the ball from the string-bed. This may justify assumptions in analytical models, which have shown the string-bed as a simple linear spring with no damping [9].

2.3. Injury

The following section on injury is included to show how the string-bed may or may not influence injury in terms of its resonant frequency. The following studies have sought to establish the range of harmful frequencies and look at how the frame and the string-bed, with and without a string damper, contribute to it.

The string-bed is the sub-system in contact with the ball and influences the transfer of energy to the racket head and handle. Other studies have investigated how to reduce the vibrations of the string-bed by attaching polymer materials called string dampers directly to the string-bed in order to minimise the harmful effect they have on the hand and arm. An investigation into the effect of vibration dampers [15], describes a study by Tomosue et al. which reports that string dampers decrease racket handle accelerations. However, it points out that no link is made with discomfort from damped and undamped impacts and therefore no evidence exists to support dampers reduce hand and arm discomfort. It concludes that whilst dampers did attenuate the presence of high-frequency vibrations measured at the racket handle by an accelerometer, they did not affect the lower-frequency, higher-amplitude racket vibrations that are transmitted from the racket head to the shaft and handle of the racquet. In another study [13], twenty participants held a racket stationary whilst balls were fired at the node (or geometric centre) and the dead spot. The resonant frequency of the racket was found to be approximately 120 Hz and the frequencies recorded on the elbow and wrist did not significantly deviate from this with either a damped or undamped string-bed deducing vibration dampers do not attenuate the dominant, lower frequencies of around 120 Hz, which are believed to contribute to injuries. In another experiment [14], an individual's subjective response to hand-held induced vibrations was recorded to establish an annoyance level. The annoyance level is shown to increase consistently across the participants when frequencies below 200 Hz were applied.

Understanding how the string-bed responds to an impact has been the focus of a study into the infamous condition 'tennis elbow' or as they referred to it radiohumeral bursitis [2]. Tennis elbow is a painful condition that affects the tendinous tissue of the origins of the wrist extensor muscles at the lateral epicondyle of the humerus, it is therefore, also known as lateral epicondylitis [8]. An analytical model was developed of the racket to simulate the magnitude and nature of the torque and force at the handle. The transfer of vibrations to a player's hand from a ball impact is simulated as a sinusoidal force decaying from 45 N at a frequency of 100 Hz ± 10 giving a corresponding oscillating torque decaying from 22 N [2]. The impact time is plotted against tension for two simulated impacts of 20 and 40 m/s. At a string tension of 250 N the impact time corresponds to 5 x 10-3 s rising to approximately 5.8 x 10-3 s at 150 N.

The CoR of the ball is regulated by the International Tennis Federation (ITF) with a rule which requires the ball to have a rebound height between 1350mm and 1470mm from a dropped height of 2540mm onto a concrete surface [12].

2.4. Dwell time

The time a ball spends in contact with the strings is known as the dwell or contact time. The dwell time is important in tennis because it relates to the accuracy of stroke: a short dwell time releases the ball off the stings more closely to the direction at which the racket impacts it, whereas a longer dwell time allows the racket to rotate through a larger angle about the player and away from the line of stroke. However, whilst a shorter dwell time gives more accuracy it has the undesirable effect of less power. This is because as the ball displaces the string-bed's energy is stored by the strings, which is then returned to the ball as kinetic energy. The tension at which a racquet is strung influences the dwell time so a less taut string-bed generates more power but at the expense of control over the direction of the rebounding ball. This explanation is a rule of thumb only because there is a limit to which the string-bed tension can be reduced before it begins to lose energy and can no longer return it perpendicularly to the plane of the string-bed. Interestingly, reducing the tension increases the dwell time which moves the oscillation of the impact closer to the racket oscillation period [3]. If these frequencies can be matched then the energy stored in the deflected racket can be returned to ball before it leaves the strings. Another way of matching these two is by making the racquet stiffer. The use of composites over organic materials helps to achieve this whilst controlling the overall weight and bulk of the racquet, for example, a Slazenger Challenge (wood) and Prince TT Viper vibrates at 87 Hz and 227 Hz, respectively.

One method of measuring the dwell time uses a laser, photo detector, mirrors and an oscilloscope. The beam of light is positioned parallel to the plane of the string-bed at a distance equal to the diameter of the ball. If the ball-string system is assumed to be a, linear simple harmonic oscillator then the dwell time is half of a sinusoidal period (Brody, 1979). Brody measured string-bed stiffness to be between 2 and 3.5×10^4 N⁄m giving a frequency of oscillation of 100 Hz with a 0.06 kg ball and a dwell time of 4.5m s.

Table 1 Typical dwell times in milliseconds [3]

	Type of ball		
Rebounding surface	Penn	Spalding Australian	Tretorn (pressureless)
Lead brick	3.9	4.5	4.5
Prince (70lb)	6.1	6.3	6.3
Prince (50lb)	6.5	6.7	6.4
Spalding Smasher	6.4	6.8	6.6

The calculation for dwell times where the deflection of the string-bed is relatively small, up to 1.5 cm, correlates well to measured values because the stiffness is relatively linear in this range. However, for larger deflections a cubic term in force-deflection relationship becomes important and the strings tend to act as if they are stiffer increasing the effective 'k' and deflecting less than the theoretical SHM value [3].

The effect of the ball's impact velocity and, therefore, string-bed displacement on the dwell time was investigated by experiments that fired different types of balls from an air canon perpendicularly at a rigidly clamped racquet [10]. The contact time was measured with a Phantom v4, high-speed video camera running at 30,000 frames per second recording both the impact and a reflection of the impact in a mirror mounted to the frame at an angle so the line of sight was along the string-bed to the contact point. Although one might expect that at higher incident velocities strings deflect more (along with the ball) resulting in longer impact durations, the results show, for example, a pressurised ball impacting with a dwell time which reduces from 4.8 seconds at 5 ms^{-1} down to 2.9 seconds at 40 ms^{-1}. In every case, higher velocities result in shorter contact times due to the dynamic stiffness of the string-bed.

However, for a given velocity, reducing the string-bed tension increases the dwell time and the CoR due to a reduction in stiffness of the ball-racquet system. The effect of increasing the velocity with which a ball impacts a rigid (concrete) surface reduces the contact time non-linearly as a result of the ball's dynamic stiffness.

3. FREQUENCY

The performance of a string or string-bed is shown to be measured in various ways. One of the measurements is frequency and this is useful because it relates to tension, which is what players use to predict how the string-bed will perform during play.

In the string-bed the individual strings are woven together and so vibrate with the same frequency. This frequency is essentially the average frequency of all the strings which if free would otherwise vibrate at different frequencies according to the varying lengths.

An experiment which measured how the fundamental frequency changed over a nine-day period, used a piezo sensor and a microphone [7]. The piezo was attached to the string-bed using Blu-Tack and used throughout the test. The microphone was used occasionally as a check for the effect of the mass of the piezo attachment. It was found that a correction of 15.7% needed to be made to the measurements for the 2.2g attachment.

Although the microphone recordings were not used continuously being described as noisy and of a small 0.5mV amplitude, with suitable filtering and instrumentation it may be a suitable method for measuring frequency.

It describes how the tension can be determined by comparing the observed string vibrational frequency with an elastic membrane model, in which the fundamental frequency of the string-bed is closely approximated by

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \qquad (1)$$

Where T is the average string tension, μ is the mass per unit length and L is a length of a single string equal to $\sqrt{A}$, and A is the area of the string-bed. The result of this vibration frequency of the whole string-bed is described as being surprisingly close to that of a single string of length L.

4. EXPERIMENT

4.1. Introduction

This section discusses an experiment which records using a high-speed camera (HSC) the full response of a tennis string-bed to a tennis ball impact at 30 mph. The data from the HSC is then analysed to give an approximation of the string-bed's damping characteristic.

4.2. Experimental setup

The racquet was clamped to the front of a steel, rectangular fixture by two bars across the face at the head and the throat, see figure. The steel fixture was bolted to a concrete wall so it is assumed that all of the kinetic energy from the impacting ball remains in the ball-string interaction. Tennis balls were projected from a BOLA machine to impact the string-bed perpendicular to its face at the geometric centre. A Photron Fastcam 1280 high-speed camera was positioned on the wall-side of the racket to record the displacement of the string-bed, see figures 5 and 6. The Photron camera was focused on a central point of one of the two longest main strings known also as the geometric centre.

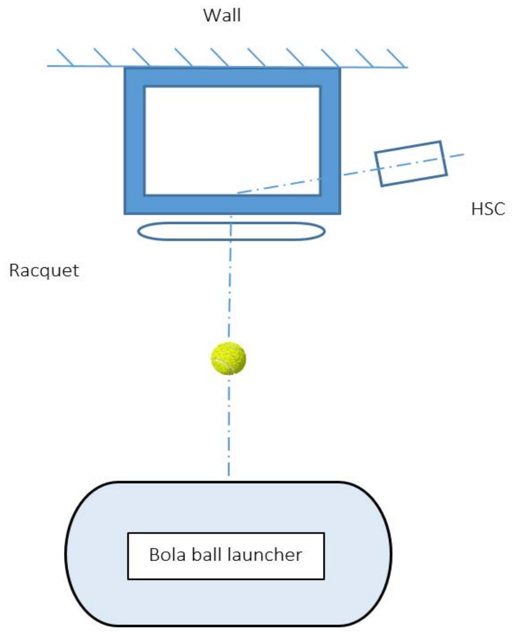

Figure 3 Layout of the experiment showing the position of the high-speed camera to the racquet

The high-speed camera recording of the impact and following string-bed response was then processed for the displacement data.

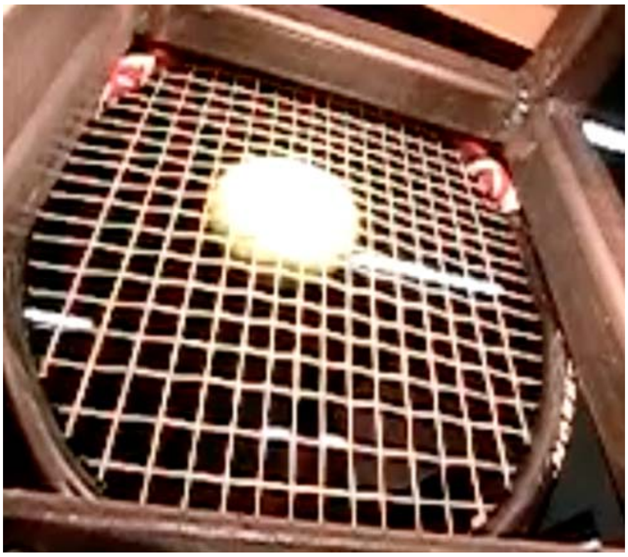

Figure 4. String-bed deflecting under a ball impact

Figure 5. Experimental set-up showing a head-clamped racquet and HSC

4.3. Frequency

The images recorded by the HSC provide a non-contact method of measuring the response and therefore an advantage over previous methods which attached sensors to the string. The fundamental frequencies were extracted from the data using FFT and then plotted against tension. An example of an FFT plot of the frequency spectrum is shown in figure 6. This tension refers to the setting of the stringing machine.

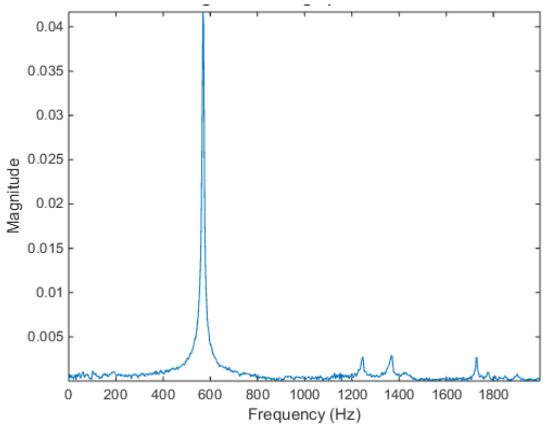

Figure 6 FFT of recorded string-bed response

A Wilson racquet of 95 sq.in with 16 mains strings and 19 crosses was strung across a range of tensions with different materials. The high-speed camera recordings of the first ball impact occurred quickly after stringing. Figures 7, 8 and 9 show the response of the string-bed when strung with a low-cost nylon, polyester and natural gut, respectively. The results show the vibration frequency of the string-bed increasing with strung tension, although the actual string-bed tension after stringing is generally accepted to have dropped by 30%.

A study describes a technique to determine the actual string tension from the stringing tension or nominal tension by measuring the vibration

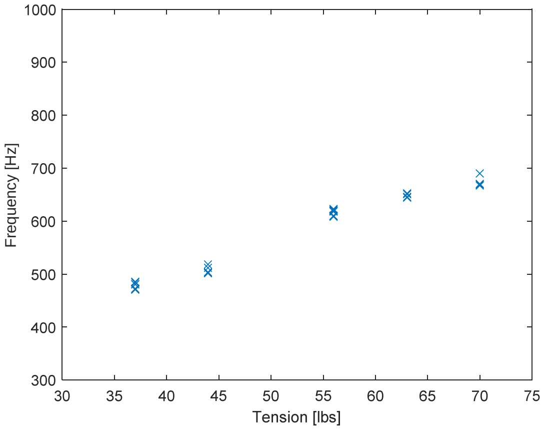

Figure 7 Effect of string tension on frequency of a racquet with 16 mains strings strung with a 'tournament' nylon.

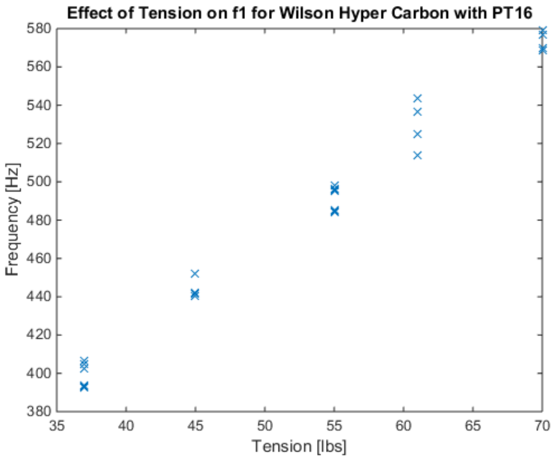

Figure 8 Effect of string tension on frequency of a racquet with 16 mains strings strung with polyester.

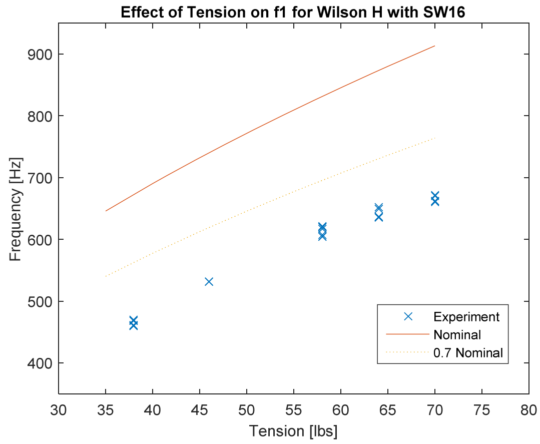

Figure 10 Comparison of theoretical frequency with no tension loss and a 30% tension drop after stringing against experimental results for synthetic gut, 16 gauge

frequency [7]. The string-bed frequency is also shown to be primarily dependent on the string-bed area and not the shape. Figure 10 shows how the frequency, calculated from tennis ball impacts on a 95 sq.in racquet strung with synthetic gut varies with nominal tension. Using equation (1) with both the nominal tension and a 30% tension drop, predictions of how the frequencies vary are plotted. Figure 10 also shows that the residual tension after stringing is in fact lower than 30% of the strung or nominal tension.

A comparison is made between the fundamental frequencies of 16 gauge and 17 gauge synthetic gut in a Wilson racquet of 95 sq.in, the results of which are shown in figure 11. The 16 gauge is represented by circles and the 17 gauge by asterisks. From the single string analogy, the frequency is inversely proportional to mass and string length and will be, therefore, higher for a thinner gauge of string. The 17 gauge string is shown to vibrate at a fundamental frequency which is approximately 11% higher than the 16 gauge material.

Each test recorded by the HSC contains two parts. The first part is the interaction between the ball and the string-bed, from which data, such as, dwell time may be determined. The second part is the free resonse of the string-bed, from which dynamical propertical properties, such as , the damping time and damping coefficient. An example of the free response test is shown in figure 12. This response is seen to be underdamped, in which case it is possible to determine the damping ratio by taking the time period, T, over several

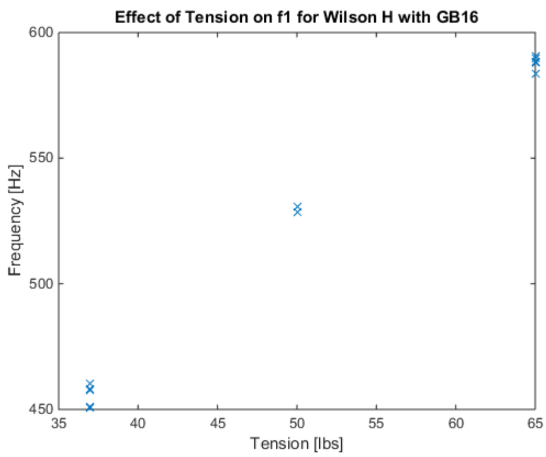

Figure 9 Effect of string tension on frequency of a racquet with 16 mains strings strung with natural gut.

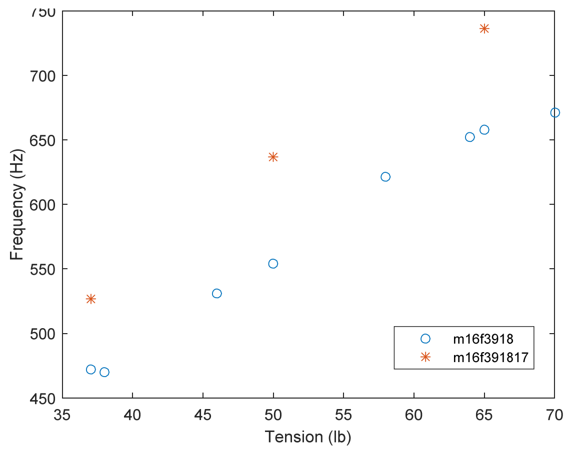

Figure 11 Effect of string tension and string gauge on frequency

oscillations. With n osillations the accuracy of the logarithmic decrement, δ, is greater

$$\delta = \frac{1}{n} ln\left(\frac{x(t)}{x(t+nT)}\right) \quad (2)$$

The damping ratio is then calculated by

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad (3)$$

The damping coefficient or, alternatively, the damping ratio is the most difficult quantity to determine. Unlike the mass and stiffness which can be determined by static tests, damping requires a dynamic test to measure [11]. The response in figure 12 is observed to have a damping ratio of $\zeta<1$ and is therefore under-damped. The values for m and ω_n are known so the damping coefficient may be calculated also.

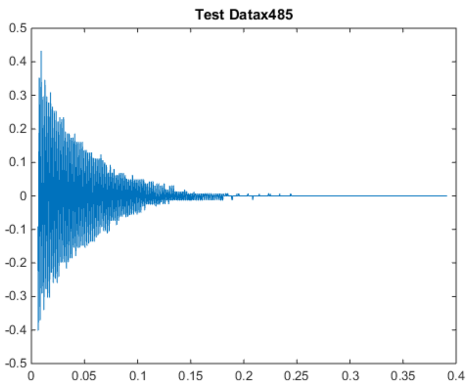

Figure 12 A string-bed deflection-time response for a mid-plus racquet with 16 mains

With a time-displacement plot, as shown in figure 12, comes an ability to test for differences in a string-bed' response. It is difficult to know at this stage how significant these measurements will be beyond the previous boule-drop experiments which show the string-bed to be highly elastic. However, knowing damping coefficients of different string-bed enables earlier models [9] to now be represented as a spring-damper.

4.4 Dwell time

The 'used' pressurised tennis ball was launched at the geometric centre to deflect the string-bed, see figure 4. The measurement of dwell time is calculated from the response of a point on one of the mains centre string close to the geometric centre.

Previous studies, described in section 2, show dwell times from experiments where string tensions are 50lb and 70lb. In each of the three cases with a Prince racquet it shows the dwell time reducing with increasing tension [3]. A sample of the dwell time results from this study, given in figure 13, show dwell times recorded by the HSC to lie in the same band of 6 to 7 milliseconds. The dwell times show a change across the three tests from 6.7 ms reducing to 6.1 ms for tension 50lb and 70lb, respectively. The dwell times recorded by the HSC in this study show a good correlation to these.

4.5 Dynamic stiffness

The dynamic stiffness is the most physical property of a string because it determines how far the string materials extend when the tension is increased [5]. Tennis strings are manufactured from polymers which are visco-elastic, so the stiffness of the strings is a function of velocity. In laboratory tests, standard tensile testing equipment may apply loads up to strain rates of a few hundred mm/min, the strain rate of a tennis ball impact may be hundred thousand mm/min or more.

From the experiments here, the dynamic stiffness is approximated using the distributed force generated from the impact of a 'used' pressurised ball to deflect the string-bed, which is captured simultaneously by the high-speed camera.

Figure 14 shows how the dynamic stiffness varies according to different stringing tensions in a mid-size racquet with 16 mains and strung with a synthetic gut of 16 gauge. The effect of the different initial tensions, 37, 50 and 65lb on dynamic stiffness is shown broadly to increase from an average of about 15 kN/m up to 28 kN/m.

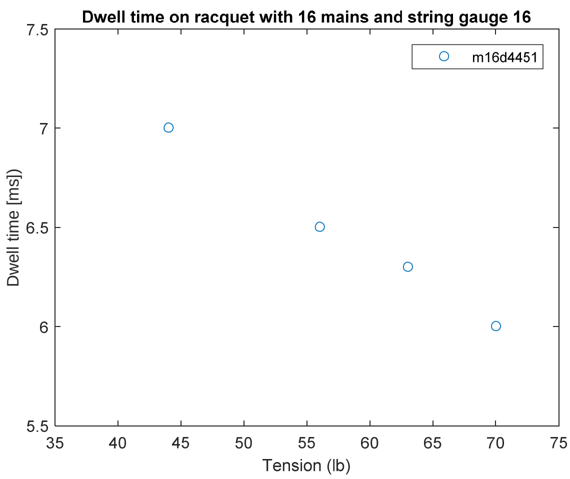

Figure 13 Dwell time between a 'tournament' nylon string and a 'used' pressured ball

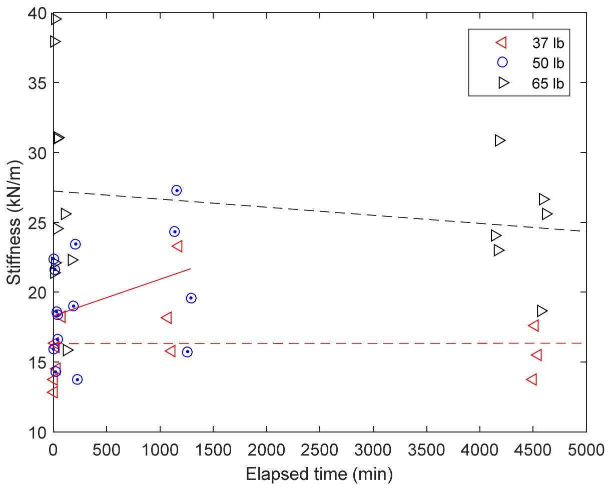

Figure 14 Dynamic stiffness over time since being strung for a mid-size racquet at three different initial tensions.

These tests contain variations in the calculated dynamic stiffness, which are most likely from the ball impacting at slightly different points on the string-bed. The 'best fit' lines through the points suggest the dynamic stiffness might not diminish significantly with elapsed time following initial stringing. This tends to agree with the observation in the single string tests which recorded strings recovering tension after being impacted by the hammer pendulum [6].

5. CONCLUSIONS

A non-contact method with a high-speed camera is used to capture the full response of the string-bed from a ball impact. The data from the recordings are used to determine the vibration frequency of the string-bed for a range of stringing tensions and with several different materials.

A comparison is made between theoretical frequency and those from the experiments to indicate how much the tension may have dropped from the stringing tension. Notwithstanding the tension drop, it is shown how the frequency varies between identical synthetic string materials of different string gauges.

From the ball-string interaction, which occurs over a few milliseconds, the dynamic stiffness of the string-bed is approximated. The racquet was strung with synthetic gut at stringing tensions of 37, 50 and 65lbs.

The mechanical properties of single strings are known, whereas little is known about the string-bed itself. It is shown that the string-bed's response may be captured and therefore analysed for its damping properties which would help to better understand the different effects from different string types and string patterns.

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